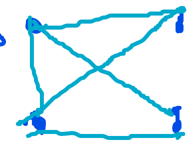
are edges matched?

PROBLEM OF THE DAY 13
Chapter 6: Graphs

(3) b c (2)



G

(3) a d (2)

INSTRUCTIONS: Solve each of the exercises below.

Exercise 1: Consider the graph $G = (V, E)$ whose set of vertices and edges are denoted by

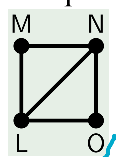
$$V = \{a, b, c, d\}$$
$$E = \{\{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}\}$$

to match, G' neighbors w/ 2 degree, G'' must have it

(a) Draw the graph *G*.
(b) Is G isomorphic to either of the two graphs below, i.e., can the vertices in either graph be relabeled to produce the graph *G*? Explain why or why not.

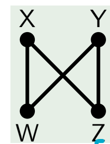
a → L
b → N
c → M
d → O

(2) (3)



(3) (2)

G' yes



G''

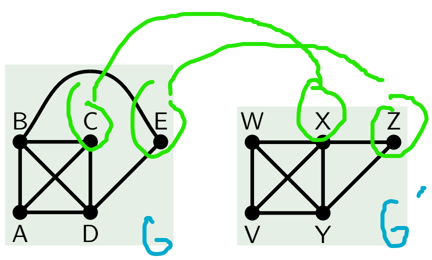
e = 5
e'' = 4

no b/c # edges different

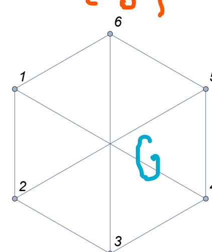
Exercise 2:
(a) Are the two graphs in part (i) isomorphic? How about part (ii)? Explain why or why not.

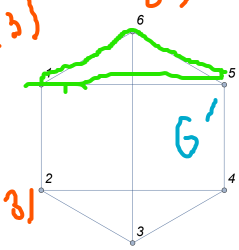
(i)

yes



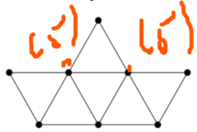
(ii)

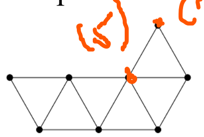


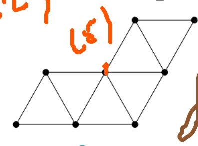


(b) In three graphs below, are any two isomorphic to each other? Explain why or why not.

G ⇔ G' NO
G ⇔ G'' NO
G' ⇔ G'' NO







shapes don't match to shapes, so no

Exercise 3: Consider the graph $G = (V, E)$ whose set of vertices and edges are denoted by

$$V = \{1, 2, 3, 4, 5, 6, 7\}$$
$$E = \{\{1,2\}, \{1,3\}, \{2,3\}, \{3,4\}, \{5,6\}\}$$

(a) Draw the graph *G*.
(b) Compute the degree of each vertex, i.e., the number of neighbors adjacent to each vertex.
(c) Sum up all the degrees computed in part (b). Do you observe any relation between this total and the number of vertices or edges? Verify whether your relationship holds in general by examining many different kinds of graphs.
(d) Write out a formula to mathematical to describe the relationship found in part (c). Then prove your formula.

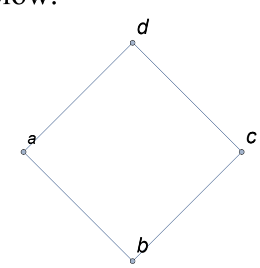
(Continued on other side)

Exercise 4: Suppose a graph G has 20 edges.
(a) Suppose it is known that G has 10 vertices. What is the average degree of each vertex?
(b) Suppose it is known that every vertex in G has at least 5 neighbors. What is the maximum number of vertices that G can have?

Exercise 5: Prove that any graph G must have an even number of vertices of odd degree.

Exercise 6: Let K_n denote the complete graph on n vertices, i.e., a graph where every vertex is adjacent to every other vertex.
(a) Draw K_n for $n = 1,2,3,4,5$ and compute the size of each graph, i.e., its number of edges.
(b) Conjecture a formula for the size of K_n. Then prove that your formula is correct.

Exercise 7: Let G denote the graph below.



(a) Draw all possible spanning subgraphs of G, i.e., subgraphs obtained by deleting any number of edges from G (but not vertices). NOTE: Vertex labels are taken into account here.
(b) How many spanning subgraphs did you find in part (a)?
(c) Conjecture a formula for the number of spanning subgraphs of a given graph G in terms of its number of edges. Then prove that your formula is correct.