AP Calculus
Chapter 6 Review

Name: ____________________

Block: ______

For questions 1 – 21, integrate each of the following indefinite integrals.

1. $\int \frac{3x}{\sqrt[3]{x^2+3}}\,dx$

2. $\int \sec(5x)\,dx$

3. $\int \frac{x-1}{x+1}\,dx$

4. $\int 9\sin x\,dx$

5. $\int 7^{3x}\,dx$

6. $\int 2\tan(x)\,dx$

7. $\int \cot^2 x\,dx$

8. $\int \frac{(x+1)^2}{x^{1/3}}\,dx$

9. $\int \cos^2(8x)\,dx$

10. $\int \frac{dx}{4+9x^2}$

11. $\int \left(10\cos t+\sin^2(10t)\right)dt$

12. $\int \sec^2 x\,dx$

13. $\int \frac{dx}{x\sqrt{x^2-4}}$

14. $\int 5e^{3x}\cot\left(e^{3x}\right)dx$

15. $\int \frac{1}{\sec(12x)}\,dx$

16. $\int 7\csc\left(\frac{x}{4}\right) dx$

17. $\int \tan^2\left(\frac{x}{5}\right) dx$

18. $\int \frac{x}{x^2-4} dx$

19. $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$

20. $\int \csc^2 x \, dx$

21. $\int xe^{-x^2+3} \, dx$

For questions 22 – 26, evaluate each definite integral without a calculator. Check your answer with your calculator.

22. $\int_{-1}^{1} \left(x^2-5\right)^2 dx$

23. $\int_{-4}^{-2} \frac{dx}{x^2+6x+10}$

24. $\int_{0}^{1} \frac{3+x}{x^2+1} dx$

25. $\int_{0}^{6} \frac{dx}{7-x}$

26. $\int_{4}^{9} \sqrt{x}\left(3-4x\right) dx$

27. Derive (SHOW EVERY STEP) $y = y_0 e^{kt}$ from $\frac{dy}{dt} = ky$ and $y(0) = y_0$.

28. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2 + 1}{2y}$.

a) Find the slope of the graph of f at the point where $x = 1$.

b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$ → underestimate? overestimate?

c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.

d) Use your solution from part c to find $f(1.2)$

29. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

A $e^{\tan x} + 4$

B $e^{\tan x} + 5$

C $5e^{\tan x}$

D $\tan x + 5$

E $\tan x + 5e^x$

30. [No Calculator] If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

A) $-\frac{1}{2}\ln 2$

B) $-\frac{1}{4}$

C) $\frac{1}{2}\ln 2$

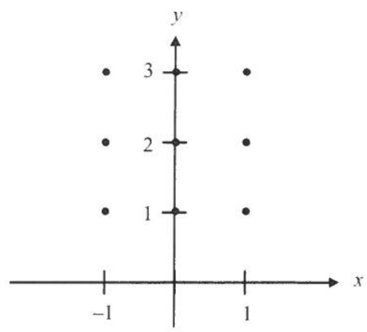
D) $\frac{\sqrt{2}}{2}$

E) $\ln 2$

31. Consider the differential equation given by $\dfrac{dy}{dx} = \dfrac{xy}{2}$.

a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

b) Draw a particular solution if $f(0) = 3$



c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

32. [Calculator] During a certain epidemic, the number of people that are infected at any time increases at rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

Quick Review 7.1

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–8, determine whether or not the function y satisfies the differential equation.

1. $\dfrac{dy}{dx} = y \qquad y = e^x$
2. $\dfrac{dy}{dx} = 4y \qquad y = e^{4x}$
3. $\dfrac{dy}{dx} = 2xy \qquad y = x^2e^x$
4. $\dfrac{dy}{dx} = 2xy \qquad y = e^{x^2}$
5. $\dfrac{dy}{dx} = 2xy \qquad y = e^{x^2} + 5$
6. $\dfrac{dy}{dx} = \dfrac{1}{y} \qquad y = \sqrt{2x}$
7. $\dfrac{dy}{dx} = y \tan x \qquad y = \sec x$
8. $\dfrac{dy}{dx} = y^2 \qquad y = x^{-1}$

In Exercises 9–12, find the constant C.

