On the Design of Some Feedback Circuits for Loudspeakers*

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Some basic relationships of feedback systems are reviewed. In particular, attention is paid to an optimum choice of controller. The general optimization technique of Kessler is described and applied to control systems with only one dominant time constant. It is shown that this general procedure can also be used for a loudspeaker system. An equivalent control circuit for a loudspeaker is derived, and the state-variable feedback design may be applied. A practical loudspeaker feedback system is described. Attention is paid to the general concept of this system and to the Kessler optimization technique. The circuits used in this system, the system design, the measurements, and the results are discussed.

1 CONTROL SYSTEM THEORY

In this section some general concepts about a class of control systems are discussed, and the Kessler optimization method is shown for this class of systems.

1.1 General Concept

The general block diagram of a control system is given in Fig. 1. In this system several basic blocks may be distinguished: the plant or system to be controlled $G(s)$, the feedback circuit $H(s)$, and the controller $C(s)$. In this system class the feedback circuit is used only to transpose the output variable into an equivalent variable, which may be compared with the reference input variable. The feedback circuit is supposed to have only small time constants with respect to the frequencies that are normally used in driving the plant. Thus, the dominant time constant should be found in $G(s)$.

In this case only the controller $C(s)$ may be used to influence system behavior with respect to stability, steady-state characteristics, speed, and accuracy. For this purpose the use of stability analysis by the root-locus method or Routh or Nyquist criteria, compensation techniques, steady-state analysis, and global error evaluation is well known [1].

For the class of control systems to be studied here the circuit of Fig. 1 may be redrawn as shown in Fig. 2, and optimization techniques may be applied to the circuit inside the dashed box.

This circuit is known as a control system with unity feedback, and the steady-state error is zero if the closed-loop gain equals 1.

1.2 Optimization

Kessler [2] proposed in his work an optimization method for systems with one or more dominant time constants. Two different methods are known: an optimum amount for systems with time constants and a symmetrical optimum for systems with a dominant integrating action. In the present study, we consider only the optimum amount. The reader is referred to [2] for further details.

1.2.1 Optimum Amount

Consider the dashed-box circuit in Fig. 2. The open-loop transfer function is given by

$$F_0(s) = C(s) \cdot G(s) \cdot H(s)$$

and the closed-loop transfer function is given by

$$F_c(s) = \frac{F_0(s)}{1 + F_0(s)} .$$

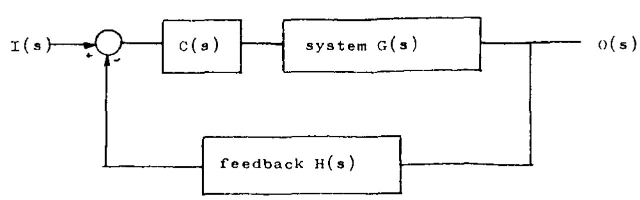

Fig. 1. Control system.

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An optimally controlled system should be characterized by $F_c(j\omega) = 1$ for all frequencies. However, no system can reach this condition. A more realistic condition is that $\| F_c(j\omega) \|$ should be 1 as far as possible in frequency greater than or equal to the highest frequency. Mathematically this may be expressed by the following conditions:

$$\| F_c(j\omega) \| = 1, \qquad \omega = 0$$

and

$$\frac{d^n \| F_c \|}{d\omega^n} = 0, \qquad \omega = 0$$

or

$$\frac{d^n \| F_c \|^2}{d\omega^n} = 0, \qquad \omega = 0 \,.$$

This means that the amplitude of the transfer function should be maximally flat, as shown in Fig. 3.

It must be remarked that some of the derivative conditions are trivial and do not lead to to useful equations for conditioning the controller parameters. When the entire system contains more than one integrator (for instance, a Proportional Integral (PI) controller and an integrating action of the system), all derivative conditions are automatically fulfilled, and no equations for calculating the controller parameters are obtained. So the method using a symmetrical optimum should be applied [2].

