Direct-Radiator Loudspeaker System Analysis*

RICHARD H. SMALL

School of Electrical Engineering, University of Sydney, Sydney, N. S. W., Australia

The low-frequency performance of direct-radiator loudspeaker systems can be accurately specified and is quantitatively related to the basic parameters of the system components. These systems function at low frequencies as low-efficiency electroacoustic high-pass filters; the frequency-dependent behavior is described by rational polynomial functions whose coefficients contain basic component parameters. These basic parameters, which are simple to evaluate, determine the system low-frequency response, efficiency, and power ratings.

Editor's Note:

This is the first of a series of papers by R. H. Small which will have a long-term impact on direct-radiator loudspeaker theory. This paper is mainly concerned with terminology, definitions, and setting a thorough background for the following papers on specific kinds of loudspeaker systems.

The work on efficiency, power considerations, and large-signal effects is the most accurate that I know of. The appendix contains the only derivation I know of in print for Thiele's methods of driver-parameter measurement.

J. R. Ashley

* Reprinted with permission from *IEEE Transections on Audio and Electroacoustics*, vol. AU-19, pp. 269-281 (Dec. 1971).

GLOSSARY OF SYMBOLS

B magnetic flux density in driver air gap

c velocity of sound in air (=345 m/s)

C_{AB} acoustic compliance of air in enclosure

C_{AP} acoustic compliance of passive radiator suspension

C_{AS} acoustic compliance of driver suspension

C_{MS} mechanical compliance of driver suspension ($=C_{AS}/S_D^2$)

C_{MES} electrical capacitance due to driver mass ($=M_{AS}S_D^2/B^2l^2$)

e_g open-circuit output voltage of source

f natural frequency variable

f_{CT} resonance frequency of driver in closed test box

f_S resonance frequency of driver

$G(s)$ response function

k_x system displacement constant

l length of voice-coil conductor in magnetic field

L_{CES}	electrical inductance due to driver compliance ($=C_{AS}B^2l^2/S_D^2$)
M_{ACT}	acoustic mass of driver in closed test box including air load
M_{AP}	acoustic mass of port or passive radiator including air load
M_{AS}	acoustic mass of driver diaphragm assembly including air load
M_{MS}	mechanical mass of driver diaphragm assembly including air load ($=M_{AS}S_D^2$)
P_A	acoustic output power
P_{AR}	displacement-limited acoustic power rating
P_E	nominal electrical input power
P_{ER}	displacement-limited electrical power rating
$P_{E(max)}$	thermally limited maximum input power
Q	ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)
Q_E	Q of driver at f_S considering system electrical resistance ($R_g + R_E$) only
Q_{ECT}	Q of driver at f_{CT} considering electrical resistance R_E only
Q_{ES}	Q of driver at f_S considering electrical resistance R_E only
Q_M	Q of driver at f_S considering system nonelectrical resistances only
Q_{MCT}	Q of driver at f_{CT} considering nonelectrical resistances only
Q_{MS}	Q of driver at f_S considering driver nonelectrical resistances only
Q_T	total Q of driver at f_S including all system resistances
R_{AB}	acoustic resistance of enclosure losses due to internal energy absorption
R_{AL}	acoustic resistance of enclosure losses due to leakage
R_{AP}	acoustic resistance of port or passive radiator losses
R_{AS}	acoustic resistance of driver suspension losses
R_{AT}	acoustic resistance of total driver-circuit losses
R_E	dc resistance of driver voice coil
R_{ES}	electrical resistance due to driver suspension losses ($=B^2l^2/S_D^2R_{AS}$)
R_g	output resistance of source or amplifier
R_{MS}	mechanical resistance of driver suspension losses ($= R_{AS}S_D^2$)
R_{AR}	acoustic radiation resistance
s	complex frequency variable ($=\sigma + j\omega$)
S_D	effective projected surface area of driver diaphragm
T	time constant ($=1/2\pi f$)
u	linear velocity
U	volume velocity
V_{AS}	volume of air having same acoustic compliance as driver suspension ($=\rho_0c^2C_{AS}$)
V_D	peak displacement volume of driver diaphragm ($=S_Dx_{max}$)
x	linear displacement
x_{max}	peak displacement limit of driver diaphragm
$X(s)$	driver diaphragm displacement function
$Z_{VC}(s)$	voice-coil impedance function
η	efficiency
η_0	reference efficiency
ρ_0	density of air ($=1.18$ kg/m^3)
$\sigma_{x(P)}$	static displacement sensitivity of unenclosed driver expressed in meters per watt$^{1/2}$
ω	radian frequency variable ($=2\pi f$)

INTRODUCTION: It is quite possible that the vagueness which infuses many discussions of loudspeakers has its roots in the chaotic terminology of the subject. The word "loudspeaker" itself long ago lost any specific meaning. Despite conflicting attempts by various nationalities to define it as a driver unit or as a complete system, the word retains value only as a general term and as an adjective. For the sake of clarity, this paper uses the common but more specific terms below.

A *source* is a device, usually an electronic power amplifier, which supplies electrical energy at a specified voltage or power level.

A loudspeaker *driver* is a transducer mechanism which converts electrical energy into mechanical and/or acoustical energy. The most common type of driver and the one dealt with in this paper is the moving-coil or electrodynamic driver consisting of a voice coil located in a permanently magnetized air gap and attached to a suspended diaphragm or "cone."

A *baffle* is a structure used to support a driver and to reduce or prevent cancellation of radiation from the front of the driver diaphragm by antiphase radiation from the rear.

An *enclosure* is a cabinet or box in which a driver is mounted for the purpose of radiating sound. The enclosure forms a closed geometrical surface except for the driver mounting aperture or other specified apertures.

A loudspeaker *system* is the combination of a driver (or drivers) with a structural radiation aid such as a horn, baffle, or enclosure which is used to convert electrical energy from a specified source into sound.

A *direct-radiator loudspeaker system* is a loudspeaker system which couples acoustical energy directly to the air from the driver diaphragm and/or simple enclosure apertures without the use of horns or other acoustical impedance-matching devices.

The *piston range* of a loudspeaker driver is that range of frequencies for which the wavelength of sound is longer than the driver diaphragm circumference. In this frequency range, a direct-radiator system using the driver in an enclosure will have an acoustic output which is essentially nondirectional.

Loudspeaker System Design

Direct-radiator loudspeaker systems have been in use for about half a century. During this time, much knowledge of the behavioral properties of various types of direct-radiator systems has been accumulated, but this knowledge is still uneven and incomplete. For example, closed-box systems are much better understood than vented-box systems, while quantitative design information for passive-radiator systems cannot be found in published form.

The design of a loudspeaker system is traditionally a trial-and-error process guided by experience: a likely driver is chosen and various enclosure designs are tried until the system performance is found to be satisfactory. In sharp contrast to this empirical design process is the synthesis of many other engineering systems. This begins with the desired system performance specifications and leads directly to specification of system components.

The latter approach requires the engineer to have precise knowledge of the relationships between system performance and component specifications. The method of analysis described in this paper is a means of obtaining this knowledge for the low-frequency performance of all types of direct-radiator loudspeaker systems; it is based on the high-pass-filter behavior of these systems.

Loudspeaker System Sensitivity and Efficiency

An ideal microphone converts sound pressure into voltage with equal sensitivity at all frequencies. Recording and reproducing systems are designed to process signal voltages representing sound pressure without distortion. To complete the sound reproduction process, an ideal loudspeaker system should convert voltage into sound pressure with equal sensitivity at all frequencies.

In practice, all loudspeaker systems have limited bandwidth. In the low-frequency region, they act as high-pass filters. The low-frequency design of a loudspeaker system may thus be regarded as the design of a high-pass filter [1], [2]. The principal difference is that the loudspeaker system designer has very limited control over the "circuit" configuration; his design freedom is limited to obtaining the best possible performance by manipulation of the system component values.

