FR

Introduction to Integrals Free Response Calculator Review

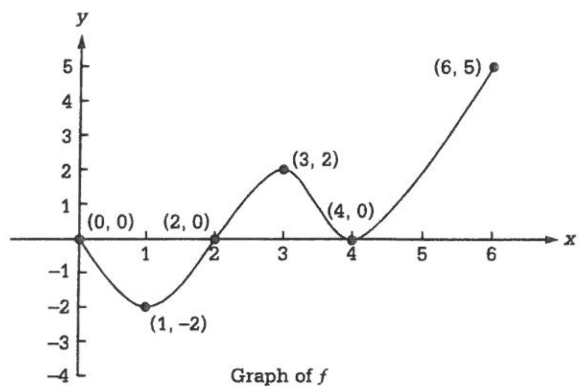
1. For $2 \leq t \leq 6$, the temperature in an art museum varies by $T(t) = 20 + 5\cos(\frac{\pi}{12}t)$, where $T(t)$ is measured in C° and t is measured in hours.

   (a) Using correct units, find the average temperature over the interval $2 \leq t \leq 6$.

   (b) Using the correct units, find the time t for which the temperature in the museum is equal to the average temperature over the interval $2 \leq t \leq 6$.

Module 4: Justifying Properties and Behaviors of Functions

The figure to the right shows the graph of f, a twice differentiable function, on the interval $[0, 6]$. The graph of f has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 4$. The areas of the regions bounded by the x-axis and the graph of f on the intervals $[0, 2]$ and $[2, 4]$ are 2.5 and 2, respectively. Let $H(x) = \int_0^x f(t)dt$.



Graph of f

a) On what open intervals contained in $(0, 6)$ is the graph of H both concave up and increasing? Give a reason for your answer.

b) Find the x-coordinates of all points of inflection for the graph of H. Give a reason for your answer.

c) Let $G(x) = 2x + \int_0^x f(t)dt$. Find the critical points of G and classify each as corresponding to a local maximum value, a local minimum value, or neither. Justify your answer.

d) Let $J(x) = x \cdot \int_0^x f(t)dt$. Find $J'(2)$.

Intro. to Int. Quiz Review

You may use a graphing calculator to solve the following problems.

NO calc

1. Multiple Choice The function f is continuous on the closed interval $[1, 7]$ and has values that are given in the table below.

x	1	4	6	7
$f(x)$	10	30	40	20

Using the subintervals $[1, 4]$, $[4, 6]$, and $[6, 7]$, what is the trapezoidal approximation of $\int_1^7 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

2. Multiple Choice Let $F(x)$ be an antiderivative of $\sin^3 x$. If $F(1) = 0$, then $F(8) =$

(A) 0.00 (B) 0.021 (C) 0.373 (D) 0.632 (E) 0.968

3. Multiple Choice Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

calc

4. Free Response Let $F(x) = \int_0^x \sin(t^2)\,dt$ for $0 \le x \le 3$.

no

(a) Use the Trapezoidal Rule with four equal subdivisions of the closed interval $[0, 2]$ to approximate $F(2)$.

(b) On what interval or intervals is F increasing? Justify your answer.

(c) If the average rate of change of F on the closed interval $[0, 3]$ is k, find $\int_0^3 \sin(t^2)\,dt$ in terms of k.

34. Rubber-Band-Powered Sled A sled powered by a wound rubber band moves along a track until friction and the unwinding of the rubber band gradually slow it to a stop. A speedometer in the sled monitors its speed, which is recorded at 3-second intervals during the 27-second run.

Time (sec)	Speed (ft/sec)
0	5.30
3	5.25
6	5.04
9	4.71
12	4.25
15	3.66
18	2.94
21	2.09
24	1.11
27	0

(a) Give an upper estimate and a lower estimate for the distance traveled by the sled.

(b) Use the Trapezoidal Rule to estimate the distance traveled by the sled.

51. Fuel Efficiency An automobile computer gives a digital readout of fuel consumption in gallons per hour. During a trip, a passenger recorded the fuel consumption every 5 minutes for a full hour of travel.

time	gal/h	time	gal/h
0	2.5	35	2.5
5	2.4	40	2.4
10	2.3	45	2.3
15	2.4	50	2.4
20	2.4	55	2.4
25	2.5	60	2.3
30	2.6		

(a) Use the Trapezoidal Rule to approximate the total fuel consumption during the hour.

In Exercises 15–24, evaluate the integral analytically by using the Integral Evaluation Theorem (Part 2 of the Fundamental Theorem, Theorem 4).

