S I X

Stability

SOLUTIONS TO CASE STUDIES CHALLENGES

**Antenna Control: Stability Design via Gain**

From the antenna control challenge of Chapter 5,

T(s) =

Make a Routh table:

|  |  |  |
| --- | --- | --- |
| s3 | 1 | 198 |
| s2 | 151.32 | 76.39K |
| s1 |  | 0 |
| s0 | 76.39K | 0 |

From the s1 row, K<392.2. From the s0 row, 0<K. Therefore, 0<K<392.2.

**UFSS Vehicle: Stability Design via Gain**













|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 1 | 3.465 | 0.10925K1 |
| s3 | 3.483 | 0.25(K1+2.4288) | 0 |
| s2 |  | 0.10925K1 | 0 |
| s1 | 0.25 | 0 | 0 |
| s0 | 0.10925K1 | 0 | 0 |

For stability : 0 < K1 < 26.42

Answers to Review Questions

**1.** Natural response

**2.** It grows without bound

**3.** It would destroy itself or hit limit stops

**4.** Sinusoidal inputs of the same frequency as the natural response yield unbounded responses even though the sinusoidal input is bounded.

**5.** Poles must be in the left-half-plane or on the j axis, but not multiple.

**6.** The number of poles of the closed-loop transfer function that are in the left-half-plane, the right-half-plane, and on the j axis.

**7.** If there is an even polynomial of second order and the original polynomial is of fourth order, the original polynomial can be easily factored.

**8.** Just the way the arithmetic works out

**9.** The presence of an even polynomial that is a factor of the original polynomial

**10.** For the ease of finding coefficients below that row

**11.** It would affect the number of sign changes

**12.** Seven

**13.** No; it could have quadrantal poles.

**14.** None; the even polynomial has 2 right-half-plane poles and two left-half-plane poles.

**15.** Yes

**16.** Det (s**I**-**A**) = 0

SOLUTIONS to Problems

**1.**

|  |  |  |  |
| --- | --- | --- | --- |
| s5 | 1 | 4 | 2 |
| s4 | 4 | 5 | 2 |
| s3 |  |  | 0 |
| s2 |  | 2 | 0 |
| s1 | - | 0 | 0 |
| s0 | 2 | 0 | 0 |

3 LHP, 2 RHP, 0 axis

**2.**

The Routh array for  is:

|  |  |  |  |
| --- | --- | --- | --- |
| s5 | 1 | 6 | 8 |
| s4 | ~~0~~ ε | 5 | 20 |
| s3 |  |  |  |
| s2 | 5 | 20 |  |
| s | ~~0~~  10 |  |  |
| 1 | 20 |  |  |

The auxiliary polynomial for row 4 is , with , so there are two roots on the -axis. The first column shows two sign changes so there are two roors on the right half-plane. The balance, one root must be in the left half-plane.

**3.**

|  |  |  |  |
| --- | --- | --- | --- |
| s5 | 1 | 4 | 2 |
| s4 | -1 | -4 | -3 |
| s3 |  |  | 0 |
| s2 |  | -3 | 0 |
| s1 | 1 | 0 | 0 |
| s0 | -3 | 0 | 0 |

2 LHP, 3 RHP, 0 axis

**4.**

|  |  |  |  |
| --- | --- | --- | --- |
| s5 | 1 | 3 | 2 |
| s4 | -1 | -3 | -2 |
| s3 | -2 | -3 | ROZ |
| s2 | -3 | -4 |  |
| s1 | -1/3 |  |  |
| s0 | -4 |  |  |

Even (4): 4 jRest(1): 1 rhp; Total (5): 1 rhp; 4 j

**5.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| s4 | 1 | 11 | 18 |  |
| s3 | 3 | 27 | 0 |  |
| s2 | 2 | 18 | 0 |  |
| s1 | 4 | 0 | 0 | ROZ |
| s0 | 18 | 0 | 0 |  |

Even (2): 2 j; Rest (2): 2 lhp; Total: 2 j; 2 lhp

**6.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| s6 | 1 | -6 | 1 | -6 |
| s5 | 1 | 0 | 1 |  |
| s4 | -6 | 0 | -6 |  |
| s3 | -24 | 0 | 0 | ROZ |
| s2 |  | -6 |  |  |
| s1 | -144/ | 0 |  |  |
| s0 | -6 |  |  |  |

Even (4): 2 rhp; 2 lhp; Rest (2): 1 rhp; 1 lhp; Total: 3 rhp; 3 lhp

**7.**

The characteristic equation is:

Or

|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 1 | 71 | 704 |
| s3 | 14 | 154 | 0 |
| s2 | 60 | 704 | 0 |
| s1 | -10.27 | 0 | 0 |
| s0 | 704 | 0 | 0 |

The system has two closed loop poles in the rhp, so it is closed-loop unstable.

