SAMPLE-EFFICIENT LEARNING OF RIGID BODY DYNAMICS

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MOTIVATION

- Current robots are stuck in repetitive, predictable environments
- Want to enable dynamic interaction with objects
- Frictional contact is fundamental to robot manipulation, but difficult to model
  - Sudden changes in dynamics when making/breaking contact
  - Inconsistencies with Coulomb friction (Painlevé paradox)
  - Many simultaneous contacts
  - Stick/slip transitions

PRIOR WORK

Learned

- Often in context of policy learning
- Slow and data inefficient
- Doesn't use existing knowledge of contact dynamics

Hybrid

- Best of both worlds

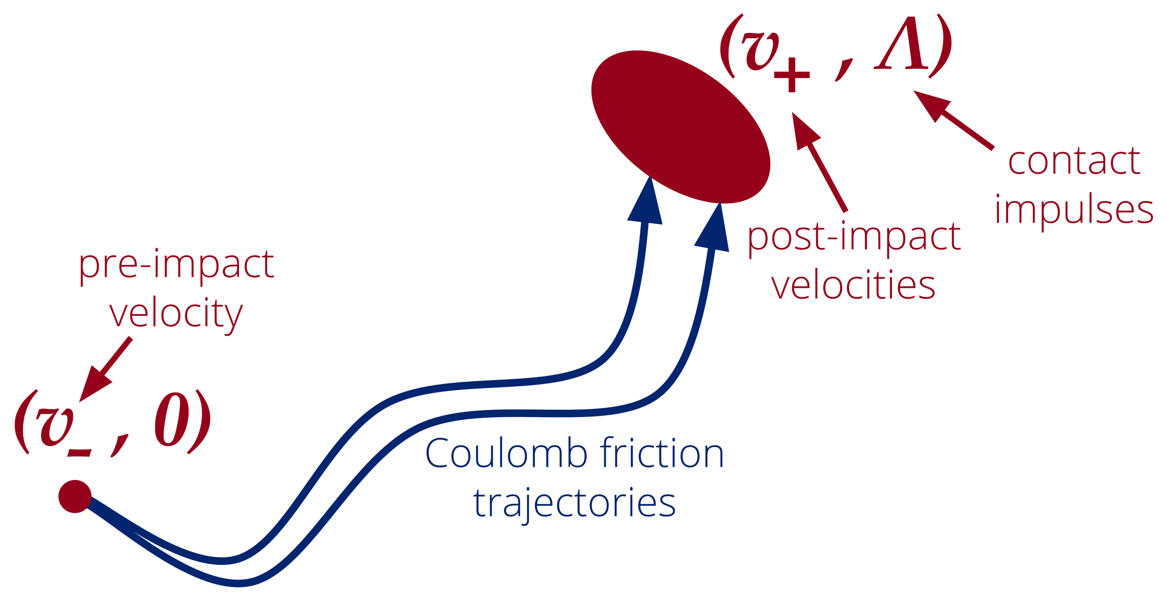
Approaches:

- Sim-to-real
- Residual physics
- **Differentiation through contact problem**

Analytical

- Only an approximation
- Doesn't fully capture real-world phenomena

METHOD



- Finds change in velocity and impulses via Newton's second law:

$$M(v_+ - v_-) = J^T\Lambda$$

- Extension of Routh's 1891 model to multiple contacts [?]
- v_+ and Λ determined *incrementally*:
  1. Increase normal impulses with slopes $\lambda_{n,i}$ such that
  $$\sum_i \lambda_{n,i} = 1$$
  2. Increment each friction impulse via Coulomb friction:
  $$\|\lambda_{t,i}\|_2 \leq \mu_i \|\lambda_{n,i}\|, \qquad \lambda_{t,i} \in \arg\min_{\lambda_{t,i}} \lambda_{t,i}^T v_i$$
  3. Terminate when $v = v_- + M^{-1}J^T\Lambda$ no longer penetrates
- Formulation as a *differential inclusion*

$$\frac{\mathrm{d}}{\mathrm{d}s} v(s) \in D(v(s))$$

[1] Michael Posa, Twan Koolen, and Russ Tedrake. Balancing and Step Recovery Capturability via Sums-of-Squares Optimization. In *Robotics: Science and Systems*, 2017.

[2] Michael Posa, Mark Tobenkin, and Russ Tedrake. Lyapunov Analysis of Rigid Body Systems with Impacts and Friction via Sums-of-Squares. In *Proceedings of the 16th International Conference on Hybrid Systems: Computation and Control (HSCC 2013)*, pages 63–72. ACM, apr 2013.

