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QUANTUM DISCORD AND QUANTUM ENTANGLEMENT IN THE PRESENCE OF AN ASYMPTOTICALLY FLAT STATIC BLACK HOLE

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In this work, the quantum discord and tripartite entanglement in the presence of an asymptotically flat static black hole are discussed. The total correlation, quantum discord, and classical correlation are found to exhibit decreasing behavior with increasing Hawking temperature. It is shown that the classical correlation is less than the quantum discord in the full range of Hawking temperature. The tripartite entanglements for Greenberger–Horne–Zeilinger (GHZ) and W-states also exhibit decreasing behavior with increasing Hawking temperature. When the Hawking temperature approaches the infinite limit, the tripartite entanglements of the GHZ and W-states reduce, in terms of the π-tangle, to 52% and 33% of the corresponding values in the flat space limit, respectively.

Keywords: Quantum discord; quantum entanglement; asymptotically flat black hole.

1. Introduction

Recently, quantum information theories in the relativistic framework have attracted considerable interest.[1–19] The most remarkable fact about inertial frames is that the entanglement of a given multipartite quantum state is conserved even though the entanglement between some degrees of freedom can be transferred to others.[4–7] Contrastively, in non-inertial frames, the entanglement is, in general, degraded, which implies that the quantum correlation between observers at rest and accelerating observers is further reduced with an increasing acceleration.[8] The main reason for reduction in the quantum correlation is that the accelerating observer located at one Rindler wedge loses information arising from the other Rindler wedge because the wedges are causally disconnected from each other. This implies that some

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quantum information is leaked into another causally disconnected Rindler space, which results in a reduction in the quantum correlation. In fact, this is the main feature of the well-known Unruh effect.[20,21] Recently, the Unruh-type decoherence effect occurring beyond the single-mode approximation was discussed in the context of quantum information theories.[22]

More recently, quantum entanglement in a black hole background was examined.[23,24] In particular, in Ref. 23, the Hawking temperature-dependence of bipartite entanglement was studied for an arbitrary, spherically symmetric and asymptotically flat black hole background.

The purpose herein is to investigate quantum discord[25,26] and tripartite entanglement in the presence of the same black hole. In Sec. 2, we introduce the spacetime background and the interrelation between the corresponding vacua, which was explicitly derived in Ref. 23. In Sec. 3, we compute the quantum discord, classical correlation, and total correlation for the spacetime background. It is found that the classical correlation is less than the quantum discord in the entire range of Hawking temperature. In Sec. 4, we discuss tripartite entanglement in the spacetime background. Degradation of the tripartite entanglement is found to occur in the presence of the black hole. However, the entanglement does not completely vanish, even at the infinite Hawking temperature limit. In Sec. 5, a brief conclusion is presented.

2. Spacetime Background

Throughout this paper, we use $G = c = \hbar = k_B = 1$. The metric used in this paper is

$$ds^2 = f(r)dt^2 - \frac{1}{h(r)}dr^2 - R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where the functions $f(r)$, $h(r)$, and $R(r)$ satisfy $f(\infty) = h(\infty) = 1$, $R(\infty) = r$, and $f(r_H) = h(r_H) = 0$, respectively. This is a general static and spherically symmetric line element, which includes Schwarzschild, Reissner–Nordström, Garfinkle–Horowitz–Strominger dilaton,[27] and Casadio–Fabbri–Mazzacurati brane[28] black holes, as indicated in Table 1. The Hawking temperature for this metric is $T_H = \kappa/2\pi$, where κ is the surface gravity defined as $\kappa = \sqrt{f'(r_H)h'(r_H)}/2$.

Table 1. Relation of Eq. (1) with other black hole metrics.

