**Chapter 11 – Solutions to Instructor Reserve Problems**

**I-1 Instructor**

**a**. A 20% overshoot corresponds to a damping factor of  and from equation 10.73 to a . The bode plot of  with  is



We search on the Bode diagram for the frequency at which the phase is . This occurs at a frequency of 1.37 rad/sec. At this frequency  dB. Thus the magnitude characteristic must be raised by this quantity;  .

**b**.

>> s=tf('s');

>> G=2.267\*(s+10)\*(s+15)/s/(s+2)/(s+5)/(s+20);

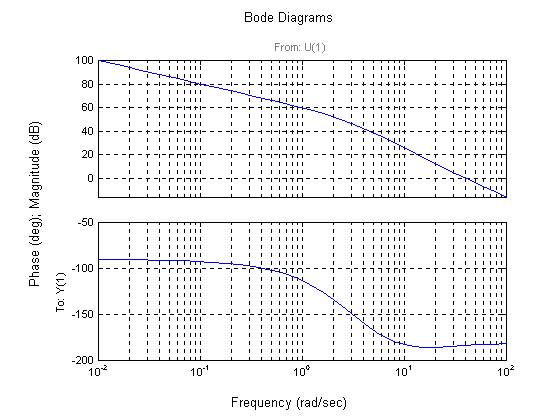
>> T=feedback(G,1);

>> step(T)



**I-2 Instructor**

**a.** For Kv = 1000, K = 1473. Plotting the Bode for this value of K:



Using Eqs. (4.39) and (10.73) a percent overshoot = 15 is equivalent to a  = 0.517 and M = 53.17. Using an extra 10o, the phase margin is 63.17o. The phase-margin frequency = 1.21 rad/s. At this frequency, the magnitude = 57.55 dB = 754.2. Hence the lag compensator K = 1/754.2 = 0.001326. Following Steps 3 and 4 of the lag compensator design procedure in Section 11.3,

Glag(s) = 0.001326

**b.**

**Program**:

%Input system

numg=1473\*poly([-10 -11]);

deng=poly([0 -3 -6 -9]);

G=tf(numg,deng);

numc=0.001326\*[1 0.121];

denc=[1 0.0001604];

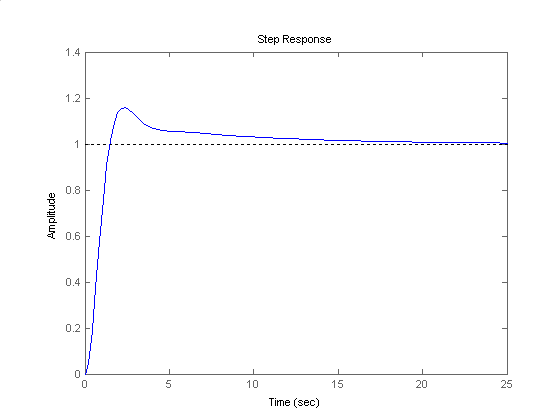
Gc=tf(numc,denc);

Ge=G\*Gc;

T=feedback(Ge,1);

step(T)

**Computer response:**



**I-3 Instructor**

For , *K* = 2400. Plotting the Bode plot for this gain,



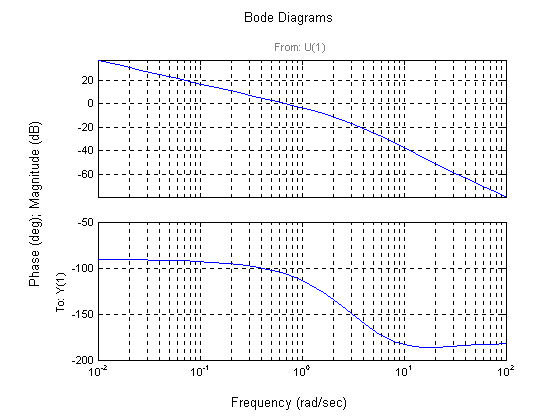
We will design the system for a phase margin 100 larger than the specification. Thus *m* = 550. The phase margin frequency is where the phase angle is –1800 + 550 = -1250. From the Bode plot this frequency is  rad/s. At this frequency the magnitude is 23.37 dB. Start the magnitude of the lag compensator at –23.37 dB and draw it to 1 decade below , or 1.1 rad/s. Then begin a +20 dB/dec climb until 0 dB is reached. Read the break frequencies as 0.0746 rad/s and 1.1 rad/s from the Bode plot as shown below.



