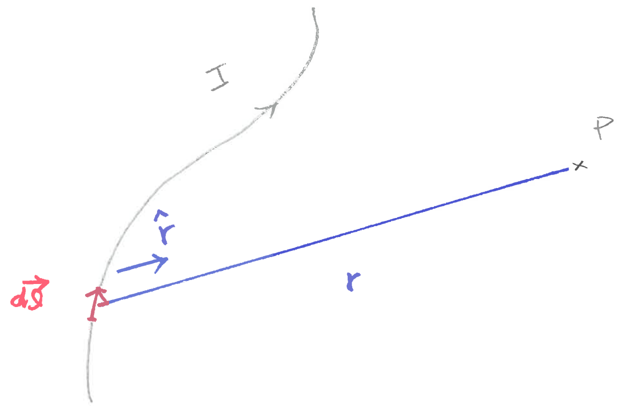
CH. 정상자계 (Static magnetic system)

§ Biot-Savart's law



$$d\vec{H} = \frac{I\, d\vec{s} \times \hat{r}}{4\pi r^2}$$

$$\Rightarrow \vec{H} = \int_c \frac{I\, d\vec{l} \times \hat{r}}{4\pi r^2}$$ Biot-Savart 법칙

c : 전류가 흐르는 선

① $\vec{H}$: 자계세기 (magnetic field intensity)

$[\vec{H}] = A/m$

② If c is a closed contour,

$$\vec{H} = \oint_c \frac{I\, d\vec{l} \times \hat{r}}{4\pi r^2}$$

ⓑ $\vec{J}$: current density (단위 면적당 전계)

Since $I\,d\vec{l} = \vec{J}\,dv$, $\vec{H}$ can be written as

$$\vec{H} = \int_v \frac{\vec{J}\times\hat{r}}{4\pi r^2}\,dv$$ v: 전류가 존재하는 volume

ⓒ For surface current

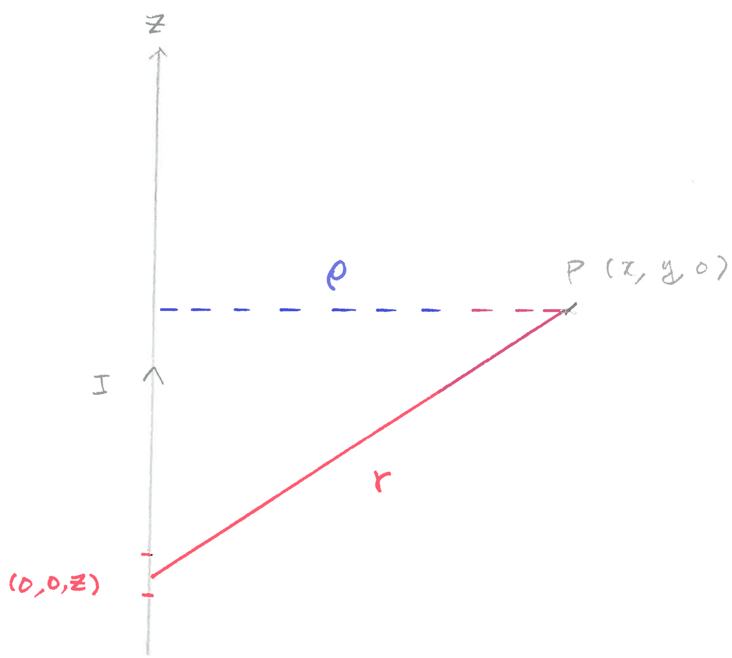
$\vec{K}$: current density (단위 길이당 current)

$[\vec{K}] = A/m$

$I\,d\vec{l} = \vec{K}\,ds$

$$\Rightarrow \vec{H} = \int_S \frac{\vec{K}\times\hat{r}}{4\pi r^2}\,ds$$ S: 전류가 존재하는 면

Ex)



$$d\vec{l} = dz\,\hat{z}$$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{\rho^2+z^2}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{1}{\sqrt{\rho^2+z^2}}(x\hat{x}+y\hat{y}-z\hat{z})$$

$$= \frac{1}{\sqrt{\rho^2+z^2}}\left[\rho\cos\phi\,(\cos\phi\,\hat{\rho}-\sin\phi\,\hat{\phi})+\rho\sin\phi\,(\sin\phi\,\hat{\rho}+\cos\phi\,\hat{\phi})-z\hat{z}\right]$$

$$= \frac{1}{\sqrt{\rho^2+z^2}}\left[\rho\hat{\rho}-z\hat{z}\right]$$

$$d\vec{l}\times\hat{r} = \frac{dz}{\sqrt{\rho^2+z^2}}\left[\rho\,\underbrace{\hat{z}\times\hat{\rho}}_{\hat{\phi}} - z\,\underbrace{\hat{z}\times\hat{z}}_{=0}\right]$$

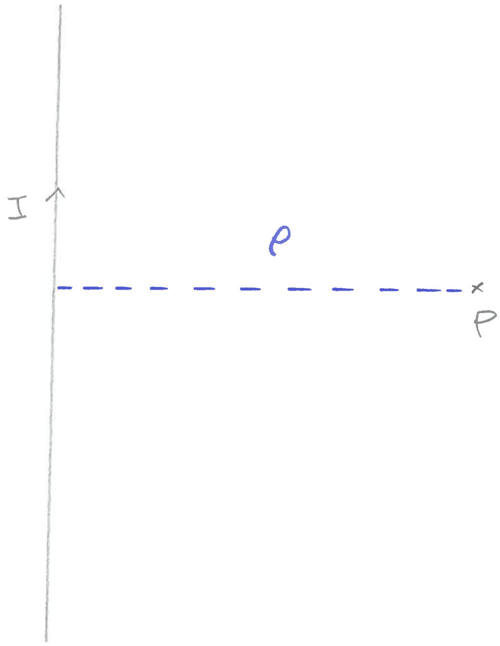
$$= \frac{\rho\hat{\phi}}{\sqrt{\rho^2+z^2}}dz$$

$$\Rightarrow \vec{H} = \frac{I}{4\pi}\int_{-\infty}^{\infty}\frac{\rho\hat{\phi}}{(\rho^2+z^2)^{\frac{3}{2}}}dz$$

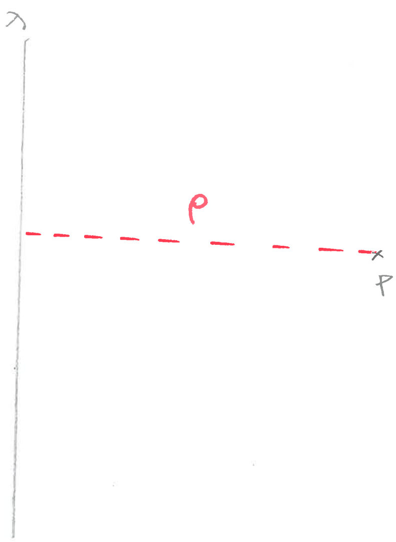
$$= \hat{\phi}\frac{I\rho}{2\pi}\int_0^{\infty}\frac{dz}{(\rho^2+z^2)^{\frac{3}{2}}} \qquad z=\rho\tan\theta$$

$$= \frac{I}{2\pi\rho}\hat{\phi}$$

※

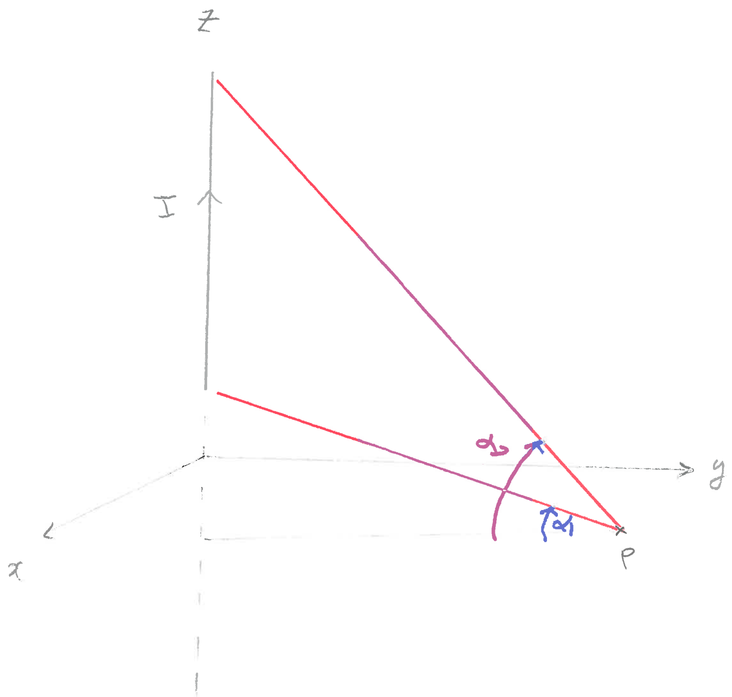


$$\vec{H} = \frac{I}{2\pi\rho}\,\hat{\phi}$$



$$\vec{D} = \frac{\lambda}{2\pi\rho}\,\hat{\rho}$$

Ex)



$$\vec{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{\phi}$$

무한도선

$\alpha_2 = \frac{\pi}{2}$, $\alpha_1 = -\frac{\pi}{2}$

$$\Rightarrow \vec{H} = \frac{I}{2\pi\rho}\hat{\phi}$$

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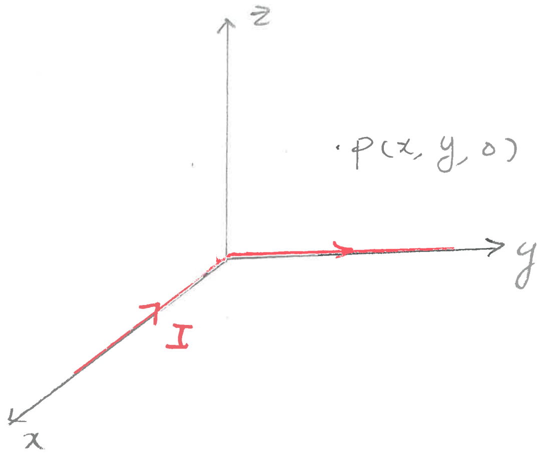
(예제 7.1)

