Collaboration-Based Learning Environments using Augmented Reality

by

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Submitted to the Department of Creative Technologies
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY IN CREATIVE TECHNOLOGIES

at the

AUCKLAND UNIVERSITY OF TECHNOLOGY

November 2025

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ABSTRACT

The developments of the "kinetic theory" of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.[1]

Thesis supervisor: Stefan Marks
Title: Associate Professor

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[1] Text from Holman (1876): doi:10.2307/25138434.

Acknowledgments

Write your acknowledgments here.

Biographical Sketch

Silas Whitcomb Holman was born in Harvard, Massachusetts on January 20, 1856. He received his S.B. degree in Physics from MIT in 1876, and then joined the MIT Department of Physics as an Assistant. He became Instructor in Physics in 1880, Assistant Professor in 1882, Associate Professor in 1885, and Full Professor in 1893. Throughout this period, he struggled with increasingly severe rheumatoid arthritis. At length, he was defeated, becoming Professor Emeritus in 1897 and dying on April 1, 1900.

Holman's light burned brilliantly before his tragic and untimely death. He published extensively in thermal physics, and authored textbooks on precision measurement, fundamental mechanics, and other subjects. He established the original Heat Measurements Laboratory. Holman was a much admired teacher among both his students and his colleagues. The reports of his department and of the Institute itself refer to him frequently in the 1880's and 1890's, in tones that gradually shift from the greatest respect to the deepest sympathy.

Holman was a student of Professor Edward C. Pickering, then head of the Physics department. Holman himself became second in command of Physics, under Professor Charles R. Cross, some years later. Among Holman's students, several went on to distinguish themselves, including: the astronomer George E. Hale ('90) who organized the Yerkes and Mt. Wilson observatories and who designed the 200 inch telescope on Mt. Palomar; Charles G. Abbot ('94), also an astrophysicist and later Secretary of the Smithsonian Institution; and George K. Burgess ('96), later Director of the Bureau of Standards.

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Chapter 1

Introduction

1.1 A section discussing the first issue: J/ψ

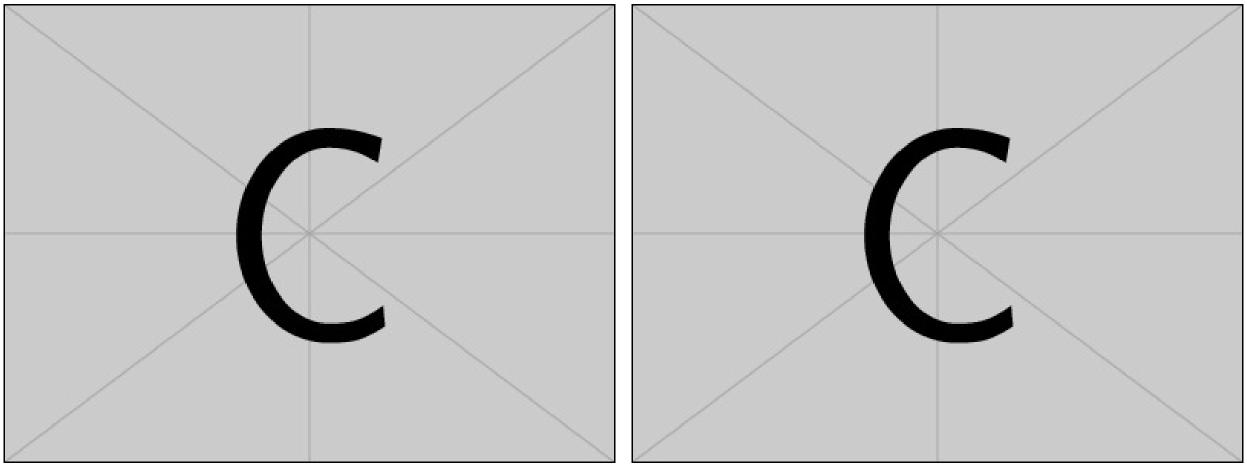
We begin with some ideas from the literature [1,2].

$$\frac{\partial}{\partial t}\left[\rho\big(e+|\vec{u}|^2/2\big)\right]+\nabla\cdot\left[\rho\big(h+|\vec{u}|^2/2\big)\vec{u}\right]=-\nabla\cdot\vec{q}+\rho\vec{u}\cdot\vec{g}+\frac{\partial}{\partial x_j}\big(d_{ji}u_i\big) \tag{1.1}$$

