SAMPLE-EFFICIENT LEARNING OF RIGID BODY DYNAMICS

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MOTIVATION

- Current robots are stuck in repetitive, predictable environments
- Want to enable dynamic interaction with objects
- Frictional contact is fundamental to robot manipulation, but difficult to model
  - Sudden changes in dynamics when making/breaking contact
  - Inconsistencies with Coulomb friction (Painlevé paradox)
  - Many simultaneous contacts
  - Stick/slip transitions

PRIOR WORK

Purely learned
- Often in context of policy learning
- Slow and data inefficient
- Doesn't use existing knowledge of contact dynamics

Hybrid
- Best of both worlds

Approaches:
- Sim-to-real
- Residual physics
- **Differentiation through contact problem**

Analytical
- Only an approximation
- Doesn't fully capture real-world phenomena

METHOD

1. Formulate base contact model as fusion of Drumwright [1] and MuJoCo [2]
   Phase 1: Solver for normal forces with no friction:
$$\begin{aligned}\underset{\lambda_n \geq 0}{\arg\min} \quad & \lambda_n^T J_n M^{-1} J_n^T \lambda_n + J_n f \lambda \\ & J_n M^{-1} J_n^T \lambda_n \Delta t + (J_n f)\phi \geq 0\end{aligned}$$
   Phase 2: Compute $\kappa = e^T \lambda_n$ from phase 1. Then solve frictional contact:
$$\begin{aligned}\underset{\lambda}{\arg\min} \quad & \lambda^T J M^{-1} J^T \lambda + J f \lambda_n + \lambda_n^T diag(exp(\phi))\lambda_n \\ & J M^{-1} J^T \lambda \Delta t + (Jf)\phi \geq 0 \\ & \lambda_n \geq 0 \\ & e^T \lambda_n \leq \kappa \\ & \lambda_t \leq \mu \lambda_n\end{aligned}$$
2. Penalize deviations from measured data in subproblem
   - Incorporate L_2 state penalty in contact model objectives
   - Introduces **tradeoff** between satisfying model and matching experimental observations to avoid nonexistant parameter gradients
3. Optimize model parameter set with respect to summed error over all data points
$$\begin{aligned}\underset{\theta}{\arg\min} \quad & \sum_i (Dynamics(q_i, u_i, \lambda) - \bar{q}_i)^2 \\ s.t. \quad & \lambda = Sol(q_i, u_i)\end{aligned}$$

q – configuration | u control input | λ_n – normal forces | λ_t – frictional forces | J_n – normal contact Jacobian | J_t – tangential contact Jacobian | f – no contact dynamics | ϕ – gap function | μ – Coulomb friction coefficient | M - inertia matrix

[1] Evan Drumwright and Dylan A Shell. Modeling Contact Friction and Joint Friction in Dynamic Robotic Simulation using the Principle of Maximum Dissipation. Technical report.

