Real Analysis of Phenomenological Velocity

by Parker Emmerson

$$\left(-\sqrt{c^2} < \frac{\sqrt{-a^2 c^2 l^2 + c^2 q^2 - 2 c^2 q s + c^2 s^2 + a^2 c^2 l^2 Sin[b]^2}}{\sqrt{-1. a^2 l^2 + q^2 - 2. q s + s^2 + a^2 l^2 Sin[b]^2}} < \sqrt{c^2} \&\&$$

$$q > s \&\& l > 0 \&\& a > \frac{q-s}{l} \&\& Sin[b] = \sqrt{\frac{a^2 l^2 - q^2 + 2 q s - s^2}{a^2 l^2}} \&\& c > 0$$

Abstract: Performing this real analysis of the Phenomenological Velocity shows that the computed solution to the phenomenological velocity, $v = \frac{\sqrt{-a^2 \ c^2 \ l^2 + c^2 \ q^2 - 2 \ c^2 \ q \ s + c^2 \ s^2 + a^2 \ c^2 \ l^2 \ Sin[b]^2}}{\sqrt{-1. \ a^2 \ l^2 + q^2 - 2. \ q \ s + s^2 + a^2 \ l^2 \ Sin[b]^2}}$ from solving the equality:

$$h = \frac{\sqrt{-q^2 + 2 \ q \ s - s^2 + l^2 \ \alpha^2}}{\alpha} = = \frac{\sqrt{-(q - s - l \ \alpha)}}{\alpha} \sqrt{(q - s + l \ \alpha)}}{\alpha} = \frac{\sqrt{(l \ \alpha + x \ \gamma - r \ \theta)} \sqrt{1 - \frac{v^2}{c^2}}}{\alpha} \sqrt{(l \ \alpha - x \ \gamma + r \ \theta) / \sqrt{1 - \frac{v^2}{c^2}}}$$

within the Lorentz Coefficient satisfies the conditions placed upon it by a full Real Analysis of the form found when not using a specified constant for c. Therefore, the computed phenomenological velocity is a true solution.

$$\log \left[\frac{\sqrt{-(q-s-l\alpha)} \sqrt{1-\frac{v^2}{c^2}}}{\alpha} \sqrt{(q-s+l\alpha) / \sqrt{1-\frac{v^2}{c^2}}} \right] = l \sin[\beta], \text{ Reals}$$

$$\left\{ \left\{ \beta \rightarrow -\text{ArcSin} \left[\frac{\sqrt{-c^2 + 1 \, \alpha}}{\sqrt{1 - \frac{v^2}{c^2}}} \right] \sqrt{\sqrt{c^2 - v^2}} \right. \\ \left. \left(-\text{q} + \text{s} + \text{l} \, \alpha \right) \right. \right. \\ \left. \left(1 > 0 \, \&\& \, \alpha \ge \frac{\mathsf{q} - \mathsf{s}}{1} \, \&\& \, \mathsf{c} < 0 \, \&\& \, -\sqrt{\mathsf{c}^2} \, \&\& \, \mathsf{c}_1 \in \mathbb{Z} \, \&\& \, \mathsf{s} < \mathsf{q} \right) \mid \mid \\ \left. \left(\text{s} > \mathsf{q} \, \&\& \, 1 > 0 \, \&\& \, \alpha \ge \frac{-\mathsf{q} + \mathsf{s}}{1} \, \&\& \, \mathsf{c} < 0 \, \&\& \, -\sqrt{\mathsf{c}^2} \, \&\& \, \mathsf{c}_1 \in \mathbb{Z} \, \&\& \, \mathsf{c}_1 \in \mathbb{Z} \right) \mid \mid \\ \left. \left(\text{s} > \mathsf{q} \, \&\& \, \mathsf{c} < 0 \, \&\& \, -\sqrt{\mathsf{c}^2} \, \&\& \, \mathsf{c}_1 \in \mathbb{Z} \, \&\& \, \mathsf{l} < 0 \, \&\& \, \alpha \le \frac{-\mathsf{q} + \mathsf{s}}{1} \right) \mid \mid \\ \left. \left(\text{c} < 0 \, \&\& \, -\sqrt{\mathsf{c}^2} \, < v < \sqrt{\mathsf{c}^2} \, \&\& \, \mathsf{c}_1 \in \mathbb{Z} \, \&\& \, \mathsf{l} < 0 \, \&\& \, \alpha \le \frac{\mathsf{q} - \mathsf{s}}{1} \right) \mid \mid \\ \left. \left(\text{c} < 0 \, \&\& \, -\sqrt{\mathsf{c}^2} \, < v < \sqrt{\mathsf{c}^2} \, \&\& \, \mathsf{c}_1 \in \mathbb{Z} \, \&\& \, \mathsf{l} < 0 \, \&\& \, \alpha \le \frac{\mathsf{q} - \mathsf{s}}{1} \right) \right. \right\}$$