1.1.1 Subsection eqn. (1.2)

A subsubsection

$$L(\mathbf{A})=\begin{pmatrix}
\frac{\varphi}{(\varphi_1,\varepsilon_1)} & 0 & \dots & \dots & \dots & 0\\
\frac{\varphi k_{2,1}}{(\varphi_2,\varepsilon_1)} & \frac{\varphi}{(\varphi_2,\varepsilon_2)} & 0 & \dots & \dots & 0\\
\frac{\varphi k_{3,1}}{(\varphi_3,\varepsilon_1)} & \frac{\varphi k_{3,2}}{(\varphi_3,\varepsilon_2)} & \frac{\varphi}{(\varphi_3,\varepsilon_3)} & 0 & \dots & 0\\
\vdots & \vdots & & \ddots & & \vdots\\
\frac{\varphi k_{n-1,1}}{(\varphi_{n-1},\varepsilon_1)} & \frac{\varphi k_{n-1,2}}{(\varphi_{n-1},\varepsilon_2)} & \dots & \frac{\varphi k_{n-1,n-2}}{(\varphi_{n-1},\varepsilon_{n-2})} & \frac{\varphi}{(\varphi_{n-1},\varepsilon_{n-1})} & 0\\
\frac{\varphi k_{n,1}}{(\varphi_n,\varepsilon_1)} & \frac{\varphi k_{n,2}}{(\varphi_n,\varepsilon_2)} & \dots & \dots & \frac{\varphi k_{n,n-1}}{(\varphi_n,\varepsilon_{n-1})} & \frac{\varphi}{(\varphi_n,\varepsilon_n)}
\end{pmatrix} \tag{1.2}$$



(a) First subfigure (b) Second subfigure

Figure 1.1: A figure with two subfigures: (a) first subfigure; (b) second subfigure.

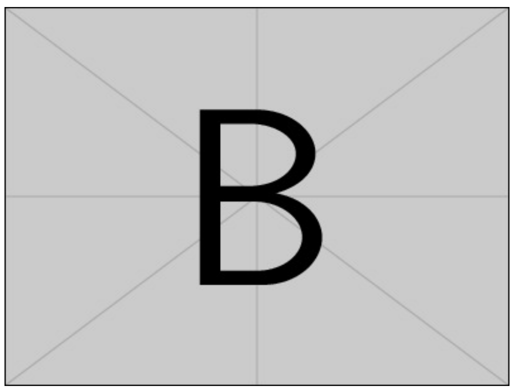


Figure 1.2: Caption text [3].

1.2 Description our paradigm

1.2.1 Conversion to a metaheuristic

1.3 Other generalizations

1.3.1 The most general case

$$x\,\mathrm{Na(NH_4)HPO_4} \xrightarrow{\Delta} \mathrm{(NaPO_3)}_x + x\,\mathrm{NH_3}\uparrow + x\,\mathrm{H_2O} \tag{1.3}$$

$$^{234}_{90}\mathrm{Th} \longrightarrow {}^{0}_{-1}\beta + {}^{234}_{91}\mathrm{Pa} \tag{1.4}$$

$$\mathrm{SO_4}^{2-} + \mathrm{Ba}^{2+} \longrightarrow \mathrm{BaSO_4}\downarrow \tag{1.5}$$

$$\mathrm{Zn}^{2+} \underset{+2\,\mathrm{H}^+}{\overset{+2\,\mathrm{OH}^-}{\rightleftharpoons}} \underset{\text{amphoteric hydroxide}}{\mathrm{Zn(OH)_2}\downarrow} \underset{+2\,\mathrm{H}^+}{\overset{+2\,\mathrm{OH}^-}{\rightleftharpoons}} \underset{\text{tetrahydroxozincate}}{[\mathrm{Zn(OH)_4}]^{2-}} \tag{1.6}$$

These examples of chemical formulæ are copied directly from the documentation of the `mhchem` package, which was used to typeset them.