The frequency response of an electrical filter is normally described in terms of a dimensionless voltage or power ratio. Because a loudspeaker system is a transducer, its sensitivity versus frequency response is the ratio of two unlike quantities, sound pressure and voltage. However, the loudspeaker system response can also be defined in terms of a dimensionless power ratio which is proportional to the square of the above sensitivity ratio.

In the frequency range for which the system radiation is nondirectional, the free-field sound pressure at a fixed distance is proportional to the square root of the acoustic power radiated by the system [3, p. 189]. The electrical power delivered into a fixed resistance by the source is proportional to the square of the source output voltage. Thus the ratio of *the actual system acoustic output power* to *the electrical power delivered into a fixed resistance by the same source* represents exactly the square of the system sensitivity ratio (i.e., the system frequency response), except for a constant factor. If the fixed resistance is chosen to fairly represent the input impedance of the loudspeaker system, the value of the power ratio in the system passband is the nominal electroacoustic conversion efficiency of the system.

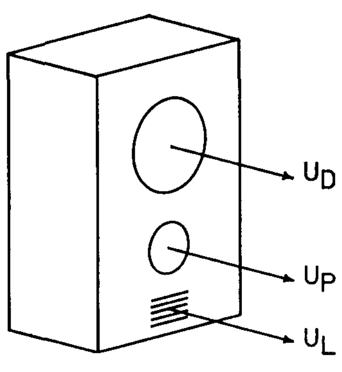

Fig. 1. Generalized direct-radiator loudspeaker system.

This method of defining loudspeaker efficiency is quite similar in principle to the power available efficiency definition used by Beranek [3, p. 190] in that both reveal the exact frequency response of the system. The principal advantage of the method used here is that the calculated passband efficiency of the system is independent of generator output resistance and realistically relates the acoustic power capability of the system to the electrical power rating of its source.

SMALL-SIGNAL PERFORMANCE RELATIONSHIPS

Acoustic Output Power

A generalized direct-radiator loudspeaker system [4, Fig. 1] is illustrated in Fig. 1. The system enclosure has apertures for a driver, a port (or passive radiator), and leakage. Electrical input to the driver produces air movement at the driver diaphragm, port, and leak; this air movement is shown in Fig. 1 as the acoustic volume velocities U_D, U_P, and U_L.

At very low frequencies, where the dimensions of and spacings between the enclosure apertures are much less than a wavelength, the system can be regarded as a combination of coincident simple sources [3, p. 93]. The acoustic output is thus nondirectional and is equivalent to that of a single simple source having a strength U_0 equal to the vector sum of the individual aperture volume velocities, i.e.,

$$U_0 = U_D + U_P + U_L. \tag{1}$$

The acoustic power radiated by the system is then

$$P_A = |U_0|^2 \mathcal{R}_{AR} \tag{2}$$

where

P_A acoustic output power
$\mathcal{R}_{AR}$ resistive part of radiation load on system.

Eq. (2) is generally valid to the upper limit of the driver piston range because the driver is normally the only significant radiator at frequencies high enough for the aperture spacings to become important.

In a recent paper [5], Allison and Berkovitz have demonstrated that the low-frequency load on a loudspeaker system in a typical listening room is essentially that for one side of a piston mounted in an infinite baffle. The resistive part of this radiation load [3, p. 216] is

$$\mathcal{R}_{AR} = \rho_0\omega^2/(2\pi c) \tag{3}$$

where

ρ_0 density of air
ω steady-state radian frequency
c velocity of sound in air.

Eq. (3) is valid only in the system piston range, but within this range the value of $\mathcal{R}_{AR}$ is independent of the size of the enclosure or its apertures.

Because mass cannot be created or stored at the en-

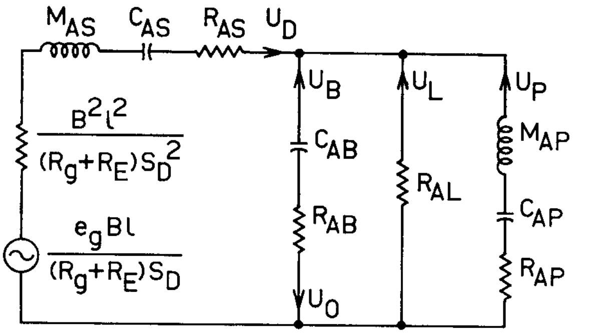

Fig. 2. Acoustical analogous circuit of generalized direct-radiator loudspeaker system.

closure boundaries, and because the sound pressure is normally much less than the atmosphere pressure, conservation of mass requires that

$$U_0 = -U_B \tag{4}$$

where U_B is the total volume velocity entering the enclosure. Eq. (4) holds even if the enclosure is internally divided. If the enclosure contains several cavities, then

$$U_B = U_{B1} + U_{B2} + U_{B3} + \cdots, \tag{5}$$

where each term on the right-hand side of Eq. (5) represents the net volume velocity entering each individual cavity.

Eqs. (1), (4), and (5) are general and hold for any number of cavities and apertures and any interconnection of these. They are vector equations which require that the relative phase of the various components be taken into account.

Although Eq. (4) is very simple, it is of key importance in the analysis of direct-radiator loudspeaker systems using an enclosure. In combination with Eq. (2), it reveals that the acoustic power radiated by the system is directly related to the volume velocity compressing and expanding air within the enclosure. This fact has been noted for bass-reflex enclosures by Beranek [3, p. 244], de Boer [1], and others; it is equally true for *all* direct-radiator system enclosures [4, eq. (72) ff].

Electrical Input Power

The nominal electrical input power to a loudspeaker system is defined here as the power delivered by the source into a resistor having the same value as the driver voice-coil resistance [2, eq. (10)]. Thus

$$P_E = \left[\frac{e_g}{R_g + R_E}\right]^2 R_E \tag{6}$$

where

- P_E nominal electrical input power
- e_g open-circuit output voltage of source
- R_g output resistance of source
- R_E dc resistance of driver voice coil.

The value of R_E is typically about 80% of the rated driver voice-coil impedance.

American [6], British [7], and international [8] standards make use of variously defined rating impedances in calculating the nominal input power to a loudspeaker driver. Because the calculated acoustic output power of the system depends on R_E and not on the fictitious rating impedance, the definition used here simplifies the expression for theoretical system efficiency derived below. This difference must be remembered if the computed piston-range reference efficiency of a system is to be compared with the efficiency measured according to the methods of one of the above standards.

EFFICIENCY

From Eqs. (2) and (6), the nominal power transfer ratio or efficiency η of a loudspeaker system is

$$\eta = \frac{P_A}{P_E} = |U_0|^2 R_{AR} \frac{(R_g + R_E)^2}{e_g^2 R_E}. \tag{7}$$

The evaluation of this efficiency expression for a given system requires a knowledge of the relationship between U_0 and e_g. This relationship is found by examining the acoustical circuit of the system.

The development of acoustical circuits is described in excellent detail by Olson [9] and Beranek [3, ch. 3]. Fig. 2 is the impedance-type acoustical analogous circuit for the generalized loudspeaker system of Fig. 1 [4, Fig. 15]. In Fig. 2,

- B magnetic flux density in driver air gap
- l length of voice-coil conductor in magnetic field of air gap
- S_D effective projected surface area of driver diaphragm
- M_{AS} acoustic mass of driver diaphragm assembly including voice coil and air load
- C_{AS} acoustic compliance of driver suspension
- R_{AS} acoustic resistance of driver suspension losses
- C_{AB} acoustic compliance of air in enclosure
- R_{AB} acoustic resistance of enclosure losses due to internal energy absorption
- R_{AL} acoustic resistance of enclosure losses due to leakage
- M_{AP} acoustic mass of port or passive radiator including air load
- C_{AP} acoustic compliance of passive radiator suspension
- R_{AP} acoustic resistance of port or passive radiator losses.