$$\left\{\beta \to \sqrt{\frac{q-s+l\,\alpha}{\sqrt{1-\frac{v^2}{c^2}}}} \ \sqrt{\sqrt{c^2-v^2}} \ (-q+s+l\,\alpha) \right\},$$

$$if \ \left(l > 0 \&\&\,\alpha \ge \frac{q-s}{l} \&\&\,c < 0 \&\&-\sqrt{c^2} < v < \sqrt{c^2} \&\&\,c_1 \in \mathbb{Z} \&\&\,s < q \right) \mid \mid$$

$$\left(s > q \&\&\,l > 0 \&\&\,\alpha \ge \frac{-q+s}{l} \&\&\,c < 0 \&\&-\sqrt{c^2} < v < \sqrt{c^2} \&\&\,c_1 \in \mathbb{Z} \right) \mid \mid$$

$$\left(s > q \&\&\,c < 0 \&\&-\sqrt{c^2} < v < \sqrt{c^2} \&\&\,c_1 \in \mathbb{Z} \right) \mid \mid$$

$$\left(s > q \&\&\,c < 0 \&\&-\sqrt{c^2} < v < \sqrt{c^2} \&\&\,c_1 \in \mathbb{Z} \right) \mid \mid$$

$$\left(c < 0 \&\&-\sqrt{c^2} < v < \sqrt{c^2} \&\&\,c_1 \in \mathbb{Z} \&\&\,l < 0 \&\&\,\alpha \le \frac{-q+s}{l} \right) \mid \mid$$

$$\left(c < 0 \&\&-\sqrt{c^2} < v < \sqrt{c^2} \&\&\,c_1 \in \mathbb{Z} \&\&\,l < 0 \&\&\,s < q \&\&\,\alpha \le \frac{q-s}{l} \right)$$

$$\left\{\beta \to \frac{\sqrt{\frac{q-s+1}{\sqrt{1-\frac{v^2}{c^2}}}}}{\sqrt{c}} \sqrt{\sqrt{c^2-v^2}} \left(-q+s+l\,\alpha\right)} \right\}$$

$$\left\{\phi \to \frac{\sqrt{c}\,l\,\alpha}{\sqrt{c}\,l\,\alpha}\right\} + 2\,\pi\,c_1 \quad \text{if} \quad \left\{c > 0\,\&\&\,l > 0\,\&\&\,\alpha \ge \frac{q-s}{l}\,\&\&\,-\sqrt{c^2}\,<\,v < \sqrt{c^2}\,\&\&\,c_1 \in \mathbb{Z}\,\&\&\,s < q\right\} \mid l$$

$$\left\{c > 0\,\&\&\,s > q\,\&\&\,l > 0\,\&\&\,\alpha \ge \frac{-q+s}{l}\,\&\&\,-\sqrt{c^2}\,<\,v < \sqrt{c^2}\,\&\&\,c_1 \in \mathbb{Z}\right\} \mid l$$

$$\left\{c > 0\,\&\&\,s > q\,\&\&\,-\sqrt{c^2}\,<\,v < \sqrt{c^2}\,\&\&\,c_1 \in \mathbb{Z}\,\&\&\,l < 0\,\&\&\,\alpha \le \frac{-q+s}{l}\right\} \mid l$$

$$\left\{c > 0\,\&\&\,-\sqrt{c^2}\,<\,v < \sqrt{c^2}\,\&\&\,c_1 \in \mathbb{Z}\,\&\&\,l < 0\,\&\&\,\alpha \le \frac{q-s}{l}\right\}$$

