

P289 Ch9 시간에 따라 변화하는 전자기학

Maxwell Equation

§ Faraday 법칙

Faraday 유도법칙

여연 개로면에 자기선 (magnetic flux) 가 시간에 따라 변화하면, 개로면의 boundary 를 통과하는 페르室友
유도 기전류가 발생하여 유도전류가 흐른다

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

$$\mathcal{E} = \oint_C \vec{E} \cdot d\vec{s} : \text{기전류}$$

$$\Phi_B = \int_S \vec{B} \cdot \hat{\vec{u}}_N ds : \text{자기선류}$$

$$\Rightarrow \underline{\oint_C \vec{E} \cdot d\vec{s} = - \frac{d}{dt} \int_S \vec{B} \cdot \hat{\vec{u}}_N ds} \quad \text{Faraday 법칙}$$

[1] 이론적

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

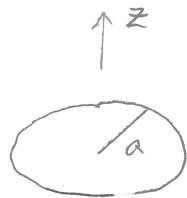
[2] Θ 의 의미 : Lenz's 법칙

회로가 시간에 따라 증가하면 감소하는 방향으로,

감소하면 증가하는 방향으로 유도전류가 발생

한다는 의미

(Ex)



$$\vec{B} = \begin{cases} B_0 e^{kt} \hat{z} & 0 \leq r \leq b \\ 0 & b < r \end{cases}$$

 $a < b$ case

$$\bar{\Phi}_B = \int_S \vec{B} \cdot \hat{n} ds = \pi a^2 B_0 e^{kt}$$

$$\mathcal{E} = -\frac{d\bar{\Phi}_B}{dt} = -\pi a^2 B_0 K e^{kt}$$

유도 전기장 $\vec{E} = ?$

$$R \rightarrow P$$

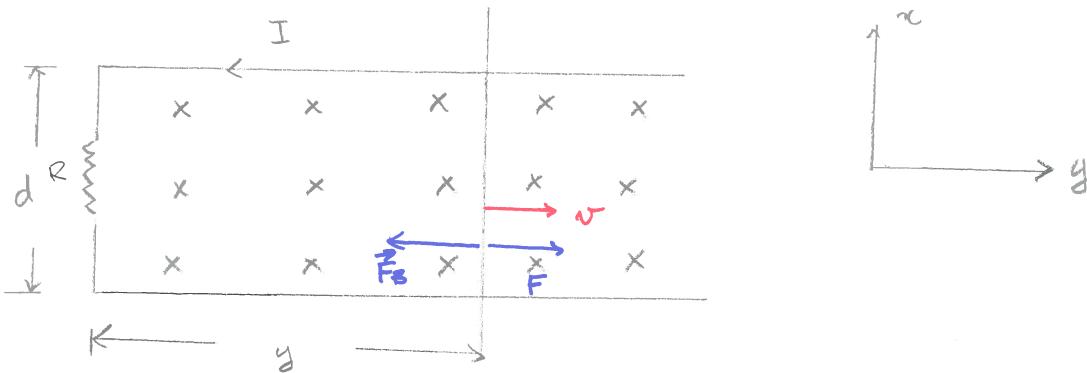
$$\mathcal{E} = -\pi p^2 K B_0 e^{kt} = \oint_C \vec{E} \cdot d\vec{s} = E \cdot 2\pi p$$

$$E = -\frac{1}{2} K B_0 p e^{kt}$$

$$\vec{E} = -\frac{1}{2} K B_0 e^{kt} p \hat{\phi}$$

$$\text{학: } \vec{B} \times \vec{E} = -K B_0 e^{kt} \hat{z} = -\frac{\vec{B}}{\partial t}$$

(Ex)



$$\vec{B} = B \hat{z} \quad (B: \text{const})$$

$$\vec{\epsilon}_B = B dy$$

$$\mathcal{E} = -\frac{d\vec{\epsilon}_B}{dt} = -Bd \frac{dy}{dt} = -Bd\omega \quad : \text{유도 기전류}$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{Bd\omega}{R} \quad : \text{유도 전류}$$

* 예상치 근정

$$\vec{F}_B = I \vec{l} \times \vec{B}, \quad F_B = IdB$$

도선이 등속 운동을 하므로 위의 $\vec{F} = -\vec{F}_B$ 가 주어져야 한다.

$$F = IdB$$

P (전력): 힘의 단위 시간당 한울

$$P = F\omega = IdB\omega \quad I = \frac{Bd\omega}{R}$$

$$= \frac{B^2 d^2 \omega^2}{R} = \frac{\mathcal{E}^2}{R}$$

$$\mathcal{E}^2 = (Bd\omega)^2$$

$$|\mathcal{E}| = Bd\omega$$

* 운동 전자의 관점

운동 전자가 받는 강

$$\vec{F}_B = q(\vec{v} \times \vec{B}) = q \vec{E}_m$$

$$\vec{E}_m = \vec{v} \times \vec{B}$$

$$\mathcal{E} = \oint_c \vec{E}_m \cdot d\vec{l} \quad c: \text{시계방향} \quad (-\text{를 얻기 위해 })$$

$$= \oint_c (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$= \int_d^0 N B \, dz$$

$$= -NBd$$

※

(9.1)

$$\epsilon = 10^{-11} \text{ (F/m)} \quad \mu = 10^5 \text{ (H/m)}$$

$$\vec{B} = 2 \times 10^{-4} \cos(10^5 t) \sin(10^3 y) \hat{x} \quad (\text{T})$$

$$(a) \vec{H} = \frac{\vec{B}}{\mu} = \frac{2 \times 10^{-4}}{\mu} \cos(10^5 t) \sin(10^3 y) \hat{z} \quad (\text{A/m})$$

$$\vec{V} \times \vec{H} = \frac{2 \times 10^{-4}}{\mu} \cos(10^5 t) \left| \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin(10^3 y) & 0 & 0 \end{array} \right|$$