$\vec{H} = \vec{H}_1 + \vec{H}_2$

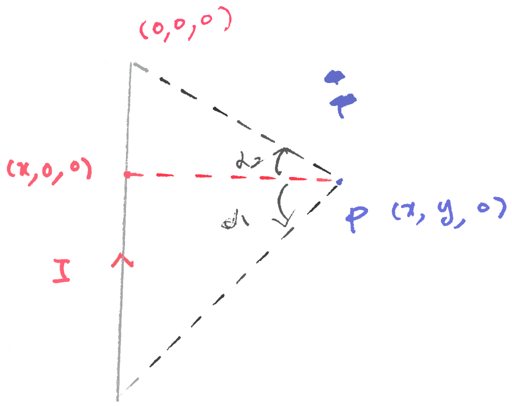


$I = 8\ (A)$

$x = 0.4$

$y = 0.3$

(i) $\vec{H}_1$



$\rho_1 = y = 0.3$

$\sin\alpha_2 = \dfrac{x}{\sqrt{x^2 + y^2}} = \dfrac{4}{5}$

$\alpha_1 = -\dfrac{\pi}{2}$

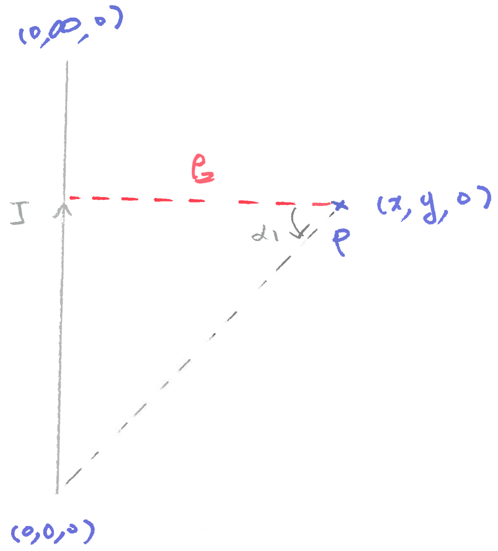
$\sin\alpha_1 = -1$

$\vec{H}_1 = \dfrac{I}{4\pi\rho_1}\left(\dfrac{4}{5} + 1\right)(-\hat{z})$

$= \dfrac{9I}{20\pi\rho_1}(-\hat{z})$

$= -\dfrac{12}{\pi}\hat{\phi}\ \hat{z}$

(ii) $\vec{H_2}$



$$\rho_2 = x = 0.4$$

$$\alpha_2 = \frac{\pi}{2}$$

$$\sin\alpha_1 = -\frac{y}{\sqrt{x^2+y^2}} = -\frac{3}{5}$$

$$\vec{H_2} = \frac{I}{4\pi\rho_2}\left(1+\frac{3}{5}\right)(-\hat{z})$$

$$= \frac{8I}{20\pi\rho_2}(-\hat{z})$$

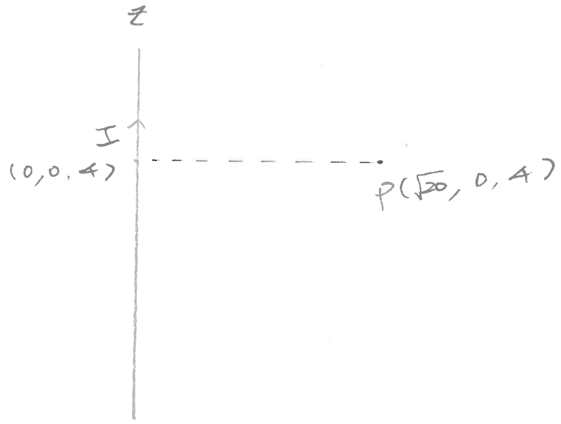
$$= -\frac{8}{\pi}\hat{z}$$

$$\Rightarrow \vec{H} = \vec{H_1} + \vec{H_2} = -\frac{20}{\pi}\hat{z} \quad (A/m) \quad \divideontimes$$

(응용예제 7.2)

$I = 15\ (A)$

(a)



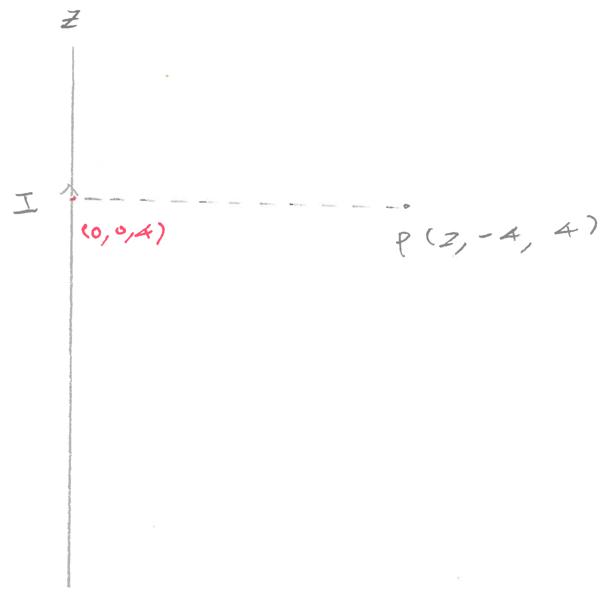
$\rho = \sqrt{20}$

$$\vec{H} = \frac{I}{2\pi\rho}\,\hat{y}$$

$$= \frac{15}{2\pi\sqrt{20}}\,\hat{y}$$

$$= 0.5338\,\hat{y}\quad (A/m)$$

(b)



$$\rho = \sqrt{2^2 + (-4)^2} = \sqrt{20}$$

$$\hat{\phi} = ?$$

$$\hat{z} \times \left(\frac{1}{\sqrt{5}}\right)(\hat{x} - 2\hat{y})$$

$$= \frac{1}{\sqrt{5}}(\hat{y} + 2\hat{x})$$

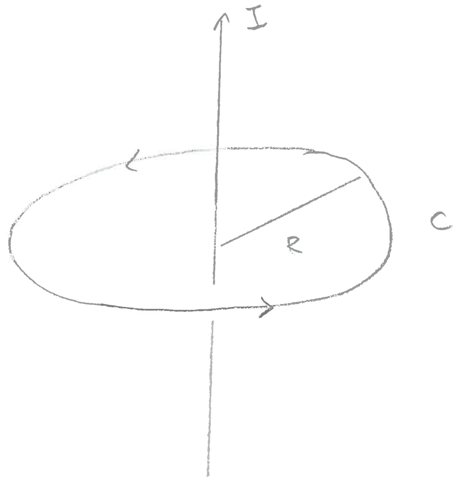
$$\Rightarrow \hat{\phi} = \frac{1}{\sqrt{5}}(2\hat{x} + \hat{y})$$

$$\vec{H} = \frac{I}{2\pi\rho}\hat{\phi}$$

$$= \frac{15}{2\pi\sqrt{20}}\frac{1}{\sqrt{5}}(2\hat{x} + \hat{y})$$

$$= 0.4775\hat{x} + 0.2387\hat{y} \quad (A/m)$$

8 암페어의 주회법칙 (Ampere's circuital law)



$$\oint_c \vec{H}\cdot d\vec{l} = \frac{I}{2\pi R} \times 2\pi R = I$$

$$\oint_c \vec{H}\cdot d\vec{l} = I_{inside}$$

Ampere 법칙

Ampere 의 주회법칙

Ampere 법칙을 이용한 $\vec{H}$ 계산

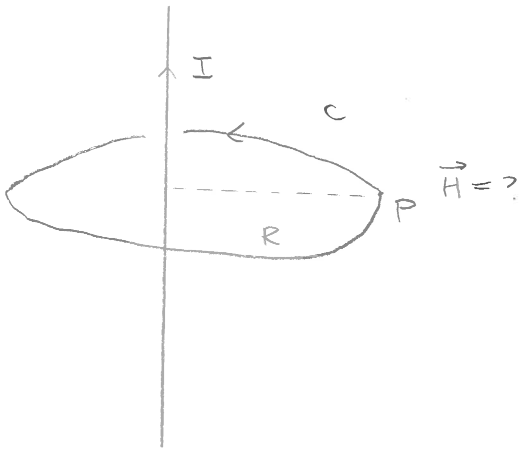
[1] 전류분포의 대칭성을 이용하여 적당히 닫힌선 c (Ampere line) 을 선택한다.

