
AP Calculus AB

Sample Student Responses and Scoring Commentary

Inside:

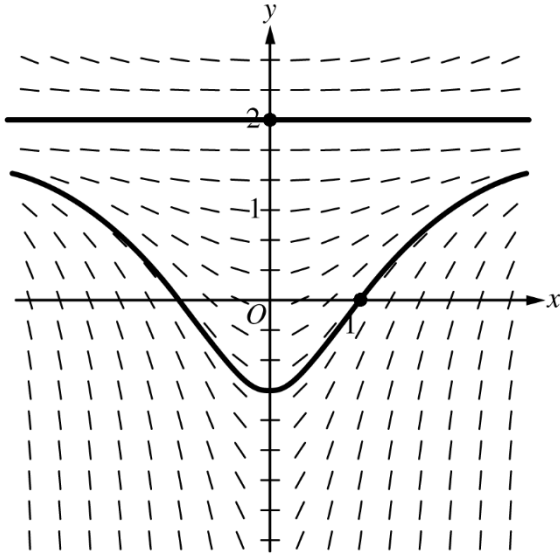
Free Response Question 6

- ☒ Scoring Guideline
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AP[®] CALCULUS AB
2018 SCORING GUIDELINES

Question 6

(a)



2 : $\begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) $\left. \frac{dy}{dx} \right|_{(x, y)=(1, 0)} = \frac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ is $y = \frac{4}{3}(x - 1)$.

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$$

2 : $\begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$
 $\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$
 $\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$$

$$y = 2 - \frac{6}{x^2 + 2}$$

5 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

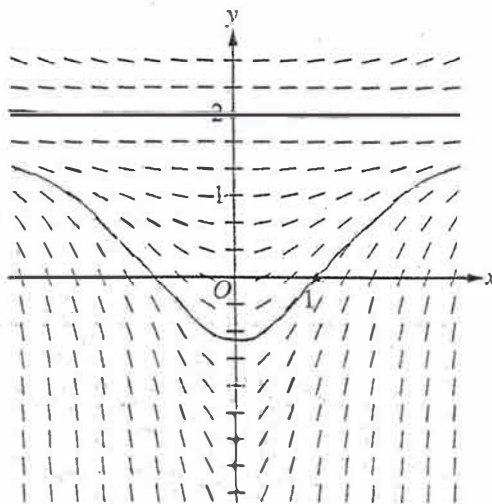
Note: this solution is valid for $-\infty < x < \infty$.

NO CALCULATOR ALLOWED

6A
1 of 2

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

- (a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



- (b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

$$y = m_T(x-1) + 0$$

$$m_T = \frac{1}{3} \cdot 1 \cdot (0-2)^2$$

$$y = \frac{4}{3}(x-1)$$

$$= \frac{1}{3}(4)$$

$$f(0.7) \approx \frac{4}{3}(0.7-1)$$

$$= \frac{4}{3}$$

$$\approx \frac{4}{3}\left(-\frac{3}{10}\right)$$

$$\approx -\frac{4}{10} = -\frac{2}{5}$$

NO CALCULATOR ALLOWED

6A
2 of 2

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$$u = y - 2$$

$$du = dy$$

$$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3} x dx$$

$$\int \frac{du}{u^2} = \frac{1}{3} \int x dx$$

$$-\frac{1}{u} = \frac{1}{3} \left(\frac{x^2}{2} \right) + C$$

$$-\frac{1}{(y-2)} = \frac{x^2}{6} + C$$

point: $x = 1$
 $y = 0$

$$C = \frac{1}{2} - \frac{1}{6} = \frac{3}{6} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$\therefore -\frac{1}{y-2} = \frac{x^2}{6} + \frac{1}{3}$$

$$\frac{1}{y-2} = -\frac{x^2}{6} - \frac{1}{3}$$

$$y-2 = \left(\frac{1}{-\frac{x^2}{6} - \frac{2}{6}} \right)^{\frac{6}{6}}$$

$$y-2 = \frac{6}{-(x^2+2)}$$

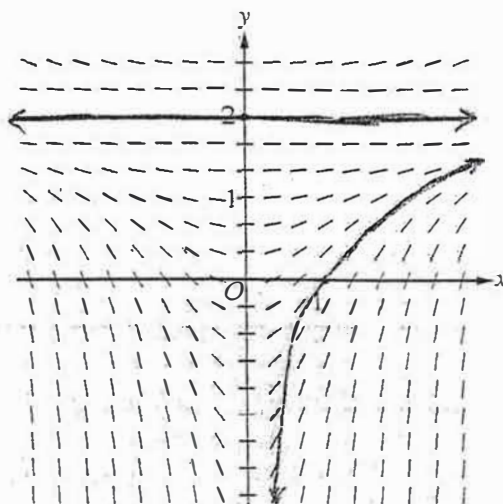
$$y = \frac{6}{-(x^2+2)} + 2$$

NO CALCULATOR ALLOWED

6B
1x2

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

- (a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



- (b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{3}x(y-2)^2 \\ &= \frac{1}{3}(1)(-2)^2 \\ &= \frac{4}{3}\end{aligned}$$