1.4 Baroclinic generation of vorticity

Substitution of the particle acceleration and application Stokes theorem leads to the *Kelvin-Bjerknes circulation theorem*, for $\rho \neq \mathrm{fn}(p)$:

$$\frac{d\Gamma}{dt} = \frac{d}{dt}\int_{\mathcal{C}} \mathbf{u}\cdot d\mathbf{r} \tag{1.7}$$

$$= \int_{\mathcal{C}} \frac{D\mathbf{u}}{Dt}\cdot d\mathbf{r} + \underbrace{\int_{\mathcal{C}} \mathbf{u}\cdot d\left(\frac{d\mathbf{r}}{dt}\right)}_{=0} \tag{1.8}$$

$$= \iint_{\mathcal{S}} \nabla \times \frac{D\mathbf{u}}{Dt}\cdot d\mathbf{A} \tag{1.9}$$

$$= \iint_{\mathcal{S}} \nabla p \times \nabla\left(\frac{1}{\rho}\right)\cdot d\mathbf{A} \tag{1.10}$$

Baroclinic generation of vorticity accounts for the sea breeze and various other atmospheric currents in which temperature, rather than pressure, creates density gradients. Further, this phenomenon accounts for ocean currents in straits joining more and less saline seas, with surface currents flowing from the fresher to the saltier water and with bottom current going oppositely.

Nomenclature for Chapter 1

Roman letters

$\mathcal{C}$	material curve
$\mathbf{r}$	material position [m]
$\mathbf{u}$	velocity [m s^{-1}]

Greek letters

Γ	circulation [m^2 s^{-1}]
ρ	mass density [kg m^{-3}]
$\boldsymbol{\omega}$	vorticity [s^{-1}]

Table 1.1: The error function and complementary error function

x	erf(x)	erfc(x)	x	erf(x)	erfc(x)
0.00	0.00000	1.00000	1.10	0.88021	0.11980
0.05	0.05637	0.94363	1.20	0.91031	0.08969
0.10	0.11246	0.88754	1.30	0.93401	0.06599
0.15	0.16800	0.83200	1.40	0.95229	0.04771
0.20	0.22270	0.77730	1.50	0.96611	0.03389
0.30	0.32863	0.67137	1.60	0.97635	0.02365
0.40	0.42839	0.57161	1.70	0.98379	0.01621
0.50	0.52050	0.47950	1.80	0.98909	0.01091
0.60	0.60386	0.39614	1.8214	0.99000	0.01000
0.70	0.67780	0.32220	1.90	0.99279	0.00721
0.80	0.74210	0.25790	2.00	0.99532	0.00468
0.90	0.79691	0.20309	2.50	0.99959	0.00041
1.00	0.84270	0.15730	3.00	0.99998	0.00002

Appendix A

Code listing

This example uses the `listings` package.

```
function print_rate(kappa,xMin,xMax,npoints,option)
      local c = 1-kappa*kappa
      local croot = (1-kappa*kappa)^(1/2)
      local logx = math.log(xMin)
      local psi = 0

      local xstep = (math.log(xMax)-math.log(xMin))/(npoints-1)

      arg0 = math.sqrt(xMin/c)
      psi0 = (1/c)*math.exp((kappa*arg0)^2)*(erfc(kappa*arg0)-erfc(arg0))

      if option~=[[]] then
        tex.sprint("\\addplot+["..option.."] coordinates{")
        -- addplot+ for color cycle to work
      else
        tex.sprint("\\addplot+ coordinates{")
      end
      tex.sprint("("..xMin..","..psi0..")")

      for i=1, (npoints-1) do
        x = math.exp(logx + xstep)
        arg = math.sqrt(x/c)
        karg = kappa*arg
        if karg<5 then
      -- this break compensates for exp(karg^2), which multiplies the error in the erf approximation...
          logpsi = -math.log(croot) + karg^2 + math.log(erfc(karg)-erfc(arg))
          psi = math.exp(logpsi)
        else
```

```
          psi = (1/(karg) - 1/(2*(karg^3)) + 3/(4*(arg^5)) )/(1
             .77245385*croot)
          -- this is the large x asymptote of the reaction rate
       end
       logx = math.log(x)
       tex.sprint("("..x..","..psi..")")
     end
     tex.sprint("}")
end
\end{luacode*}
```

Appendix B

One-term coefficients for heat conduction

B.1 A multipage table of numbers

This example uses the `longtable` package: $\theta = A_1 f_1 \exp(-\lambda_1^2 \mathrm{Fo})$, $\overline{\theta} = D_1 \exp(-\lambda_1^2 \mathrm{Fo})$.