- $10^{-3} \cos(10^3 y) \hat{z}$

$$= - \frac{2 \times 10^{-7}}{\mu} \cos(10^5 t) \cos(10^3 y) \hat{z} \equiv \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \vec{E} = E \hat{z}$$

$$\frac{\partial \vec{E}}{\partial t} = - \frac{2 \times 10^{-7}}{\epsilon \mu} \cos(10^5 t) \cos(10^3 y)$$

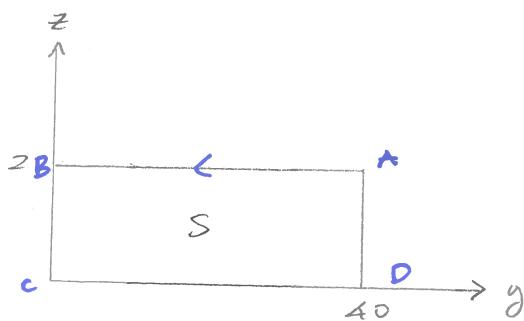
$$E = - \frac{2 \times 10^{-7}}{\epsilon \mu} \frac{1}{10^5} \sin(10^5 t) \cos(10^3 y)$$

$$= - \frac{2 \times 10^{-12}}{\epsilon \mu} \sin(10^5 t) \cos(10^3 y) \quad \epsilon \mu = 10^{-16}$$

$$= - 20000 \sin(10^5 t) \cos(10^3 y)$$

$$\vec{E} = - 20000 \sin(10^5 t) \cos(10^3 y) \hat{z}$$

(b)



$$\Phi_B = \int_S \vec{B} \cdot \hat{n} \, ds \quad \hat{u}_n = \hat{z}$$

$$ds = dy \, dz$$

$$= \int_0^{40} dy \int_0^2 dz \quad (z \times 10^{-4}) \cos(10^5 t) \sin(10^3 y)$$

$$= 0.4 [1 - \cos(0.04)] \cos(10^5 t)$$

$$\Phi_B (t = 10^{-6})$$

$$= 0.4 [1 - \cos(0.04)] \cos(0.1)$$

$$= 0.318359 \times 10^{-3} \text{ (Wb)}$$

$$= 0.318359 \text{ (mWb)}$$

$$(c) \quad \Phi_B = 0.4 [1 - \cos(0.04)] \cos(10^5 t)$$

$$-\frac{\partial \Phi_B}{\partial t} = 40000 [1 - \cos(0.04)] \sin(10^5 t)$$

$$\Rightarrow \oint_C \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t} = 40000 [1 - \cos(0.04)] \sin(10^5 t)$$

$$\text{at } t = 10^{-6} \text{ (sec)}$$

$$\oint_C \vec{E} \cdot d\vec{s} = 40000 [1 - \cos(0.04)] \sin(0.1)$$

$$= 3.19424 \text{ (V)}$$

* (a) 결과는 이용해서 구할 수 있다!!

$$\vec{E} = -20000 \sin(10^5 t) \cos(10^3 y) \hat{z}$$

$$\oint \vec{E} \cdot d\vec{s}$$

$$= \underbrace{\int_A^B \vec{E} \cdot d\vec{s}}_{d\vec{s} = dy \hat{y}} + \underbrace{\int_B^C \vec{E} \cdot d\vec{s}}_{d\vec{s} = dz \hat{z}} + \underbrace{\int_C^D \vec{E} \cdot d\vec{s}}_{d\vec{s} = dy \hat{y}} + \underbrace{\int_D^A \vec{E} \cdot d\vec{s}}_{d\vec{s} = dz \hat{z}}$$

$$y=0 \qquad \qquad \qquad y=40$$

$$= \int_z^0 [-20000 \sin(10^5 t)] dz$$

$$+ \int_0^2 [-20000 \sin(10^5 t) \cos(0.04z)] dz$$

$$= 40000 \sin(10^5 t) [1 - \cos(0.04z)]$$

*

(용융예제 9-2)

$$d = 0.09 \text{ cm}$$

$$\vec{B} = 0.3 \hat{z} \text{ T}$$

$$\vec{v} = 0.1 e^{20t} \hat{y} \text{ (m/sec)}$$

(a) $v(t=0) = v(y=0) = 0.1 \text{ (m/sec)}$

(b) $v = \frac{dy}{dt} = 0.1 e^{20t} \text{ (단위는 m/sec)}$

$$y = -\frac{1}{20} \ln(1-2t)$$

$$y(t=0.1) = -\frac{1}{20} \ln(0.8) = 0.011157 \text{ (m)} = 1.1157 \text{ (cm)}$$

(c) $v(t=0.1)$

$$= v(y=0.011157)$$

$$= 0.1 e^{20 \times 0.011157}$$

$$= 0.125 \text{ (m/sec)}$$

(d) $V_{12} = \mathcal{E} = -BdV$

$$V_{12}(t=0.1) = -0.3 \times 0.09 \times 0.125$$

$$= -0.002625 \text{ (V)}$$

$$= -2.625 \text{ (mV)}$$

XX

는 어떤 것인가?