9. $y = 3x^2 + 4x + C$ and $y = 2$ when $x = 1$
10. $y = 2 \sin x - 3 \cos x + C$ and $y = 4$ when $x = 0$
11. $y = e^{2x} + \sec x + C$ and $y = 5$ when $x = 0$
12. $y = \tan^{-1} x + \ln(2x - 1) + C$ and $y = \pi$ when $x = 1$

Section 7.1 Exercises

In Exercises 1–10, find the general solution to the exact differential equation.

1. $\dfrac{dy}{dx} = 5x^4 - \sec^2 x$
2. $\dfrac{dy}{dx} = \sec x \tan x - e^x$
3. $\dfrac{dy}{dx} = \sin x - e^{-x} + 8x^3$
4. $\dfrac{dy}{dx} = \dfrac{1}{x} - \dfrac{1}{x^2}\ (x > 0)$
5. $\dfrac{dy}{dx} = 5^x \ln 5 + \dfrac{1}{x^2 + 1}$
6. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 - x^2}} - \dfrac{1}{\sqrt{x}}$
7. $\dfrac{dy}{dt} = 3t^2 \cos(t^3)$
8. $\dfrac{dy}{dt} = (\cos t)\, e^{\sin t}$
9. $\dfrac{du}{dx} = (\sec^2 x^5)(5x^4)$
10. $\dfrac{dy}{du} = 4\, (\sin u)^3(\cos u)$

In Exercises 11–20, solve the initial value problem explicitly.

11. $\dfrac{dy}{dx} = 3 \sin x$ and $y = 2$ when $x = 0$
12. $\dfrac{dy}{dx} = 2e^x - \cos x$ and $y = 3$ when $x = 0$
13. $\dfrac{du}{dx} = 7x^6 - 3x^2 + 5$ and $u = 1$ when $x = 1$
14. $\dfrac{dA}{dx} = 10x^9 + 5x^4 - 2x + 4$ and $A = 6$ when $x = 1$
15. $\dfrac{dy}{dx} = -\dfrac{1}{x^2} - \dfrac{3}{x^4} + 12$ and $y = 3$ when $x = 1$
16. $\dfrac{dy}{dx} = 5 \sec^2 x - \dfrac{3}{2}\sqrt{x}$ and $y = 7$ when $x = 0$
17. $\dfrac{dy}{dt} = \dfrac{1}{1 + t^2} + 2^t \ln 2$ and $y = 3$ when $t = 0$
18. $\dfrac{dx}{dt} = \dfrac{1}{t} - \dfrac{1}{t^2} + 6$ and $x = 0$ when $t = 1$
19. $\dfrac{dv}{dt} = 4 \sec t \tan t + e^t + 6t$ and $v = 5$ when $t = 0$
20. $\dfrac{ds}{dt} = t(3t - 2)$ and $s = 0$ when $t = 1$

In Exercises 21–24, solve the initial value problem using the Fundamental Theorem. (Your answer will contain a definite integral.)

21. $\dfrac{dy}{dx} = \sin(x^2)$ and $y = 5$ when $x = 1$
22. $\dfrac{du}{dx} = \sqrt{2 + \cos x}$ and $u = -3$ when $x = 0$
23. $F'(x) = e^{\cos x}$ and $F(2) = 9$
24. $G'(s) = \sqrt[3]{\tan s}$ and $G(0) = 4$

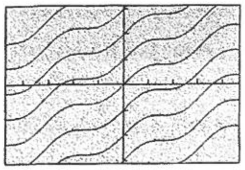
In Exercises 25–28, match the differential equation with the graph of a family of functions (a)–(d) that solve it. Use slope analysis, not your graphing calculator.

25. $\dfrac{dy}{dx} = (\sin x)^2$
26. $\dfrac{dy}{dx} = (\sin x)^3$
27. $\dfrac{dy}{dx} = (\cos x)^2$
28. $\dfrac{dy}{dx} = (\cos x)^3$

match with fields on back

match these with #25 to #28 on previous page

see previous



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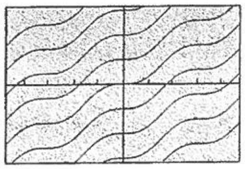

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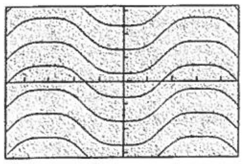

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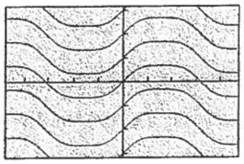

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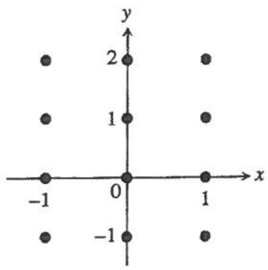
In Exercises 29–34, construct a slope field for the differential equation. In each case, copy the graph at the right and draw tiny segments through the twelve lattice points shown in the graph. Use slope analysis, not your graphing calculator.