Starting from the circuit of Fig. 2, the acoustical analogous circuits of most common direct-radiator systems can be obtained by removing or short-circuiting appropriate elements. Note that for the analogy used in this circuit, voltages represent acoustic pressures and currents

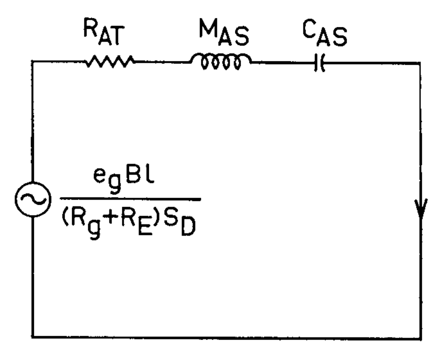

Fig. 3. Acoustical analogous circuit of infinite-baffle loudspeaker system.

represent volume velocities. The method of obtaining the system efficiency expression from analysis of the system acoustical circuit is illustrated below for the simple infinite-baffle system.

The acoustical analogous circuit of an infinite-baffle loudspeaker system is derived from the general circuit of Fig. 2 by removing the branches representing the passive radiator and enclosure leakage and short-circuiting the branch representing the interior of the enclosure to make the enclosure dissipation zero and the enclosure compliance infinite. The resulting circuit is shown in Fig. 3. A simplification has been made in this circuit by combining the remaining series resistances to form the total acoustic resistance

$$R_{AT} = R_{AS} + \frac{B^2l^2}{(R_g + R_E) S_D{}^2}. \tag{8}$$

From circuit analysis of Fig. 3,

$$U_0 = \frac{e_g Bl}{(R_g + R_E) S_D s M_{AS}} \cdot G(s) \tag{9}$$

where

$$G(s) = \frac{s^2 C_{AS} M_{AS}}{s^2 C_{AS} M_{AS} + s C_{AS} R_{AT} + 1} \tag{10}$$

and s is the complex frequency variable.

For steady-state sinusoidal excitation $s = j\omega$, and Eqs. (5) and (9) may be combined with Eq. (7) to yield the infinite-baffle efficiency expression

$$\eta(j\omega) = \frac{\rho_0}{2\pi c} \frac{B^2l^2}{R_E S_D{}^2 M_{AS}{}^2} |G(j\omega)|^2 \tag{11}$$

where $G(j\omega)$ is $G(s)$ from Eq. (10) with $s = j\omega$. Note that $G(j\omega)$ contains all the frequency-dependent terms of Eq. 11); the remainder of the expression contains only physical, numerical, and driver constants.

The last part of Eq. (11), i.e., the squared magnitude of $G(j\omega)$, is the infinite-baffle system frequency response expressed as a normalized power ratio. The normalized ratio of sound pressure to source voltage, i.e., the normalized sensitivity or sound pressure frequency response, is thus simply $|G(j\omega)|$; it can be seen from Eq. (10) that this is a second-order (12-dB per octave cutoff) high-pass filter function.

For any direct-radiator system using an enclosure, the expressions for total volume velocity and efficiency have the same form as Eqs. (9) and (11); only the function $G(s)$ is different for each system.

The system response function $G(s)$ contains complete information about the amplitude and phase versus frequency responses and the transient response of the system. $G(s)$ is always a high-pass filter function with a value of unity in the passband. Thus the constant part of Eq. (11) is the system passband efficiency.

ASSUMPTIONS AND APPROXIMATIONS

The acoustical analogous circuits of Figs. 2 and 3 are valid only for frequencies within the piston range of the driver; the circuit components are assumed to have values which are independent of frequency within this range.

Circuit elements which do not contribute enough impedance to affect the analysis are neglected. One of these elements is the radiation resistance. Although this resistance is responsible for the radiated power and is therefore included in Eq. (2), it is in fact quite small compared to the other impedances in the acoustical circuit [2, p. 489]. This is fortunate for purposes of analysis because the radiation resistance is not constant but varies with frequency squared. Also neglected is the driver voice-coil inductance which usually has negligible effects in the limited frequency range of this analysis.

The treatment of acoustical masses is simplified by adding together all masses appearing in series in the same branch of the analogous circuit. This means that physical and air-load masses are lumped together. While the resulting total mass is essentially constant with frequency, it may vary, in the case of the driver, with mounting location or mounting conditions. This must be remembered when dealing with the actual system and measuring its parameters.

SMALL-SIGNAL PARAMETERS

The response function and other describing equations of a loudspeaker system generally contain driver, enclosure, and source parameters. Knowledge of these relationships for a particular system is of practical use only if the parameter values are known or can be measured.

One key to the identification and measurement of the system parameters lies in the system electrical equivalent circuit. This is the dual of the system acoustical analogous circuit and may be derived from it; its formation is well explained in [9] and [3, ch. 3]. Once the circuit is determined, straightforward circuit analysis yields the relationship between the impedance measured at the voice-coil terminals of the actual system and the physical components which constitute the system. It is thus possible to determine the system parameters from measurement of the voice-coil circuit impedance.

Driver Parameters

The fundamental electromechanical driver parameters which control system small-signal performance are R_E, (Bl), S_D, C_{MS}, M_{MS}, and R_{MS}, where

C_{MS} mechanical compliance of driver suspension $(= C_{AS}/S_D{}^2)$

M_{MS} mechanical mass of driver diaphragm assembly including voice coil and air load $(= M_{AS} S_D{}^2)$

R_{MS} mechanical resistance of driver suspension losses $(= R_{AS} S_D{}^2)$.

These parameters are fundamental because each can be set independently of the others, and each has some effect on the system small-signal performance.

For purposes of analysis and design, it is advantageous to describe the driver in terms of the four basic parameters used by Thiele [2] which are related to those above but are easier to measure and to work with. These are as follows.

f_S resonance frequency of moving system of driver, defined by Eq. (12) and usually specified for driver in air with no baffle (f_{SA}) or on a specified baffle (f_{SB})

V_{AS} acoustic compliance of driver, expressed as an equivalent volume of air according to Eq. (15)

Q_{MS} ratio of driver electrical equivalent frictional resistance to reflected motional reactance at f_S, defined by Eq. (13).

Q_{ES} ratio of voice-coil dc resistance to reflected motional reactance at f_S, defined by Eq. (14).

The parameters Q_{MS} and Q_{ES} correspond to Thiele's Q_a and Q_e. They have been given the extra subscript S to make it clear that they apply to the driver alone and to prevent confusion with the *system* parameters Q_M and Q_E, corresponding to Thiele's Q_a and Q_e (total), defined at the end of this section.

Driver Electrical Equivalent Circuit

The electrical equivalent circuit of a driver in air or mounted on an infinite baffle is shown in Fig. 4. In this circuit,

C_{MES} electrical capacitance due to driver mass ($= M_{AS}S_D^2/B^2l^2$)

L_{CES} electrical inductance due to driver compliance ($= C_{AS}B^2l^2/S_D^2$)

R_{ES} electrical resistance due to driver suspension losses ($= B^2l^2/S_D^2R_{AS}$).

The circuit of Fig. 4 is the dual of Fig. 3. An important difference is that the real voice-coil terminals are available in Fig. 4.

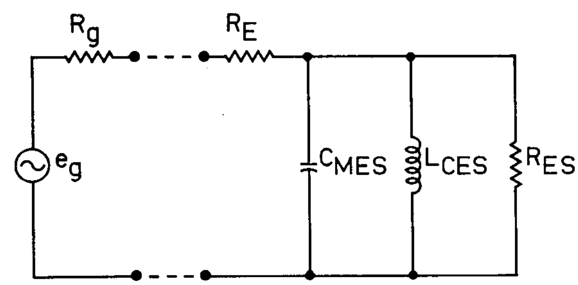

Fig. 4. Electrical equivalent circuit of moving-coil electrodynamic driver.

