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Aharonov–Bohm-like scattering in the generalized uncertainty principle-corrected quantum mechanics

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We discuss classical electrodynamics and the Aharonov–Bohm effect in the presence of the minimal length. In the former, we derive the classical equation of motion and the corresponding Lagrangian. In the latter, we adopt the generalized uncertainty principle (GUP) and compute the scattering cross-section up to the first-order of the GUP parameter β. Even though the minimal length exists, the cross-section is invariant under the simultaneous change $\phi \to -\phi$, $\alpha' \to -\alpha'$, where ϕ and α' are azimuthal angle and magnetic flux parameter. However, unlike the usual Aharonv–Bohm scattering, the cross-section exhibits discontinuous behavior at every integer α'. The symmetries, which the cross-section has in the absence of GUP, are shown to be explicitly broken at the level of $\mathcal{O}(\beta)$.

Keywords: Generalized uncertainty principle; Aharonov-Bohm effect.

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1. Introduction

Most theories of quantum gravity predict the existence of a minimal length[1–4] at the Planck scale. It appears as various different expressions in loop quantum gravity,[5,6] string theory,[7,8] path-integral quantum gravity,[9–11] and black hole physics.[12] From the aspect of quantum mechanics the existence of a minimal length results in the modification of the Heisenberg uncertainty principle (HUP)[13,14] $\Delta P \Delta Q \geq \frac{\hbar}{2}$, because ΔQ should be larger than the minimal length. Various modifications of HUP, called the generalized uncertainty principle (GUP), were suggested in Refs. 15 and 16. The GUP has been used to explore the various branches of physics such as micro-black hole,[17] gravity,[18] cosmological constant,[19] and classical central potential problem.[20] It is also used in the low-energy regime[21] and the emergence of

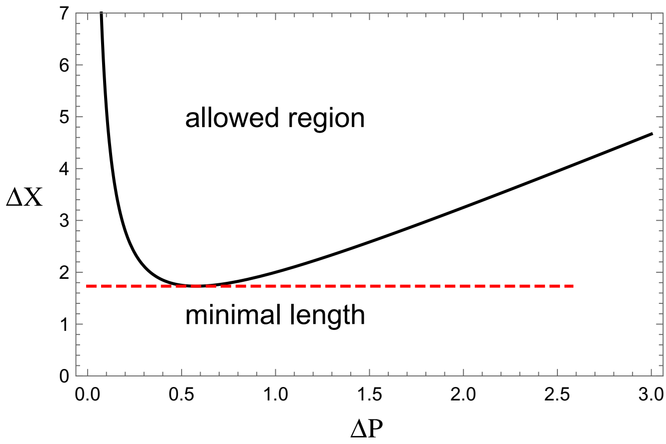

Fig. 1. The minimal length and allowed region of one-dimensional GUP (1.3) when $\hbar = \beta = 1$.

(doubly) special relativity.[22] The experimental detection of GUP was emphasized in Ref. 23, where GUP is directly linked to the deformation of the space–time metric. In this way, the existence of GUP can be experimentally verified by measuring the light deflection and perihelion procession. As we will show in this paper, the existence of GUP also can be verified by measuring the cross-section of the Aharonov–Bohm-like scattering.

In this paper,[a] we will choose the d-dimensional GUP in a form

$$\Delta P_i \Delta X_i \geq \frac{\hbar}{2}[1 + \beta(\Delta \mathbf{P}^2 + \langle \mathbf{P} \rangle^2) + 2\beta(\Delta P_i^2 + \langle P_i \rangle^2)], \quad (i = 1, 2, \ldots, d), \tag{1.1}$$

where β is a GUP parameter, which has a dimension $(\text{momentum})^{-2}$. Using $\Delta A \Delta B \geq \frac{1}{2}|\langle [A, B] \rangle|$, Eq. (1.1) induces the modification of the commutation relations as[b]

$$\begin{aligned} [X_i, P_j] &= i\hbar(\delta_{ij} + \beta \delta_{ij} \mathbf{P}^2 + 2\beta P_i P_j), \\ [X_i, X_j] &= [P_i, P_j] = 0. \end{aligned} \tag{1.2}$$

[a] Although the effect of the presence of a minimal length should be discussed in a relativistic fashion, we will examine it in the nonrelativistic quantum mechanics when the Aharonov–Bohm potential is involved. Therefore, the results presented in this paper should be modified when the relativistic effect is included.

[b] One may wonder whether the commutation relations (1.2) are inconsistent with each other. If, in fact, we impose $[P_i, P_j] = 0$, the Jacobi identity determines $[X_i, X_j]$ in a form

$$[X_i, X_j] = \frac{4\beta^2 \mathbf{P}^2}{1 + \beta \mathbf{P}^2}(P_i X_j - P_j X_i) = \mathcal{O}(\beta^2).$$

Since we will explore the AB-like phenomenon up to first of β, Eq. (1.2) is valid for this reason.

The existence of the minimal length is easily shown at $d = 1$. In this case, Eq. (1.1) is expressed as

$$\Delta P \Delta X \geq \frac{\hbar}{2}(1 + 3\beta \Delta P^2) \tag{1.3}$$

if $\langle P \rangle = 0$. Then, the equality of Eq. (1.3) yields

$$\Delta X^2 \geq \Delta X^2_{\text{min}} = 3\beta\hbar^2. \tag{1.4}$$

In Fig. 1, the allowed region and the minimal length of Eq. (1.3) are plotted when $\hbar = \beta = 1$.

If β is small, Eq. (1.2) can be solved as

$$P_i = p_i(1 + \beta \mathbf{p}^2) + \mathcal{O}(\beta^2), \quad X_i = x_i, \tag{1.5}$$

where p_i and x_i obey the usual HUP. Using Eq. (1.5) and Feynman's path-integral technique,[24,25] the Feynman propagator (or kernel) was exactly derived up to $\mathcal{O}(\beta)$ for $d = 1$ free particle case.[26,27] Also, the propagator for d-dimensional simple harmonic oscillator system was also derived recently in Refs. 28 and 29.

The main purpose of this paper is to examine how the Aharonov–Bohm (AB) effect[30,31] is modified when the GUP (1.1) is introduced. The AB effect is a pure quantum mechanical phenomenon, which predicts that the electromagnetic vector potential plays a role of observable at the quantum level when the charged particle moves around an infinitely thin magnetic flux tube. The experimental realization of this effect was discussed in Ref. 32. The effect of the particle spin in the AB-scattering was examined a few years ago in Refs. 31, 33 and 34. In particular, when the spin is 1/2, the corresponding Schrödinger-like equation derived from Dirac equation involves the δ-function potential[31,35] as a Zeeman interaction. In order to make the theory finite a mathematically-oriented self-adjoint extension[36] or the physically-oriented renormalization[37] can be adopted. The equivalence of both methods was discussed in Refs. 35 and 38.