Ensuring unity dc gain, the transfer function of the lag is . The compensated forward-path transfer function is thus the product of the plant and the compensator, or

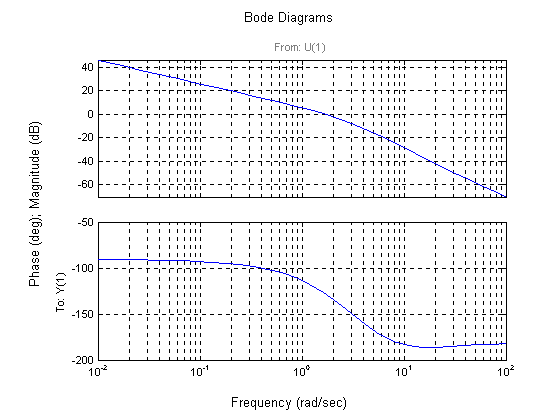


**I-4 Instructor**

Bode plots for K = 1:Using Eqs. (4.39) and (10.73) a percent overshoot = 15 is equivalent to a  = 0.517 and M = 53.17o. The phase-margin frequency = 1.66 rad/s. The magnitude = -9.174 dB = 0.3478. Hence K = 1/ 0.3478 = 2.876.

**b.**

Bode plots for K = 2.876.



Adding 10o to the phase margin yields 63.17. Thus, the required phase is –1800 + 63.170 = -116.830, which occurs at a frequency of 1.21 rad/s. The magnitude = 3.366 dB = 1.473. Hence, the lag compensator K = 1/ 1.473 = =0.6787. Selecting the break a decade below the phase-margin frequency,

Gc(s) = 0.6787

**c.**

**Program:**

%Input system

numg=2.876\*poly([-10 -11]);

deng=poly([0 -3 -6 -9]);

G=tf(numg,deng);

numc=0.6787\*[1 0.121];

denc=[1 0];

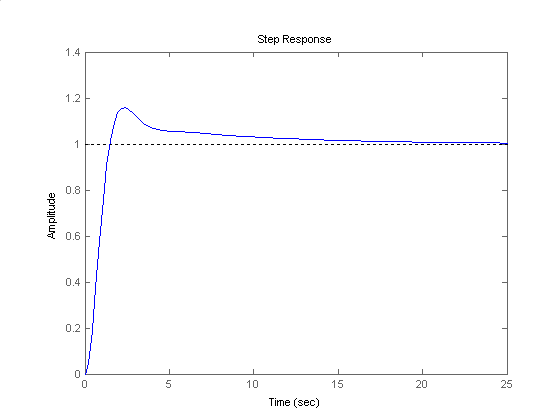
Gc=tf(numc,denc);

Ge=G\*Gc;

T=feedback(Ge,1);

step(T)

**Computer response:**



**I-5 Instructor**

If G(s) = , Kv = 40. Also, for a 0.1 second peak time, and = 0.456 (20% overshoot), Eq. (10.56) yields a required bandwidth of 46.59 rad/s. Using Eq. (10.73), the required phase margin is 48.15o. Let us assume that we raise the phase margin frequency to 39 rad/s. The phase angle of the uncompensated system at this frequency is -158.6o. To obtain the required phase margin, the compensator must contribute 26.75o more at 39 rad/s. Assume the following form for the compensator: Gc(s) = K'KD(s+). The angle contributed by the compensator is

c = tan-1  = 26.75o. Letting  = 39 rad/s, KD = 0.0129. Hence, the compensator is

Gc(s) = 0.0129 (s+77.37). The compensated forward path is



The closed-loop bandwidth is approximately 50 rad/s, which meets the requirements.

The lag compensated forward path is

G(s) = 7.759

**16. 11**

**a.** Bode plots and specifications for gain compensated system are the same as Problem 13. Required phase margin and required bandwidth is the same as Problem 13. Select a phase margin frequency 7 rad/s higher than the bandwidth = 9 + 7 = 16 rad/s. The phase angle at the new phase-margin frequency is -201.60. The phase contribution required from the compensator is –1800 + 201.60 + 58.590 = 80.20 at the phase-margin frequency. Using the geometry below:



tan (80.2) = . Therefore, zc = 2.76. Thus, Gc (s) = .