1.2.2 Example

Consider a system with one dominant time constant T:

$$G(s) \cdot H(s) = \frac{K}{(1 + sT)\pi_i(1 + s\tau_i)} \,, \quad T >> \tau_i \,.$$

This expression may be approximated by

$$G(s) \cdot H(s) = \frac{K}{(1 + sT)(1 + s\sigma)} \,, \quad \sigma = \sum_i \tau_i \,.$$

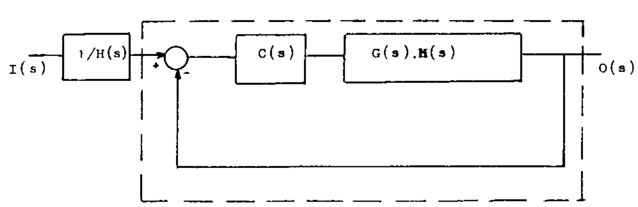

Fig. 2. Unity feedback control system.

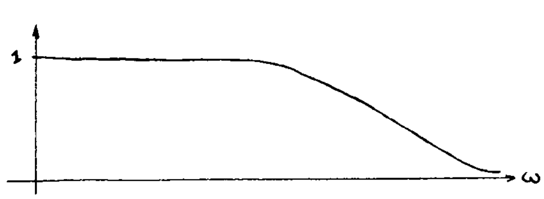

Fig. 3. Maximally flat frequency response.

If we chose a PI controller $C(s) = (K_I + sK_p)/s$, the system transfer functions are given by

$$F_o(s) = \frac{KK_I(1 + sK_p/K_I)}{s(1 + sT)(1 + s\sigma)}$$

$$F_c(s) = \frac{KK_I(1 + sK_p/K_I)}{KK_I(1 + sK_p/K_I) + s(1 + sT)(1 + s\sigma)} \,.$$

If we now apply the Kessler conditions, the controller parameters are found to be $K_I = 1/2K\sigma$ and $K_p/K_I = T$, and the system becomes

$$F_c = \frac{1}{1 + 2s\sigma + 2s^2\sigma^2} \,.$$

$F_c(s)$ is a second-order system with natural frequency $\omega_n = 1/\sqrt{2}\sigma$ and damping coefficient $\zeta = 1/\sqrt{2}$. The natural frequency seems to be very high, and the system is critically damped.

The system has a very well-known behavior, and is very stable. As an example, the typical step response is given in Fig. 4. Typical characteristics are: overshoot $\approx 4\%$; rise time: 4.7σ; and settling time: 10σ.

2 LOUDSPEAKER EQUIVALENT CIRCUITS

2.1 Circuit Theory

In a first-order approximation, a loudspeaker may be described by the following well-known equations:

mechanical: $Bll = sMV + \frac{S}{s}V + KV + Z_{rad} \cdot V$

electrical: $E = R_eI + sL_eI + BlV$

where

- Bl = force coefficient
- M = cone mass
- V = cone velocity
- S = cone stiffness
- K = mechanical friction
- R_e, L_e = coil resistance and inductance
- Z_{rad} = radiation impedance,

$Z_{rad} = \pi a^2 \cdot \rho_0 c(R_{rad} + jX_{rad})$ as sketched in Fig. 5.

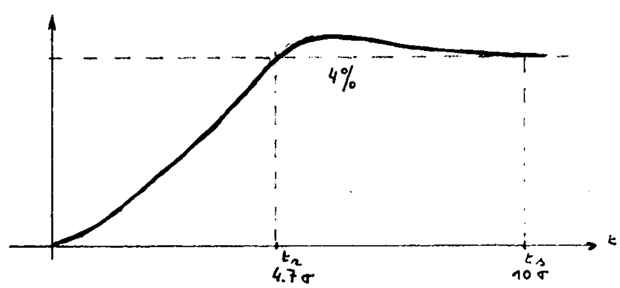

Fig. 4. Step response of critically damped second-order system.

E = electrical voltage applied to the loudspeaker
I = corresponding current through the loudspeaker

An equivalent circuit may be found as shown in Fig. 6. The motional circuit elements are given by

$$L = \frac{B^2l^2}{S}$$

$$C = \frac{M + X_{\mathrm{rad}}/\omega}{B^2l^2}$$

$$R = \frac{B^2l^2}{K + R_{\mathrm{rad}}} .$$

Only the power dissipated in R_{rad} is useful since it is the radiated acoustic power.

Because of the relationship between acoustic pressure and air velocity ($P/V = \rho_0 c$ in air), linearization of the loudspeaker response may be obtained by linearizing the cone velocity V. In practice this may be carried out by building an amplifier with a negative output resistance ($-R_e - sL_e$). Thus the amplifier voltage E_a seems to be placed directly over the motional impedance and BlV equals E_a. Normally this is realized by mounting the loudspeaker in a bridge circuit and using positive feedback.