$$\left\{ \beta \rightarrow \begin{array}{l} ArcSin \bigg[\frac{\sqrt{\frac{q-s+l\,\alpha}{\sqrt{1-\frac{v^2}{c^2}}}}}{\sqrt{1-\frac{v^2}{c^2}}} \ \sqrt{\sqrt{c^2-v^2}} \ (-q+s+l\,\alpha) \\ \hline \sqrt{c} \ l\,\alpha \\ \\ \bigg(c > 0 \,\&\&\, l > 0 \,\&\&\, \alpha \geq \frac{q-s}{l} \,\&\&\, -\sqrt{c^2} \,<\, v <\, \sqrt{c^2} \,\&\&\, c_1 \in \mathbb{Z} \,\&\&\, s < q \,\Big) \,|\,| \\ \\ \bigg(c > 0 \,\&\&\, s > q \,\&\&\, l > 0 \,\&\&\, \alpha \geq \frac{-q+s}{l} \,\&\&\, -\sqrt{c^2} \,<\, v <\, \sqrt{c^2} \,\&\&\, c_1 \in \mathbb{Z} \,\Big) \,|\,| \\ \\ \bigg(c > 0 \,\&\&\, s > q \,\&\&\, -\sqrt{c^2} \,<\, v <\, \sqrt{c^2} \,\&\&\, c_1 \in \mathbb{Z} \,\&\&\, l < 0 \,\&\&\, \alpha \leq \frac{-q+s}{l} \,\Big) \,|\,| \\ \\ \bigg(c > 0 \,\&\&\, -\sqrt{c^2} \,<\, v <\, \sqrt{c^2} \,\&\&\, c_1 \in \mathbb{Z} \,\&\&\, l < 0 \,\&\&\, \alpha \leq \frac{q-s}{l} \,\Big) \,|\,| \\ \\ \bigg(c > 0 \,\&\&\, -\sqrt{c^2} \,<\, v <\, \sqrt{c^2} \,\&\&\, c_1 \in \mathbb{Z} \,\&\&\, l < 0 \,\&\&\, \alpha \leq \frac{q-s}{l} \,\Big) \\ \end{array} \right)$$

$$\left\{ \text{l} \rightarrow \boxed{\text{0 if } \left(c > \text{0 \&\& - } \sqrt{c^2} \, < \, \text{v} < \, \sqrt{c^2} \, \right) \, | \, \left(c < \text{0 \&\& - } \sqrt{c^2} \, < \, \text{v} < \, \sqrt{c^2} \, \right)} \right. ,$$

$$s \rightarrow \boxed{ \text{q if } \left(c > 0 \, \&\& - \, \sqrt{c^2} \, < \, v < \, \sqrt{c^2} \, \right) \, | \, \left| \, \left(c < 0 \, \&\& - \, \sqrt{c^2} \, < \, v < \, \sqrt{c^2} \, \right) \, \right| },$$

$$\left\{ s \to \boxed{ \mbox{q if $\left(c > 0 \&\& l > 0 \&\& \alpha > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \&\& \, c_1 \in \mathbb{Z}\right) \mid \mbox{\mid}} , \\ \left(c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \&\& \, c_1 \in \mathbb{Z} \&\& \, l < 0 \&\& \, \alpha < 0 \right) } \right.$$

$$\beta \to \begin{bmatrix} \sqrt{1 \sqrt{c^2 - v^2} \alpha} & \sqrt{\frac{1 \alpha}{\sqrt{1 - \frac{v^2}{c^2}}}} \\ \sqrt{c} & 1 \alpha \end{bmatrix} + 2 \pi c_1 \\ \text{if } \left(c > 0 \&\& 1 > 0 \&\& \alpha > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \&\& c_1 \in \mathbb{Z}\right) \mid \mid \\ \left(c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \&\& c_1 \in \mathbb{Z} \&\& 1 < 0 \&\& \alpha < 0\right) \end{bmatrix}$$

$$\left\{ s \to \boxed{ \mbox{q if $\left(c > 0 \&\& l > 0 \&\& \alpha > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \&\& \, c_1 \in \mathbb{Z}\right) \mid \mbox{\mid}} , \\ \left(c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \&\& \, c_1 \in \mathbb{Z} \&\& \, l < 0 \&\& \, \alpha < 0 \right) } \right. ,$$