In Fig. 4, the driver reactances form a resonant circuit which has a resonance frequency $\omega_S = 2\pi f_S$, or a characteristic time constant T_S, given by

$$T_S^2 = 1/\omega_S^2 = C_{MES}L_{CES} = C_{AS}M_{AS}. \qquad (12)$$

The Q of the driver resonant circuit with R_{ES} acting alone is

$$Q_{MS} = \omega_S C_{MES} R_{ES} = 1/(\omega_S C_{AS} R_{AS}). \qquad (13)$$

Similarly, the Q with R_E acting alone, i.e., with $R_g = 0$, is

$$Q_{ES} = \omega_S C_{MES} R_E = \omega_S R_E M_{AS} S_D^2/(B^2l^2). \qquad (14)$$

The parameter V_{AS} is a volume of air having the same acoustic compliance as the driver suspension. Thus [3, p. 129]

$$V_{AS} = \rho_0 c^2 C_{AS}. \qquad (15)$$

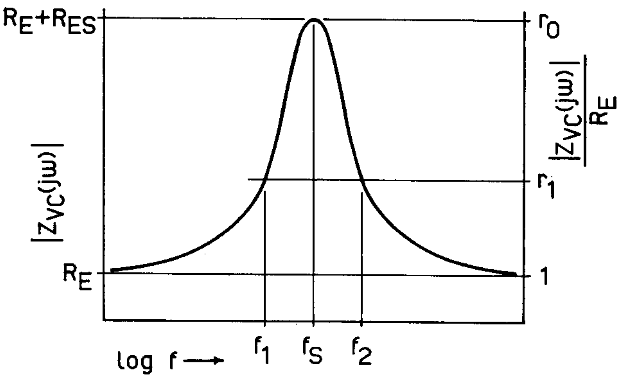

Fig. 5. Driver voice-coil impedance magnitude.

Driver Voice-Coil Impedance Function

The impedance of the circuit to the right of the voice-coil terminals in Fig. 4 is

$$Z_{VC}(s) = R_E + R_{ES}\left[\frac{sT_S/Q_{MS}}{s^2T_S^2 + sT_S/Q_{MS} + 1}\right]. \qquad (16)$$

The steady-state magnitude $|Z_{VC}(j\omega)|$ of Eq. (16) is plotted in Fig. 5; this has the form of a resonance curve which is displaced upward by an amount R_E.

Measurement of Driver Parameters

If the voice-coil impedance of an actual driver is plotted against frequency with the driver in air or on a simple test baffle, the resulting plot will have the same shape as Fig. 5. The driver resonance frequency f_S is easily located where the measured impedance is a maximum. If the ratio of the maximum voice-coil impedance to the dc resistance R_E is defined as r_0, and the two frequencies $f_1 < f_S$ and $f_2 > f_S$ are found where the impedance magnitude is $\sqrt{r_0}R_E$, then as shown in the Appendix,

$$Q_{MS} = \frac{f_S\sqrt{r_0}}{f_2 - f_1} \qquad (17)$$

and

$$Q_{ES} = \frac{Q_{MS}}{r_0 - 1}. \qquad (18)$$

To obtain the value of V_{AS}, a known compliance is added to the moving system by mounting the driver in a small unlined test box which is closed except for the driver aperture. The above driver parameters are then remeasured and values obtained for the new resonance frequency f_{CT} and the electrical Q, Q_{ECT}. Then, as shown in the Appendix,

$$V_{AS} = V_T\left[\frac{f_{CT}Q_{ECT}}{f_S Q_{ES}} - 1\right] \qquad (19)$$

where V_T is the net internal volume of the test box.

Source Parameters

The amplifier specifications that affect the small-signal performance of a loudspeaker system are frequency response and output resistance.

The frequency response of a good audio amplifier is usually wider and flatter than that of the loudspeaker system, and thus the frequency response function obtained from the system efficiency expression effectively describes the overall low-frequency response from the amplifier input terminals. The overall response may be modified or adjusted if desired by the addition to the amplifier of supplementary electrical filters [2].

The amplifier output resistance R_g is in series with the driver voice-coil resistance R_E and therefore affects the system behavior by influencing the total Q in the driver branch. Most modern amplifiers are designed to have a high damping factor, which means that R_g is made small compared to any expected value of R_E. This condition is usually assumed in the design of general-purpose loudspeaker systems, and the driver parameters are adjusted to give the required total Q.

If an amplifier and loudspeaker system are designed as a unit, extra design freedom may be gained by adjusting R_g to provide the desired total Q. Using suitable feedback techniques, R_g may be made positive, zero, or negative.

Measurement of Amplifier Source Resistance

The value of R_g may be found by driving the amplifier with a sinusoidal signal and measuring the amplifier output voltage under conditions of no load and rated load. If the no-load output voltage is e_0, the loaded output voltage is e_L, and the load resistance is R_L, then

$$R_g = R_L \frac{e_0 - e_L}{e_L}. \qquad (20)$$

If there is no measurable difference between e_0 and e_L, R_g may be considered zero as far as its effect on total Q is concerned. Accurate measurement is not required in this case, as it is the *total* resistance $(R_g + R_E)$ that is important.

Amplifier specifications often give the value of R_g (or the damping factor for rated load) measured at 1 kHz. For purposes of calculating system Q at low frequencies, the value measured at 50 Hz is more meaningful.

Enclosure Parameters

The enclosure parameters vary in number according to the type of system. Referring to Fig. 2, all of the vertical branches on the right of the figure contain enclosure components.

The most important property of the enclosure is its physical volume V_B which determines the compliance C_{AB}. If the component M_{AP} is present in the system, with or without C_{AP}, the enclosure will exhibit a resonance frequency f_B (or time constant T_B). If C_{AP} is present, an additional resonance frequency f_P (or time constant T_P) is introduced. The enclosure or aperture losses may be accounted for by defining Q for the various branches at specified frequencies (f_B or f_P).

Measurement of Enclosure Parameters

In general, the change in the driver voice-coil impedance which occurs when the driver is placed in the enclosure permits identification of the enclosure parameters. Because the relationships are different for every type of enclosure, they are not presented here but will be included in later papers describing each type of system.

Composite System Parameters

In the analysis of direct-radiator loudspeaker systems, certain combinations of the component parameters occur naturally, and consistently, in the system-describing functions. One of these is the ratio of driver compliance to enclosure compliance C_{AS}/C_{AB}. This parameter, the *system compliance ratio*, is of fundamental importance to direct-radiator systems using an enclosure. It appears in the analyses published by Beranek [3, ch. 8] and Thiele [2], and in the equivalent stiffness ratio form S_B/S_S used by Novak [10]. The importance of this parameter to system performance justifies giving it a simplified symbol; in later papers the symbol α introduced by Benson [4, eq. (91)] will be used.

In tuned-enclosure systems, the frequency ratio f_B/f_S occurs naturally in the analysis. This is the *system tuning ratio*; Novak [10] has given it the symbol h.

In every type of system, the driver parameter Q_{ES} is altered by the presence of the source parameter R_g to form a system parameter

$$Q_E = Q_{ES} \frac{R_g + R_E}{R_E}. \qquad (21)$$

The effective value of $(R_g + R_E)$ includes any significant resistance present in connecting leads and crossover inductors.

Similarly, the driver parameter Q_{MS} is modified if the system acoustical analogous circuit has an acoustic resistance in series with R_{AS}. The new system parameter Q_M is usually found by measurement.

The total Q of the driver branch of the system is then given by a composite system parameter

$$Q_T = \frac{Q_E Q_M}{Q_E + Q_M}. \qquad (22)$$

FREQUENCY RESPONSE

Response Function

The response function $G(s)$ of a loudspeaker system may be obtained from the complete efficiency expression as illustrated earlier or by a simpler general method which provides only the response function. In Fig. 6 the acoustical analogous circuit of Fig. 2 is reduced to only four essential components:

p_g acoustic driving pressure given by

$$p_g = \frac{e_g Bl}{(R_g + R_E) S_D} \qquad (23)$$

Z_{AS} impedance of driver branch, normally given by

$$Z_{AS}(s) = R_{AT} + sM_{AS} + \frac{1}{sC_{AS}} \qquad (24)$$

Z_{AB} impedance of branch representing enclosure interior, normally given by

$$Z_{AB}(s) = R_{AB} + \frac{1}{sC_{AB}} \qquad (25)$$

Z_{AA} impedance of all enclosure apertures (except that for the driver) which contribute to total output volume velocity. Note that U_A in Fig.

