* Method 1: $n + (n-1) + (n-2) + \ldots + (n+1)$ (LHS)

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Method 2: Handshake = 2 el subset $\{A, B\} \rightarrow \binom{n+1}{2} \rightarrow \frac{(n+1)!}{2!(n-1)!} \rightarrow \frac{(n+1)(n)(n-1)!}{2!(n-1)!} \rightarrow \frac{n+1(n)}{2}$ (RHS)

PROBLEM OF THE DAY 9

Chapter 4: Proof by Smallest Counterexample and Mathematical Induction

INSTRUCTIONS: For each of the exercises below, prove the given statement using the indicated method of proof or if no method is indicated, then use any of the methods discussed in the course.

Proof by Smallest Counterexample: To prove the statement $\forall n \in \mathbb{N}(p(n))$, we proceed as follows:

1. Assume the existence of a smallest counterexample m so that ON THE ONE HAND $p(m)$ is *false*.
2. Using the fact that $p(m')$ must be true for all $m' < m$, choose a specific value for m', say $m' = m - k$ (e.g. $m' = m - 1$) where the choice of k depends on the problem.
3. Prove the implication $p(m-k) \Rightarrow p(m)$ so that ON THE OTHER HAND $p(m)$ is *true*.
4. Conclude by contradiction that $p(n)$ is true for all $n \in \mathbb{N}$.

Principle of Mathematical Induction: Suppose the two statements below are true:

I. (Base case) $p(b)$

II. (Inductive step) $p(n-1) \Rightarrow p(n)$ for all $n > b$

Then $p(n)$ is true for all integers $n \geq b$.

both methods solve the same problem

for you LHS RHS

Exercise 1: (Induction) Prove the formula $0 + 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$ for all integers $n \geq 0$.

Algebra

0 to n is n+1

$S = 0 + 1 + 2 + \ldots + n$

$S = n + (n-1) + (n-1) + \ldots 0$

$2S = n + n + \ldots + n = (n+1)n$

$S = \frac{n(n+1)}{2}$

Combinatorial Proof

How many handshakes take place @ party w/ n+1 people if every person shakes hands w/ everyone else? *

Exercise 2: (Smallest counterexample) Prove the formula $1 + 3 + 5 + \ldots + (2n-1) = n^2$ for all positive integers n.

Exercise 3: (Induction) Prove the formula $2^0 + 2^1 + \ldots + 2^n = 2^{n+1} - 1$ for all integers $n \geq 0$.

Exercise 4: (Induction) Prove that $6^n - 1$ is divisible by 5 for all positive integers *n*. HINT: For the inductive step, add and subtract 6^{n-1} and regroup.

Exercise 5: Prove the formula $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers *n*.

Exercise 6: (Induction) Let *n* a positive integer. Prove that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$. HINT: Use Pascal's relationship and the fact that $\binom{n}{j} = 0$ for $j < 0$ or $j > n$.

Exercise 7: Prove that every positive integer *n* can be expressed as the sum of distinct powers of 2. For example, we have $11 = 2^3 + 2^1 + 2^0$. HINT: Use strong induction.