Name____ Date________________________ Period______

Worksheet 2.8—Inverse & Inverse Trig Functions

Show all work. No calculator unless otherwise stated.

Short Answer

1. Find the derivative with respect to the appropriate variable. **Simplify your expression.**

(a) $y=\sec^{-1}\left(x^2\right)$ (b) $y=s\sqrt{1-s^2}+\arccos s$ (c) $y=\dfrac{1}{\arcsin(2x)}$

(d) $y=\cot^{-1}\sqrt{t-1}$ (e) $y=\csc^{-1}\dfrac{x}{2}$ (f) $y=\sin(\arccos t)$

(g) $y=m\arctan m$ (h) $y=x\arcsin x+\sqrt{1+x^2}$ (i) $y=\dfrac{\arcsin 3x}{x}$

(j) $y=\dfrac{1}{2}\left(x\sqrt{4-x^2}+4\arcsin\left(\dfrac{x}{2}\right)\right)$ (k) $y=\sin^{-1}t+\cos^{-1}t$

2. If a particle's position is given by $x(t) = \tan^{-1}(t^2)$, find the particle's velocity at $t = 1$.

3. (Unit Circle Time!) Find the equation for the tangent line to the graph of y at the indicated point.

(a) $y = \sec^{-1} x$ at $x = 2$, $0 \le y \le \dfrac{\pi}{2}$

(b) $y = \sin^{-1}\left(\dfrac{x}{2}\right)$ at $x = \sqrt{3}$, $0 \le y \le \dfrac{\pi}{2}$

4. A particle moves along the x-axis so that its position at any time $t \ge 0$ is given by $x(t) = \arctan t$.

(a) Prove that the particle is always moving to the right by analyzing the velocity function.

(b) Prove that the particle's velocity is always decreasing by analyzing the acceleration function.

(c) What is the limiting position of the particle as t approaches infinity.

5. If $f(x) = x^5 + 2x^3 + x - 1$ and $f(g(x)) = x = g(f(x))$ find

(a) $f(1)$

(b) $g'(3)$.

6. If $h(x) = \cos x + 3x$, find $\left(h^{-1}\right)'(h(0))$.

7. Find the equation of the tangent line to the graph of $x^2 + x\arctan y = y - 1$ at $\left(-\frac{\pi}{4}, 1\right)$.

8. If $f(x) = \frac{1}{8}x - 3$, find

(a) $f^{-1}(-3)$

(b) $\left(f^{-1}\right)'(3.14159)$

(c) $\left(f^{-1}\right)''(3.14159)$

Multiple Choice

_____ 9. Find the value of $f(1)$ when $f(x)=5\sin^{-1}x+6\tan^{-1}x$.

(A) 3π (B) 2π (C) 4π (D) $\dfrac{7\pi}{2}$ (E) $\dfrac{5\pi}{2}$

_____ 10. If $k(j(t))=j(k(t))$ and $j(-3)=2$, $k(-3)=4$, $k(2)=-3$, $k'(-3)=\frac{2}{5}$, $k'(2)=\frac{4}{3}$, then $j'(-3)=$

(A) $\dfrac{3}{4}$ (B) $-\dfrac{1}{3}$ (C) $\dfrac{5}{2}$ (D) $-\dfrac{5}{2}$ (E) $-\dfrac{3}{4}$

