

90 Minutes—No Calculator

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

1.
$$\int_{1}^{2} x^{-3} dx = \frac{x^{-2}}{2} \Big|_{0}^{2} = -\frac{1}{2} \Big|_{0}^{2} = -\frac{1}{8} + \frac{1}{2} \frac{4}{8} = \frac{3}{8}$$

- (A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$
- If $f(x) = (2x+1)^4$, then the 4th derivative of f(x) at x = 0 is $f'(x) = 4(2x+1)^3 \cdot 2 = 8(2x+1)^3$ $f''(x) = 24(2x+1)^3 \cdot 2 = 48(2x+1)^3$

- (A) 0 (B) 24 (C) 48 (D) 240 (E) 384 $f'''(x) = \frac{3}{4+x^2}, \text{ then } \frac{dy}{dx} = y = 3(4+x^2)^{-1}$ $den + use quotient value not needed (A) (C) <math>\frac{6x}{(4+x^2)^2}$ (C) $\frac{6x}{(4+x^2)^2}$ (D) $\frac{-3}{(4+x^2)^2}$ (E) $\frac{3}{2x}$

(A)
$$\frac{-6x}{(4+x^2)^2}$$
 (B) $\frac{3x}{(4+x^2)^2}$ (C) $\frac{6x}{(4+x^2)^2}$ (D) $\frac{-3}{(4+x^2)^2}$ (E) $\frac{3}{2x}$

- 4. If $\frac{dy}{dx} = \cos(2x)$, then $y = \frac{1}{2} \int \cos(2x) (2) dx$
- (A) $-\frac{1}{2}\cos(2x) + C$ (B) $-\frac{1}{2}\cos^2(2x) + C$ (C) $\frac{1}{2}\sin(2x) + C$
- (D) $\frac{1}{2}\sin^2(2x) + C$ (E) $-\frac{1}{2}\sin(2x) + C$
- $\lim_{n\to\infty} \frac{4n^2}{n^2 + 10,000n} \text{ is } \begin{cases} \text{Limb approaches coefficients} &= \frac{4}{1} = 4 \\ \text{local is each term by } n^2 + \text{Simplify} \end{cases}$

- nonexistent

6. If
$$f(x) = x$$
, then $f'(5) = f'(x) = 1$ (at all values)

- (A) 0

7. Which of the following is equal to ln 4?

- (A) $\ln 3 + \ln 1$

- (A) $\ln 3 + \ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_{1}^{4} e^{t} dt$ (D) $\int_{1}^{4} \ln x dx$ (E) $\ln (3 \cdot 1)$ \ln 8.

- (E)

9. If
$$\int_{-1}^{1} e^{-x^2} dx = k$$
, then $\int_{-1}^{0} e^{-x^2} dx = \frac{\text{can do integration algebraically}}{|OP|}$ realize e^{-x^2} is symmetric to y exis, so $\Rightarrow \int e^{-x^2} dx$

- (B) -k (C) $-\frac{k}{2}$

10. If
$$y = 10^{(x^2-1)}$$
, then $\frac{dy}{dx} = 10^{(x^2-1)} (\ln 10)(\partial x)$

(A) $(\ln 10)10^{(x^2-1)}$

- (B) $(2x)10^{(x^2-1)}$
- (C) $(x^2-1)10^{(x^2-2)}$

 $2x(\ln 10)10^{\left(x^2-1\right)}$

(E) $x^2 (\ln 10) 10^{(x^2-1)}$

11. The position of a particle moving along a straight line at any time
$$t$$
 is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$? $\alpha(t) = \alpha$

- (A) 0
- (C)
- (D)
- 12 (E)

12. If
$$f(g(x)) = \ln(x^2 + 4)$$
, $f(x) = \ln(x^2)$ and $g(x) > 0$ for all real x, then $g(x) = 1$

- (A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$
- (C) $\sqrt{x^2+4}$
- (E)