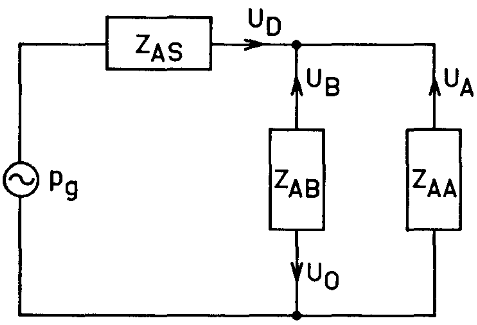

Fig. 6. Simplified acoustical analogous circuit corresponding to Fig. 2.

6 is equal to the sum of U_L and U_P in Fig. 2. Z_{AA} is determined by the specific enclosure design.

The response function is then in all cases

$$G(s) = sM_{AS}\frac{U_0}{p_g} = \frac{sM_{AS}}{Z_{AB} + Z_{AS} + Z_{AB}Z_{AS}/Z_{AA}}. \quad (26)$$

Simplifying the Response Function

The response function obtained from the system acoustical analogous circuit is always a normalized high-pass filter function which is in the form of the ratio of two polynomials in s. The polynomial coefficients contain various combinations of the acoustical masses, compliances, and resistances contained in the system.

The response function is easier to interpret if the acoustical quantities in the coefficients are replaced by the simpler system parameters described in the previous section. Because the coefficients must have dimensions of time only, it is always possible to redefine them in terms of system time constants (or resonance frequencies) together with such dimensionless quantities as Q, compliance ratios, mass ratios, and resistance ratios. These variables are easier for the electrical engineer to interpret than the unfamiliar acoustical quantities.

For the infinite-baffle system analyzed earlier, the response function $G(s)$ is given by Eq. (10). This expression is simplified by substituting

$$T_S^2 = C_{AS}M_{AS} \quad (12)$$

and

$$Q_T = 1/(\omega_S C_{AS} R_{AT}) \quad (27)$$

where Q_T is the total Q (at f_S) of the driver connected to the source. This is the same parameter defined for the general case in Eq. (22). Then

$$G(s) = \frac{s^2T_S^2}{s^2T_S^2 + sT_S/Q_T + 1}. \quad (28)$$

Using the Response Function

Once the system response function is known, the response of any specific system design can be determined if the system parameters are known or are measured so that the corresponding response function coefficients can be calculated. This process is useful in determining the response of existing or proposed systems but gives little insight into the means of improving such systems.

A more useful approach is to explore the behavior of the system response function to determine which coefficient values (i.e., parameter values) produce the most desirable response characteristics. This sounds like a formidable and time-consuming task suitable for computer application, but fortunately the response shapes of greatest interest to the loudspeaker system designer, e.g., those providing flat response in the passband, have already been studied extensively by filter designers.

Because loudspeaker systems have minimum-phase behavior at low frequencies, the amplitude, phase, delay, and transient responses are all related and cannot be specified independently. The most common criterion for optimum response in audio systems is flatness of the amplitude response over a maximum bandwidth, but there may be cases where the designer requires an optimized transient response or delay characteristic. Whatever criterion is used, it is translated into a set of optimum polynomial coefficients so that the system parameter values can be specified or adjusted accordingly.

The adjustment of loudspeaker system response is clearly analogous to the alignment of conventional types of filters. This is particularly apparent where the adjustment goal is the achievement of a predetermined response condition, rather than trial-and-error optimization.

Consider again the infinite-baffle system which has the response function given by Eq. (28). The general form of this class of response function as used by filter designers is

$$G(s) = \frac{s^2T_0^2}{s^2T_0^2 + a_1sT_0 + 1} \quad (29)$$

where

T_0 nominal filter time constant
a_1 damping, or shape, coefficient.

The behavior of Eq. (29) is well known and thus reveals the behavior of the infinite-baffle system when $T_S = T_0$ and $Q_T = 1/a_1$. Using standard curves for Eq. (29), the steady-state magnitude $|G(j\omega)|$ of Eq. (28) is plotted in Fig. 7 for several values of Q_T. The curve for $Q_T = 0.50$ corresponds to the condition for critical damping of the resonant circuit. The curve for $Q_T = 0.71$ is a maximally flat (Butterworth) alignment which has no amplitude peaking. The curves for $Q_T = 1.0$, 1.4, and 2.0 have amplitude peaks of approximately 1 dB, 3½ dB, and 6 dB, respectively, but provide extensions of half-power bandwidth as compared to the maximally flat alignment.

For this simple system, the design engineer can choose the response shape he desires and specify the system parameters accordingly; he can also see at a glance the effects of parameter tolerances.

REFERENCE EFFICIENCY

The first part of the efficiency expression (11) for a loudspeaker system contains only physical constants and driver parameters, while the last part, the system response function squared, is always unity for the portion of the piston range above system cutoff. Thus the first part of the expression is the passband or reference efficiency of the system. This reference efficiency, designated η_0, is given by

$$\eta_0 = \frac{\rho_0}{2\pi c}\cdot\frac{B^2l^2}{R_ES_D{}^2M_{AS}{}^2}. \quad (30)$$

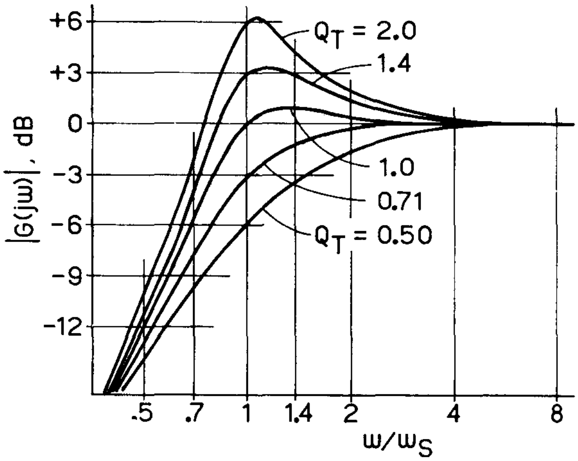

Fig. 7. Normalized frequency response of infinite baffle loudspeaker system.

In terms of the fundamental electromechanical driver parameters, this is

$$\eta_0 = \frac{\rho_0}{2\pi c} \cdot \frac{B^2 l^2}{R_E} \cdot \frac{S_D{}^2}{M_{MS}{}^2}. \tag{31}$$

It must be remembered that M_{AS} and M_{MS} include relevant air-load masses and any deliberate mass loading imposed by the enclosure.

Combining Eqs. (12), (14), and (15) with Eq. (30), the expression for reference efficiency becomes

$$\eta_0 = \frac{4\pi^2}{c^3} \cdot \frac{f_S{}^3 V_{AS}}{Q_{ES}}. \tag{32}$$

The reference efficiency of the system can thus be calculated from the basic driver parameters discussed in Section "Small-Signal Parameters." This result is surprising at first, because these parameters can be determined from simple electrical measurements. This means that the system piston-range electroacoustic efficiency can be found without any direct mechanical, magnetic, or acoustical measurements.

Note that Eq. (32) yields an efficiency twice as large as [2, eq. (76)]. This is because Thiele's expression is derived for the radiation load of a 4π-sr free field, while Eq. (32) assumes the radiation load of a 2π-sr free field. The latter is used here because it is more nearly representative of the radiation load presented to a loudspeaker system by a typical listening room [5].

The physical constants in Eq. (32) have a value of 9.6×10^{-7} in the International System, and this value may be used to compute efficiency if f_S is expressed in hertz and V_{AS} is expressed in cubic meters. However, the value of V_{AS} for most drivers is more conveniently expressed in liters (one liter $= 10^{-3}$ cubic meters). Thus for V_{AS} in liters,

$$\eta_0 = 9.6 \times 10^{-10} \frac{f_S{}^3 V_{AS}}{Q_{ES}}. \tag{33}$$

Alternatively, if V_{AS} is expressed in cubic feet,

$$\eta_0 = 2.7 \times 10^{-8} \frac{f_S{}^3 V_{AS}}{Q_{ES}}. \tag{34}$$

The calculated value of efficiency may be converted into decibels ($10 \log_{10}\eta_0$) or percent ($100\ \eta_0$). The reference efficiency of direct-radiator systems is quite low, typically of the order of one percent.

