Convex Optimization for Control and Stability Analysis of Dynamic Interaction

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Motivation

- Robotics requires interacting with a complex world to effect positive change
- **Dynamic interaction**, necessary for faster and efficient task completion, poses notable challenges
  - Dynamics are discontinuous due to Coulomb friction and impulsive impact forces
  - Fundamental questions of uniqueness and existence–modeling and prediction of dynamics is hard or impossible
  - Hybrid models scale poorly (exponentially) in the number of contacts
- Robots, individually or as teams, must be robust to unexpected contact and must embrace interaction
- Research focuses on numerical algorithms to enable dynamic interaction between robots and the world

Contact-invariant Control and Stability [2, 3]

- Most approaches to control and analysis rely upon derivatives, while contact dynamics are discontinuous.
- Hybrid models decompose states into continuous regions, but the number of hybrid modes is combinatoric in number of contacts
- **Contribution: tractable control design and verification through contact**

Method

- Exploit compatibility between sums-of-squares optimization and the *measure differential inclusion* formulation of nonsmooth dynamics
- For positions $\mathbf{q}$, velocities $\mathbf{v}$, and (possibly impulsive) forces $\boldsymbol{\lambda}$, prove stability for **all physically possible** $(\mathbf{q}, \mathbf{v}, \boldsymbol{\lambda})$ pairings
- Admissibility of $(\mathbf{q}, \mathbf{v}, \boldsymbol{\lambda})$ naturally generates a basic semialgebraic set

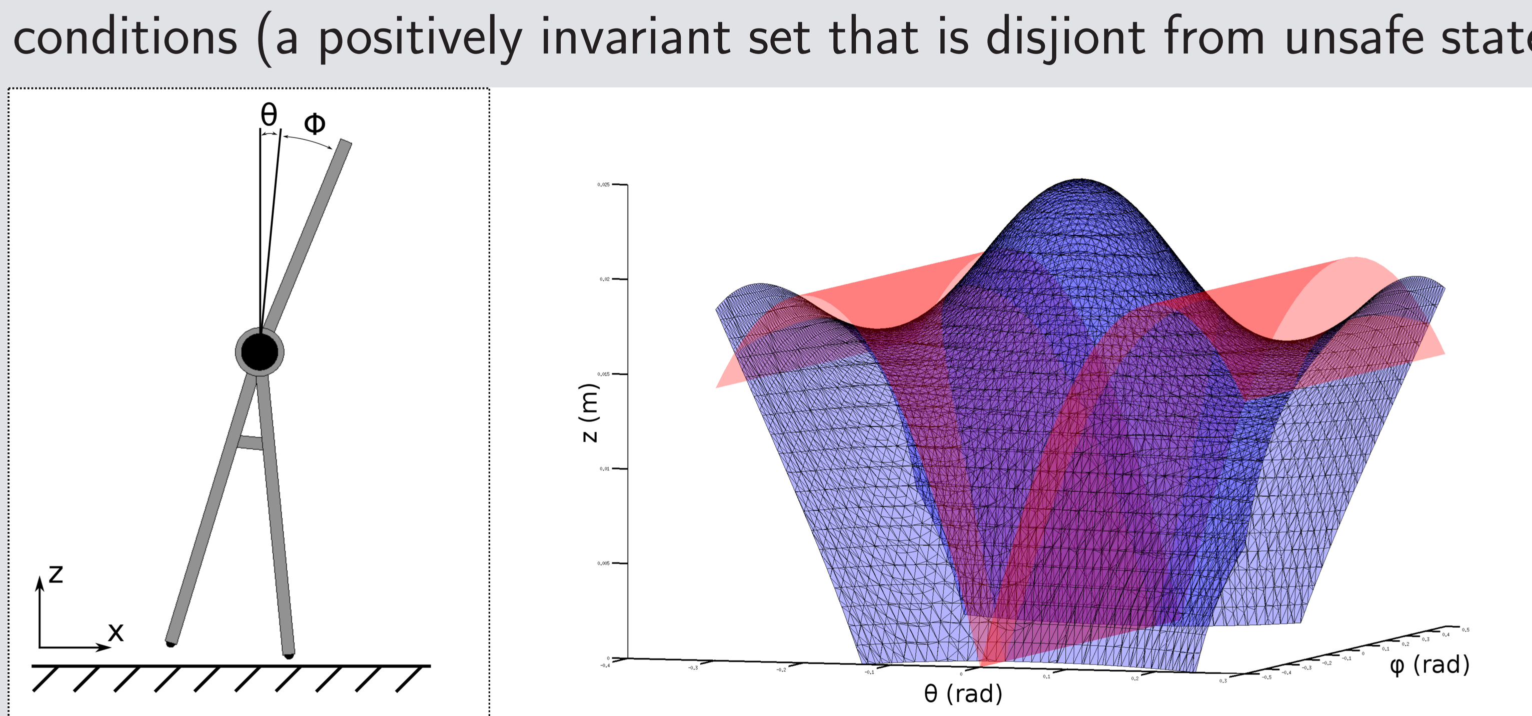
  $0 \leq \phi(\mathbf{q}) \perp \lambda_N \geq 0$ Non-penetration & no force at a distance