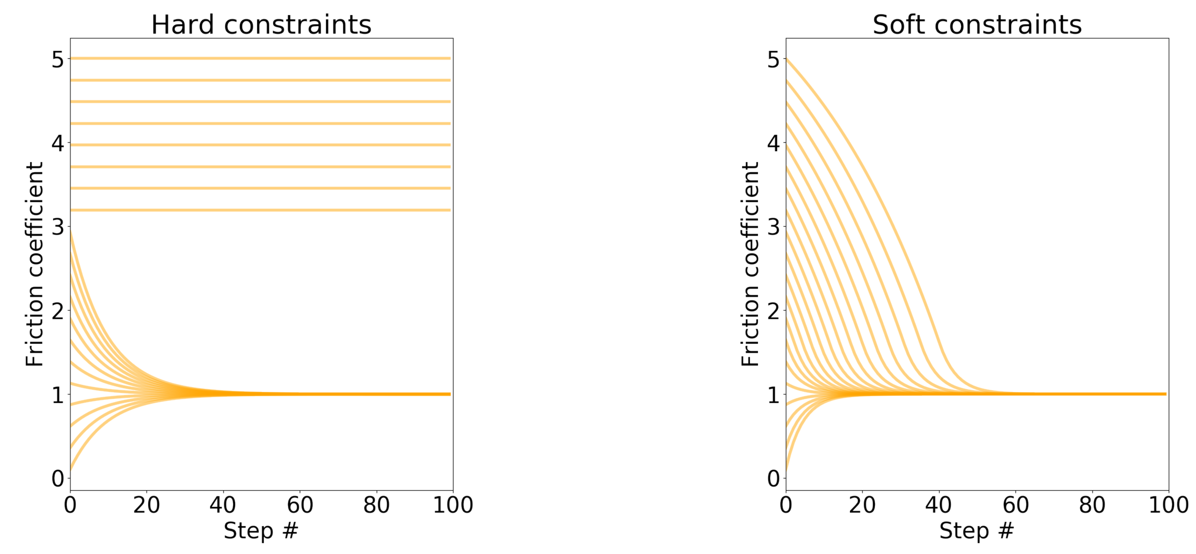
[2] Emanuel Todorov, Tom Erez, and Yuval Tassa. MuJoCo: A physics engine for model-based control. Technical report.

THEORETICAL RESULTS

Model is proven to be well behaved:

- Dissipation of kinetic energy $K(s)$, **but** no guaranteed rate $\frac{d}{ds}K < -\varepsilon K$
$$K(s+k) \leq K(s),\ \forall k > 0$$
- Homogeneity of impact map
$$(v_- \to v_+) \implies (kv_- \to kv_+, \forall k \geq 0)$$
- **Existence of solutions** to every initial value problem

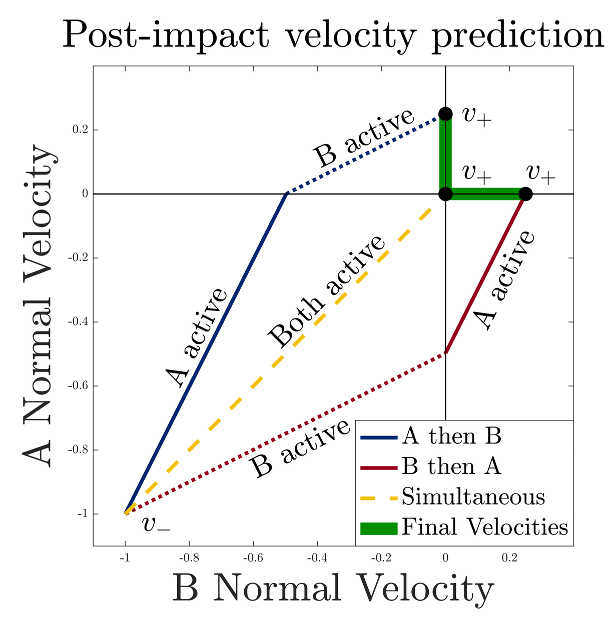
Antagonistic scenarios may prevent finding valid post-impact state:



Theorem. For non-jammed systems, impact terminates linearly in $\|v(0)\|$.