No calc

15. $\int_{-2}^{2} 5\,dx$

16. $\int_{2}^{5} 4x\,dx$

17. $\int_{0}^{\pi/4} \cos x\,dx$

18. $\int_{-1}^{1} (3x^2 - 4x + 7)\,dx$

19. $\int_{0}^{1} (8s^3 - 12s^2 + 5)\,ds$

20. $\int_{1}^{2} \frac{4}{x^2}\,dx$

21. $\int_{1}^{27} y^{-4/3}\,dy$

22. $\int_{1}^{4} \frac{dt}{t\sqrt{t}}$

23. $\int_{0}^{\pi/3} \sec^2 \theta\,d\theta$

24. $\int_{1}^{e} (1/x)\,dx$

In Exercises 25–29, evaluate the integral.

No calc

25. $\int_{0}^{1} \frac{36}{(2x+1)^3}\,dx$

26. $\int_{1}^{2} \left(x + \frac{1}{x^2}\right) dx$

27. $\int_{-\pi/3}^{0} \sec x \tan x\,dx$

28. $\int_{-1}^{1} 2x \sin(1 - x^2)\,dx$

29. $\int_{0}^{2} \frac{2}{y+1}\,dy$

38. Find the average value of

(a) $y = \sqrt{x}$ over the interval $[0, 4]$.

(b) $y = a\sqrt{x}$ over the interval $[0, a]$.

In Exercises 39–42, find dy/dx.

39. $y = \int_{2}^{x} \sqrt{2 + \cos^3 t}\,dt$

40. $y = \int_{2}^{7x^2} \sqrt{2 + \cos^3 t}\,dt$

41. $y = \int_{x}^{1} \frac{6}{3 + t^4}\,dt$

42. $y = \int_{x}^{2x} \frac{1}{t^2 + 1}\,dt$

43. ***Printing Costs*** Including start-up costs, it costs a printer \$50 to print 25 copies of a newsletter, after which the marginal cost at x copies is

$$\frac{dc}{dx} = \frac{2}{\sqrt{x}} \text{ dollars per copy.}$$

Find the total cost of printing 2500 newsletters.

AP Calculus
Chapter 5 Review

Name: ____________________

Block: ____

1. [No Calculator] Evaluate using the FTOC (the evaluation part)

a) $\int_{2}^{7}\left(\frac{8}{x}+7\sqrt[3]{x}+x^{-4}\right)dx$

b) $\int_{4}^{9}\left(\frac{5}{x^3}+7\sqrt{x}+\frac{1}{x}\right)dx$

2. [No Calculator] Evaluate using geometry

a) $\int_{-2}^{3}\sqrt{25-(x+2)^2}\,dx$

c) $\int_{-6}^{1}|8+2x|\,dx$

3. [No Calculator] Evaluate each derivative.

a) $\frac{d}{dx}\left[\int_{10}^{x}\tan\left(3t^2+9\right)dt\right]$

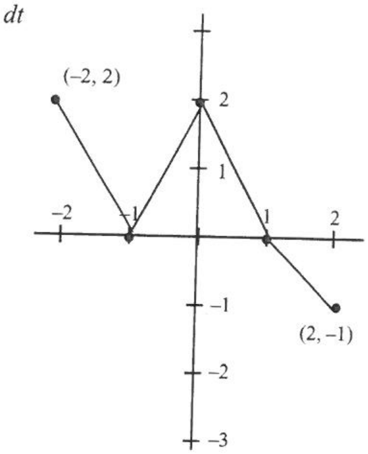
b) Find $h'(x)$ if $h(x)=\int_{5x^4}^{\sec x}\sqrt{4t-9}\,dt$.

c) $\frac{d}{dx}\left[\int_{8}^{x}\ln\left(3t^2+9\right)dt\right]$

d) Find $h'(x)$ if $h(x)=\int_{3x^6}^{\tan x}\frac{9}{x^2-1}\,dt$.

4. [No Calculator] Given the graph of $f(x)$ as shown and the definition of $g(x)=\int_{0}^{x}f(t)\,dt$

a) Find $g(-1)$, $g'(-1)$, $g''(-1)$



Graph of f

b) Over what interval is $g(x)$ increasing.
Show your work and explain your reasoning.

c) Over what interval is $g(x)$ concave up? Show your work and explain your reasoning.

d) Graph $g(x)$

7. [Calculator] For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

a) Show that the number of mosquitoes is increasing at time $t = 6$.

b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

8. [No Calculator] Suppose $\int_1^2 f(x)\,dx = 3$, $\int_1^5 f(x)\,dx = -13$, and $\int_1^5 g(x)\,dx = 7$. Find each of the following:

a) $\int_3^3 g(x)\,dx$

b) $\int_5^1 f(x)\,dx$

c) $\int_1^5 [g(x) - f(x)]\,dx$

d) $\int_2^5 f(x)$

e) $\int_1^5 [3f(x) - g(x)]\,dx$

f) $\int_1^5 \frac{g(x)}{4}\,dx$

9. [No Calculator] Suppose $H(x)=\int_2^x \ln(t+5)\,dt$ for the interval [2, 10].

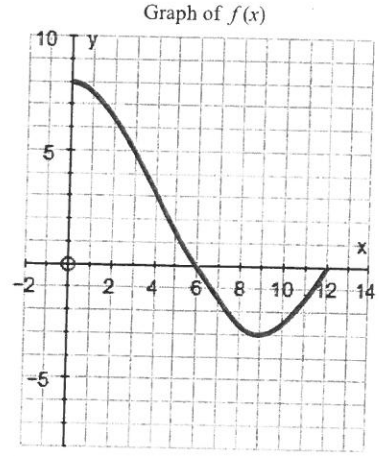
a) Use MRAM to approximate $H(10)$ using 4 equal subdivisions.

b) When is $H(x)$ decreasing? Justify your response.

c) If the average rate of change of $H(x)$ on [2, 10] is k, what is the value of $\int_2^{10} \ln(t+5)\,dt$ in terms of k.

10. [No Calculator] Let $H(x)=\int_0^x f(t)\,dt$, where f is the continuous function with domain [0, 12] shown below.

Graph of $f(x)$



a) Find $H(0)$

b) Is $H(12)$ positive or negative? Explain.

c) Find $H'(x)$ and use it to evaluate $H'(0)$.

d) When is $H(x)$ increasing? Justify your answer.

e) Find $H''(x)$.

f) When is $H(x)$ concave up? Justify your answer.

g) At what x-value does $H(x)$ achieve its maximum value? Justify your answer.

11. [No Calculator] If $\int_a^b f(x)\,dx = a+2b$, then $\int_a^b [f(x)+3]\,dx =$

A $a + 2b + 3$

B $3b - 3a$

C $4a - b$

D $5b - 2a$

E $5b - 3a$

12. [No Calculator] Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

A none

B 0.5

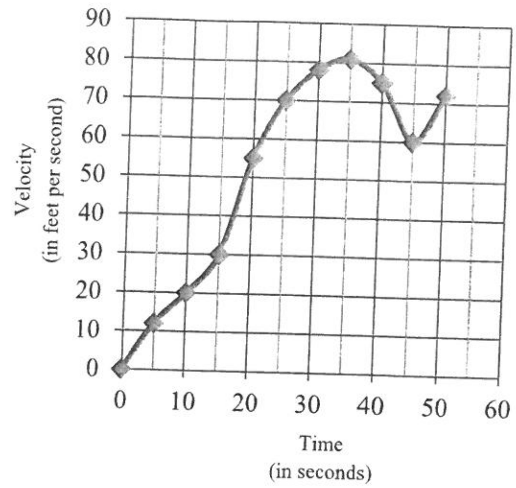
C 1.5

D 2

E 3

13. [Calculator] If $f(x) = \int_a^x \ln(2+\sin t)\,dt$, and $f(3) = 4$, what does $f(5) = ?$

Time (in seconds)	$v(t)$ (in ft/sec)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72



14. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

c) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

Module 6: Interpreting Notational Expressions

Suppose f and g are continuous functions.

(a) Suppose the interval $[1, 3]$ is divided into n subintervals, each of width Δx_i, and let x_i^* be a point in the ith subinterval. Express the following limit as a definite integral:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\cdot f(x_i^*)\cdot\Delta x_i$$

(b) Suppose $\int_0^4 f(x)dx = 5$ and $\int_0^4 g(x)dx = -7$. Find $\int_0^4 [2f(x)-3g(x)]dx$.

(c) Suppose $\int_{10}^{14} f(x)dx = 11$ and $\int_{10}^{20} f(x)dx = 8$. Find $\int_{14}^{20} f(x)dx$.

(d) Let $H(x)=\int_{x^2}^{4} f(t)g(t)\,dt$. Find an expression for $H'(x)$.