area
volume
net
change test
review

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

   (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

   (b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

   (c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

   (d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

---

a. $W'(12) = \dfrac{W(15) - W(9)}{15 - 9} = 1.017$ or 1.016

The water temperature is increasing at a rate of 1.017 °F per minute at t=12 minutes

c. $\frac{1}{20}\cdot\left((4\cdot W(0)) + (5\cdot W(4)) + (6\cdot W(9)) + (5\cdot W(15))\right)$

$\frac{1}{20}\int_0^{20} W'(t) = \frac{1}{20}\cdot 1215.8 = 60.79$

The approximation is an underestimate because W is increasing.

GO ON TO THE NEXT PAGE.

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

   (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

   (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

   (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

   (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

Amount: $A(t) = 500 + \int_0^t G(s) - 100\, ds$

$A'(t) = G(t) - 100 = 0$

$t = 4.92348$ (calculator)

t	$A(t)$
0	500
4.923	635.376
8	525.551

The maximum amount of unprocessed gravel is 635.376 tons

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	−120	−150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

(c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

(d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

---

a. average: $\dfrac{V_A(8) - V_A(2)}{8-2} = \dfrac{-120-100}{6} = \dfrac{-110}{3}$ m/min²

b. V_A is differentiable, so V_A is continuous

$-120 < -100 < 40$

$V_A(5) < -100 < V_A(8)$ must write

There fore by the IVT there is a time t, $5<t<8$, such that $V_A(t) = -100$

2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

---

a. $v(4) = 2.978716$

$v'(4) = -1.164$

The particle is slowing down because velocity and accleration have oppisite signs.

END OF PART A OF SECTION II

b. $v(t) = 0$ when $t = 2.707468$

$v(t)$ changes from positive to negative at $t = 2.707$.

~~Therefore it changes direction~~

Therefore the particle changes direction.

c. $x(0) = x(4) + \int_4^0 v(t)\,dt$

$x(0) = 2 + \int_4^0 v(t)\,dt = 2 - 5.815$

$= -3.815$

d. $\int_0^3 |v(t)|\,dt$

$= \boxed{5.301}$

7. [Calculator] For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

a) Show that the number of mosquitoes is increasing at time $t = 6$.

$R(6) = 4.438$ since $R(6) > 0$ the number of mosquitoes is increasing

b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

$R'(6) = -1.913$ since $R'(6) < 0$ the number of mosquitoes is increasing at a decreasing rate ($R(6) > 0$) ($R'(6) < 0$)

c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

Total = starting + change

$\boxed{\text{Total} = 1000 + \int_0^{31} R(t)\,dt = 964.335}$

964 total mosquitoes

d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

$T(x) = 1000 + \int_0^{x} R(t)\,dt$

$T'(x) = R(x) = 0$ when $x = 0, 7.85398, 23.561945$

x	$T(x)$
0	1000
7.85398	1039
23.561945	842
31	964

The maximum number of mosquitoes is 1039 after 7 or 8 days

8. [No Calculator] Suppose $\int_1^2 f(x)\,dx = 3$, $\int_1^5 f(x)\,dx = -13$, and $\int_1^5 g(x)\,dx = 7$. Find each of the following:

a) $\int_3^3 g(x)\,dx$

b) $\int_5^1 f(x)\,dx$

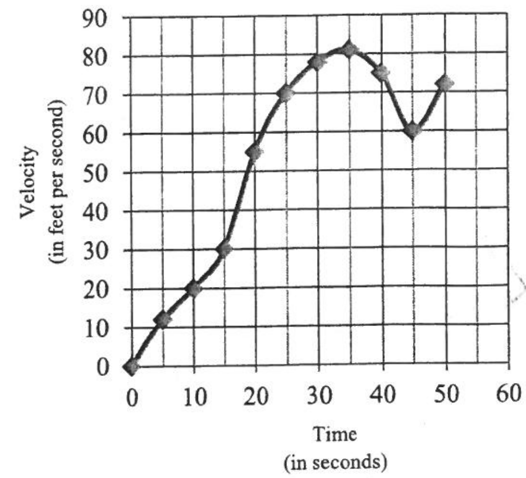
c) $\int_1^5 [g(x) - f(x)]\,dx$

d) $\int_2^5 f(x)$

e) $\int_1^5 [3f(x) - g(x)]\,dx$

f) $\int_1^5 \frac{g(x)}{4}\,dx$

Time (in seconds)	$v(t)$ (in ft/sec)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72



14. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

$a(t) = v'(t)$ when $v(t)$ is increasing, then $a(t) > 0$

$a(t) > 0$ on $(0, 35)$ and $(45, 50)$

b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

$$\frac{v(50) - v(0)}{50 - 0} = \frac{72 - 0}{50} = 14.4 \text{ ft/sec}^2$$

c) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

$$\int_0^{50} v(t)\,dt = 10(12) + 10(30) + 10(70) + 10(81) + 10(60)$$
$$= 120 + 300 + 700 + 810 + 600$$
$$= 2{,}530$$

~~The car tra~~

The car traveled 2,530 feet from $t = 0$ to $t = 50$.

Application of Integrals Review

In Exercises 1–8, the function $v(t)$ is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to do each of the following:

(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval. If $s(0) = 3$, what is the particle's final position?

(c) Find the total distance traveled by the particle.

1. $v(t) = 5\cos t, \quad 0 \le t \le 2\pi$
2. $v(t) = 6\sin 3t, \quad 0 \le t \le \pi/2$
3. $v(t) = 49 - 9.8t, \quad 0 \le t \le 10$
4. $v(t) = 6t^2 - 18t + 12, \quad 0 \le t \le 2$
5. $v(t) = 5\sin^2 t \cos t, \quad 0 \le t \le 2\pi$
6. $v(t) = \sqrt{4 - t}, \quad 0 \le t \le 4$
7. $v(t) = e^{\sin t}\cos t, \quad 0 \le t \le 2\pi$
8. $v(t) = \dfrac{t}{1 + t^2}, \quad 0 \le t \le 3$

→ see other document

In Exercises 6–19, find the area of the region enclosed by the lines and curves. You may use a graphing calculator to graph the functions.

9. $x = 2y^2, \quad x = 0, \quad y = 3$
10. $4x = y^2 - 4, \quad 4x = y + 16$
11. $y = \sin x, \quad y = x, \quad x = \pi/4$
12. $y = 2\sin x, \quad y = \sin 2x, \quad 0 \le x \le \pi$
13. $y = \cos x, \quad y = 4 - x^2$

9. $\int_0^3 2y^2\,dy = 18$

10. $\int_{-4}^{5} \left(\frac{y}{4} + 4\right) - \left(\frac{y^2}{4} - 1\right) dy = 30.375$

11. $\int_0^{\pi/4} x - \sin x = 0.0155$

12. $\int_0^{\pi} 2\sin x - \sin 2x\,dx = 4$

13. $\int_{-2.1281}^{2.1281} 4 - x^2 - \cos x\,dx = 8.9023$

a. $\pi\int_0^4 (2\sqrt{x})^2 - x^2\,dx = \frac{32\pi}{3}$ b. $\pi\int_0^4 y^2 - \left(\frac{y^2}{4}\right)^2 dy = \frac{128\pi}{15}$

$y = 2\sqrt{x}$ or $x = y^2/4$

21. Find the volume of the solid generated by revolving the region enclosed by the parabola $y^2 = 4x$ and the line $y = x$ about

(a) the x-axis. **(b)** the y-axis.

~~**(c)** the line~~ **(d)** the line $y = 4$.

(d.) $\pi\int_0^4 (4-x)^2 - (4-2\sqrt{x})^2\,dx = \frac{32\pi}{3}$

24. The base of a solid is the region enclosed between the graphs of $y = \sin x$ and $y = -\sin x$ from $x = 0$ to $x = \pi$. Each cross section perpendicular to the x-axis is a semicircle with diameter connecting the two graphs. Find the volume of the solid.

$d = 2\sin x$ $r = \sin x$

24. $\int_0^{\pi} \frac{\pi}{2}(\sin x)^2$

$= 2.4674$

You may use a graphing calculator to solve the following problems.

53. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of $y = 2 + \sin x$ and $y = \sec x$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares.

