Name:

Review for Applications of Derivatives Quiz (No Calculator!)

Section I: Free Response

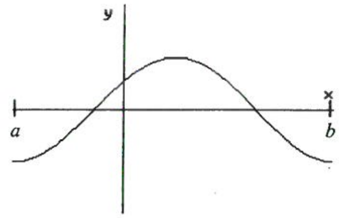
1. Let $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + 4$

   (a) For what x-coordinate does $f(x)$ have a relative maximum? Justify your answer.

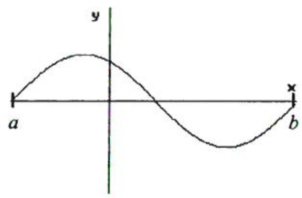
   (b) For what x-coordinate does $f(x)$ have an inflection point? Justify your answer.

application of derivatives quiz

11. The graph of f is shown below.



Which of the following could be the graph of the derivative of f?

A.


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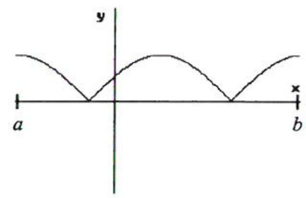

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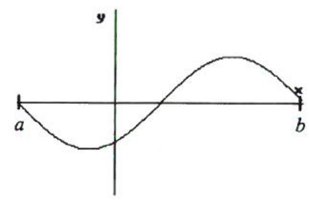

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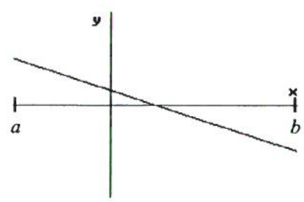

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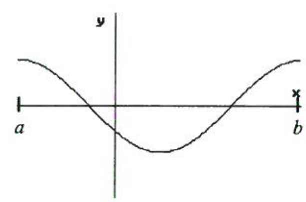
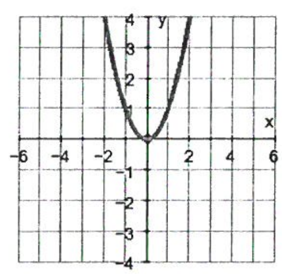
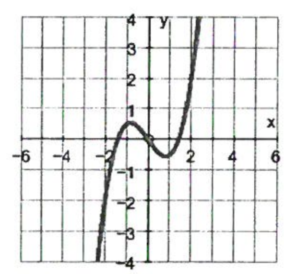
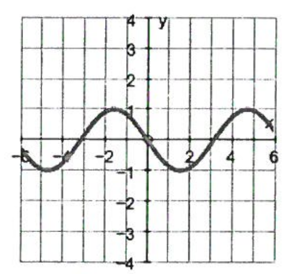
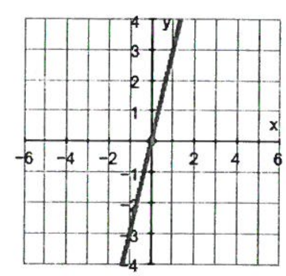
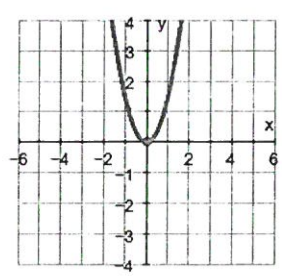
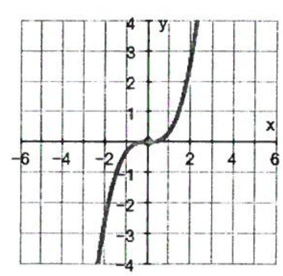
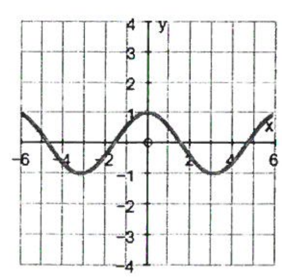
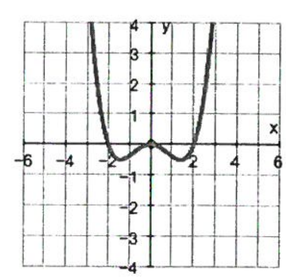


12. The graphs in the first row are the derivatives. Match them with the graph of their function shown in the second row.

