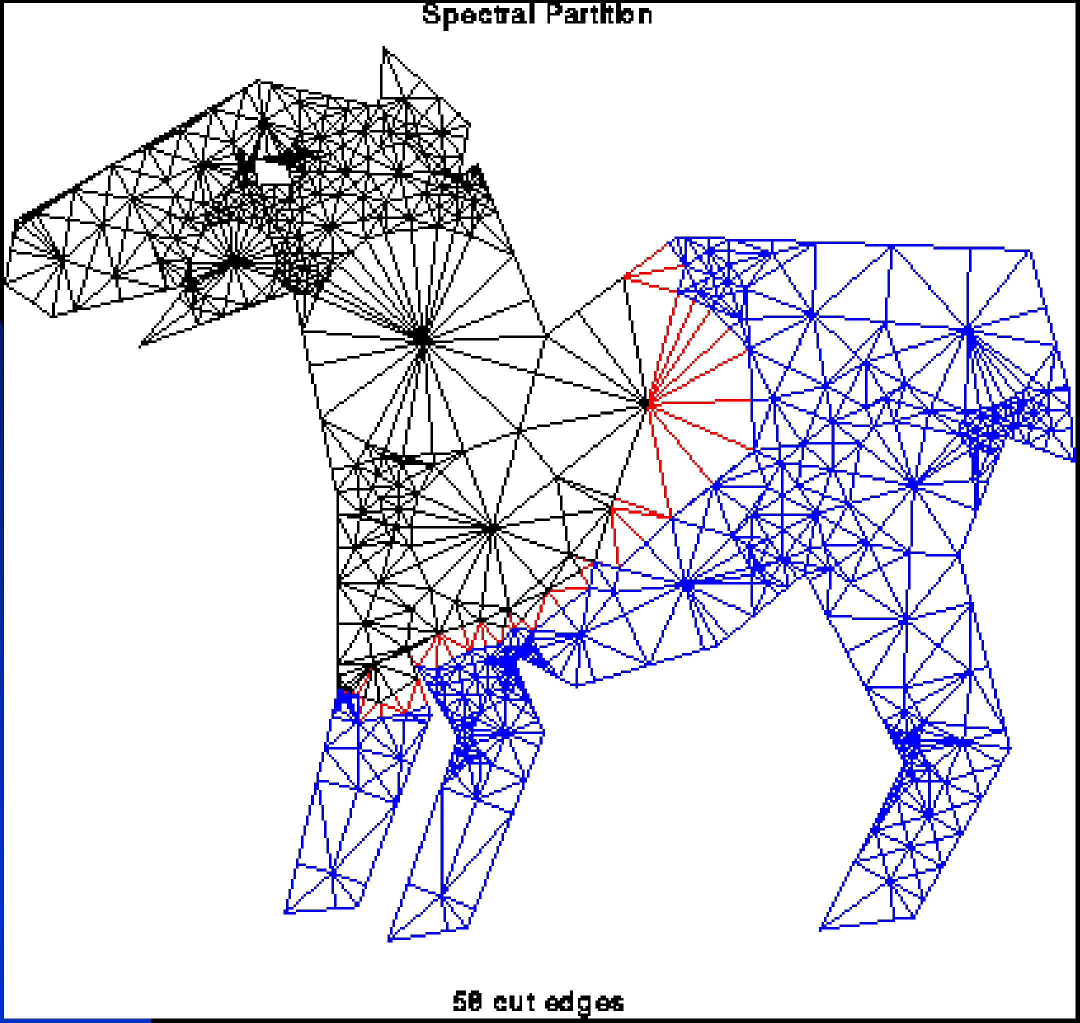
Introduction to FEM

X

- Finite Element Method:
- $-d^2u(x)/d^2x = f(x)$, $0<x<1$
- $u(0)=u(1)=0$
- Variation formulation:
- Min $\frac{1}{2} \int v'.v'\, dx - \int f.v\, dx$
- $V(x)= \sum \eta_i \phi_i(x)$
  - where η_i are the unknowns, and $\phi_i(x)$ the so-called basis functions
- $\sum \eta_i \int \phi_i'\phi_j'\, dx = \int f\, \phi_j\, dx$, $j=1,\ldots,N$
  - With $a(i,j)= \int \phi_i'\phi_j'\, dx\ !=0$ if node i and j are in the same element, otherwise =0 (I and j not in the same element)

- $\Rightarrow A\underline{\eta} = \underline{b}$
  - With $a(i,j)= \int \phi_i' \phi_j' dx$!=0 if node i and j are in the same element, otherwise =0 (i and j not in the same element)
- Comparison of matrix structure:
  - 1-D --- both FD and FE: tri-diagonal
  - 2-D --- FD: pental-diagonal, FE: less structured sparse matrix

A grid with traingular elements



FEM exercise 2

Two programs: GridDist.c and MPI_Fempois.c

- Partition&Distribution: GridDist.c
  - Generate a FE mesh (triangular elements, grid.c)
  - Divide in subdomains
- OUTPUT:
  - file inputX_Y.dat (X=#grid points, Y=process/subdomain number)
  - file mappingP.dat: topology between the processes (connectivity between subdomains)

- Format of “inputX_Y.dat”:
  - List of elements in subdomain
  - #neighboring subdomains: 2
  - From 2: 2550 2551
  - To 2: 2499 2500
  - From 1: 50 101 152
  - To 1: 49 100 151
- MPI_Fempois.c: the parallel FEM solver

- Communication:
  - Point-to-point: between neighbors defined in inputX_Y.dat
  - Global: MPI_Allreduce for computing the in-product