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad (?)$$

$$\underline{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J}}$$

$$\vec{\nabla} \cdot \vec{J} = 0$$

$$\text{However } \vec{\nabla} \cdot \vec{J} = - \frac{\partial \rho}{\partial t} \quad (\text{연속방정식})$$

Something wrong!!

$$\Rightarrow \vec{\nabla} \times \vec{H} = \vec{J} + \vec{G}$$

$$0 = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{G} = - \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{G}$$

$$\text{Since } \vec{\nabla} \cdot \vec{D} = \rho, \quad \vec{G} = \frac{\partial \vec{D}}{\partial t}$$

$$\Rightarrow \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}$$

\vec{J} : 전도 전류 밀도

$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$: 이동 전류 밀도 (displacement current density)

* 적분학

$$\oint_C \vec{H} \cdot d\vec{s}$$

$$= \int_S (\vec{J} \times \vec{H}) \cdot \hat{\vec{u}}_N ds$$

$$= \int_S \left(\vec{J} + \frac{\partial \vec{P}}{\partial t} \right) \cdot \hat{\vec{u}}_N ds$$

$$= \underline{\int_S \vec{J} \cdot \hat{\vec{u}}_N ds} + \int_S \frac{\partial \vec{P}}{\partial t} \cdot \hat{\vec{u}}_N ds$$

I

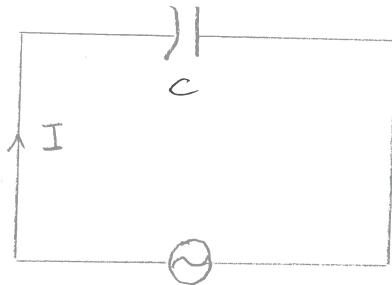
$$= I + \int_S \frac{\partial \vec{P}}{\partial t} \cdot \hat{\vec{u}}_N ds$$

$$\oint_C \vec{H} \cdot d\vec{s} = I + \int_S \frac{\partial \vec{P}}{\partial t} \cdot \hat{\vec{u}}_N ds$$

I : 전도전류

$\int_S \frac{\partial \vec{P}}{\partial t} \cdot \hat{\vec{u}}_N ds$: 변화전류

* 电容 C 的充放电



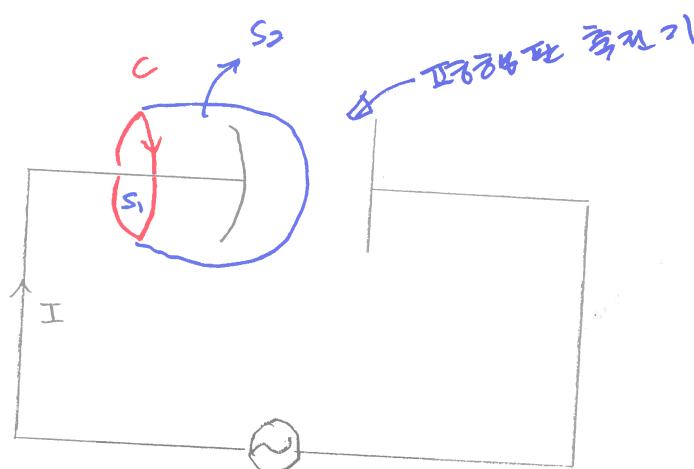
$$V = V_0 \cos \omega t$$

起始电量：

$$\frac{q}{c} = V_0 \cos \omega t$$

$$q = CV_0 \cos \omega t$$

$$I = \frac{dq}{dt} = -CV_0 \omega \sin \omega t$$



$$V = V_0 \cos \omega t$$

$S_1:$

$$\oint_C \vec{H} \cdot d\vec{l} = I + \int_{S_1} \frac{\partial \vec{D}}{\partial \tau} \cdot \hat{u}_N ds$$

$$I = -CV_0\omega \sin \omega t$$

$$\vec{B} = 0$$

$$\oint_c \vec{H} \cdot d\vec{l} = -CV_0\omega \sin \omega t = -\omega \frac{\epsilon S}{d} V_0 \sin \omega t$$

$$S_2: \oint_c \vec{H} \cdot d\vec{l} = I + \int_{S_2} \frac{\partial \vec{D}}{\partial \tau} \cdot \hat{u}_N ds$$

$$I = 0$$

$$\oint \vec{E} \cdot d\vec{l} = V$$

$$\Rightarrow Ed = V_0 \cos \omega t$$

$$\Rightarrow E = \frac{V_0}{d} \cos \omega t$$

$$\Rightarrow D = \epsilon E = \frac{\epsilon V_0}{d} \cos \omega t$$

$$\frac{\partial D}{\partial \tau} = -\frac{\epsilon V_0 \omega}{d} \sin \omega t$$

$$\int_{S_2} \frac{\partial \vec{D}}{\partial \tau} \cdot \hat{u}_N ds$$

$$= -\frac{\epsilon V_0 \omega}{d} \sin \omega t S$$

$$= -\omega \frac{\epsilon S}{d} V_0 \sin \omega t$$

$$\Rightarrow \oint_c \vec{H} \cdot d\vec{l} = -\omega \frac{\epsilon S}{d} V_0 \sin \omega t$$

Zgq !!.