The PD compensated Bode plots:



Compensated phase margin is 62.942o.

**b.**

**Program:**

numg=1000;

deng=poly([0 -5 -20]);

G=tf(numg,deng);

numc=(1/2.76)\*[1 2.76];

denc=1;

Gc=tf(numc,denc);

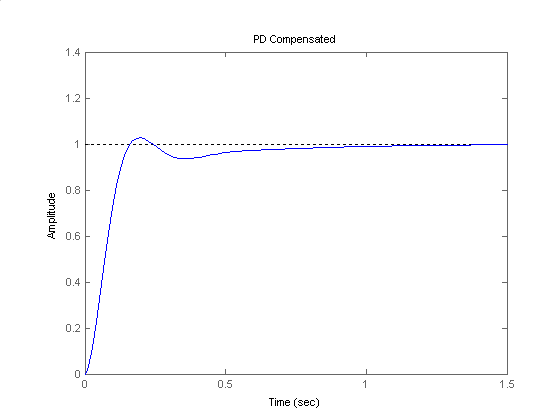
Ge=G\*Gc;

T=feedback(Ge,1);

step(T)

title('PD Compensated')

**Computer response:**



**I-6 Instructor**

**Program:**

%PD Compensator Design via Frequency Response

%Input system

%Uncompensated system

K=input('Type K to meet steady-state error ');

numg=K\*[1 1];

deng=poly([0 -2 -6]);

G=tf(numg,deng);

T=feedback(G,1);

step(T)

title('Gain Compensated')

'Open-loop system'

'G(s)'

Gzpk=zpk(G)

%Input transient response specifications

Po=input('Type %OS ');

%Ts=input('Type settling time ');

Tp=input('Type peak time ');

%Determine required bandwidth

z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));

%wn=4/(z\*Ts);

wn=pi/(Tp\*sqrt(1-z^2));

wBW=wn\*sqrt((1-2\*z^2)+sqrt(4\*z^4-4\*z^2+2));

%Make a Bode plot and get Bode data

%Get Bode data

bode(G)

title('Gain Compensated')

w=0.01:0.1:100;

[M,P]=bode(G,w);

%Find current phase margin

[Gm,Pm,wcp,wcg]=margin(M,P,w);

%Calculate required phase margin

z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));

Pmreq=atan(2\*z/(sqrt(-2\*z^2+sqrt(1+4\*z^4))))\*(180/pi)+20;

%Determine a phase margin frequency

wpm=wBW+7;

%Find phase angle at new phase margin frequency and

%calculate phase required from the compensator

for i=1:1:length(w);

if w(i)-wpm>=0;

wpm=w(i);

Pwpm=P(i);

break

end

end

%Design PD compensator

Pc=Pmreq-(180+Pwpm);

zc=wpm/tan(Pc\*pi/180);

Kc=1/zc;

numc=Kc\*[1 zc];

denc=1;

'Gc(s)'

Gc=tf(numc,denc);

Gczpk=zpk(Gc)

%Display data

fprintf('\nK = %g',K)

fprintf(' Percent Overshoot = %g',Po)

fprintf(', Damping Ratio = %g',z)

%fprintf(', Settling Time = %g',Ts)

fprintf(', Peak Time = %g',Tp)

fprintf(', Current Phase Margin = %g',Pm)

fprintf(', Required Phase Margin = %g',Pmreq)

fprintf(', Required Bandwidth = %g',wBW)

fprintf(', New phase margin frequency = %g',wpm)

fprintf(', Phase angle at new phase margin frequency = %g',Pwpm)

fprintf(', Required Phase Contribution from Compensator = %g',Pc)

fprintf(' Compensator gain, Kc = %g',Kc)

fprintf(' Compensator zero,= %g',-zc)

pause

%Generate compensated Bode plots

%Make a Bode plot and get Bode data

%Get Bode data

'G(s)Gc(s)'

Ge=G\*Gc;

Gezpk=zpk(Ge)

bode(Ge)

title('PD Compensated')

w=0.01:0.1:100;

[M,P]=bode(Ge,w);

