PROBLEM OF THE DAY 15
Shortest Path Problem: Dijkstra's Algorithm

Dijkstra's Algorithm: Suppose we want to find a shortest path from a given node (vertex) *s* to other nodes in a network (one-to-all shortest path problem)
• Node *s* is called a starting node (or initial) node
• Dijkstra's algorithm starts by assigning some initial values for the distances from node s and to every other node in the network.
• It operates in steps, where at each step the algorithm improves the distance values.
• At each step, the shortest distance from node *s* to another node is determined

Step 1. Initialization
• Assign distance zero to node s, and label it as Permanent. [The state of node *s* is (0, p).]
• Assign to every other node a distance value of ∞ and label them as Temporary. [The state of every other node is (∞, t).]
• Designate the node *s* as the current node

Step 2. Distance Value Update and Current Node Designation Update
Let *i* be the current node.
(1) Find the set *J* of nodes with temporary labels that can be reached from the current node *i* by a link (*i*, *j*) (edge). Update the distance values of these nodes.
• For each $j \in J$, the distance value d_j of node *j* (distance from node *i*) is updated as follows:

$$d_j = \min\{d_j, d_i + c_{ij}\},$$

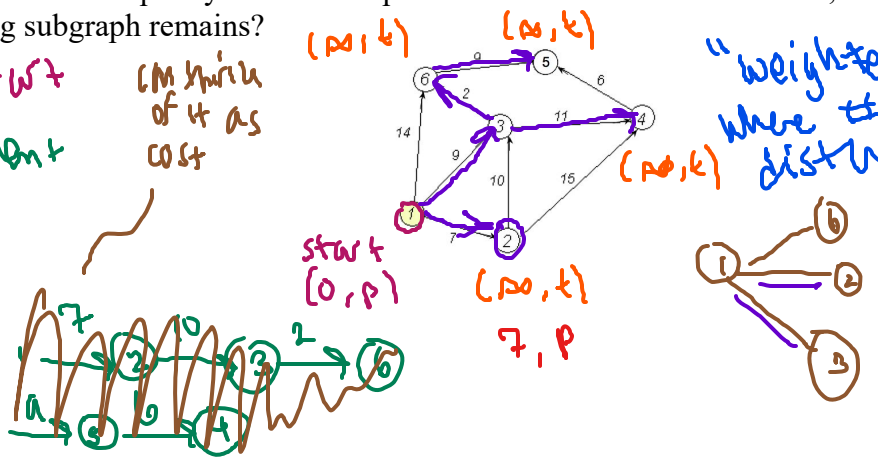
where c_{ij} is the cost (or length) of link (i, j), as given in the graph network.

(2) Determine a node j^* that has the smallest distance value d_{j^*} among all temporary nodes.

(3) Change the label of node j^* to permanent and designate this node as the current node.

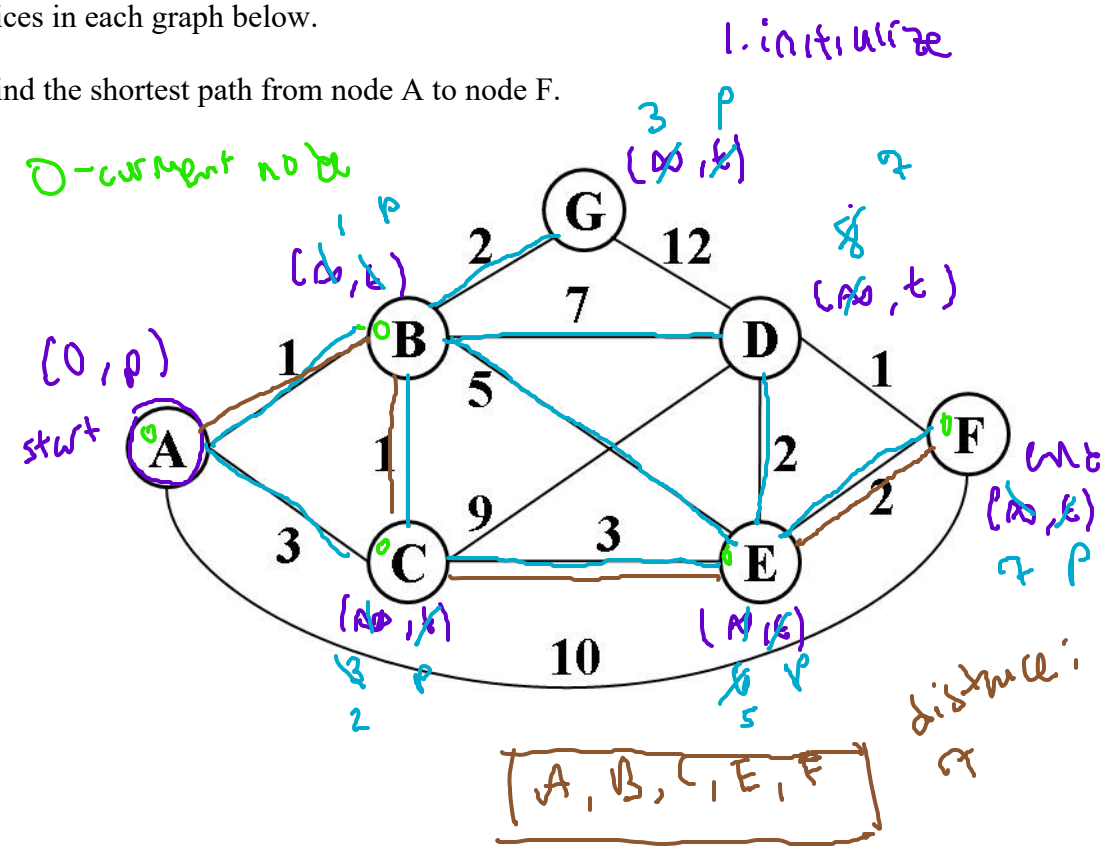
Step 3. Termination Criterion
• If all nodes that can be reached from node *s* are labeled permanent, then stop - we are done.
• If we cannot reach any temporary labeled node from the current node, then all the temporary labels become permanent - we are done.
• Otherwise, go to Step 2.

Question: In Dijkstra's algorithm, if we delete those links (edges) from our graph network that are not needed to specify the shortest path from node *s* to all other nodes, then what kind of spanning subgraph remains?



INSTRUCTIONS: Use Dijkstra's algorithm to find the shortest path between two specified vertices in each graph below.

1. Find the shortest path from node A to node F.



2. Find the shortest path from node 1 to node 9.