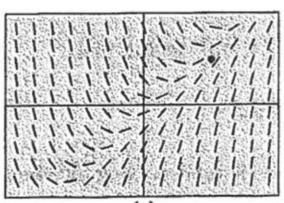


29. $\frac{dy}{dx} = x$ **30.** $\frac{dy}{dx} = y$ **31.** $\frac{dy}{dx} = 2x + y$

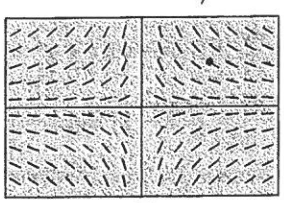
32. $\frac{dy}{dx} = 2x - y$ **33.** $\frac{dy}{dx} = x + 2y$ **34.** $\frac{dy}{dx} = x - 2y$

In Exercises 35–40, match the differential equation with the appropriate slope field. Then use the slope field to sketch the graph of the particular solution through the highlighted point (3, 2). (All slope fields are shown in the window $[-6, 6]$ by $[-4, 4]$.)

↓ 35-40



(a)



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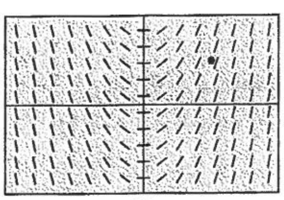

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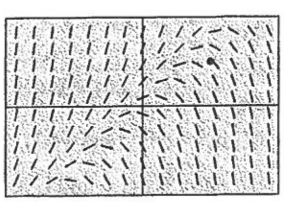

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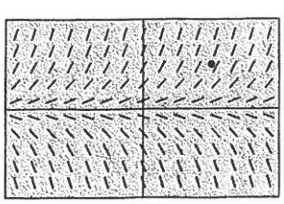

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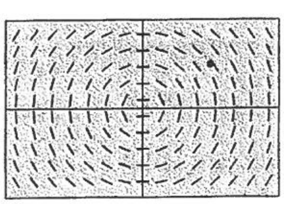

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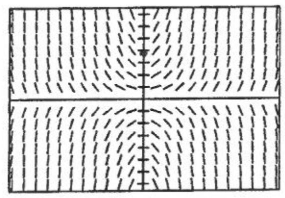
35. $\frac{dy}{dx} = x$ **36.** $\frac{dy}{dx} = y$

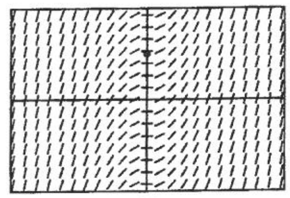
37. $\frac{dy}{dx} = x - y$ **38.** $\frac{dy}{dx} = y - x$

39. $\frac{dy}{dx} = -\frac{y}{x}$ **40.** $\frac{dy}{dx} = -\frac{x}{y}$

In Exercises 41–46, match the differential equation with the appropriate slope field. Then use the slope field to sketch the graph of the particular solution through the highlighted point (0, 2). (All slope fields are shown in the window $[-6, 6]$ by $[-4, 4]$.)