53. intersect at $x = 1.22378$

a. $\int_0^{1.22378} 2 + \sin x - \sec x\,dx$
$= 1.366$

b. $\int_0^{1.22378} \pi(2+\sin x)^2 - (\sec x)^2\,dx$
$= 16.404$

c. $\int_0^{1.22378} (2+\sin x - \sec x)^2\,dx$
$= 1.629$

54. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right), 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(b) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(c) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

You may use a graphing calculator to solve the following problems.

1. **Multiple Choice** The base of a solid is the region in the first quadrant bounded by the x-axis, the graph of $y = \sin^{-1} x$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume?

(A) 0.117 (B) 0.285 (C) 0.467 (D) 0.571 (E) 1.571

2. **Multiple Choice** Let R be the region in the first quadrant bounded by the graph of $y = 3x - x^2$ and the x-axis. A solid is generated when R is revolved about the vertical line $x = -1$. Set up, but do not evaluate, the definite integral that gives the volume of this solid.

(A) $\int_0^3 2\pi(x + 1)(3x - x^2)\, dx$

(B) $\int_{-1}^3 2\pi(x + 1)(3x - x^2)\, dx$

(C) $\int_0^3 2\pi(x)(3x - x^2)\, dx$

(D) $\int_0^3 2\pi(3x - x^2)^2\, dx$

(E) $\int_0^3 (3x - x^2)\, dx$

$\int_0^1 (\sin^{-1}(x))^2\, dx = 0.467$

3. Multiple Choice A developing country consumes oil at a rate given by $r(t) = 20e^{0.2t}$ million barrels per year, where t is time measured in years, for $0 \le t \le 10$. Which of the following expressions gives the amount of oil consumed by the country during the time interval $0 \le t \le 10$?

net change integrate the rate

(A) $r(10)$

(B) $r(10) - r(0)$

(C) $\int_0^{10} r'(t)\,dt$

(D) $\boxed{\int_0^{10} r(t)\,dt}$

(E) $10 \cdot r(10)$

4. Free Response Let R be the region bounded by the graphs of $y = \sqrt{x}$, $y = e^{-x}$, and the y-axis.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = -1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle whose diameter runs from the graph of $y = \sqrt{x}$ to the graph of $y = e^{-x}$. Find the volume of this solid.

intersect at ~~x = 1.22378~~

$x = 0.426303$

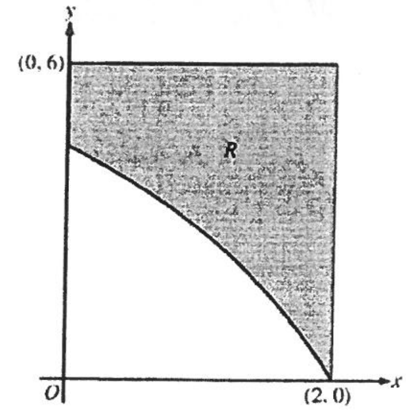
$$\text{a. } \boxed{\int_0^{0.426303} e^{-x} - \sqrt{x}\,dx = 0.162}$$

$$\text{b. } \boxed{\int_0^{0.426303} \pi(e^{-x}+1)^2 - (\sqrt{x}+1)^2\,dx = 1.631}$$

$$\text{c. } \boxed{\int_0^{0.426303} \frac{\pi}{2}\left(\frac{e^{-x} - \sqrt{x}}{2}\right)^2 dx = 0.035}$$

AP Calculus
Chapter 7 Review WS

1. [Calculator] In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.



a) Find the volume of the solid generated when R is revolved about the line $y = 8$.

$$\pi\int_0^2 (18 - 4\ln(3-x))^2 - (4)\,dx$$

$$= 168.179 \text{ or } 168.180$$

~~b) Find the volume of the solid generated when R is revolved about the line $x = 3$.~~

~~c) Find the volume of the solid generated when R is revolved about the line $x = 5$.~~