Black holes	$f(r)$	$h(r)$	$R^2(r)$
Schwarzschild	$1 - \frac{r_H}{r}$	$1 - \frac{r_H}{r}$	r^2
Reissner–Nordström	$\left(1 - \frac{r_H}{r}\right)\left(1 - \frac{r_-}{r}\right)$ $(r_H > r_-)$	$\left(1 - \frac{r_H}{r}\right)\left(1 - \frac{r_-}{r}\right)$ $(r_H > r_-)$	r^2
Dilaton	$1 - \frac{r_H}{r}$	$1 - \frac{r_H}{r}$	$r\left[r - \frac{2Q^2 e^{-2\phi_0}}{r_H}\right]$
Brane	$1 - \frac{r_H}{r}$	$\frac{\left(1-\frac{r_H}{r}\right)\left[1-\frac{r_H}{4r}(4\beta-1)\right]}{1-\frac{3r_H}{4r}}$	r^2

As demonstrated in Ref. 23, three different vacuum states, $|0\rangle_{\text{in}}$, $|0\rangle_{\text{out}}$, and $|0\rangle_K$, can be considered to exist in this background. The first two vacuum states are the Fock vacua located inside and outside the horizon, respectively, and the last one is the Kruskal vacuum located outside the event horizon. The interrelation between these vacua[23] is

$$|0\rangle_K = \sqrt{1 - e^{-\omega/T_H}} \sum_{n=0}^{\infty} e^{-n\omega/2T_H} |n\rangle_{\text{in}} \otimes |n\rangle_{\text{out}}, \tag{2}$$

where $|n\rangle_{\text{in}}$ and $|n\rangle_{\text{out}}$ are, respectively, the n-particle states constructed from $|0\rangle_{\text{in}}$ and $|0\rangle_{\text{out}}$ by operating the corresponding creation operators n times, and ω is the frequency of the scalar field. When deriving Eq. (2), we assume that the particle detector is sensitive to only the particles, whose energy is $\hbar\omega$. This is why the right-hand side of Eq. (2) is monochromatic. By applying the creation operator of the Kruskal spacetime in Eq. (2) and using the Bogoliubov coefficients, one can construct $|1\rangle_K$ as follows:

$$|1\rangle_K = (1 - e^{-\omega/T_H}) \sum_{n=0}^{\infty} \sqrt{n+1} e^{-n\omega/2T_H} |n\rangle_{\text{in}} \otimes |n+1\rangle_{\text{out}}. \tag{3}$$

3. Quantum Discord

Quantum discord[25,26] is a measure for the *quantumness* of a given bipartite quantum state. Generally, the two parties each consist of a system and corresponding apparatus. In this paper, however, we refer to these parties as Alice and Bob. We discuss, in this section, how quantum discord is changed in the presence of a black hole (1).

The physical situation is exactly the same with that of Ref. 23. Let Alice and Bob share a maximally entangled state

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}} (|1\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B), \tag{4}$$

at the same initial point in flat Monkowski space before the black hole formed. After sharing, Alice remains at the asymptotically flat region, but Bob freely falls in toward the big mass with his monochromatically sensitive detector and hovers outside of it before it collapses to form a black hole. Now, let it collapse to form a black hole. Then, Bob's detector registers only thermally excited monochromatic particles due to the Hawking effect.[23] Since Bob believes in his particle detector which shows a Kruskal particle spectrum, it is reasonable to assume that the Bob's state in Eq. (4) is Kruskal state.

However, since the inside region of the black hole is causally disconnected from Alice and Bob, we need to perform a partial trace over the in-state. Then, the state between Alice and Bob becomes a mixed state with the following density matrix

$$\rho_{AB} = \frac{1}{2}|0\rangle_A\langle 0| \otimes M_{00} + \frac{1}{2}|1\rangle_A\langle 1| \otimes M_{11} + \frac{1}{2}|0\rangle_A\langle 1| \otimes M_{01} + \frac{1}{2}|1\rangle_A\langle 0| \otimes M_{10}, \tag{5}$$