[2] $\oint_c \vec{H}\cdot d\vec{l}$ 을 계산한다.

[3] I_{inside} 를 계산한다.

[4] [2] 결과 = [3] 결과로 부터 $\vec{H}$ 를 계산한다.

(ex) 무한 도선



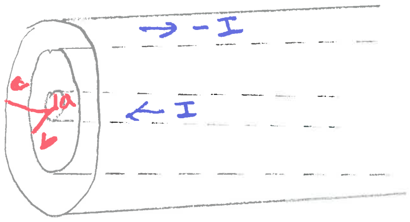
$$\oint_c \vec{H} \cdot d\vec{l} = H \cdot 2\pi R$$

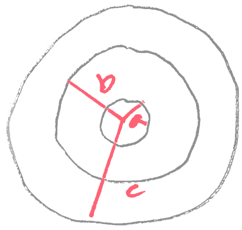
$$I_{inside} = I$$

$$\Rightarrow \quad H \cdot 2\pi R = I$$

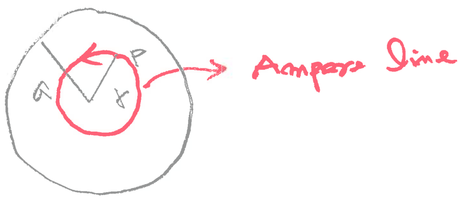
$$H = \frac{I}{2\pi R}$$

(Ex) 동축 cable





① $r \le a$

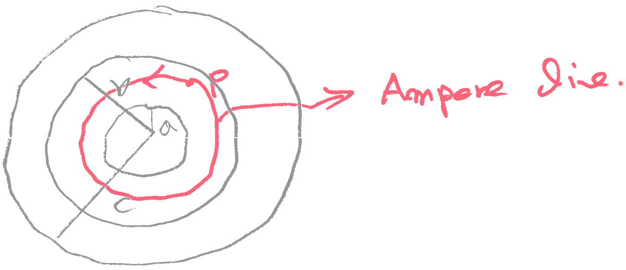


$$\oint_C \vec{H} \cdot d\vec{l} = H \; 2\pi r$$

$$I_{inside} = I \; \frac{\pi r^2}{\pi a^2}$$

$$\underline{H = \frac{I r}{2\pi a^2}}$$

② $a \le r \le b$

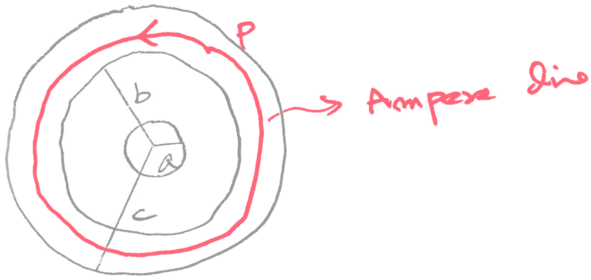


$$\oint_C \vec{H} \cdot d\vec{l} = H \cdot 2\pi r$$

$$I_{inside} = I$$

$$\underline{H = \frac{I}{2\pi r}}$$

③ $b \le r \le c$

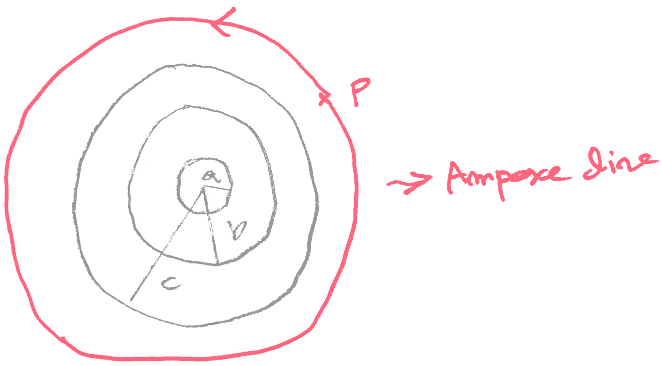


$$\oint_c \vec{H} \cdot d\vec{l} = H\, 2\pi r$$

$$I_{inside} = I - I \frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2} = I \frac{c^2 - r^2}{c^2 - b^2}$$

$$\Rightarrow \quad H = \frac{I}{2\pi r} \frac{c^2 - r^2}{c^2 - b^2}$$

④ $c \le r$

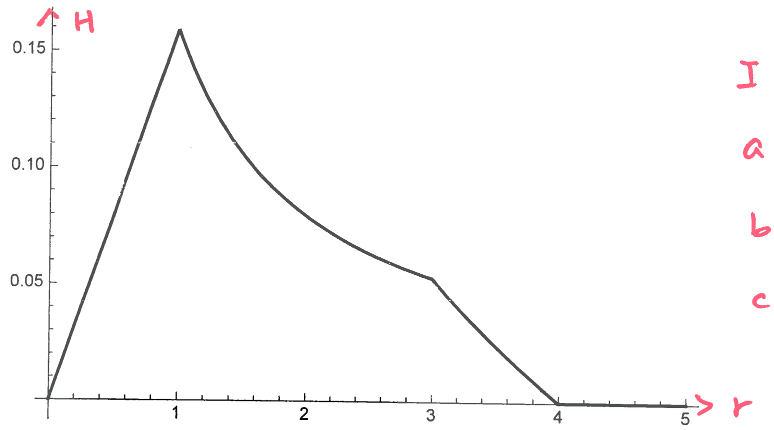


$$\oint_c \vec{H} \cdot d\vec{l} = H\, 2\pi r$$

$$I_{inside} = I - I = 0$$

$$H = 0.$$

```
current = 1; a = 1; b = 3; c = 4;
H[r_] := If[r ≤ a, current r / (2 Pi a^2), If[a < r ≤ b, current / (2 Pi r),
    If[b < r ≤ c, current (c^2 - r^2) / (2 Pi r (c^2 - b^2)), 0]]];
Plot[H[r], {r, 0, 5}, PlotStyle → Black]
```



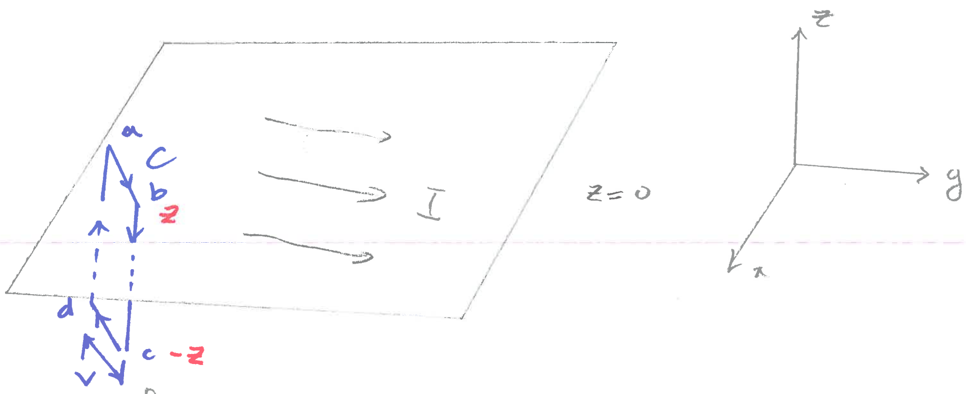
$I = 1$

$a = 1$

$b = 3$

$c = 4$

(Ex) 표면전류 (판전류, surface current)



From Biot-Savart law

$$\vec{H} = H(z)\,\hat{x}$$

$$H(-z) = -H(z)$$

$$\oint_C \vec{H}\cdot d\vec{l}$$

$$= \underbrace{\int_a^b \vec{H}\cdot d\vec{l}}_{H(z)\,L} + \underbrace{\int_b^c \vec{H}\cdot d\vec{l}}_{=0\ (d\vec{l}=-dz\,\hat{z})} + \overbrace{\int_c^d \vec{H}\cdot d\vec{l}}^{-H(-z)\,L} + \underbrace{\int_d^a \vec{H}\cdot d\vec{l}}_{=0\ (\because d\vec{l}=dz\,\hat{z})}$$

$$= [H(z) - H(-z)]\,L$$

$$= 2H(z)\,L$$

K: surface current density (단위 길이당 전류)

$$I_{inside} = K\,L$$

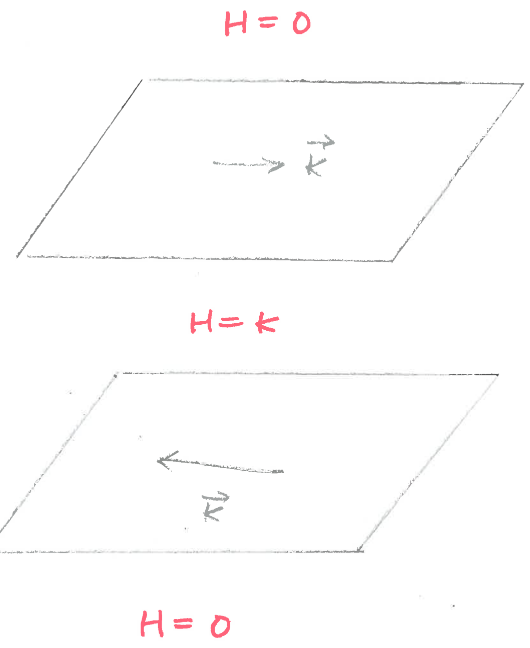
$$\Rightarrow H(z) = \frac{K}{2}$$

$$\vec{H} = \begin{cases} \frac{K}{2}\,\hat{x} & (z > 0) \\ -\frac{K}{2}\,\hat{x} & (z < 0) \end{cases}$$