**8.**

**Program:**

numg=584;

deng=poly([-2 -3 -4 -5]);

'G(s)'

G=tf(numg,deng)

'Poles of G(s)'

pole(G)

'T(s)'

T=feedback(G,1)

'Poles of T(s)'

pole(T)

**Computer response:**

ans =

'G(s)'

G =

584

-----------------------------------

s^4 + 14 s^3 + 71 s^2 + 154 s + 120

Continuous-time transfer function.

ans =

'Poles of G(s)'

ans =

-5.0000

-4.0000

-3.0000

-2.0000

ans =

'T(s)'

T =

584

-----------------------------------

s^4 + 14 s^3 + 71 s^2 + 154 s + 704

Continuous-time transfer function.

ans =

'Poles of T(s)'

ans =

-7.0657 + 3.3858i

-7.0657 - 3.3858i

0.0657 + 3.3858i

0.0657 - 3.3858i

System is unstable, since two closed-loop poles are in the right half-plane.

**9.**

The characteristic equation is:

Or

|  |  |  |
| --- | --- | --- |
| s3 | 1 |  |
| s2 | 5 |  |
| s1 |  | 0 |
| s0 |  | 0 |

Therefore .

**10.**



|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 2 | 1 | 1 |
| s3 | 5 | 2 | 0 |
| s2 | 1 | 5 |  |
| s1 | -23 | 0 |  |
| s0 | 5 |  |  |

Total: 2 lhp, 2 rhp

System is unstable

**11.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| s7 | 1 | -3 | -1 | 3 |  |
| s6 | 2 | -10 | -2 | 10 |  |
| s5 | 2 | 0 | -2 | 0 |  |
| s4 | -10 | 0 | 10 | 0 |  |
| s3 | -40 | 0 | 0 | 0 | ROZ |
| s2 |  | 10 | 0 | 0 |  |
| s1 |  | 0 | 0 | 0 |  |
| s0 | 10 | 0 | 0 | 0 |  |

Even (4): 1 rhp,1 lhp, 2jω axis. Tot: 2 rhp, 3 lhp, 2jω axis

**12.**

Even (6): 1 rhp, 1 lhp, 4 j; Rest (1): 1 lhp; Total: 1 rhp, 2 lhp, 4 j

**13.**

The characteristic equation is or

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| s5 | 1 | 0 | 64 |  |
| s4 | 1 | 0 | 64 |  |
| s3 | 4 | 0 | 0 | ROZ |
| s2 |  | 64 | 0 |  |
| s1 |  | 0 | 0 |  |
| s0 | 64 | 0 | 0 |  |

Even (4): 2rhp, 2 lhp, 0 jω axis. Tot: 2rhp, 3lhp, 0 jω axis

**14.**

The characteristic equation is

|  |  |  |
| --- | --- | --- |
| s3 | 1 | 6 + K |
| s2 | 5 | 10K |
| s1 | 6 -K | 0 |
| s0 | 10K | 0 |

Stable for 0 < K < 6

**15.**

The characteristic equation for all cases is  or . The Routh array is

|  |  |  |
| --- | --- | --- |
|  | 1 |  |
|  |  |  |
| 1 |  |  |

a) 

b) 

c) 

d) No solution

**16.**



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | For 1st column negative | For 1st column positive |
| s2 | 1+K | 8+15K | K < -1 | K > -1 |
| s1 | 8K-6 | 0 | K < 6/8 | K > 6/8 |
| s0 | 8+15K | 0 | K < -8/15 | K > -8/15 |

Stable for K > 6/8

**17.**

**Program:**

%-det([si() si();sj() sj()])/sj()

%Template for use in each cell.

syms K %Construct a symbolic object for

%gain, K.

s2=[(1+K) (8+15\*K) 0]; %Create s^2 row of Routh table.

s1=[(8\*K-6) 0 0]; %Create s^1 row of Routh table.

s0=[-det([s2(1) s2(2);s1(1) s1(2)])/s1(1)...

-det([s2(1) s2(3);s1(1) s1(3)])/s1(1) 0 0];

%Create s^0 row of Routh table.