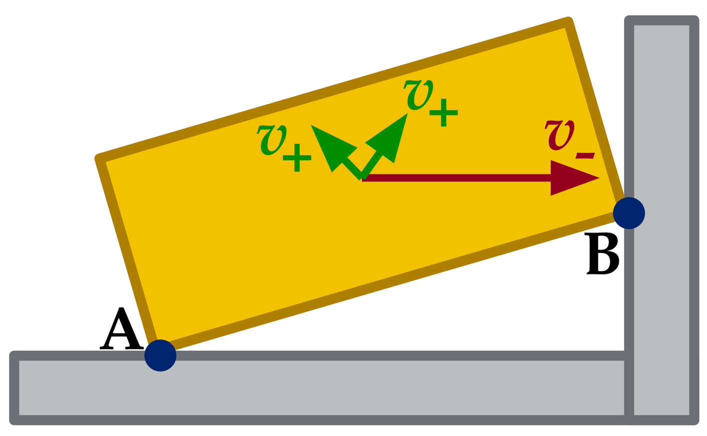
APPLICATION: RIMLESS WHEEL

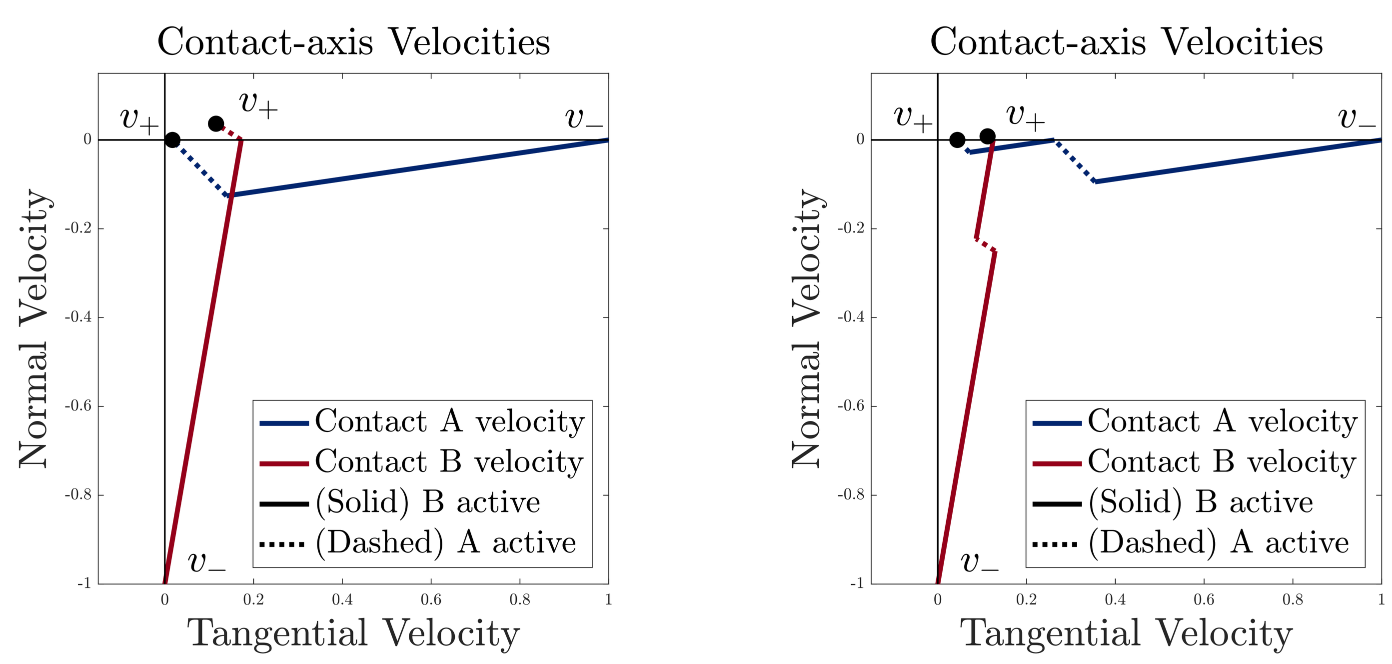
Impact model not only gives each of the three first-principles results, but also returns every reasonable intermediate result.



APPLICATION: MANIPULATION

Non-uniqueness emerges even without simultaneous impact. A block slid into a wall (right) will have sensitive behaviors due to propagation of shockwaves through the body.





SUMMARY

Contributions
- Derivation of a simultaneous inelastic impact model
- Proven characterization of model properties
- Guarantees for existence of solutions and impact termination

Ongoing Work
- Modeling of elastic impacts
- Embedding impact model into full dynamics
- Time-stepping simulation through impact
- Algorithms for approximating post-impact set

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