Because of the specific situation that R_{rad} is very small, this method has a tendency of being unstable under disturbed conditions, and another method should be applied to obtain a rigid control system.

2.2 Control Theory [3]

From the set of loudspeaker equations given in Sec. 2.1, the block diagram of Fig. 7 may be obtained. The following remarks have to be taken into account:

1) There is a feedback path in the loudspeaker equivalent circuit. This means that a new open-loop circuit has to be calculated before Kessler optimization is considered.

2) The loudspeaker behavior is strongly frequency dependent, with a resonance effect in it. This natural frequency depends also on the dimensions of the loudspeaker box.

3) A number of loudspeaker parameters have to be measured, such as mass, stiffness, friction, and the force factor Bl.

4) Consideration has to be given to the kind of controller to be chosen, and how to choose the feedback circuit $H(s)$ so that the cone velocity can be measured.

5) There exists a point between I where the loudspeaker current is obtained. This current can be measured and a state-space-like feedback circuit used.

2.3 Equivalent Circuits

As mentioned, a new open-loop circuit has to be calculated. The equivalent open-loop expression is given by

$$\frac{Bl}{B^2l^2 + (R_e + sL_e)(sM + S/s + K + Z_{\mathrm{rad}})} .$$

It should now be controlled whether or not the denumerator can be factored into useful factors so that we can choose the type of controller (PI, PID, PID_2, etc.). Therefore the equations to be used can be determined depending on the frequency domain under consideration. The following set of transfer functions is obtained:

below resonance: $$\frac{Bls}{R_eS + (L_eS + B^2l^2)s}$$

at resonance: $$\frac{Bl}{B^2l^2 + (K + R_{\mathrm{rad}})(R_e + sL_e)}$$

above resonance: $$\frac{Bl}{B^2l^2 + sMR_e + s^2ML_e} .$$

It seems that below resonance the system reduces to a system with only one dominant pole, but with a differentiating action. It could be compensated considerably by using a PI controller. At resonant frequency the system is determined only by the electrical char-

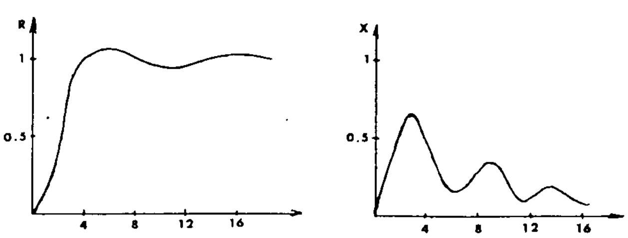

Fig. 5. Radiation impedance of a piston.

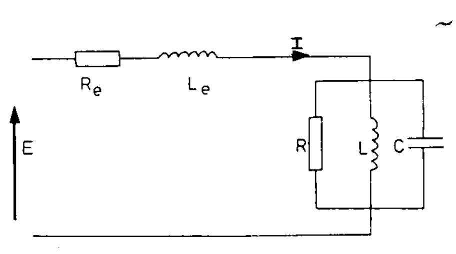

Fig. 6. Equivalent circuit of a loudspeaker.

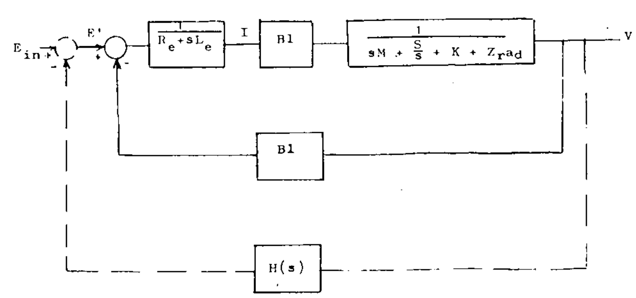

Fig. 7. Block diagram of a loudspeaker.

acteristics of the coil impedance. Above resonant frequency the system may be approximated by a second-order system. For the latter case it should be known whether poles are real or complex conjugate so that the choice of controller can be made: either two PI controllers or one PID controller.