%Find compensated phase margin

[Gm,Pm,wcp,wcg]=margin(M,P,w);

fprintf('\nCompensated Phase Margin,= %g',Pm)

pause

%Generate step response

T=feedback(Ge,1);

step(T)

title('PD Compensated')

**Computer response:**

Type K to meet steady-state error 360

ans =

Open-loop system

ans =

G(s)

Zero/pole/gain:

360 (s+1)

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s (s+6) (s+2)

Type %OS 10

Type peak time 0.1

ans =

Gc(s)

Zero/pole/gain:

0.05544 (s+18.04)

K = 360 Percent Overshoot = 10, Damping Ratio = 0.591155, Peak Time = 0.1, Current Phase Margin = 21.0851, Required Phase Margin = 78.5931, Required Bandwidth = 45.1795, New phase margin frequency = 52.21, Phase angle at new phase margin frequency = -172.348, Required Phase Contribution from Compensator = 70.9409 Compensator gain, Kc = 0.0554397 Compensator zero,= -18.0376

ans =

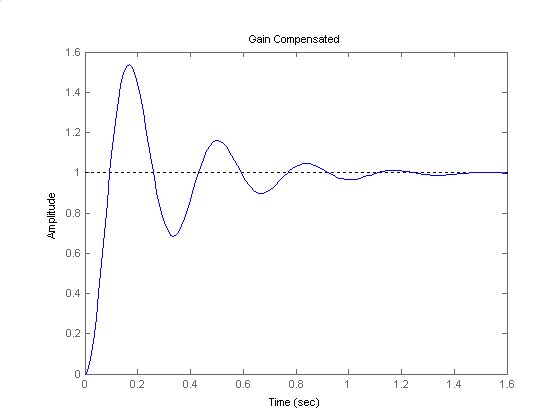
G(s)Gc(s)

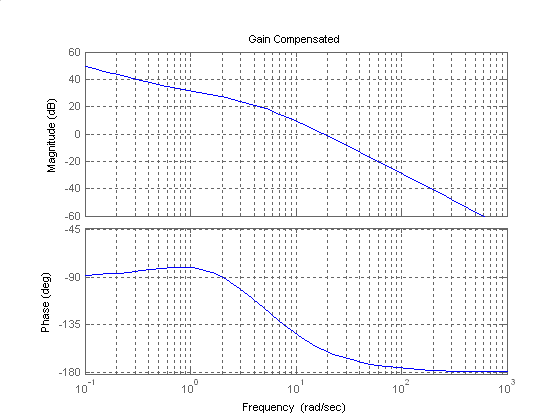
Zero/pole/gain:

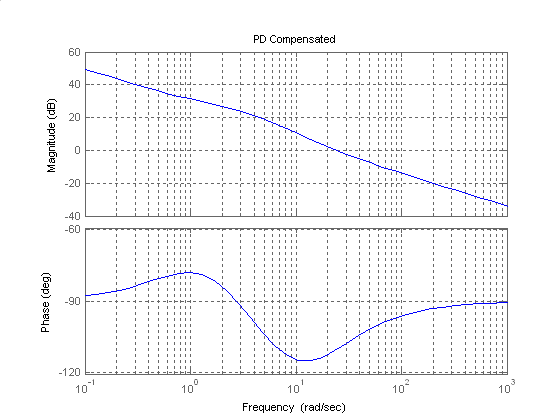
19.9583 (s+18.04) (s+1)

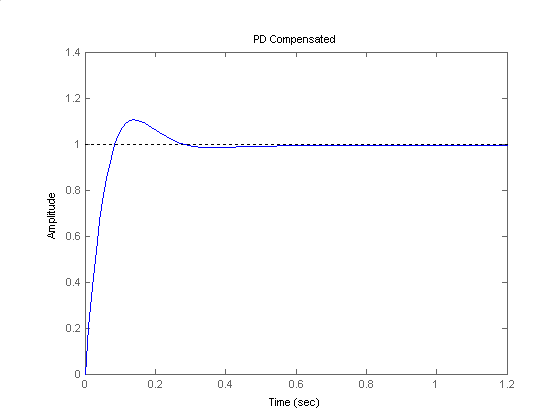
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s (s+6) (s+2)

Compensated Phase Margin,= 69.546 







**SOLUTIONS TO DESIGN PROBLEMS**

**I-7 Instructor**

**a**. The bode plot for the open loop transmission is obtained and shown next:

>> syms s

>> s=tf('s');