13. If
$$x^2 + xy + y^3 = 0$$
, then, in terms of x and y, $\frac{dy}{dx} =$

$$+ \times dy + y + 3y^{2} dy = 0$$

 $dy \left[\times + 3y^{2} \right] = -y - 2x$

$$(A) - \frac{2x + y}{x + 3y^2}$$

$$(B) \quad -\frac{x+3y^2}{2x+y}$$

(C)
$$\frac{-2x}{1+3y^2}$$

$$(D) \quad \frac{-2x}{x+3y^2}$$

$$\frac{2x+y}{x+3y^2}$$
 (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{1}{2}}$ meters per second. How many meters did the particle travel from t = 0 to t = 4?

| Solve | Indicate | Indicate

- (A)

15. The domain of the function defined by $f(x) = \ln(x^2 - 4)$ is the set of all real numbers x such that

- (B) $|x| \le 2$ (C) |x| > 2 (D) $|x| \ge 2$
- (E) x is a real number

16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at x =

- (D) 2
- $f'(x) = 3x^2 6x$ $= 3 \times (x - 2)$

retimez

 $\int_0^1 x e^{-x} dx =$

- (E) 2e - 1

18. If $y = \cos^2 x - \sin^2 x$, then y' =double < formula (= cos 2x)

 $y = \cos(2x)$ $y' = -\sin(ax) \cdot 2$

- (C) $-2\sin(2x)$
- (D) $-2(\cos x + \sin x)$
- (E) $2(\cos x \sin x)$

19. If $f(x_1) + f(x_2) = f(x_1 + x_2)$ for all real numbers x_1 and x_2 , which of the following could precalc. define f?

$$f(x) = x + 1 \qquad \text{(B)} \quad f(x) = 2x \qquad \text{(C)} \quad f(x) = \frac{1}{x} \qquad \text{(D)} \quad f(x) = e^x$$

$$f(x_1) + f(x_2)$$

f(x) + f(x2) X, +1 + X2+1

= XI+X2+2 <

AP Calculus Multiple-Choice Question Collection

Copyright © 2005 by College Board. All rights reserved. Available at apcentral collegeboard.com. $f(x_1 + x_2)$

$$=\frac{f(x_1+x_2)}{2(x_1+x_2)}=2x_1+3x_2$$

20. If
$$y = \arctan(\cos x)$$
, then $\frac{dy}{dx} =$

$$(A) \frac{-\sin x}{1+\cos^2 x}$$

(B)
$$-(\arccos(\cos x))^2 \sin x$$

(C)
$$\left(\operatorname{arcsec}\left(\cos x\right)\right)^2$$

(D)
$$\frac{1}{\left(\arccos x\right)^2 + 1}$$

$$(E) \quad \frac{1}{1+\cos^2 x}$$

21. If the domain of the function f given by $f(x) = \frac{1}{1-x^2}$ is $\{x:|x|>1\}$, what is the range of f?

$$(A) \quad \left\{ x : -\infty < x < -1 \right\}$$

(B)
$$\{x : -\infty < x < 0\}$$

(B)
$$\{x:-\infty < x < 0\}$$

$$x : -\infty < x < 0\}$$

$$x^{2} \mid y \circ (C) \quad \{x:-\infty < x < 1\}$$

$$(D) \quad \left\{ x : -1 < x < \infty \right\}$$

(E)
$$\{x: 0 < x < \infty\}$$

22.
$$\int_{1}^{2} \frac{x^{2} - 1}{x + 1} dx = \frac{(x + 1)(x - 1)}{x + 1}$$

(D)
$$\{x:-1 < x < \infty\}$$
 (E) $\{x:0 < x < \infty\}$ $\xrightarrow{\times = 1, -1}$ $\xrightarrow{\text{doment indicates}}$ $\xrightarrow{\text{doment indicates}}$ $\xrightarrow{\text{doment indicates}}$ $\xrightarrow{\text{doment indicates}}$ $\xrightarrow{\text{doment indicates}}$ $\xrightarrow{\text{doment indicates}}$ $= \frac{x^2 - x}{a} - x$ $= \frac{x^2 - x}$

$$(A) \frac{1}{2}$$

(D)
$$\frac{5}{2}$$

$$23. \quad \frac{d}{dx} \left(\frac{1}{x^3} - \frac{1}{x} + \frac{1}{x} \right)$$

