MODELING AND ANALYSIS OF NON-UNIQUE BEHAVIORS IN MULTIPLE FRICTIONAL IMPACTS

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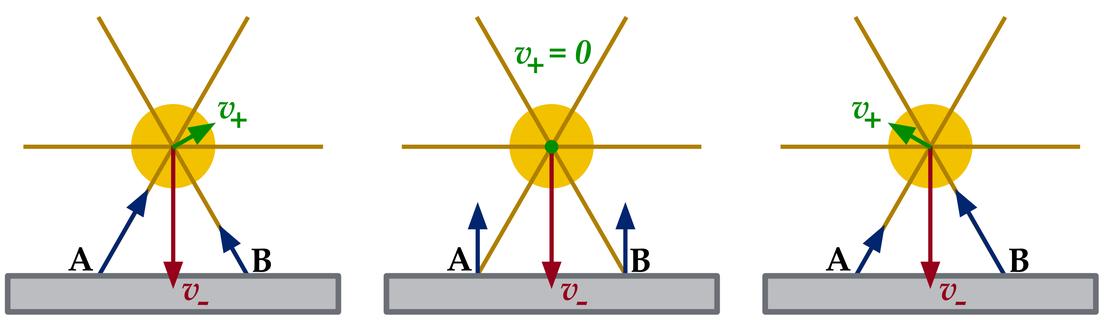
MOTIVATION

- Frictional contact is **the fundamental behavior** of robot locomotion and manipulation
- Simultaneous frictional impacts between rigid bodies are **pervasive, extremely sensitive, and not well understood**
- **We develop a simultaneous impact model that enables algorithms to reason about impacts' ambiguity**

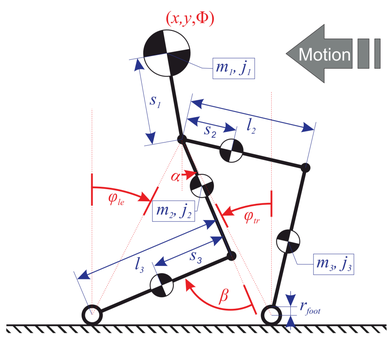
KEY EXAMPLES

Simple Model: Rimless Wheel



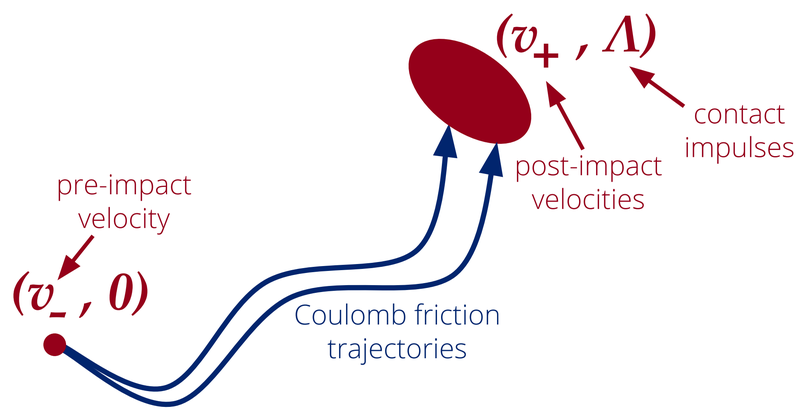
A then B | Simultaneous | B then A

Complex Model: RAMone [1]



Real System: Billiards

METHOD



- Finds change in velocity and impulses via Newton's second law:

$$M(v_+ - v_-) = J^T \Lambda$$

- Extension of Routh's 1891 model to multiple contacts [2]
- v_+ and Λ determined *incrementally*:
  1. Increase normal impulses with slopes $\lambda_{n,i}$ such that

$$\sum_i \lambda_{n,i} = 1$$

  2. Increment each friction impulse via Coulomb friction:

$$\|\lambda_{t,i}\|_2 \leq \mu_i \|\lambda_{n,i}\|, \qquad \lambda_{t,i} \in \arg\min_{\lambda_{t,i}} \lambda_{t,i}^T v_i$$

  3. Terminate when $v = v_- + M^{-1} J^T \Lambda$ no longer penetrates
- Formulation as a *differential inclusion*

$$\frac{\mathrm{d}}{\mathrm{d}s} v(s) \in D(v(s))$$

[1] C David Remy. Ambiguous collision outcomes and sliding with infinite friction in models of legged systems. *The International Journal of Robotics Research*, 36(12):1252–1267, oct 2017.

[2] E J Routh. *Dynamics of a system of rigid bodies*. MacMillan and co. London, 1891.