where

$$\begin{aligned}
M_{00} &= (1 - e^{-\omega/T_H}) \sum_{n=0}^{\infty} e^{-n\omega/T_H} |n\rangle\langle n|, \\
M_{11} &= (1 - e^{-\omega/T_H})^2 \sum_{n=0}^{\infty} (n+1) e^{-n\omega/T_H} |n+1\rangle\langle n+1|, \\
M_{01} &= (1 - e^{-\omega/T_H})^{3/2} \sum_{n=0}^{\infty} \sqrt{n+1}\, e^{-n\omega/T_H} |n\rangle\langle n+1|, \\
M_{10} &= (1 - e^{-\omega/T_H})^{3/2} \sum_{n=0}^{\infty} \sqrt{n+1}\, e^{-n\omega/T_H} |n+1\rangle\langle n|.
\end{aligned} \tag{6}$$

It is worthwhile to note that $\mathrm{Tr}_B M_{00} = \mathrm{Tr}_B M_{11} = 1$ and that $\mathrm{Tr}_B M_{01} = \mathrm{Tr}_B M_{10} = 0$.

Now, we discuss the quantum discord. We assume that Alice performs a projective measurement with a complete set of measurement operators $\{\Pi_j^A\}$. The usual mutual information between Alice and Bob is

$$I(A:B) = S(A) + S(B) - S(A,B), \tag{7}$$

where S denotes the von Neumann entropy $S(\rho) = \mathrm{Tr}(\rho \log \rho)$. In our paper, all logarithms are taken to base 2. The classical analogue of Eq. (7) is $I_{cl}(A:B) = H(A) + H(B) - H(A,B)$, where H denotes the Shannon entropy. In classical information theories, a different representation for the mutual information is $I_{cl}(A:B) = H(A) - H(A|B) = H(B) - H(B|A)$, where $H(X|Y)$ is the conditional entropy of X given Y. The quantum analogue of this representation[25] is

$$J(A:B)_{\{\Pi_j^A\}} = S(B) - \sum_j p_j S(B|\Pi_j^A), \tag{8}$$

where[a] $\{\Pi_j^A\}$ denotes a complete set of measurement operators prepared by party A and $S(B|\Pi_j^A)$ is a von Neumann entropy of party B after party A obtains a measurement outcome j. As obvious, p_j is the probability of obtaining outcome j in the quantum measurement. The general quantum mechanical postulates[31] imply that

$$p_j = \mathrm{Tr}_{A,B}(\Pi_j^A \rho_{AB} \Pi_j^A), \quad S(B|\Pi_j^A) = S(\rho(B|\Pi_j^A)), \tag{9}$$

where $\rho(B|\Pi_j^A) = \mathrm{Tr}_A(\Pi_j^A \rho_{AB} \Pi_j^A)/p_j$. Hence, unlike $I(A:B)$, $J(A:B)$ is dependent on the complete set of measurement operators. The quantum discord is defined as:

$$\begin{aligned}
\mathcal{D}(A:B) &= \min[I(A:B) - J(A:B)] \\
&= \min\left[S(A) - S(A,B) + \sum_j p_j S(B|\Pi_j^A)\right],
\end{aligned} \tag{10}$$

[a] Depending on the specific rules about the local operations and classical communication (LOCC) between Alice and Bob, one can define several different generalizations of $I_{cl}(A:B)$.[29,30] Thus, several different quantum discords can be defined. Our definition (8) corresponds to the optimal efficiency of a one-way purification strategy.[29]

where the minimum is taken over all possible choice of the complete set of measurement operators.[b]

Now, we compute the quantum discord in the black hole background. From Eq. (5), it is easy to show that $\rho_A \equiv \mathrm{Tr}_B \rho_{AB}$ is a completely mixed state and that

$$S(A) = 1. \tag{11}$$

Further, it is easy to show that

$$\begin{aligned} S(A,B) &= -\sum_{n=0}^{\infty} \Lambda_n \log \Lambda_n \\ \Lambda_n &= \frac{1}{2} e^{-n\omega/T_H}(1 - e^{-\omega/T_H})[1 + (n+1)(1 - e^{-\omega/T_H})]. \end{aligned} \tag{12}$$