23.
$$\frac{d}{dx} \left(\frac{1}{x^3} - \frac{1}{x} + x^2 \right)$$
 at $x = -1$ is $\begin{cases} -1 & x = -1 \\ y & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ y & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -1 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1 & x = -3 \\ -1 & -3 = -3 \end{cases} + \begin{cases} -1$

$$3x^{-4} + x^{-2} + 2$$

$$0 \times -1 \Rightarrow -3(\frac{1}{6})$$

(E)
$$6 -3 + 1 - 2$$

24. If
$$\int_{-2}^{2} (x^7 + k) dx = 16$$
, then $k = 16 = \frac{x^8}{8} + k \times \frac{2}{3} = \frac{16 = (2^8 + 2^8) - (2^8 + 2^8)}{2^3 + 2^8} + 2^8 = \frac{2}{3}$

$$16 = \left(\frac{2}{\lambda^3} + 2K\right) - \left(\frac{-2}{\lambda^3} + -2K\right)$$

$$16 = 2K + 2K$$

$$(A) -12$$

$$(C)$$
 0

25. If
$$f(x) = e^x$$
, which of the following is equal to $f'(e)$?

(A)
$$\lim_{h\to 0} \frac{e^{x+h}}{h}$$

(B)
$$\lim_{h \to 0} \frac{e^{x+h} - e^e}{h}$$

(C)
$$\lim_{h \to 0} \frac{e^{e+h} - e^{r}}{h}$$

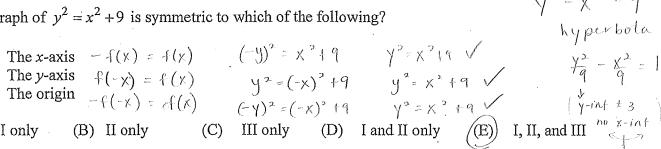
(D)
$$\lim_{h \to 0} \frac{e^{x+h} - 1}{h}$$

$$\underbrace{\text{(E)}}_{h\to 0} \frac{\lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$$

$$f'(x) = \lim_{x \to \infty} \frac{f(x+h) - f(x)}{e^{x+h} - e^{x}} = \lim_{x \to \infty} \frac{e^{x+h} - e^{x}}{e^{x+h} - e^{x}}$$
 at $x=e = \lim_{x \to \infty} \frac{e^{x+h} - e^{x}}{e^{x+h} - e^{x}}$



26. The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?



I. The x-axis
$$-f(x) = f(x)$$

II. The y-axis
$$f(-x) =$$

$$y^2 = (-x)^2 +$$

III. The origin
$$-f(-x) = f(x)$$

$$(-Y)^2 = (-X)^2 + 9$$

(B)
$$\frac{3}{2}$$

$$\bigcirc \frac{5}{2}$$

28. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle = aug. rate of x(t) = x(3) - x(0)for $0 \le t \le 3$ is

$$(D)$$
 -10

29. Which of the following functions are continuous for all real numbers x?

$$\frac{-5(\frac{3}{9})}{3} + 6$$

I.
$$y = x^{\frac{2}{3}}$$
 $\sqrt[3]{x}$ or even for zero, negatives, etc.

III. $y = e^x$

III. $y = \tan x$



- (B) I only
- II only



I and II

(E) I and III

30.
$$\int \tan(2x) dx = \frac{1}{\text{quotient}} \int \int \tan x \cdot \ln |\sec x| + C$$

(A)
$$-2\ln|\cos(2x)|+C$$

(B)
$$-\frac{1}{2}\ln\left|\cos(2x)\right| + C$$
 (C) $\frac{1}{2}\ln\left|\cos(2x)\right| + C$