The paper is organized as follows. In Sec. 2, we derive the classical equation of motion up to $\mathcal{O}(\beta)$ by making use of the Poisson bracket formalism when the minimal length (1.4) exists. Also, the classical Lagrangian is explicitly derived in this section. In Sec. 3, we discuss the AB-scattering in the presence of GUP (1.5). Unlike the usual AB-effect with HUP, it is shown that the irregularity at the origin cannot be avoided because of the effect of GUP. The scattering cross-section is shown to be discontinuous at every integer $\alpha' = \alpha/\hbar$, where α is a magnetic flux parameter. The various symmetries of the cross-section in the usual AB-scattering are explicitly broken. In Sec. 4, a brief conclusion is given.

2. Classical Electrodynamics in the Presence of Minimal Length

In this section, we discuss how the classical electrodynamics is modified if the minimal length (1.4) exists. We start with a classical Hamiltonian

$$H_{\text{cl}} = \frac{1}{2M}(\boldsymbol{P} - q\boldsymbol{A})^2 + qV = H_{0,\text{cl}} + \frac{\beta}{M}\boldsymbol{p}^2(\boldsymbol{p} - q\boldsymbol{A}) \cdot \boldsymbol{p} + \mathcal{O}(\beta^2), \tag{2.1}$$

where $\boldsymbol{A}$ and V are the vector and scalar potentials, and $H_{0,\mathrm{cl}}$ is the classical Hamiltonian when there is no minimal length, which is explicitly expressed by

$$H_{0,\mathrm{cl}} = \frac{1}{2M}(\boldsymbol{p} - q\boldsymbol{A})^2 + qV. \tag{2.2}$$

In Eq. (2.1), we used Eq. (1.5). Of course, we have not considered the ordering problem of $\boldsymbol{p}$ and $\boldsymbol{x}$ because we deal with the classical Hamiltonian.

In order to derive a classical equation of motion, we use the Poisson bracket

$$\dot{x}_j \equiv \{H_{\mathrm{cl}}, x_j\} = \frac{\partial H_{cl}}{\partial p_j}\frac{\partial x_j}{\partial x_j} - \frac{\partial H_{cl}}{\partial x_j}\frac{\partial x_j}{\partial p_j}, \tag{2.3}$$

which yields

$$M\dot{\boldsymbol{x}} = \boldsymbol{p} - q\boldsymbol{A} + \beta[4\boldsymbol{p}^2\boldsymbol{p} - 2q(\boldsymbol{A}\cdot\boldsymbol{p})\boldsymbol{p} - q\boldsymbol{p}^2\boldsymbol{A}] + \mathcal{O}(\beta^2). \tag{2.4}$$

Also, one can compute $\dot{p}_j = \{H_{\mathrm{cl}}, p_j\}$, which gives

$$\dot{p}_j = \left[\frac{q}{M}(\boldsymbol{p} - q\boldsymbol{A})\cdot\frac{\partial \boldsymbol{A}}{\partial x_j} - q\frac{\partial V}{\partial x_j}\right] + \frac{\beta q}{M}\boldsymbol{p}^2\left(\frac{\partial \boldsymbol{A}}{\partial x_j}\cdot\boldsymbol{p}\right) + \mathcal{O}(\beta^2). \tag{2.5}$$

Combining Eqs. (2.4) and (2.5) with long and tedious calculation, it is possible to derive

$$M\ddot{x}_j = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})_j + \beta q\Gamma_j + \mathcal{O}(\beta^2), \tag{2.6}$$

where $\boldsymbol{E} = -\boldsymbol{\nabla}V - \frac{\partial}{\partial t}\boldsymbol{A}$ and $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$, which are the usual electric and magnetic fields. The correction term Γ_j at the first-order of β is expressed as

$$\begin{aligned}\Gamma_j = {}& \boldsymbol{H}_{11}^2(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})_j + 2H_{11,j}(\boldsymbol{H}_{11}\cdot\boldsymbol{E}) \\ & - 2mv_j(\boldsymbol{H}_{32}\cdot\boldsymbol{\nabla}V) - 2qA_j(\boldsymbol{H}_{21}\cdot\boldsymbol{\nabla}V) \\ & - \frac{\partial V}{\partial x_j}(\boldsymbol{H}_{11}\cdot\boldsymbol{H}_{31}) + 2qH_{32,j}(\boldsymbol{A}\cdot(\boldsymbol{v}\times\boldsymbol{B})) + 2mH_{32,j}(\boldsymbol{G}\cdot\boldsymbol{v}) \\ & + 2qH_{21,j}(\boldsymbol{G}\cdot\boldsymbol{A}) - q(\boldsymbol{v}\cdot\boldsymbol{H}_{11})\frac{\partial}{\partial x_j}\boldsymbol{A}^2,\end{aligned} \tag{2.7}$$

where

$$\boldsymbol{H}_{ab} = am\boldsymbol{v} + bq\boldsymbol{A}, \quad \boldsymbol{G} = \sum_i v_i\frac{\partial \boldsymbol{A}}{\partial x_i}. \tag{2.8}$$

The equation of motion (2.6) can be derived as a Euler–Lagrange equation from the Lagrangian

$$L(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \frac{1}{2}m\dot{\boldsymbol{x}}^2 + q\dot{\boldsymbol{x}}\cdot\boldsymbol{A} - qV - \beta\boldsymbol{H}_{11}^2(\dot{\boldsymbol{x}}\cdot\boldsymbol{H}_{11}) + \mathcal{O}(\beta^2), \tag{2.9}$$

where $\boldsymbol{v}$ in $\boldsymbol{H}_{ab}$ should be replaced by $\dot{\boldsymbol{x}}$ in Eq. (2.9). Unlike the classical equation of motion in the absence of the minimal length, the scalar and vector potentials explicitly appear in Eq. (2.6) at the first-order of β. This means that if the minimal length exists, the potentials V and $\boldsymbol{A}$ are not merely mathematical tools for the derivation of $\boldsymbol{E}$ and $\boldsymbol{B}$ even at the classical level. Of course, the classical equation of motion (2.6) indicates that the usual gauge symmetry $\mathbf{A} \to \mathbf{A} + \frac{1}{q}\nabla\Lambda$ and $V \to V - \frac{1}{q}\frac{\partial\Lambda}{\partial t}$ does not hold at the classical level. However, one can show that this theory has a modified symmetry up to $\mathcal{O}(\beta)$ in the form

$$\mathbf{A} \to \mathbf{A} + \frac{1}{q}(\nabla\Lambda + \beta\mathbf{F}), \quad V \to V - \frac{1}{q}\left(\frac{\partial\Lambda}{\partial t} + \beta F_0\right), \tag{2.10}$$

where $\mathbf{F}$ and F_0 satisfy

$$\mathbf{F} - 2(\mathbf{H}_{11}\cdot\nabla\Lambda)\mathbf{H}_{11} - \mathbf{H}_{11}^2\nabla\Lambda \equiv \nabla\Lambda_1, \quad F_0 = \frac{\partial\Lambda_1}{\partial t}. \tag{2.11}$$