Next, we introduce the complete set of projective measurement operators $\{\Pi_1^A, \Pi_2^A\}$, given by

$$\Pi_1^A = \frac{I_2 + \boldsymbol{x} \cdot \boldsymbol{\sigma}}{2}, \quad \Pi_2^A = \frac{I_2 - \boldsymbol{x} \cdot \boldsymbol{\sigma}}{2}. \tag{13}$$

In Eq. (13), $\boldsymbol{\sigma}$ denotes the Pauli matrix, and $x_1^2 + x_2^2 + x_3^2 = 1$. Then, it is straightforward to show that $p_1 = p_2 = 1/2$ and that

$$\begin{aligned} \rho(B|\Pi_1^A) &= \frac{1}{2}[(1 + x_3)M_{00} + (1 - x_3)M_{11} + (x_1 + ix_2)M_{01} + (x_1 - ix_2)M_{10}], \\ \rho(B|\Pi_2^A) &= \frac{1}{2}[(1 - x_3)M_{00} + (1 + x_3)M_{11} - (x_1 + ix_2)M_{01} - (x_1 - ix_2)M_{10}]. \end{aligned} \tag{14}$$

Since it is impossible to compute the eigenvalues of $\rho(B|\Pi_j^A)$ $(j = 1, 2)$, we need to compute $S(\rho(B|\Pi_j^A))$ numerically. One can perform this numerical calculation by parameterizing $x_1 = \sin\theta\cos\phi$, $x_2 = \sin\theta\sin\phi$, and $x_3 = \cos\theta$. Then, it is possible to show that the eigenvalues of $\rho(B|\Pi_j^A)$ are independent of ϕ.

The $(T_H/\omega, \theta)$-dependence of $I(A:B) - J(A:B)$ is plotted in Fig. 1. As this figure shows, the minimum value occurs at $\theta = \pi/2$. Therefore, as defined, the quantum discord $\mathcal{D}(A:B)$ is obtained as the value of $I(A:B) - J(A:B)$ at $\theta = \pi/2$. If we assume that the total correlation is the mutual information $I(A:B)$, it is possible to compute the classical correlation $\mathcal{C}(A:B)$ by

$$\mathcal{C}(A:B) = I(A:B) - \mathcal{D}(A:B). \tag{15}$$

In Fig. 2, we plot the Hawking temperature-dependence of the total correlation, quantum discord, and classical correlation. As Fig. 2 shows, all correlations exhibit a decreasing behavior with increasing T_H. At the $T_H \to 0$ limit, all correlations

[b] Although the authors in Ref. 25 consider the projective measurement, the authors in Ref. 26 consider the general measurement including positive operator-valued measure (POVM). Thus, the quantum discord in Ref. 26 is the lower bound of that in Ref. 25.