$$\beta \to \text{ArcSin} \left[\frac{\sqrt{\mathsf{L} \cdot \mathsf{V}^2} \ \alpha}{\sqrt{\mathsf{L} \cdot \mathsf{L} \cdot \mathsf{V}^2}} \right] + 2 \, \pi \, \mathfrak{C}_1 \quad \text{if} \\ \left(\mathsf{C} > 0 \, \&\& \, \mathsf{L} > 0 \, \&\& \, \alpha > 0 \, \&\& \, - \, \sqrt{\mathsf{C}^2} \, < \mathsf{V} < \, \sqrt{\mathsf{C}^2} \, \&\& \, \mathfrak{C}_1 \in \mathbb{Z} \right) \mid \mid \\ \left(\mathsf{C} > 0 \, \&\& \, - \, \sqrt{\mathsf{C}^2} \, < \mathsf{V} < \, \sqrt{\mathsf{C}^2} \, \&\& \, \mathfrak{C}_1 \in \mathbb{Z} \, \&\& \, \mathsf{L} < 0 \, \&\& \, \alpha < 0 \right)$$

$$\left\{ s \to \boxed{ \mbox{q if $\left(l > 0 \&\& \alpha > 0 \&\& c < 0 \&\& - \sqrt{c^2} \ < v < \sqrt{c^2} \&\& \, c_1 \in \mathbb{Z} \right) \mid \mbox{\mid}} , \\ \left(c < 0 \&\& - \sqrt{c^2} \ < v < \sqrt{c^2} \&\& \, c_1 \in \mathbb{Z} \&\& \, l < 0 \&\& \, \alpha < 0 \right) } \right. ,$$

$$\beta \to -\text{ArcSin}\Big[\frac{\sqrt{-c} \ \sqrt{l \ \sqrt{c^2 - v^2} \ \alpha}}{c \ l \ \alpha}\Big] + 2 \ \pi \ c_1} \\ \text{if} \ \left(l > 0 \ \&\& \ \alpha > 0 \ \&\& \ c < 0 \ \&\& \ - \sqrt{c^2} \ < v < \sqrt{c^2} \ \&\& \ c_1 \in \mathbb{Z}\right) \mid \mid \\ \left(c < 0 \ \&\& \ - \sqrt{c^2} \ < v < \sqrt{c^2} \ \&\& \ \alpha < 0\right)$$

$$\left\{ s \to \boxed{ \begin{array}{l} q \quad \text{if} \quad \left(l > 0 \&\& \alpha > 0 \&\& c < 0 \&\& - \sqrt{c^2} \ < v < \sqrt{c^2} \&\& \, c_1 \in \mathbb{Z} \right) \mid \mid} \\ \\ \left(c < 0 \&\& - \sqrt{c^2} \ < v < \sqrt{c^2} \&\& \, c_1 \in \mathbb{Z} \&\& \, l < 0 \&\& \, \alpha < 0 \right) \end{array} } \right. },$$

$$\beta \rightarrow \boxed{ \pi + \text{ArcSin} \Big[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Big] + 2 \pi c_1 } \\ \text{if } \Big(1 > 0 \&\& \alpha > 0 \&\& c < 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \&\& c_1 \in \mathbb{Z} \Big) \mid | \\ \Big(c < 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \&\& c_1 \in \mathbb{Z} \Big) \otimes | | \\ \text{descending the second second$$

In[*]:= Reduce[

$$Out[\circ]= \left(q < s \&\&\right)$$

$$\left(\left[1 < 0 \, \&\& \left(\left[a < \frac{-q+s}{l} \, \&\& \, Sin[b] \right. \right] \right. = \sqrt{\frac{a^2 \, l^2 - q^2 + 2 \, q \, s - s^2}{a^2 \, l^2}} \, \&\& \left(\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right) \right] + \left[\left(a = \frac{-q+s}{l} \, \&\& \, Sin[b] \right. = 0 \, \&\& \left(\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right) \right] \right) + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] \right) \right) + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c > 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right) \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \, \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] + \left[\left(c < 0 \,$$

$$\left(1 = 0 \&\& \left(\left[c < 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \mid \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right) \mid \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right) \mid \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \mid \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \right) \mid \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \right) \mid \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \right) \mid \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \right) \mid \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \right) \mid \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \right] \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \right] \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[\left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \right] \left[c > 0 \&\& - \sqrt{c^2} < v < \sqrt{c^2} \right] \left[c > 0 \&\& - \sqrt{c^2} \right] \left[c > 0 \&$$