  $\dot{\psi}(\mathbf{q})\lambda_T \leq 0$ Frictional dissipation

  $\dot{\psi}(\mathbf{q})(\mu^2\lambda_N^2 - \lambda_T^2) = 0$ Stick-slip Coulomb friction
- Verify Lyapunov stability over semialgebraic set in polynomial time
- Bilinear alternations enable **control synthesis**: find $\mathbf{u}(\mathbf{q}, \mathbf{v})$ to maximize region of attraction

Examples

For a glider perching against a wall, we compute and verify safe initial conditions (a positively invariant set that is disjiont from unsafe states)



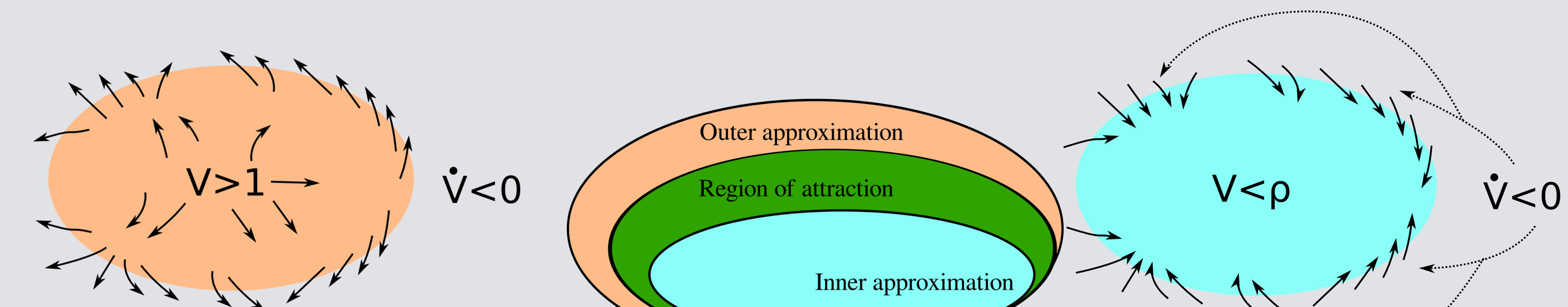
A polynomial feedback policy is designed to maximize the verified region of attraction (blue). The policy and verification work through contact between the feet and the ground. Contact surfaces (red) prevent ground penetration and cause frictional and impulsive forces.

Reduced-order Model Analysis [1]

- Simple models widely used in bipedal walking. Designed with expert intuition, they enable robust strategies and simple analysis
- Full dynamical models cannot be formally analyzed $\Rightarrow$ difficult to know limits of simple strategies and models
- **Contribution: sums-of-squares algorithms to formally compare capabilities of reduced-order models**

Algorithms

Compute outer and inner bounds on regions of attraction



- outer bound (left)
  - Prove that **no control policy** can stabilize $\mathbf{x_0}$ outside bound

$$\mathbf{x} \in \mathcal{X}, \mathbf{u} \in \mathcal{U} \Rightarrow \dot{\mathbf{V}}(\mathbf{t}, \mathbf{x}) < 0$$
$$\mathbf{x} \in \mathcal{X}_{\mathbf{goal}} \Rightarrow \mathbf{V}(0, \mathbf{x}) > 1$$

