Name KEY Date_____ Period_____

Worksheet 1.3—Limits at Infinity

Show all work. No calculator

Short Answer:

On problems 1 – 6, find
(a) $\lim_{x\to\infty} f(x)$
(b) $\lim_{x\to-\infty} f(x)$
(c) the equations of any horizontal or slant asymptotes.

1. $f(x)=\dfrac{3x^3-4x^2-x-1}{x^2+x-13}$

(a) $\lim_{x\to\infty} f(x)=\infty$
(b) $\lim_{x\to-\infty} f(x)=-\infty$
(c) Slant Asymptote @ $y=3x-7$

$$\begin{array}{r} 3x-7 \\ x^2+x-13\overline{)\,3x^3-4x^2-x-1} \\ -3x^3+3x^2+39x \\ \hline -7x^2+38x-1 \\ +7x^2+7x+91 \\ \hline 45x-92 \end{array}$$

So $f(x)=3x-7+\dfrac{45x-92}{x^2+x-13}$ goes to zero as $x\to\pm\infty$

2. $f(x)=\dfrac{4x^2-3x+5}{2x^3+x-1}$

(a) $\lim_{x\to\infty} f(x)=0$
(b) $\lim_{x\to-\infty} f(x)=0$
(c) f has a HA @ $y=0$

3. $f(x)=\dfrac{3x+1}{x-4}$

(a) $\lim_{x\to\infty} f(x)=3$
(b) $\lim_{x\to-\infty} f(x)=3$
(c) Horizontal Asymptote @ $y=3$

4. $f(x)=\dfrac{\sin 3x}{x}$

(a) $\lim_{x\to\infty} f(x)=0$
(b) $\lim_{x\to-\infty} f(x)=0$
(c) f has a HA @ $y=0$

5. $f(x)=\dfrac{-2x^2+4}{\sqrt{4x^4+8x^2+1}}$

(a) $\lim_{x\to\infty} f(x)=-\frac{2}{2}=-1$
(b) $\lim_{x\to-\infty} f(x)=-\frac{2}{2}=-1$
(c) Horizontal Asymptote @ $y=-1$

6. $f(x)=\dfrac{5x^3+1}{\sqrt{3x^6+x^2+4}}$

(a) $\lim_{x\to\infty} f(x)=\frac{5}{\sqrt{3}}$
(b) $\lim_{x\to-\infty} f(x)=-\frac{5}{\sqrt{3}}$
(c) f has HA's at $y=\frac{5}{\sqrt{3}}$ and $y=-\frac{5}{\sqrt{3}}$

Multiple Choice

D 7. $\lim_{x\to\infty} x\sin(x) = (\lim_{x\to\infty} x)(\lim_{x\to\infty} \sin x) = \infty \cdot \sin\infty = \infty \cdot DNE = DNE$ (by oscillation) (Entire function oscillates between ∞ and $-\infty$)

Amplitude grows ∞ large

(A) 1 (B) 0 (C) 7 (D) DNE (E) -1

E 8. $\lim_{x\to-\infty} \dfrac{-2\sqrt{9x^{10}+2x^8+5}}{-12x^5+4x^3-2x^2-1} = \dfrac{-2\sqrt{9}}{+12} = -\dfrac{1}{2}$

(A) 0 (B) $-\dfrac{1}{6}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{2}$ (E) $-\dfrac{1}{2}$

B 9. $\lim_{x\to-\infty} \dfrac{2x^3+4-7x-5x^4}{x^3+x^2+2x-5} = \lim_{x\to-\infty} \left(\dfrac{-5x^4+\cdots}{x^3+\cdots}\right) \approx \lim_{x\to-\infty} (-5x) = \infty$

(A) $-\infty$ (B) ∞ (C) 1 (D) -2 (E) 2

D 10. $\lim_{x\to\infty} \dfrac{4-x^2}{x^2-1} = \dfrac{-1}{1} = -1$

(A) 1 (B) ∞ (C) 0 (D) -1 (E) -4

A 11. $\lim_{x\to-\infty} \dfrac{5x^3+27}{20x^2+10x+9} = -\infty$

(A) $-\infty$ (B) ∞ (C) 3 (D) -1 (E) 0

C 12. $\lim_{x\to\infty} \dfrac{3x^2+27}{x^3-27} \approx \lim_{x\to\infty} \dfrac{3}{x} = 0$

(A) $-\infty$ (B) 1 (C) 0 (D) -1 (E) 3

C 13. $\lim_{x\to\infty} \dfrac{2^{-x}}{3^x} = \lim_{x\to\infty} \dfrac{1}{2^x \cdot 3^x} = 0$

(A) $-\infty$ (B) 1 (C) 0 (D) ∞ (E) $\dfrac{2}{3}$

B 14. $\lim_{x\to\infty} \dfrac{5+e^{-x}}{1-e^{-x}} = \dfrac{5+0}{1-0} = \dfrac{5}{1} = 5$

(A) $-\infty$ (B) 5 (C) -5 (D) ∞ (E) -1

E 15. $\lim_{x\to-\infty} \dfrac{5+e^{-x}}{1-e^{-x}} = \dfrac{5+e^{\infty}}{1-e^{\infty}} = \lim_{x\to\infty} \dfrac{e^x+5}{-e^x+1} = -1$

(A) $-\infty$ (B) 5 (C) -5 (D) ∞ (E) -1