$$\left[-\sqrt{c^2} < v < \sqrt{c^2} & \& \& q < s & \& \& 1 > 0 & \& a > \frac{-q+s}{1} & \& \& \\ Sin[b] = \sqrt{\frac{a^2 \, 1^2 - q^2 + 2 \, q \, s - s^2}{a^2 \, 1^2}} & \& \& c < 0 \right] | | \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& \& \\ q < s & \& \& 1 > 0 & \& a > \frac{-q+s}{1} & \& \& Sin[b] = \sqrt{\frac{a^2 \, 1^2 - q^2 + 2 \, q \, s - s^2}{a^2 \, 1^2}} & \& \& c > 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& \& q = s & \& \& 1 < 0 & \& a < 0 & \& Sin[b] = 1 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& \& q = s & \& \& 1 < 0 & \& a < 0 & \& Sin[b] = 1 & \& c > 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& \& q = s & \& \& 1 = 0 & \& a < 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& \& q = s & \& & 1 = 0 & \& a < 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& k q = s & \& & 1 = 0 & \& a > 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& k q = s & \& & 1 = 0 & \& a > 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& k q = s & \& & 1 = 0 & \& a > 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& k q = s & \& & 1 > 0 & \& a > 0 & \& Sin[b] = 1 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& k q = s & \& & 1 > 0 & \& a > 0 & \& Sin[b] = 1 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& k q = s & \& & 1 < 0 & \& a > 0 & \& Sin[b] = 1 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& k q > s & \& 1 < 0 & \& a > 0 & \& Sin[b] = \sqrt{\frac{a^2 \, 1^2 - q^2 + 2 \, q \, s - s^2}{a^2 \, 1^2}} & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& a > s & \& 1 < 0 & \& a = \frac{q - s}{1} & \& Sin[b] = 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& a > s & \& 1 < 0 & \& a = \frac{q - s}{1} & \& Sin[b] = 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& a > s & \& 1 > 0 & \& a = \frac{q - s}{1} & \& Sin[b] = 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& a > s & \& 1 > 0 & \& a = \frac{q - s}{1} & \& Sin[b] = 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& a > s & \& 1 > 0 & \& a = \frac{q - s}{1} & \& Sin[b] = 0 & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& a > s & \& 1 > 0 & \& a > \frac{q - s}{1} & \& Sin[b] = \sqrt{\frac{a^2 \, 1^2 - q^2 + 2 \, q \, s - s^2}{a^2 \, 1^2}} & \& c < 0 \right] | | \\ \left[-\sqrt{c^2} < v < \sqrt{c^2} & \& a > s & a > 0 & \& a > 0 & a > \frac{q - s}{1} & \& Sin[b] = \sqrt{\frac{a^2 \, 1^2 - q^2 + 2 \, q \,$$

$$\log \left[l \sin \left[\beta \right] \right] = \frac{\sqrt{(l \alpha + x \gamma - r \theta) \sqrt{1 - \frac{v^2}{c^2}}} \sqrt{(l \alpha - x \gamma + r \theta) / \sqrt{1 - \frac{v^2}{c^2}}}}, v \right]$$

$$\log \left[\left\{ v \right\} \right]$$

$$- \left(\left(1. \sqrt{(-8.98755 \times 10^{16} l^2 \alpha^2 + 8.98755 \times 10^{16} x^2 \gamma^2 - 1.79751 \times 10^{17} r x \gamma \theta + 8.98755 \times 10^{16} r^2 \theta^2 + 8.98755 \times 10^{16} l^2 \alpha^2 \sin \left[\beta \right]^2) \right) / \left(\sqrt{-1. l^2 \alpha^2 + x^2 \gamma^2 - 2. r x \gamma \theta + r^2 \theta^2 + l^2 \alpha^2 \sin \left[\beta \right]^2} \right) \right) \right\},$$

$$\left\{ v \rightarrow \left(\sqrt{(-8.98755 \times 10^{16} l^2 \alpha^2 + 8.98755 \times 10^{16} x^2 \gamma^2 - 1.79751 \times 10^{17} r x \gamma \theta + 8.98755 \times 10^{16} l^2 \alpha^2 \sin \left[\beta \right]^2} \right) \right) / \left(\sqrt{-1. l^2 \alpha^2 + x^2 \gamma^2 - 2. r x \gamma \theta + r^2 \theta^2 + l^2 \alpha^2 \sin \left[\beta \right]^2} \right) \right) \right\}$$

