CH 8 자기력, 자성체, 인덕턴스

8 운동하는 전하에 작용하는 힘

$$\vec{F}_B = q\,\vec{v}\times\vec{B} \qquad \vec{F}_E = q\vec{E} \text{ (전기력)}$$

자기력

① $F_B = qvB\sin\theta$

② $\vec{F}_B \perp \vec{B}$ and $\vec{F}_B \perp \vec{v}$

③ if $\vec{v} \parallel \vec{B}$, $\vec{F}_B = 0$

④ $W_B = \int \vec{F}_B \cdot d\vec{s} = 0$

* 자기력이 전하에 한 일은 "0"이다.

※ 일반적으로 $\vec{E}$ 와 $\vec{B}$ 가 공존하는 계에서 전하가 받는 힘

$$\vec{F} = \vec{F}_E + \vec{F}_B = q(\vec{E} + \vec{v}\times\vec{B})$$ Lorentz force

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(응용예제 8.1)

$q = 18\times10^{-9}$ (C)

$\vec{v} = (5\times10^{6})(0.6\hat{x} + 0.75\hat{y} + 0.3\hat{z})$ (m/sec)

(a) $\vec{B} = 10^{-3}(-3\hat{x} + 4\hat{y} + 6\hat{z})$ (T)

$\vec{F}_B = q\,\vec{v}\times\vec{B}$

$$= 18\times10^{-9}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 5\times0.6\times10^{6} & 5\times0.75\times10^{6} & 5\times0.3\times10^{6} \\ -3\times10^{-3} & 4\times10^{-3} & 6\times10^{-3} \end{vmatrix}$$

$$= 18\times10^{-9}\left[\begin{array}{l} \hat{x}\,[30\times0.75\times10^{3} - 20\times0.3\times10^{3}] \\ +\hat{y}\,[-15\times0.3\times10^{3} - 30\times0.6\times10^{3}] \\ +\hat{z}\,[20\times0.6\times10^{3} + 15\times0.75\times10^{3}] \end{array}\right]$$

$$= 18\times10^{-6}[16.5\hat{x} - 22.5\hat{y} + 23.25\hat{z}] \quad (N)$$

$$= 297\hat{x} - 405\hat{y} + 418.5\hat{z} \quad (\mu N)$$

$$F_B = \sqrt{(297)^2 + (405)^2 + (418.5)^2} = 653.74\ (\mu N)$$

(b) $\vec{E} = 10^3(-3\hat{x} + 4\hat{y} + 6\hat{z})$ (V/m)

$$\vec{F}_E = q\vec{E}$$

$$= 18 \times 10^{-6}(-3\hat{x} + 4\hat{y} + 6\hat{z}) \quad (N)$$

$$= -54\hat{x} + 72\hat{y} + 108\hat{z} \quad (\mu N)$$

$$F_E = \sqrt{54^2 + 72^2 + 108^2} = 140.584 \quad (\mu N)$$

(c) $\vec{F}_B + \vec{F}_E$

$$= (297-54)\hat{x} + (-405+72)\hat{y} + (418.5+108)\hat{z} \quad (\mu N)$$

$$= 243\hat{x} - 333\hat{y} + 526.5\hat{z} \quad (\mu N)$$

$$F = \sqrt{243^2 + 333^2 + 526.5^2} = 668.685 \quad (\mu N)$$

※

§ 전체에 작용하는 자기력

미소 전하가 받는 미소힘

$$d\vec{F} = dq\ \vec{v} \times \vec{B}$$

$$dq = \rho\, dv \qquad (dv:\ \text{미소 부피})$$

$$\Rightarrow\ d\vec{F} = \rho\, \vec{v} \times \vec{B}\, dv$$

$$\vec{J} = \rho \vec{v}$$ p177 (2)

$$\Rightarrow\ d\vec{F} = \vec{J} \times \vec{B}\, dv$$

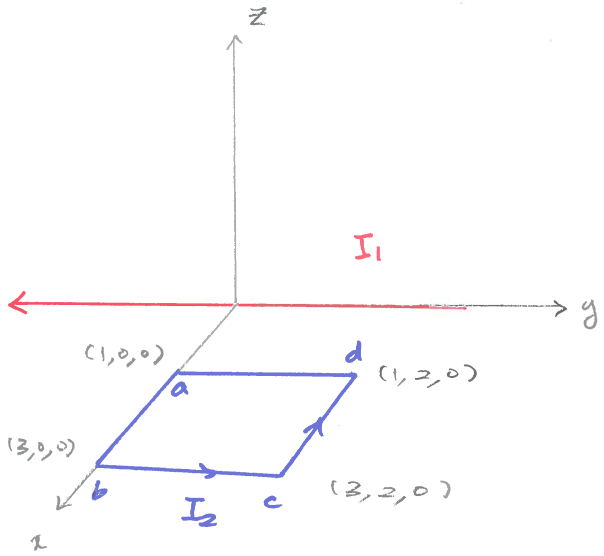
$$\vec{J}\, dv = I\, d\vec{l} = \vec{K}\, ds$$

$$\Rightarrow\ d\vec{F} = I\, d\vec{l} \times \vec{B} = \vec{K} \times \vec{B}\, ds$$

$$\boxed{\vec{F} = \int_C I\, d\vec{l} \times \vec{B} = \int_S \vec{K} \times \vec{B}\, ds = \int_v \vec{J} \times \vec{B}\, dv}$$

선전류　　면전류　　부피전류

(예제 8.1)



$I_1 = 15\ (A)$

$I_2 = 2\ (mA)$

폐회로에 작용하는 힘?

I_1 이 만드는 $\vec{B}$ ⇒

$$\vec{B} = \frac{\mu_0 I_1}{2\pi x}\hat{z} = \frac{3\times10^{-6}}{x}\hat{z}\ (T) \quad - ①$$

$$\vec{F} = I_2 \oint d\vec{l}\times\vec{B}$$

$$= I_2\int_a^b d\vec{l}\times\vec{B} + I_2\int_b^c d\vec{l}\times\vec{B} + I_2\int_c^d d\vec{l}\times\vec{B} + I_2\int_d^a d\vec{l}\times\vec{B} \quad - ②$$

$$\vec{F}_1 = I_2\int_a^b d\vec{l}\times\vec{B}$$

$$= 6\times10^{-9}\int_a^b d\vec{l}\times\frac{1}{x}\hat{z} \qquad d\vec{l} = dx\,\hat{x}$$

$$= 6\times10^{-9}\int_1^3 \frac{1}{x}dx\ (-\hat{y})$$

$$= -6\ln 3\times10^{-9}\,\hat{y}\quad (N)$$

$$= -6\ln 3\,\hat{y}\quad (nN) \quad - ③$$

$$\vec{F}_2 = I_2 \int_b^c d\vec{l} \times \vec{B}$$

$$= 6\times10^{-9} \int_b^c d\vec{l} \times \frac{1}{x}\hat{z} \qquad (x=3,\ d\vec{l} = dy\,\hat{y})$$

$$= 2\times10^{-9} \int_0^2 dy \quad \hat{x}$$

$$= 4\times10^{-9}\ \hat{x} \qquad (N)$$

$$= 4\hat{x} \quad (mN) \qquad — ④$$

$$\vec{F}_3 = I_2 \int_c^d d\vec{l} \times \vec{B}$$

$$= 6\times10^{-9} \int_c^d d\vec{l} \times \frac{1}{x}\hat{z} \qquad (d\vec{l} = dx\,\hat{x})$$

$$= 6\times10^{-9} \int_3^1 \frac{dx}{x} \quad (-\hat{y})$$

$$= +6\times10^{-9} \int_1^3 \frac{dx}{x} \quad \hat{y}$$

$$= 6\ln 3\times10^{-9}\ \hat{y} \qquad (N)$$

$$= 6\ln 3\ \hat{y} \quad (mN) \qquad — ⑤$$

$$\vec{F}_4 = I_2 \int_d^a d\vec{l} \times \vec{B}$$

$$= 6\times10^{-9} \int_d^a d\vec{l} \times \frac{1}{x}\hat{z} \qquad (x=1,\ d\vec{l} = dy\,\hat{y})$$

$$= 6\times10^{-9} \int_2^0 dy \quad \hat{x}$$

$$= -12\times10^{-9}\ \hat{x} \qquad (N)$$

$$= -12\hat{x} \quad (mN) \qquad — ⑥$$

③, ④, ⑤, ⑥ → ②

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = -8\,\hat{x} \quad (mN)$$ ※

(응용예제 8.2)

$$I = 12\ (A), \quad \vec{B} = 10^{-3}(-2\hat{x} + 3\hat{y} + 4\hat{z})\ (T)$$

(a) $\vec{l} = \overrightarrow{AB} = \hat{x}$

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

$$= 12 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ -2\times10^{-3} & 3\times10^{-3} & 4\times10^{-3} \end{vmatrix}$$

$$= 10^{-3}\left[-48\hat{y} + 36\hat{z}\right] \quad (N)$$

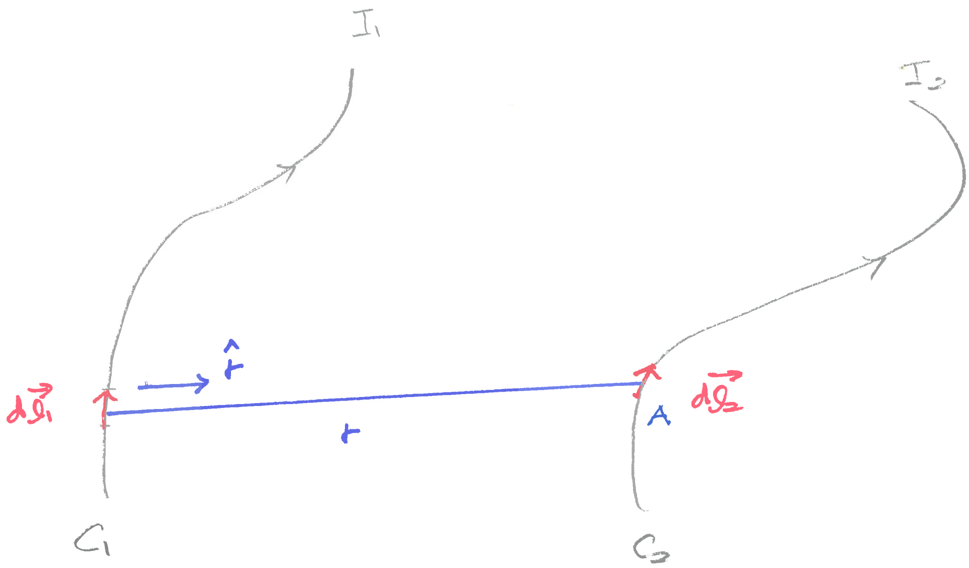
$$= -48\hat{y} + 36\hat{z} \quad (mN)$$

(b) $\vec{l} = \overrightarrow{AB} = 2\hat{x} + 4\hat{y} + 5\hat{z}$

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

$$= 12\hat{x} - 216\hat{y} + 168\hat{z} \quad (mN)$$

§ 전류 사이에 작용하는 자기력



$\vec{F}_{12}$: I_1 에 의하여 I_2 가 받는 힘

A 에서의 $\vec{B}$: From Biot – Savart law

$$\vec{B} = \int_{C_1} \frac{\mu_0 I_1 \, d\vec{l}_1 \times \hat{r}}{4\pi r^2}$$

$\vec{B}$ 에 의하여 $d\vec{l}_2$ 가 받는 힘 : $d\vec{F}_{12}$

$$d\vec{F}_{12} = I_2 \, d\vec{l}_2 \times \vec{B}$$

$$= I_2 \, d\vec{l}_2 \times \int_{C_1} \frac{\mu_0 I_1 \, d\vec{l}_1 \times \hat{r}}{4\pi r^2}$$

