Learning Contact Dynamics with LCP Constraint Relaxations

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Overview

- Problems with manipulation
- Existing approaches
- How are we learning?
- What are we learning?

Problems with manipulation

- Sudden changes in dynamics when making/breaking contact
- Inconsistencies with Coulomb friction (Painlevé paradox)
- Many simultaneous contacts
- Stick/slip transitions

The Linear Complimentarity Problem (LCP)

- Given matrix M and vector q, find vector λ such that:

$$\begin{aligned} \lambda &\geq 0 \\ M\lambda + q &\geq 0 \quad \text{nonnegativity} \\ \lambda^T(M\lambda + q) &= 0 \quad \text{complimentarity} \end{aligned}$$

Shorthand:

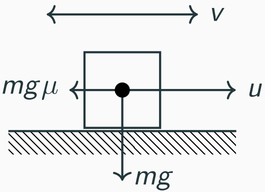
$$0 \leq \lambda \perp M\lambda + q \geq 0$$

- If M positive definite, Lemke's algorithm always finds λ
- Can formulate contact problems using complimentarity
  - Either separation distance is zero, or normal force is
  - Either tangential velocity is zero, or friction force is at boundary of cone (uses slack variables)
  - Solve an LCP for each time step
  - Described by Stewart, Anitescu

A simple example

- 1 DOF block with friction and external force
- Encodes stick/slip, no separation

$$\begin{aligned} v_{k+1} &= v_k + \lambda^+_{k+1} - \lambda^-_{k+1} + u_{k+1} && \text{dynamics equation} \\ 0 \leq \lambda^+_{k+1} &\perp \gamma_{k+1} + v_{k+1} \geq 0 && \text{positive friction if block moving left} \\ 0 \leq \lambda^-_{k+1} &\perp \gamma_{k+1} - v_{k+1} \geq 0 && \text{negative friction if block moving right} \\ 0 \leq \gamma_{k+1} &\perp mg\mu - \lambda^+_{k+1} - \lambda^-_{k+1} \geq 0 && \text{max friction if block is moving} \end{aligned}$$



A simple example (matrix form)

$$0 \le \underbrace{\begin{bmatrix} \lambda^+_{k+1} \\ \lambda^-_{k+1} \\ \gamma_{k+1} \end{bmatrix}}_{\lambda} \perp \underbrace{\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}}_{\mathrm{M}} \begin{bmatrix} \lambda^+_{k+1} \\ \lambda^-_{k+1} \\ \gamma_{k+1} \end{bmatrix} + \underbrace{\begin{bmatrix} v_k + u_{k+1} \\ -v_k - u_{k+1} \\ mg\mu \end{bmatrix}}_{\mathrm{q}} \ge 0$$

- M is not PSD, but with some small adjustments Anitescu shows Lemke always finds solution
- Not necessarily unique
- Generalizes to friction cone approximation

Existing approaches

Learned

- Often in context of policy learning
- Slow and data innefficient
- Doesn't leverage existing understanding of contact dynamics

Hybrid

- Best of both worlds
- Sim-to-real
- Residual physics
- **Differentiation through LCPs**

Analytical

- Only an approximation
- Doesn't fully capture real-world phenomena

LCP differentiation (Belbute-Peres / Amos / Kolter)

- Similar to previous work on QPs
  - KKT conditions for QP are an LCP
- Gives gradients of LCP solutions with respect to M and q
- Forms bilevel optimization problem:

$$
\begin{aligned}
\min_{M,q} \quad & \sum_i (\text{Dynamics}(x_i,\lambda_i) - \bar{x}_i)^2 \\
\text{subject to} \quad & \lambda_i \in LCP(M, q)
\end{aligned}
$$

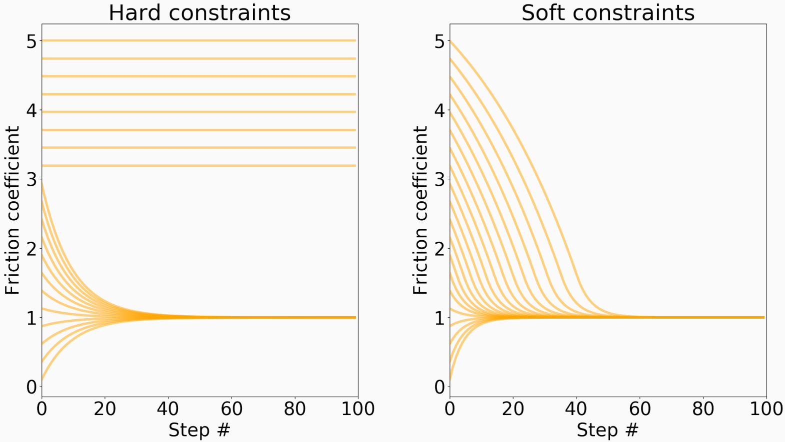
- **Problem:** bad priors can cause zero gradients
- **Cause:** hard constriants in LCP

Reformulating the optimization problem

- Move cost into subproblem
- Soften hard LCP constraints
- Allow unphysical behavior to reduce prediction error

$$
\begin{aligned}
\min_{M,q} \quad & \sum_i c_i \\
\text{subject to} \quad & c_i = \min_{\lambda_i \geq 0} \quad (\text{Dynamics}(x_i, \lambda_i) - \bar{x}_i)^2 \\
& \qquad\qquad + \lambda_i^T (M\lambda_i + q) + \text{hinge}(M\lambda_i + q)
\end{aligned}
$$

Learning $\mu = 1$



The lower level optimization problem

- Can be written in quadratic form:

$$(\mathrm{Dynamics}(x_i,\lambda_i) - \bar{x}_i)^2 = \left(v_{k+1} - (v_k + \lambda_i^+ - \lambda_i^- + u_i)\right)^2$$

$$\text{Write } \beta_i = v_{k+1} - v_k - u_i\text{:}$$

$$= \lambda_i^T \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{D} \lambda_i + \underbrace{\begin{bmatrix} -2\beta_i & 2\beta_i & 0 \end{bmatrix}}_{b} \lambda_i$$

- Get lower-level QP (with hard nonnegativity constraint):

$$\begin{aligned} \min_{\lambda_i \geq 0} \quad & \frac{1}{2}\lambda_i^T (Q + Q^T)\lambda_i + (q^T + b)\lambda_i \\ & M\lambda_i + q \geq 0 \\ & Q = M + D \end{aligned}$$

Convexity guarantees

- Want symmetric part of $M + D$ to have nonnegative eigenvalues
- Eigenvalues of D nonnegative, real
- Eigenvalues of M complex with nonnegative real part
- Extend analysis to full Anitescu formulation
- Scaling D with sufficiently large constant?

Structured learning

- Elementwise learning of M, q undesirable
  - Can prove an ϵ error in single q element breaks sticking
  - Coefficient antisymmetry across rows breaks sticking
- How much structure do we need to impose?
  - Learning physical parameters intuitive but restrictive
  - Eventually progress to noninterpretable parameters

Thank you!

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