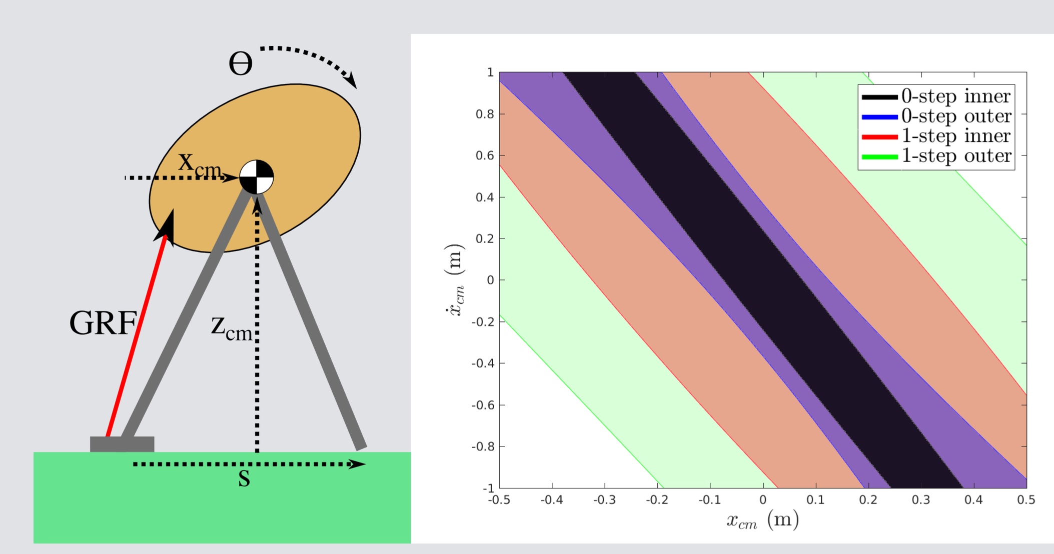
- inner bound (right)
  - Find stabilizing control policy and barrier (Lyapunov) function

$$\mathbf{V}(\mathbf{x}, \mathbf{t}) = \rho(\mathbf{t}) \Rightarrow \dot{\mathbf{V}}(\mathbf{t}, \mathbf{x}) \leq \dot{\mathbf{r}}(\mathbf{t})$$

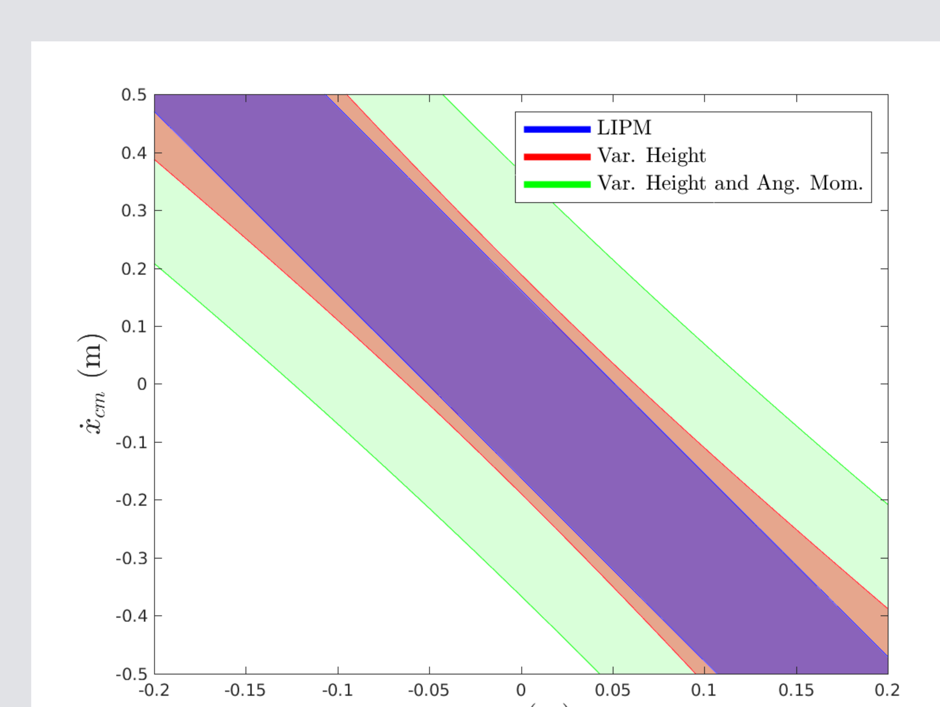
- Iterate calculations through impacts as robot takes multiple steps

Examples

(Left) A model captures angular momentum and vertical COM motion. (Right) Explicit bounds on the capability to reject disturbances via balancing and with a single step.



Three different reduced-order models of increasing complexity are compared with one another. The figure demonstrates the explicit limitations of control policies based on the simplest models.



References

[1] Michael Posa, Twan Koolen, and Russ Tedrake. Balancing and Step Recovery Capturability via Sums-of-Squares Optimization. In *Robotics: Science and Systems*, 2017.

[2] Michael Posa, Mark Tobenkin, and Russ Tedrake. Lyapunov Analysis of Rigid Body Systems with Impacts and Friction via Sums-of-Squares. In *Proceedings of the 16th International Conference on Hybrid Systems: Computation and Control (HSCC 2013)*, pages 63–72. ACM, apr 2013.

[3] Michael Posa, Mark Tobenkin, and Russ Tedrake. Stability analysis and control of rigid-body systems with impacts and friction. *IEEE Transactions on Automatic Control (TAC)*, 61(6):1423–1437, jun 2016.

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