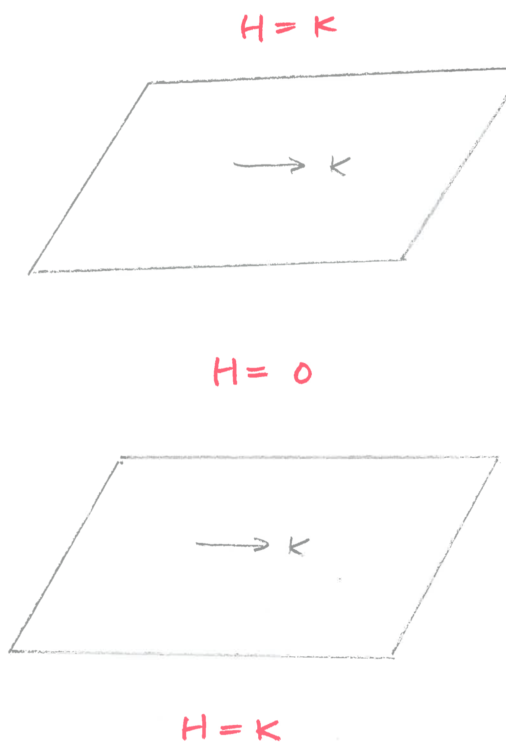
$$\Rightarrow \boxed{\begin{array}{l} \vec{H} = \frac{1}{2}\vec{K} \times \hat{a}_N \\ \\ \hat{a}_N : \text{面에서 P 점을 향한 단위 법선 vector} \end{array}}$$

(Ex) 평행한 두 표면전류

(i)

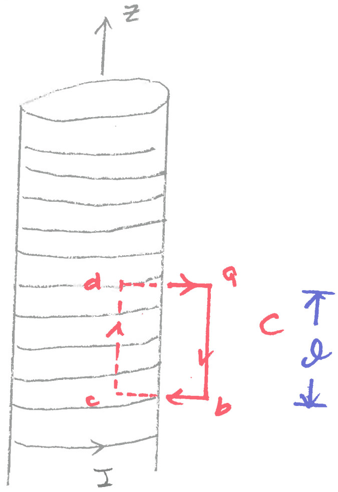


(ii)



(Ex) Solenoid

① infinite solenoid



n : 단위 길이당 turn 수

$$\vec{H} = \begin{cases} z \text{ 방향} & \text{solenoid 내부} \\ 0 & \text{외부.} \end{cases}$$

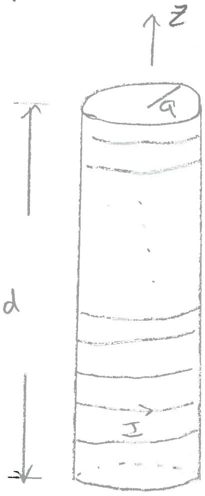
$$\oint_C \vec{H} \cdot d\vec{l} = l H$$

$$I_{inside} = I n l$$

$$\Rightarrow \quad H = n I$$

$$\Rightarrow \quad \vec{H} = \begin{cases} n I \hat{z} & \text{solenoid 내부} \\ 0 & \text{외부} \end{cases}$$

② finite solenoid



N : turn 수

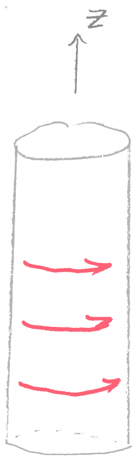
일반적으로 finite solenoid 인 경우 $\vec{H}$ 를 계산할 수 없다.

$d \gg a$ 근사적으로 infinite solenoid 로 취급

$$n = \frac{N}{d}$$

$$\vec{H} = \begin{cases} \dfrac{NI}{d}\hat{z} & \text{solenoid 내부} \\ 0 & \text{외부} \end{cases}$$

③ 표면 전류

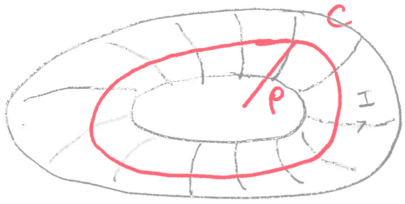


$K = nI$

$$\vec{H} = \begin{cases} K\hat{z} & \text{solenoid 내부} \\ 0 & \text{외부} \end{cases}$$

(Ex) Toroid

①



N: turn 수

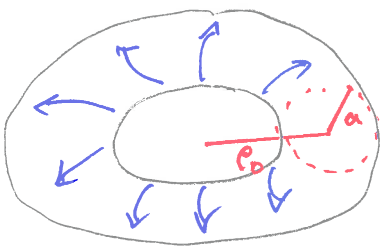
$$\vec{H} = \begin{cases} \hat{\phi} \text{ 방향} & : \text{toroid 내부} \\ 0 & : \text{외부} \end{cases}$$

$$\oint_C \vec{H} \cdot d\vec{l} = H\, 2\pi\rho$$

$$I_{inside} = NI$$

$$\vec{H} = \begin{cases} \dfrac{NI}{2\pi\rho}\,\hat{\phi} & \text{toroid 내부} \\ 0 & \text{외부} \end{cases}$$

② 표면전류



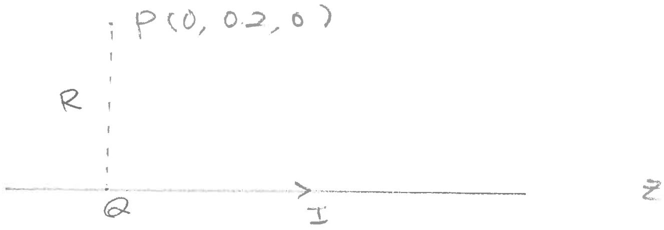
$$NI : 2\pi(\rho_0 - a) = K : 1$$

$$\Rightarrow NI = 2\pi(\rho_0 - a)K$$

$$\vec{H} = \begin{cases} \dfrac{\rho_0 - a}{\rho} K\,\hat{\phi} & \text{toroid 내부} \\ 0 & \text{외부.} \end{cases}$$

응용예제 7.3

(a)



$x = 0.1, \quad y = 0.3$

$I = 2.5\ (A)$

$Q(0.1, 0.3, 0)$

$$R = \overline{PQ} = \sqrt{(0.1)^2 + (0.1)^2}$$

$$H = \frac{I}{2\pi R} = \frac{2.5}{2\pi\sqrt{(0.1)^2+(0.1)^2}} = 2.81349$$

방향:

$$\hat{z} \times \overrightarrow{QP}$$

$$= \hat{z} \times [-0.1\hat{x} - 0.1\hat{y}]$$

$$= -0.1\hat{y} + 0.1\hat{x}$$

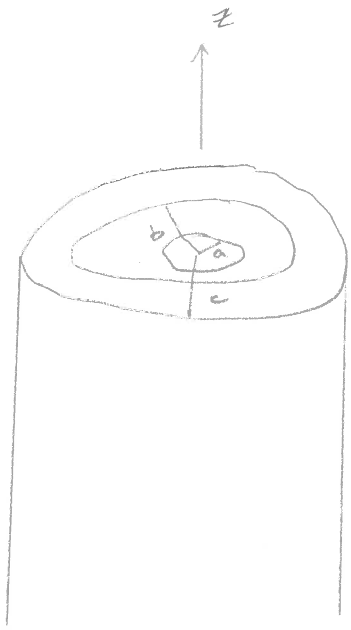
$$= 0.1(\hat{x} - \hat{y})$$

$$\hat{\varphi} = \frac{1}{\sqrt{2}}(\hat{x} - \hat{y})$$

$$\vec{H} = \frac{I}{2\pi R}\frac{1}{\sqrt{2}}(\hat{x} - \hat{y}) = 1.989(\hat{x} - \hat{y}) \quad (A/m)$$

//

(b)