THEORETICAL RESULTS

Model is proven to be well behaved:

- Dissipation of kinetic energy $K(s)$, but no guaranteed rate $\frac{\mathrm{d}}{\mathrm{d}s} K < -\varepsilon K$

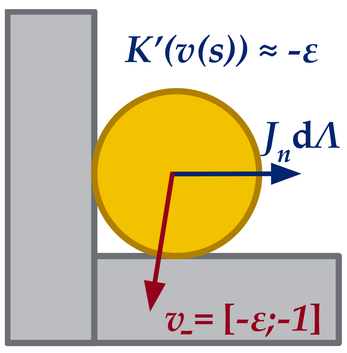
$$K(s + k) \leq K(s), \forall k > 0$$

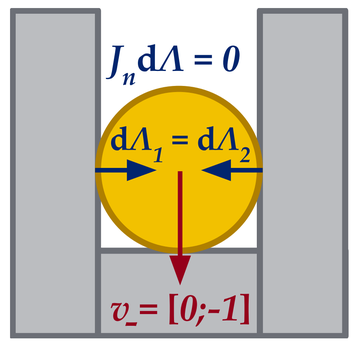
- Homogeneity of impact map

$$(v_- \to v_+) \implies (kv_- \to kv_+, \forall k \geq 0)$$

- **Existence of solutions** to every initial value problem

Antagonistic scenarios may prevent finding valid post-impact state:

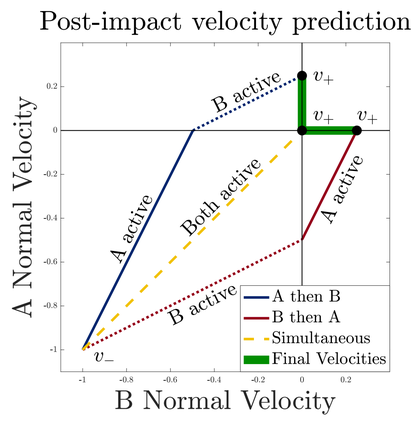




Theorem. For non-jammed systems, impact terminates linearly in $\|v(0)\|$.

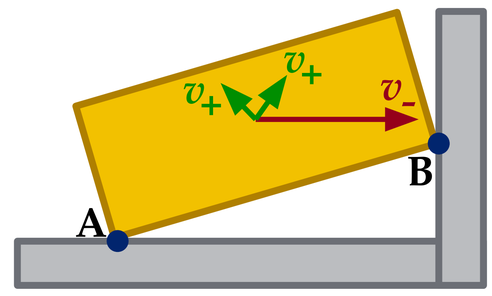
APPLICATION: RIMLESS WHEEL

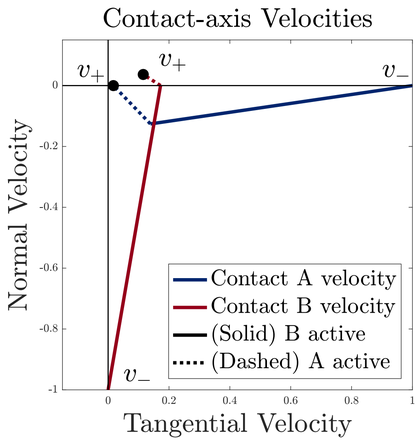
Impact model not only gives each of the three first-principles results, but also returns every reasonable intermediate result.

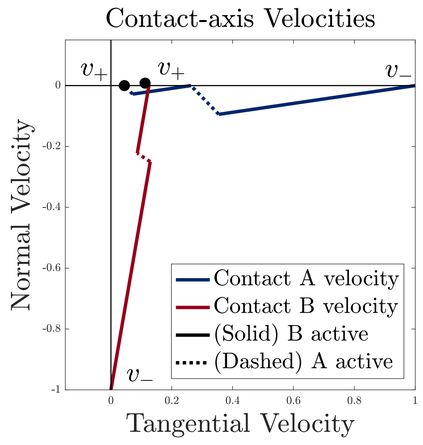


APPLICATION: MANIPULATION

Non-uniqueness emerges even without simultaneous impact. A block slid into a wall (right) will have sensitive behaviors due to propagation of shockwaves through the body.







SUMMARY

Contributions

- Derivation of a simultaneous inelastic impact model
- Proven characterization of model properties
- Guarantees for existence of solutions and impact termination

Ongoing Work

- Modeling of elastic impacts
- Embedding impact model into full dynamics
- Time-stepping simulation through impact
- Algorithms for approximating post-impact set

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