The poles of the system above resonant frequency are found to be

$$p_{1,2} = \frac{R_e M \pm R_e M\sqrt{1 - 4B^2l^2ML_e/M^2R_e^{\,2}}}{2ML_e} .$$

These poles are real numbers and may be sketched as shown in Fig. 8. If $4B^2l^2ML_e << R_e^2M^2$, the square-root expression may be approximated by a linear expression, and the poles reduce to

$$p_{1,2} = \frac{R_e M \pm R_e M(1 - 2B^2l^2ML_e/M^2R_e^2)}{2ML_e} .$$

Because of the fact that $4B^2l^2ML_e/M^2R_e^2$ should be considered as very small, this holds even for

$$\frac{2B^2l^2ML_e}{M^2R_e^{\,2}} .$$

Thus the poles may be approximated very well by

$$p_1 = \frac{R_e}{L_e}$$

$$p_2 = \frac{B^2l^2}{R_e M} .$$

The loudspeaker may now be given by the equivalent circuit of Fig. 9. Comparing this block diagram with that in Fig. 7, the point between I is again the representation of the current that flows through the loudspeaker. This current can be measured, and the general remarks about a possible structure of the feedback circuit remain valid. Since for all frequencies the loudspeaker may be described by a system having real dominant poles, Kessler optimization can be applied.

3 KESSLER OPTIMIZATION

It should first be noted that all equivalent circuits given above are only first-order approximations where the dominant poles are considered. However, some smaller time constants are present in the circuit, but most of them depend directly on the way the appropriate output parameter is measured and on the kind of feedback circuit used. In the following two sections, small time constants are disregarded, although they are present.

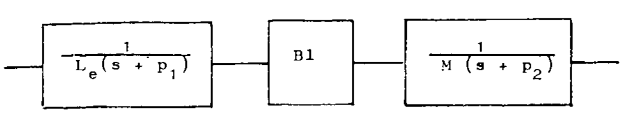

Fig. 8. Loudspeaker above resonant frequency.

3.1 Below Resonant Frequency

Below resonant frequency the loudspeaker can be described by one dominant pole. Thus a control circuit with only one PI controller is needed. This is shown in Fig. 10. This optimization of the controller leads to the following conditions:

$$K_I = \frac{R_e S}{Bl}$$

$$K_p = \frac{B^2l^2 + SL_e}{Bl} .$$

3.2 Above Resonant Frequency

For the case where the loudspeaker is driven at frequencies above resonant frequency, the control circuit sketched in Fig. 11 is applicable.

The optimization procedure has to be performed in two steps. The first step concerns the system enclosed in a dashed box. Here optimization may be accomplished for a system having a dominant pole $T = L_e/R_e$ and by using a feedback circuit that measures the current through the loudspeaker coil. Hence this may be considered an electric system.

When optimization of this electric system is achieved, only some smaller time constants define the behavior of the dashed box. Thus for the entire system there again remains only one dominant pole. Optimization may now be done for the entire system by adjusting the parameters of a second PI controller to this dominant pole,

$$T' = \frac{R_e M}{B^2l^2} .$$

4 PRACTICAL CIRCUITS AND RESULTS

As an example, consider a loudspeaker with the fol-

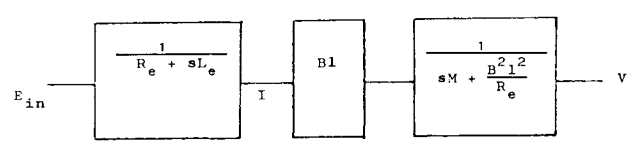

Fig. 9. Loudspeaker equivalent circuit.

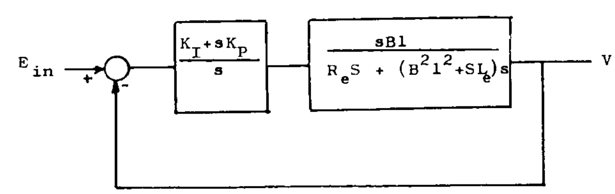

Fig. 10. Control circuit for loudspeaker below resonance.

lowing parameters:

$R_e = 5\Omega$
$L_e = 0.35$ mH
$l = 10.35$ m
$B = 1.45T$
$Bl = 4.94$ N/A
$M = 9.35 \times 10^{-3}$ kg
$S = 525$ N/m
$K \rightarrow 0$
$f_{res} = 36$ Hz ($\omega_{res} = 226$) .