(Graphs of Derivative)

(Graphs of Function)

8. If f is a continuous, decreasing function on [0, 10] with a critical point at (4, 2), which of the following statements MUST BE FALSE?

A $f(10)$ is an absolute minimum of f on [0, 10]
B $f(4)$ is neither a relative maximum nor a relative minimum
C $f'(4)$ does not exist.
D $f'(4)=0$
E $f'(4)<0$

9. Find the extrema on each interval and where they occur … Use a "candidates test".

a) $f(x)=\frac{1}{x}+\ln x$ when $0.5\le x\le 4$

b) $g(x)=\ln(x+1)$ when $0\le x\le 3$

c) $k(x)=x^{\frac{2}{5}}$ when $-3\le x<1$

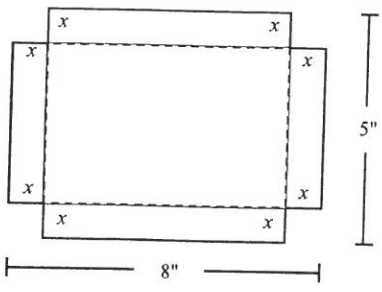
10. Find the extrema of $h(\theta)=2\sin\theta-\cos(2\theta)$ for $0\le\theta\le 2\pi$. Use your graphing calculator to investigate first.

11. [No Calculator] An open-top box is to be made by cutting congruent squares of side length x from the corners of a 5- by 8-inch sheet of tin and bending up the sides (see figure below).

a) Write an equation for the Volume of the box.

b) What is the domain of this function? _____

c) How large should the squares be to maximize the volume?



d) What are the dimensions of the box with maximum volume? What is the maximum volume?

AP Calculus
4.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Complete each statement with the correct word.

   a) When f' is _____, the graph of f is increasing,.

   b) When f' is _____, the graph of f is decreasing,.

   c) When f'' is _____, the graph of f is concave upward.

   d) When f'' is _____, the graph of f is concave downward.

   e) When f' is _____, the graph of f is concave upward.

   f) When f' is _____, the graph of f is concave downward.

2. Use the function $y = 3x - x^3 + 5$. [No calculator allowed]

   a) Where is the function increasing? Justify your response.

   b) Where is the function decreasing? Justify your response.

   c) Where is the function concave up? Justify your response.

   d) Where is the function concave down? Justify your response.

   e) Where are the point(s) of inflection? Justify your response.

   f) Find ALL extrema and justify your response.

   g) Create a *sketch* of the function using the information you have found from $a - f$.

3. Find all local extrema and justify your response for each function:

a) $y = -2x^3 + 6x^2 - 3$

b) $y = xe^{1/x}$

4. Determine the intervals on which the graph of each function is concave up or concave down and determine all points of inflection. Justify your responses.

a) $y = \frac{1}{20}x^5 + \frac{1}{4}x^4 - \frac{3}{2}x^3 - \frac{27}{2}x^2 + x - 4$

b) $y = 2x^{1/5} + 3$

5. If f is continuous on [0, 3] and satisfies the following:

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3
$f(x)$	0	+	2	+	0	–	–2
$f'(x)$	3	+	0	–	DNE	–	–3
$f''(x)$	0	–	–1	–	DNE	–	0

a) Find the absolute extrema of f and where they occur. Justify your response.

b) Find any points of inflection. Justify your response.

c) Sketch a possible graph of f.