Table B.1: One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [4].

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
0.01	0.09983	1.0017	1.0000	0.14124	1.0025	1.0000	0.17303	1.0030	1.0000
0.02	0.14095	1.0033	1.0000	0.19950	1.0050	1.0000	0.24446	1.0060	1.0000
0.03	0.17234	1.0049	1.0000	0.24403	1.0075	1.0000	0.29910	1.0090	1.0000
0.04	0.19868	1.0066	1.0000	0.28143	1.0099	1.0000	0.34503	1.0120	1.0000
0.05	0.22176	1.0082	0.9999	0.31426	1.0124	0.9999	0.38537	1.0150	1.0000
0.06	0.24253	1.0098	0.9999	0.34383	1.0148	0.9999	0.42173	1.0179	0.9999
0.07	0.26153	1.0114	0.9999	0.37092	1.0173	0.9999	0.45506	1.0209	0.9999
0.08	0.27913	1.0130	0.9999	0.39603	1.0197	0.9999	0.48600	1.0239	0.9999
0.09	0.29557	1.0145	0.9998	0.41954	1.0222	0.9998	0.51497	1.0268	0.9999
0.10	0.31105	1.0161	0.9998	0.44168	1.0246	0.9998	0.54228	1.0298	0.9998
0.15	0.37788	1.0237	0.9995	0.53761	1.0365	0.9995	0.66086	1.0445	0.9996
0.20	0.43284	1.0311	0.9992	0.61697	1.0483	0.9992	0.75931	1.0592	0.9993
0.25	0.48009	1.0382	0.9988	0.68559	1.0598	0.9988	0.84473	1.0737	0.9990
0.30	0.52179	1.0450	0.9983	0.74646	1.0712	0.9983	0.92079	1.0880	0.9985
0.40	0.59324	1.0580	0.9971	0.85158	1.0931	0.9970	1.05279	1.1164	0.9974
0.50	0.65327	1.0701	0.9956	0.94077	1.1143	0.9954	1.16556	1.1441	0.9960
0.60	0.70507	1.0814	0.9940	1.01844	1.1345	0.9936	1.26440	1.1713	0.9944
0.70	0.75056	1.0918	0.9922	1.08725	1.1539	0.9916	1.35252	1.1978	0.9925
0.80	0.79103	1.1016	0.9903	1.14897	1.1724	0.9893	1.43203	1.2236	0.9904
0.90	0.82740	1.1107	0.9882	1.20484	1.1902	0.9869	1.50442	1.2488	0.9880
1.00	0.86033	1.1191	0.9861	1.25578	1.2071	0.9843	1.57080	1.2732	0.9855
1.10	0.89035	1.1270	0.9839	1.30251	1.2232	0.9815	1.63199	1.2970	0.9828
1.20	0.91785	1.1344	0.9817	1.34558	1.2387	0.9787	1.68868	1.3201	0.9800
1.30	0.94316	1.1412	0.9794	1.38543	1.2533	0.9757	1.74140	1.3424	0.9770
1.40	0.96655	1.1477	0.9771	1.42246	1.2673	0.9727	1.79058	1.3640	0.9739
1.50	0.98824	1.1537	0.9748	1.45695	1.2807	0.9696	1.83660	1.3850	0.9707

Table B.1: (continued)