Under the transformation, the Lagrangian (2.9) transforms $L \to L + \frac{d}{dt}(\Lambda + \beta\Lambda_1)$. Of course, the symmetry (2.10) reduces to the usual gauge symmetry at $\beta = 0$. However, this symmetry is completely different from the usual one because the vector $\mathbf{H}_{11}$ contains not only particle's velocity but also the vector potential itself. The Lagrangian (2.9) can be used to explore the quantum electrodynamics in the presence of the minimal length by applying the path-integral technique.[24,25]

3. AB-Like Phenomena with GUP

In this section, we examine how the AB effect[30] is modified when the GUP (1.5) is introduced. The Hamiltonian with AB system can be written as

$$\begin{aligned}\hat{H} &= \frac{1}{2M}(\boldsymbol{P} - e\boldsymbol{A})^2 \\ &= \frac{1}{2M}[(1+\beta\boldsymbol{p}^2)\boldsymbol{p} - e\boldsymbol{A}]^2 + \mathcal{O}(\beta^2) = \hat{H}_0 + \beta\hat{H}_1 + \mathcal{O}(\beta^2),\end{aligned} \tag{3.1}$$

where e is an particle charge and

$$\hat{H}_0 = \frac{1}{2M}(\boldsymbol{p} - 2\boldsymbol{A})^2 \quad \hat{H}_1 = \frac{1}{M}\left[(\boldsymbol{p}^2)^2 - \frac{e}{2}\{(\boldsymbol{A}\cdot\boldsymbol{p})\boldsymbol{p}^2 + \boldsymbol{p}^2(\boldsymbol{p}\cdot\boldsymbol{A})\}\right]. \tag{3.2}$$

If we represent the energy eigenvalue in terms of the wave number as $E = E_0 + \beta E_1 + \mathcal{O}(\beta^2)$ with $E_0 = \hbar^2k^2/(2M)$ and $E_1 = \hbar^4k^4/M$, it is straightforward to show that the Schrödinger equation $\hat{H}\psi = E\psi$ can be written as

$$\begin{aligned}(-i\hbar\boldsymbol{\nabla} - e\boldsymbol{A})^2\psi + 2\beta\hbar^4\left[(\boldsymbol{\nabla}^2)^2 - \frac{ie}{2\hbar}(\boldsymbol{A}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}^2 + \boldsymbol{\nabla}^2\boldsymbol{\nabla}\cdot\boldsymbol{A})\right]\psi \\ + \mathcal{O}(\beta^2) = (\hbar^2k^2 + 2\beta\hbar^4k^4)\psi.\end{aligned} \tag{3.3}$$

We assume that there is a thin magnetic flux tube along the z-axis, which gives the vector potential in the form

$$eA_i = \frac{\alpha\epsilon_{ij}x_j}{r^2}, \tag{3.4}$$

where ϵ_{ij} is an antisymmetric tensor with $\epsilon_{01} = -\epsilon_{10} = 1$. In the usual electromagnetic theory, the choice of the vector potential (3.4) is not unique due to the gauge symmetry. Thus, the choice of Eq. (3.4) corresponds to the Coulomb gauge

$\nabla \cdot \mathbf{A} = 0$. However, the gauge symmetry is modified to Eq. (2.10) for our case, which contains the particle's velocity. Thus, our results presented in the paper are valid only for the particular choice of $\mathbf{A}$ given in Eq. (3.4). Then, the corresponding magnetic field is $eB = -(\alpha/r)\delta(r)$. Using Eq. (3.4) explicitly, one can show

$$\boldsymbol{\nabla}^2 \boldsymbol{\nabla} \cdot (e\boldsymbol{A}) = (e\boldsymbol{A}) \cdot \boldsymbol{\nabla}\boldsymbol{\nabla}^2 + \frac{4\alpha}{r^3}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\frac{\partial}{\partial \phi}. \tag{3.5}$$

Then, the Schrödinger equation (3.3) reduces to

$$\begin{aligned}&\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\left(\frac{\partial}{\partial \phi} + i\alpha'\right)^2 + k^2\right]\psi - 2\beta\hbar^2 \\ &\quad \times \left[(\boldsymbol{\nabla}^2)^2 + \frac{i\alpha'}{r^2}\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{2}{r^2}\right)\frac{\partial}{\partial \phi} - k^4\right]\psi + \mathcal{O}(\beta^2) = 0,\end{aligned} \tag{3.6}$$

where $\alpha' = \alpha/\hbar$. One can show that the Schrödinger equation (3.6) is invariant under the simultaneous operations $\alpha \to -\alpha$, $\phi \to -\phi$.

If one imposes

$$\psi(\boldsymbol{r}) = \sum_{m=-\infty}^{\infty} e^{im\phi} f_m(r), \tag{3.7}$$

the radial equation of (3.6) can be written as

$$(\widehat{S}_1 - 2\beta\hbar^2 \widehat{S}_2) f_m(r) + \mathcal{O}(\beta^2) = 0, \tag{3.8}$$

where

$$\begin{aligned}\widehat{S}_1 &= \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{(m+\alpha')^2}{r^2} + k^2, \\ \widehat{S}_2 &= \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{m^2 - \alpha'(m+\alpha')}{r^2} - k^2\right)\widehat{S}_1 \\ &\qquad - \frac{2\alpha'(3m+2\alpha')}{r^2}\left(\frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} + \frac{k^2}{2}\right) + \frac{\alpha'^2(m+\alpha')(2m+\alpha')}{r^4}.\end{aligned} \tag{3.9}$$

The symmetry of the simultaneous operations $\alpha \to -\alpha$, $\phi \to -\phi$ is represented in the radial equation as the simultaneous changes $\alpha \to -\alpha$, $m \to -m$. If we set

$$f_m(r) = f_{0,m}(r) + \beta f_{1,m}(r) + \mathcal{O}(\beta^2), \tag{3.10}$$

within $\mathcal{O}(\beta)$ the radial equation is represented as the following two equations:

$$\begin{aligned}&\widehat{S}_1 f_{0,m}(r) = 0, \\ &\widehat{S}_1 f_{1,m}(r) = 2\hbar^2\left[-\frac{2\alpha'(3m+2\alpha')}{r^2}\left(\frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} + \frac{k^2}{2}\right)\right. \\ &\qquad\qquad\qquad \left. + \frac{\alpha'^2(m+\alpha')(2m+\alpha')}{r^4}\right] f_{0,m}(r).\end{aligned} \tag{3.11}$$