$a = 0.3$

$b = 0.5$

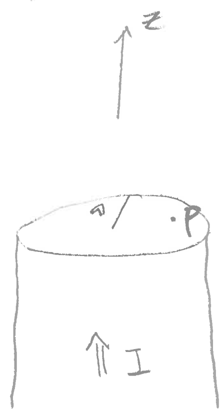
$c = 0.6$

$p(0, 0.2, 0)$

$I = 2.5\,(A)$

$r = 0.2 < a$

$$H = \frac{Ir}{2\pi a^2}$$



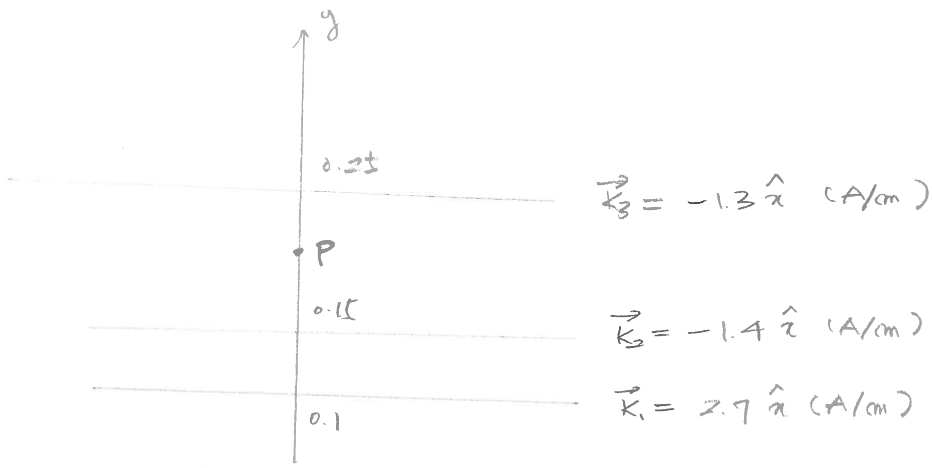
$\vec{H}$의 방향

$$\hat{z} \times \hat{y} = -\hat{x}$$

$$\vec{H} = -\frac{Ir}{2\pi a^2}\hat{x}$$

$$= -0.884194\,\hat{x} \quad (A/m)$$

(c)



$$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3$$

$$\vec{H}_1 = \frac{1}{2}\vec{K}_1 \times \hat{y} = \frac{2.7}{2}\,\hat{x} \times \hat{y} = 1.35\,\hat{z}$$

$$\vec{H}_2 = \frac{1}{2}\vec{K}_2 \times \hat{y} = -0.7\,\hat{x} \times \hat{y} = -0.7\,\hat{z}$$

$$\vec{H}_3 = \frac{1}{2}\vec{K}_3 \times (-\hat{y}) = \frac{1.3}{2}\,\hat{z} = 0.65\,\hat{z}$$

$$\Rightarrow \vec{H} = 1.3\,\hat{z} \quad (A/m)$$

$\vec{\nabla} \times \vec{H}$

① 직각좌표계

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \hat{x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \hat{y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{z}$$

② 원통좌표계

$$\vec{\nabla} \times \vec{H} = \frac{1}{\rho} \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$

$$= \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \hat{\rho} + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) \hat{\phi} + \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho H_\phi) - \frac{1}{\rho} \frac{\partial H_\rho}{\partial \phi} \right) \hat{z}$$

③ 구좌표계

$$\vec{\nabla} \times \vec{H} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\, H_\phi \end{vmatrix}$$

$$= \frac{1}{r\sin\theta} \left[\frac{\partial}{\partial \theta} (\sin\theta H_\phi) - \frac{\partial H_\theta}{\partial \phi} \right] \hat{r} + \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial}{\partial r} (rH_\phi) \right] \hat{\theta}$$

$$+ \frac{1}{r} \left[\frac{\partial}{\partial r} (rH_\theta) - \frac{\partial H_r}{\partial \theta} \right] \hat{\phi}$$

(Ex) $\vec{H} = y\hat{x} + x\hat{y} + z\hat{z}$

① 직각좌표계

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & z \end{vmatrix} = 0$$

② 원통좌표계

$$\vec{H} = \rho \sin 2\phi \, \hat{\rho} + \rho \cos 2\phi \, \hat{\phi} + z\hat{z}$$

$$\vec{\nabla}\times\vec{H} = 0$$

③ 구좌표계

$$\vec{H} = r(\sin^2\theta \sin 2\phi + \cos^2\theta)\hat{r} - \frac{1}{2} r \sin 2\theta (1 - \sin 2\phi)\hat{\theta}$$

$$+ r \sin\theta \cos 2\phi \, \hat{\phi}$$

$$\vec{\nabla}\times\vec{H} = 0$$

※

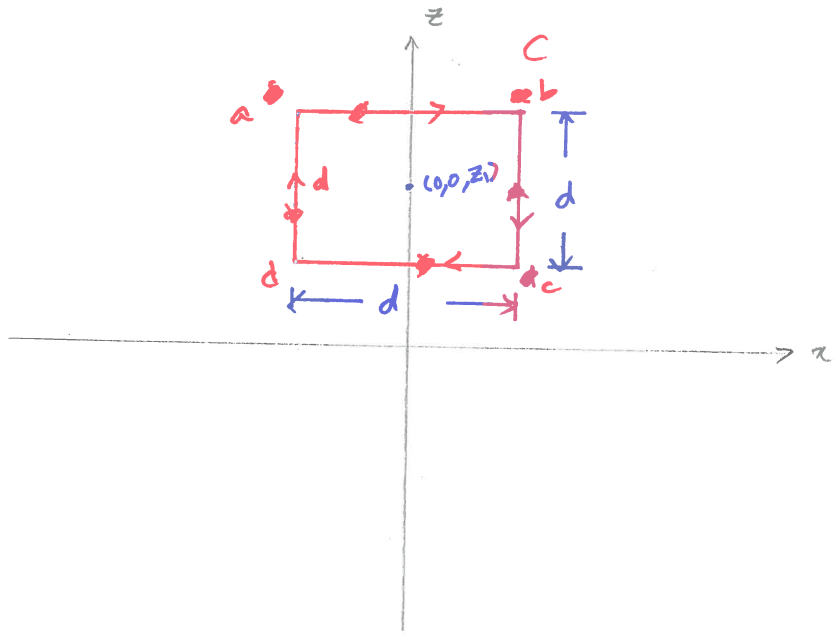
Stoke's theorem

$$\oint_C \vec{H}\cdot d\vec{l} = \int_S (\vec{\nabla}\times\vec{H})\cdot\hat{n}\, ds$$

S : C 내부의 면적

$\hat{n}$: C의 오른나사 법칙을 만족하는 S의 단위 법선 vector

(예제 7.2)



$\vec{H} = 0.2 z^2 \hat{x}$

$\oint_C \vec{H} \cdot d\vec{l}$

$= \underbrace{\int_a^b \vec{H} \cdot d\vec{l}}^{d\vec{l} = dx\,\hat{x}} + \underbrace{\int_b^c \vec{H} \cdot d\vec{l}}^{=0} + \underbrace{\int_c^d \vec{H} \cdot d\vec{l}}^{d\vec{l} = dx\,\hat{x}} + \underbrace{\int_d^a \vec{H} \cdot d\vec{l}}^{=0}$

$= \int_a^b 0.2 \left(z_1 + \frac{d}{2}\right)^2 dx \quad + \int_c^d 0.2 \left(z_1 - \frac{d}{2}\right)^2 dx$

$= 0.2 \left(z_1 + \frac{d}{2}\right)^2 \int_{-\frac{d}{2}}^{\frac{d}{2}} dx \quad + 0.2 \left(z_1 - \frac{d}{2}\right)^2 \int_{\frac{d}{2}}^{-\frac{d}{2}} dx$

$= 0.2d \left[\left(z_1 + \frac{d}{2}\right)^2 - \left(z_1 - \frac{d}{2}\right)^2 \right]$

$= 0.4 z_1 d^2.$

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0.2z^2 & 0 & 0 \end{vmatrix} = 0.4z\,\hat{y}$$

$$\hat{n} = \hat{y}$$

$$(\vec{\nabla}\times\vec{H})\cdot\hat{n} = 0.4z$$

$$\int_S (\vec{\nabla}\times\vec{H})\cdot\hat{n}\,dS$$

$$= \int_S (0.4z)\,dS \qquad (dS = dx\,dz)$$

$$= 0.4 \underbrace{\int_{-\frac{d}{2}}^{\frac{d}{2}} dx}_{d} \underbrace{\int_{z_1-\frac{d}{2}}^{z_1+\frac{d}{2}} z\,dz}_{z_1 d}$$

$$= 0.4 z_1 d^2$$

※

Ampere 법칙

$$\oint_C \vec{H}\cdot d\vec{l} = I_{inside}$$

$$\Rightarrow \int_S (\vec{\nabla}\times\vec{H})\cdot\hat{n}\, ds = I_{inside} = \int_S \vec{J}\cdot\hat{n}\, ds$$

$$\Rightarrow \quad \underline{\vec{\nabla}\times\vec{H} = \vec{J}}$$

P213

(응용예제 7.5)

(a) $\vec{H} = x^2 z\,\hat{y} - y^2 x\,\hat{z}$

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x^2 z & -y^2 x \end{vmatrix}$$

$$= -(2xy + x^2)\,\hat{x} + y^2\,\hat{y} + 2xz\,\hat{z}$$

P(2, 3, 4)

$$\vec{J} = \vec{\nabla}\times\vec{H} = -16\,\hat{x} + 9\,\hat{y} + 16\,\hat{z} \quad (A/m^2)$$

(b) $P(\rho=1.5,\ \phi=90^\circ,\ z=0.5)$

$$\vec{H} = \frac{2}{\rho}\cos 0.2\phi$$

$$\vec{\nabla}\times\vec{H} = \frac{0.4}{\rho^2}\sin 0.2\phi\ \hat{z}$$

$$\vec{J} = \frac{0.4}{1.5^2}\sin\frac{\pi}{10}\ \hat{z} = 0.0549 \quad (A/m^2)$$

