CHAPTER IV

ELECTRICAL, MECHANICAL AND ACOUSTICAL ANALOGIES

4.1. Introduction. — Systems in use to-day for sound reproduction include mechanical, acoustical, electro-acoustical, electro-mechanical, mechano-acoustical and electro-mechano-acoustical systems. Almost any work involving mechanical or acoustical systems also includes electrical systems and electric circuit theory. Acoustical measurements are usually made with electrical instruments and in terms of electrical quantities. Ultimately the electrical units must be compared with acoustical and mechanical units. In general, anyone in any manner connected with sound reproduction must be familiar with electric circuit theory. Therefore, it is logical, whenever possible and convenient, to treat mechanical and acoustical problems by the same mathematical theory as is used in electric circuit problems.

Electric circuit theory is the branch of electromagnetic theory which deals with electrical oscillations in linear electrical networks. An electrical network is a connected set of separate circuits termed branches or meshes. These branches or meshes are composed of elements. Resistance, inductance and capacitance are termed elements.

Mathematically the elements in an electrical circuit are the coefficients in the differential equations. The equations of electric circuit theory may be based upon Maxwell's dynamical theory. In this case the network forms a dynamical system in which the currents play the role of velocities. In the same way the coefficients in the differential equations of a mechanical or acoustical system may be looked upon as mechanical or acoustical elements. In other words, every electrical, mechanical or acoustical system may be considered as a combination of electrical, mechanical or acoustical elements. Therefore, any mechanical or acoustical system may be reduced to an electrical network. Then the problem may be solved by electric circuit theory.

It is the purpose of this chapter to consider electrical, mechanical and acoustical elements and the combination of elements and to indicate analogies between the elements and connections in the three systems.

4.2. Resistance. — A. *Electrical Resistance.* — Resistance is the circuit element which causes dissipation.

Electrical resistance r_E, in abohms, is defined by Ohm's law

$$r_E = \frac{e}{i} \qquad 4.1$$

where e = voltage across the resistance, in abvolts,
i = current through the resistance, in abamperes.

B. *Mechanical Resistance.* — In a mechanical system dissipation is due to friction. Mechanical resistance r_M, in mechanical ohms, is defined as

$$r_M = \frac{f_M}{u} \qquad 4.2$$

where f_M = applied mechanical force, in dynes, and
u = velocity at the point of application of the force, in centimeters per second.

C. *Acoustical Resistance.* — In an acoustical system dissipation may be due to fluid resistance or radiation resistance. Fluid resistance is due to viscosity. See Sec. 5.2. In the case of fluid flowing through a pipe with a velocity of one cubic centimeter per second, resistance is represented by the pressure drop along the pipe. The case of radiation resistance is discussed in Sec. 5.7.

Acoustical resistance r_A, in acoustical ohms, is defined as,

$$r_A = \frac{p}{U} \qquad 4.3$$

where p = pressure, in dynes per square centimeter, and
U = volume current, in cubic centimeters per second.

Volume current (sometimes termed volume velocity) is the rate of change of volume displacement with time. In other words, volume current is the linear velocity over an area multiplied by the area.

4.3. Inductance, Inertia, Inertance. — A. *Inductance.* — Inductance is the electrical circuit element which opposes a change in current. Inductance is defined as

$$e = L\frac{di}{dt} \qquad 4.4$$

where L = inductance, in abhenries,
e = electromotive or driving force, in abvolts, and
$\frac{di}{dt}$ = rate of change of current, in abamperes per second

Equation 4.4 states that the driving force or electromotive force across an inductance is proportional to the inductance and the rate of change of current.

B. *Inertia.* — Mass is the mechanical element which opposes a change in velocity. Mass is defined as

$$f_M = m \frac{du}{dt} \tag{4.5}$$

where m = mass in grams,

$\frac{du}{dt}$ = acceleration, in centimeters per second per second, and

f_M = driving force, in dynes.

Equation 4.5 states that the driving force applied to the mass is proportional to the mass and the rate of change of velocity.

C. *Inertance.* — Inertance is the acoustical element which opposes a change in volume current. Inertance is defined as

$$p = M \frac{dU}{dt} \tag{4.6}$$

where M = inertance, in grams per square centimeter per square centimeter,

$\frac{dU}{dt}$ = change in volume current, in cubic centimeters per second per second, and

p = driving pressure, in dynes per square centimeter.