The resonance frequency of a loudspeaker driver is usually measured with the driver mounted on a standard test baffle having an area of a few square meters [7, sec. 3b], [8, sec. 4.4.1]. Alternatively, some manufacturers prefer to use an effectively infinite baffle, or no baffle at all. Because most drivers are ultimately used in enclosures, the system designer is most interested in the resonance frequency, Q and reference efficiency for an air-load mass equivalent to that of an enclosure; this condition is most nearly approached by a finite "standard" baffle.

If deliberate mass loading of the driver is employed in the system, e.g., placing a restricted aperture in front of the driver, the system reference efficiency will be less than the basic efficiency of the driver. The system efficiency can still be found from Eq. (32) if the values of f_S and Q_{ES} are measured under mass-loaded conditions. The efficiency reduction will be proportional to the square of the mass increase, as shown by Eq. (30).

LARGE-SIGNAL PERFORMANCE

Power Ratings and Large-Signal Parameters

Loudspeaker standards such as [6]–[8] provide only a general guide for the establishment of loudspeaker (driver) power ratings: the input power rating should be such that an amplifier of equivalent undistorted output power rating can be used with the loudspeaker without causing damage or excessive distortion.

At moderately high frequencies, where little diaphragm displacement is required of the driver, the power handling capability of a loudspeaker system is limited by the ability of the driver voice coil to dissipate heat. This leads to a thermally limited absolute maximum input power rating for the *driver*, regardless of the system design. This input power rating is designated $P_{E(max)}$.

At low frequencies much more diaphragm displacement is required of the driver, and it is necessary to establish an input power rating which ensures that the diaphragm is not driven beyond a specified displacement limit. This displacement-limited input power rating is often less than $P_{E(max)}$. Because diaphragm displacement is a function of enclosure design, the displacement-limited power rating is a property of the *system*, not the driver, although it depends on the driver displacement limit.

The displacement limit of a particular driver may be determined by any of a number of criteria. Among these are

1) prevention of suspension damage,
2) limitation of frequency-modulation distortion [11],
3) limitation of nonlinear (harmonic and amplitude-modulation) distortion [12].

For the purpose of this paper it is assumed that a peak displacement limit can be established; this limit is designated x_{max}.

The fundamental large-signal parameter of a driver at low frequencies is then

$$V_D = S_D x_{max}. \tag{35}$$

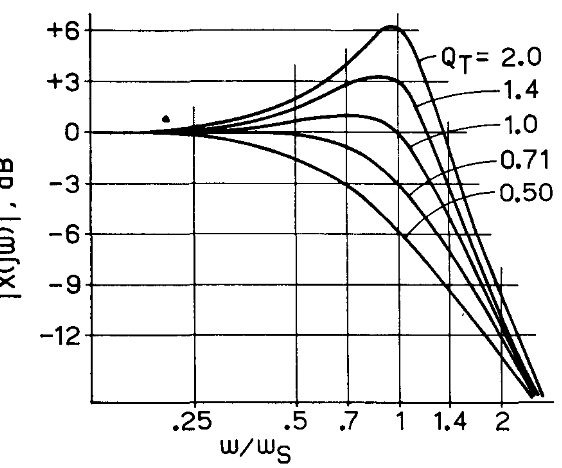

Fig. 8. Normalized diaphragm displacement of driver mounted on infinite baffle.

This parameter, the *diaphragm peak displacement volume*, is the volume of air displaced by the driver diaphragm in moving from rest to its peak displacement limit. It describes the volume displacement limitation and therefore the volume velocity versus frequency limitation of the driver. The practical usefulness of this parameter is illustrated in the following section.

Thus, in addition to the driver small-signal parameters discussed earlier, the system designer must know (or specify) the large-signal parameters $P_{E(\max)}$ and V_D.

Diaphragm Displacement

The small-signal diaphragm displacement of a loudspeaker system driver is determined from the system acoustical analogous circuit. The circuit is first analyzed to obtain the diaphragm volume velocity U_D. Division by S_D then gives the diaphragm velocity u_D, and a further division by s (i.e., integration) yields the diaphragm displacement x_D. The diaphragm displacement expression is always of the form

$$x_D = P_E^{1/2}\sigma_{x(P)}k_x X(s) \tag{36}$$

where

P_E nominal input power defined by Eq. (6)

$\sigma_{x(P)}$ static (dc) displacement sensitivity of unenclosed driver, expressed in meters per watt$^{1/2}$ and given by

$$\sigma_{x(P)} = \left[\frac{C_{MS}^2B^2l^2}{R_E}\right]^{1/2} = \left[\frac{V_{AS}}{2\pi\rho_0c^2f_SQ_{ES}S_D^2}\right]^{1/2} \tag{37}$$

k_x system displacement constant of unity or less

$X(s)$ normalized system displacement function.

$X(s)$ is always a *low-pass* filter function which has a value of unity at zero frequency.

For a particular system, the product of the displacement constant k_x and the displacement function $X(s)$ is evaluated by either of two methods. In the first method, the displacement expression (36) is established as described above and divided by $P_E^{1/2}\sigma_{x(P)}$ using Eqs. (6) and (37). In the second method, the acoustical analogous circuit is analyzed for the admittance seen by the generator, and this quantity is divided by sC_{AS}; referring to Fig. 6, this means that in all cases

$$k_xX(s) = \frac{1}{sC_{AS}}\cdot\frac{1+Z_{AB}/Z_{AA}}{Z_{AB}+Z_{AS}+Z_{AB}Z_{AS}/Z_{AA}}. \tag{38}$$

The resulting expression is then split into a constant factor k_x and a frequency-dependent factor $X(s)$ normalized to unity at zero frequency.

For the infinite-baffle system, circuit analysis of Fig. 3 reveals that the displacement constant is unity and the displacement function is

$$X(s) = \frac{1}{s^2T_S^2+sT_S/Q_T+1}. \tag{39}$$

The steady-state magnitude $|X(j\omega)|$ of this function is plotted against normalized frequency in Fig. 8. For this simple system, the curves are exact mirror images of those of Fig. 7.

DISPLACEMENT-LIMITED POWER RATINGS

Electrical Power Rating

A useful indication of the sinusoidal steady-state displacement-limited electrical input power capacity of a loudspeaker system is obtained by assuming linear diaphragm displacement for large input signals and limiting the peak value of x_D in Eq. (36) to $x_{\max}$. Thus

$$P_{ER} = \frac{1}{2}\left[\frac{x_{\max}}{\sigma_{x(P)}k_x|X(j\omega)|_{\max}}\right]^2 \tag{40}$$

where

P_{ER} displacement-limited electrical input power rating in watts

$|X(j\omega)|_{\max}$ maximum magnitude attained by system displacement function, i.e., its value at the frequency of maximum diaphragm displacement.

Substituting Eqs. (35) and (37) into Eq. (40),

$$P_{ER} = \pi\rho_0c^2\frac{f_SQ_{ES}V_D^2}{V_{AS}k_x^2|X(j\omega)|_{\max}^2}. \tag{41}$$

Acoustic Power Rating

The displacement-limited electrical power rating of a loudspeaker system places a limitation on the continuous power rating of the amplifier to be used with the system. This power rating, together with the reference efficiency of the system, then determines the maximum continuous acoustic power that can be radiated in the flat (upper) region of the system passband. Thus, using Eqs. (32) and (41), the steady-state displacement-limited acoustic power rating P_{AR} of the loudspeaker system is

$$P_{AR} = \frac{4\pi^3\rho_0}{c}\cdot\frac{f_S^4V_D^2}{k_x^2|X(j\omega)|_{\max}^2}. \tag{42}$$

This rating may easily be converted into a sound pressure level rating for standardized radiation and measurement conditions, e.g., [8, sec. 3.16]. The factor $4\pi^3\rho_0/c$ has the value 0.42 for SI units, i.e., for f_S in Hz and V_D in m^3.