(c) $P(r=2,\ \theta=30^\circ,\ \phi=20^\circ)$

$$\vec{H} = \frac{1}{\sin\theta}\hat{\theta}$$

$$\vec{\nabla}\times\vec{H} = \frac{1}{r\sin\theta}\hat{\phi}$$

$$\vec{J} = \vec{\nabla}\times\vec{H} = \frac{1}{2\sin\frac{\pi}{6}}\hat{\phi} = \hat{\phi} \quad (c/m^2)$$ ※

p214

(예제 7.3)

$$\vec{H} = 6r\sin\phi\,\hat{r} + 18r\sin\theta\cos\phi\,\hat{\phi}$$

$$C = C_1 + C_2 + C_3$$

C_1: $r=4$, $0 \le \theta \le 0.1\pi$, $\phi = 0$

C_2: $r=4$, $\theta = 0.1\pi$, $0 \le \phi \le 0.3\pi$

C_3: $r=4$, $0 \le \theta \le 0.1\pi$, $\phi = 0.3\pi$

$$\oint_C \vec{H}\cdot d\vec{l}$$

$$= \int_{C_1} \vec{H}\cdot d\vec{l} + \int_{C_2} \vec{H}\cdot d\vec{l} + \int_{H_3} \vec{H}\cdot d\vec{l} \qquad -①$$

$$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{H}\cdot d\vec{l} = 6r\sin\phi\,dr + 18r^2\sin^2\theta\cos\phi\,d\phi$$

Since $r=4$, $dr=0$

$$\vec{H}\cdot d\vec{l} = 288\sin^2\theta\cos\phi\,d\phi \qquad -②$$

Since $\phi = 0$ at C_1, $d\phi = 0 \Rightarrow \int_{C_1} \vec{H}\cdot d\vec{l} = 0$

Since $\phi = 0.3\pi$ at C_3, $d\phi = 0 \Rightarrow \int_{C_3} \vec{H}\cdot d\vec{l} = 0$

$$\oint_C \vec{H} \cdot d\vec{l}$$

$$= \int_{C_2} \vec{H} \cdot d\vec{l}$$

$$= 288 \sin^2(0.1\pi) \int_0^{0.3\pi} \cos\phi \, d\phi$$

$$= 288 \sin^2(0.1\pi) \sin(0.3\pi)$$

$$= 22.2 \ (A)$$

$$\vec{\nabla} \times \vec{H} = 36 \cos\theta \cos\phi \ \hat{r} + \left(\frac{6\cos\phi}{\sin\theta} - 36 \sin\theta \cos\phi \right) \hat{\theta}$$

$$\hat{n} = \hat{r}$$

$$(\vec{\nabla} \times \vec{H}) \cdot \hat{n} = 36 \cos\theta \cos\phi$$

$$\int_S (\vec{\nabla} \times \vec{H}) \cdot \hat{n} \, ds$$

$$= 36 \int \cos\theta \cos\phi \, ds \qquad ds = r^2 \sin\theta \, d\theta \, d\phi = 16 \sin\theta \, d\theta \, d\phi$$

$$= 576 \int_0^{0.1\pi} d\theta \int_0^{0.3\pi} d\phi \ \sin\theta \cos\theta \cos\phi$$

$$= 576 \underbrace{\int_0^{0.1\pi} \sin\theta \cos\theta \, d\theta}_{\frac{1}{2} \sin^2\theta \Big|_{\theta=0}^{\theta=0.1\pi} = \frac{1}{2}\sin^2(0.1\pi)} \ \underbrace{\int_0^{0.3\pi} \cos\phi \, d\phi}_{\sin(0.3\pi)}$$

$$= 288 \sin^2(0.1\pi) \sin(0.3\pi)$$

$$= 22.2 \ (A)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) = 0$$

vector identity

$\Rightarrow$ $$\vec{\nabla}\times\vec{H} = \vec{J}$$

$\rightarrow$ $$\vec{\nabla}\cdot\vec{J} = 0$$ 정상계에서의 연속방정식

note)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

정상계 에서는 $\frac{\partial \rho}{\partial t} = 0$.

$\Rightarrow$ $\vec{\nabla}\cdot\vec{J} = 0$.

※

§ 자장과 자속밀도

$\vec{B} = \mu_0 \vec{H}$ 자속밀도 (magnetic flux density)

magnetic induction

(i) $[\vec{B}] = \frac{N}{Am} = T$ (tesla)

$1\,(T) = 10^4\,(G)$ gauss

$Wb = T\,m^2 = J/A$ (weber)

(ii) μ_0 : 진공투자율

$\mu_0 = 4\pi \times 10^{-7}$ (Tm/A)

$\frac{Tm}{A} = \frac{N}{A^2}$

$H = \frac{V\,sec}{A} = \frac{J}{A^2}$: Henry $\Rightarrow H = \cancel{Wb \cdot A}\,\frac{Wb}{A}$

$\Rightarrow \frac{Tm}{A} = H/m$

$\mu_0 = 4\pi \times 10^{-7}$ (H/m)

○ 자기선속 (magnetic flux)

$$\Phi = \int_s \vec{B} \cdot \hat{u}_N \, ds \quad \leftrightarrow \quad \Psi = \int_s \vec{E} \cdot \hat{u}_N \, ds$$ 전기선속

$$[\Phi] = [B][S] = Wb$$

$$\oint_s \vec{B} \cdot \hat{u}_N \, ds = 0 \quad \leftrightarrow \quad \oint_s \vec{D} \cdot \hat{u}_N \, ds = Q_{inside}$$

자기에 관한 Gauss법칙　　　　전기에 관한 Gauss 법칙

Using divergence theorem $\oint_s \vec{F} \cdot \hat{u}_N \, ds = \int_v (\vec{\nabla} \cdot \vec{F}) \, dv$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \leftrightarrow \quad \vec{\nabla} \cdot \vec{D} = \rho$$

⇒ 정상계에서의 Maxwell 방정식

(i) 미분형

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = 0 \quad \leftarrow \quad \vec{F} = q\vec{E}$$ 보존력

$$\vec{\nabla} \times \vec{H} = J$$

$$* \quad \vec{D} = \epsilon_0 \vec{E}, \quad \vec{B} = \mu_0 \vec{H}, \quad \vec{E} = -\vec{\nabla} V$$

(ii) 적분형

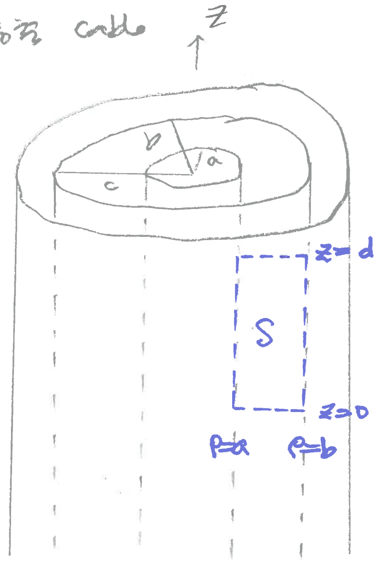
$$\oint_S \vec{D} \cdot \hat{u}_N \, ds = Q_{inside} = \int_v \rho \, dv$$

$$\oint_S \vec{B} \cdot \hat{u}_N \, ds = 0$$

$$\oint_c \vec{H} \cdot d\vec{l} = I_{inside} = \int_s \vec{J} \cdot \hat{u}_N \, ds$$

$$\oint_c \vec{E} \cdot d\vec{l} = 0$$

(Ex) 동축 cable



$$\vec{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2}\,\hat{\phi} & 0 \le \rho \le a \\[2ex] \dfrac{I}{2\pi\rho}\,\hat{\phi} & a \le \rho \le b \\[2ex] \dfrac{I}{2\pi\rho}\,\dfrac{c^2-\rho^2}{c^2-b^2}\,\hat{\phi} & b \le \rho \le c \\[2ex] 0 & c \le \rho \end{cases}$$

In S $\quad \vec{H} = \dfrac{I}{2\pi\rho}\hat{\phi} \Rightarrow \vec{B} = \mu_0\vec{H} = \dfrac{\mu_0 I}{2\pi\rho}\hat{\phi}$

$$\Phi = \int_S \vec{B}\cdot\hat{u}_N\, ds \qquad \left(\begin{array}{c} \hat{u}_N = \hat{\phi} \\ ds = d\rho\, dz \end{array} \right)$$

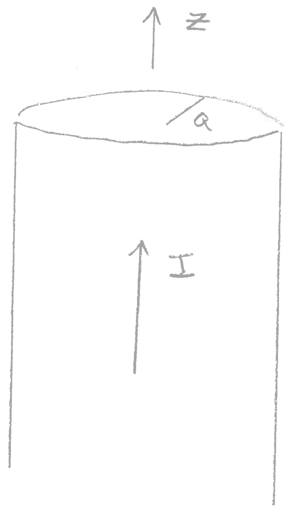
$$= \int_a^b d\rho \int_0^d dz \; \frac{\mu_0 I}{2\pi\rho}$$

$$= \frac{\mu_0 I d}{2\pi} \ln\frac{b}{a}$$

※

p220

(응용예제 7.7)



