Review report on "Approximate Nearest Neighbors: Towards Removing the curse of Dimensionality"

Introduction

This paper aims to make a short review on the essay- Approximate Nearest Neighbors: Towards Removing the curse of Dimensionality. This paper first introduces the content of the essay. The second part is the innovation part, in this part, the improved algorithm is presented, including its propositions and techniques. The third part is the technical quality part, which mainly shows the rigor of the author's argument. The fourth part introduces the application of the improved algorithm in multimedia databases. The last part evaluates the essay's logic and proposes a piece of advice.

Content

The research is about the improvement over the nearest neighbor search (NNS) problem. In the case of the d-dimensional Euclidean space where metrix space X = $\mathcal{R}^d$ under some l_p norm, the low-dimensional case is well-solved. However, when dealing with the issue of the "curse of dimensionality", the current solutions and algorithms are far from satisfactory. The author proposes an algorithm by reducing ϵ-approximate nearest neighbor search (ϵ-NNS) to a new problem—point location in equal balls to avoid the curse of dimensionality and improve the known bounds of the nearest neighbor search problem.

It is easy to conclude that the key idea of the improved algorithm is to reduce the ϵ-NNS to the problems of point location in equal balls (PLEB) and ϵ-point location in equal balls ($\epsilon - \text{PLEB}$). Therefore, the author introduces a data structure called a ring-cover tree, which the reduction relies on. The constructing of a ring-cover tree is recursive: For any given P at the root, P can be decomposed into some smaller sets and these sets are assigned to the children of the node for P. Moreover, the ring-cover tree can be constructed in deterministic $\tilde{O}(n^2)$ time. At last, after establishing the validity of the search procedure, the author analyzes the ring-cover tree construction.

In the 4th section, the author gives two solutions to the point location problem and also presents the proof sketch of relative propositions and theorems, one is based on a method akin to the Elias bucketing algorithm and the other one applies the technique of locality-sensitive hashing with preprocessing cost only linear in d and sublinear in n. After introducing the two solutions, the author presents further applications of PLEB algorithms, that is, the PLEB procedures described above can also be used in cases where points are being inserted and deleted over time. It is worth noting that to apply the procedures, the researcher should assume that the points have integer coordinates with absolute value bounded.

Innovation

The basic problem of the nearest neighbor is to perform indexing or similarity searching for query objects. The number of features of query objects range from tens to thousands. However, when the dimension d of metrix space X is relatively large, the solutions provide little improvement over a brute-force algorithm in theory or in practice.

To improve the known bounds of the approximate version, the author reduces the optimization problem ϵ-NNS to its decision version, vie., ϵ-point location in equal balls (ϵ-PLEB) by means of a novel data structure named ring-cover trees. It is known that PLEB (ϵ-PLEB) can be reduced to NNS (ϵ-NNS) with the same preprocessing and query costs. ϵ-PLEB can also be reduced to ϵ-NNS with only a small overhead in preprocessing and query costs. Then, the author presents two solutions to the point location problem. One is based on a method akin to the Elias bucketing algorithm and works for any l_p norm. This solution decomposes each ball into a bounded number of cells and store them in a dictionary. This decomposition has the property that while searching a point set P, it is possible to quickly restrict the search to one of the decomposed sets. By doing so, $\tilde{O}(d)$ query time is achieved and the preprocessing time exponential in d.

The other solution introduces the technique of locality-sensitive hashing and applies directly only to Hamming spaces. However, by exploiting some facts in appendix, the locality-sensitive hashing can be used for any l_p norm where pϵ[1,2].The solution's key idea is to use hash functions so that the probability of collision is much higher for objects that are close to each other than for those that are far apart. Then the author proves that the existence of such functions for any domain (not necessarily a metric space) implies the existence of fast ϵ-NNS algorithms for that domain, with preprocessing cost only linear in d and sublinear in n.

Technical quality

The technical quality is of high quality. The authors support their theory using mathmatical proof. After presenting the improved method, the authors first introduce the data structure the improved method relies on as well as some necessary definitions to make the proof clear, such as the definition of a(γ, δ)-cluster and a (b, c, d)-cover. Then the authors introduce the corresponding propositions of the point set P under different cases.