$$= \int_{C_1} \frac{\mu_0 I_1 I_2 \, d\vec{l}_2 \times (d\vec{l}_1 \times \hat{r})}{4\pi r^2}$$

도선 I_2 가 받는 total 자기력 : $\vec{F}_{12}$

$$\boxed{\vec{F}_{12} = \int_{C_2} d\vec{F}_{12} = \int_{C_2} \int_{C_1} \frac{\mu_0 I_1 I_2 \, d\vec{l}_2 \times (d\vec{l}_1 \times \hat{r})}{4\pi r^2}}$$

note) $\vec{F}_{21} = -\vec{F}_{12}$

note)

$$d^2\vec{F}_{12} \neq -\, d^2\vec{F}_{21}$$

$$d^2\vec{F}_{12} = \frac{\mu_0 I_1 I_2 \, d\vec{l}_2 \times (d\vec{l}_1 \times \hat{r})}{4\pi r^2}$$

$$d^2\vec{F}_{21} = \frac{\mu_0 I_1 I_2 \, d\vec{l}_1 \times [d\vec{l}_2 \times (-\hat{r})]}{4\pi r^2}$$

$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$ 을 이용해서 증명하라!!

(예제 8.2) 두 미소전류 사이의 자기력.

$$d^2\vec{F} = \frac{\mu_0 I_1 I_2\, d\vec{l}_2 \times (d\vec{l}_1 \times \hat{r})}{4\pi r^2}$$

$$I_1 d\vec{l}_1 = -3\hat{y}\ (\mathrm{Am}) \ : \ P_1(5, 2, 1)$$

$$I_2 d\vec{l}_2 = -4\hat{z}\ (\mathrm{Am}) \ : \ P_2(1, 8, 5)$$

$$\vec{r} \equiv \overrightarrow{P_1P_2} = -4\hat{x} + 6\hat{y} + 4\hat{z}$$

$$r = \sqrt{4^2 + 6^2 + 4^2} = 2\sqrt{17}$$

$$\hat{r} = \frac{1}{2\sqrt{17}}(-4\hat{x} + 6\hat{y} + 4\hat{z}) = \frac{1}{\sqrt{17}}(-2\hat{x} + 3\hat{y} + 2\hat{z})$$

$$I_1 I_2\, d\vec{l}_2 \times (d\vec{l}_1 \times \hat{r})$$

$$= -4\hat{z} \times \left[-3\hat{y} \times \frac{1}{\sqrt{17}}(-2\hat{x} + 3\hat{y} + 2\hat{z})\right]$$

$$= \frac{12}{\sqrt{17}}\hat{z} \times (2\hat{z} + 2\hat{x})$$

$$= \frac{24}{\sqrt{17}}\hat{y}$$

$$d^2\vec{F}_{12} = \frac{4\pi \times 10^{-7}}{4\pi \times 68}\ \frac{24}{\sqrt{17}}\hat{y}$$

$$= 8.56 \times 10^{-9}\ \hat{y} \quad (N)$$

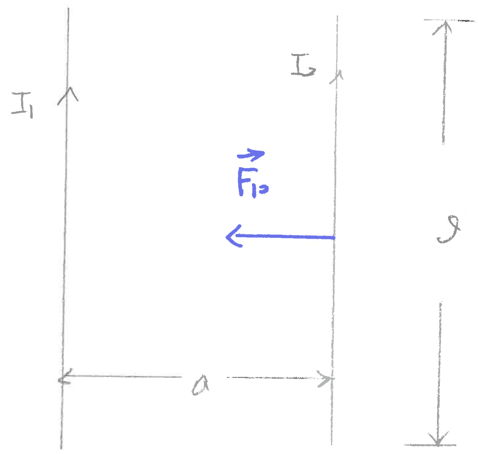
$$= 8.56\ \hat{y} \quad (nN) \quad ※$$

note)

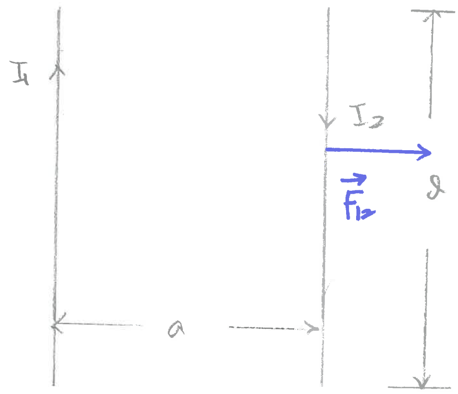
$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$ 를 이용하여

다시 풀어 주어라

* 평행 도선 사이의 자기력



$$\frac{F_{12}}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$



$$\frac{F_{12}}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

* 같은 방향의 전류는 서로 끌어당기고, 반대 방향의 전류는 서로 민다.

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(응용예제 8.4)

$I_1 \Delta \vec{l}_1 = 3\times 10^{-6}\ \hat{y}$ (Am) $\quad P_1(1,0,0)$

$I_2 \Delta \vec{l}_2 = 3\times 10^{-6}\ (-0.5\hat{x} + 0.4\hat{y} + 0.3\hat{z})$ (Am) $\quad P_2(2,2,2)$

(a)

$$d^2\vec{F}_{12} = \frac{\mu_0 I_1 I_2\, d\vec{l}_2 \times (d\vec{l}_1 \times \hat{r})}{4\pi r^2}$$

$$r = \sqrt{1^2 + 2^2 + 2^2} = 3$$

$$\hat{r} = \frac{1}{3}\overrightarrow{P_1P_2} = \frac{1}{3}(\hat{x} + 2\hat{y} + 2\hat{z})$$

$$I_1 I_2\, d\vec{l}_2 \times (d\vec{l}_1 \times \hat{r})$$

$$= 3\times 10^{-6}(-0.5\hat{x} + 0.4\hat{y} + 0.3\hat{z}) \times \left[(3\times 10^{-6})\hat{y} \times \frac{1}{3}(\hat{x} + 2\hat{y} + 2\hat{z})\right]$$

$$= 3\times 10^{-12}(-0.5\hat{x} + 0.4\hat{y} + 0.3\hat{z}) \times (2\hat{x} - \hat{z})$$

$$= 3\times 10^{-12} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -0.5 & 0.4 & 0.3 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= 3\times 10^{-12}\left[-0.4\hat{x} + 0.1\hat{y} - 0.8\hat{z}\right]$$

$$\Rightarrow d^2\vec{F}_{12} = \frac{4\pi\times 10^{-7}}{4\pi\times 9}\, 3\times 10^{-12}\left[-0.4\hat{x} + 0.1\hat{y} - 0.8\hat{z}\right]$$

$$= 10^{-20}\times\left[-1.333\hat{x} + 0.333\hat{y} - 2.667\hat{z}\right] \text{ (N)}$$

(b)

$$d^2\vec{F}_{21} = -\frac{\mu_0 I_1 I_2 \, d\vec{l}_1 \times (d\vec{l}_2 \times \hat{r})}{4\pi r^2}$$

$$I_2 d\vec{l}_2 \times \hat{r}$$

$$= 10^{-6} \times \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -0.5 & 0.4 & 0.3 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= 10^{-6} \times [0.2\hat{x} + 1.3\hat{y} - 1.4\hat{z}]$$

$$I_1 I_2 \, d\vec{l}_1 \times (d\vec{l}_2 \times \hat{r})$$

$$= 3 \times 10^{-12} \; \hat{y} \times (0.2\hat{x} + 1.3\hat{y} - 1.4\hat{z})$$

$$= 3 \times 10^{-12} (-1.4\hat{x} - 0.2\hat{z})$$

$$= -3 \times 10^{-12} (1.4\hat{x} + 0.2\hat{z})$$

$$d^2\vec{F}_{21} = -\frac{4\pi \times 10^{-7}}{4\pi \times 9} (-3 \times 10^{-12}) (1.4\hat{x} + 0.2\hat{z})$$

$$= 10^{-20} \times [4.667\hat{x} + 0.667\hat{z}] \quad (N)$$

※

5 폐회로에 작용하는 힘과 회전력 (torque)

$$\vec{F} = I \oint_c d\vec{s} \times \vec{B}$$

$$= -I \oint \vec{B} \times d\vec{s}$$

If $\vec{B}$ = constant vector,

$$\vec{F} = -I\,\vec{B} \times \underbrace{\oint d\vec{s}}_{=0} = 0$$

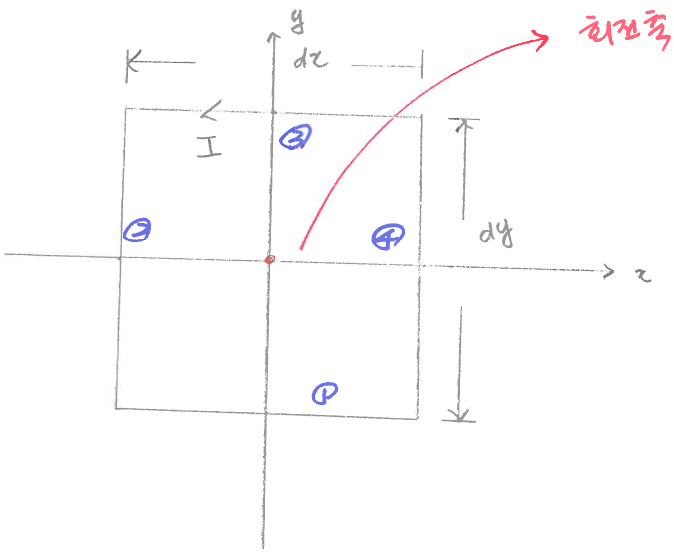
* 균일한 자계내의 폐회로에 작용하는 자기력은 0 이다.

* Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$\vec{r}$: 회전축으로 부터 위치 까지의 vector

$\vec{F}$: 위치가 받는 힘



① 이 받는 힘 $d\vec{F}_1 = I\,dx\,\hat{x} \times \vec{B}$

$$= I\,dx\,(-B_z\,\hat{y} + B_y\,\hat{z})$$

① 이 받는 torque $d\vec{\tau}_1 = \left(-\frac{1}{2}dy\,\hat{y}\right) \times d\vec{F}_1$

$$= -\frac{I}{2}\,dx\,dy\,B_y\,\hat{x}$$

③ 이 받는 힘 $d\vec{F}_3 = -I\,dx\,\hat{x} \times \vec{B} = -d\vec{F}_1$

③ 이 받는 torque $d\vec{\tau}_3 = \left(\frac{1}{2}dy\,\hat{y}\right) \times d\vec{F}_3 = -\frac{I}{2}\,dx\,dy\,B_y\,\hat{x}$

$$\Rightarrow d\vec{\tau}_1 + d\vec{\tau}_3 = -I\,dx\,dy\,B_y\,\hat{x}$$

③ 가 받는 힘: $d\vec{F_3} = I(-dy\,\hat{y}) \times \vec{B} = -I\,dy\,(B_z\,\hat{x} - B_x\,\hat{z})$

③ 가 받는 torque $d\vec{\tau_3} = -\frac{dx}{2}\,\hat{x} \times d\vec{F_3} = \frac{I}{2}\,dx\,dy\,B_x\,\hat{y}$

같은 방법으로 $d\vec{\tau_4} = d\vec{\tau_3}$

$$d\vec{\tau_3} + d\vec{\tau_4} = I\,dx\,dy\,B_x\,\hat{y}$$

$$\Rightarrow d\vec{\tau} = d\vec{\tau_1} + d\vec{\tau_2} + d\vec{\tau_3} + d\vec{\tau_4} = I\,\underbrace{dx\,dy}_{ds}\,(-B_y\,\hat{x} + B_x\,\hat{y})$$

$$= I\,ds\,\hat{z} \times (B_x\,\hat{x} + B_y\,\hat{y} + B_z\,\hat{z})$$

$$= I\,ds\,\hat{z} \times \vec{B}$$

$$d\vec{m} = I\,ds\,\hat{z} = I\,ds\,\hat{u}_{n} \quad \Rightarrow \quad d\vec{\tau} = d\vec{m} \times \vec{B}$$

$$\Rightarrow \quad \underline{\vec{m} = \int_S I\,ds\,\hat{u}_{n}}$$

자기 쌍극자 모멘트 (magnetic dipole moment)

① $[\vec{m}] = A\,m^2$ $\quad [\vec{p}] = C\,m$

② 평면 폐회로 : $\vec{m} = I\,S\,\hat{u}_{n}$

Then

$$\underline{\vec{\tau} = \vec{m} \times \vec{B}}$$

균일한 자기장 내에서 폐회로가 받는 torque

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(예제 8.3)

$S = 1 \times 2 = 2\ (cm^2)$

$I = 4\ (mA) = 4 \times 10^{-2}\ (A)$

$\hat{u}_N = \hat{z}$

$\vec{m} = I S\ \hat{u}_N = 8 \times 10^{-3}\ \hat{z}\ (Am^2)$

$\vec{B} = -0.6\hat{y} + 0.8\hat{z}\ (T)$

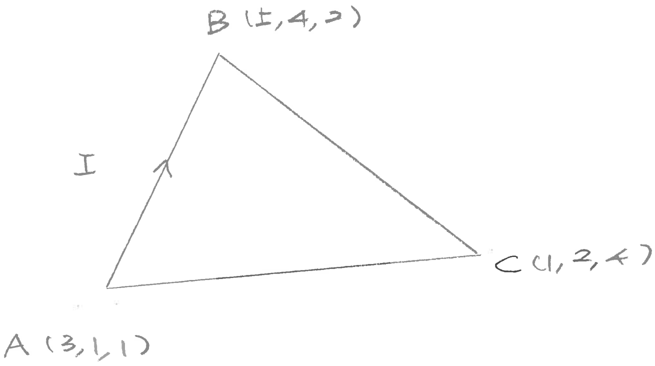
$\vec{\tau} = m \times \vec{B}$

$= 8 \times 10^{-3}\ \hat{z} \times (-0.6\hat{y} + 0.8\hat{z})$

$= 4.8 \times 10^{-3}\ \hat{x}\quad (Nm)$

$= 4.8\ \hat{x}\ (mNm)$ ※

(응용예제 8.5)



$I = 0.2\ (A)$

$$\vec{B} = 0.2\hat{x} - 0.1\hat{y} + 0.3\hat{z} \quad (T)$$

(a) $\vec{\ell} = \vec{BC} = -4\hat{x} - 2\hat{y} + 2\hat{z}$

$$\vec{F}_{BC} = I\,\vec{\ell} \times \vec{B}$$

$$= 0.2 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -4 & -2 & 2 \\ 0.2 & -0.1 & 0.3 \end{vmatrix}$$

$$= 0.2\left[-0.4\hat{x} + 1.6\hat{y} + 0.8\hat{z}\right]$$

$$= -0.08\hat{x} + 0.32\hat{y} + 0.16\hat{z} \quad (N)$$

(b) * AB

$$\vec{\ell} = \vec{AB} = 2\hat{x} + 3\hat{y} + \hat{z}$$

$$\vec{F}_{AB} = 0.2 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 1 \\ 0.2 & -0.1 & 0.3 \end{vmatrix}$$

$$= 0.2\left(\hat{x} - 0.4\hat{y} - 0.8\hat{z}\right)$$

$$= 0.2\hat{x} - 0.08\hat{y} - 0.16\hat{z} \quad (N)$$

* CA

$$\vec{l} = \vec{CA} = 2\hat{x} - \hat{y} - 3\hat{z}$$

$$\vec{F}_{CA} = 0.2 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & -1 & -3 \\ 0.2 & -0.1 & 0.3 \end{vmatrix}$$

$$= 0.2 \left[-0.6\hat{x} - 1.2\hat{y} \right]$$

$$= -0.12\hat{x} - 0.24\hat{y} \qquad (N)$$

$$\vec{F} = \vec{F}_{AB} + \vec{F}_{BC} + \vec{F}_{CA} = 0$$

(c)

* AB

$$\vec{r}_{AB} = \frac{1}{2}\vec{AB} = \frac{1}{2}(2\hat{x} + 3\hat{y} + \hat{z})$$

$$\vec{F}_{AB} = 0.2\hat{x} - 0.08\hat{y} - 0.16\hat{z}$$

$$\vec{\tau}_{AB} = \vec{r}_{AB} \times \vec{F}_{AB} = -0.2\hat{x} + 0.26\hat{y} - 0.38\hat{z} \quad (Nm)$$

* AC

$$\vec{r}_{AC} = \frac{1}{2}\vec{AC} = \frac{1}{2}(-2\hat{x} + \hat{y} + 3\hat{z})$$

$$\vec{F}_{CA} = -0.12\hat{x} - 0.24\hat{y}$$

$$\vec{\tau}_{AC} = \vec{r}_{AC} \times \vec{F}_{CA} = 0.36\hat{x} - 0.18\hat{y} + 0.3\hat{z} \quad (Nm)$$

* BC

$$D = \frac{B+C}{2} = D(3,3,3)$$

$$\vec{r}_{BC} = \vec{AD} = 2\hat{y} + 2\hat{z}$$

$$\vec{F}_{BC} = -0.08\hat{x} + 0.32\hat{y} + 0.16\hat{z}$$

$$\vec{\tau}_{BC} = \vec{r}_{BC} \times \vec{F}_{BC} = -0.32\hat{x} - 0.16\hat{y} + 0.16\hat{z} \quad (Nm)$$

$$\vec{\tau}_{A} = \vec{\tau}_{AB} + \vec{\tau}_{AC} + \vec{\tau}_{BC} = -0.16\hat{x} - 0.08\hat{y} + 0.08\hat{z} \quad (Nm)$$

(d) 질량중심에 대한 torque

$\vec{m} = I\, S\, \hat{u}_A$

$\vec{AB} \times \vec{AC}$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 1 \\ -2 & 1 & 3 \end{vmatrix}$$

$= 8\hat{x} - 8\hat{y} + 8\hat{z}$

$S\hat{u}_A = \frac{1}{2}(8\hat{x} - 8\hat{y} + 8\hat{z}) = 4(\hat{x} - \hat{y} + \hat{z}) \quad (m^2)$

$\vec{m} = 0.8(\hat{x} - \hat{y} + \hat{z}) \quad (Am^2)$

$\vec{\tau} = \vec{m} \times \vec{B}$

$$= 0.8 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & -1 & 1 \\ 0.2 & -0.1 & 0.3 \end{vmatrix}$$

$= 0.8(-0.2\hat{x} - 0.1\hat{y} + 0.1\hat{z})$

$= -0.16\hat{x} - 0.08\hat{y} + 0.08\hat{z} \quad (Nm)$ ※

5. 자성체의 성질

○ 반자성 (diamagnetic material)

외부에서 자기장을 가하면 반대방향으로 자기장이 생기도록 전자가 회전하는 물체

B_{appl} : 외부자기장

B_{int} : 물체 내의 자기장

$$B_{int} < B_{appl}$$

○ 상자성 (paramagnetic material)

외부에서 자기장을 가하면 같은 방향으로 자기장이 생기도록 전자가 회전하는 물체

$$B_{int} > B_{appl}$$

참고)
반자성이나 상자성 물체는 외부자기장이 없으면 자기 쌍극자 모멘트가 0 이다. 그러므로 외부에 자기장 $\vec{B}$ 를 주지 못한다.