$$v = \frac{\sqrt{-c^2 l^2 \alpha^2 + c^2 x^2 \gamma^2 - 2 c^2 r x \gamma \theta + c^2 r^2 \theta^2 + c^2 l^2 \alpha^2 \sin \left[\beta \right]^2}}{\sqrt{-1. l^2 \alpha^2 + x^2 \gamma^2 - 2. r x \gamma \theta + r^2 \theta^2 + l^2 \alpha^2 \sin \left[\beta \right]^2}}$$

$$(1)$$

Modus ponens substitutions for the respective arc lengths and imaginary arc lengths.

$$V = \frac{\sqrt{-c^2 w^2 + c^2 q^2 - 2 c^2 s q + c^2 s^2 + c^2 w^2 Sin[\beta]^2}}{\sqrt{-1 \cdot w^2 + q^2 - 2 \cdot s q + s^2 + w^2 Sin[\beta]^2}}$$

Rewrite variables $\alpha = a$, $b = \beta$

$$\begin{split} & \mathit{In[s]} = \ \mathbf{V} := \frac{\sqrt{-c^2 \, l^2 \, a^2 + c^2 \, q^2 - 2 \, c^2 \, s \, q + c^2 \, s^2 + c^2 \, l^2 \, a^2 \, Sin[b]^2}}{\sqrt{-1 \cdot 1^2 \, a^2 + q^2 - 2 \cdot 1 \cdot s \, q + s^2 + l^2 \, a^2 \, Sin[b]^2}} \\ & \mathit{Out[s]} = \left(-\sqrt{c^2} \, < \, \frac{\sqrt{-a^2 \, c^2 \, l^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, l^2 \, Sin[b]^2}}{\sqrt{-1 \cdot a^2 \, l^2 + q^2 - 2 \cdot q \, s + s^2 + a^2 \, l^2 \, Sin[b]^2}} \, < \, \sqrt{c^2} \, \&\& \\ & q < s \, \&\& \, l < 0 \, \&\& \, a < \, \frac{-q + s}{l} \, \&\& \, Sin[b] = \sqrt{\frac{a^2 \, l^2 - q^2 + 2 \, q \, s - s^2}{a^2 \, l^2}} \, \&\& \, c < 0 \right) \, | \, | \\ & \left(-\sqrt{c^2} \, < \, \frac{\sqrt{-a^2 \, c^2 \, l^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, l^2 \, Sin[b]^2}}{\sqrt{-1 \cdot a^2 \, l^2 + q^2 - 2 \cdot q \, s + s^2 + a^2 \, l^2 \, Sin[b]^2}} \, < \, \sqrt{c^2} \, \&\& \, q < s \, \&\& \\ & l < 0 \, \&\& \, a < \, \frac{-q + s}{l} \, \&\& \, Sin[b] = \sqrt{\frac{a^2 \, l^2 - q^2 + 2 \, q \, s - s^2}{a^2 \, l^2}} \, \&\& \, c > 0 \right) \, | \, | \\ & \left(-\sqrt{c^2} \, < \, \frac{\sqrt{-a^2 \, c^2 \, l^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, l^2 \, Sin[b]^2}}{\sqrt{-1 \cdot a^2 \, l^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, l^2 \, Sin[b]^2}} \, < \, \sqrt{c^2} \, \&\& \\ & q < s \, \&\& \, l < 0 \, \&\& \, a = \, \frac{-q + s}{l} \, \&\& \, Sin[b] = 0 \, \&\& \, c < 0 \, \right) \, | \, | \, | \end{aligned}$$

