$T(n) =$ sum of works at all levels
$= n + 2n + 4n + 8n + \dots + n*n$
$= n(1+2+4+\dots+n)$
$= n\,(2^0 + 2^1 \dots 2^k)$

PROBLEM OF THE DAY 11
Chapter 4: Recursion Trees

EXERCISES:

1. Complete the recursion tree table for the following recurrence and use it to determine an explicit formula for $T(n)$:

$$T(n) = \begin{cases} 4T(n/2) + n & \text{if } n > 1 \\ T(1) = 1 & \text{if } n = 1 \end{cases}$$

\# of parts to divide into; problem size; work it takes to merge

Level	Number of Problems	Problem Size	Tree Diagram	Work per Problem	Work per Level
0	$4^0 = 1$	n	•	n	n
1	4^1	$n/2$	• • • •	$n/2$	$2n$
2	4^2	$n/2^2$	• • • • • • • • • • • • • • • •	$n/2^2$	$4n$
3	4^3	$n/2$	64 dots	$n/2^3$	$8n$
…					
k (Assume $n = 2^k$)	4^k	$1 = n/2^k$		$n/2^k$	n^2

$4(n/2)$

$1 = n*n$

geometric formula; also n^2; always $= 1$

geometric $\frac{1-r^{n+1}}{1-r}$ $\quad T(1) = 1$

$= n(2^{k+1} - 1)$
$= n(2n-1)$
$= 2n^2 - n$
$= O(n^2)$

2. Complete the recursion tree table for the following recurrence and use it to determine an explicit formula for $T(n)$:

$$T(n) = \begin{cases} 4T(n/3) + n & \text{if } n > 1 \\ T(1) = 1 & \text{if } n = 1 \end{cases}$$

Level	Number of Problems	Problem Size	Tree Diagram	Work per Problem	Work per Level
0	1	n		n	n
1	4^1	$n/3$		$n/3$	$4n/3$
2	4^2	$n/3^2$		$n/3^2$	$16n/9$
3	4^3	$n/3^3$		$n/3^3$	$64n/27$
…					
k (Assume $n = 3$^k)	4^k	$1 = n/3^k$		1	

$n = 3^k \; \left(\frac{4}{3}\right)^k$
$= 4^k$