$$y = \frac{4}{3}(x-1)$$

$$y = \frac{4}{3}(0.7-1)$$

$$y = \frac{4}{3}(-.3)$$

$$= \frac{4}{9}$$

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$$\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$$

$$\int (y-2)^{-2} dy = \int \frac{1}{3}x dx$$

$$-(y-2)^{-1} = \frac{1}{6}x^2 + C$$

$$-\frac{1}{y-2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C$$

$$\frac{3}{6} - \frac{1}{6} = C$$

$$\frac{1}{3} = C$$

$$-\frac{1}{y-2} = \frac{1}{6}x^2 + \frac{1}{3}$$

$$-1 = \frac{1}{6}x^2 + \frac{1}{3}(y-2)$$

$$-6x^2 - 3 = y - 2$$

$$-6x^2 - 1 = y$$

$$\boxed{y = -6x^2 - 1}$$

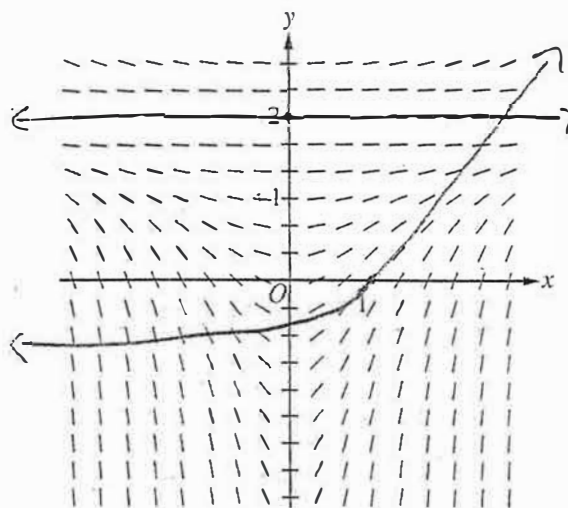
NO CALCULATOR ALLOWED

6C

1 + 2

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

- (a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



- (b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{3}x(y-2)^2 \\ \frac{1}{3}(1)(0-2)^2 &= \frac{1}{3}(-2)^2 \\ \frac{1}{3}(4) &= \frac{4}{3} \\ f(0.7) &= -\frac{2}{5}\end{aligned}$$

$$\begin{aligned}0 &= \frac{4}{3}(1) + b \\ 0 &= \frac{4}{3} + b \\ -\frac{4}{3} &= b\end{aligned}$$

$$\begin{aligned}y &= \frac{4}{3}x - \frac{4}{3} \\ y &= \frac{4}{3}(0.7) - \frac{4}{3} \\ y &= \frac{28}{30} - \frac{4}{3} \\ y &= \frac{28}{30} - \frac{40}{30} \\ y &= -\frac{12}{30} \\ y &= -\frac{2}{5}\end{aligned}$$

NO CALCULATOR ALLOWED

6C
2 of 2

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$$\frac{dy}{dx} = \frac{1}{3} x (y-2)^2$$