○ 강자성 (ferromagnetic material)

외부 자기장이 없어도 자기 쌍극자를 갖고 있는 물질

(Ex) 철, 코발트, 니켈 ...

$$B_{int} \gg B_{appl}$$

참고)
외부온도가 높아지면 강자성은 상자성으로 변화한다.
이 온도를 퀴리온도 (Curie temperature) 라 한다.

강자성체	퀴리온도 (°K)
철	1043
코발트	1394
니켈	631

⑤ 자화

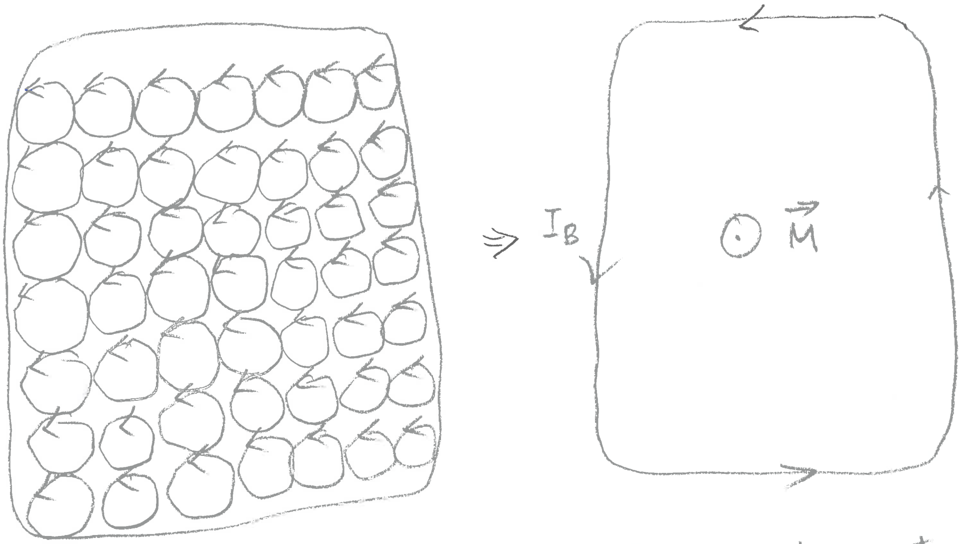
$\vec{M}$ (자화, magnetization)

: 단위 부피당 자기 쌍극자 모멘트

$$[\vec{M}] = A/m = [\vec{H}]$$

외부에서 자성체에 자기장을 가하면 전자 궤도가 배열하여

자화 $\vec{M}$이 발생한다.



I_B: bounded current

(속박 전류)

* I_B와 $\vec{M}$ 사이의 관계

$$I_B = \oint_c \vec{M} \cdot d\vec{l}$$

C: 자성체 내외의 닫힌 contour

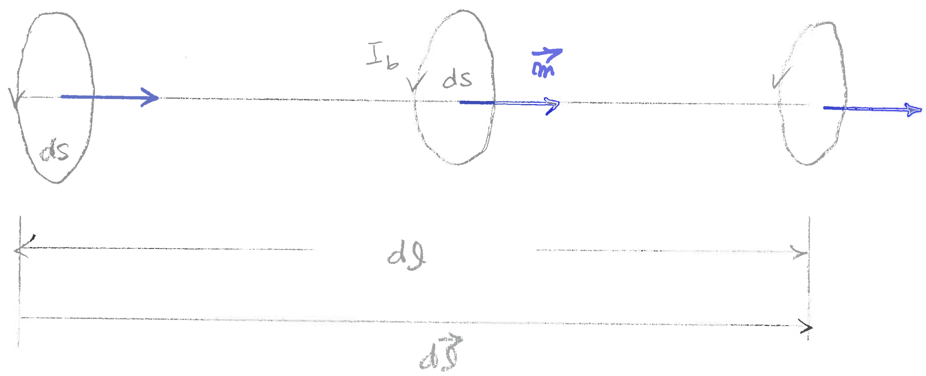
* $\vec{J}_B = \nabla \times \vec{M}$

(증명)

n : 단위 부피당 원자의 갯수

$\vec{m}$: 원자 한 개당 magnetic dipole moment

닫힌 contour C 의 작은 일부분을 생각하자



$m = I_b \, ds$

$M = n m = n I_b \, ds$

$dV = ds \, dl$ 원통의 부피

dV 내의 원자수 $= n dV = n \, ds \, dl$

dV 내의 total 자기 쌍극자 모멘트 $m_t = n dV \, m = ds \, dI_B$

$$\Rightarrow \quad dI_B = n \, ds \, dl \, I_b$$

$$= n m \, dl$$

$$= M \, dl$$

일반적으로

$$dI_B = \vec{M} \cdot d\vec{l}$$

$$I_B = \oint_C dI_B = \oint_C \vec{M} \cdot d\vec{l}$$

$$\int_S \vec{J}_B \cdot \hat{u}_N \, ds = \oint_C \vec{M} \cdot d\vec{l} = \int_S (\vec{\nabla} \times \vec{M}) \cdot \hat{u}_N \, ds$$

$$\Rightarrow \quad \vec{J}_B = \vec{\nabla} \times \vec{M}$$

⁂

$$\Rightarrow \oint_c \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$\Rightarrow \oint_c \vec{B} \cdot d\vec{s} = \mu_0 (I_F + I_B)$$

I_F : free current (자유 전류)

I_B : bounded current (속박 전류)

$I_T = I_F + I_B$ (total 전류)

$$\Rightarrow \oint_c \vec{B} \cdot d\vec{s} = \mu_0 I_T$$

$$I_F = I_T - I_B$$

$$= \frac{1}{\mu_0} \oint_c \vec{B} \cdot d\vec{s} - \oint_c \vec{M} \cdot d\vec{s}$$

$$= \oint_c \left(\frac{1}{\mu_0} \vec{B} - \vec{M}\right) \cdot d\vec{s}$$

$$\vec{H} \equiv \frac{1}{\mu_0} \vec{B} - \vec{M}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{E} = \frac{1}{\epsilon_0} (\vec{D} - \vec{P})$$

$$\Rightarrow \oint_c \vec{H} \cdot d\vec{s} = I_F$$

$$\boxed{\begin{array}{lll} I_B = \oint_c \vec{M} \cdot d\vec{l} & \Rightarrow & \vec{J}_B = \nabla \times \vec{M} \\ I_T = \oint_c \frac{\vec{B}}{\mu_0} \cdot d\vec{l} & \Rightarrow & \vec{J}_T = \frac{1}{\mu_0} \nabla \times \vec{B} \\ I_F = \oint_c \vec{H} \cdot d\vec{l} & \Rightarrow & \vec{J}_F = \nabla \times \vec{H} \\ I_T = I_F + I_T & & \\ \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} & & \end{array}}$$

$\vec{P} = \chi_e \epsilon_0 \vec{E}$

* $\vec{M} = \chi_m \vec{H}$

χ_e : 전하율

χ_m : 자화율 (magnetic susceptibility)

$$\Rightarrow \vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$= \mu_0 (1 + \chi_m) \vec{H}$$

$$= \mu_0 \mu_r \vec{H}$$

μ_r : 비투자율 (relative permeability) $\mu_r = 1 + \chi_m$

$$= \mu \vec{H}$$

$\mu = \mu_0 \mu_r$: 투자율. (permeability)

$$\Rightarrow \boxed{\begin{array}{l} \vec{B} = \mu \vec{H} \\ \mu = \mu_0 \mu_r \\ \mu_r = 1 + \chi_m \end{array}}$$

p581 표 C.3 참조

p281

(예제8.3)

$B = 0.05\ (T)$

$\mu_r = 50$

$\chi_m = \mu_r - 1 = 49$

$B = \mu H = \mu_0 \mu_r H$

$$H = \frac{B}{\mu_0 \mu_r} = \frac{0.05}{50 \times 4\pi \times 10^{-7}} = 796\ (A/m)$$

$M = \chi_m H = 49 \times 796 = 39000\ (A/m)$ ※

P263

(응용 예제 8.6)

(a)

$\mu = \mu_0 \mu_r = 1.8 \times 10^{-5}$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.8 \times 10^{-5}}{4\pi \times 10^{-7}} = \frac{180}{4\pi}$$

$$\chi_m = \mu_r - 1 = \frac{180 - 4\pi}{4\pi}$$

$$M = \chi_m H = \frac{180 - 4\pi}{4\pi} \times 120 = 1598.87 \text{ (A/m)}$$

(b) $\mu_r = 22$

$n = 8.3 \times 10^{28}$ $(1/m^3)$

$m = 4.5 \times 10^{-27}$ (Am^2)

$M = n \times m = 8.3 \times 45 = 373.5$ (A/m^2)

(c) $B = 300 \times 10^{-6}$ (T)

$\chi_m = 15$

$\mu_r = 16$

$\mu = \mu_0 \mu_r = 16 \times 4\pi \times 10^{-7}$ (H/m)

$$H = \frac{B}{\mu} = \frac{300 \times 10^{-6}}{16 \times 4\pi \times 10^{-7}} = \frac{3000}{16 \times 4\pi}$$

$$M = \chi_m H = \frac{15 \times 3000}{16 \times 4\pi} = 223.8 \text{ (A/m)}$$

p263

(응용예제 8.7)

$\vec{M} = 150 z^2 \hat{x}$ (A/m)

$\chi_m = 8$

(a) $\vec{J}_T = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B}$

$\vec{H} = \frac{1}{\chi_m} \vec{M} = \frac{150}{8} z^2 \hat{x}$ (A/m)

$\mu = \mu_0 \mu_r = 9\mu_0$

$\vec{B} = \mu \vec{H} = \frac{150 \times 9}{8} \mu_0 z^2 \hat{x}$ (T)

$\vec{\nabla} \times \vec{B} = \frac{150 \times 9}{4} \mu_0 z \hat{y}$

$\vec{J}_T = \frac{150 \times 9}{4} z \hat{y}$ (A/m^2)

$\vec{J}_T (z = 0.04) = \frac{150 \times 9}{4} \times 0.04 \hat{y} = 13.5 \hat{y}$ (A/m^2)

(b) $\vec{J}_F = \vec{\nabla} \times \vec{H} = \frac{150}{4} z \hat{x}$ (A/m^2)

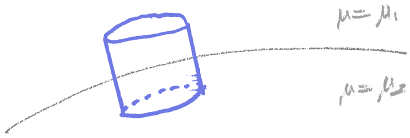
$\vec{J}_F (z = 0.04) = \frac{150}{4} \times 0.04 \hat{x} = 1.5 \hat{x}$ (A/m^2)

(c) $\vec{J}_B = \vec{\nabla} \times \vec{M} = 300 z \hat{x}$ (A/m^2)

$\vec{J}_B (z = 0.04) = 12 \hat{x}$ (A/m^2)

※.