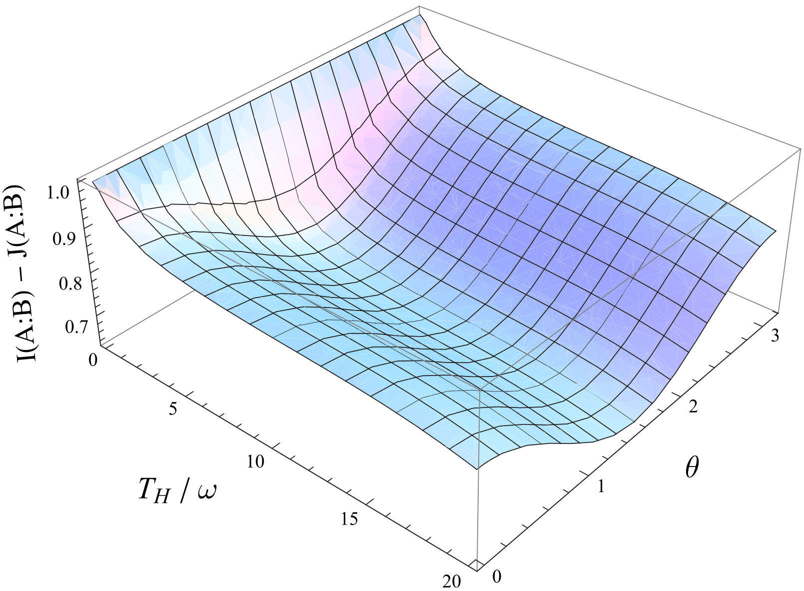

Fig. 1. The θ- and Hawking temperature-dependence of $I(A:B) - J(A:B)$. The minimum value of $I(A:B) - J(A:B)$ occurs at $\theta = \pi/2$ in the full range of Hawking temperature.

approach the values observed in the absence of the black hole. At the opposite limit, i.e. for $T_H \to \infty$, $I(A:B)$, $\mathcal{D}(A:B)$, and $\mathcal{C}(A:B)$ approach 1.0, 0.6, and 0.4, respectively. A remarkable fact in this regard is that the classical correlation is less than the quantum discord in the full range of Hawking temperature. Similar behavior was

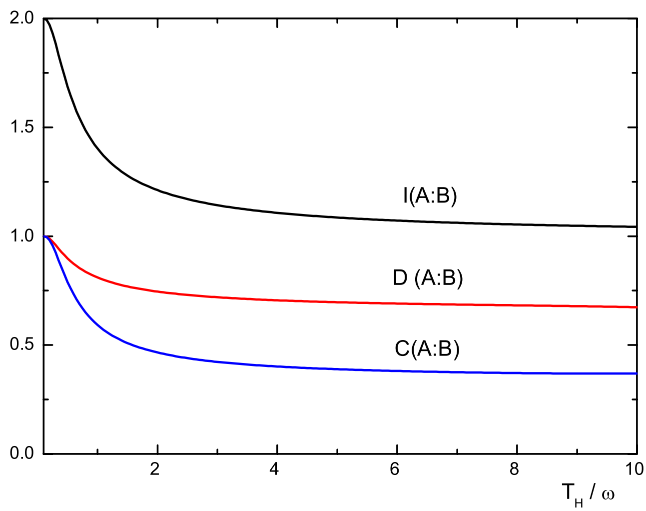

Fig. 2. The Hawking temperature-dependence of total correlation, quantum discord, and classical correlation. All correlations show decreasing behavior with increasing temperature and reduce to 50%, 60% and 40% of the corresponding values in the flat space limit at $T_H = \infty$.

observed in the case of the classical correlation and quantum discord sharing the Dirac field in the non-inertial frame.[12] In the next section, we discuss the tripartite entanglement in the presence of a black hole (1).

4. Tripartite Entanglement Degradation

The most well-known measure for tripartite entanglement is a three-tangle.[32] However, as the three-tangle is not defined in the qudit system, we cannot use it, because of Eqs. (2) and (3). Hence, instead of the three-tangle, we use the π-tangle[33] as a measure of the tripartite entanglement in this work.

4.1. *Greenberger–Horne–Zeilinger state*

The physical situation is exactly the same with that of the previous section. The only difference is that Bob's role in the previous section is changed into Charlie's role. Let Alice, Bob, and Charlie share the Greenberger–Horne–Zeilinger (GHZ) state

$$|\mathrm{GHZ}\rangle_{ABC} = \frac{1}{\sqrt{2}}[|000\rangle + |111\rangle]_{ABC}, \tag{16}$$

at the same initial point in flat Minkowski space before the black hole formed. Since Charlie feels a Hawking radiation eventually, Charlie's state should be the Kruskal state. Then using Eqs. (2) and (3), and by tracing over Charlie's in-state, we obtain

$$\begin{aligned} |GHZ\rangle_{ABC} \to \rho_{ABC} = \frac{1}{2}\sum_{n=0}^{\infty} e^{-n\omega/T_H}[\nu|00n\rangle\langle 00n| + \nu^2(n+1)|11(n+1)\rangle\langle 11(n+1)| \\ + \nu^{3/2}\sqrt{n+1}\{|00n\rangle\langle 11(n+1)| + |11(n+1)\rangle\langle 00n|\}, \end{aligned} \tag{17}$$

