

p289 CH4 시간에 따라 변하는 전자기학

Maxwell Equation

Faraday 법칙

Faraday 유도법칙

어떤 개곡면에 자기선속 (magnetic flux) 가 시간에 따라
변화하면, 개곡면의 boundary를 형성하는 폐곡선에
유도기전력이 발생하여 유도전류가 흐른다

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

$$\mathcal{E} = \oint_C \vec{E} \cdot d\vec{l} : \text{기전력}$$

$$\Phi_B = \int_S \vec{B} \cdot \hat{u}_N dS : \text{자기선속}$$

$$\Rightarrow \oint_C \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_S \vec{B} \cdot \hat{u}_N dS \quad \text{Faraday 법칙}$$

[1] 미분형

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

[2] (-)의 의미 : Lenz's 법칙

Φ_B가 시간에 따라나 증가하면 감소하는 방향으로,

감소하면 증가하는 방향으로 유도전류가 발생

한다는 의미

(Ex)



$$\vec{B} = \begin{cases} B_0 e^{kt} \hat{z} & 0 \leq \rho \leq b \\ 0 & b < \rho \end{cases}$$

 $a < b$ case

$$\Phi_B = \int_S \vec{B} \cdot \hat{n} ds = \pi a^2 B_0 e^{kt}$$

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = - \pi a^2 B_0 k e^{kt}$$

유한 크기일 때 $\vec{E} = ?$ $a \rightarrow \rho$

$$\mathcal{E} = - \pi \rho^2 k B_0 e^{kt} = \oint_C \vec{E} \cdot d\vec{l} = E \cdot 2\pi \rho$$

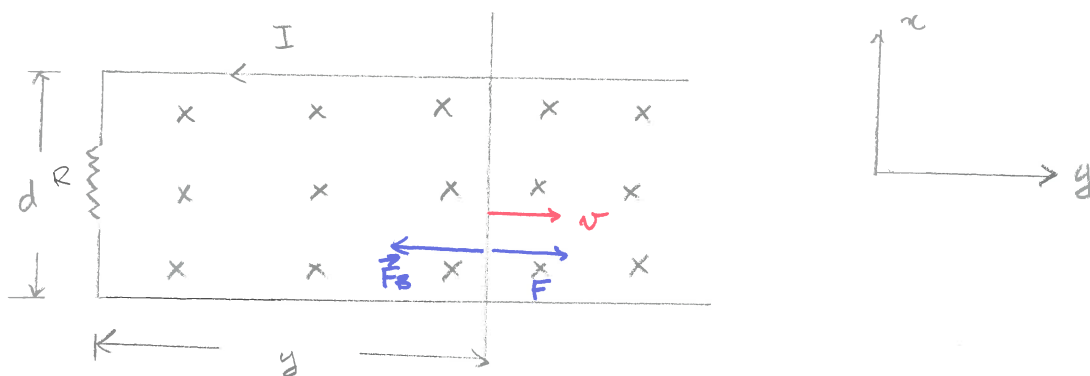
$$E = - \frac{1}{2} k B_0 \rho e^{kt}$$

$$\vec{E} = - \frac{1}{2} k B_0 e^{kt} \rho \hat{\phi}$$

$$\text{또는 } \vec{\rho} \times \vec{E} = - k B_0 e^{kt} \hat{z} = - \frac{\partial \vec{B}}{\partial t}$$

*

(ex)



$$\vec{B} = B \hat{z} \quad (B: \text{const})$$

$$\Phi_B = B d y$$

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = - B d \frac{dy}{dt} = - B d v \quad : \text{유도 기전력}$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{B d v}{R} \quad : \text{유도 전류}$$

* 에너지 보존

$$\vec{F}_B = I \vec{L} \times \vec{B}, \quad F_B = I d B$$

회선이 등속운동을 하므로 $\vec{F} = -\vec{F}_B$ 가 주어져야 한다.

$$F = I d B$$

(P (일률)): 회선이 한 단위 시간당 한 일

$$P = F v = I d B v \quad I = \frac{B d v}{R}$$

$$= \frac{B^2 d^2 v^2}{R} = \frac{\mathcal{E}^2}{R}$$

$$\mathcal{E}^2 = (B d v)^2$$

$$|\mathcal{E}| = B d v$$

* 유도 전자의 관점

유도 전자가 받는 힘

$$\vec{F}_L = q(\vec{v} \times \vec{B}) = q \vec{E}_m$$

$$\vec{E}_m = \vec{v} \times \vec{B}$$

$$\mathcal{E} = \oint_C \vec{E}_m \cdot d\vec{l}$$

C: 시계방향 (→를 얻기 위하여)

$$= \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$= \int_d^0 NB \, dz$$

$$= -NBd$$

✕

(문제 9.1)

$$\epsilon = 10^{-11} \text{ (F/m)} \quad \mu = 10^{-5} \text{ (H/m)}$$

$$\vec{B} = 2 \times 10^{-4} \cos(10^5 t) \sin(10^{-3} y) \hat{z} \text{ (T)}$$

$$(a) \quad \vec{H} = \frac{\vec{B}}{\mu} = \frac{2 \times 10^{-4}}{\mu} \cos(10^5 t) \sin(10^{-3} y) \hat{z} \text{ (A/m)}$$

$$\vec{\nabla} \times \vec{H} = \frac{2 \times 10^{-4}}{\mu} \cos(10^5 t) \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin(10^{-3} y) & 0 & 0 \end{vmatrix}$$

$$-10^{-3} \cos(10^{-3} y) \hat{z}$$

$$= - \frac{2 \times 10^{-7}}{\mu} \cos(10^5 t) \cos(10^{-3} y) \hat{z} \equiv \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \vec{E} = E \hat{z}$$

$$\frac{\partial E}{\partial t} = - \frac{2 \times 10^{-7}}{\epsilon \mu} \cos(10^5 t) \cos(10^{-3} y)$$

$$E = - \frac{2 \times 10^{-7}}{\epsilon \mu} \frac{1}{10^5} \sin(10^5 t) \cos(10^{-3} y)$$