$$u = y - 2$$

$$e = \frac{1}{3} x (u)^2$$

$$\frac{\frac{1}{3} x (u)^2}{3} \cdot u'$$

$$\frac{\frac{1}{3} x (y-2)^2}{3} \cdot \frac{y^2}{2} - 2x$$

$$y = \frac{\frac{1}{3} x (y-2)^3}{3} \cdot \frac{y^2}{2} - 2x + 2$$

$$y = \frac{\frac{1}{3} x (y-2)^3}{3} \cdot \frac{y^2}{2} - 2x + C$$

$$0 = \frac{\frac{1}{3} (1) (0-2)^3}{3} \cdot \frac{0}{2} - 2(1) + C$$

$$0 = \frac{\frac{1}{3} (-8)}{3} \cdot 0 - 2$$

$$0 = -2 + C$$

$$C = 2$$

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2018 SCORING COMMENTARY

Question 6

Overview

This problem deals with the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$. In part (a) students were given a slope field for the differential equation and asked to sketch solution curves corresponding to solutions that pass through the points $(0, 2)$ and $(1, 0)$. A correct response should be two sketched curves that pass through the indicated points, follow the given slope lines, and extend to the boundaries of the provided slope field. In part (b) students were given that a solution f satisfies $f(1) = 0$ and asked to supply an equation for the line tangent to the graph of f at $x = 1$. Students were then to use this equation to approximate $f(0.7)$. A correct response should use the fact that the slope of the tangent line is the value of the derivative of f at the indicated point, and this value can be computed from substitution of $(x, y) = (1, 0)$ in the differential equation. Combining the slope and the point $(1, 0)$ gives the tangent line equation $y = \frac{4}{3}(x - 1)$; substituting $x = 0.7$ into this equation gives the requested approximation for $f(0.7)$. In part (c) students were asked to find the particular solution $y = f(x)$ to the given differential equation that satisfies $f(1) = 0$. A correct response should employ the method of separation of variables and use the initial condition $f(1) = 0$ to resolve the constant of integration to arrive at the solution $f(x) = 2 - \frac{6}{x^2 + 2}$.

For part (a) see LO 2.3F/EK 2.3F1. For part (b) see LO 2.3B/EK 2.3B2. For part (c) see LO 3.3B(a)/EK 3.3B5, LO 3.5A/EK 3.5A2. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

Sample: 6A

Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), and 5 points in part (c). In part (a) the response earned the first point with the solution curve $y = 2$ drawn through the point $(0, 2)$. The response earned the second point with the solution curve through $(1, 0)$ that follows the slope lines and extends to both boundaries of the slope field. In part (b) the response earned the first point with the tangent line equation $y = \frac{4}{3}(x - 1)$. The response would have earned the second point for $f(0.7) \approx \frac{4}{3}(0.7 - 1)$ with no simplification. In this case, correct simplification to $-\frac{2}{5}$ earned the point. In part (c) the response earned the first point with a correct separation of variables in line 1, $\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$. The response earned the second and third points with the correct antiderivatives $-\frac{1}{(y - 2)} = \frac{x^2}{6} + C$ in line 4. The response earned the fourth point by both including C with the antiderivatives in line 4 and for substituting 1 for x and 0 for y in line 5. The response earned the fifth point by presenting a correct solution, $y = \frac{6}{-(x^2 + 2)} + 2$.

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Question 6 (continued)

Sample: 6B

Score: 6

The response earned 6 points: 1 point in part (a), 1 point in part (b), and 4 points in part (c). In part (a) the response earned the first point with the solution curve $y = 2$ drawn through the point $(0, 2)$. The response did not earn the second point because the curve through $(1, 0)$ does not follow the slope lines correctly from the left boundary to the right boundary of the slope field. In part (b) the response earned the first point with the tangent line equation $y = \frac{4}{3}(x - 1)$ in line 1 on the right. The response would have earned the second point for the approximation $y = \frac{4}{3}(.7 - 1)$ in line 2 on the right with no simplification. In this case, incorrect simplification to $\frac{4}{9}$ did not earn the second point. In part (c) the response earned the first point with a correct separation of variables in line 2, $\int (y - 2)^{-2} dy = \int \frac{1}{3}x dx$. The response earned the second and third points with the correct antiderivatives $-(y - 2)^{-1} = \frac{1}{6}x^2 + C$ in line 3. The response earned the fourth point with both the constant of integration in line 3 and for substituting 1 for x and 0 for y resulting in $\frac{1}{2} = \frac{1}{6} + C$ in line 5. The response does not correctly solve for $y = f(x)$, so the fifth point was not earned.

Sample: 6C

Score: 3

The response earned 3 points: 1 point in part (a), 2 points in part (b), and no points in part (c). In part (a) the response earned the first point with the solution curve $y = 2$ drawn through the point $(0, 2)$. The response did not earn the second point because the curve through $(1, 0)$ does not follow the slope lines correctly. In part (b) the response earned the first point with the tangent line $y = \frac{4}{3}x - \frac{4}{3}$ in the box on the right. The response would have earned the second point for $y = \frac{4}{3}(.7) - \frac{4}{3}$ below the box on the right with no simplification. In this case, correct simplification of the approximation to $f(.7) = -\frac{2}{5}$ earned the second point. In part (c) the response does not separate the variables. The response is not eligible for any points in part (c).