Equation 4.6 states that the driving pressure applied to an inertance is proportional to the inertance and the rate of change of volume current.

4.4. Capacitance, Compliance, Acoustic Capacitance. — A. *Electrical Capacitance.* — Capacitance is the electrical circuit element which opposes a change in voltage. Capacitance is defined as

$$i = C_E \frac{de}{dt} \tag{4.7}$$

where C_E = capacitance, in abfarads,

$\frac{de}{dt}$ = rate of change in voltage, in abvolts per second, and

i = current, in abamperes.

Equation 4.7 may be written

$$e = \frac{1}{C_E}\int idt = \frac{q}{C_E} \tag{4.8}$$

where q = charge on the capacitance, in abcoulombs.

Equation 4.8 states that the charge on a condenser is proportional to the capacitance and the applied electromotive force.

B. *Mechanical Compliance.* — Compliance is the mechanical element which opposes a change of the applied force. Compliance may be defined as

$$f_M = \frac{x}{C_M} \tag{4.9}$$

where C_M = compliance, in centimeters per dyne,
x = displacement, in centimeters, and
f_M = applied force, in dynes.

Equation 4.9 states that the displacement of a compliance is proportional to the compliance and the applied force.

C. *Acoustical Capacitance.* — Acoustical capacitance is the acoustic element which opposes a change in the applied pressure. The pressure, in dynes per square centimeter, in terms of the condensation, from equation 1.21 is

$$p = c^2\rho s \tag{4.10}$$

where c = velocity, in centimeters per second,
ρ = density, in grams per cubic centimeter, and
s = condensation.

The condensation in a volume V due to a change of volume dV is

$$s = \frac{dV}{V} \tag{4.11}$$

The change in volume is

$$dV = Sx \tag{4.12}$$

where x = displacement, in centimeters, over the area S, in square centimeters.

The volume displacement, in cubic centimeters, is,

$$X = Sx \tag{4.13}$$

From equations 4.10, 4.11, 4.12 and 4.13 the pressure is

$$p = \frac{\rho c^2}{V} X \tag{4.14}$$

From the definition of acoustic capacitance, equation 4.14, the acoustic capacitance of a volume is,

$$C_A = \frac{V}{\rho c^2} \tag{4.15}$$

Then equation 4.14 may be written,

$$p = \frac{X}{C_A} \tag{4.16}$$

Equation 4.16 states that the volume displacement in an acoustic capacitance is proportional to the pressure and the acoustic capacitance.

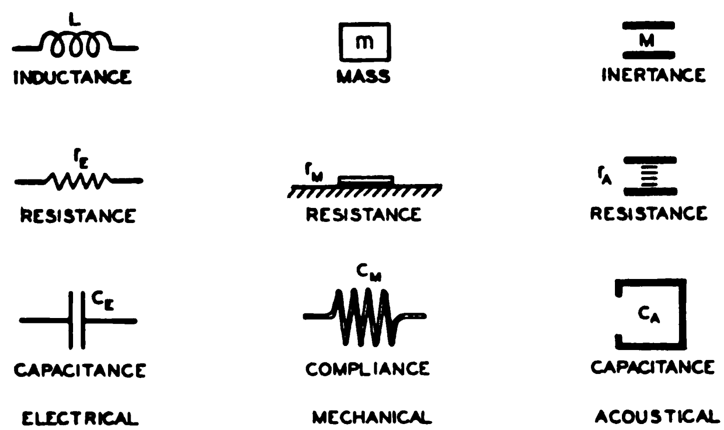

FIG. 4.1. Graphical representation of the three basic elements in electrical, mechanical and acoustical systems.

4.5. Representation of Electrical, Mechanical and Acoustical Elements. — Electrical, mechanical and acoustical elements have been defined in the preceding sections. Figure 4.1 illustrates schematically the three elements in each of the three systems.

Mechanical resistance is represented by sliding friction which causes dissipation. Acoustic resistance is represented by narrow slits which cause dissipation due to viscosity when fluid is forced through the slits. These elements are analogous to resistance in the electrical system.

Inertia in the mechanical system is represented by a mass. Inertance