where $\nu = 1 - e^{-\omega/T_H}$. Since Charlie's out-state is a qudit state, it is impossible to compute the genuine tripartite entanglement measure known as three-tangle.[32] Hence, as discussed previously, we select the π-tangle[33] as a tripartite measure, which is defined as:

$$\pi = \frac{1}{3}(\pi_A + \pi_B + \pi_C), \tag{18}$$

and the use of which leads to a more tractable computation. The terms π_A, π_B and π_C used in Eq. (18) are defined as:

$$\begin{aligned} &\pi_A = \mathcal{N}^2_{A(BC)} - \mathcal{N}^2_{AB} - \mathcal{N}^2_{AC}, \quad \pi_B = \mathcal{N}^2_{B(AC)} - \mathcal{N}^2_{AB} - \mathcal{N}^2_{BC}, \\ &\pi_C = \mathcal{N}^2_{C(AB)} - \mathcal{N}^2_{AC} - \mathcal{N}^2_{BC}, \end{aligned} \tag{19}$$

where $\mathcal{N}_{\alpha(\beta\gamma)} = \|\rho^{T_\alpha}_{ABC}\| - 1$ and $\mathcal{N}_{\alpha\beta} = \|(\mathrm{Tr}_\gamma \rho^{T_\alpha}_{ABC}\| - 1$, with T_α being a partial transposition over the α-state, and where $\|A\| = \mathrm{Tr}\sqrt{AA^\dagger}$. It is easy to show that $\pi_{GHZ} = 1$ in the absence of a black hole background.

Now, we compute the one-tangle $\mathcal{N}_{A(BC)}$. From Eq. (17), it is easy to show that $(\rho^{T_A}_{ABC})(\rho^{T_A}_{ABC})^\dagger$ is a diagonal matrix. Therefore, the eigenvalues of $(\rho^{T_A}_{ABC})(\rho^{T_A}_{ABC})^\dagger$

can be easily computed. Since $\|\rho_{ABC}^{T_A}\|$ is the sum of the square root of the eigenvalues, one can derive $\mathcal{N}_{A(BC)}$ as:

$$\mathcal{N}_{A(BC)} = \nu^{3/2} e^{\omega/T_H} Li_{-1/2}(e^{-\omega/T_H}), \tag{20}$$

Where $Li_n(z)$ is a polylogarithm function defined as

$$Li_n(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^n} = \frac{z}{1^n} + \frac{z^2}{2^n} + \frac{z^3}{3^n} + \cdots . \tag{21}$$

By using a property of the polylogarithm function, one can show that $\mathcal{N}_{A(BC)}$ approaches $\sqrt{\pi}/2$ when $T_H \to \infty$. From the symmetry of the GHZ state, it is also easy to show that $\mathcal{N}_{B(AC)} = \mathcal{N}_{A(BC)}$.

Now, let us compute the last one-tangle $\mathcal{N}_{C(AB)}$. It should be noted that $(\rho_{ABC}^{T_C})(\rho_{ABC}^{T_C})^\dagger$ becomes

$$(\rho_{ABC}^{T_C})(\rho_{ABC}^{T_C})^\dagger = D + F, \tag{22}$$

where D and F are expressed as:

$$\begin{aligned} D = \frac{1}{4}\sum_{n=0}^{\infty} e^{-2n\omega/T_H}[&\nu^2|00n\rangle\langle 00n| + \nu^4(n+1)^2|11(n+1)\rangle\langle 11(n+1)| \\ &+ \nu^3(n+1)\{|00(n+1)\rangle\langle 00(n+1)| + |11n\rangle\langle 11n|\}], \end{aligned} \tag{23}$$

$$\begin{aligned} F = \frac{1}{4}\sum_{n=0}^{\infty} e^{-(2n+1)\omega/T_H}[&\nu^{5/2}\sqrt{n+1}\{|11n\rangle\langle 00(n+1)| + |00(n+1)\rangle\langle 11n|\} \\ &+ \nu^{7/2}(n+1)\sqrt{n+2}\{|11(n+1)\rangle\langle 00(n+2)| + |00(n+2)\rangle\langle 11(n+1)|\}]. \end{aligned}$$