Power Ratings of Infinite-Baffle System

The displacement-limited acoustic power rating of a driver mounted on an infinite baffle is found by setting $k_x = 1$ in Eq. (42). Thus,

$$P_{\mathrm{AR(IB)}} = \frac{4\pi^3\rho_0}{c} \cdot \frac{f_S{}^4 V_D{}^2}{|X(j\omega)|_{\max}{}^2}. \tag{43}$$

For a given value of V_D, the acoustic power rating is a strong function of the driver resonance frequency. It is also sensitive to Q_T through $|X(j\omega)|_{\max}$ (see Fig. 8), but is maximized for $Q_T \leq 0.71$.

As an example, consider an infinite-baffle system having a resonance frequency of 50 Hz and a second-order Butterworth response. If the driver is a 12-inch unit (effective radius 0.12 m) capable of $\pm$ 4 mm peak displacement, then $V_D = 0.18$ dm^3, and the acoustic power rating is $P_{\mathrm{AR}} = 0.086$ watt. This is equivalent to a sound pressure level rating of 101.5 dB at a distance of 1 meter [3, p. 14].

Setting k_x equal to unity in Eq. (41), the displacement-limited electrical power rating of the infinite-baffle system is

$$P_{\mathrm{ER(IB)}} = \pi\rho_0 c^2 \frac{f_S Q_{\mathrm{ES}} V_D{}^2}{V_{\mathrm{AS}}\,|X(j\omega)|_{\max}{}^2}. \tag{44}$$

This equation demonstrates quantitatively the well-known fact that a woofer designed for acoustic-suspension use (i.e., with very low resonance and high compliance) has a low (input) power handling capacity, compared to that of a conventional woofer, if it is operated in air or on an infinite baffle.

The electrical power rating of the system in the above numerical example depends on the value of driver compliance. If the total moving mass of the driver has a typical value of 30 grams, the driver compliance, from Eq. (12), must be $V_{\mathrm{AS}} = 0.1$ m^3. Ignoring mechanical losses and taking $Q_{\mathrm{ES}} = Q_T = 0.71$, the electrical power rating from Eq. (44) is then $P_{\mathrm{ER}} = 5$ watts. Comparing P_{AR} with P_{ER}, or using Eq. (33), the reference efficiency of the driver is $\eta_0 = 1.7\%$.

Note that the same ratings also apply to an infinite-baffle system using an 8-inch driver (effective radius 0.08 m) capable of $\pm$ 9-mm peak displacement (so that $V_D = 0.18$ dm^3) and having the same resonance frequency, acoustic compliance, and Q.

Assumptions and Corrections

The accuracy of the calculated displacement-limited power ratings depends on the assumptions that the diaphragm displacement is linear up to $x_{\max}$ and that the source power bandwidth extends down to the frequency of maximum displacement. Both assumptions may lead to conservative ratings.

For example, the infinite-baffle system described above reaches maximum displacement only at very low frequencies. This system might typically be driven by an amplifier with a low-frequency power bandwidth (-3 dB) of 30 Hz. If the plot of $|X(j\omega)|$ (with constant voltage drive) for $Q_T = 0.71$ in Fig. 8 is multiplied by the normalized power output curve of this amplifier, the resulting maximum value of $|X(j\omega)|$ falls from unity to about 0.7. A more realistic set of power ratings for this loudspeaker system would thus be $P_{\mathrm{ER}} = 10$ watts and $P_{\mathrm{AR}} = 0.17$ watt.

Similarly, if $x_{\max}$ is defined at a displacement beyond the linear range of the driver, then the actual input power required to reach this peak displacement will be higher than the calculated value. A correction factor can easily be computed from the actual displacement-versus input characteristic of the driver.

CONCLUSION

The low-frequency response, efficiency, and power ratings of a direct-radiator loudspeaker system are determined by the parameters of the system components. These relationships are reciprocal; specification of the system performance places definite requirements on the component parameters. The most important system component is the driver, which is completely described only when a sufficient number of small-signal and large-signal parameters are specified.

An interesting result of the analysis in this paper is that the driver diaphragm area S_D does not appear explicitly in the small-signal response, small-signal efficiency, or displacement-limited power ratings of a loudspeaker system. This means that it is theoretically possible to design drivers of different diameter with identical values of the parameters f_S, Q_{MS}, Q_{ES}, V_{AS}, and V_D. Used in identical enclosures, these drivers must give identical small-signal performance and displacement-limited power capacity. The principal differences are that the larger driver will cost more but require less diaphragm displacement and thus produce less modulation distortion for a given acoustic output [11], [12].

Although the electrodynamic moving-coil driver has been manufactured throughout the world for decades, hardly a single manufacturer provides complete low-frequency parameter information with his products, or has ever been asked to do so. In the future, trial-and-error design of loudspeaker systems using available drivers will increasingly be replaced by system synthesis based on final performance specifications and resulting in specific driver parameter requirements. Driver manufacturers must be ready to meet demands of this kind and to provide complete parameter information with their products.

The parameters used to describe driver behavior in this paper are not the only consistent set that can be used. However, they do have the advantage of being easy to measure and to comprehend, and, as later papers will show, they are well suited for use in the analysis and design of complete systems.

APPENDIX

DRIVER PARAMETER MEASUREMENTS

Driver Q

From Eqs. (13) and (14),

$$\frac{Q_{\mathrm{MS}}}{Q_{\mathrm{ES}}} = \frac{R_{\mathrm{ES}}}{R_E}. \tag{45}$$

The ratio of voice-coil maximum impedance to dc resistance, from Fig. 5, is therefore

$$r_0 = \frac{R_{ES} + R_E}{R_E} = 1 + \frac{Q_{MS}}{Q_{ES}} \tag{46}$$

from which

$$Q_{ES} = \frac{Q_{MS}}{r_0 - 1}. \tag{18}$$

Also, the total driver Q with a zero-impedance source ($R_g = 0$) is given by

$$Q_{TS} = \frac{Q_{MS} Q_{ES}}{Q_{MS} + Q_{ES}} = \frac{Q_{MS}}{r_0}. \tag{47}$$

Eq. (16) now becomes

$$Z_{VC}(s) = R_E \frac{r_0 + Q_{MS}(sT_S + 1/sT_S)}{1 + Q_{MS}(sT_S + 1/sT_S)} \tag{48}$$

and

$$|Z_{VC}(j\omega)|^2 = R_E^2 \frac{r_0^2 + Q_{MS}^2(\omega/\omega_S - \omega_S/\omega)^2}{1 + Q_{MS}^2(\omega/\omega_S - \omega_S/\omega)^2}. \tag{49}$$

At any two frequencies $\omega_1 < \omega_2$ such that $\omega_1\omega_2 = \omega_S^2$, it can be shown using (49) that the impedance magnitudes will be equal. Let this magnitude be defined by

$$|Z_{VC}(j\omega_1)| = |Z_{VC}(j\omega_2)| = r_1 R_E. \tag{50}$$

Then

$$\begin{aligned} |Z_{VC}(j\omega_{1,2})|^2 &= r_1^2 R_E^2 \\ &= R_E^2 \frac{r_0^2 + Q_{MS}^2[(\omega_2 - \omega_1)/\omega_S]^2}{1 + Q_{MS}^2[(\omega_2 - \omega_1)/\omega_S]^2} \end{aligned} \tag{51}$$

and therefore

$$Q_{MS} = \frac{\omega_S}{\omega_2 - \omega_1}\sqrt{\frac{r_0^2 - r_1^2}{r_1^2 - 1}}. \tag{52}$$

If $r_1 = \sqrt{r_0}$, Eq. (52) reduces to

$$Q_{MS} = \frac{f_S\sqrt{r_0}}{f_2 - f_1}. \tag{17}$$

Choosing $r_1 = \sqrt{r_0}$ not only makes the calculation simple but provides good measurement accuracy because f_1 and f_2 are reasonably well separated and are located in regions of high slope on the impedance curve.