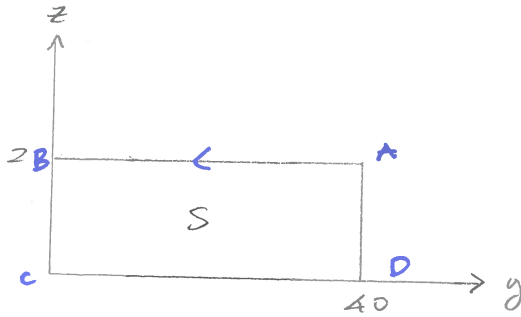
$$= - \frac{2 \times 10^{-12}}{\epsilon \mu} \sin(10^5 t) \cos(10^{-3} y)$$

$$\epsilon \mu = 10^{-16}$$

$$= - 20000 \sin(10^5 t) \cos(10^{-3} y)$$

$$\vec{E} = - 20000 \sin(10^5 t) \cos(10^{-3} y) \hat{z}$$

(b)



$$\Phi_B = \int_S \vec{B} \cdot \hat{u}_n dS \quad \hat{u}_n = \hat{z}$$

$$dS = dy dz$$

$$= \int_0^{40} dy \int_0^2 dz \quad (2 \times 10^{-4}) \cos(10^5 t) \sin(10^{-3} y)$$

$$= 0.4 [1 - \cos(0.04)] \cos(10^5 t)$$

$$\Phi_B (t = 10^{-6})$$

$$= 0.4 [1 - \cos(0.04)] \cos(0.1)$$

$$= 0.318359 \times 10^{-3} \text{ (Wb)}$$

$$= 0.318359 \text{ (mWb)}$$

$$(c) \quad \Phi_B = 0.4 [1 - \cos(0.04)] \cos(10^5 t)$$

$$- \frac{\partial \Phi_B}{\partial t} = 40000 [1 - \cos(0.04)] \sin(10^5 t)$$

$$\Rightarrow \oint_C \vec{E} \cdot d\vec{l} = - \frac{\partial \Phi_B}{\partial t} = 40000 [1 - \cos(0.04)] \sin(10^5 t)$$

$$\text{at } t = 10^{-6} \text{ (sec)}$$

$$\oint \vec{E} \cdot d\vec{l} = 40000 [1 - \cos(0.04)] \sin(0.1)$$

$$= 3.19424 \text{ (V)}$$

* (a) 결과를 이쯤해서 구할 수 있다!!

$$\vec{E} = -20000 \sin(10^5 t) \cos(10^3 y) \hat{z}$$

$$\oint \vec{E} \cdot d\vec{s}$$

$$= \underbrace{\int_A^B \vec{E} \cdot d\vec{s}}_{d\vec{s} = dy \hat{y}} + \underbrace{\int_B^C \vec{E} \cdot d\vec{s}}_{\substack{d\vec{s} = dz \hat{z} \\ y=0}} + \underbrace{\int_C^D \vec{E} \cdot d\vec{s}}_{d\vec{s} = dy \hat{y}} + \underbrace{\int_D^A \vec{E} \cdot d\vec{s}}_{\substack{d\vec{s} = dz \hat{z} \\ y=40}}$$

$$= \int_z^0 [-20000 \sin(10^5 t)] dz$$

$$+ \int_0^2 [-20000 \sin(10^5 t) \cos(10.04)] dz$$

$$= 40000 \sin(10^5 t) [1 - \cos(0.04)]$$

*

(응용예제 9.2)

$$d = 0.09 \text{ (cm)}$$

$$\vec{B} = 0.3 \hat{z} \text{ (T)}$$

$$\vec{v} = 0.1 e^{20y} \hat{y} \text{ (cm/sec)}$$

$$(a) v(t=0) = v(y=0) = 0.1 \text{ (cm/sec)}$$

$$(b) v = \frac{dy}{dt} = 0.1 e^{20y} \text{ (미분방정식)}$$

$$y = -\frac{1}{20} \ln(1-2t)$$

$$y(t=0.1) = -\frac{1}{20} \ln(0.8) = 0.0111572 \text{ (cm)} = 1.11572 \text{ (mm)}$$

$$(c) v(t=0.1)$$

$$= v(y=0.0111572)$$

$$= 0.1 e^{20 \times 0.0111572}$$

$$= 0.125 \text{ (cm/sec)}$$

$$(d) V_L = \mathcal{E} = -B d v$$

$$V_L(t=0.1) = -0.3 \times 0.09 \times 0.125$$

$$= -0.002625 \text{ (V)}$$

$$= -2.625 \text{ (mV)}$$

X

이걸까? 아니

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad (?)$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J}$$

$$\vec{\nabla} \cdot \vec{J} = 0$$

$$\text{However } \vec{\nabla} \cdot \vec{J} = - \frac{\partial \rho}{\partial t} \quad (\text{연속 방정식})$$