$I = 20\ (A)$

$a = 1\ (mm) = 10^{-3}\ (m)$

(a) $\rho = 0.5\ (mm) = 0.5 \times 10^{-3}\ (m)$

$$H = \frac{I\rho}{2\pi a^2} = 1591.55 \quad (A/m)$$

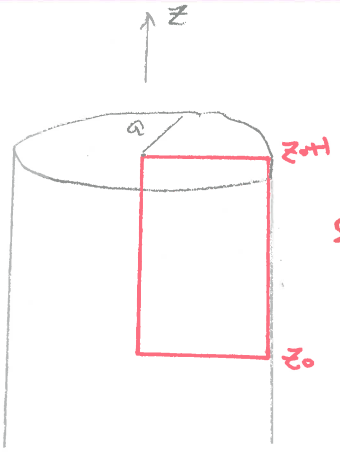
$$\vec{H} = 1591.55\ \hat{\phi} \quad (A/m)$$

(b) $\rho = 0.8\ (mm) = 0.8 \times 10^{-3}\ (m)$

$$H = \frac{I\rho}{2\pi a^2}$$

$$B = \mu_0 H = \frac{\mu_0 I \rho}{2\pi a^2} = 3.2 \times 10^{-3}\ (T) = 3.2\ (mT)$$

(c)



$$\vec{B} = \frac{\mu_0 I \rho}{2\pi a^2}\hat{\phi}$$

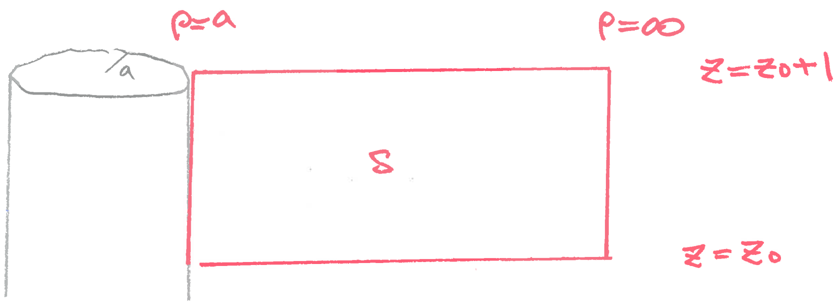
$$\Phi = \int_S \vec{B}\cdot\hat{u}_N\, ds \qquad \hat{u}_N = \hat{\phi}, \quad ds = d\rho\, dz$$

$$= \int_{z_0}^{z_0+1} dz \int_0^a d\rho\ \frac{\mu_0 I \rho}{2\pi a^2}$$

$$= \frac{\mu_0 I}{2\pi a^2}\cdot\frac{1}{2}a^2$$

$$= \frac{\mu_0 I}{4\pi} = 2\times 10^{-6}\ (\mathrm{Wb/m}) = 2\ (\mu\mathrm{Wb/m})$$

(d)



$$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\hat{\phi}$$

$$\Phi = \int_S \vec{B}\cdot\hat{u}_N\, ds$$

$$= \int_{z_0}^{z_0+1} dz \int_a^{\infty} d\rho\ \frac{\mu_0 I}{2\pi\rho} = \infty$$

//

※ 스칼라 자위와 vector 자위

$\vec{E} = -\vec{\nabla} V$ V : 전위 (electric potential)

V와 같은 역할을 하는 magnetic potential을 만들 수 있을까?

$\vec{\nabla} \times \vec{H} = \vec{J}$

(i) $\vec{J} = 0$ case

$\vec{\nabla} \times \vec{H} = 0$

$\vec{\nabla} \times (\vec{\nabla} \varphi) = 0$ vector identity

$\vec{H} = -\vec{\nabla} V_m$

$\vec{E} = -\vec{\nabla} V$

V_m : scalar magnetic potential

(스칼라 자위)

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

① $[V_m] = A$

② $\vec{\nabla} \cdot \vec{B} = \mu_0 \vec{\nabla} \cdot \vec{H} = 0$

$\Rightarrow \vec{\nabla}^2 V = -\frac{\rho}{\varepsilon}$ Poisson Equation

If $\rho = 0$,

$\Rightarrow \underline{\vec{\nabla}^2 V_m = 0}$ $(\vec{J} = 0)$

$\vec{\nabla}^2 V = 0$ Laplace Equation

Laplace Equation

③ $V_m(A) = -\int_{C,\text{기준점}}^{A} \vec{H} \cdot d\vec{l}$

$V(A) = -\int_{\text{기준점}, C}^{A} \vec{E} \cdot d\vec{l}$

$\oint \vec{H} \cdot d\vec{l} = I_{inside}$

$\oint \vec{E} \cdot d\vec{l} = 0$

V(A)가 C에 무관

경로 C가 I_{inside}를 갖는다면

$V_m(A)$는 경로에 의존

Ex) 동축 cable

$$\vec{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2}\,\hat{\phi} & 0 \le \rho \le a \\[2ex] \dfrac{I}{2\pi\rho}\,\hat{\phi} & a \le \rho \le b \\[2ex] \dfrac{I}{2\pi\rho}\,\dfrac{c^2-\rho^2}{c^2-b^2} & b \le \rho \le c \\[2ex] 0 & c \le \rho \end{cases}$$

Now, we consider $a \le \rho \le b$ region.

$\vec{J} = 0$, $\vec{\nabla}\times\vec{H} = 0$

$$\vec{H} = -\vec{\nabla} V_m$$

$$\frac{I}{2\pi\rho}\,\hat{\phi} = -\left[\frac{\partial V_m}{\partial \rho}\,\hat{\rho} + \frac{1}{\rho}\frac{\partial V_m}{\partial \phi}\,\hat{\phi} + \frac{\partial V_m}{\partial z}\,\hat{z}\right]$$

$\Rightarrow$
$$\frac{\partial V_m}{\partial \rho} = 0$$

$$\frac{\partial V_m}{\partial \phi} = -\frac{I}{2\pi}$$

$$\frac{\partial V_m}{\partial z} = 0$$

$\Rightarrow$ $V_m = -\dfrac{I}{2\pi}\phi$ (기준점을 $\phi = 0$ 으로 선택)

$\phi = \frac{\pi}{4}$ 에서의 scalar 자위 ?

$$V_m\left(\phi = \frac{\pi}{4}\right) = -\frac{I}{8}$$

$$V_m\left(\phi = \frac{9}{4}\pi\right) = -\frac{9I}{8}$$

$$\vdots$$

$$V_m\left(\phi = 2n\pi + \frac{\pi}{4}\right) = -\frac{I}{2}\left(2n + \frac{1}{4}\right) \qquad (n = 0, \pm 1, \pm 2, \cdots)$$

Unique 한 V_m 을 갖기 위해서는 ϕ 의 범위를 제한

$-\pi < \phi \le \pi$ $\qquad$ $\phi = \pi$ 에서 "branch cut"

$$V_m\left(\phi = \frac{\pi}{4}\right) = -\frac{I}{8}$$ ※

$\vec{E} = -\vec{\nabla} V$

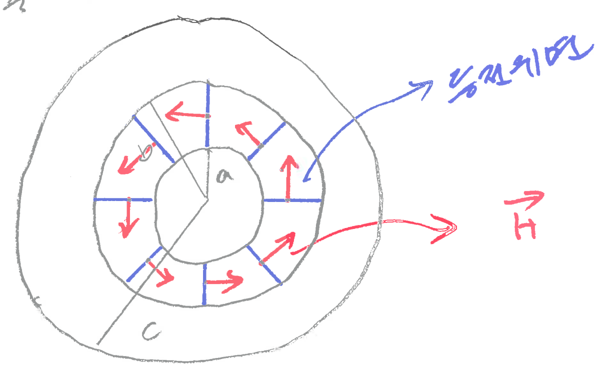
(4) $\vec{H} = -\vec{\nabla} V_m$

$(V_m)_{AB} \equiv V_m(A) - V_m(B) = \int_A^B \vec{H} \cdot d\vec{l}$ $\qquad$ $V_{AB} = V(A) - V(B) = \int_A^B \vec{E} \cdot d\vec{l}$

If $V_m(A) = V_m(B)$, $\vec{H} \cdot d\vec{l} = 0$ $\qquad$ If $V(A) = V(B)$, $\vec{E} \cdot d\vec{l} = 0$

등자위면과 $\vec{H}$ 가 수직 $\qquad$ 등전위면과 $\vec{E}$ 가 수직

(ex) 동축 cable



※

(ii) $\vec{J} \neq 0$ case

$$\vec{\nabla} \cdot \vec{B} = 0$$

$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$

vector identity

$$\boxed{\begin{array}{l} \vec{B} = \vec{\nabla} \times \vec{A} \\ \vec{A} : \text{vector magnetic potential} \end{array}}$$

① $[\vec{A}] = Wb/m$

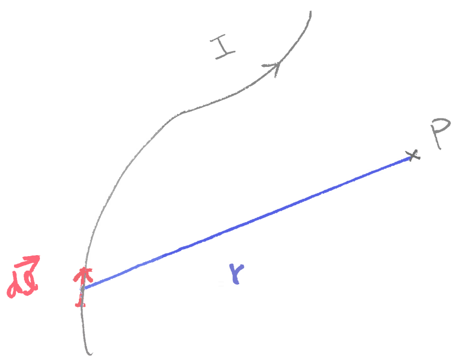
② $\vec{B} = \mu_0 \vec{H}$

$\Rightarrow \vec{H} = \frac{1}{\mu_0} \vec{B}$

$\Rightarrow \vec{\nabla} \times \vec{H} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} = \frac{1}{\mu_0} \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{J}$

$\vec{A}$를 알면 $\vec{H}$와 $\vec{J}$를 구할수 있다.