PB00

(문제 93)

$$(a) \vec{H} = 0.15 \cos [3.12(3 \times 10^8 t - y)] \hat{x} \text{ (A/m)}$$

$$\vec{J}_d = \vec{\nabla} \times \vec{H}$$

$$= -0.15 \times 3.12 \sin [3.12(3 \times 10^8 t - y)] \hat{z}$$

$$\text{진폭} = 0.15 \times 3.12 = 0.468 \text{ (A/m²)}$$

$$(b) \vec{B} = 0.8 \cos [1.257 \times 10^{-6} (3 \times 10^8 t - x)] \hat{y} \text{ (T)}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{0.8}{\mu_0} \cos [1.257 \times 10^{-6} (3 \times 10^8 t - x)] \hat{y}$$

$$\vec{J}_d = \vec{\nabla} \times \vec{H}$$

$$= \hat{z} \frac{0.8}{\mu_0} \times 1.257 \times 10^{-6} \sin []$$

$$\text{진폭} = \frac{0.8 \times 1.257 \times 10^{-6}}{4\pi \times 10^{-7}} = 0.800231 \text{ (A/m²)}$$

(c) $\epsilon_r = 5$

$$\vec{E} = 0.9 \times 10^6 \cos [1.257 \times 10^6 (3 \times 10^8 t - \sqrt{5} z)] \hat{x} \text{ (V/m)}$$

$$\vec{D} = \epsilon \vec{E}$$

$$= \epsilon_r \epsilon_0 \vec{E}$$

$$\epsilon_0 = 8.84 \times 10^{-12} \text{ (C}^2/\text{Nm}^2)$$

$$= \epsilon_r \epsilon_0 \times 0.9 \times 10^6 \cos [1.257 \times 10^6 (3 \times 10^8 t - \sqrt{5} z)] \hat{x} \text{ (C/m}^2\text{)}$$

$$\vec{J}_d = \frac{\partial \vec{B}}{\partial t} = - \frac{\epsilon_r \epsilon_0 \times 0.9 \times 10^6 \times 1.257 \times 10^6 \times 3 \times 10^8}{0.0150} \sin [] \hat{x}$$

$$\text{진폭} = 0.0150 \text{ (A/m}^2\text{)}$$

$$(d) \epsilon = \epsilon_0, \mu = \mu_0, \sigma = 5.8 \times 10^7 \text{ (1/2m)}$$

$$\vec{J} = 10^6 \sin(377t - 117.1z) \hat{x} \text{ (A/m}^2\text{)}$$

$$\vec{J} = \sigma \vec{E}$$

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{10^6}{\sigma} \sin(377t - 117.1z) \hat{x} \text{ (V/m)}$$

$$\vec{D} = \epsilon_0 \vec{E} = \frac{\epsilon_0}{\sigma} \times 10^6 \sin(377t - 117.1z) \hat{x} \text{ (C/m}^2\text{)}$$

$$\vec{J}_d = \frac{\partial \vec{B}}{\partial t} = \frac{\epsilon_0}{\sigma} \times 377 \times 10^6 \cos(377t - 117.1z) \hat{x} \text{ (A/m}^2\text{)}$$

$$\text{진폭} = \frac{\epsilon_0}{\sigma} \times 377 \times 10^6 = 57.5 \times 10^{-12} \text{ (A/m}^2\text{)}$$

$$= 57.5 \text{ (PA/m}^2\text{)}$$

**

§ Maxwell Equations

$$\vec{\nabla} \cdot \vec{D} = \rho \quad \text{electric Gauss law}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{magnetic "}$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \text{Faraday "}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t} \quad \text{Maxwell-Ampere "}$$

Maxwell Equation

$$* \quad \vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E} \quad (\vec{P} = \chi_e \epsilon_0 \vec{E})$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu \vec{H} \quad (\vec{M} = \chi_m \vec{H})$$

$$\vec{J} = \sigma \vec{E} = \rho \vec{v}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

P302

(으뜸 9.4)

$$\mu = 10^{-5} \text{ (H/cm)} \quad \epsilon = 4 \times 10^{-9} \text{ (F/cm)} \quad \sigma = 0 \quad \rho = 0$$

(a) $\vec{D} = 10^{-9} \times (6\hat{x} - 2y\hat{y} + 2z\hat{z}) \text{ (C/m}^2\text{)}$

$$\vec{H} = kx\hat{x} + 10y\hat{y} - 25z\hat{z} \text{ (A/m)}$$

$$[k] = \frac{\text{A}}{\text{m}} \cdot \frac{1}{[\epsilon]} = \text{A/m}^2$$

$$\vec{D} \cdot \vec{H} = 0$$

$$k = 15 \text{ (A/m}^2\text{)}$$

(b) $\vec{E} = (20y - kx)\hat{x} \text{ (V/m)} \quad \vec{H} = (y + 2 \times 10^6 z)\hat{z} \text{ (A/m)}$

$$[k] = \frac{\text{V}}{\text{m}} \cdot \frac{1}{[\epsilon]} = \text{V/msec}$$

$$\vec{D} \times \vec{H} = \hat{z}$$

$$\vec{D} = \epsilon \vec{E} = \epsilon (20y - kx)\hat{x}$$

$$\frac{\partial \vec{D}}{\partial x} = -k\epsilon \hat{z}$$

$$\text{From } \vec{D} \times \vec{H} = \frac{\partial \vec{D}}{\partial x},$$

$$-k\epsilon = 1$$

$$k = -\frac{1}{\epsilon} = -\frac{1}{4 \times 10^{-9}} = -2.5 \times 10^{10} \text{ (V/msec)} \times$$