's2' %Display label.

s2=simplify(s2); %Simplify terms in s^2 row.

pretty(s2) %Pretty print s^2 row.

's1' %Display label.

s1=simplify(s1); %Simplify terms in s^1 row.

pretty(s1) %Pretty print s^1 row.

's0' %Display label.

s0=simplify(s0); %Simplify terms in s^0 row.

pretty(s0) %Pretty print s^0 row.

**Computer response:**

ans =

s2

[1 + K 8 + 15 K 0]

ans =

s1

[8 K - 6 0 0]

ans =

s0

[8 + 15 K 0 0 0]

**18.**

. For positive coefficients in the denominator, . Hence marginal stability only for this range of K.

**19.**

The closed-loop transfer function:



is always unstable since *s*3 and *s*2 terms are missing.

**20.**



|  |  |  |
| --- | --- | --- |
| s3 | K | 2K |
| S2 |  |  |
| s1 |  | 0 |
| S0 | 12-40K |  |

For stability, 

**21.**

T(s) =

|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 1 | - 3 | 2K - 4 |
| s3 | 3 | K+ 3 | 0 |
| s2 |  | 2K - 4 | 0 |
| s1 |  | 0 | 0 |
| s0 | 2K - 4 | 0 | 0 |

Conditions state that K < -12, K > 2, and K > -33. These conditions cannot be met simultaneously. System is not stable for any value of K.

**22.**



|  |  |  |
| --- | --- | --- |
| s3 | 1 | 6031 |
| s2 | 142 | K+79002 |
| s1 | (777400-K)/142 | 0 |
| s0 | K+79002 | 0 |

There will be a row of zeros at *s*1 row if *K* = 777400. The previous row, *s*2, yields the auxiliary equation, . Thus, s = ±j77.6595. Hence, *K* = 777400 yields an oscillation of 77.6595 rad/s.

**23.**

T(s) = 

Since all coefficients must be positive for stability in a second-order polynomial, -1 < K < 1;

- ∞< K < 1; -1 < 2K < ∞. Hence, - < K < 1.

**24.**

The characteristic equation is

which can be expressed as a polynomial as

The Routh array is

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1 |  |  |
|  | 12 |  |  |
|  |  |  |  |
| s |  |  |  |
| 1 |  |  |  |

To make all the entries of the first column >0, the dominant requirement is the one in the fifth row Thus for closed loop stability .

**25.**

The characteristic equation is

|  |  |  |
| --- | --- | --- |
| s3 | 1 | 3 + K |
| s2 | 4 | 5K |
| s1 |  | 0 |
| s0 | 5K | 0 |

1. Stable for 0 < K < 12.
2. The system will oscillate when K=12.
3. When K=12 the third row becomes a row of zeros, the auxiliary equation is . The poles on the axis are so the oscillation frequency is rad/sec.

**26.**

The characteristic equation is

|  |  |  |
| --- | --- | --- |
| s3 | 1 | 22 |
| s2 | 10 | K |
| s1 |  | 0 |
| s0 | K | 0 |

The system is closed-loop stable for . The system will oscillate when . When the third row becomes a row of zeros, yielding an auxiliary equation . The poles on the axis are so the oscillation frequency is rad/sec.

**27.**

T(s) =

|  |  |  |
| --- | --- | --- |
| s3 | 1 | 2K-4 |
| s2 | K-1 | 24 |
| s1 |  | 0 |
| s0 | 24 | 0 |

For stability, K > 5; Row of zeros if K = 5. Therefore, 4s2 + 24 = 0. Hence,  = for

oscillation.

**28.**

The characteristic equation is

|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 1 | 18 |  |
| s3 | 8 | 16 | 0 |
| s2 | 16 |  | 0 |
| s1 |  | 0 | 0 |
| s0 |  | 0 | 0 |

The system is closed-loop stable for . The system will oscillate when . When the fourth row becomes a row of zeros, yielding an auxiliary equation . The poles on the axis are so the oscillation frequency is rad/sec.

**29.**



|  |  |  |
| --- | --- | --- |
| s3 | 1 | 201 |
| s2 | 53 | K+245 |
| s1 | 10408-K | 0 |
| s0 | K+245 | 0 |

**a.** System is stable for -245 < K < 10408.

**b.** Row of zeros when K = 10408. Therefore, 53s2 + 10653. Thus, s =  , or = 14.18 rad/s.

**30.**



|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 1 | 18 | K |
| s3 | 9 | 12 | 0 |
| s2 |  | K | 0 |
| s1 | K + 12 | 0 | 0 |
| s0 | K | 0 | 0 |

**a.** For stability 0 < K < 22.22.

**b.** Row of zeros when K = 22.22. Therefore, s2 + 22.22. Thus, s = ± j, or *ω* = 1.1547 rad/s.

**c.** The denominator of the closed-loop transfer function is: . Substituting K = 22.22 and solving for the roots yields: s = ± j 1.1547, – 6.393, and – 2.607.