6. Let f be a function that is continuous on the interval $[0, 4]$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

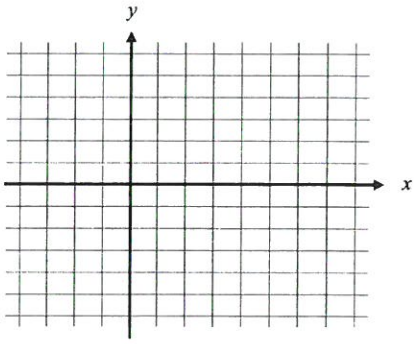
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

a) Describe the behavior of $f(x)$ in each interval using the information above.

x	$0 < x < 1$	$1 < x < 2$	$2 < x < 3$	$3 < x < 4$
$f(x)$				

b) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

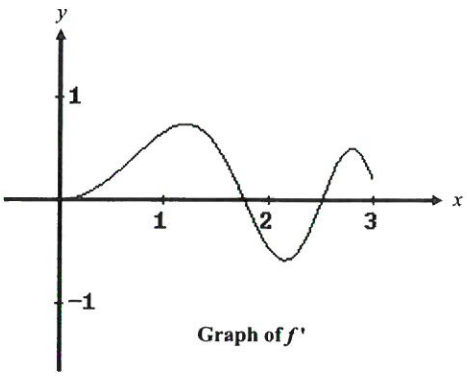
c) On the axes provided, sketch the graph of a function that has all the characteristics of f.



7. [Calculator Required] Let f be a function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown below.

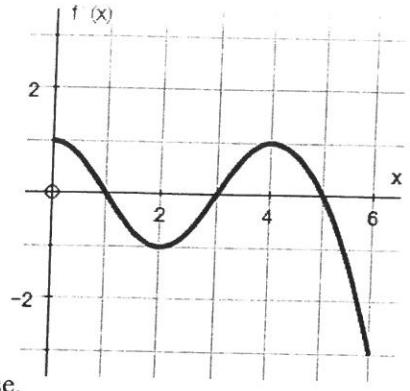
a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

b) Write an equation for the line tangent to the graph of f at the point (2, 5.623).



Graph of f'

8. Use the graph of $f'(x)$ defined on $[0, 6]$ provided below to estimate the following:



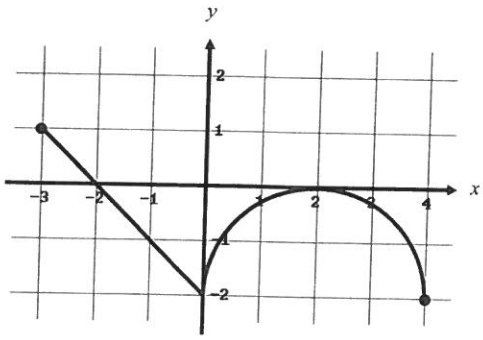
a) When is f increasing? When is f decreasing? Justify your response.

b) Determine the x-coordinates of all local extrema. Justify your response.

c) Determine when f is concave up and concave down. Justify your response.

d) Determine whether f has any points of inflection. Justify your response.

9. [No Calculator Allowed] Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.



Graph of f'

a) On what intervals, if any, is f increasing? Decreasing? Justify your answer.

b) Find all values of x for which f assumes a relative maximum. Justify your answer.

c) Where is the graph of f concave up? concave down? Justify your answers.

d) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

e) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

f) Sketch a possible graph of f.

10. If g is a differentiable function such that $g(x) < 0$ for all real numbers x, and if $f'(x) = (x^2 - 9)g(x)$, which of the following is true?

A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 3$.

B) f has a relative minimum at $x = -3$ and a relative maximum at $x = 3$.

C) f has relative minima at $x = -3$ and at $x = 3$.

D) f has relative maxima at $x = -3$ and at $x = 3$.

E) It cannot be determined if f has any relative extrema.

11. Suppose that at any time t (sec) the current I (amp) in an alternating current circuit is $I = 2\cos t + 2\sin t$. What is the peak (largest magnitude) current for this circuit? Justify your response.

12. A rectangle is inscribed between the parabolas $y = 4x^2$ and $y = 30 - x^2$ as shown in the picture. What is the maximum area of such a rectangle? Justify your response.