Math 03.160 Discrete Structures Date: 10-2-2020

PROBLEM OF THE DAY 8
Chapter 3: Methods of Proof

INSTRUCTIONS: For each of the exercises below, prove the given statement using the indicated method of proof or if no method is indicated, then use any of the methods discussed in the course.

Methods of Proof:
I. Direct: To prove "If *p*, then *q*": Assume *p* and prove *q*.
II. Proof by Contrapositive: To prove "If *p*, then *q*": Assume (not *q*) and prove (not *p*).
III. Proof by Contradiction: To prove "If *p*, then *q*": Assume *p* and suppose on the contrary (not *q*). Argue until a contradiction is reached (i.e., a situation where a statement *r* and its negation (not *r*) are both true), which proves that (not *q*) is IMPOSSIBLE.

Definitions:
(a) An integer n is called *even* if it can be written as $n = 2k$ for some integer k.
(b) An integer n is called *odd* if it can be written as $n = 2k+1$ for some integer k.
(c) A positive integer n is called *prime* if it has exactly two divisors, namely 1 and itself.
(d) Two integers *a* and *b* are congruent modulo *n*, denoted by $a \equiv b \bmod n$, if $n \mid (a-b)$ or equivalently, $a - b = k \cdot n$ for some integer *k*.

know definitions

Exercise 1: (Direct) If m is an even integer and n is an odd integer, then $m + n \equiv 1 \bmod 2$ ~~is an odd integer~~. HINT: First prove that $m+n$ is odd.

m even $\Rightarrow m = 2k$
n odd $\Rightarrow n = 2l+1,\ l \in \mathbb{Z}$] odd $\rightarrow$ $m+n = 2k+2l+1$
$m+n = 2(k+l)+1$
$m+n \equiv 1 \bmod 2 \Leftarrow (m+n) - 1 = 2j$ m odd $\longrightarrow$ $= 2j+1,\ j = k+l$

Exercise 2: (Direct, contrapositive and contradiction) If $x > 1$, then $x > 0$.

Direct: Assume $x>1$. Since $1>0$, follows by transitivity that $x>0$

Contrapositive: Assume $x \le 0$ (not q), since $0<1$, follows by transitivity that $x \le 1$ (not p)

Contradiction: Assume $x>1$ and $x \le 0$. Since $0<1$, follows by transitivity that $x \le 1$. This contradicts assumption that $x>1$. And not p.

Exercise 3: (Contrapositive) If the sum of two prime integers is prime, then one of the primes must be equal to 2.

Exercise 4: (Contradiction) There is no greatest even integer.

Exercise 5: (Contradiction) An integer n cannot be both even and odd. HINT: Rephrase this statement in "If-then" form.

Exercise 6: (Contrapositive and contradiction) If n^2 is an odd integer, then n is an odd integer.

Exercise 7: If $x^2 - 6x + 5$ is even, then x is odd.

Exercise 8: If $a \equiv 1 \bmod 3$ and $b \equiv 2 \bmod 3$, then $a + b \equiv 0 \bmod 3$.

Exercise 9: There does not exist a right triangle with all three sides of odd length.

Exercise 10: If $\frac{x}{2} + \frac{y}{3} = 1$, then $x^2 + y^2 > 1$.

Exercise 11: If $a, b \in \mathbb{Z}$, then $a^2 - 4b \neq 2$.