_____ 11. The expression $f(x)=\sin(\tan^{-1}x)$ is equivalent to which algebraic identity?

(A) $f(x)=\dfrac{1}{\sqrt{1+x^2}}$

(B) $f(x)=\sqrt{1+x^2}$

(C) $f(x)=\dfrac{x}{\sqrt{1-x^2}}$

(D) $f(x)=\dfrac{x}{\sqrt{1+x^2}}$

(E) $f(x)=\dfrac{1}{\sqrt{1-x^2}}$

_____ 12. Determine $f'(x)$ when $f(x)=\sin^{-1}\left(\dfrac{x}{\sqrt{6}}\right)$.

(A) $f'(x)=\dfrac{\sqrt{6}}{\sqrt{6+x^2}}$

(B) $f'(x)=\dfrac{x}{x^2+6}$

(C) $f'(x)=\dfrac{x}{\sqrt{x^2-6}}$

(D) $f'(x)=\dfrac{\sqrt{6}}{\sqrt{6-x^2}}$

(E) $f'(x)=\dfrac{1}{\sqrt{6-x^2}}$

_____ 13. Find the derivative of f when $f(x)=5\arcsin\frac{x}{5}+\sqrt{25-x^2}$ (this one requires a bit of algebraic finagling after you get the derivative, including factoring the radicand.)

(A) $f'(x)=\sqrt{\frac{5-x}{5+x}}$

(B) $f'(x)=\frac{x}{\sqrt{25-x^2}}$

(C) $f'(x)=\frac{5}{\sqrt{25-x^2}}$

(D) $f'(x)=\frac{1}{\sqrt{5+x}}$

(E) $f'(x)=\sqrt{\frac{5+x}{5-x}}$

_____ 14. Find the derivative of f when $f(x)=3\left(\sin^{-1}x\right)^2$.

(A) $f'(x)=\frac{6\sin^{-1}x}{\sqrt{1-x^2}}$

(B) $f'(x)=\frac{3\sin^{-1}x}{\sqrt{1-x^2}}$

(C) $f'(x)=\frac{6\sin^{-1}x}{1+x^2}$

(D) $f'(x)=\frac{6\cos^{-1}x}{\sqrt{1-x^2}}$

(E) $f'(x)=\frac{3\cos^{-1}x}{\sqrt{1-x^2}}$

_____ 15. Determine if $\lim\limits_{x\to\infty}\sin^{-1}\left(\frac{\sqrt{3}+x}{2+2x}\right)$ exists, and if it does, find its value.

(A) $\frac{\pi}{6}$ (B) DNE (C) 0 (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{4}$

_____ 16. Determine if $\lim\limits_{x\to 0}\sin^{-1}\left(\frac{\sqrt{3}+x}{2+2x}\right)$ exists, and if it does, find its value.

(A) $\frac{\pi}{6}$ (B) DNE (C) 0 (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{4}$

_____ 17. Let f be a twice-differentiable function and let g be its inverse. Consider the following equations:

I. $g(f(x)) = x,\ f(g(x)) = x$

II. $f''(g(x))(g'(x))^2 + f'(g(x))g''(x) = 0$

III. $g'(x) = \dfrac{1}{f'(g(x))}$

Which of the three above do both f and g satisfy?

(A) I and III only (B) I only (C) III only (D) I and III only (E) I, II, and III

_____ 18. Find the value of $g'(1)$ when g is the inverse of the function $f(x) = 2\sin x,\ -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.

(A) $\dfrac{1}{\sqrt{3}}$ (B) -1 (C) $\dfrac{1}{\sqrt{2}}$ (D) $-\dfrac{1}{\sqrt{3}}$ (E) 1

_____ 19. Suppose g is the inverse function of a differentiable function f and $G(x) = \dfrac{1}{g(x)}$. If $f(3) = 7$ and $f'(3) = \dfrac{1}{9}$, find $G'(7)$.

(A) -5 (B) 4 (C) 6 (D) -1 (E) -4

_____ 20. Find $\dfrac{dy}{dx}$ when $\tan(2x - y) = 2x$ (Preview your answer choices first. Notice there is no trig!)

(A) $\dfrac{dy}{dx} = \dfrac{8x^2}{1+4x^2}$

(B) $\dfrac{dy}{dx} = -\dfrac{8x^2}{1+4x^2}$

(C) $\dfrac{dy}{dx} = -\dfrac{4y^2}{2+x^2}$

(D) $\dfrac{dy}{dx} = \dfrac{4y^2}{2+x^2}$

(E) $\dfrac{dy}{dx} = -\dfrac{8x^2}{1+4y^2}$