※ 자계의 경계조건



$$\vec{\nabla}\cdot\vec{B}=0 \quad \Rightarrow \quad \oint_S \vec{B}\cdot\hat{u}_N\, dS = 0$$

$$\Rightarrow \quad B_{1\perp}\Delta S - B_{2\perp}\Delta S = 0$$

$$\Rightarrow \quad \underline{B_{1\perp} = B_{2\perp}} \qquad -\ (1)$$

$$\left.\begin{aligned}\vec{B}_1 &= \mu_1 \vec{H}_1\\ \vec{B}_2 &= \mu_2 \vec{H}_2\end{aligned}\right\} \quad (2)$$

(2) → (1)

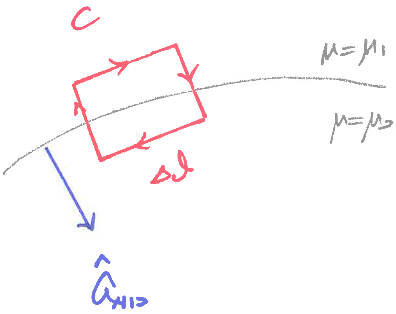
$$\mu_1 H_{1\perp} = \mu_2 H_{2\perp}$$

$$\Rightarrow \quad \underline{\frac{H_{1\perp}}{H_{2\perp}} = \frac{\mu_2}{\mu_1}} \qquad (3)$$

$$\left.\begin{aligned}\vec{M}_1 &= \chi_1 \vec{H}_1\\ \vec{M}_2 &= \chi_2 \vec{H}_2\end{aligned}\right\} \quad (4)$$

(4) → (3)

$$\underline{\frac{M_{1\perp}}{M_{2\perp}} = \frac{\mu_2 \chi_1}{\mu_1 \chi_2}} \qquad (4)$$



$$\oint_C \vec{H} \cdot d\vec{l} = I_F$$

$$(H_{1,//} - H_{2,//})\,\Delta l = I_F = K\,\Delta l$$

$$H_{1,//} - H_{2,//} = K$$ K : 표면 전류 밀도

$$\Rightarrow (\vec{H_1} - \vec{H_2}) \times \hat{a}_{N12} = \vec{K} \quad \text{or} \quad \vec{H}_{1,//} - \vec{H}_{2,//} = \hat{a}_{N12} \times \vec{K}$$

$\hat{a}_{N12}$: 영역 1 에서 영역 2 로 향하면서 경계면 에서 수직한 단위 vector

$$\Rightarrow \frac{B_{1,//}}{\mu_1} - \frac{B_{2,//}}{\mu_2} = K$$

$$\frac{M_{1,//}}{\chi_1} - \frac{M_{2,//}}{\chi_2} = K$$

* 정리

$$B_{1,\perp} = B_{2,\perp}$$

$$H_{1,\parallel} - H_{2,\parallel} = K$$

$$\vec{H}_{1,\parallel} - \vec{H}_{2,\parallel} = \hat{a}_{n12} \times \vec{K}$$

$$\frac{H_{1,\perp}}{H_{2,\perp}} = \frac{\mu_2}{\mu_1}$$

$$\frac{B_{1,\parallel}}{\mu_1} - \frac{B_{2,\parallel}}{\mu_2} = K$$

$$\frac{M_{1,\perp}}{M_{2,\perp}} = \frac{\mu_2 \chi_1}{\mu_1 \chi_2}$$

$$\frac{M_{1,\parallel}}{\chi_1} - \frac{M_{2,\parallel}}{\chi_2} = K$$

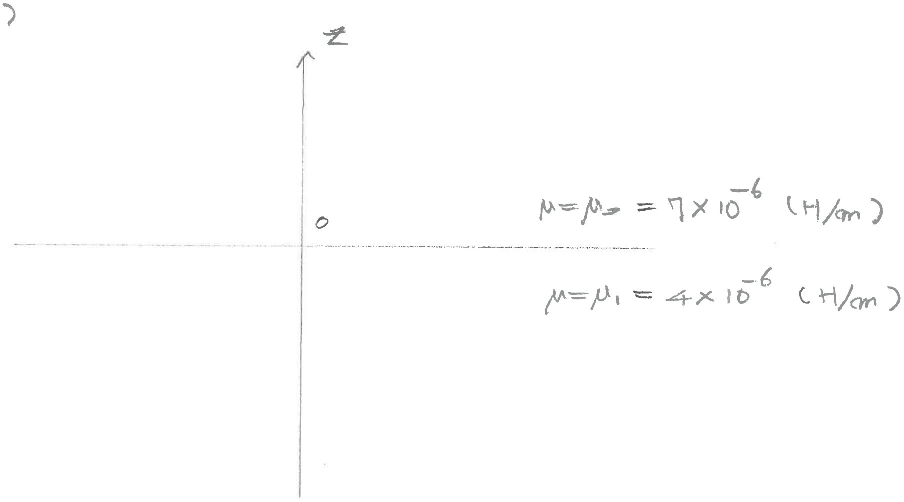
(cf)

$$E_{1,\parallel} = E_{2,\parallel} \qquad \frac{D_{1,\parallel}}{D_{2,\parallel}} = \frac{\epsilon_1}{\epsilon_2}$$

$$\epsilon_1 E_{1,\perp} = \epsilon_2 E_{2,\perp} \qquad D_{1,\perp} = D_{2,\perp}$$

p265

(예제8.6)



$$\vec{K} = 80\,\hat{x} \quad (A/m)$$

$$\vec{B}_1 = 2\hat{x} - 3\hat{y} + \hat{z} \quad (mT)$$

$$\vec{B}_{1,//} = (2\hat{x} - 3\hat{y}) \times 10^{-3} \quad (T)$$

$$\vec{B}_{1,\perp} = 10^{-3}\,\hat{z} \quad (T)$$

$$\Rightarrow \vec{B}_{2,\perp} = 10^{-3}\,\hat{z} \quad (T) \qquad -①$$

$$\vec{H}_{1,//} = \frac{1}{\mu_1}\vec{B}_{1,//} = \frac{10^{-3}}{4\times10^{-6}}(2\hat{x} - 3\hat{y}) \quad (A/m)$$

$$= 250 \times (2\hat{x} - 3\hat{y}) \quad (A/m)$$

$$\vec{H}_{2,//} = \vec{H}_{1,//} - \hat{a}_{n12} \times \vec{K} \qquad (\hat{a}_{n12} = \hat{z})$$

$$= 250 \times (2\hat{x} - 3\hat{y}) - \hat{z} \times 80\,\hat{x}$$

$$= 500\,\hat{x} - 750\,\hat{y} - 80\,\hat{y}$$

$$= 500\,\hat{x} - 830\,\hat{y} \qquad (A/m)$$

$$\vec{B}_{2,\parallel} = \mu_2 \vec{H}_{2,\parallel}$$

$$= 7\times10^{-6} \times (500\hat{x} - 830\hat{y})$$

$$= 35\times10^{-4}\hat{x} - 581\times10^{-5}\hat{y}$$

$$= (3.5\hat{x} - 5.81\hat{y})\times10^{-3} \quad (T)$$

$$= 3.5\hat{x} - 5.81\hat{y} \quad (mT)$$

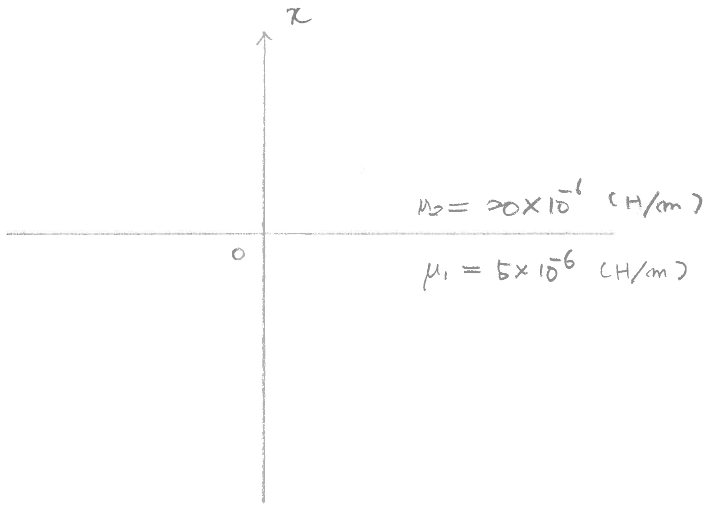
$$\vec{B}_2 = \vec{B}_{2,\parallel} + \vec{B}_{2,\perp}$$

$$= (3.5\hat{x} - 5.81\hat{y} + \hat{z}) \quad (mT)$$

\#

p266

(응용예제 8.8)



$$\vec{K} = 150\hat{y} - 200\hat{z} \quad (A/m)$$

$$\hat{a}_{n12} = \hat{x}$$

$$\vec{H}_1 = 300\hat{x} - 400\hat{y} + 500\hat{z} \quad (A/m)$$

(a) $$\vec{H}_{1,//} = -400\hat{y} + 500\hat{z} \quad (A/m)$$

$$H_{1,//} = \sqrt{(400)^2 + (500)^2} = 640.312 \quad (A/m)$$

(b) $$\vec{H}_{1,\perp} = 300\hat{x} \quad (A/m)$$

$$H_{1,\perp} = 300 \quad (A/m)$$

(c)

$$\vec{H}_{2,\parallel} = \vec{H}_{1,\parallel} - \hat{a}_{N12} \times \vec{K}$$

$$= -400\hat{y} + 500\hat{z} - \hat{x} \times (150\hat{y} - 200\hat{z})$$

$$= -600\hat{y} + 350\hat{z} \quad (A/m)$$

$$H_{2,\parallel} = \sqrt{600^2 + 350^2} = 694.6 \quad (A/m)$$

(d) $\vec{H}_{2,\perp} = \frac{\mu_1}{\mu_2} \vec{H}_{1,\perp}$

$$= \frac{5}{20} \times 300\hat{x}$$

$$= 75\hat{x} \quad (A/m)$$

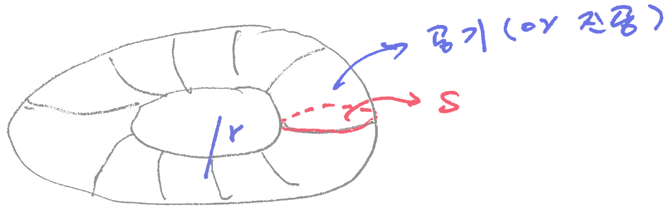
$$H_{2,\perp} = 75 \quad (A/m)$$

※

§ 자기회로 (magnetic circuit)