**31.**

T(s) =

|  |  |  |
| --- | --- | --- |
| s3 | 1 | K+1 |
| s2 | 2 | - K |
| s1 |  | 0 |
| s0 | - K | 0 |

Stability if - < K < 0.

**32.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| s8 | 1 | 1.18E+03 | 2.15E+03 | -1.06E+04 | -415 |
| s7 | 103 | 4.04E+03 | -8.96E+03 | -1.55E+03 | 0 |
| s6 | 1140.7767 | 2236.99029 | -10584.951 | -415 | 0 |
| s5 | 3838.02357 | -8004.2915 | -1512.5299 | 0 | 0 |
| s4 | 4616.10784 | -10135.382 | -415 | 0 | 0 |
| s3 | 422.685462 | -1167.4817 | 0 | 0 | 0 |
| s2 | 2614.57505 | -415 | 0 | 0 | 0 |
| s1 | -1100.3907 | 0 | 0 | 0 | 0 |
| s0 | -415 | 0 | 0 | 0 | 0 |

**a.** From the first column, 1 rhp, 7 lhp, 0 j

**b.** G(s) is not stable because of 1 rhp.

**33.**

The characteristic equation for this system is:

 or 

The Routh array is:

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1 |  |  |
|  | ~~0~~  ~~4~~ | ~~0~~  2 |  |
|  | -2 |  |  |
|  |  |  |  |
| 1 |  |  |  |

The second row of zeros was substituted with the coefficients resulting from differentiating the characteristic equation:  and .

Since all the plant parameters are positive, there are two sign changes in the first column of the Routh array. So there are two poles in the RHP, two must be in the LHP.

**34.**

**Program:**

A=[0 1 0;0 1 -4;-1 1 8];

eig(A)

**Computer response:**

ans =

7.4641

0.5359

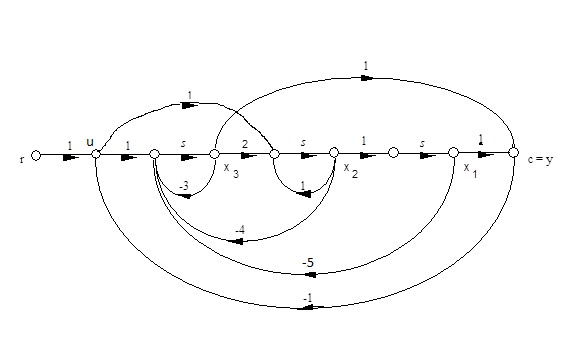
1.0000

**35.**

Writing the open-loop state and output equations we get**,**

****

Drawing the signal-flow diagram and including the unity feedback path yields,



In closed loop , substituting

In matrix form the closed loop system is

The characteristic equation is

The Routh array:

|  |  |  |
| --- | --- | --- |
|  | 1 | -2 |
|  | 2 | 8 |
| s | -6 |  |
| 1 | 8 |  |

Since there are two sign changes in the first column the system is closed loop unstable.

**36.**

**Program:**

A=[0,1,0;0,1,2;-5,-4,-3];

B=[0;1;0];

C=[1,0,1];

D=0;

'G'

G=ss(A,B,C,D)

'T'

T=feedback(G,1)

'Eigenvalues of T'

ssdata(T);

eig(T)

**Computer response:**

ans =

G

G =

a =

x1 x2 x3

x1 0 1 0

x2 0 1 2

x3 -5 -4 -3

b =

u1

x1 0

x2 1

x3 0

c =

x1 x2 x3

y1 1 0 1

d =

u1

y1 0

Continuous-time state-space model.

ans =

T

T =

a =

x1 x2 x3

x1 0 1 0

x2 -1 1 1

x3 -5 -4 -3

b =

u1

x1 0

x2 1

x3 0

c =

x1 x2 x3

y1 1 0 1

d =

u1

y1 0

Continuous-time state-space model.

ans =

Eigenvalues of T

ans =

0.2442 + 1.7764i

0.2442 - 1.7764i

-2.4883 + 0.0000i

**37.**

The matrix **A** was found to be:



Using MATLAB gives:

**Program:**

A=[0 1 0 0;29.8615 0 0 0;0 0 0 1;-0.9401 0 0 0];

eig(A)

**Computer response:**

ans =

0

0

5.4646

-5.4646

With one pole only in the left half-plane, one in the right half-plane, and two on the jω-axis (in this case, at the origin), we conclude that this unit is unstable and requires stabilization.

