

must alternate between colors

contradiction  
 $B \sim W \sim B \sim W \sim \dots \sim B \sim W$   
 $\sim B$   
 $\uparrow$  last B  
 $\uparrow$  last W

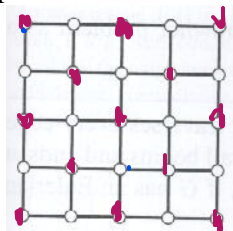
# PROBLEM OF THE DAY 16 Chapter 6: Eulerian and Hamiltonian Paths/Circuits

1. Is it possible to walk the seven bridges of Konisburg so that you cross every bridge exactly twice?
2. For which values of  $n$  does the complete graph  $K_n$  have a Eulerian circuit?
3. Determine whether or not the graph below has a Hamiltonian circuit.

$$V_B = 13$$

$$V_W = 12$$

$v = 25$   
 color-vertices

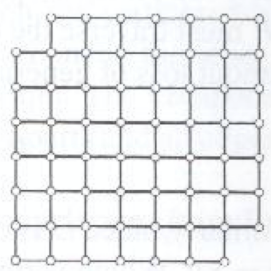


NO  
 no way to get back to start



must go back to start but impossible to move between same color

4. Determine whether or not the graph below has a Hamiltonian path.



for white's case, you have 2Bs at the end

5. For which values of  $n$  does the complete graph  $K_n$  have a Hamiltonian path?
6. Mouse and Cheese: A block of cheese is made up of  $3 \times 3 \times 3$  cubes as shown in the figure below. Is it possible for a mouse to tunnel its way through this block of cheese by (a) starting at a corner, (b) eating its way from cube to adjacent cube, (c) never passing through any cube twice, and, finally, (d) finishing at the center cube? Prove your answer.