Because of the very low natural frequency, for all normal audio frequencies only the case above resonant frequency has to be taken into account. Thus the condition of Sec. 2.3 has to be checked:

$$4B^2l^2ML_e = 310 \times 10^{-6}$$

$$R_e^2M^2 = 2185 \times 10^{-6} .$$

The condition mentioned is satisfied and optimization is possible using two PI controllers. The resultant poles are $p_1 = 68$ μs and $p_2 = 1.9$ ms, which corresponds to −3-dB points of 2340 Hz and 83 Hz.

4.1 Practical Circuits

The loudspeaker may be driven by a power amplifier. The PI controllers are built with low-noise high-gain operational amplifiers. The current through the loudspeaker is measured as the voltage over a small resistor (0.1 Ω), which is placed in series with the loudspeaker.

In the previous sections it was supposed that the cone velocity is known. But this velocity is not directly measurable. Several methods can be used. For instance, the acceleration of the cone could be measured and integrated so that the velocity is obtained. Another possibility is to build a bridge around the loudspeaker so that the induced voltage BlV may be detected.

For the present experiments the displacement of the cone was measured. By differentiating this signal, the velocity was obtained. Therefore a capacitor was built in the center of the cone, so that a displacement of the cone introduces a variation of the capacitor value. This capacitor has a value of about 4 pF, and the variation is about 2 pF. To facilitate detecting these variations, this capacitor is used as the tuning element of a Clapp oscillator. This circuit is shown in Fig. 12, and the tuning capacitor is C_3.

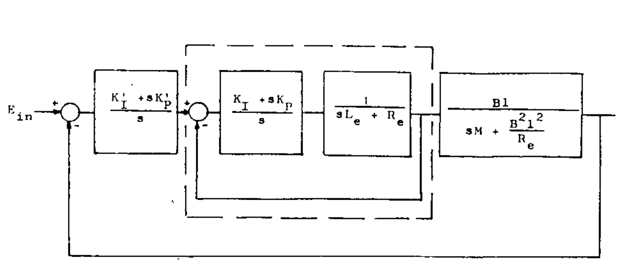

Fig. 11. Control circuit for loudspeaker above resonance.

A frequency-modulated signal is obtained in this manner. The carrier frequency is about 2 MHz. The demodulation is done with a phase-locked-loop detector (NE 562). By differentiating this demodulated signal the cone velocity is obtained. For the given loudspeaker and the circuitry used, valid measurements of the velocity are obtained up to 500 Hz. The circuit is sketched in Fig. 13.

Tuning of the PI controllers is accomplished with the aid of a square wave. The tuning is first carried out on the current feedback circuit and then on the mechanical feedback circuit. Each time the controller parameters are tuned to give the well-known step response, with an overshoot of about 4%.

4.2 Results

After tuning, the loudspeaker system was fed with pink noise, and the frequency response was measured in an anechoic room. The frequency response above 500 Hz may be disregarded since the mechanical feedback loop was no longer effective. But for low frequencies a very flat frequency response was obtained. Fig. 14 gives the frequency responses for the loudspeaker with and without the tuned feedback circuitry. Fig. 15 shows the picture of some feedback signals due to a square-wave input signal.

The results of these experiments can be considered as very good. Further research is now concentrated on obtaining linearization over a large frequency band by altering the feedback circuit and the displacement transducer. Subsequent research may be concerned with the digital realization of the PI controllers. Algorithms

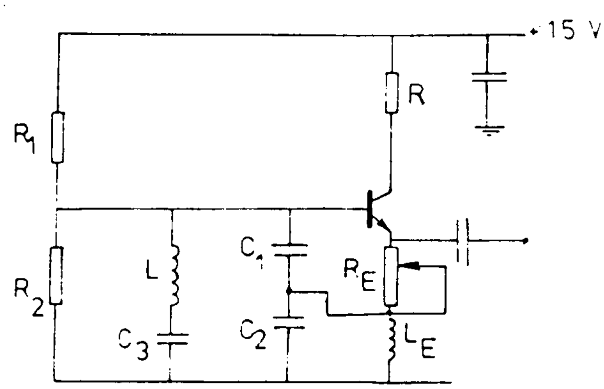

Fig. 12. Clapp oscillator.