In section 3.1, the authors let $\beta = 2\left(1+\frac{1}{\varepsilon}\right), b = 1 + \frac{1}{log^2 n}, \alpha = \frac{1-1/logn}{2}$, in this case, it is easy to see that $0 < \alpha < 1, \beta > 1, b > 1$, therefore, according to the corollary 1, P can be a ring node or a cover node. Then the author introduces the time complexity and space complexity of the construction of the ring-cover tree. The ring cover tree can be constructed in deterministic $\tilde{O}(n^2)$ time. Also, when searching the ring-cover tree, for any node P, it is achievable to restrict the search to one of its children using a small number of tests.

In section 4, which is the most important section, the authors present two techniques for solving the ϵ-PLEB problem. The first is based on a method similiar to the Elias bucketing algorithm and works for any l_p norm. The second uses locality-sensitive hashing and applies directly to Hamming spaces. However, it is worth noting that by exploiting some facts in appendix, the locality-sensitive hashing can be used for any l_p norm where p$\in$[1,2]. In the bucketing method part, the authors assume for now that p = 2 and impose a uniform grid of spacing $s = \epsilon/\sqrt{d}$ on $\Re^d$. Then they make the claimation that for $0 < \epsilon < 1, \overline{|B|} = O(1/\epsilon)^d$, for general l_p norms, they modify s to $\epsilon/d^{1/p}$ and the bound on $\overline{|B|}$ applies unchanged. In the locality-sensitive hashing part, the author introduces the notion of locality-sensitive hashing and apply it to sublinear-time similarity searching. After defining a ball for a similarity measure D and generalizing the notion of ϵ-PLEB to $(r_1, r_2) - \text{PLEB}$, the authors propose the existence of an algorithm for $(r_1, r_2) - \text{PLEB}$ under measure D which uses $O(\text{dn} + n^{1+p})$ space and $O(n^p)$ evaluations of the hash function for each query. Then the authors apply the theorem to two measures: the Hamming metric and set resemblance. For the first measure, they apply a family of projections for fast hashing with AC^0 operations. For the second measure, they use sketch functions to make estimation of the resemblance between the given pair of sets. At last, the authors state that by combing both techniques, they obtain a method for dynamic estimation of closest pair, which is the first algorithm solving this problem in subquadratic time for any d.

Application and X-factor

I find the proposal in the paper promising that the hierarchical agglomerative clustering, greedy matching and other problems can be solved more efficiently. For example, in multimedia applications such as IBM's QBIC (Query by image content), the number of features could be several hundreds, so it is considered to be a dramatic improvement that dimension -reduction techniques can reduce the dimensionality to a mere few hundreds. The examples also include fingding the best matches for local image features in large datasets (Lowe, 2004), or clustering local features into visual words using the k-means or similar algorithms (Sivic and Zisserman, 2003).

Hakan and Ertem (Ferhatosmanoglu et al., 2001) evaluate the performance of a simple progressive approaches-KLT (Kanade-Lucas-Tomasi Tracking Method) based approximate k-NN searching and the result shows that even simple technique can be extended and adapted for approximate searching in a very effective way. However, it is worth noting that they propose a cluster-based approach which allows a user to progressively explore the approximate results with increasing accuracy.

Presentation

The overall structure is clear. I found that reading is easy. The authors first introduce the improved method- ϵ-point location in equal balls as the beginning. Then they present the key point of the improved method, that is, how to reduce the ϵ-NNS problem to the ϵ-point location in equal balls. In this section, the authors give the procedure of constructing ring-cover trees and prove some propositions of the searching procedue. In the next section, the authors present two techniques for solving the ϵ-PLEB problem, the bucketing method and locality-sensitive hashing. At last, the authors show readers some further applications of PLEB algorithms as the end.

However, if one wants to prove that his method is better than the other one, then it is necessary to show readers the experimental results. So the paper could have been more attractive and convincing if the authors had provided experimental results' comparsion between the improved method and the original nearest neighbor search.

Reference

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