The off-diagonal part F makes it difficult to compute the eigenvalues of $(\rho_{ABC}^{T_C})(\rho_{ABC}^{T_C})^\dagger$. However, one can convert $(\rho_{ABC}^{T_C})(\rho_{ABC}^{T_C})^\dagger$ into a block-diagonal matrix by ordering the basis vectors as $\{|000\rangle, |110\rangle, |001\rangle, |111\rangle, |002\rangle, |112\rangle, \ldots\}$. Thus, one can compute the eigenvalues of $(\rho_{ABC}^{T_C})(\rho_{ABC}^{T_C})^\dagger$ analytically, which are $\{\nu^2/4, \Lambda_n^\pm|_{n=0,1,2,\ldots}\}$. Here, $\Lambda_n^\pm$ are the eigenvalues of each block and are given by

$$\Lambda_n^\pm = \frac{\nu^2}{8} e^{-2n\omega/T_H}[(\mu_n^2 + 2\nu) \pm \mu_n\sqrt{\mu_n^2 + 4\nu}], \tag{24}$$

where $\mu_n = n e^{\omega/T_H}\nu + e^{-\omega/T_H}$. Therefore, $\mathcal{N}_{C(AB)}$ reduces to

$$\mathcal{N}_{C(AB)} = \frac{\nu}{2} + \sum_{n=0}^{\infty}(\sqrt{\Lambda_n^+} + \sqrt{\Lambda_n^-}) - 1. \tag{25}$$

Finally, one can show that all two-tangles, $\mathcal{N}_{AB}$, $\mathcal{N}_{AC}$ and $\mathcal{N}_{BC}$, are identically zero.

The one-tangles and the π-tangle are plotted in Fig. 3 as a function of Hawking temperature. As this figure shows, the π-tangle decreases with increasing Hawking temperature, and eventually reduces to $\pi/6 \sim 0.524$ at $T_H \to \infty$. At $T_H = 0$, the

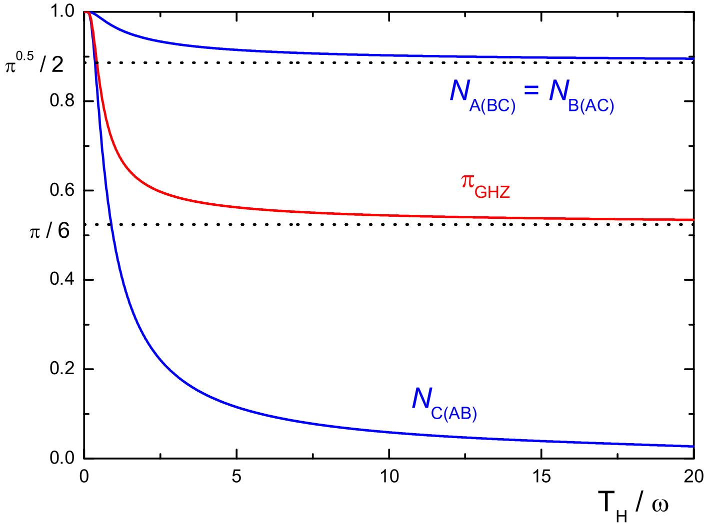

Fig. 3. The Hawking temperature-dependence of one tangles and $\pi_{\rm GHZ}$. The π-tangle decreases with increasing T_H, and eventually reduces to $\pi/6 \sim 0.524$ at $T_H = \infty$.

π-tangle exactly coincides with that in the absence of the black hole. Thus, as expected, the tripartite entanglement is degraded when Charlie moves to the near-horizon region from the asymptotic region with his own particle detector.