자기회로	전기회로
$\vec{H} = -\vec{\nabla} V_m$ $(\vec{J} = 0)$ V_m : 자위 or 기자력 $[V_m] = A = At$ (Ampere turn수) At : 암페어 회선수	$\vec{E} = -\vec{\nabla} V$ V : 전위 or 기전력 $[V] = \text{Volt}$
$\vec{B} = \mu \vec{H}$	$\vec{J} = \sigma \vec{E}$ σ : 전도율
$\Phi = \int_S \vec{B} \cdot \hat{u}_N \, ds$: 자속	$I = \int_S \vec{J} \cdot \hat{u}_N \, ds$: 전류
$V_m = \Phi \mathcal{R}$ $\mathcal{R}$: reluctance (리럭턴스) $[\mathcal{R}] = \frac{A}{Wb} = \frac{At}{Wb}$	$V = IR$ R : 저항
$\mathcal{R} = \frac{d}{\mu S}$	$R = \frac{d}{\sigma S}$ S : 단면적 d : 길이
$\oint_c \vec{H} \cdot d\vec{l} = I_{inside}$ or $\oint_c \vec{H} \cdot d\vec{l} = NI$ N : turn 수	$\oint_c \vec{E} \cdot d\vec{l} = 0$

(Ex) toroid : toroid 내부에서 $H = ?$



$N = 500$ 회

$S = 6 \times 10^{-4}\ (m^2)$

$r = 0.15\ (m)$

$I = 4\ (A)$

$$V_m \equiv H \cdot 2\pi r = \oint_c \vec{H} \cdot d\vec{l} = NI = 2000\ (At)$$

$$\mathcal{R} = \frac{d}{\mu_0 S} = \frac{2\pi r}{4\pi \times 10^{-7} \times 6 \times 10^{-4}} = \frac{2\pi \times 0.15}{24\pi \times 10^{-11}} = 1.25 \times 10^{9}\ (At/Wb)$$

$$\Phi = \frac{V_m}{\mathcal{R}} = \frac{2000}{1.25 \times 10^{9}} = 1.6 \times 10^{-6}\ (Wb)$$

$$B = \frac{\Phi}{S} = \frac{1.6 \times 10^{-6}}{6 \times 10^{-4}} = 2.67 \times 10^{-3}\ (T)$$

$$H = \frac{B}{\mu_0} = \frac{2.67 \times 10^{-3}}{4\pi \times 10^{-7}} = 2120\ (At/m)$$

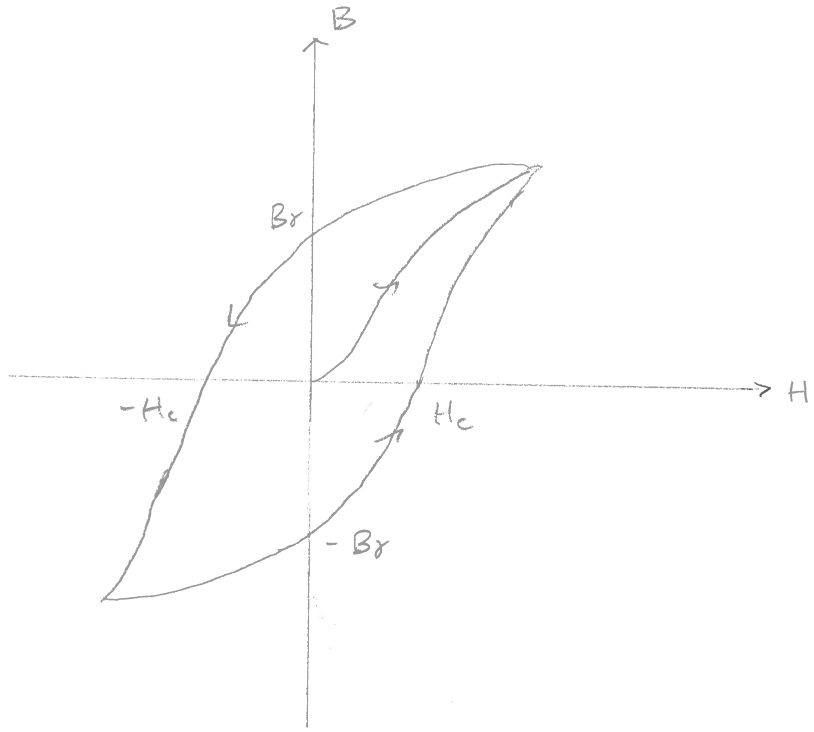
* Ampere 법칙을 사용하면

$$H\, 2\pi r = NI$$

$$H = \frac{NI}{2\pi r} = 2120\ (A/m)$$

※

∘ 자기 이력 곡선 (hysteriesis loop)



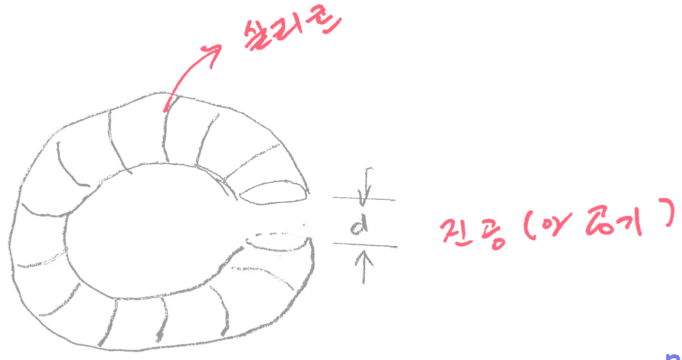
강자성체에서 B와 H의 관계 그림

B_r : H=0 에서의 자속밀도 (잔류 자속밀도)

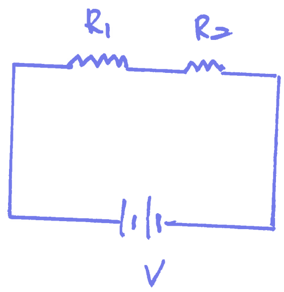
H_c : B=0 에서의 자계세기 (보자력 coericive force)

P270

(예제 8.17)



⇔ 직렬회로



$d = 2\times10^{-3}$ (m)

$N = 500$

$S = 6\times10^{-4}$ (m^2)

$r = 0.15$ (m)

실리콘 내에서 $B = 1$ (T)

$I = ?$

$$(R)_{공기} = \frac{d}{\mu_0 S} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times6\times10^{-4}} = 2.65\times10^{6} \ (At/Wb)$$

$$\Phi = BS = 1\times6\times10^{-4} = 6\times10^{-4} \ (Wb)$$

$$(V_m)_{공기} = \Phi\,(R)_{공기} = 6\times10^{-4}\times2.65\times10^{6} = 1590 \ (At)$$

그림 8.11

$B = 1$ (T) $\Rightarrow$ $H = 200$ (A/m)

$$(V_m)_{실리콘} \fallingdotseq H\cdot(2\pi r - d) \approx 188 \ (At)$$

$$(V_m)_{total} = (V_m)_{공기} + (V_m)_{실리콘} = 1778 \ (A) \fallingdotseq NI$$

$$I = \frac{(V_m)_{total}}{N} = \frac{1778}{500} = 3.56 \ (A)$$

※

(예제 8.8)

$I = 4\ (A)$

실리콘 내의 자속밀도 $B = ?$

그림 8.11

$$(B)_{실리콘} \cong \frac{(H)_{실리콘}}{200}$$

공기 (가 진공)

$$B = \mu_0 H$$

$$(\mu)_{실리콘} \approx \frac{1}{200}$$

$$(R)_{실리콘} = \frac{2\pi r - d}{(\mu)_{실리콘} S} = \frac{2\pi \times 0.15 - 2\times10^{-3}}{\frac{1}{200} \times 6\times10^{-4}} \approx 0.314 \times 10^{6}\ (At/Wb)$$

$$(R)_{공기} = \frac{d}{\mu_0 S} = \frac{2\times10^{-3}}{4\pi\times10^{-7} \times 6\times10^{-4}} = 2.65 \times 10^{6}\ (At/Wb)$$

$$(R)_{total} = (R)_{실리콘} + (R)_{공기} = 2.964 \times 10^{6}\ (At/Wb)$$

$$V_m \cong H \cdot 2\pi r = NI = 2000\ (At)$$

$$\Phi = \frac{V_m}{(R)_{total}} = \frac{2000}{2.964\times10^{6}} = 6.741 \times 10^{-4}\ (Wb)$$

$$B = \frac{\Phi}{S} = \frac{6.741 \times 10^{-4}}{6 \times 10^{-4}} = 1.1235\ (T)$$

※.