The general solutions of Eq. (3.11) are

$$\begin{aligned} f_{0,m}(r) &= A_m J_{|m+\alpha'|}(z) + B_m J_{-|m+\alpha'|}(z), \\ f_{1,m}(r) &= C_m J_{|m+\alpha'|}(z) + D_m J_{-|m+\alpha'|}(z) \\ &\quad + u_m(z) J_{|m+\alpha'|}(z) + v_m(z) J_{-|m+\alpha'|}(z), \end{aligned} \tag{3.12}$$

where $z = kr$ and $J_\nu(z)$ is usual Bessel function of the first kind. In Eq. (3.12), u_m and v_m are

$$\begin{aligned} u_m(z) &= \frac{\pi\hbar^2 k^2}{\sin(|m+\alpha'|\pi)} \\ &\quad \times [A_m\{2\xi_m F_2(z; -|m+\alpha'|, |m+\alpha'|+1) \\ &\quad + \xi_{m,-} F_3(z; -|m+\alpha'|, |m+\alpha'|) - \xi_m F_1(z; -|m+\alpha'|, |m+\alpha'|)\} \\ &\quad + B_m\{2\xi_m F_2(z; -|m+\alpha'|, -|m+\alpha'|+1) + \xi_{m,+} F_3(z; -|m+\alpha'|, \\ &\quad - |m+\alpha'|) - \xi_m F_1(z; -|m+\alpha'|, -|m+\alpha'|)\}], \\ v_m(z) &= -\frac{\pi\hbar^2 k^2}{\sin(|m+\alpha'|\pi)} \\ &\quad \times [A_m\{2\xi_m F_2(z; |m+\alpha'|, |m+\alpha'|+1) + \xi_{m,-} F_3(z; |m+\alpha'|, |m+\alpha'|) \\ &\quad - \xi_m F_1(z; |m+\alpha'|, |m+\alpha'|)\} + B_m\{2\xi_m F_2(z; |m+\alpha'|, \\ &\quad - |m+\alpha'|+1) + \xi_{m,+} F_3(z; |m+\alpha'|, \\ &\quad - |m+\alpha'|) - \xi_m F_1(z; |m+\alpha'|, -|m+\alpha'|)\}], \end{aligned} \tag{3.13}$$

where

$$\xi_m = \alpha'(3m + 2\alpha') \quad \xi_{m,\pm} = \alpha'^2(m+\alpha')(2m+\alpha') + 2\xi_m(1 \pm |m+\alpha'|) \tag{3.14}$$

and

$$F_n(z; \mu, \nu) \equiv \int \frac{J_\mu(z) J_\nu(z)}{z^n} dz. \tag{3.15}$$

In order to escape the infinity at $r = 0$, we should choose $B_m = 0$. Therefore, the wave function can be written in the form

$$\begin{aligned} \psi(\boldsymbol{r}) &= \sum_{m=-\infty}^{\infty} e^{im\phi} A_m J_{|m+\alpha'|}(z) + \beta \sum_{m=-\infty}^{\infty} e^{im\phi} (C_m J_{|m+\alpha'|}(z) \\ &\quad + D_m J_{-|m+\alpha'|}(z) + u_m(z) J_{|m+\alpha'|}(z) + v_m(z) J_{-|m+\alpha'|}(z)) + \mathcal{O}(\beta^2), \end{aligned} \tag{3.16}$$

where

$$\begin{aligned} u_m(z) &= \frac{A_m \pi\hbar^2 k^2}{\sin(|m+\alpha'|\pi)} \{2\xi_m F_2(z; -|m+\alpha'|, |m+\alpha'|+1) \\ &\quad + \xi_{m,-} F_3(z; -|m+\alpha'|, |m+\alpha'|) - \xi_m F_1(z; -|m+\alpha'|, |m+\alpha'|)\}, \end{aligned}$$

$$v_m(z) = -\frac{A_m \pi \hbar^2 k^2}{\sin(|m+\alpha'|\pi)}\{2\xi_m F_2(z;|m+\alpha'|,|m+\alpha'|+1)$$
$$+\xi_{m,-}F_3(z;|m+\alpha'|,|m+\alpha'|) - \xi_m F_1(z;|m+\alpha'|,|m+\alpha'|)\}. \tag{3.17}$$

If we choose

$$A_m = (-i)^{|m+\alpha'|} = e^{-i\pi|m+\alpha'|/2}, \tag{3.18}$$

one can show[31]

$$\lim_{r\to\infty}\sum_{m=-\infty}^{\infty} e^{im\phi}A_m J_{|m+\alpha'|}(z) = e^{-iz\cos\phi} + \frac{e^{ikr}}{\sqrt{r}}f_0(\phi), \tag{3.19}$$

where[c] $f_0(\phi)$ is

$$f_0(\phi) = \frac{1}{\sqrt{2\pi ik}}\left[-2\pi\delta(\phi-\pi)(1-\cos\pi\alpha') - ie^{-iN(\phi-\pi)}\frac{\sin\pi\alpha' e^{-i\phi/2}}{\cos\frac{\phi}{2}}\right]. \tag{3.20}$$

In Eq. (3.20), we used $\alpha' = N+\gamma$, where N is integer and $0 \le \gamma < 1$.

Using

$$\frac{1}{z}J_\nu(z) = \frac{1}{2\nu}[J_{\nu-1}(z) + J_{\nu+1}(z)], \tag{3.21}$$

it is possible to show

$$\begin{aligned} F_2(z;\mu,\nu) &= \frac{1}{2\nu}t[F_1(z;\mu,\nu-1) + F_1(z;\mu,\nu+1)],\\ F_3(z;\mu,\nu) &= \frac{1}{4\mu\nu}[F_1(z;\mu-1,\nu-1) + F_1(z;\mu-1,\nu+1)\\ &\quad + F_1(z;\mu+1,\nu-1) + F_1(z;\mu+1,\nu+1)]. \end{aligned} \tag{3.22}$$

Then, $u_m(z)$ and $v_m(z)$ can be expressed as

$$u_m(z) = \frac{A_m\pi\hbar^2 k^2}{\sin(|m+\alpha'|\pi)}\left[-\frac{|m+\alpha'|\xi_m}{|m+\alpha'|+1}F_1(z;-|m+\alpha'|,|m+\alpha'|)\right.$$
$$+\frac{\xi_m}{|m+\alpha'|+1}F_1(z;-|m+\alpha'|,|m+\alpha'|+2)$$
$$-\frac{\xi_{m,-}}{4|m+\alpha'|^2}\{F_1(z;-|m+\alpha'|-1,|m+\alpha'|-1)$$
$$+F_1(z;-|m+\alpha'|+1,|m+\alpha'|+1)$$

[c]The incident wave derived by Ref. 30 is $e^{-iz\cos\phi - i\alpha\phi}$, which is different from that of Eq. (3.19). The authors in this reference derived it by solving the appropriate differential equation. It was argued[39] that this discrepancy is originated from the fact that the long-range nature of the vector potential does not allow the interchange of the summation over m with the taking of the $r\to\infty$ limit in the partial-wave analysis.