SOLUTIONS to DESIGN Problems

**38.**

T(s) =

|  |  |  |
| --- | --- | --- |
| s3 | 1 | 11.91+11K |
| s2 | 5.45+K | 43.65+10K |
| s1 |  | 0 |
| s0 | 43.65+10K | 0 |

For stability, - 0.36772 < K < ∞. Stable for all positive K.

**39.**



|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 1 | 1.37 | 0.07K+0.015 |
| s3 | 2.3 | 0.265 | 0 |
| s2 | 1.2548 | 0.07K+0.015 | 0 |
| s1 | 0.23751 - 0.12831K | 0 | 0 |
| s0 | 0.07K+0.015 | 0 | 0 |

For stability, – 0.21429 < K < 1.85106

**40.**

The characteristic equation is given by:



Or



The corresponding Routh array is:

|  |  |  |
| --- | --- | --- |
|  | 1 |  |
|  |  |  |
|  |  |  |
| 1 |  |  |

For stability row 2 requires  and row 4 requires . The dominant requirement being the latter. It is clear also that when , the first element on row 3 is positive. So the overall requirement for stability is .

**41.**

The characteristic equation of the system is given by:

**** or

**** or

**** or

**** or

****Substituting numerical values the equation becomes:

****

The Routh array is given by

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
| 1 |  |  |

To obtain positive quantities on the first column it is required:









or



or



or



or



So either  and 

or  and 

The most dominant requirement is given by the fourth row. We conclude requiring .

**42.**

For simplification we substitute parameter values into the open loop transfer function. It becomes:

The characteristic equation becomes:

=0 or

Or

The Routh array becomes:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

It is clear from the array that the entries in the first column will be positive for all

**43.**

First, find the transfer function of the internal (flow-control) loop, , then the overall transfer function 

The transfer function of the internal (flow-control) feedback loop in figure 2 is:



Thus, the overall system transfer function is:



= 

= 

The characteristic polynomial is, therefore:

 =

= 

Hence, theRouth-Hurwitz array for the system is given by:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *s5* | 250 | 213 | 3.5 + K | 0 |
| *s4* | 335 | 45.5 + 4K | 0.5 | 0 |
| *s3* | [[1]](#footnote-2)# |  | 0 |  |
| *s2* | *C* | 0.5 | 0 |  |
| *s1* |  | 0 |  |  |
| *s0* | 0.5 |  |  |  |

From the *s3* row: , therefore K < 59.98 (4)

From the *s2* row: C =  > 0.

Therefore:  > 0.

This inequality may be expanded to:  =

, or which shows that for stability:

*K* < 36.73

For *C > 0*, the *s1* row  is greater than 0 if:

 > 0

This inequality may be re-written as: > 0

After algebraic manipulations this can be rewritten as:

or

From the previous, and for positive K, we conclude that the system will be stable only if level controller’s derivative gain, *KD*LC is within the range: 0 *< KD*LC< 36.7.

**44.**

****

****

****

****

The Routh array is:

|  |  |  |  |
| --- | --- | --- | --- |
| s4 |  |  |  |
| s3 |  |  |  |
| s2 |  |  |  |
| s |  |  |  |
| 1 |  |  |  |

Row 3 is positive if 

Rows 4 and 5 are positive if 

So the system is closed loop stable if .

**45.**

The characteristic equation for this system is

or

The Routh array is:

|  |  |  |  |
| --- | --- | --- | --- |
|  | 1 | 13537.5 | 25000 |
|  | 1013.5 | 37525 | 1022.5K |
|  | 13500.5 | 25000-1.009K |  |
|  | 0.07574K+35648.22 | 1022.5K |  |
| s |  |  |  |
| 1 | 1022.5K |  |  |

For closed loop stability the last row requires K>0, which means that the fourth row will also be positive. This makes the denominator of the 5th row negative which requires a negative numerator. The numerator of the 5th row can be expressed as , which will be negative as long as K<64.4. Therefore the range for closed loop stability is 0<K<64.4.