8 정부정 Maxwell Equation

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$$\oint_S \vec{D} \cdot \hat{u}_N ds = Q_{\text{inside}}$$

자기 Gauss 법칙

$$\oint_S \vec{B} \cdot \hat{u}_N ds = 0$$

자기 "

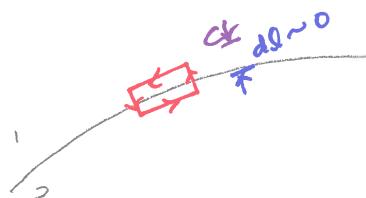
$$\oint_C \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_S \vec{B} \cdot \hat{u}_N ds$$

Faraday "

$$\oint_C \vec{H} \cdot d\vec{l} = I_{\text{inside}} + \frac{d}{dt} \int_S \vec{D} \cdot \hat{u}_N ds$$

Maxwell-Ampere "

정부정 Maxwell Equation

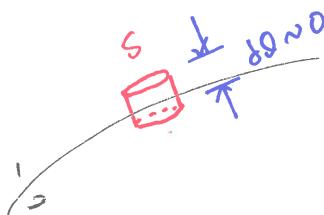


Faraday 법칙 사용하면 $E_{1,\parallel} = E_{2,\parallel}$

p140 (22)

Maxwell-Ampere 법칙을 사용 ($K=0$), $H_{1,\parallel} = H_{2,\parallel}$ p264 (25)

$$H_{1,\parallel} - H_{2,\parallel} = K$$



자기 Gauss 법칙을 사용하면 $B_{1,\perp} = B_{2,\perp}$

p264 (22)

자기 Gauss 법칙을 사용하면 $D_{1,\perp} - D_{2,\perp} = \sigma$

p141 (34)

* 안전도체 ($\sigma(\text{전도도}) = \infty$) : \vec{J} 자유한

$$\vec{J} = \sigma \vec{E}$$

$\vec{E} = 0$: 안전도체 내부에서 $\vec{E} = 0$ 이다.

$$\vec{B} = 0 : " \quad \vec{B} = 0 " \quad "$$

From $\vec{\nabla} \cdot \vec{B} = \rho$

$$\rho = 0 : " \quad \text{부피전하밀도가 } 0 \text{ 이다.}$$

\Rightarrow 전하가 항상 표면에 있다.

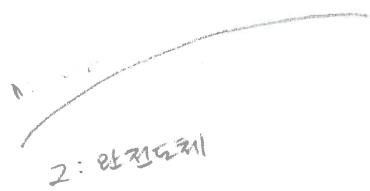
From $\vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\vec{B} = \vec{H} = 0 : " \quad \vec{B} = \vec{H} = 0 \quad \text{이다.}$$

From $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}$

$$\vec{J} = 0 : " \quad \text{부피전류밀도는 } 0 \text{ 이다.}$$

\Rightarrow 전류는 항상 표면으로 흐른다. (Skin effect)



$$E_{\perp\parallel} = 0$$

$$H_{\perp\parallel} = \kappa \quad \left\{ \begin{array}{l} \vec{H}_{\perp\parallel} = \vec{k} \times \hat{u}_N \\ \hat{u}_N : \text{면에 표면 밖으로} \text{ 향하는} \\ \text{단위 법선 vector} \end{array} \right.$$

$$D_{\perp\parallel} = \sigma$$

$$B_{\perp\parallel} = 0$$

(※ 용예제 9.5)

$$\hat{u}_N = 0.64\hat{x} + 0.6\hat{y} - 0.48\hat{z}$$

↑

1 $\epsilon_{r,1}=4, \mu_{r,1}=2, \sigma_1=0$

2 $\epsilon_{r,2}=2, \mu_{r,2}=3, \sigma_2=0$

$$\vec{B}_1 = (\hat{x} - 2\hat{y} + 3\hat{z}) \sin(300t) \text{ (T)}$$

(a) $\vec{B}_{1,\perp} = (\vec{B}_1 \cdot \hat{u}_N) \hat{u}_N$

$$= -2 \sin(300t) (0.64\hat{x} + 0.6\hat{y} - 0.48\hat{z})$$

$$|\vec{B}_{1,\perp} \text{의 진폭}| = \rightarrow \text{(T)}$$

(b) $\vec{B}_{1,\parallel} = \vec{B}_1 - \vec{B}_{1,\perp}$

$$= (2.28\hat{x} - 0.8\hat{y} + 2.04\hat{z}) \sin(300t)$$

$$|\vec{B}_{1,\parallel} \text{의 진폭}| = \sqrt{2.28^2 + 0.8^2 + 2.04^2} = 3.16 \text{ (T)}$$