$$
\begin{aligned}
&+F_1(z; -|m+\alpha'|-1, |m+\alpha'|+1) \\
&+F_1(z; -|m+\alpha'|+1, |m+\alpha'|-1)\}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
v_m(z) = -\frac{A_m \pi \hbar^2 k^2}{\sin(|m+\alpha'|\pi)}\Bigg[&-\frac{|m+\alpha'|\xi_m}{|m+\alpha'|+1} F_1(z; |m+\alpha'|, |m+\alpha'|) \\
&+\frac{\xi_m}{|m+\alpha'|+1} F_1(z; |m+\alpha'|, |m+\alpha'|+2) \\
&+\frac{\xi_{m,-}}{4|m+\alpha'|^2}\{F_1(z; |m+\alpha'|-1, |m+\alpha'|-1) \\
&+F_1(z; |m+\alpha'|+1, |m+\alpha'|+1) \\
&+2F_1(z; |m+\alpha'|-1, |m+\alpha'|+1)\}\Bigg].
\end{aligned}
\tag{3.23}
$$

Now, let us examine the behavior of $\psi(\boldsymbol{r})$ around $r \sim 0$. We use the following indefinite integral formula:

$$
\begin{aligned}
F_1(z; \mu, \nu) = &-\frac{z}{\mu^2 - \nu^2}[J_{\mu+1}(z)J_\nu(z) - J_\mu(z)J_{\nu+1}(z)] \\
&+\frac{1}{\mu+\nu} J_\mu(z)J_\nu(z).
\end{aligned}
\tag{3.24}
$$

If we take $\nu \to \pm\mu$ limit in Eq. (3.24), one can also derive

$$
\begin{aligned}
F_1(z; \mu, \mu) &= \frac{2^{-1-2\mu} z^{2\mu}}{\mu \Gamma^2(1+\mu)} {}_2F_3\left(\mu, \mu + \frac{1}{2} : 1+\mu, 1+\mu, 1+2\mu : -z^2\right), \\
F_1(z; \mu, -\mu) &= -\frac{z^2}{4\Gamma(2-\mu)\Gamma(2+\mu)} {}_3F_4 \\
&\quad \times \left(1, 1, \frac{3}{2} : 2-\mu, 2+\mu, 2, 2 : -z^2\right) + \frac{\ln z}{\Gamma(1-\mu)\Gamma(1+\mu)},
\end{aligned}
\tag{3.25}
$$

where $\Gamma(z)$ and ${}_pF_q(a_1, \ldots, a_p : b_1, \ldots, b_q : z)$ are the usual gamma and generalized hypergeometric functions. Using the limiting form

$$
\lim_{z \to 0} J_\nu(z) = \frac{1}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu,
\tag{3.26}
$$

one can show

$$
\begin{aligned}
\lim_{r \to 0} F_1(z; \mu, \nu) &= \frac{1}{(\mu+\nu)\Gamma(\mu+1)\Gamma(\nu+1)}\left(\frac{z}{2}\right)^{\mu+\nu}, \\
\lim_{r \to 0} F_1(z; \mu, \mu) &= \frac{1}{2\mu \Gamma^2(1+\mu)}\left(\frac{z}{2}\right)^{2\mu}, \\
\lim_{r \to 0} F_1(z; \mu, -\mu) &= \frac{\ln z}{\Gamma(1-\mu)\Gamma(1+\mu)}.
\end{aligned}
\tag{3.27}
$$

Then the dominant terms in u_m and v_m at $r \sim 0$ are

$$\begin{aligned}\lim_{r\to 0} u_m(z) &= -\frac{A_m \hbar^2 k^2}{8} \frac{\xi_{m,-}}{|m+\alpha'|} \left(\frac{z}{2}\right)^{-2}, \\ \lim_{r\to 0} v_m(z) &= \frac{A_m \hbar^2 k^2}{8} \frac{\Gamma(-|m+\alpha'|)\xi_{m,-}}{(|m+\alpha'|-1)\Gamma(1+|m+\alpha'|)} \left(\frac{z}{2}\right)^{2(|m+\alpha'|-1)},\end{aligned} \tag{3.28}$$

which yield at $\mathcal{O}(\beta)$

$$\begin{aligned}\lim_{r\to 0} \psi(\boldsymbol{r}) = \beta \sum_{m=-\infty}^{\infty} e^{im\phi} \Bigg[& \frac{D_m}{\Gamma(1-|m+\alpha'|)} \left(\frac{z}{2}\right)^{-|m+\alpha'|} \\ & - \frac{A_m h^2 k^2}{8} \frac{\xi_{m,-}}{(|m+\alpha'|-1)\Gamma(1+|m+\alpha'|)} \left(\frac{z}{2}\right)^{|m+\alpha'|-2} \Bigg].\end{aligned} \tag{3.29}$$

Since we cannot make $\lim_{r\to 0} \psi(\boldsymbol{r})$ regular by choosing D_m appropriately, unlike the AB-scattering in the usual quantum mechanics the AB-like scattering with GUP (1.5) should allow the irregular solution at the origin.

Now, let us examine the behavior of $\psi(\boldsymbol{r})$ around $r \sim \infty$. Using the limiting behavior of the Bessel function

$$\begin{aligned}\lim_{r\to\infty} J_\nu(z) &= \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) \\ &= \frac{1}{\sqrt{2\pi z}} [(-i)^{\nu+1/2} e^{iz} + (i)^{\nu+1/2} e^{-iz}],\end{aligned} \tag{3.30}$$

it is straightforward to show

$$\begin{aligned}\lim_{r\to\infty} F_1(z;\mu,\nu) = -\frac{1}{2\pi(\mu^2-\nu^2)} [& \{(-i)^{\mu-\nu+1} - (-i)^{\nu-\mu+1}\} \\ & + \{(-i)^{\nu-\mu-1} - (-i)^{\mu-\nu-1}\}].\end{aligned} \tag{3.31}$$

When $\nu = \pm\mu$ in Eq. (3.31), one can derive the following asymptotic formula by making use of Eq. (3.25):

$$\begin{aligned}\lim_{r\to\infty} F_1(z;\mu,\mu) &= \frac{1}{2\mu} \\ \lim_{r\to\infty} F_1(z;\mu,-\mu) &= \frac{-\gamma - \psi\left(\frac{1}{2}\right) + \psi(1-\mu) + \psi(1+\mu)}{2\Gamma(1-\mu)\Gamma(1+\mu)},\end{aligned} \tag{3.32}$$

where γ and $\psi(z)$ are Euler number and digamma function. Using $\psi(z+1) = \psi(z) + 1/z$ explicitly, one can show

$$\lim_{r\to\infty} u_m(z) = g_{1,m}, \qquad \lim_{r\to\infty} v_m(z) = g_{2,m}, \tag{3.33}$$

where

$$g_{1,m} = -\frac{A_m \hbar^2 k^2}{2(1+|m+\alpha'|)} \Bigg[\left(\xi_m + \frac{\xi_{m,-}}{2|m+\alpha'|(1-|m+\alpha'|)} \right) \Bigg\{ -\gamma - \psi\left(\frac{1}{2}\right) + \psi(|m+\alpha'|) + \psi(1-|m+\alpha'|) \Bigg\} + \frac{1+2|m+\alpha'|}{|m+\alpha'|(1+|m+\alpha'|)} \xi_m - \frac{1-|m+\alpha|-3|m+\alpha'|^2+|m+\alpha|^3}{2|m+\alpha'|^3(1+|m+\alpha'|)(1-|m+\alpha'|)^2} \xi_{m,-} \Bigg],$$

$$g_{2,m} = \frac{A_m \hbar^2 k^2 \Gamma(|m+\alpha'|)\Gamma(1-|m+\alpha'|)}{2(1+|m+\alpha'|)} \left(\xi_m + \frac{\xi_{m,-}}{2|m+\alpha'|(1-|m+\alpha'|)} \right). \tag{3.34}$$