Something wrong!!

$$\Rightarrow \vec{\nabla} \times \vec{H} = \vec{J} + \vec{G}$$

$$0 = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{G} = - \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{G}$$

$$\text{Since } \vec{\nabla} \cdot \vec{D} = \rho, \quad \vec{G} = \frac{\partial \vec{D}}{\partial t}$$

$$\Rightarrow \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

\vec{J} : 전하 전류 밀도

$\vec{J}_d \equiv \frac{\partial \vec{D}}{\partial t}$: 변위 전류 밀도 (displacement current density)

* 정리

260

$$\oint_C \vec{H} \cdot d\vec{r}$$

$$= \int_S (\vec{\nabla} \times \vec{H}) \cdot \hat{u}_N ds$$

$$= \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \hat{u}_N ds$$

$$= \underbrace{\int_S \vec{J} \cdot \hat{u}_N ds}_I + \int_S \frac{\partial \vec{D}}{\partial t} \cdot \hat{u}_N ds$$

I

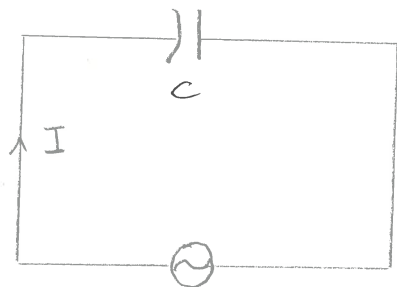
$$= I + \int_S \frac{\partial \vec{D}}{\partial t} \cdot \hat{u}_N ds$$

$$\oint_C \vec{H} \cdot d\vec{r} = I + \int_S \frac{\partial \vec{D}}{\partial t} \cdot \hat{u}_N ds$$

I : 전류

$\int_S \frac{\partial \vec{D}}{\partial t} \cdot \hat{u}_N ds$: 변위전류

* 교류 C 회로

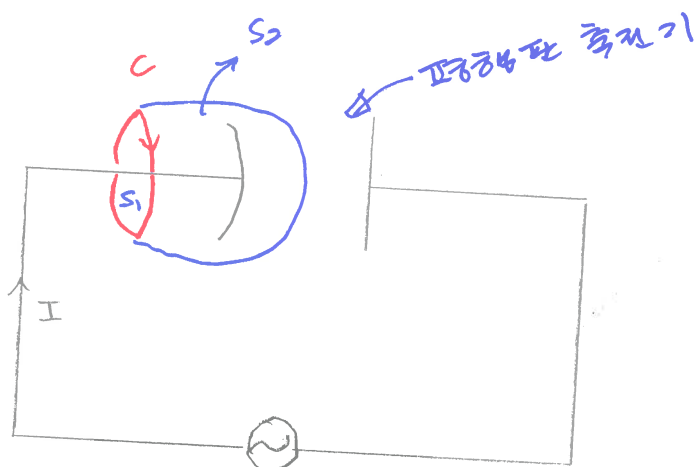


$$V = V_0 \cos \omega t$$

키르호프: $\frac{q}{C} = V_0 \cos \omega t$

$$q = CV_0 \cos \omega t$$

$$I = \frac{dq}{dt} = -CV_0 \omega \sin \omega t$$



$$V = V_0 \cos \omega t$$

S_1 :

$$\oint_C \vec{H} \cdot d\vec{l} = I + \int_{S_1} \frac{\partial \vec{D}}{\partial t} \cdot \hat{u}_N dS$$

$$I = -cV_0\omega \sin\omega t$$

$$\vec{B} = 0$$

$$\oint_C \vec{H} \cdot d\vec{l} = -cV_0\omega \sin\omega t = -\omega \frac{\epsilon S}{d} V_0 \sin\omega t$$

$$S_2: \oint_C \vec{H} \cdot d\vec{l} = I + \int_{S_2} \frac{\partial \vec{D}}{\partial t} \cdot \hat{u}_N dS$$

$$I = 0$$

$$\int \vec{E} \cdot d\vec{l} = V$$

$$\Rightarrow Ed = V_0 \cos\omega t$$

$$\Rightarrow E = \frac{V_0}{d} \cos\omega t$$

$$\Rightarrow D = \epsilon E = \frac{\epsilon V_0}{d} \cos\omega t$$

$$\frac{\partial D}{\partial t} = -\frac{\epsilon V_0 \omega}{d} \sin\omega t$$

$$\int_{S_2} \frac{\partial \vec{D}}{\partial t} \cdot \hat{u}_N dS$$

$$= -\frac{\epsilon V_0 \omega}{d} \sin\omega t S$$

$$= -\omega \frac{\epsilon S}{d} V_0 \sin\omega t$$

$$\Rightarrow \oint_C \vec{H} \cdot d\vec{l} = -\omega \frac{\epsilon S}{d} V_0 \sin\omega t$$

eq. 11.

