PROBLEM OF THE DAY 4b **Counting Multisets**

Definition: We denote the number of k-element multisets formed by choosing elements from an

n-element set by
$$\binom{n}{k}$$
 ('*n* choose *k* repeated').

Theorem: Let $n, k \in \mathbb{N}$. Then

$$\begin{pmatrix} n \\ k \end{pmatrix} = \begin{pmatrix} n+k-1 \\ k \end{pmatrix} = \frac{(n+k-1)!}{k!(n-1)!}$$
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Solve the following counting problems:

Exercise 1: How many ways are there to arrange 5 identical books on 3 bookshelves? Assume that each shelf may contain zero, one, or more than one book. 37 W

Exercise 2: How many ways are there to pass and child is allowed to zero, one, or more than one apple. Exercise 2: How many ways are there to pass out 6 identical apples to 4 children? Assume each

Exercise 3: How many ways are there to pass out 10 identical apples to 4 children assuming that

each child gets at least one apple?

Exercise 4: How many ways are there to write the integer 6 as a sum of 2 non-negative integers (repetition allowed and order matters)? Three examples are 0+6=6, 6+0=6, and 3+3=6.

What if order does NOT matter?

Exercise 5: How many ways are there to write the integer 10 as a sum of 4 non-negative integers (repetition allowed and order does matter)? Two examples are 0+0+3+7=10 and 1+2+2+5=10. What if order does NOT matter?

Exercise 6: How many ways are there to write the integer 10 as a sum of 4 positive integers (repetition allowed and order does matter)? What if order does NOT matter?

Exercise 7: How many solutions are there to the equation $x_1 + x_2 + ... + x_5 = 9$ where each x_i is a non-negative integer?