Using Eq. (3.33), it is straightforward to compute $\lim_{r\to\infty} f_{1,m}(r)$ explicitly. Since $f_{1,m}(r)$ should be outgoing wave at $r = \infty$, we should impose the coefficient of e^{-ikr} to be zero, which gives

$$C_m + g_{1,m} = -e^{-i\pi|m+\alpha'|}(D_m + g_{2,m}). \tag{3.35}$$

Then, $f_{1,m}(r \to \infty)$ reduces to

$$\lim_{r\to\infty} f_{1,m}(r) = \frac{e^{ikr}}{\sqrt{2\pi i k r}} e^{i\pi|m+\alpha'|/2}(1 - e^{-2i\pi|m+\alpha'|})(D_m + g_{2,m}). \tag{3.36}$$

Thus, the asymptotic behavior of the wave function given in Eq. (3.16) can be written as a standard from

$$\lim_{r\to\infty} \psi(\boldsymbol{r}) = e^{-ikr\cos\phi} + \frac{e^{ikr}}{\sqrt{r}} f(\phi), \tag{3.37}$$

where the scattering amplitude $f(\phi)$ is

$$f(\phi) = f_0(\phi) + \beta f_1(\phi) + \mathcal{O}(\beta^2). \tag{3.38}$$

In Eq. (3.38), $f_0(\phi)$ is given in Eq. (3.20) and $f_1(\phi)$ is

$$f_1(\phi) = \frac{1}{\sqrt{2\pi i k}} \sum_{m=-\infty}^{\infty} e^{im\phi} e^{i|m+\alpha'|\pi/2}(1 - e^{-2i\pi|m+\alpha'|})(D_m + g_{2,m}). \tag{3.39}$$

Here, we consider a special case $D_m = 0$ for all m. Inserting ξ_m and $\xi_{m,-}$ given in Eq. (3.14) into $g_{2,m}$, one can express $g_{2,m}$ in the form

$$g_{2,m} = -\frac{(-i)^{|m+\alpha'|}\hbar^2 k^2}{4} \Gamma(m+\alpha')\Gamma(-m-\alpha') \times \left[-2\alpha'^2 + 6\alpha'(m+\alpha') + \alpha'^2(m+\alpha') \left\{ \frac{1-\alpha'/2}{1-m-\alpha'} - \frac{1+\alpha'/2}{1+m+\alpha'} \right\} \right]. \tag{3.40}$$

Here, we used $A_m = (-i)^{|m+\alpha'|}$. It is worthwhile noting that except A_m there is no absolute value in Eq. (3.40). Inserting Eq. (3.40) into Eq. (3.39), we get

$$\begin{aligned} f_1(\phi) = &-\frac{\hbar^2 k^2}{4}\frac{1}{\sqrt{2\pi i k}}\sum_{m=-\infty}^{\infty} e^{im\phi}(1-e^{-2i\pi|m+\alpha'|}) \\ &\times\Bigg[-2\alpha'^2\Gamma(m+\alpha')\Gamma(-m-\alpha')+6\alpha'\Gamma(1+m+\alpha')\Gamma(-m-\alpha') \\ &+\alpha'^2\left(1-\frac{\alpha'}{2}\right)\frac{\Gamma(1+m+\alpha')\Gamma(-m-\alpha')}{1-m-\alpha} \\ &-\alpha'^2\left(1+\frac{\alpha'}{2}\right)\frac{\Gamma(1+m+\alpha')\Gamma(-m-\alpha')}{1+m+\alpha}\Bigg]. \end{aligned} \tag{3.41}$$

Let us express α' as $\alpha' = N+\gamma$, where N is integer and $0 \le \gamma < 1$. Then, $f_1(\phi)$ in Eq. (3.41) can be written in the following form:

$$\begin{aligned} f_1(\phi) = &-\frac{i\hbar^2k^2}{2}\sin(\pi\gamma)\frac{e^{-i\pi\gamma}}{\sqrt{2\pi i k}} \\ &\times\left[-2\alpha'^2 J_1 + 6\alpha' J_2 + \alpha'^2\left(1-\frac{\alpha'}{2}\right)J_3 - \alpha'^2\left(1+\frac{\alpha'}{2}\right)J_4\right] \\ &+\frac{i\hbar^2k^2}{2}\sin(\pi\gamma)\frac{e^{i\pi\gamma}}{\sqrt{2\pi i k}} \\ &\times\left[-2\alpha'^2 K_1 + 6\alpha' K_2 + \alpha'^2\left(1-\frac{\alpha'}{2}\right)K_3 - \alpha'^2\left(1+\frac{\alpha'}{2}\right)K_4\right], \end{aligned} \tag{3.42}$$

where

$$\begin{aligned} J_1 &= \sum_{m=-N}^{\infty} e^{im\phi}\Gamma(m+\alpha')\Gamma(-m-\alpha'), \\ J_2 &= \sum_{m=-N}^{\infty} e^{im\phi}\Gamma(1+m+\alpha')\Gamma(-m-\alpha'), \\ J_3 &= \sum_{m=-N}^{\infty} e^{im\phi}\frac{\Gamma(1+m+\alpha')\Gamma(-m-\alpha')}{1-m-\alpha'}, \\ J_4 &= \sum_{m=-N}^{\infty} e^{im\phi}\frac{\Gamma(1+m+\alpha')\Gamma(-m-\alpha')}{1+m+\alpha'} \end{aligned} \tag{3.43}$$

and K_j $(j = 1, 2, 3, 4)$ are similar to J_j. The only difference is the summation range, which is from $-\infty$ to $-N-1$. It is straightforward to show

$$\begin{aligned} K_1 &= J_1|_{\phi\to-\phi,\alpha'\to-\alpha'}, \quad K_2 = -J_2|_{\phi\to-\phi,\alpha'\to-\alpha'}, \\ K_3 &= -J_4|_{\phi\to-\phi,\alpha'\to-\alpha'}, \quad K_4 = -J_3|_{\phi\to-\phi,\alpha'\to-\alpha'}. \end{aligned} \tag{3.44}$$