(응답예제 92)

$$(a) \quad \vec{H} = 0.15 \cos [3.12 (3 \times 10^8 t - y)] \hat{x} \quad (\text{A/m})$$

$$\vec{J}_d = \vec{\nabla} \times \vec{H}$$

$$= -0.15 \times 3.12 \sin [3.12 (3 \times 10^8 t - y)] \hat{z}$$

$$\text{진폭} = 0.15 \times 3.12 = 0.468 \quad (\text{A/m}^2)$$

$$(b) \quad \vec{B} = 0.8 \cos [1.257 \times 10^{-6} (3 \times 10^8 t - x)] \hat{y} \quad (\text{T})$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{0.8}{\mu_0} \cos [1.257 \times 10^{-6} (3 \times 10^8 t - x)] \hat{y}$$

$$\vec{J}_d = \vec{\nabla} \times \vec{H}$$

$$= \hat{z} \frac{0.8}{\mu_0} \times 1.257 \times 10^{-6} \sin [\quad]$$

$$\text{진폭} = \frac{0.8 \times 1.257 \times 10^{-6}}{4\pi \times 10^{-7}} = 0.800231 \quad (\text{A/m}^2)$$

$$(c) \epsilon_r = 5$$

$$\vec{E} = 0.9 \times 10^6 \cos [1.257 \times 10^6 (3 \times 10^8 t - \sqrt{5} z)] \hat{x} \quad (V/m)$$

$$\vec{D} = \epsilon \vec{E}$$

$$= \epsilon_r \epsilon_0 \vec{E}$$

$$\epsilon_0 = 8.84 \times 10^{-12} (C^2/Nm^2)$$

$$= \epsilon_r \epsilon_0 \times 0.9 \times 10^6 \cos [1.257 \times 10^6 (3 \times 10^8 t - \sqrt{5} z)] \hat{x} \quad (C/m^2)$$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = - \frac{\epsilon_r \epsilon_0 \times 0.9 \times 10^6 \times 1.257 \times 10^6 \times 3 \times 10^8 \sin [\quad] \hat{x}}{0.0150}$$

$$0.0150$$

$$\text{rms } J_d = 0.0150 (A/m^2)$$

$$(d) \epsilon = \epsilon_0, \mu = \mu_0, \sigma = 5.8 \times 10^7 (1/\Omega m)$$

$$\vec{J} = 10^6 \sin(377t - 117.1z) \hat{x} \quad (A/m^2)$$

$$\vec{J} = \sigma \vec{E}$$

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{10^6}{5.8} \sin(377t - 117.1z) \hat{x} \quad (V/m)$$

$$\vec{D} = \epsilon_0 \vec{E} = \frac{\epsilon_0}{5.8} \times 10^6 \sin(377t - 117.1z) \hat{x} \quad (C/m^2)$$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \frac{\epsilon_0}{5.8} \times 377 \times 10^6 \cos(377t - 117.1z) \hat{x} \quad (A/m^2)$$

$$\text{rms } J_d = \frac{\epsilon_0}{5.8} \times 377 \times 10^6 = 57.5 \times 10^{-12} (A/m^2)$$

$$= 57.5 (pA/m^2)$$

*

§ Maxwell Equation

$$\vec{\nabla} \cdot \vec{D} = \rho$$

electric Gauss eq³³

$$\vec{\nabla} \cdot \vec{B} = 0$$

magnetic "

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

Faraday "

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Maxwell - Ampere "

Maxwell Equation

$$* \quad \vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E} \quad (\vec{P} = \chi_e \epsilon_0 \vec{E})$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu \vec{H} \quad (\vec{M} = \chi_m \vec{H})$$

$$\vec{J} = \sigma \vec{E} = \rho \vec{v}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

(9.4)

$$\mu = 10^{-5} \text{ (H/m)} \quad \epsilon = 4 \times 10^{-9} \text{ (F/m)} \quad \sigma = 0 \quad \rho = 0$$

$$(a) \quad \vec{D} = 10^{-9} \times (6\hat{x} - 2y\hat{y} + 2z\hat{z}) \quad (\text{C/m}^2)$$

$$\vec{H} = kx\hat{x} + 10y\hat{y} - 25z\hat{z} \quad (\text{A/m})$$

$$[k] = \frac{\text{A}}{\text{m}} \frac{1}{[\text{z}]} = \text{A/m}^2$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

$$k = 15 \text{ (A/m}^2\text{)}$$

$$(b) \quad \vec{E} = (20y - kt)\hat{x} \quad (\text{V/m}) \quad \vec{H} = (y + 2 \times 10^6 t)\hat{z} \quad (\text{A/m})$$

$$[k] = \frac{\text{V}}{\text{m}} \frac{1}{[t]} = \text{V/msec}$$

$$\vec{\nabla} \times \vec{H} = \hat{x}$$

$$\vec{D} = \epsilon \vec{E} = \epsilon (20y - kt)\hat{x}$$

$$\frac{\partial \vec{D}}{\partial t} = -k\epsilon \hat{x}$$

$$\text{From } \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t},$$

$$-k\epsilon = 1$$

$$k = -\frac{1}{\epsilon} = -\frac{1}{4 \times 10^{-9}} = -2.5 \times 10^8 \text{ (V/msec)} \quad \times$$

$$\oint_S \vec{D} \cdot \hat{u}_N dS = Q_{\text{inside}}$$

전기 Gauss 법칙

$$\oint_S \vec{B} \cdot \hat{u}_N dS = 0$$

자기 "