(c) $\vec{B}_{2,\perp} = \vec{B}_{1,\perp} = -2 \sin(300t) (0.64\hat{x} + 0.6\hat{y} - 0.48\hat{z})$

$$|\vec{B}_{2,\perp} \text{의 진폭}| = \rightarrow \text{(T)}$$

$$(d) \quad \vec{H}_{z,||} = \vec{H}_{y,||} \quad (\vec{K} = 0)$$

$$\frac{1}{\mu_2} \vec{B}_{z,||} = \frac{1}{\mu_1} \vec{B}_{y,||}$$

$$\vec{B}_{z,||} = \frac{\mu_2}{\mu_1} \vec{B}_{y,||}$$

$$= \frac{3}{2} (2.28 \hat{x} - 0.8 \hat{y} + 2.04 \hat{z}) \sin(300t)$$

$$\vec{B}_z = \vec{B}_{z,\perp} + \vec{B}_{z,||}$$

$$= (2.14 \hat{x} - 2.4 \hat{y} + 4.02 \hat{z}) \sin(300t)$$

$$|\vec{B}_z| = \sqrt{2.14^2 + 2.4^2 + 4.02^2} = 5.15 \text{ (T)}$$

(응용 예제 9.6)

$$\epsilon_r = 5,$$

$$\mu_r = 3,$$

$$\sigma = 0$$

$$\epsilon_0 = 8.84 \times 10^{-12} \text{ (C}^2/\text{Nm}^2\text{)}$$



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$$t = 6 \times 10^9 \text{ (sec)} \quad P(2, 0, 0.3) : \text{포인트}$$

$$\vec{E} = 20 \cos[(2 \times 10^8)t - 2.58z] \hat{j} \text{ (V/m)}$$

$$(a) \vec{B} = \epsilon \vec{E}$$

$$= 20 \epsilon_r \epsilon_0 \cos[(2 \times 10^8)t - 2.58z] \hat{j} \text{ (C/m}^2\text{)}$$

$$\sigma = D_{z,z}$$

$$= 20 \epsilon_r \epsilon_0 \cos[(2 \times 10^8)t - 2.58z]$$

$$= 20 \times 5 \times 8.84 \times 10^{-12} \cos[1.2 - 0.3 \times 2.58]$$

$$= 0.805 \times 10^{-9} \text{ (C/m}^2\text{)}$$

$$= 0.805 \text{ (mC/m}^2\text{)}$$

$$(b) \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -20 \times 2.58 \sin[(2 \times 10^8)t - 2.58z] \hat{x}$$

$$\Rightarrow \frac{\partial \vec{H}}{\partial t} = \frac{-20 \times 2.58}{\mu} \sin[(2 \times 10^8)t - 2.58z] \hat{x}$$

$$\Rightarrow \vec{H} = - \frac{\frac{20 \times 2.58}{2 \times 10^8 \times \mu}}{0.0624} \cos[(2 \times 10^8)t - 2.58z] \hat{x}$$

$$= -0.0624 \cos(1.2 - 0.3 \times 2.58) \hat{x}$$

$$= -0.0623 \hat{x} \text{ (A/m)}$$

$$= -62.3 \hat{x} \text{ (m A/m)}$$

$$(c) \vec{H}_{\perp, \parallel}$$

$$= -62.3 \hat{x} = \vec{k} \times \hat{y}$$

$$\vec{k} = 62.3 \hat{z} \text{ (mA/m)}$$

p204

↳ retarded electric and magnetic potentials

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = -\vec{\nabla} \times (\vec{\nabla} V) = 0$$

From Maxwell Eq.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \vec{E} = -\vec{\nabla} V \text{ is something wrong !!}$$

$$\vec{E} = -\vec{\nabla} V + \vec{\mathcal{N}}$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = \vec{\nabla} \times \vec{\mathcal{N}} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{A}) = \vec{\nabla} \times \left(-\frac{\partial \vec{A}}{\partial t} \right)$$

$$\Rightarrow \vec{\mathcal{N}} = -\frac{\partial \vec{A}}{\partial t}$$

$$\Rightarrow \boxed{\begin{aligned} \vec{E} &= -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \\ \vec{B} &= \vec{\nabla} \times \vec{A} \end{aligned}}$$

note) $\{V, \vec{A}\}$ 은 미지 $\{\vec{E}, \vec{B}\}$ 를 구할 수 있다.

* gauge symmetry

$$\vec{A}' = \vec{A} - \vec{\nabla} \Lambda, \quad v' = v + \frac{\partial \Lambda}{\partial t} \approx \text{같은 } \vec{E} \text{와 } \vec{B} \text{ 는 같다}$$

즉 같은 \vec{E} 와 \vec{B} 를 주는 $\{v, \vec{A}\}$ 는 유일하지 않다.

(증명)

$$\vec{B}' = \vec{\nabla} \times \vec{A}'$$

$$= \vec{\nabla} \times (\vec{A} - \vec{\nabla} \Lambda)$$

$$= \vec{\nabla} \times \vec{A} - \underline{\vec{\nabla} \times (\vec{\nabla} \Lambda)} \stackrel{=} 0$$

$$= \vec{\nabla} \times \vec{A}$$

$$= \vec{B}$$

$$\vec{E}' = -\vec{\nabla} v' - \frac{\partial \vec{A}'}{\partial t}$$

$$= -\vec{\nabla} \left(v + \frac{\partial \Lambda}{\partial t} \right) - \frac{\partial}{\partial t} (\vec{A} - \vec{\nabla} \Lambda)$$

$$= -\vec{\nabla} v - \frac{\partial \vec{A}}{\partial t} + \underline{\left(-\vec{\nabla} \frac{\partial \Lambda}{\partial t} + \frac{\partial}{\partial t} \vec{\nabla} \Lambda \right)} \stackrel{=} 0$$

$$= -\vec{\nabla} v - \frac{\partial \vec{A}}{\partial t}$$

$$= \vec{E}$$

\times

* 그러므로 $\{v, \vec{A}\}$ 를 유일하게 결정하기 위해서는

1번 추가 조건이 더 필요하다. (gauge fixing).

