Name: key

Class:

Free Response No Calculator: Practice

1. Consider the function $f(x) = \frac{x^2-x-6}{x^2+x-2}$ $\quad \frac{(x-3)(x+2)}{(x-1)(x+2)} = \frac{x-3}{x-1}$

   (a) Give the zeros of $f(x)$.

   (b) Give the equation(s) of any vertical asymptotes. Justify using limits.

   (c) Give the equation(s) of any horizontal asymptotes. Justify using limits.

   (d) List all points where $f(x)$ is discontinuous. Justify your answer using the definition of continuity.

a. $f(x) = 0$

$x - 3 = 0$

$x = 3$ (1)

b. $f(x) = \frac{x-3}{x-1}$

$x - 1 = 0$

$x = 1$ (1)

c. $\lim_{x\to\infty} \frac{x-3}{x-1}$

(1) needs both

$\lim_{x\to\infty} \frac{x}{x} = 1$

$\lim_{x\to-\infty} \frac{x}{x} = 1$

$y = 1$ (1)

(1) needs both

$\lim_{x\to1^-} f(x) = +\infty$

$\lim_{x\to1^+} f(x) = -\infty$

d. $\lim_{x\to-2} \frac{x-3}{x-1} = \frac{-5}{-3} = \frac{5}{3}$

(1) $f(-2)$ DNE

(1) $\lim_{x\to-2} f(x) \neq f(-2)$

(1) $\lim_{x\to1^-} f(x) \neq \lim_{x\to1^+} f(x)$ or $f(1)$ DNE

(1) $\lim_{x\to1} f(x) \neq f(1)$

(1) Therefore $f(x)$ is discontinuous at $x = -2, 1$

Free Response No Calculator: Practice

$$f(x) = \begin{cases} x+1, & x \leq 1 \\ 3x-1, & x > 1 \end{cases}$$

2. Is $f(x)$ continuous at $x = 1$? Justify your answer

(1) $\lim_{x \to 1^-} x+1 = 1+1 = 2$

(1) $\lim_{x \to 1^+} 3x-1 = 3-1 = 2$

(1) $f(1) = 1+1 = 2$

(1) $\lim_{x \to 1} f(x) = f(1)$

(1) $\therefore f(x)$ is cont. at $x=1$

OR

$\lim_{x \to 1^-} g(x) = 3$ (1)

$\lim_{x \to 1^+} g(x) = 5$ (1)

$\lim_{x \to 1} g(x) = DNE$ (1)

$\lim_{x \to 1} g(x) \neq g(1)$ (1)

$\therefore g(x)$ is not cont. at $x=1$ (1)

Free Response Calculator: Practice

3. Consider the function $f(x) = 2x^3$.

   (a) What is the average rate of change of f on the interval $[0.75, 1.25]$?

   (b) Find an equation for the line tangent to f at $x = 1$.

a. $\dfrac{f(1.25) - f(0.75)}{1.25 - 0.75} = 6.125$ (1) (1)

b. $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

$\lim_{h \to 0} \dfrac{2(a+h)^3 - 2a^3}{h}$

$\lim_{h \to 0} \dfrac{2(a^3 + 3a^2h + 3ah^2 + h^3) - 2a^3}{h}$

$\lim_{h \to 0} \dfrac{2a^3 + 6a^2h + 6ah^2 + 2h^3 - 2a^3}{h}$

$\lim_{h \to 0} \dfrac{6a^2h + 6ah^2 + 2h^3}{h}$

$\lim_{h \to 0} \dfrac{h(6a^2 + 6ah + 2h^2)}{h}$

$\lim_{h \to 0} 6a^2 + 6ah + 2h^2 = 6a^2$

(1)

$(1, 2)$ (1)

$y - 2 = 6(x - 1)$ (1)

(2)

$a = 1 \quad m = 6$