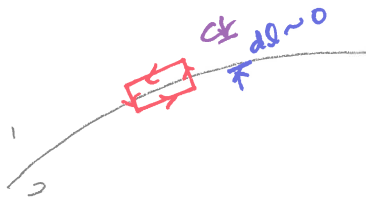
$$\oint_C \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_S \vec{B} \cdot \hat{u}_N dS$$

Faraday "

$$\oint_C \vec{H} \cdot d\vec{l} = I_{\text{inside}} + \frac{d}{dt} \int_S \vec{D} \cdot \hat{u}_N dS$$

Maxwell-Ampere "

정자기학 Maxwell Equation

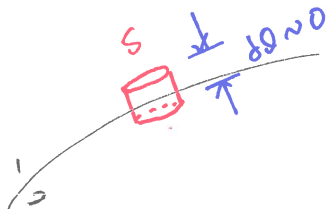


Faraday 법칙을 사용하면 $E_{1,\parallel} = E_{2,\parallel}$

p140 (22)

Maxwell-Ampere 법칙을 사용 ($K=0$), $H_{1,\parallel} = H_{2,\parallel}$ p264 (25)

$$H_{1,\parallel} - H_{2,\parallel} = K$$



자기 Gauss 법칙을 사용하면

$$B_{1,\perp} = B_{2,\perp}$$

p264 (22)

전기 Gauss 법칙을 사용하면

$$D_{1,\perp} - D_{2,\perp} = \sigma$$

p141 (34)

* 안전도체 ($\sigma(\text{전도체}) = \infty$) : \vec{J} 가 유한

$$\vec{J} = \sigma \vec{E}$$

$\vec{E} = 0$: 안전도체 내부에서 $\vec{E} = 0$ 이다.

$\vec{D} = 0$: " $\vec{B} = 0$ "

$$\text{From } \vec{\nabla} \cdot \vec{D} = \rho$$

$\rho = 0$: " 외부전하밀도가 0 이다.

\Rightarrow 전하는 항상 표면에 있다.

$$\text{From } \vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$\vec{B} = \vec{H} = 0$: " $\vec{B} = \vec{H} = 0$ 이다.

$$\text{From } \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$\vec{J} = 0$: " 외부전류 밀도는 0 이다

\Rightarrow 전류는 항상 표면으로 흐른다. (Skin effect)

2: 안전도제

$$E_{\perp} = 0$$

$$H_{\perp} = K \left\{ \begin{array}{l} \vec{H}_{\perp} = \vec{K} \times \hat{u}_N \\ \hat{u}_N : \text{도체 표면 밖으로 향하는} \\ \text{단위 법선 vector} \end{array} \right.$$

$$D_{\perp} = 0$$

$$B_{\perp} = 0$$

(응용 예제 9.5)

$$\begin{array}{l}
 1 \quad \epsilon_{r,1} = 4, \mu_{r,1} = 2, \sigma_1 = 0 \\
 2 \quad \epsilon_{r,2} = 2, \mu_{r,2} = 3, \sigma_2 = 0
 \end{array}$$

↑ $\hat{u}_N = 0.64\hat{x} + 0.6\hat{y} - 0.48\hat{z}$

$$\vec{B}_1 = (\hat{x} - 2\hat{y} + 3\hat{z}) \sin(300t) \text{ (T)}$$

$$(a) \vec{B}_{1,\perp} = (\vec{B}_1 \cdot \hat{u}_N) \hat{u}_N$$

$$= -2 \sin(300t) (0.64\hat{x} + 0.6\hat{y} - 0.48\hat{z})$$

$$|\vec{B}_{1,\perp} \text{의 진폭}| = 2 \text{ (T)}$$

$$(b) \vec{B}_{1,\parallel} = \vec{B}_1 - \vec{B}_{1,\perp}$$

$$= (2.28\hat{x} - 0.8\hat{y} + 2.04\hat{z}) \sin(300t)$$

$$|\vec{B}_{1,\parallel} \text{의 진폭}| = \sqrt{2.28^2 + 0.8^2 + 2.04^2} = 3.16 \text{ (T)}$$

$$(c) \vec{B}_{2,\perp} = \vec{B}_{1,\perp} = -2 \sin(300t) (0.64\hat{x} + 0.6\hat{y} - 0.48\hat{z})$$

$$|\vec{B}_{2,\perp} \text{의 진폭}| = 2 \text{ (T)}$$

$$(d) \quad \vec{H}_{2,||} = \vec{H}_{1,||} \quad (\vec{K} = 0)$$

$$\frac{1}{\mu_2} \vec{B}_{2,||} = \frac{1}{\mu_1} \vec{B}_{1,||}$$

$$\vec{B}_{2,||} = \frac{\mu_2}{\mu_1} \vec{B}_{1,||}$$

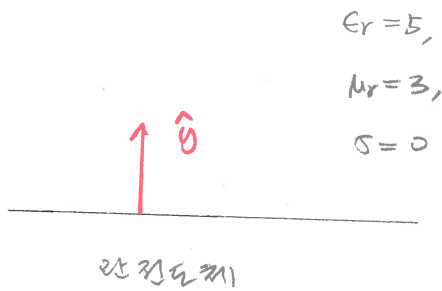
$$= \frac{3}{2} (2.28 \hat{x} - 0.8 \hat{y} + 2.04 \hat{z}) \sin(300\pi t)$$