(응용예제 8.9)

$B = 1\ (T)$

$S_1 = 5 \times 10^{-4}\ (m^2)$

$S_2 = 3 \times 10^{-4}\ (m^2)$

$L_1 = 8 \times 10^{-2}\ (m)$

$L_2 = 16 \times 10^{-2}\ (m)$

$d = 0.5 \times 10^{-2}\ (m)$

(a) $(\Phi)_{공기} = B S_2$

$$(\mathcal{R})_{공기} = \frac{d}{\mu_0 S_2}$$

$$(V_m)_{공기} = (\Phi)_{공기} (\mathcal{R})_{공기} = \frac{Bd}{\mu_0} = 3978.87\ (At)$$

(b) $B = 1\ (T) \Rightarrow H = 200\ (A/m)$

$$(V_m)_{실리콘} = H(L_1 + 2L_2) = 80\ (A/m)$$

(c) $V_m = (V_m)_{공기} + (V_m)_{실리콘} = 4059\ (At)$

$$I = \frac{V_m}{N} = \frac{4059}{1300} = 3.12231\ (A)$$

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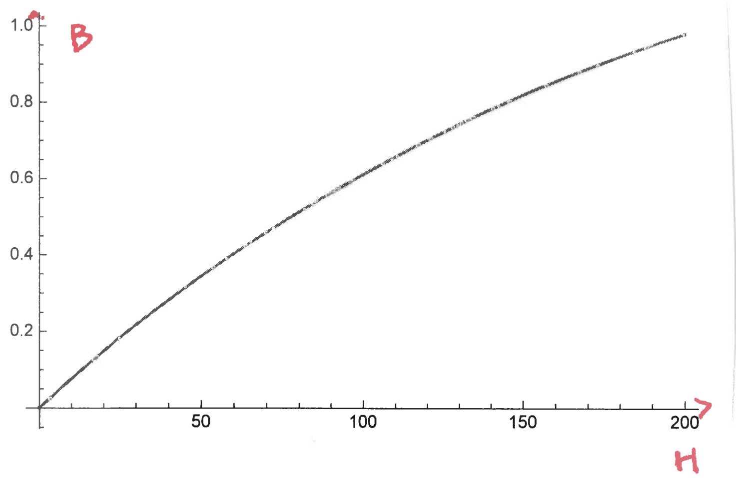
(응용예제 8.10)

$$B = \frac{H}{160}\left(0.25 + e^{-\frac{H}{320}}\right)$$

```
b[h_] := (h / 160) (0.25 + Exp[-h / 320]);
Plot[b[h], {h, 0, 200}]
```



$L = 12 \times 10^{-2}$ (m), $d = 0.25 \times 10^{-3}$ (m), $S = 2.5 \times 10^{-4}$ (m²)

(a) $\Phi = 10^{-5}$ (Wb) $= BS$

$$B = \frac{\Phi}{S} = \frac{10^{-5}}{2.5 \times 10^{-4}} = 0.04 \text{ (T)}$$

$H = 5.18671$ (A/m)

$(V_m)_x = HL = 0.622405$ (At)

$$(V_m)_{공기} = \frac{Bd}{\mu_0} = 7.95775 \text{ (At)}$$

$V_m = (V_m)_x + (V_m)_{공기} = 8.58$ (At)

(b) $\Phi = 10^{-4}$ (Wb)

$$B = \frac{10^{-4}}{S} = 0.4 \text{ (T)}$$

$H = 59.2$ (A/m)

$(V_m)_x = HL = 7.104$ (At)

$$(V_m)_{\text{공기}} = \frac{Bd}{\mu_0} = 79.6 \text{ (At)}$$

$V_m = (V_m)_x + (V_m)_{\text{공기}} = 86.7$ (At)

§ 자성체 에서의 potential energy 의 정

$$u_E = \frac{1}{2}\vec{D}\cdot\vec{E}$$ 전기 에너지 밀도

$$W_E = \int_V u_E \, dv$$ V 내의 전기에너지

$$u_B = \frac{1}{2}\vec{B}\cdot\vec{H}$$ 자기 에너지 밀도 (11장)

$$W_B = \int_V u_B \, dv$$ V 내의 자기 에너지

Since $\vec{B} = \mu\vec{H}$, $u_B = \frac{1}{2}\mu H^2$

$$\Rightarrow \underline{W_B = \frac{1}{2}\int_V \mu H^2 \, dv = \frac{1}{2}\int_V \frac{B^2}{\mu} \, dv}$$

§ inductance

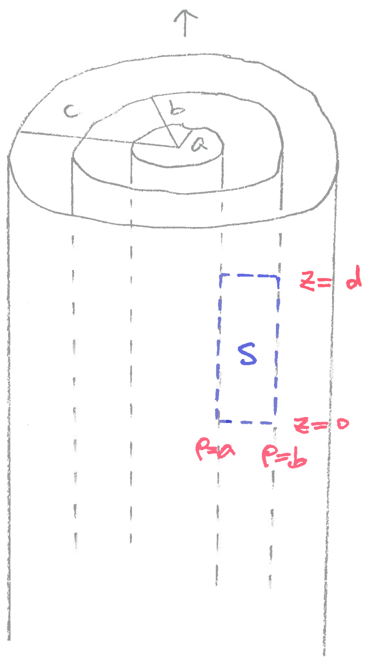
$$L = \frac{\Phi_{total}}{I} \quad : \text{inductance}$$

$$[L] = Wb/A = \frac{V\,sec}{A} \equiv H$$

$$\left(\because \frac{Wb}{A} = \frac{T\,m^2}{A} = \frac{N}{Am}\frac{m^2}{A} = \frac{J}{A^2} = \frac{1}{A}\frac{J}{\frac{C}{sec}} = \frac{1}{A}\frac{J\,sec}{C} \right.$$

$$\left. = \frac{V\,sec}{A} \right)$$

Ex) 동축 cable



$$\vec{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2}\hat{\phi} & 0 \le \rho \le a \\ \dfrac{I}{2\pi\rho}\hat{\phi} & a \le \rho \le b \\ \dfrac{I}{2\pi\rho}\dfrac{c^2-\rho^2}{c^2-b^2}\hat{\phi} & b \le \rho \le c \\ 0 & c \le \rho \end{cases}$$

$$\vec{H} = \frac{I}{2\pi\rho}\hat{\phi}$$

$$\vec{B} = \mu_0\vec{H} = \frac{\mu_0 I}{2\pi\rho}\hat{\phi}$$

$$\Phi = \int_s \vec{B} \cdot \hat{u}_N \, ds \qquad \left(\begin{array}{l} \hat{u}_N = \hat{\phi} \\ ds = d\rho \, dz \end{array} \right.$$

$$= \int_a^b d\rho \int_0^d dz \; \frac{\mu_0 I}{2\pi\rho}$$

$$= \frac{\mu_0 I d}{2\pi} \ln \frac{b}{a}$$

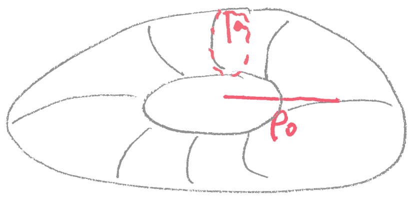
$$L = \frac{\Phi}{I} = \frac{\mu_0 d}{2\pi} \ln \frac{b}{a}$$ 동축 cable 의 inductance

단위길이당 inductance

$$\frac{L}{d} = \frac{\mu_0}{2\pi} \ln \frac{b}{a} \qquad (H/m)$$

※

(Ex) toroid



$$B_\phi = \frac{\mu_0 N I}{2\pi\rho}$$

If $a \ll \rho_0$, $\quad B_\phi \cong \dfrac{\mu_0 N I}{2\pi \rho_0}$

$$\Phi_{total} = N B_\phi S = \frac{\mu_0 N^2 S}{2\pi\rho_0} I$$ ($S = \pi a^2$ 단면적)

$$L = \frac{\Phi_{total}}{I} = \frac{\mu_0 N^2 S}{2\pi\rho_0} \quad \#$$

(note)

$$L = \frac{\mu_0 N^2 S}{2\pi\rho_0}$$

$$= \underbrace{\frac{\mu_0 N I}{2\pi\rho_0}}_{B} \frac{N S}{I}$$

$$= B \frac{N S}{I} \underbrace{\frac{N I}{2\pi\rho_0}}_{H} \frac{2\pi\rho_0}{N I}$$

$$= \underbrace{BH}_{2u_B} \frac{2\pi\rho_0 S}{I^2}$$ ($2\pi\rho_0 S = V$: toroid 의 부피)

$$= 2u_B \frac{V}{I^2}$$ ($u_B V = W_B$: toroid 내의 자기 에너지)

$$= \frac{2W_B}{I^2}$$

일반적으로

$$L = \frac{1}{I^2}\int_V \vec{B}\cdot\vec{H}\,dv$$

Since $\vec{B} = \vec{\nabla}\times\vec{A}$,

$$L = \frac{1}{I^2}\int_V \vec{H}\cdot(\vec{\nabla}\times\vec{A})\,dv$$

Vector identity

$$\vec{\nabla}\cdot(\vec{A}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{H})$$

$$\Rightarrow\quad L = \frac{1}{I^2}\left[\int_V \vec{\nabla}\cdot(\vec{A}\times\vec{H})\,dv + \int_V \vec{A}\cdot(\vec{\nabla}\times\vec{H})\,dv\right]$$

$$= \underbrace{\frac{1}{I^2}\oint_S (\vec{A}\times\vec{H})\cdot\hat{a}_N\,ds}_{=0\ (V\text{를 크게})} + \frac{1}{I^2}\int_V \vec{A}\cdot\underbrace{(\vec{\nabla}\times\vec{H})}_{\vec{J}}\,dv$$

$$= \frac{1}{I^2}\int_V \vec{A}\cdot\vec{J}\,dv$$

$$\vec{A} = \int_V \frac{\mu\vec{J}}{4\pi r}\,dv$$

$$\Rightarrow\quad L = \frac{1}{I^2}\int_V dv\int_V dv\left(\frac{\mu\vec{J}}{4\pi r}\right)\cdot\vec{J}$$

$$\vec{J}\,dv = I\,d\vec{l}$$

$$= \frac{\mu}{4\pi}\oint\left(\oint\frac{d\vec{l}}{r}\right)\cdot d\vec{l}$$

$$= \frac{\mu}{4\pi}\oint_{C_1}\oint_{C_2}\frac{d\vec{l}_1\cdot d\vec{l}_2}{r}$$

$$\Rightarrow\quad L = \frac{\mu}{4\pi}\oint_{C_1}\oint_{C_2}\frac{d\vec{l}_1\cdot d\vec{l}_2}{r}$$

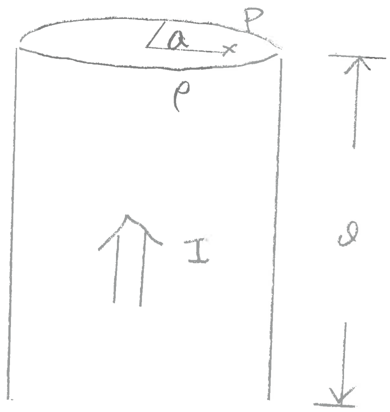
* inductance 는 기하학적 구조에 관련이 있다.

note)