(a)



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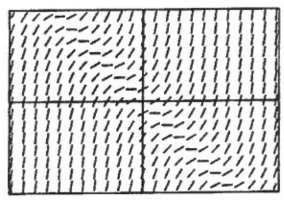

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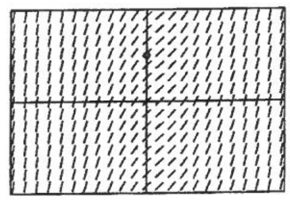

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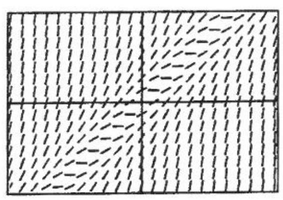

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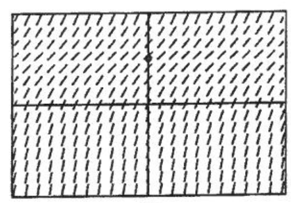


(f)

41. $\frac{dy}{dx} = \sqrt{x^2 - x + 1}$ **42.** $\frac{dy}{dx} = \sqrt{y^2 - 4y + 5}$

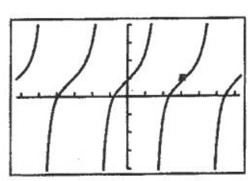
43. $\frac{dy}{dx} = |x + y|$ **44.** $\frac{dy}{dx} = |x - y|$

45. $\frac{dy}{dx} = |x|$ **46.** $\frac{dy}{dx} = xy$

47. **(a)** Sketch a graph of the solution to the initial value problem

$$\frac{dy}{dx} = \sec^2 x \text{ and } y = 1 \text{ when } x = \pi.$$

(b) Writing to Learn A student solved part (a) and used a graphing calculator to produce the following graph:



$[-2\pi, 2\pi]$ by $[-4, 4]$

How would you explain to this student why this graph is *not* the correct answer to part (a)?

AP Calculus
6.4 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Suppose the rate of change of ☺ is proportional to the amount of ☺ present.

   a) Write the differential equation that this statement represents.

   b) Solve the differential equation from part *a* … do not skip ANY steps.

2. Find the particular solution $y = f(x)$ to each differential equation using the given initial value.

   a) $\dfrac{dy}{dx} = (y+5)(x+2)$ and $y = 1$ when $x = 0$.

   b) $\dfrac{dy}{dx} = \dfrac{1}{5}(8-y)$ and $y = 6$ when $x = 0$

   c) $\dfrac{dy}{dx} = \cos^2 y$ and $y = 0$ when $x = 0$.

   d) $\dfrac{dy}{dx} = e^{x-y}$ and $y = 2$ when $x = 0$.

3. [No Calculator] The rate of change in the population of a group of elk in the local national forest is proportional to the difference between the maximum number of elk the forest can support and the number of elk currently present. At time $t = 0$, when the number of elk are first counted, there are 40 elk. If $L(t)$ is the number of elk at time t years after they are first counted, then

$$\frac{dL}{dt} = \frac{1}{4}(500 - L)$$

a) Are the elk increasing in number faster when there are 160 or when there are 360? Explain and use correct notation.

b) Find an equation for $\frac{d^2L}{dt^2}$ in terms of L. What does $\frac{d^2L}{dt^2}$ tell you about the graph of L ?

c) Use separation of variables to find the particular solution to $\frac{dL}{dt} = \frac{1}{4}(500 - L)$ if $L(0) = 40$.

4. [No Calculator] If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

A) $-\frac{1}{2}\ln 2$
B) $-\frac{1}{4}$
C) $\frac{1}{2}\ln 2$
D) $\frac{\sqrt{2}}{2}$
E) $\ln 2$

5. [Calculator] A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

A) 4.2 pounds
B) 4.6 pounds
C) 4.8 pounds
D) 5.6 pounds
E) 6.5 pounds

→ very similar to FR on test

6. [Calculator] During the zombie invasion of a small town the number of infected people is proportional to the difference between the town's population and the number of zombies currently roaming around. There are 8 zombies roaming around when they are first discovered (call this time $t = 0$ hours). If $Z(t)$ represents the number of zombies roaming the town at time t, then

$$\frac{dZ}{dt} = 0.05(1100 - Z)$$

a) Find a tangent line to the graph of Z when $t = 0$.

b) Find $\frac{d^2Z}{dt^2}$ in terms of Z.

c) Use your tangent line from part a to estimate the number of zombies roaming the town 24 hours after they are first discovered ($t = 24$). Is this an over approximation or an under approximation? Explain.

d) Use separation of variables to find the particular solution for $Z(t)$ if $Z(0) = 8$.

see #5

7. [Calculator] A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

A) 4.2 pounds
B) 4.6 pounds
C) 4.8 pounds
D) 5.6 pounds
E) 6.5 pounds