$$\vec{B}_2 = \vec{B}_{2,\perp} + \vec{B}_{2,||}$$

$$= (2.14 \hat{x} - 2.4 \hat{y} + 4.02 \hat{z}) \sin(300\pi t)$$

$$|\vec{B}_2 \text{의 진폭}| = \sqrt{2.14^2 + 2.4^2 + 4.02^2} = 5.15 \text{ (T)}$$

(응용예제 9.6)



$$\epsilon_r = 5,$$

$$\mu_r = 3,$$

$$\sigma = 0$$

$$\epsilon_0 = 8.84 \times 10^{-12} \text{ (C}^2/\text{Nm}^2\text{)}$$

$$t = 6 \times 10^{-9} \text{ (sec)} \quad P(2, 0, 0.2) : \text{점}$$

$$\vec{E} = 20 \cos[(2 \times 10^8)t - 2.58z] \hat{y} \text{ (V/m)}$$

$$(a) \vec{D} = \epsilon \vec{E}$$

$$= 20 \epsilon_r \epsilon_0 \cos[(2 \times 10^8)t - 2.58z] \hat{y} \text{ (C/m}^2\text{)}$$

$$\sigma = D_{y,z}$$

$$= 20 \epsilon_r \epsilon_0 \cos[(2 \times 10^8)t - 2.58z]$$

$$= 20 \times 5 \times 8.84 \times 10^{-12} \cos[1.2 - 0.2 \times 2.58]$$

$$= 0.805 \times 10^{-9} \text{ (C/m}^2\text{)}$$

$$= 0.805 \text{ (nC/m}^2\text{)}$$

$$(b) \quad \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -20 \times 2.58 \sin[(2 \times 10^8)t - 2.58z] \hat{x}$$

$$\Rightarrow \frac{\partial \vec{H}}{\partial t} = \frac{20 \times 2.58}{\mu} \sin[(2 \times 10^8)t - 2.58z] \hat{x}$$

$$\Rightarrow \vec{H} = - \frac{20 \times 2.58}{2 \times 10^8 \times \mu} \cos[(2 \times 10^8)t - 2.58z] \hat{x}$$

0.0624

$$= -0.0624 \cos(1.2 - 0.3 \times 2.58) \hat{x}$$

$$= -0.0623 \hat{x} \quad (\text{A/m})$$

$$= -62.3 \hat{x} \quad (\text{mA/m})$$

$$(c) \quad \vec{H}_{1//}$$

$$= -62.3 \hat{x} = \vec{K} \times \hat{y}$$

$$\vec{K} = 62.3 \hat{z} \quad (\text{mA/m})$$

§ retarded electric and magnetic potentials

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = -\vec{\nabla} \times (\vec{\nabla} V) = 0$$

From Maxwell Eq.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$\Rightarrow \vec{E} = -\vec{\nabla} V$ is something wrong!!

$$\vec{E} = -\vec{\nabla} V + \vec{N}$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = \vec{\nabla} \times \vec{N} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{A}) = \vec{\nabla} \times \left(-\frac{\partial \vec{A}}{\partial t} \right)$$

$$\Rightarrow \vec{N} = -\frac{\partial \vec{A}}{\partial t}$$

$$\Rightarrow \boxed{\begin{aligned} \vec{E} &= -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \\ \vec{B} &= \vec{\nabla} \times \vec{A} \end{aligned}}$$

note) $\{V, \vec{A}\}$ 은 스칼라 $\{\vec{E}, \vec{B}\}$ 은 벡터이다.

* gauge symmetry

$$\vec{A}' = \vec{A} - \vec{\nabla}\Lambda, \quad V' = V + \frac{\partial\Lambda}{\partial t} \quad \text{같은 } \vec{E} \text{ 와 } \vec{B} \text{ 는 된다}$$

즉 같은 \vec{E} 와 \vec{B} 를 주는 $\{V, \vec{A}\}$ 는 유일하지 않다.

(증명)

$$\vec{B}' = \vec{\nabla} \times \vec{A}'$$

$$= \vec{\nabla} \times (\vec{A} - \vec{\nabla}\Lambda)$$

$$= \vec{\nabla} \times \vec{A} - \underbrace{\vec{\nabla} \times (\vec{\nabla}\Lambda)}_{=0}$$

$$= \vec{\nabla} \times \vec{A}$$

$$= \vec{B}$$

$$\vec{E}' = -\vec{\nabla}V' - \frac{\partial\vec{A}'}{\partial t}$$

$$= -\vec{\nabla}\left(V + \frac{\partial\Lambda}{\partial t}\right) - \frac{\partial}{\partial t}(\vec{A} - \vec{\nabla}\Lambda)$$

$$= -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t} + \underbrace{\left(-\vec{\nabla}\frac{\partial\Lambda}{\partial t} + \frac{\partial}{\partial t}\vec{\nabla}\Lambda\right)}_{=0}$$

$$= -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$$

$$= \vec{E}$$

✕

* 그러므로 $\{V, \vec{A}\}$ 는 유일하게 결정하기 위해서는

1번 $\pi\pi$ 이 더 필요하다. (gauge fixing)