As shown above, the frequencies f_1 and f_2 where the the measured voice-coil impedance magnitude is $\sqrt{r_0}R_E$ should satisfy the condition

$$\sqrt{f_1 f_2} = f_S. \tag{53}$$

For most real drivers this is not precisely so because the fundamental driver parameters, particularly compliance and mechanical resistance, vary slightly with frequency or diaphragm excursion. Also, the voice-coil inductance, if large, will skew the curve slightly. However, for most well-designed drivers, the result computed from (53) is within about 1 Hz of the measured value. Eq. (53) is thus a useful check to catch measurement errors or to identify drivers which cannot be represented accurately by a set of constant-value parameters.

Driver Compliance

A simple unlined test enclosure at atmospheric pressure has an acoustic compliance C_{AB} related to its net internal volume V_T by [3, p. 129]

$$C_{AB} = V_T/\rho_0 c^2. \tag{54}$$

A driver having total acoustical mass M_{AS} and compliance C_{AS} has a self-resonance defined by

$$T_S^2 = 1/\omega_S^2 = M_{AS}C_{AS}. \tag{12}$$

When this driver is mounted in the closed test box, a new resonance will be measured which is given by

$$T_{CT}^2 = 1/\omega_{CT}^2 = M_{ACT}\frac{C_{AB}C_{AS}}{C_{AB} + C_{AS}} \tag{55}$$

where M_{ACT} is the new total moving mass resulting from any change in the value of the diaphragm air load mass. Then

$$\frac{\omega_{CT}^2}{\omega_S^2} = \frac{M_{AS}}{M_{ACT}}\left[1 + \frac{C_{AS}}{C_{AB}}\right]. \tag{56}$$

From Eq. (14),

$$Q_{ES} = \omega_S R_E M_{AS} S_D^2/(B^2 l^2). \tag{57}$$

Similarly,

$$Q_{ECT} = \omega_{CT} R_E M_{ACT} S_D^2/(B^2 l^2). \tag{58}$$

Therefore,

$$\frac{M_{AS}}{M_{ACT}} = \frac{\omega_{CT} Q_{ES}}{\omega_S Q_{ECT}} \tag{59}$$

and combining Eqs. (56) and (59),

$$1 + \frac{C_{AS}}{C_{AB}} = \frac{\omega_{CT} Q_{ECT}}{\omega_S Q_{ES}}. \tag{60}$$

From Eqs. (15) and (54),

$$\frac{C_{AS}}{C_{AB}} = \frac{V_{AS}}{V_T} \tag{61}$$

and therefore

$$\frac{V_{AS}}{V_T} = \frac{\omega_{CT} Q_{ECT}}{\omega_S Q_{ES}} - 1 \tag{62}$$

or

$$V_{AS} = V_T\left[\frac{f_{CT} Q_{ECT}}{f_S Q_{ES}} - 1\right]. \tag{19}$$

The initial driver measurements (f_S and Q_{ES}) may be made with a baffle of any size or with no baffle. It is advisable, however, especially with low-resonance drivers, that the driver have its axis horizontal for both sets of measurements to avoid excessive static diaphragm displacement due to gravity.

Energy absorption in the test enclosure walls affects only the measured value of Q_{MCT} and thus has no effect on the compliance calculation. However, absorbing material placed inside the enclosure can affect the value of C_{AB} and should therefore not be used.

It is particularly important to avoid leaks in the test enclosure because these can also change the effective value of C_{AB} and seriously reduce the accuracy of the measurement. The test enclosure must be constructed carefully, and the driver under test must be checked for a tight seal at the mounting gasket. Some drivers have a built-in leakage path around the voice coil, others through a porous edge-suspension material. Measurements on these drivers must be used with caution. To test for leakage, apply an input signal of about 10 Hz at moderate level and listen carefully all around the enclosure and driver for "breathing" indicative of a leak.

Measurement Technique

Loudspeaker impedance measurements are commonly taken with either constant-voltage [3, p. 503] or constant-current [10, p. 13] drive. If the driver is perfectly linear or the measuring level is low enough, the two methods should give the same result. The constant-voltage method has the advantage of more nearly duplicating the usual operating conditions of the driver.

Accurate measurement of small-signal parameters requires a signal level that is small enough for all voltage and current waveforms to be undistorted sinusoids. Use an oscilloscope to observe waveforms and adjust the signal level accordingly. It is often necessary, particularly with unloaded high-compliance drivers, to measure parameters at an input level of 0.1 watt or less.

Measure the driver voice-coil resistance accurately with a dc bridge. A dummy resistance of the same value can then be made up and used as a calibrating load on the equipment for measuring impedance.

Do not trust the frequency scale of audio-sweep type beat frequency oscillators. For maximum accuracy, take frequency readings with a frequency or period counter or from the scale of a stable, accurately calibrated sine-wave generator.

ACKNOWLEDGMENT

This paper is part of the result of a program of post-graduate research into the low-frequency performance of direct-radiator electrodynamic loudspeaker systems. I am indebted to the School of Electrical Engineering of The University of Sydney for providing research facilities, supervision, and assistance, and to the Australian Commonwealth Department of Education and Science for financial support.

Numerous authors have contributed through their published works to the basic ideas which are developed here. I am particularly indebted to A. N. Thiele for having originated both the filter-oriented approach to analysis and the simple methods of parameter measurement which are described here, and to J. E. Benson for originating the simple generalized loudspeaker system concept, for contributing many hours to the discussion of terminology and symbols, and for carefully checking the equations and computations in the manuscript.

REFERENCES

[1] E. de Boer, "Acoustic Interaction in Vented Loudspeaker Enclosures," *J. Acoust. Soc. Am.*, vol. 31, p. 246 (Feb. 1959).

[2] A. N. Thiele, "Loudspeakers in Vented Boxes," *Proc. IREE (Australia)*, vol. 22, p. 487 (Aug. 1961); also, *J. Audio Eng. Soc.*, vol. 19, pp. 382-392 (May 1971), and pp. 471-483 (June 1971).

[3] L. L. Beranek, *Acoustics* (McGraw-Hill, New York, 1954).

[4] J. E. Benson, "Theory and Design of Loudspeaker Enclosures," *A.W.A. Tech. Rev.*, vol. 14, p. 1 (1968); also, *Proc. IREE (Australia)*, vol. 30, p. 261 (Sept. 1969).

[5] R. F. Allison and R. Berkovitz, "The Sound Field in Home Listening Rooms," presented at the 39th Convention of the Audio Engineering Society, October 1970, Preprint 779.

[6] American National Standards Institute, "American Standard Recommended Practices for Loudspeaker Measurements," Standard S1.5-1963 (1963).

[7] British Standards Institution, "British Standard Recommendations for Ascertaining and Expressing the Performance of Loudspeakers by Objective Measurements," B.S. 2498 (1954).

[8] International Electrotechnical Commission, "IEC Recommendation, Methods of Measurement for Loudspeakers," IEC Publ. 200 (1966).

[9] H. F. Olson, *Dynamical Analogies* (Van Nostrand, New York, 1943).

[10] J. F. Novak, "Performance of Enclosures for Low-Resonance High-Compliance Loudspeakers," *IRE Trans. Audio*, vol. AU-7, p. 5 (Jan.-Feb. 1959); also, *J. Audio Eng. Soc.*, vol. 7, p. 29 (Jan. 1959).

[11] P. W. Klipsch, "Modulation Distortion in Loudspeakers," *J. Audio Eng. Soc.*, vol. 17, p. 194 (Apr. 1969).

[12] P. W. Klipsch, "Modulation Distortion in Loudspeakers: Part II," *J. Audio Eng. Soc.*, vol. 18, p. 29 (Feb. 1970).

Note: Mr. Small's biography appeared in the January/February issue.