4.2. *W-state*

Let Alice, Bob, and Charlie share the W-state[34]

$$|W\rangle_{ABC} = \frac{1}{\sqrt{3}}[|001\rangle + |010\rangle + |100\rangle]_{ABC}, \tag{26}$$

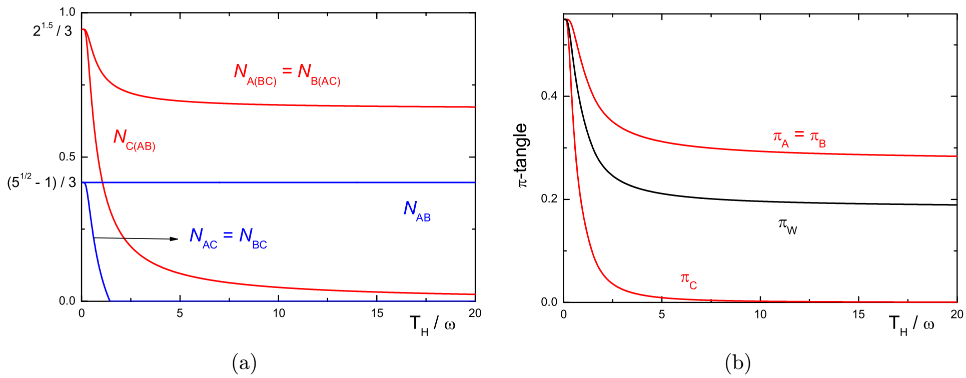

Fig. 4. (a) The Hawking temperature-dependence of one- and two-tangles. (b) The Hawking temperature-dependence of π_W. As in the case of the GHZ state, π_W decreases with an increasing temperature, and eventually reduces to 0.18 at $T_H = \infty$.

in the asymptotic flat region. It is easy to show that the π-tangle for the W-state is $\pi_W = 4(\sqrt{5}-1)/9 \sim 0.55$ in the flat space limit.

By following a similar calculation as that used for the GHZ state, one can compute the Hawking temperature-dependence of π_W in the presence of a black hole. Instead of repeating the computational procedure here, we present Fig. 4, which shows one-tangles, two-tangles, and π_W as a function of Hawking temperature. In Fig. 4(a), the plot of one- and two-tangles is shown. All tangles except $\mathcal{N}_{AB}$, which is independent of T_H, exhibit decreasing behavior with increasing Hawking temperature. At $T_H \to \infty$, $\mathcal{N}_{A(BC)}$ and $\mathcal{N}_{B(AC)}$ approach 0.659, whereas $\mathcal{N}_{C(AB)}$ has a vanishing limit. Remarkably, the two-tangles $\mathcal{N}_{AC}$ and $\mathcal{N}_{BC}$ abruptly become zero in the region $T_H > 1.45\omega$. This resembles concurrence, which is a bipartite entanglement measure. In Fig. 4(b), we plot π_W as a function of T_H. At $T_H = 0$, π_W in the flat space is recovered. However, it exhibits a decreasing behavior with increasing T_H and eventually reduces to 0.18 at the $T_H = \infty$ limit.

5. Conclusion

In this paper, the quantum discord and tripartite entanglement in the presence of an asymptotically flat static black hole are discussed. Both the quantum discord and the tripartite entanglement exhibit decreasing behavior with increasing Hawking temperature. This implies that the presence of a black hole reduces the quantum correlation when one party moves from the asymptotic to the near-horizon region with their own particle detector.

Although we have not discussed this, the tripartite entanglement of Alice, Bob, and Charlie's in-state (or AntiCharlie) does not completely vanish. This fact implies that some quantum information processes can probably be performed partially across the black hole horizon. To confirm this, it seems to be important to compute the teleportation fidelity by making use of the tripartite teleportation scheme.[35,36] If tripartite teleportation is possible, even partially, across the horizon, what implications would this have in the context of causality? The answer to this may be important in the context of quantum gravity. We plan to explore this issue in the future.

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