$C_1 = C_2$ L: self inductance

$C_1 \neq C_2$ L: mutual inductance

(Ex)



$$\vec{H} = \frac{I\rho}{2\pi a^2}\hat{\phi}$$

$$\vec{B} = \mu\vec{H} = \frac{\mu I\rho}{2\pi a^2}\hat{\phi}$$

$$\vec{H}\cdot\vec{B} = \frac{\mu I^2\rho^2}{4\pi^2 a^4}$$

$$u_B = \frac{1}{2}\vec{H}\cdot\vec{B} = \frac{\mu I^2\rho^2}{8\pi^2 a^4}$$

$$W_B = \int_V u_B\, dv$$

$$= \frac{\mu I^2}{8\pi^2 a^4}\int_V \rho^2\, dv \qquad dv = \rho\, d\rho\, d\phi\, dz$$

$$= \frac{\mu I^2}{8\pi^2 a^4}\underbrace{\int_0^l dz\int_0^{2\pi} d\phi\int_0^a \rho^3\, d\rho}_{2\pi l\,\frac{1}{4}a^4}$$

$$= \frac{\mu l}{16\pi} I^2$$

$$L = \frac{2W_B}{I^2} = \frac{\mu}{8\pi} l$$

단위 길이당 inductance

$$\frac{L}{l} = \frac{\mu}{8\pi} \quad (H/m)$$

//

상호 inductance

$$M_{12} = \frac{(\Phi_{12})_{total}}{I_1}$$

$(\Phi_{12})_{total}$: 회로 I_1에 의하여 I_2에 생기는 total 자속

에너지 관계식

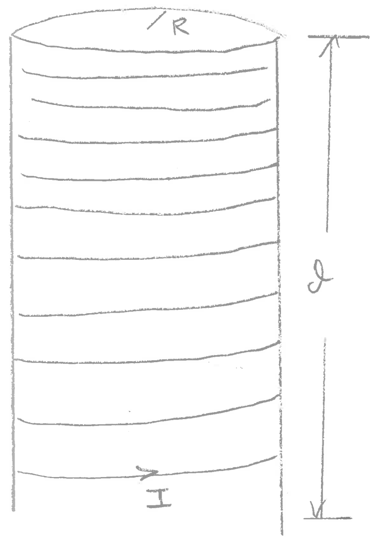
$$M_{12} = \frac{1}{I_1 I_2} \int_V (\vec{B_1} \cdot \vec{H_2})\, dv$$

$\vec{B_1}$: I_1에 의한 자속 밀도

$\vec{H_2}$: I_2에 의한 자계세기

$$M_{12} = M_{21}$$

(Ex) solenoid



n: 단위길이당 turn 수

$B = \mu n I$

$H = n I$

$u_B = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{\mu}{2} n^2 I^2$

V: solenoid volume

$V = \pi R^2 l$

$\Rightarrow W_B = u_B V = \frac{\mu}{2} n^2 \pi R^2 l I^2$

$L = \frac{2W_B}{I^2} = \mu n^2 \pi R^2 l$

단위 길이당 inductance

$\frac{L}{l} = \mu n^2 \pi R^2 \quad (H/m)$

p280

(예제 8.9)

I_1 : $\frac{L_1}{l} = \mu_0 n^2 \pi R_1^2$ (H/m)

I_2: $\frac{L_2}{l} = \mu_0 n^2 \pi R_2^2$ (H/m)

M_{12}: $\vec{B_1} = \begin{cases} \mu_0 n_1 I_1 & 0 \le \rho \le R_1 \\ 0 & 그외 \end{cases}$

$\vec{H_2} = \begin{cases} n_2 I_2 & 0 \le \rho \le R_2 \\ 0 & 그외 \end{cases}$

$\vec{B_1} \cdot \vec{H_2} = \begin{cases} \mu_0 n_1 n_2 I_1 I_2 & 0 \le \rho \le R_1 \\ 0 & 그외 \end{cases}$

$$M_{12} = \frac{1}{I_1 I_2} \int_V (\vec{B_1} \cdot \vec{H_2})\, dv$$

$$= \frac{1}{I_1 I_2} \mu_0 n_1 n_2 I_1 I_2 (\pi R_1^2 l)$$

$$= \mu_0 n_1 n_2 (\pi R_1^2 l)$$

$$\frac{M_{12}}{l} = \mu_0 n_1 n_2 \pi R_1^2$$

※

p281

(응용예제 8.12)

(a) $a = 0.8 \times 10^{-3}$ (m) , $b = 4 \times 10^{-3}$ (m) , $\mu_r = 50$, $l = 3.5$ (m)

$$L = \frac{\mu l}{2\pi} \ln \frac{b}{a}$$

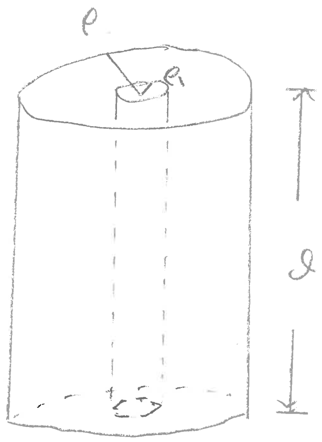
$$= \frac{50 \times 4\pi \times 10^{-7} \times 3.5}{2\pi} \ln \frac{4}{0.8}$$

$$= 56.3 \times 10^{-6} \text{ (H)}$$

$$= 56.3 \text{ (}\mu\text{H)}$$

(b)

(c)



$\rho = 2\times10^{-2}$ (m) , $N=500$, $l = 0.5$ (m) , $n = \frac{N}{l} = 1000$

$\rho_1 = 0.5\times10^{-2}$ (m) $\Rightarrow$ $\mu_r = 50$

$$B = \begin{cases} \mu_r \mu_0 n I & 0 \le r \le \rho_1 \\ \mu_0 n I & \rho_1 \le r \le \rho \\ 0 & \text{그외} \end{cases}$$

$$H = \begin{cases} n I & 0 \le r \le \rho \\ 0 & \text{그외} \end{cases}$$

$$BH = \begin{cases} \mu_r \mu_0 n^2 I^2 & 0 \le r \le \rho_1 \\ \mu_0 n^2 I^2 & \rho_1 \le r \le \rho \\ 0 & \text{그외} \end{cases}$$

$$u_B \equiv \frac{1}{2}\vec{B}\cdot\vec{H} = \begin{cases} \frac{\mu_r\mu_0}{2} n^2 I^2 & 0 \le r \le \rho_1 \\ \frac{\mu_0}{2} n^2 I^2 & \rho_1 \le r \le \rho \\ 0 & \text{그외} \end{cases}$$

$$W_B = \int_V u_B \, dv$$

$$= \frac{\mu_r \mu_0}{2} n^2 I^2 (\pi \rho_1^2) l + \frac{\mu_0}{2} n^2 I^2 \left[\pi \rho^2 l - \pi \rho_1^2 l \right]$$

$$= \frac{\mu_0}{2} n^2 I^2 \pi l \left[(\mu_r - 1) \rho_1^2 + \rho^2 \right]$$

$$L = \frac{2W_B}{I^2} = \mu_0 n^2 \pi l \left[(\mu_r - 1) \rho_1^2 + \rho^2 \right]$$

$$= 3.2 \times 10^{-3} \ (H)$$

$$= 3.2 \ (mH)$$

※.

(응용예제 8.13)

$l = 0.5$ (m)

내부: $r_1 = 0.01$ (m), $N_1 = 1500$, $n_1 = \frac{N_1}{l} = 3000$, $\mu_r = 75$

외부: $r_2 = 0.015$ (m), $N_2 = 1200$, $n_2 = \frac{N_2}{l} = 2400$

(c)

$$B_1 = \begin{cases} \mu_r \mu_0 n_1 I_1 & 0 \le \rho \le r_1 \\ 0 & \text{그외} \end{cases}$$

$$H_2 = \begin{cases} n_2 I_2 & 0 \le \rho \le r_2 \\ 0 & \text{그외} \end{cases}$$

$$\vec{B_1} \cdot \vec{H_2} = \begin{cases} \mu_r \mu_0 n_1 n_2 I_1 I_2 & 0 \le \rho \le r_1 \\ 0 & \text{그외} \end{cases}$$

$$\int_V (\vec{B_1} \cdot \vec{H_2})\, dv$$

$$= \mu_r \mu_0 n_1 n_2 I_1 I_2 (\pi r_1^2)\, l$$

$$M_{12} = \frac{1}{I_1 I_2} \int_V (\vec{B_1} \cdot \vec{H_2})\, dv$$

$$= \mu_r \mu_0 n_1 n_2 (\pi r_1^2)\, l$$

$$= 75 \times 4\pi \times 10^{-7} \times 3000 \times 2400 \times \pi \times 0.01^2 \times 0.5$$

$$= 106.6 \times 10^{-3} \text{ (H)}$$ ~~답과 다르다!~~

$$= 106.6 \text{ (mH)}$$

※

(a)

$$L = \mu n_1^2 \pi r_1^2 l$$

$$= 75 \times 4\pi \times 10^{-7} \times 3000^2 \times \pi \times 0.01^2 \times 0.5$$

$$= 133.2 \times 10^{-3} \quad (H)$$

$$= 133.2 \quad (mH)$$

(b) $B = \begin{cases} \mu_r \mu_0 n_2 I & 0 \le \rho \le r_1 \\ \mu_0 n_2 I & r_1 \le \rho \le r_2 \end{cases}$

$$H = n_2 I$$

$$BH = \begin{cases} \mu_r \mu_0 n_2^2 I^2 & 0 \le \rho \le r_1 \\ \mu_0 n_2^2 I^2 & r_1 \le \rho \le r_2 \end{cases}$$

$$2W_B = \int_V \vec{B} \cdot \vec{H} \, dv$$

$$= \mu_r \mu_0 n_2^2 I^2 (\pi r_1^2) l$$

$$+ \mu_0 n_2^2 I^2 [\pi r_2^2 - \pi r_1^2] l$$

$$= \mu_0 n_2^2 I^2 \pi l [(\mu_r - 1) r_1^2 + r_2^2]$$

$$L = \frac{2W_B}{I^2} = \mu_0 n_2^2 \pi l [(\mu_r - 1) r_1^2 + r_2^2]$$

$$= 86.6946 \times 10^{-3} \quad (H)$$

$$= 86.7 \quad (mH)$$

#