

1. (No Calculator) Evaluate each limit or explain why the limit does not exist.

a) $\lim_{x \rightarrow 5} \int(x)$

b) $\lim_{x \rightarrow \infty} \frac{x^2 + 5x - 3}{3x + 2}$

c) $\lim_{x \rightarrow \infty} \frac{x^2 + 5x - 3}{3x^2 + 2}$

d) $\lim_{x \rightarrow -2} (x^3 - 2x^2 + 1)$

e) $\lim_{x \rightarrow \infty} \frac{x^2 + 5x - 3}{3x^3 + 2}$

f) $\lim_{x \rightarrow 0} \frac{x}{\sin(2x)}$

g) $\lim_{x \rightarrow \infty} \frac{\sin x}{2x}$

h) $\lim_{x \rightarrow 0} \frac{\tan(5x)}{\sin(3x)}$

i) $\lim_{x \rightarrow 0} e^x \sin x$

j) $\lim_{x \rightarrow 1} \frac{4x^2 + 5x}{x - 3}$

k) $\lim_{x \rightarrow \frac{1}{2}} \int(2x - 1)$

l) $\lim_{x \rightarrow \infty} \frac{5x - 7x^2}{4x^2 + 1}$

m) $\lim_{x \rightarrow -3} \frac{|x + 3|}{x + 3}$

n) $\lim_{x \rightarrow \infty} \frac{x^4 + x^3}{12x^3 + 128}$

o) $\lim_{x \rightarrow 4} \sqrt{1 - 2x}$

p) $\lim_{x \rightarrow 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$

q) $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$

r) $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 + x - 6}$

2. (Calculator) Use a table of values to evaluate the following limit: $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

Recognize the number? ... Add this to your notecards under "Limits you should know". ... yeah ... those things you have to hand in before your test! ☺

3. (Calculator) Make a table of values (4 of them would work) to evaluate $\lim_{x \rightarrow 2^+} \frac{x+3}{x-2}$.

4. (No Calculator) Suppose $\lim_{x \rightarrow c} f(x)$ and $\lim_{x \rightarrow c} g(x)$ exist, and

$$\lim_{x \rightarrow c} [f(x) + g(x)] = 2$$

$$\lim_{x \rightarrow c} [f(x) - g(x)] = 1$$

Find $\lim_{x \rightarrow c} f(x)$ and $\lim_{x \rightarrow c} g(x)$.

5. (No Calculator) Find all asymptotes for each function and justify your response.

a) $y = \ln x$

b) $f(x) = \frac{(x+2)(x-3)}{(x+2)(x-1)}$

c) $f(x) = \frac{x-1}{x^2(x+2)}$

d) $f(x) = \frac{x^3 - 3x^2 + x - 1}{x^2 + x - 2}$

6. (No Calculator) Let $y = \frac{x^2 + 5x - 3}{x - 2}$.

a) Find the End Behavior Model

b) Describe the End Behavior using limits.

c) Find all asymptotes.

7. The number of bears in a federal wildlife reserve is given by the population $p(t) = \frac{200}{1 + 7e^{-0.1t}}$, where t is in years.

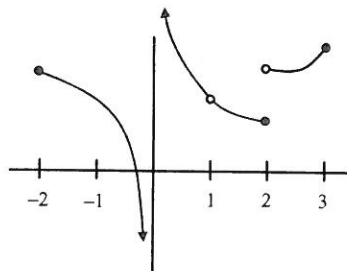
a) Find $p(0)$ and give a possible interpretation of this number.

b) Find $\lim_{t \rightarrow \infty} p(t)$ and give a possible interpretation of this number.

8. Let $h(x) = \frac{(x-1)(x+3)}{(x+3)(x-2)}$. Identify all values of c where the $\lim_{x \rightarrow c} h(x)$ EXISTS.

9. The function shown to the right is defined on $[-2, 3]$.

For what values of c , does $\lim_{x \rightarrow c} f(x)$ exist?



For questions 10 - 13, use the function shown to the right. The domain is $[-1, 6]$.