>> M=0.005/(s+0.005);

>> P=140625/(s+2.67)/(s+10);

>> set(P,'inputdelay',0.1)

>> L=M\*P

Transfer function:

703.1

exp(-0.1\*s) \* ----------------------------------

s^3 + 12.68 s^2 + 26.76 s + 0.1335

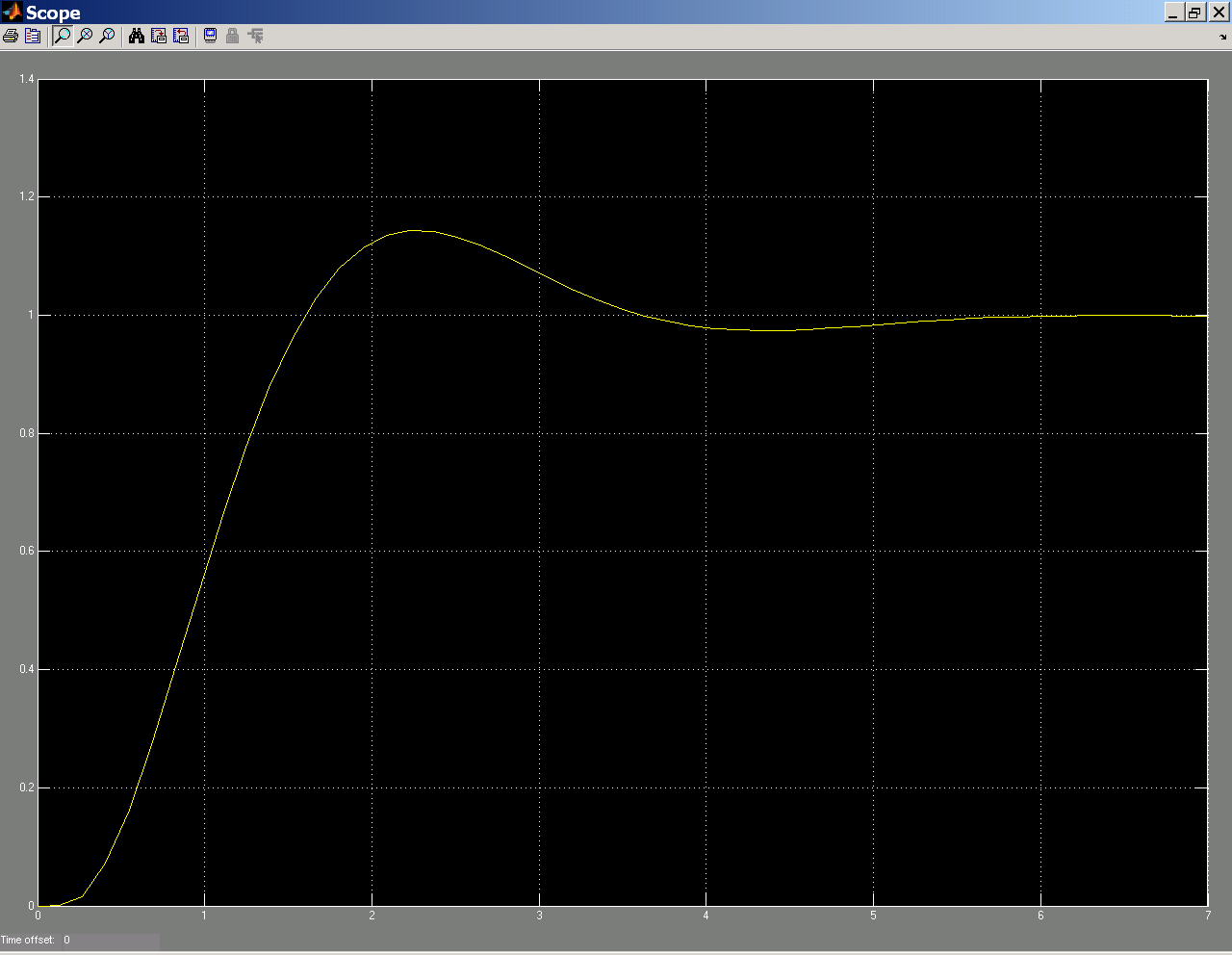
>> bode(L)



It can be seen that with , the system is closed loop unstable. The desired %OS=15% corresponds to a damping factor of . In turn this corresponds to a phase margin of . For this to occur the open loop transmission must have a phase of at the point where the open loop transmission has a magnitude of 0db. The open loop transmission attains this phase when rad/sec, and at that frequency the open loop transmission has a magnitude of 26.2db. So the open loop transmission must be decreased by this amount resulting in .