③



$$\vec{A} = \int_C \frac{\mu_0 I \, d\vec{l}}{4\pi r} \propto \frac{1}{r}$$

$$* \quad \vec{B} = \int_C \frac{\mu_0 I \, d\vec{l} \times \hat{r}}{4\pi r^2} \propto \frac{1}{r^2}$$

(증명)

$$\vec{\nabla} \times \vec{A} = \int_c \frac{\mu_0 I}{4\pi} \vec{\nabla} \times \left(\frac{d\vec{l}'}{r} \right)$$

$d\vec{l}'$: 미소전류의 좌표

r: P점의 좌표

$$\left(\vec{\nabla} \times (\varphi \vec{V}) = \varphi \vec{\nabla} \times \vec{V} + \vec{\nabla}\varphi \times \vec{V} \right)$$

$$= \int_c \frac{\mu_0 I}{4\pi} \left[\frac{1}{r} \underbrace{\vec{\nabla} \times d\vec{l}'}_{=0} + \underbrace{\vec{\nabla}\left(\frac{1}{r}\right)}_{-\frac{\hat{r}}{r^2}} \times d\vec{l} \right]$$

$$= -\int_c \frac{\mu_0 I}{4\pi} \frac{\hat{r} \times d\vec{l}}{r^2}$$

$$= \int_c \frac{\mu_0 I \, d\vec{l} \times \hat{r}}{4\pi r^2}$$

$$= \vec{B}$$

※

④ $I \, d\vec{l} = \vec{J} \, dv$

$$\vec{A} = \int_v \frac{\mu_0 \vec{J} \, dv}{4\pi r}$$

v: $\vec{J}$가 존재하는 모든 부피

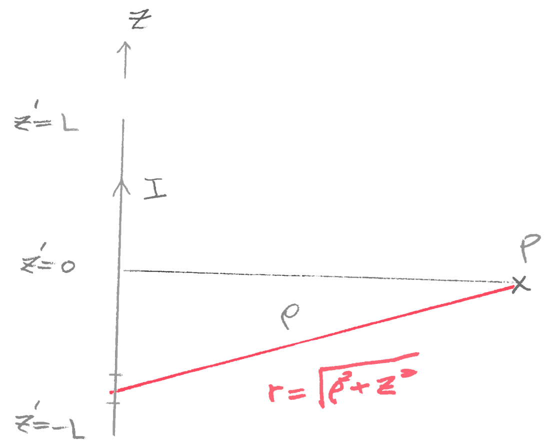
⑤ 표면전류

$I \, d\vec{l} = \vec{K} \, ds$

$$\vec{A} = \int_s \frac{\mu_0 \vec{K} \, ds}{4\pi r}$$

s: $\vec{K}$가 존재하는 모든 면

(Ex)



(i) $\vec{A}$ at P

$$d\vec{l}' = dz'\,\hat{z}$$

$$r = \sqrt{\rho^2 + z'^2}$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \int_{-L}^{L} \frac{dz'}{\sqrt{\rho^2 + z'^2}}\,\hat{z}$$

$$\int \frac{dx}{\sqrt{a + cx^2}} = \frac{1}{\sqrt{c}} \ln\left[x\sqrt{c} + \sqrt{a + cx^2}\right]$$

$$= \hat{z}\,\frac{\mu_0 I}{4\pi} \ln\left[z' + \sqrt{\rho^2 + z'^2}\right]\Bigg|_{z'=-L}^{z'=L}$$

$$= \hat{z}\,\frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{\rho^2 + L^2} + L}{\sqrt{\rho^2 + L^2} - L}$$

(ii) $\vec{B}$ at P

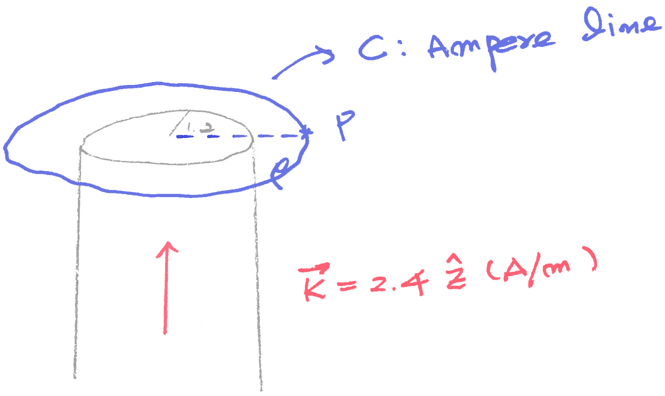
$$\vec{B} = \vec{\nabla} \times \vec{A} = -\frac{\partial}{\partial \rho}\left[\frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{\rho^2 + L^2} + L}{\sqrt{\rho^2 + L^2} - L}\right]\hat{\phi}$$

$$= \frac{\mu_0 I L}{2\pi \rho \sqrt{\rho^2 + L^2}}\,\hat{\phi}$$

If $L = \infty$, $\vec{B} = \frac{\mu_0 I}{2\pi \rho}\,\hat{\phi}$ ※

(응용예제 7.8)

(a)



$$\oint_C \vec{H} \cdot d\vec{l} = H\, 2\pi \rho$$

$$I_{inside} = K \;\; 2\pi \times 1.2$$

$$H = \frac{K \times 1.2}{\rho} = \frac{1.2 \times 2.4}{\rho} = \frac{2.88}{\rho}$$

$$\vec{H} = \frac{2.88}{\rho}\,\hat{\phi} \quad (A/m)$$

(b) $P(\rho = 1.5,\ \phi = 0.6\pi,\ z = 1)$, $\phi = \pi$ branch cut $(-\pi \le \phi < \pi)$

$$\vec{H} = \frac{2.88}{\rho}\hat{\phi} = -\vec{\nabla} V_m = -\left[\frac{\partial V_m}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial V_m}{\partial \phi}\hat{\phi} + \frac{\partial V_m}{\partial z}\hat{z} \right]$$

$$V_m(\phi) = -2.88\,\phi + C$$

$C = 0$

$$V_m(0.6\pi) = -2.88 \times 0.6 \times \pi = -5.42867 \ (A)$$

(c) $\phi = \frac{\pi}{2} = 0.5\pi$ branch cut : $c = 0$

$$V_m(\phi) = V_m(2\pi + 0.6\pi) = 2.88 \times 1.4 \times \pi = 12.6669 \ (A)$$

(d) $C = 2.88\pi$, branch cut $\phi = 0$ $(0 < \phi \le 2\pi)$

$V_{cm}(\phi) = 2.88(\pi - \phi)$

$V_{cm}(\phi) = 2.88 \times 0.4\pi = 3.6191$ (A)

(e) branch cut $\phi = 0.8\pi$

$V_{cm}(-\pi) = 2.88\pi + C = 5$

$C = 5 - 2.88\pi$

$V_{cm}(\phi) = -2.88\phi + (5 - 2.88\pi)$

$= -2.88(\pi + \phi) + 5$

$V_{cm}(\phi) = -2.88 \times 1.6\pi + 5 = -9.47646$ (A) ※

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(응용예제 7.9)

$$\vec{H} = \frac{I\rho}{2\pi a^2}\hat{\phi}$$

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I \rho}{2\pi a^2}\hat{\phi} = \vec{\nabla}\times\vec{A}$$

$$\vec{A} = A\hat{z}$$

$$\Rightarrow \frac{\partial A}{\partial \rho} = -\frac{\mu_0 I \rho}{2\pi a^2}$$

$$\Rightarrow A = -\frac{\mu_0 I \rho^2}{4\pi a^2} + C \qquad -①$$

$$A(\rho=a) = C - \frac{\mu_0 I}{4\pi} = \frac{\mu_0 I \ln 5}{2\pi}$$

$$C = \frac{\mu_0 I}{4\pi}(1 + 2\ln 5) \qquad -②$$

② → ①

$$A = \frac{\mu_0 I}{4\pi}\left(1 + 2\ln 5 - \frac{\rho^2}{a^2}\right)$$

$$= 10^{-7} I \left(1 + 2\ln 5 - \frac{\rho^2}{a^2}\right)$$

(i) $\rho = 0$. $\quad A = 10^{-7} I (1 + 2\ln 5) = 0.422\, I \quad (\mu Wb/m)$

(ii) $\rho = 0.25a$ $\quad A = 10^{-7} I (1 + 2\ln 5 - 0.25^2) = 0.416\, I \quad (\mu Wb/m)$

(iii) $\rho = 0.75a$ $\quad A = 10^{-7} I (1 + 2\ln 5 - 0.75^2) = 0.3656 \quad (\mu Wb/m)$

(iv) $\rho = a$ $\quad A = 10^{-7} I \times (2\ln 5) = 0.322 \quad (\mu Wb/m)$ ※