**46.**

The pay-loaded characteristic equation is:

,

which for convenience we express as

The corresponding Routh array is

|  |  |  |
| --- | --- | --- |
|  | 1 |  |
|  |  |  |
| s |  |  |
| 1 |  |  |

Since it is known that all , the condition for stability is

Substituting

Solving

**47.**

The characteristic equation can be obtained by calculating Δ=0 from Mason’s Rule. Namely

Substituting

or

The Routh array is

|  |  |  |
| --- | --- | --- |
|  | 1 |  |
| s | 4.2 |  |
| 1 |  |  |

For closed loop stability .

**48.**

1. Applying the feedback formula to the inner loop



Thus, the inner loop will have two equal negative real poles if *K*2 = 1

Multiplying *G*e by *K*1 and closing the loop yields 

Making a Routh table:

|  |  |  |  |
| --- | --- | --- | --- |
| s3 | 1 | 9 | 0 |
| s2 | 6 | *K*1 |  |
| s1 | (54– *K*1) / 6 | 0 |  |
| s0 | *K*1 |  |  |

For the system to be stable: *K*1 > 0 and 54– *K*1 ≥ 0.

1. For the system to oscillate, the s1 row must be a zero row; e.g., *K*1 = 54.

Hence, the frequency of oscillation is found from: 6s2 + 54 = 0 or ω = 3 rad/s.

1. If a real closed-loop pole is at s = *-* 5, the characteristic polynomial may be factored into

. Thus:

5 *b* = *K*1, 5 *a* + *b* = 9, and 5 + *a* = 6 → *a* = 1, *b* = 4, and *K*1 = 20.

Hence, the second-order factor of the characteristic polynomial is: 

This yields: &  → *ζ* = 0.25 for the dominant poles, which, therefore, are given by: . Since the pole at - 5 is 10 times farther than the dominant poles from the j axis, the step response of this system, *c(t)*, may be approximated by a second-order under-damped response.

1. Thus, at *K* = 20, the % O.S. and settling time, Ts, are:

% O.S. = 81.1% & = 8 seconds

**49.**

The transfer function of the minor loop shown in Figure P5.50 was found to be:



Thus, the transfer function of the forward path of the system shown in Figure P 5.50 is given by:



Here, *E(s)* = *ΩL*(s) – *Ωr*(s).

The characteristic polynomial for the system, hence, is given by:



Constructing the Routh table, gives:

|  |  |  |  |
| --- | --- | --- | --- |
| s4 | 1 |  | 312500 |
| s3 |  |  | 0 |
| s2 | C | 312500 | 0 |
| s1 | D | 0 | 0 |
| s0 | 312500 | 0 | 0 |

Here: **** (1)

and **** (2)

For the system to be stable, all of the entries must be positive. This means that:

*Kp* > – 0.0324, C> 0, and D> 0.

Hence, equations (1) & (2) may be re-written as:

, which gives *Kp* > – 0.0381

, which yields *Kp* > – 0.01

Reviewing the last two equations, we conclude that for this system to be stable *Kp* > – 0.01 if *KI* = 0.1, which means that this system will be stable for any positive value of *Kp* if *KI* = 0.1.

**50.**

The Characteristic Equation is given by



or



or



The Routh Array is:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
| 1 |  |  |

Thus for stability

 or 

and

 or 

The intersection of both requirements gives .

**51.**

From the block diagram it is readily obtained:

 or





Noting that the change in car speed, , we get thesystem transfer function, *T(s) = V(s)/Rv(s):*



The characteristic polynomial for that system is, therefore:



Hence, theRouth-Hurwitz for the system is given by:

|  |  |  |  |
| --- | --- | --- | --- |
| *s3* | 6 |  | 0 |
| *s2* |  | 2.65 | 0 |
| *s1* |  | 0 | 0 |
| *s0* | 2.65 | 0 | 0 |

For stability,  and

. That is:





The latter condition indicates that for stability  and 

The intersection of these two requirements shows that for stability:  or, alternatively.

**52.**

Using the suggested first order Padé approximation, the characteristic equation for the system is given by

Equivalently

The Routh array is

|  |  |  |
| --- | --- | --- |
|  | 0.4368 |  |
|  | 1.4368 |  |
|  |  |  |
| 1 |  |  |

For a positive first column in the array, the third row requires ; the fourth row . The intersection of these requirements is

1. # The *S3*row was multiplied by 335 and the *S1*row was multiplied by *C*. [↑](#footnote-ref-2)