[3] Michael Posa, Mark Tobenkin, and Russ Tedrake. Stability analysis and control of rigid-body systems with impacts and friction. *IEEE Transactions on Automatic Control (TAC)*, 61(6):1423–1437, jun 2016.

THEORETICAL RESULTS

Model is proven to be well behaved:

- Dissipation of kinetic energy $K(s)$, **but** no guaranteed rate $\frac{\mathrm{d}}{\mathrm{d}s}K < -\varepsilon K$

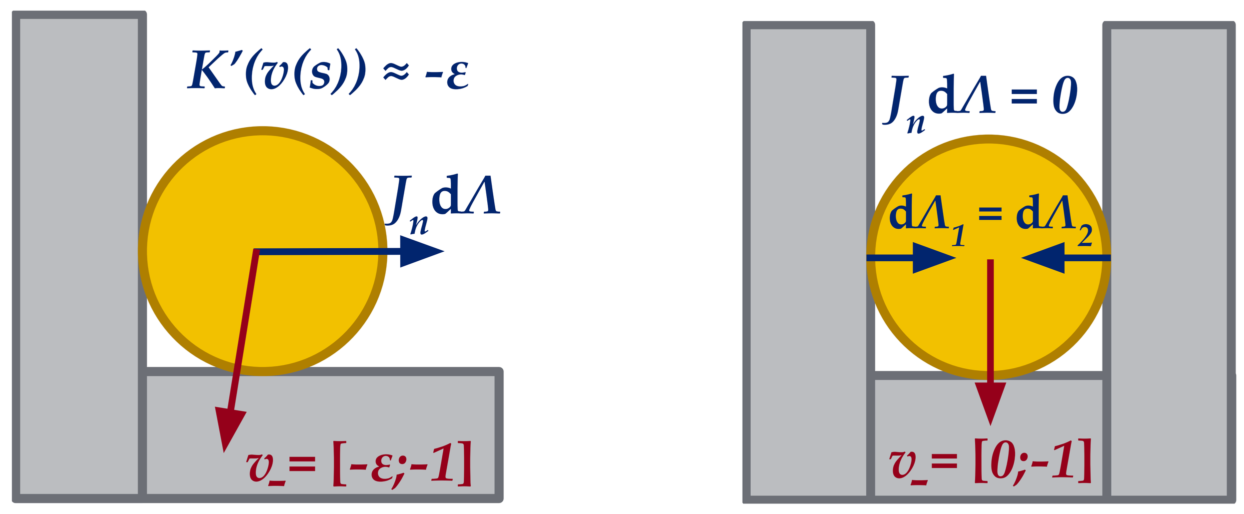
$$K(s+k) \leq K(s),\ \forall k > 0$$

- Homogeneity of impact map

$$(v_- \to v_+) \implies (kv_- \to kv_+, \forall k \geq 0)$$

- **Existence of solutions** to every initial value problem

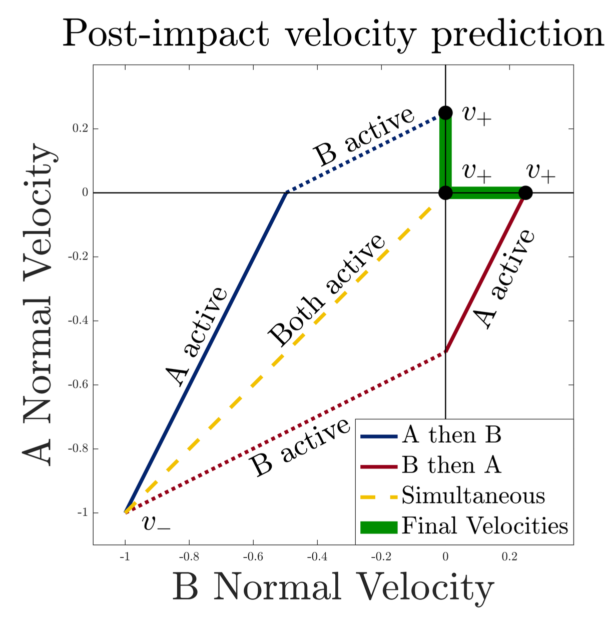
Antagonistic scenarios may prevent finding valid post-impact state:



Theorem. For non-jammed systems, impact terminates linearly in $\|v(0)\|$.