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
1.60	1.00842	1.1593	0.9726	1.48917	1.2934	0.9665	1.87976	1.4052	0.9674
1.70	1.02725	1.1645	0.9703	1.51936	1.3055	0.9633	1.92035	1.4247	0.9640
1.80	1.04486	1.1695	0.9680	1.54769	1.3170	0.9601	1.95857	1.4436	0.9605
1.90	1.06136	1.1741	0.9658	1.57434	1.3279	0.9569	1.99465	1.4618	0.9570
2.00	1.07687	1.1785	0.9635	1.59945	1.3384	0.9537	2.02876	1.4793	0.9534
2.20	1.10524	1.1864	0.9592	1.64557	1.3578	0.9472	2.09166	1.5125	0.9462
2.40	1.13056	1.1934	0.9549	1.68691	1.3754	0.9408	2.14834	1.5433	0.9389
2.60	1.15330	1.1997	0.9509	1.72418	1.3914	0.9345	2.19967	1.5718	0.9316
2.80	1.17383	1.2052	0.9469	1.75794	1.4059	0.9284	2.24633	1.5982	0.9243
3.00	1.19246	1.2102	0.9431	1.78866	1.4191	0.9224	2.28893	1.6227	0.9171
3.50	1.23227	1.2206	0.9343	1.85449	1.4473	0.9081	2.38064	1.6761	0.8995
4.00	1.26459	1.2287	0.9264	1.90808	1.4698	0.8950	2.45564	1.7202	0.8830
4.50	1.29134	1.2351	0.9193	1.95248	1.4880	0.8830	2.51795	1.7567	0.8675
5.00	1.31384	1.2402	0.9130	1.98981	1.5029	0.8721	2.57043	1.7870	0.8533
6.00	1.34955	1.2479	0.9021	2.04901	1.5253	0.8532	2.65366	1.8338	0.8281
7.00	1.37662	1.2532	0.8932	2.09373	1.5411	0.8375	2.71646	1.8673	0.8069
8.00	1.39782	1.2570	0.8858	2.12864	1.5526	0.8244	2.76536	1.8920	0.7889
9.00	1.41487	1.2598	0.8796	2.15661	1.5611	0.8133	2.80443	1.9106	0.7737
10.00	1.42887	1.2620	0.8743	2.17950	1.5677	0.8039	2.83630	1.9249	0.7607
12.00	1.45050	1.2650	0.8658	2.21468	1.5769	0.7887	2.88509	1.9450	0.7397
14.00	1.46643	1.2669	0.8592	2.24044	1.5828	0.7770	2.92060	1.9581	0.7236
16.00	1.47864	1.2683	0.8541	2.26008	1.5869	0.7678	2.94756	1.9670	0.7109
18.00	1.48830	1.2692	0.8499	2.27556	1.5898	0.7603	2.96871	1.9734	0.7007
20.00	1.49613	1.2699	0.8464	2.28805	1.5919	0.7542	2.98572	1.9781	0.6922
25.00	1.51045	1.2710	0.8400	2.31080	1.5954	0.7427	3.01656	1.9856	0.6766
30.00	1.52017	1.2717	0.8355	2.32614	1.5973	0.7348	3.03724	1.9898	0.6658
35.00	1.52719	1.2721	0.8322	2.33719	1.5985	0.7290	3.05207	1.9924	0.6579
40.00	1.53250	1.2723	0.8296	2.34552	1.5993	0.7246	3.06321	1.9942	0.6519
50.00	1.54001	1.2727	0.8260	2.35724	1.6002	0.7183	3.07884	1.9962	0.6434
60.00	1.54505	1.2728	0.8235	2.36510	1.6007	0.7140	3.08928	1.9974	0.6376
80.00	1.55141	1.2730	0.8204	2.37496	1.6013	0.7085	3.10234	1.9985	0.6303
100.00	1.55525	1.2731	0.8185	2.38090	1.6015	0.7052	3.11019	1.9990	0.6259
200.00	1.56298	1.2732	0.8146	2.39283	1.6019	0.6985	3.12589	1.9998	0.6170
∞	1.57080	1.2732	0.8106	2.40483	1.6020	0.6917	3.14159	2.0000	0.6079

References

[1] C. Fong. *Analytical Methods for Squaring the Disc. ArXiv e-prints*. Sept. 2015. arXiv: 1509.06344.

[2] M. Sharpe. *New TX Font Package*. Version 1.71. Comprehensive TeX Archive Network, Mar. 2022. URL: https://ctan.org/pkg/newtx (visited on 05/29/2023).

[3] M. Galassi, J. Davies, J. Theiler, B. Gough, G. Jungman, P. Alken, M. Booth, F. Rossi, and R. Ulerich. *GNU Scientific Library*. Version 2.4. Free Software Foundation. Boston, MA, 2017. URL: https://www.gnu.org/software/gsl/ (visited on 12/12/2018).

[4] H. D. Baehr and K. Stephan. *Heat and Mass Transfer*. Berlin: Springer-Verlag, 1998. ISBN: 3-540-63695-1.