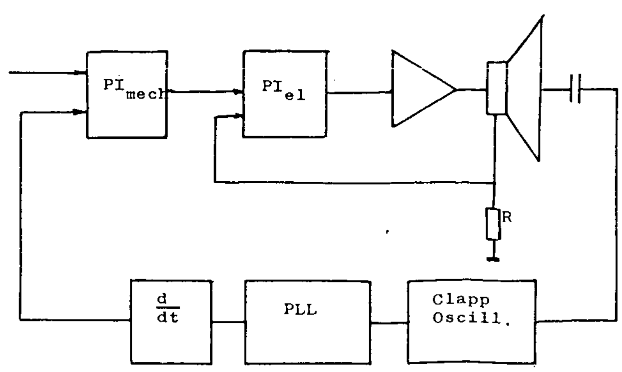

Fig. 13. Overall circuit for loudspeaker feedback.

for doing this with the aid of a microprocessor or an analog processor are well known. Then digital audio signals could be processed directly with these circuits. Reports on these topics will be published in a future paper.

5 ACKNOWLEDGMENT

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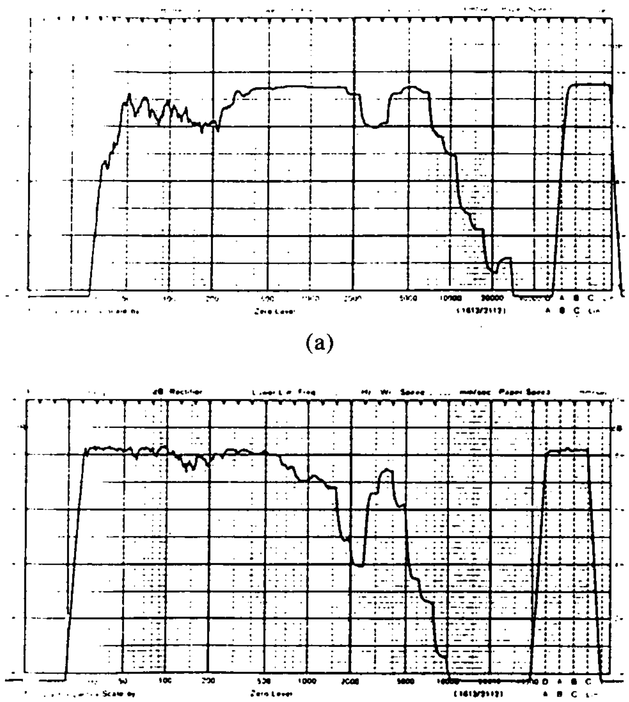

(a)

(b)

Fig. 14. Frequency response of loudspeaker.

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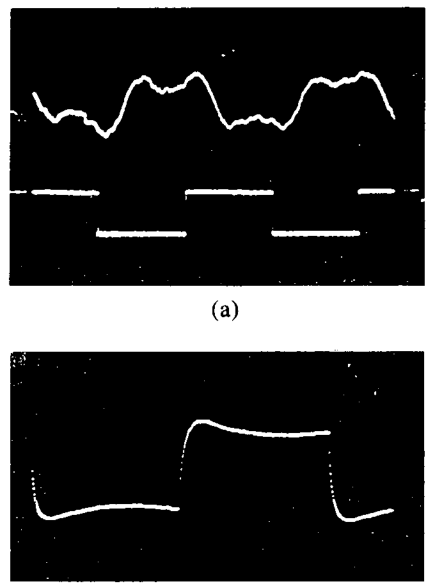

(a)

(b)

Fig. 15. (a) Displacement of the cone due to a square-wave input signal. (b) Overcompensated current response.

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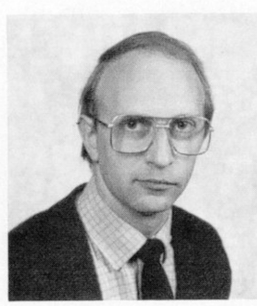

Johan Catrysse was born in Brugge, Belgium, in 1949. He received a degree in electrical engineering from the Rijksuniversiteit Ghent in 1971. From 1971 to 1974 he was an assistant at the laboratory of electromagnetism and acoustics at this university and was involved in acoustic holography and SAW design. Since 1974 he has been professor of electronic engineering at the Katholieke Industriële Hogeschool West-Vlaanderen at Oostende. He is currently responsible for the electronic department of the institute. His main interest is in network theory and CAD techniques for designing electronic circuits, digital signal processing (DSP) and control engineering. Since 1979 he has been chairman of the AES Belgian Section. He is the author of papers in the audio field published in the *Journal of the Audio Engineering Society*.