Maxwell Eq:

$$\vec{\nabla} \cdot \vec{D} = \rho, \quad \vec{\nabla} \cdot \vec{B} = 0, \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\textcircled{1} \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$$

vector identity

$$\textcircled{2} \quad \vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{D} = \epsilon \vec{\nabla} \cdot \vec{E}$$

$$= \epsilon \vec{\nabla} \cdot \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right)$$

$$= \epsilon \left[-\underbrace{\vec{\nabla} \cdot (\vec{\nabla} V)}_{\vec{\nabla} \cdot V} - \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A}) \right] = \rho$$

$$\vec{\nabla} \cdot V$$

$$\Rightarrow \quad \vec{\nabla} V + \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon}$$

$$\textcircled{3} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right)$$

$$= -\underbrace{\vec{\nabla} \times (\vec{\nabla} V)}_{=0} - \frac{\partial}{\partial t} \underbrace{(\vec{\nabla} \times \vec{A})}_{\vec{B}}$$

$$= -\frac{\partial}{\partial t} \vec{B}$$

$$\textcircled{4} \quad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{\mu} \vec{\nabla} \times \vec{B}$$

$$= \frac{1}{\mu} \underline{\vec{\nabla} \times (\vec{\nabla} \times \vec{A})}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}$$

$$= \frac{1}{\mu} [\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}]$$

$$\frac{\partial \vec{D}}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t} = \epsilon \frac{\partial}{\partial t} \left[-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right]$$

$$= -\epsilon \frac{\partial}{\partial t} \left[\vec{\nabla} V + \frac{\partial \vec{A}}{\partial t} \right]$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A} = -\mu \epsilon \left[\vec{\nabla} \frac{\partial V}{\partial t} + \frac{\partial^2 \vec{A}}{\partial t^2} \right] + \mu \vec{J}$$

그러므로 Maxwell 방정식을 만족하기 위해서는

$$\vec{\nabla}^2 V + \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A} = -\mu \epsilon \left[\vec{\nabla} \frac{\partial V}{\partial t} + \frac{\partial^2 \vec{A}}{\partial t^2} \right] + \mu \vec{J}$$

를 만족해야 한다.

Gauge fixing

$$\vec{\nabla} \cdot \vec{A} = -\mu \epsilon \frac{\partial V}{\partial t}$$

$$\Rightarrow \begin{cases} \vec{\nabla}^2 V = \mu \epsilon \frac{\partial V}{\partial t} - \frac{\rho}{\epsilon} \\ \vec{\nabla}^2 \vec{A} = \mu \epsilon \frac{\partial \vec{A}}{\partial t} - \mu \vec{J} \end{cases}$$

(i) $\frac{\rho}{\epsilon}, \mu \vec{J}$: source term

(ii) $\rho = \vec{J} = 0$ case

$$\vec{\nabla}^2 V = \mu \epsilon \frac{\partial V}{\partial t}$$

$$\vec{\nabla}^2 \vec{A} = \mu \epsilon \frac{\partial \vec{A}}{\partial t}$$

파동 방정식

$$\vec{\nabla}^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

v : 파동의 속도

전자파의 속도

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

진공

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ (m/sec)} = c \quad (\text{광속})$$

\Rightarrow 전자파 = 빛

* 전자기파의 진행 속도가 유한하므로 source 에서 거리가 멀어질 때

V 와 \vec{A} 를 계산하는데 시간이 걸린다.

이 문제를 해결하기 위하여 retarded potential 을 정의한다.

$$V_{\text{ret}}[\vec{r}, t] = V[\vec{r}, t - \frac{r}{c}]$$

r : source 와 P점 사이의 거리

$$\vec{A}_{\text{ret}}[\vec{r}, t] = \vec{A}[\vec{r}, t - \frac{r}{c}]$$

retarded potential

그러므로 P점이 source 에서 멀거나 혹은 source 가 time-dependent

하면

$$\vec{E} = -\vec{\nabla} V_{\text{ret}} - \frac{\partial \vec{A}_{\text{ret}}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}_{\text{ret}}$$

를 사용하여야 동적 장을 계산할 수 있다.

(응용예제 9.7)

$$P_+(0, 0, 1.5) \quad g_+(t) = 4 \cos(10^8 \pi t) \times 10^{-6} \text{ (C)}$$

$$P_-(0, 0, -1.5) \quad g_-(t) = -4 \cos(10^8 \pi t) \times 10^{-6} \text{ (C)}$$

$$t = 15 \times 10^{-9} \text{ (sec)}$$

$$(a) \quad P(r=450, \theta=0, \phi=0) = P(0, 0, 450)$$

$$r_+ = \overline{P_+P} = 448.5 \quad r_- = \overline{P_-P} = 451.5$$

$$V_{\text{ret}}(t) = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r_+} g_+(t - \frac{r_+}{c}) + \frac{1}{r_-} g_-(t - \frac{r_-}{c}) \right]$$

$$V(t) = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r_+} g_+(t) + \frac{1}{r_-} g_-(t) \right]$$