**b.**

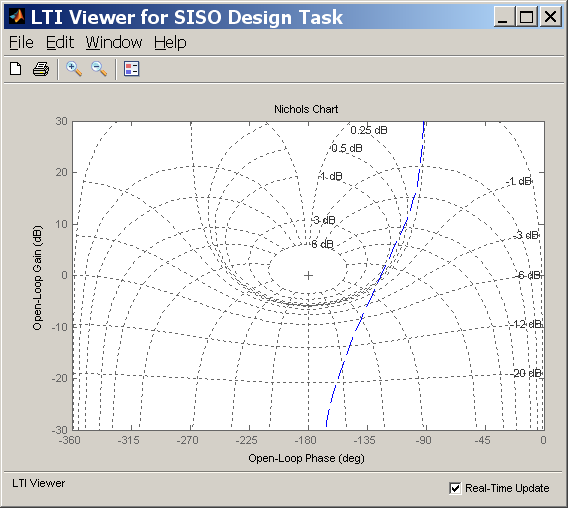




The figure shows a %OS slightly smaller than 15%.

**I-8 Instructor**

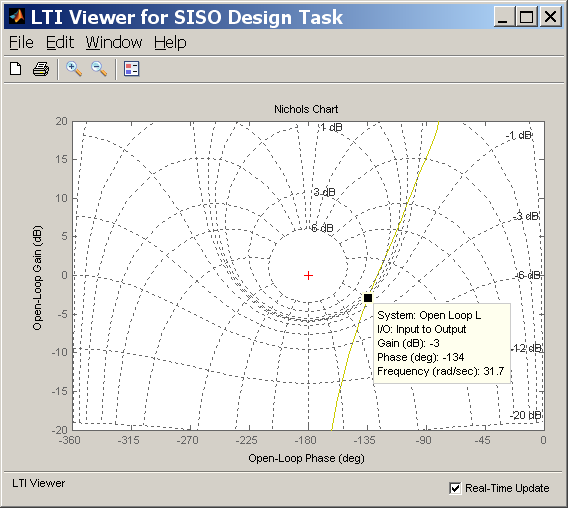
1. Using equation (4.39), x=0.7. Then from equation (10.52) Mp=1=0db. For zero steady state error we add an integrator, and then we raise the system’s gain.



Then we design an appropriate lead compensator. So far, as shown in the figure with a slight violation of the 1db requirement.

The resulting Kv=8.306 so a factor of 2.4 is needed to reach the required 20. The lag compensator is designed by arbitrarily adding a pole zero pair at 0.1 and 0.24 respectively, giving . The total compensation is

1. For this design, we start by adding an integrator to the compensator. The resulting Kv=8.14 so a K=2.46 is needed to obtain Kv=20. Then enough lead is provided to satisfy the Mp<1db requirement. The resulting . The resulting rad/sec as shown in the next figure.



1. The following figure shows the step response of both designs. The faster design corresponds to part b with the larger bandwidth.



**I-9 Instructor**

Including the additional integrator and gain the open loop transfer function becomes



Let  to achieve the steady state requirement on ramp inputs. A 13% overshoot corresponds to a  damping factor. Using equation (4.42) . The required bandwidth is obtained using equation (10.54)  rad/sec. The phase margin is obtained using equation (10.73) with an additional  ; . A new phase margin frequency is chosen close to the bandwidth frequency  rad/sec. The Bode plot for and the integrator is



At the new phase margin frequency the phase angle is  . The contribution from the lead compensator must be . Using equation (11.11) .

Lag compensator design:  ;  ;  ; 

Lead compensator design: Using equations (11.6), (11.9) and (11.12)  ;  ;  ;  ; 

A unit step input simulation is performed:



The overshoot obtained is certainly unacceptable; it can be corrected by iterating on the pole of the lag compensator:  . In this case the overshot is corrected at detriment of the settling time:

