

The graph of the function  $f$  is shown above. The domain of  $f$  is  $0 \leq x \leq 9$ .

Let  $g$  be the function defined by:

$$g(x) = \int_2^x f(t) dt$$

$$g'(x) = f(x) \quad g''(x) = f'(x)$$

a) Find the value of  $g(5)$ .  $= \int_2^5 f(t) dt = \frac{1}{2} \cdot 3 \cdot 2 = 3$

b) Find the value of  $g(0)$ .  $= \int_2^0 f(t) dt = -\int_0^2 f(t) dt = -(\frac{1}{2} \cdot 2 \cdot 2) = -2$

c) Find the value of  $g'(1)$ .  $= f(1) = 1$

d) Find the value of  $g'(5)$ .  $= f(5) = 0$

e) Find the value of  $g''(3)$ .  $= \frac{f(2) - f(5)}{2 - 5} = \frac{2 - 0}{-3} = -\frac{2}{3}$

f) Find the value of  $g''(5)$ .  $= \text{DNE}$

g) For what values of  $x$  in the interval  $[0, 9]$  is  $g$  increasing. Justify your answer.

$$f(x) > 0 \quad g'(x) = f(x)$$

$g(x)$  is increasing when  $0 < x < 5$  and  $8.5 < x < 9$

because  $g'(x)$  or  $f(x)$  is positive.

h) For what values of  $x$  in the interval  $[0, 9]$  is the graph of  $g$  concave up? Justify your answer.

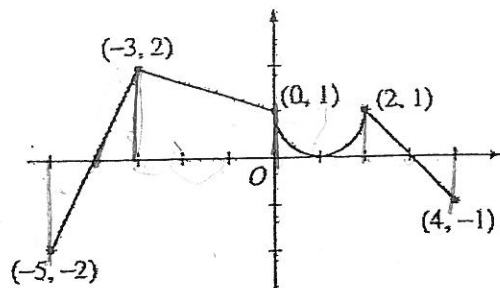
$$g''(x) = f'(x) \quad f' > 0 \quad \text{which means } f \text{ is increasing}$$

$g(x)$  is concave up when  $0 < x < 2$  and  $8 < x < 9$

because  $g''(x)$  is positive or  $g'(x)$  is increasing

i) Find the absolute minimum value of  $g$  on the interval  $[0, 9]$ . Justify your answer.

j) State all the  $x$ -coordinate of all inflection points of the function  $g$ .



Graph of  $f$

5. The graph of the function  $f$  shown above consists of a semicircle and three line segments. Let  $g$  be the function given by  $g(x) = \int_{-3}^x f(t) dt$ .

(a) Find  $g(0)$  and  $g'(0)$ .

(b) Find all values of  $x$  in the open interval  $(-5, 4)$  at which  $g$  attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of  $g$  on the closed interval  $[-5, 4]$ . Justify your answer.

(d) Find all values of  $x$  in the open interval  $(-5, 4)$  at which the graph of  $g$  has a point of inflection.

a.  $g(0) = \int_{-3}^0 f(t) dt = \frac{1}{2} \cdot 3 \cdot (2+1) = \frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2}$   
 $g'(0) = f(0) = 1$

b.  $g(x)$  has a rel max at  $x = -3$   
 bc  $g'(x)$  or  $f(x)$  goes from  $+$  to  $-$

~~cl.~~

x	y
-5	0
-4	-1
4	2

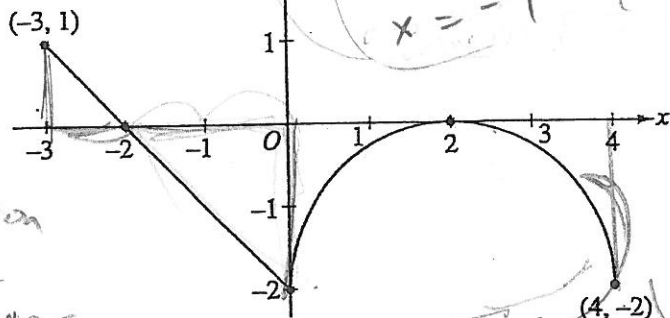
$\int_{-5}^{-3} f(x) dx = -\int_{-3}^{-5} f(x) dx = 0$

$\int_{-3}^4 f(x) dx = \frac{9}{2} + (2 - \frac{1}{2}) = 0$

$\int_{-3}^{-4} f(x) dx = -\int_{-4}^{-3} f(x) dx = -(\frac{1}{2} \cdot 1 \cdot 2) = -1$

abs min value is  $-1$  at  $x = -4$

a.  $f(x)$  is incr when  $-3 < x < -2$   
 bc  $f' > 0$



Graph of  $f'$

c.  $f'(0) = -2$   
 $y - 3 = -2x$   
 $y = -2x + 3$

d.  $f(-3) = f(0) + \int_0^{-3} f'(t) dt$

4. Let  $f$  be a function defined on the closed interval  $-3 \leq x \leq 4$  with  $f(0) = 3$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is  $f$  increasing? Justify your answer.

(b) Find the  $x$ -coordinate of each point of inflection of the graph of  $f$  on the open interval  $-3 < x < 4$ . Justify your answer.

(c) Find an equation for the line tangent to the graph of  $f$  at the point  $(0, 3)$ .

Find  $f(-3)$  and  $f(4)$ . Show the work that leads to your answers.



$f(4) = f(0) + \int_0^4 f'(t) dt$   
 $f(4) = 3 - (8 - \frac{\pi}{2} \cdot 4)$   
 $f(4) = -5 + 2\pi$

$3 - (\frac{1}{2} - \frac{1}{2})$   
 $3 - (\frac{1}{2} - \frac{1}{2})$   
 $3 - (-\frac{3}{2})$   
 $3 + \frac{3}{2} = \frac{9}{2}$

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