(C)
$$\frac{1}{2} \ln |\cos(2x)| + C$$

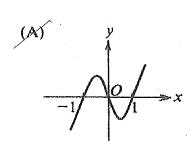
(D)
$$2\ln|\cos(2x)| + C$$

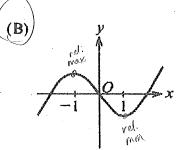
(E)
$$\frac{1}{2}\sec(2x)\tan(2x) + C$$

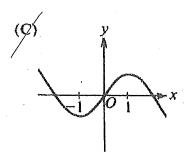
$$\int \frac{\sin 3x}{\cos 3x} dx = \iint \frac{1}{\cos 3x} (3) \sin 3x dx = -\frac{1}{2} \ln \left| \cos(3x) \right| + C$$

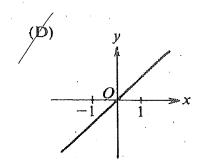
- 31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height $\frac{\partial V}{\partial t} = .5 \text{ cm/s}$ both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per $\frac{\partial V}{\partial t} = .5 \text{ cm/s}$ second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?
 - (A) $\frac{1}{2}\pi$
- (B) 10π
- (C) 24 T
- (D) 54 π
- (E) $108\pi \frac{\sqrt{= \frac{11}{3}} r^2 h}{dt = \frac{11}{3} (r^2 dt)}$
- $32.1 \int_{0}^{\frac{\pi}{3}} \sin(3x) dx = -\frac{1}{3} \cos(3x) \Big|_{0}^{\frac{\pi}{3}} = -\frac{1}{3} \cos(3x) \Big|_{0}^{\frac{\pi}{3}} = -\frac{1}{3} \cos(3x) \Big|_{0}^{\frac{\pi}{3}} = \frac{1}{3} \cos(3x) \Big|_{0}^{\frac{\pi}{3}$
- u sub wed) (A) -2
- (B) $-\frac{2}{3}$
- (C) 0
- $(D) \frac{2}{3}$
- (E) 2
- 72 17 3.

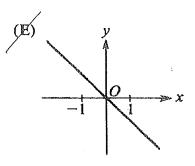
- y = f'(x) y = f'(x)
- 33. The graph of the <u>derivative</u> of f is shown in the figure above. Which of the following could be the graph of f?

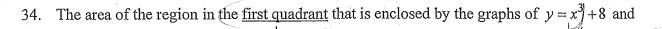










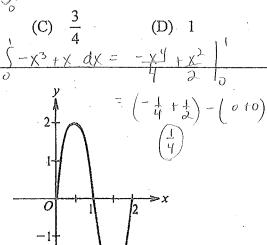


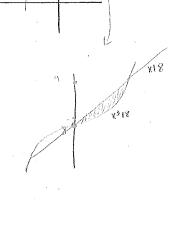


y = x + 8 is



(E)
$$\frac{65}{4}$$





. pre-fre calc!

The figure above shows the graph of a sine function for one complete period). Which of the following is an equation for the graph? 5 = 2 = ap

(A)
$$y = 2\sin\left(\frac{\pi}{2}x\right)$$

(B)
$$y = \sin(\pi x)$$

(C)
$$y = 2\sin(2x)$$
 $b = 1$

(D)
$$y = 2\sin(\pi x)$$

(E)
$$y = \sin(2x)$$

36. If f is a continuous function defined for all real numbers x and if the maximum value of f(x) is 5 and the minimum value of f(x) is -7, then which of the following must be true?

$$\max f(?) = 5$$

I. The maximum value of
$$f(x)$$
 is 5.

min
$$f(3) = -1$$

II. The maximum value of
$$|f(x)|$$
 is 7.

$$|f(x)| = |-7|$$

The minimum value of
$$f(|x|)$$
 is 0. b/c of IVT there exists at least one

III.

(E)
$$\alpha$$

famous ex of squeeze thm > lim six () = lim sinx = ()

AP Calculus Multiple-Choice Question Collection

I and II only

38. Let f and g have continuous first and second derivatives everywhere. If $f(x) \le g(x)$ for all real x, which of the following must be true?