Maxwell Eq:

$$\vec{\nabla} \cdot \vec{B} = \rho, \quad \vec{\nabla} \cdot \vec{E} = 0, \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}$$

$$\textcircled{1} \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0 \quad \text{vector identity}$$

$$\textcircled{2} \quad \vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{D} = \epsilon \vec{\nabla} \cdot \vec{E}$$

$$= \epsilon \vec{\nabla} \cdot \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right)$$

$$= \epsilon \left[-\frac{\vec{\nabla} \cdot (\vec{\nabla} V)}{\vec{\nabla} \cdot \vec{V}} - \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A}) \right] = \rho$$

$$\Rightarrow \vec{\nabla} V + \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon}$$

$$\textcircled{3} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right)$$

$$= -\frac{\partial}{\partial t} \cancel{(\vec{\nabla} V)} - \frac{\partial}{\partial t} \cancel{(\vec{\nabla} \times \vec{A})}$$

$$= -\frac{\partial}{\partial t} \vec{B}$$

$$\textcircled{4} \quad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{\mu} \vec{\nabla} \times \vec{B}$$

$$= \frac{1}{\mu} \underline{\vec{\nabla} \times (\vec{\nabla} \times \vec{A})}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}$$

$$= \frac{1}{\mu} [\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}]$$

$$\frac{\partial \vec{B}}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t} = \epsilon \frac{\partial}{\partial t} \left[-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right]$$

$$= -\epsilon \frac{\partial}{\partial t} \left[\vec{\nabla} V + \frac{\partial \vec{A}}{\partial t} \right]$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A} = -\mu \epsilon \left[\vec{\nabla} \frac{\partial V}{\partial t} + \frac{\partial^2 \vec{A}}{\partial t^2} \right] + \mu \vec{J}$$

그러므로 Maxwell 방정식을 만족하기 위해서는

$$\vec{\nabla}^2 V + \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A} = -\mu \epsilon \left[\vec{\nabla} \frac{\partial V}{\partial t} + \frac{\partial^2 \vec{A}}{\partial t^2} \right] + \mu \vec{J}$$

을 만족해야 한다.

Gauge Fixing

$$\vec{\nabla} \cdot \vec{A} = -\mu \epsilon \frac{\partial V}{\partial t}$$

$$\Rightarrow \vec{\nabla}^2 V = \mu \epsilon \frac{\partial^2 V}{\partial t^2} - \frac{\rho}{\epsilon}$$

$$\vec{\nabla}^2 \vec{A} = \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2} - \mu \vec{J}$$

(i) $\frac{\rho}{\epsilon}, \mu \vec{J}$: source term

(ii) $\rho = \vec{J} = 0$ case

$$\vec{\nabla}^2 V = \mu \epsilon \frac{\partial^2 V}{\partial t^2}$$

$$\vec{\nabla}^2 \vec{A} = \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

파동 방정식

$$\vec{\nabla}^2 f = \frac{1}{\mu \epsilon} \frac{\partial^2 f}{\partial t^2}$$

N : 파동의 속도

전자파의 속도

$$N = \sqrt{\frac{1}{\mu \epsilon}}$$

진공

$$N = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ (m/sec)} = c \quad (\text{광속})$$

$$\Rightarrow \text{전자파} = 빛$$

* 전자파의 진행 속도가 유한하므로 source에서 각 진동에
 \vec{V} 와 \vec{A} 를 측정하는데 시간이 걸린다.

이 문제를 해결하기 위하여 retarded potential을 정의한다.

$$V_{\text{ret}}[\vec{r}, t] = V[\vec{r}, t - \frac{r}{c}]$$

r: source와 P점 사이의 거리

$$\vec{A}_{\text{ret}}[\vec{r}, t] = \vec{A}[\vec{r}, t - \frac{r}{c}]$$

retarded potential

그리므로 P점의 source에서 멀거나 가까운 source가 time-dependent 하면

$$\vec{E} = -\vec{\nabla}V_{\text{ret}} - \frac{\partial \vec{A}_{\text{ret}}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}_{\text{ret}}$$

을 사용하여 P점의場을 정의하자.

(응용대지 9.7)

$$P_+(0, 0, 1.5) \quad g_+(t) = 4 \cos(10^8 \pi t) \times 10^{-6} \text{ (c)}$$

$$P_-(0, 0, -1.5) \quad g_-(t) = -4 \cos(10^8 \pi t) \times 10^{-6} \text{ (c)}$$

$$t = 15 \times 10^{-9} \text{ (sec)}$$

$$\textcircled{(1)} \quad P(r=450, \theta=0, \varphi=0) = P(0, 0, 450)$$

$$r_+ = \overline{P_+ P} = 448.5 \quad r_- = \overline{P_- P} = 451.5$$

$$V_{\text{ret}}(t) = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r_+} g_+(t - \frac{r_+}{c}) + \frac{1}{r_-} g_-(t - \frac{r_-}{c}) \right]$$

$$V(t) = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r_+} g_+(t) + \frac{1}{r_-} g_-(t) \right]$$

$$\textcircled{} \quad V_{\text{ret}}(t = 15 \times 10^{-9}) = 160.002 \text{ (V)}$$