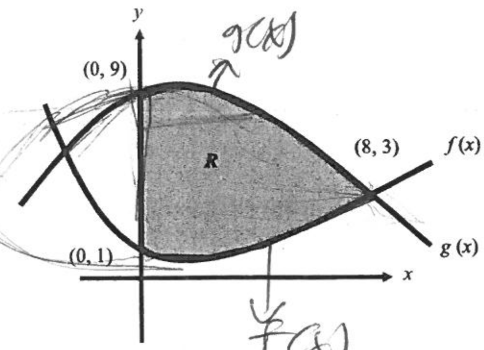
d) Find the volume of the solid generated when R is revolved about the line $y = -3$.

$$\pi\int_0^2 81 - (4\ln(3-x)+3)^2\,dx$$

$$= 302.953 \text{ or } 302.954$$

2. Complete the following questions from your textbook: (Mostly for review of 7.1)
Page 386 #8, 10
Pages 430 – 433 #1 – 5, and 54
Don't forget to review your 7.1 worksheets and the last problem on many of your other worksheets!

3. Each of the questions below refer to the region R as shown in the figure below. Simply set up the integral expression that would be used to answer each question.



a) Find the area of R.

$$\int_0^8 g(x) - f(x)\,dx$$

b) Find the volume of the solid whose base is R and where the cross sections perpendicular to the x-axis make the following shapes:

i) rectangles whose height equal 3 times its base.

$$\int_0^8 3(g(x) - f(x)) \cdot (g(x) - f(x))\,dx$$

ii) semicircles

$$\int_0^8 \frac{\pi}{2} \cdot \left(\frac{g(x) - f(x)}{2}\right)^2 dx$$

c) Find the volume of the solid formed by revolving the region R around each given axis.

i) x–axis $\pi \int_0^8 (g(x))^2 - (f(x))^2\,dx$

ii) y-axis $\pi \int_1^3 f(y)\,dy + \pi \int_3^9 g(y)\,dy$

~~iii) the line $x = 10$~~

iv) the line $y = 10$ $\pi \int_0^8 (10 - f(x))^2 - (10 - g(x))^2\,dx$

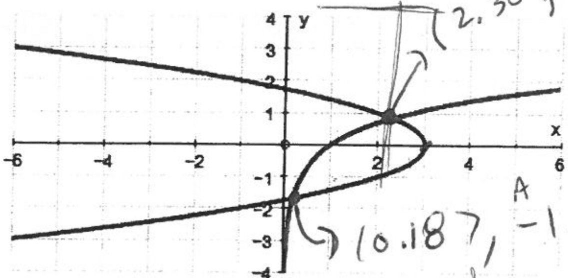
~~v) the line $x = -2$~~

vi) the line $y = -2$ $\pi \int_0^8 (g(x) + 2)^2 - (f(x) + 2)^2\,dx$

4. Each of the questions below refer to the region R, the region enclosed by the graphs of $y = \ln(x)$ and $x = 3 - y^2$. Set up an integral expression to answer each question, then use your calculator to evaluate.

a) Find the area of R.

$$\int_A^B (3-y^2) - (e^y)\, dy = 3.651 \text{ or } 3.652$$



B $(2.304, 0.834)$

A $(0.187, -1.677)$

intersects at

$y = 0.83449$ and $y = -1.67723$

$x = 0.18689$ → B

$x = 2.30363$ → A

b) Find the volume of the solid that uses R as a base and has cross sections perpendicular to the y-axis that are …

i) squares

$$\int_A^B \left[(3-y^2) - (e^y)\right]^2 = 6.358$$

ii) ~~equilateral triangles~~

c) Find the volume of the solid formed by revolving the region R around each given axis.

i) ~~the line $x = 5$~~

will be nicer on test

$$y^2 = 3 - x$$
$$y = \pm\sqrt{3-x}$$

ii) the line $y = 5$

$$\pi \int_{0.18689}^{2.30363} \underbrace{(5-(-\sqrt{3-x}))^2}_{\text{outer}} - \underbrace{(5-\ln(x))^2}_{\text{inner}}\, dx$$

iii) ~~the line $x = 3$~~

$$+\ \pi \int_{2.30363}^{3} (5-(-\sqrt{3-x}))^2 - (5-\sqrt{3-x})^2\, dx$$

$$= 123.759$$

will be nicer on test so dont worry

iv) the line $y = -3$

~~See back~~