I. $f'(x) \le g'(x)$ for all real x slopes of f(x) aren't dependent on y values

H. $f''(x) \le g''(x)$ for all real x concavity not dependent on y's $\iiint_0^1 f(x) dx \le \int_0^1 g(x) dx$

(C) III only

(A) None

- 39. If $f(x) = \frac{\ln x}{x}$, for all x > 0, which of the following is true? $f'(x) = \frac{1 + \ln x}{x}$

(E) I, II, and III

f(x)

(B) I only

- (B) f/is increasing for all x greater than 1.
- (C) f is decreasing for all x between 0 and 1.
- (D) f is decreasing for all x between 1 and e.
- (E)) f is decreasing for all x greater than e.

- Let f be a continuous function on the closed interval [0,2]. If $2 \le f(x) \le 4$, then the greatest



If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true? be a tent a little less

(A) f'(a) exists. f(a) doesn't have to = L (i.e. if graph is undefined at a pt., limit con exist the (B) f(x) is continuous at x = a. doesn't need to be contin. for timely to use is

- (C) f(x) is defined at x = a. (A)
- (D) f(a) = LSec (A)
- None of the above

$$42. \quad \frac{d}{dx} \int_{2}^{x} \sqrt{1+t^2} dt =$$

$$(A) \quad \frac{x}{\sqrt{1+x^2}}$$

(B)
$$\sqrt{1+x^2} - 5$$

$$(C) \sqrt{1+x^2}$$

(D)
$$\frac{x}{\sqrt{1+x^2}} - \frac{1}{\sqrt{5}}$$

(E)
$$\frac{1}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{5}}$$

An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) y = -6x - 6(B) y = -3x + 1(C) y = 2x + 10(D) y = 3x - 1(E) y = 4x + 1(D) y = 3x - 1(E) y = 4x + 1(E) y = 4x + 1(D) y = 3x - 1(E) y = 4x + 1(E) y = 3x - 3 + 6(E) y = 3x

(A)
$$y = -6x - 6$$

$$(B)$$
 $y = -3x + 1$

(C)
$$v = 2x + 10$$

(E)
$$v = 4x + 1$$
 we correct slope

$$y - y_1 = M(x - x_1)$$

 $y - b = -3(x + 1)$

$$(E) \quad y = 4x + 1 \qquad \qquad \omega$$

$$\frac{y = -3x - 3 + 6}{(6 + (3x + 3) + 6)}$$

$$\frac{(7 - 3x + 3)}{(6 + (3x + 3) + 6)}$$

$$\frac{(7 - 3x + 3)}{(5 + (3x + 3) + 6)}$$

44. The average value of $f(x) = x^2 \sqrt{x^3 + 1}$ on the closed interval [0,2] is $\frac{1}{b-a} \int_{a}^{b} f(x) = \frac{1}{a-0} \int_{a}^{b} (x^3 + 1)^{\frac{1}{2}} \int_{a}^{2} x^2 dx = \frac{1}{b} \int_{a}^{2} (x^3 + 1)^{\frac{1}{2}} \int_{a}^{$

$$(\widehat{A})$$
 $\frac{26}{2}$

$$(B) \frac{13}{3}$$

(C)
$$\frac{26}{3}$$

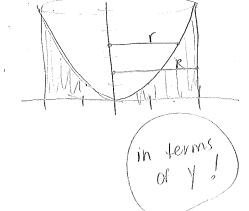
The region enclosed by the graph of $y = x^2$, the line x = 2, and the x-axis is revolved about the y-axis. The volume of the solid generated is



(B)
$$\frac{32}{5}$$

(C)
$$\frac{16}{3}\pi$$
 (D) 4π

(E)
$$\frac{8}{3}\pi$$



$$V = T \int_{0}^{4} a^{2} - Jy^{2} dy = T \int_{0}^{4} 4 - y dy$$

$$T \left(\frac{4y - y^{2}}{5} \right) \Big|_{0}^{4}$$

$$= 2$$

$$X = Jy$$

$$8TT$$

6 . ()