Of course, $\alpha' \to -\alpha'$ implies $N \to -N-1$ and $\gamma \to 1-\gamma$. Then, it is easy to show that $f_1(\phi)$ is invariant under the simultaneous change $\phi \to -\phi$, $\alpha' \to -\alpha'$, which is a symmetry of the Hamiltonian. Summing over m, one can show

$$
\begin{aligned}
J_1 &= -\frac{\pi}{\gamma \sin(\pi\gamma)} e^{-iN\phi} {}_2F_1(1, \gamma : 1+\gamma : -e^{i\phi}) \\
&= -\frac{\pi e^{-i(N+1/2)\phi}}{2\gamma \sin(\pi\gamma)\cos(\phi/2)} {}_2F_1\left(1, 1 : 1+\gamma : \frac{e^{i\phi/2}}{2\cos(\phi/2)}\right), \\
J_2 &= -\frac{\pi}{\sin(\pi\gamma)} \frac{e^{-iN\phi}}{1+e^{i\phi}} = -\frac{\pi e^{-i(N+1/2)\phi}}{2\sin(\pi\gamma)\cos(\phi/2)}, \\
J_3 &= -\frac{\pi}{(1-\gamma)\sin(\pi\gamma)} e^{-iN\phi} {}_2F_1(1, -1+\gamma : \gamma : -e^{i\phi}) \\
&= -\frac{\pi e^{-i(N+1/2)\phi}}{2(1-\gamma)\sin(\pi\gamma)\cos(\phi/2)} {}_2F_1\left(1, 1 : \gamma : \frac{e^{i\phi/2}}{2\cos(\phi/2)}\right), \\
J_4 &= -\frac{\pi}{(1+\gamma)\sin(\pi\gamma)} e^{-iN\phi} {}_2F_1(1, 1+\gamma : 2+\gamma : -e^{i\phi}) \\
&= -\frac{\pi e^{-i(N+1/2)\phi}}{2(1+\gamma)\sin(\pi\gamma)\cos(\phi/2)} {}_2F_1\left(1, 1 : 2+\gamma : \frac{e^{i\phi/2}}{2\cos(\phi/2)}\right),
\end{aligned}
\tag{3.45}
$$

where ${}_2F_1(a, b : c : z)$ is a hypergeometric function and we used the identity

$$
{}_2F_1(a, b : c : z) = (1-z)^{-a} {}_2F_1\left(a, c-b : c : \frac{z}{z-1}\right). \tag{3.46}
$$

Using Eq. (3.44) and Eq. (3.45) it is straightforward to show

$$
\begin{aligned}
K_1 &= -\frac{\pi e^{-i(N+1/2)\phi}}{2(1-\gamma)\sin(\pi\gamma)\cos(\phi/2)} {}_2F_1\left(1, 1 : 2-\gamma : \frac{e^{-i\phi/2}}{2\cos(\phi/2)}\right), \\
K_2 &= \frac{\pi e^{-i(N+1/2)\phi}}{2\sin(\pi\gamma)\cos(\phi/2)}, \\
K_3 &= \frac{\pi e^{-i(N+1/2)\phi}}{2(2-\gamma)\sin(\pi\gamma)\cos(\phi/2)} {}_2F_1\left(1, 1 : 3-\gamma : \frac{e^{-i\phi/2}}{2\cos(\phi/2)}\right), \\
K_4 &= \frac{\pi e^{-i(N+1/2)\phi}}{2\gamma\sin(\pi\gamma)\cos(\phi/2)} {}_2F_1\left(1, 1 : 1-\gamma : \frac{e^{-i\phi/2}}{2\cos(\phi/2)}\right).
\end{aligned}
\tag{3.47}
$$

Inserting Eqs. (3.45) and (3.47) into Eq. (3.42), one can show

$$
f_1(\phi) = \frac{i\pi\hbar^2 k^2 e^{-i(N+1/2)\phi}}{4\cos(\phi/2)\sqrt{2\pi i k}} G(\alpha', \phi), \tag{3.48}
$$

where

$$
G(\alpha', \phi) = 2\alpha'^2 \left[\frac{e^{i\pi\gamma}}{1-\gamma} {}_2F_1(1, 1 : 2-\gamma : x^*) - \frac{e^{-i\pi\gamma}}{\gamma} {}_2F_1(1, 1 : 1+\gamma : x)\right]
$$

$$
\begin{aligned}
&+12\alpha'\cos(\pi\gamma)+\alpha'^2\left(1-\frac{\alpha'}{2}\right)\\
&\times\left[\frac{e^{i\pi\gamma}}{2-\gamma}{}_2F_1(1,1:3-\gamma:x^*)+\frac{e^{-i\pi\gamma}}{1-\gamma}{}_2F_1(1,1:\gamma:x)\right]\\
&-\alpha'^2\left(1+\frac{\alpha'}{2}\right)\left[\frac{e^{i\pi\gamma}}{\gamma}{}_2F_1(1,1:1-\gamma:x^*)\right.\\
&\left.+\frac{e^{-i\pi\gamma}}{1+\gamma}{}_2F_1(1,1:2+\gamma:x)\right].
\end{aligned}
\tag{3.49}
$$

In Eq. (3.49), x is given by

$$
x=\frac{e^{i\phi/2}}{2\cos(\phi/2)} \tag{3.50}
$$

and x^* is its complex conjugate. From Eq. (3.48), one can show again that $f_1(\phi)$ is invariant under the simultaneous change $\phi\to-\phi$, $\alpha'\to-\alpha'$. If $\phi\neq\pi$, the scattering amplitude becomes

$$
f(\phi)=\frac{-ie^{-i(N+1/2)\phi}}{\cos(\phi/2)\sqrt{2\pi ik}}\left[\sin(\pi\gamma)-\frac{\pi\hbar^2k^2\beta}{4}G(\alpha',\phi)+\mathcal{O}(\beta^2)\right]. \tag{3.51}
$$

Then, the differential cross-section reduces to

$$
\begin{aligned}
\frac{d\sigma}{d\phi}&=\frac{1}{2\pi k\cos^2(\phi/2)}\left|\sin(\pi\gamma)-\frac{\pi\hbar^2k^2\beta}{4}G(\alpha',\phi)\right|^2+\mathcal{O}(\beta^2)\\
&=\frac{\sin(\pi\gamma)}{2\pi k\cos^2(\phi/2)}\left[\sin(\pi\gamma)-\beta\frac{\pi\hbar^2k^2}{2}\mathrm{Re}\ G(\alpha',\phi)\right]+\mathcal{O}(\beta^2).
\end{aligned}
\tag{3.52}
$$

In the usual quantum mechanics with HUP, the differential cross-section vanishes when α' is integer. This is analogous to the Ramsauer effect.[40] However, this behavior is not maintained at the first-order of β. Furthermore, discontinuity occurs at every integer of α'. For example, if $\alpha'=N^+=\lim_{\gamma\to0}(N+\gamma)$, one can show from the second expression of Eq. (3.52)

$$
\frac{d\sigma}{d\phi}=\beta\frac{\pi\hbar^2kN^2}{2}\left[2(N-2)\cos^2\frac{\phi}{2}+6-N\right]. \tag{3.53}
$$

If, however, $\alpha'=N^-=\lim_{\gamma\to1}[(N-1)+\gamma]$, one can also show

$$
\frac{d\sigma}{d\phi}=\beta\frac{\pi\hbar^2kN^2}{2}\left[N+6-2(N+2)\cos^2\frac{\phi}{2}\right]. \tag{3.54}
$$

