AP Calculus
4.2 Worksheet

similar to what you have to show on free response

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. State the hypothesis of each of the following theorems:

a) IVT

continuous on (a,b)

b) EVT

continuous on [a,b)

c) MVT

differentiable on [a,b]

2. State the MVT two different ways …

a) … in words

average rate of change equal to actual rate of change

b) … algebraically

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

3. For each of the following, (a) state whether or not the function satisfies the hypotheses of the MVT on the given interval, and (b) if it does, find the value of c that the MVT guarantees.

a) $f(x)=-2x^2+14x-12$ on the interval [1, 6]

differentiable on that interval

$$f'(c)=\frac{f(6)-f(1)}{6-1}$$

$-4c+14=\frac{0-0}{5}$

$-4c+14=0$

$-4c=-14$

$c=7/2$

b) $h(x)=x^{1/3}$ on [–1, 1]

$h'(x)=1/3\,x^{-2/3}$

$h'(x)=\frac{1}{3x^{2/3}}$

$h'(x)$ DNE at $x=0$
can't use MVT

4. When a trucker came to his second toll booth in a 169-mile stretch of road, he handed in a ticket stub that was stamped 2 hours earlier. The trucker was cited for speeding. Why?

5. Suppose $f(x)$ is a differentiable function on the interval [–7, 1] such that $f(-7)=4$ and $f(1)=-1$.

a) *Explain* why f must have at least one value in the interval (–7, 1), where the function equals 2.

Since f(x) is differentiable, then f(x) is continuous. $f(-1)<2<f(-7)$. Therefore by the IVT $f(x)=2$

b) *Explain* why there must be at least one point in the interval (–7, 1) whose derivative is $-\frac{5}{8}$.

$$\frac{f(b)-f(a)}{b-a}=\frac{4-(-1)}{-7-1}=\frac{5}{-8}$$

since f(x) is differentiable, the MVT guarantees a point c in the interval where $f'(c)=\frac{-5}{8}$

Similar to what you have to show on free response

Worksheet 4.5.
L'Hôpital's Rule

1. Evaluate the limit.

a. $\lim_{x\to 0} \frac{\sin(4x)}{\sin(3x)}$ $= \frac{0}{0}$

$\lim_{x\to 0} \sin(4x) = \sin(0) = 0$

$\lim_{x\to 0} \sin(3x) = \sin(0) = 0$

By L'Hopital's

$\lim_{x\to 0} \frac{\sin(4x)}{\sin(3x)} = \boxed{\lim_{x\to 0} \frac{4\cos(4x)}{3\cos(3x)}} = \frac{4\cos(0)}{3\cos(0)}$

$= \boxed{4/3}$

b. $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$ $= \frac{\infty}{\infty}$

$\lim_{x\to\infty} \ln x = \infty$

$\lim_{x\to\infty} \sqrt{x} = \infty$

By L'Hopital's

$\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} = \lim_{x\to\infty} \frac{1/x}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{x^{-1}}{\frac{1}{2}x^{-1/2}} = 2\lim_{x\to\infty} x^{-1/2} = \lim_{x\to\infty} \frac{2}{x^{1/2}}$

$= \boxed{0}$

c. $\lim_{x\to\infty} x^2 e^{-x/2}$

$= \lim_{x\to\infty} \frac{x^2}{e^{x/2}} = \frac{\infty}{\infty}$

d. $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

~~e.~~ $\lim_{x\to 1} (1+\ln x)^{1/(x-1)}$

c. $\lim_{x\to\infty} \frac{x^2}{e^{x/2}}$

$\lim_{x\to\infty} x^2 = \infty$

$\lim_{x\to\infty} e^{x/2} = \infty$

By L'Hopital's

$= \lim_{x\to\infty} \frac{2x}{\frac{1}{2}e^{x/2}} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} 2x = \infty$

$\lim_{x\to\infty} \frac{e^{x/2}}{2} = \infty$

By L'Hopital's

$= \lim_{x\to\infty} \frac{2}{\frac{1}{4}e^{x/2}} = \frac{2}{\infty} = \boxed{0}$

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4.2 Worksheet

key continued
see back

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a) … in words

b) … algebraically

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a) $f(x) = -2x^2 + 14x - 12$ on the interval [1, 6]

b) $h(x) = x^{1/3}$ on [–1, 1]

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key continued

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b. $\lim\limits_{x\to\infty} \dfrac{\ln x}{\sqrt{x}}$

c. $\lim\limits_{x\to\infty} x^2 e^{-x/2}$

d. $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right) = \lim\limits_{x\to 0} \dfrac{\sin x}{x\sin x} - \dfrac{x}{x\sin x} = \dfrac{\sin x - x}{x \sin x}$

$\lim\limits_{x\to 0} \dfrac{\sin x - x}{x\sin x} = 0/0$

e. $\lim\limits_{x\to 1}(1+\ln x)^{1/(x-1)}$

$\lim\limits_{x\to 0} \sin x - x = \sin 0 - 0 = 0$

$\lim\limits_{x\to 0} x\sin x = 0\cdot\sin 0 = 0$

By LHopital's

$= \lim\limits_{x\to 0} \dfrac{\cos x - 1}{x\cos x + \sin x} = 0/0$

$\lim\limits_{x\to 0} \cos x - 1 = \cos 0 - 1 = 1-1=0$

$\lim\limits_{x\to 0} x\cos x + \sin x = 0 + 0 = 0$

By LHopitals

$= \lim\limits_{x\to 0} \dfrac{-\sin x}{-x\sin x + 2\cos x}$

$= \dfrac{-\sin(0)}{0+1} = \dfrac{0}{1} = \boxed{0}$