$$V_{\text{ret}}(t = 15 \times 10^{-9}) = 160.002 \text{ (V)}$$

$$V(t = 15 \times 10^{-9}) = 0$$

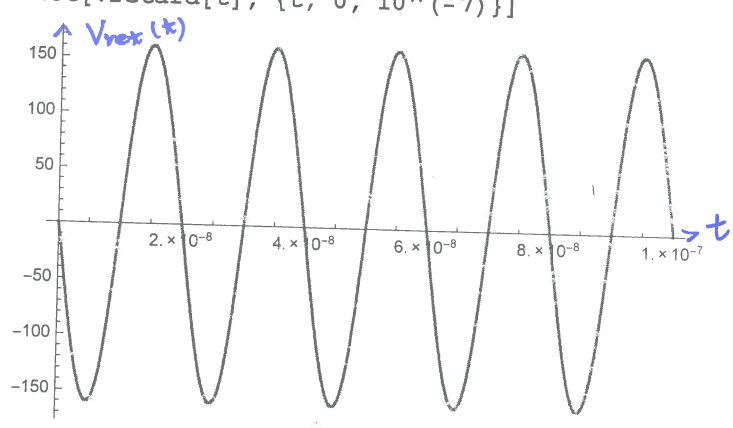
```

rp = 448.5; rm = 451.5; c = 3 × 10^8;
qp[t_] := 4 × 10^(-6) Cos[10^8 Pi t];
qm[t_] := -qp[t];
Vretard[t_] := 9 × 10^9 (qp[t - rp / c] / rp + qm[t - rm / c] / rm);
Vnormal[t_] := 9 × 10^9 (qp[t] / rp + qm[t] / rm);
a = Vretard[15 × 10^(-9)];
b = Vnormal[15 × 10^(-9)];
Print["retard = ", a]
Print["normal = ", b]
(**Plot[{Vretard[t], Vnormal[t]}, {t, 0, 10^(-7)}, PlotRange -> {0, 1}])**

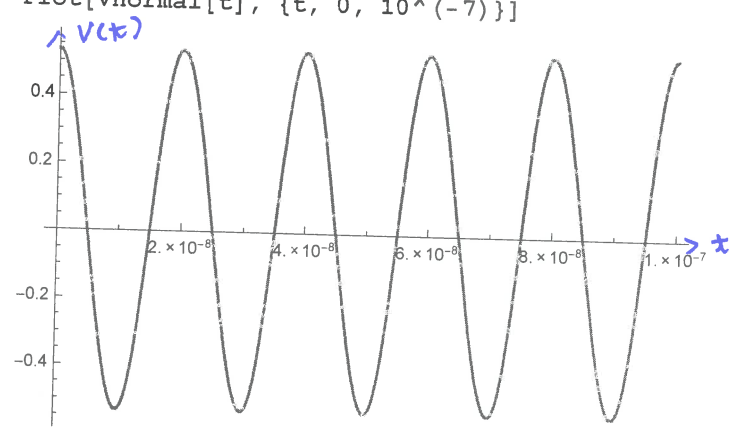
retard = 160.002
normal = 0.

```

`Plot[Vretard[t], {t, 0, 10-7}]`



`Plot[Vnormal[t], {t, 0, 10-7}]`



$$(b) P(r=450, \theta=90^\circ, \phi=0) = P(450, 0, 0)$$

$$r_+ = \sqrt{450^2 + 1.5^2} = 450.002 = r_-$$

$$V_{\text{ret}}(t = 15 \times 10^{-9}) = 0$$

$$V(t = 15 \times 10^{-9}) = 0$$

```

: rp = 450.002; rm = 450.002; c = 3 × 10^8;
qp[t_] := 4 × 10^(-6) Cos[10^8 Pi t];
qm[t_] := - qp[t];
Vretard[t_] := 9 × 10^9 (qp[t - rp / c] / rp + qm[t - rm / c] / rm);
Vnormal[t_] := 9 × 10^9 (qp[t] / rp + qm[t] / rm);
a = Vretard[15 × 10^(-9)];
b = Vnormal[15 × 10^(-9)];
Print["retard = ", a]
Print["normal = ", b]

retard = 0.
normal = 0.

```

$$c) P(r=450, \theta=45^\circ, \phi=0) = P\left(\frac{450}{\sqrt{2}}, 0, \frac{450}{\sqrt{2}}\right)$$

$$r_+ = \sqrt{\left(\frac{450}{\sqrt{2}}\right)^2 + \left(\frac{450}{\sqrt{2}} - 1.5\right)^2} = 448.941$$

$$r_- = \sqrt{\left(\frac{450}{\sqrt{2}}\right)^2 + \left(\frac{450}{\sqrt{2}} + 1.5\right)^2} = 451.062$$

$$V_{\text{ret}}(t = 15 \times 10^{-9}) = 143.351 \text{ (V)}$$

$$V(t = 15 \times 10^{-9}) = 0$$

```

rp = 448.941; rm = 451.062; c = 3 × 10^8;
qp[t_] := 4 × 10^(-6) Cos[10^8 Pi t];
qm[t_] := -qp[t];
Vretard[t_] := 9 × 10^9 (qp[t - rp / c] / rp + qm[t - rm / c] / rm);
Vnormal[t_] := 9 × 10^9 (qp[t] / rp + qm[t] / rm);
a = Vretard[15 × 10^(-9)];
b = Vnormal[15 × 10^(-9)];
Print["retard = ", a]
Print["normal = ", b]

retard = 143.351
normal = 0.

```

X