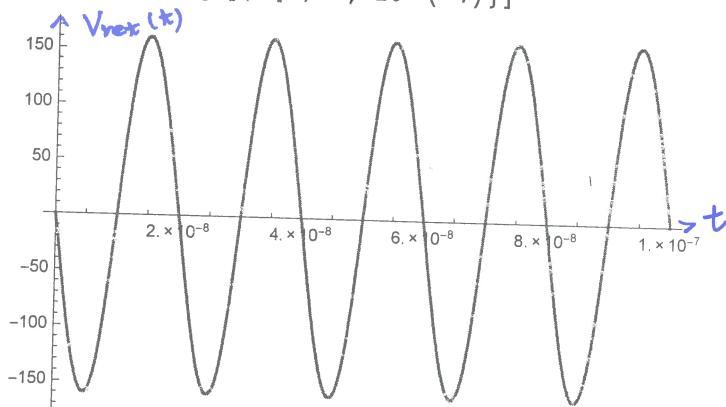
$$V(t = 15 \times 10^{-9}) = 0$$

```

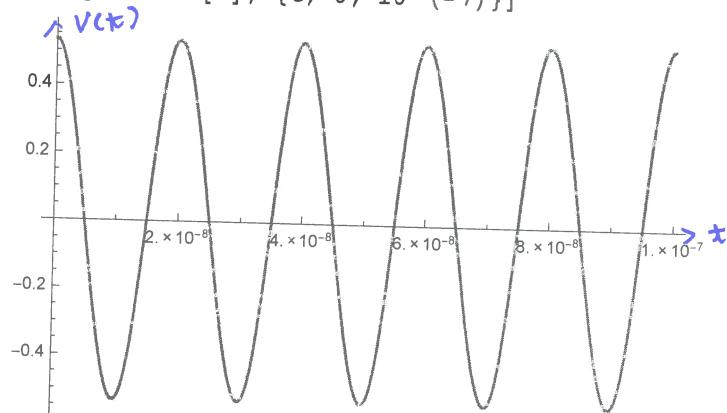
rp = 448.5; rm = 451.5; c = 3 × 10^8;
qp[t_] := 4 × 10^(-6) Cos[10^8 Pi t];
qm[t_] := -qp[t];
vretard[t_] := 9 × 10^9 (qp[t - rp/c] / rp + qm[t - rm/c] / rm);
vnormal[t_] := 9 × 10^9 (qp[t] / rp + qm[t] / rm);
a = vretard[15 × 10^(-9)];
b = vnormal[15 × 10^(-9)];
Print["retard = ", a]
Print["normal = ", b]
(**Plot[{vretard[t], vnormal[t]}, {t, 0, 10^(-7)}, PlotRange → {0, 1}]**)
retard = 160.002
normal = 0.

```

Plot[Vretard[t], {t, 0, 10⁻⁷}]



Plot[Vnormal[t], {t, 0, 10⁻⁷}]



$$(b) P(r=450, \theta=90^\circ, \phi=0) = P(450, 0, 0)$$

$$r_+ = \sqrt{450^2 + 1.5^2} = 450.002 = r_-$$

$$V_{ret} (t = 15 \times 10^{-9}) = 0$$

$$V(t = 15 \times 10^{-9}) = 0$$

```

rp = 450.002; rm = 450.002; c = 3 × 10^8;
qp[t_] := 4 × 10^(-6) Cos[10^8 Pi t];
qm[t_] := -qp[t];
Vretard[t_] := 9 × 10^9 (qp[t - rp / c] / rp + qm[t - rm / c] / rm);
Vnormal[t_] := 9 × 10^9 (qp[t] / rp + qm[t] / rm);
a = Vretard[15 × 10^(-9)];
b = Vnormal[15 × 10^(-9)];
Print["retard = ", a]
Print["normal = ", b]
retard = 0.
normal = 0.

```

$$\text{Q) } P(r=450, \theta=45^\circ, \phi=0) = P\left(\frac{450}{\sqrt{2}}, 0, \frac{45^\circ}{\sqrt{2}}\right)$$

$$r_+ = \sqrt{\left(\frac{450}{\sqrt{2}}\right)^2 + \left(\frac{450}{\sqrt{2}} - 1.5\right)^2} = 448.941$$

$$r_- = \sqrt{\left(\frac{450}{\sqrt{2}}\right)^2 + \left(\frac{450}{\sqrt{2}} + 1.5\right)^2} = 451.062$$

$$V_{ret} (t = 15 \times 10^{-9}) = 143.351 \text{ (V)}$$

$$V(t = 15 \times 10^{-9}) = 0$$

```

rp = 448.941; rm = 451.062; c = 3 × 10^8;
qp[t_] := 4 × 10^(-6) Cos[10^8 Pi t];
qm[t_] := -qp[t];
Vretard[t_] := 9 × 10^9 (qp[t - rp/c] / rp + qm[t - rm/c] / rm);
Vnormal[t_] := 9 × 10^9 (qp[t] / rp + qm[t] / rm);
a = Vretard[15 × 10^(-9)];
b = Vnormal[15 × 10^(-9)];
Print["retard = ", a]
Print["normal = ", b]
retard = 143.351
normal = 0.

```

X