10. Evaluate each of the following limits. If they do not exist, explain why.

a) $\lim_{x \rightarrow 1} f(x)$

b) $\lim_{x \rightarrow 3^-} f(x)$

c) $\lim_{x \rightarrow 3^+} f(x)$

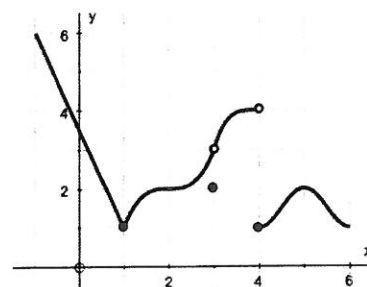
d) $\lim_{x \rightarrow 3} f(x)$

e) $\lim_{x \rightarrow 4^+} f(x)$

f) $\lim_{x \rightarrow 4^-} f(x)$

g) $\lim_{x \rightarrow 4} f(x)$

h) $\lim_{x \rightarrow 0} f(x)$



11. For what values of x is the function continuous?

12. For what values of x is the function not continuous?

13. Are any of the values you used to answer question 11 removable? If so, describe how you would make the function continuous at that point?

14. Let $f(x) = \begin{cases} 2 & \text{if } x \leq -1 \\ -x+1 & \text{if } -1 < x < 0 \\ 2 & \text{if } x = 0 \\ -x+1 & \text{if } 0 < x < 1 \\ 2 & \text{if } x \geq 1 \end{cases}$.

a) Find the right-hand and left-hand limits of f at $x = -1$, 0 , and 1 .

b) Does f have a limit as x approaches -1 ? 0 ? 1 ? If so, what is it? If not, why not?

c) Is f continuous at $x = -1$? 0 ? 1 ? Explain.

15.

Find the value of the parameter that would make each function continuous.

Justify your response using the definition of continuity.

a)

$$j(x) = \begin{cases} ax^2 & ; x < 1 \\ 4x - 2 & ; x \geq 1 \end{cases}$$

b)

$$k(x) = \begin{cases} \frac{\sin 3x}{x} & x \neq 0 \\ a & x = 0 \end{cases}$$

c)

$$f(x) = \begin{cases} \frac{x^2 - 2x}{x} & \text{if } x \neq 0 \\ b & \text{if } x = 0 \end{cases}$$

d)

$$f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x - 3} & \text{if } x < 3 \\ kx - 2 & \text{if } x \geq 3 \end{cases}$$

16. Let $k(x) = \frac{\sqrt{x} - 3}{x - 9}$. Write an extension to the function so that it is continuous at $x = 9$.

17.

[No Calculator] Find the average rate of change of $f(x) = 3 - \sin x$ over the interval $[0, \frac{\pi}{2}]$.

18. Find the rate of change of the Surface Area $A = 6s^2$ of a cube with respect to the edge length s when $s = 3$.

19. Let $g(x) = \sqrt{x}$. Find the instantaneous slope at $x = 4$.

20. Let $y = x^3 - 4x$.

(a) Find the instantaneous slope for any value of $x = a$.

(b) Use your answer in part a to find the slope at $x = -1$.

(c) Find the equation of the tangent line when $x = -1$.

(d) Find the equation of the normal line when $x = -1$.

21. Let $h(x) = x^2 + 3x - 1$.

(a) Find the slope of the curve at $x = 1$.

(b) Find the equation of the tangent line at the point when $x = 1$.

(c) Find the equation of the normal line at the point when $x = 1$.

(d) At what point(s), if any, are the tangents to the graph of $h(x)$ horizontal? ... [Use Calculus!]

22. Let $g(x) = \frac{x^2 + 5x + 6}{x^2 + 3x + 2}$.

(a) Find the domain of $g(x)$.

(b) Find the $\lim_{x \rightarrow c} g(x)$ for all values of c where $g(x)$ is not defined.

(c) Find all asymptotes and justify your response.

d) Write an extension to the function so that $g(x)$ is continuous for all $x < -1$.