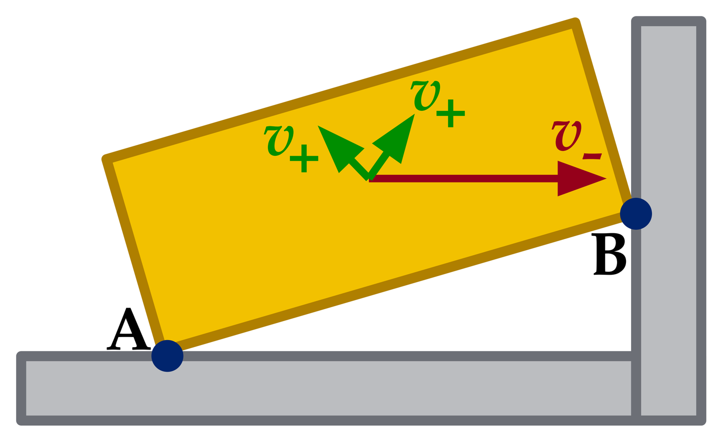
APPLICATION: RIMLESS WHEEL

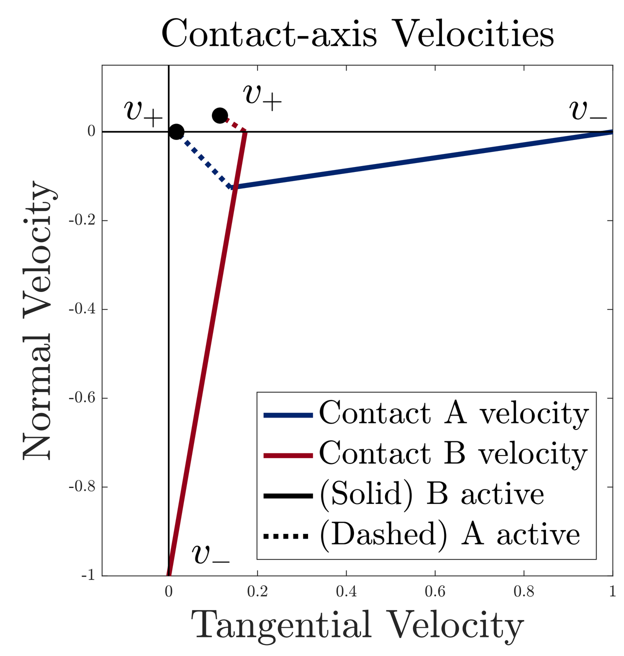
Impact model not only gives each of the three first-principles results, but also returns every reasonable intermediate result.

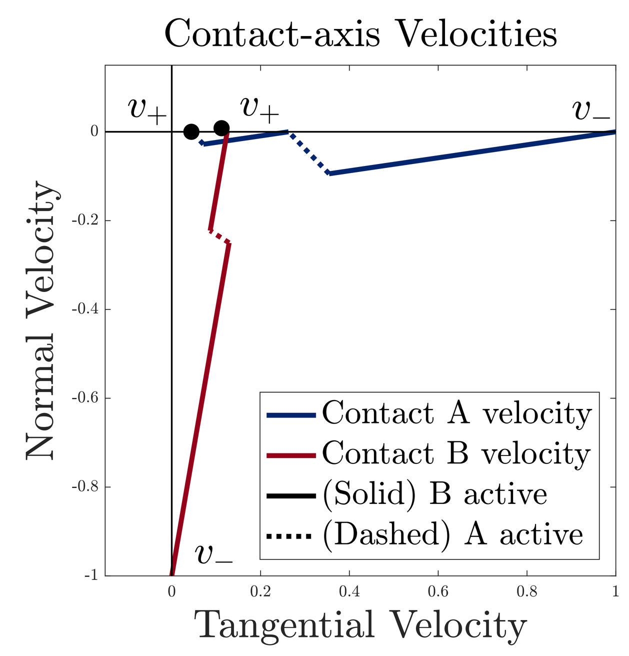


APPLICATION: MANIPULATION

Non-uniqueness emerges even without simultaneous impact. A block slid into a wall (right) will have sensitive behaviors due to propagation of shockwaves through the body.







SUMMARY

Contributions

- Derivation of a simultaneous inelastic impact model
- Proven characterization of model properties
- Guarantees for existence of solutions and impact termination

Ongoing Work

- Modeling of elastic impacts
- Embedding impact model into full dynamics
- Time-stepping simulation through impact
- Algorithms for approximating post-impact set