In Fig. 2, we plot the α'-dependence of the differential cross-section when $\beta=0$ (black solid line), $\beta=0.01$ (red dashed line), and $\beta=0.02$ (blue dotted line) for $\phi=\pi/4$ (Fig. 2(a)) and $\phi=-\pi/4$ (Fig. 2(b)). We chose $\hbar=k=1$ for simplicity. As expected, Fig. 2 exhibits discontinuous behavior at $\alpha'=1$ when $\beta\neq0$. Another

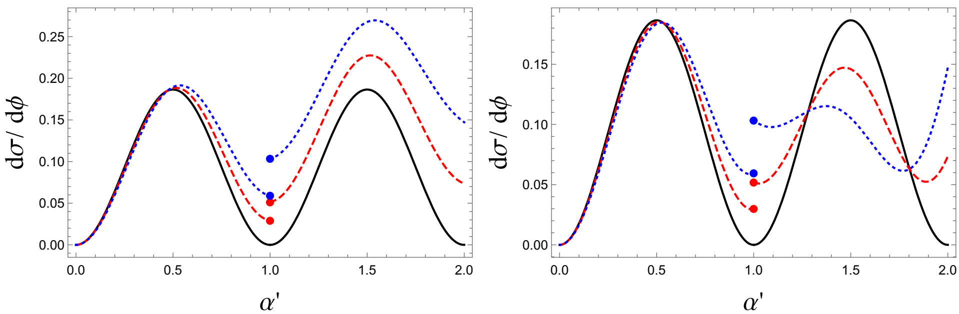

Fig. 2. (Color online) The α'-dependence of the differential cross-section when $\beta = 0$ (black solid line), $\beta = 0.01$ (red dashed line), and $\beta = 0.02$ (blue dotted line) for $\phi = \pi/4$ (Fig. 2(a)) and $\phi = -\pi/4$ (Fig. 2(b)). We chose $\hbar = k = 1$ for simplicity. As expected, Fig. 2 exhibits discontinuous behavior at $\alpha' = 1$ when $\beta \neq 0$.

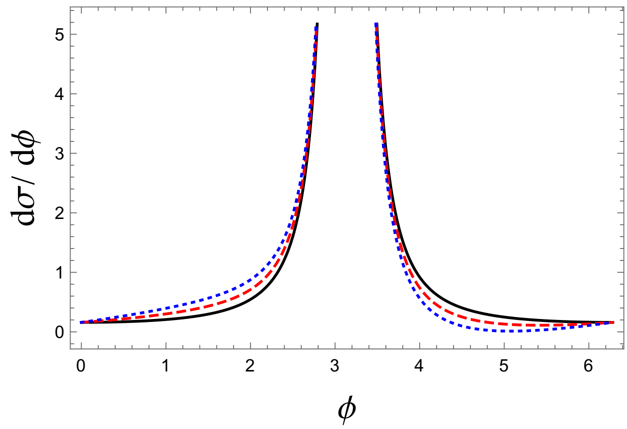

Fig. 3. (Color online) The ϕ-dependence of the differential cross-section when $\beta = 0$ (black solid line), $\beta = 0.004$ (red dashed line), and $\beta = 0.008$ (blue dotted line). We chose $\hbar = k = 1$ and $\alpha' = 2.5$ for simplicity. Figure 3 shows apparently that the symmetry (3.55) is broken when $\beta \neq 0$.

interesting behavior Fig. 2 shows is the fact that while $d\sigma/d\phi$ at $\phi = \pm\theta$ are exactly identical in the usual quantum mechanics, this symmetry is obviously broken due to $G(\alpha', \phi)$.

In usual quantum mechanics with HUP, the cross-section is symmetric at $\phi = \pi$, i.e.

$$\frac{d\sigma}{d\phi}(\phi = \pi - \theta) = \frac{d\sigma}{d\phi}(\phi = \pi + \theta). \tag{3.55}$$

However, this symmetry is also broken at the first-order of β because

$$x(\phi = \pi - \theta) = x^*(\phi = \pi + \theta) = \frac{ie^{-i\theta/2}}{2\sin(\theta/2)} \tag{3.56}$$

and $\mathrm{Re}G(\alpha', \phi)$ does not have $x \leftrightarrow x^*$ symmetry. In order to confirm the fact we plot the ϕ-dependence of the differential cross-section when $\beta = 0$ (black solid line),

Table 1. Comparison between usual and GUP-corrected AB-like effect.

	HUP	GUP
$\phi \to -\phi, \alpha' \to -\alpha'$ symmetry	Y	Y
symmetry of $\frac{d\sigma}{d\phi}$ at $\phi = \pi$	Y	N
Ramsauer effect	Y	N and discontinuous at integer α'

$\beta = 0.004$ (red dashed line), and $\beta = 0.008$ (blue dotted line) in Fig. 3. We chose $\hbar = k = 1$ and $\alpha' = 2.5$ for simplicity. This figure obviously show that the symmetry (3.55) is broken when $\beta \neq 0$.

4. Conclusion

In this paper, we explored how the Aharonov–Bohm scattering is modified in the GUP-corrected quantum mechanics. In Table 1, we compare the GUP-correct AB-like phenomenon with the usual AB-effect. The most striking difference is that the cross-section is discontinuous at every integer α' due to $G(\alpha', \phi)$ given in Eq. (3.49). From Eqs. (3.53) and (3.54), one can show that the discontinuity width at $\alpha' = N$ is

$$\Delta \frac{d\sigma}{d\phi} = \beta \pi \hbar^2 k N^3 \left| 2\cos^2 \frac{\phi}{2} - 1 \right|. \tag{4.1}$$

Thus, it is possible, in principle, to verify the presence or absence of GUP experimentally by measuring the discontinuity. Of course, it seems to be very difficult to measure it because the discontinuity arises at the order of β, and β is believed to be extremely small.

One can use the Lagrangian (2.9) to derive the Feynman propagator (or Kernel) for the spin-0 AB-system with GUP. The propagator of the usual AB-system was derived long ago in Refs. 41–43. It seems to be of interest to explore the quantum effect by deriving the Feynman propagator corresponding to the Lagrangian (2.9). In the usual quantum mechanics, it is well known that the magnetic flux trapped by a superconductor ring is quantized by $\Phi = \pi n \hbar c / e$ $(n = 0, \pm 1, \pm 2, \ldots)$. It is of interest to explore how this quantization rule is modified in the presence of GUP.

Finally, one can extend this paper to the spin-1/2 AB problem in the presence of GUP. As commented earlier, the Zeeman interaction term in this case is expressed as a two-dimensional singular δ-function potential. One-dimensional δ-function potential problem in the GUP-corrected quantum mechanics was recently discussed in Ref. 44. It was shown in this reference that unlike usual quantum mechanics, the Schrödinger and Feynman's path-integral approaches are inequivalent at the first-order of β. It seems to be of interest to examine whether the two-dimensional δ-function potential yields a similar result or not in the spin-1/2 AB problem with GUP.

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