$$\begin{aligned} & a < \theta \&\& c < \theta \\ & \| \cdot \| \left[-\sqrt{c^2} < \frac{\sqrt{-a^2 \, c^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}}{\sqrt{-1..a^2 \, t^2 + q^2 - 2..q \, s + s^2 + a^2 \, t^2 \, sin[b]^2}} < \sqrt{c^2} \, \&\& \\ & q = s \&\& 1 = \theta \&\& a < \theta \&\& c > \theta \\ & \| \cdot \| \\ & \left[-\sqrt{c^2} < \frac{\sqrt{-a^2 \, c^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, t^2 \, sin[b]^2}}{\sqrt{-1..a^2 \, t^2 + q^2 - 2..q \, s + s^2 + a^2 \, t^2 \, sin[b]^2}} < \sqrt{c^2} \, \&\& \\ & q = s \&\& 1 = \theta \&\& a > \theta \&\& c < \theta \\ & \| \cdot \| \\ & \left[-\sqrt{c^2} < \frac{\sqrt{-a^2 \, c^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}}{\sqrt{-1..a^2 \, t^2 + q^2 - 2..q \, s + s^2 + a^2 \, t^2 \, sin[b]^2}} < \sqrt{c^2} \, \&\& \\ & q = s \&\& 1 = \theta \&\& a > \theta \&\& c > \theta \\ & \| \cdot \| \\ & \left[-\sqrt{c^2} < \frac{\sqrt{-a^2 \, c^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}}{\sqrt{-1..a^2 \, t^2 + q^2 - 2..q \, s + s^2 + a^2 \, t^2 \, sin[b]^2}} < \sqrt{c^2} \, \&\& \\ & q = s \&\& 1 > \theta \&\& a > \theta \&\& sin[b] = 1 \&\& c > \theta \\ & \| \cdot \| \\ & -\sqrt{c^2} < \frac{\sqrt{-a^2 \, c^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}}{\sqrt{-1..a^2 \, t^2 + q^2 - 2..q \, s + s^2 + a^2 \, t^2 \, sin[b]^2}}} < \sqrt{c^2} \, \&\& \\ & q = s \&\& 1 > \theta \&\& a > \theta \&\& sin[b] = 1 \&\& c > \theta \\ & \| \cdot \| \\ & -\sqrt{c^2} < \frac{\sqrt{-a^2 \, c^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}}{\sqrt{-1..a^2 \, t^2 + q^2 - 2..q \, s + s^2 + a^2 \, t^2 \, sin[b]^2}}} < \sqrt{c^2} \, \&\& q > s \&\& \\ & 1 < \theta \&\& a < \frac{q - s}{1..a^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}}{\sqrt{-1..a^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}}} < \sqrt{c^2} \, \&\& q > s \&\& \\ & 1 < \theta \&\& a < \frac{q - s}{1..a^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}}{\sqrt{-1..a^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2}} < \sqrt{c^2} \, \&\& q > s \&\& \\ & 1 < \theta \&\& a < \frac{q - s}{1..a^2 \, t^2 + c^2 \, q^2 - 2 \, c^2 \, q \, s + c^2 \, s^2 + a^2 \, c^2 \, t^2 \, sin[b]^2} < \sqrt{c^2} \, \&\& c > \theta \\ & 1 < \theta \&\& a < \frac{q - s}{1..a^2 \, t^2 + c^$$

$$\begin{array}{l} q>s\,\&\&\,l<0\,\&\&\,a=\frac{q-s}{l}\,\&\&\,Sin[b]=0\,\&\&\,c<0 \end{array} |\ |\ | \\ = \sqrt{c^2}<\frac{\sqrt{-a^2\,c^2\,l^2+c^2\,q^2-2\,c^2\,q\,s+c^2\,s^2+a^2\,c^2\,l^2\,Sin[b]^2}}{\sqrt{-1.\,\,a^2\,l^2+q^2-2.\,\,q\,s+s^2+a^2\,l^2\,Sin[b]^2}}<\sqrt{c^2}\,\&\& \\ q>s\,\&\&\,l<0\,\&\&\,a=\frac{q-s}{l}\,\&\&\,Sin[b]=0\,\&\&\,c>0 \\ = \left(-\sqrt{c^2}<\frac{\sqrt{-a^2\,c^2\,l^2+c^2\,q^2-2\,c^2\,q\,s+c^2\,s^2+a^2\,c^2\,l^2\,Sin[b]^2}}{\sqrt{-1.\,\,a^2\,l^2+q^2-2.\,\,q\,s+s^2+a^2\,l^2\,Sin[b]^2}}<\sqrt{c^2}\,\&\& \\ q>s\,\&\&\,l>0\,\&\&\,a=\frac{q-s}{l}\,\&\&\,Sin[b]=0\,\&\&\,c<0 \\ = \left(-\sqrt{c^2}<\frac{\sqrt{-a^2\,c^2\,l^2+c^2\,q^2-2\,c^2\,q\,s+c^2\,s^2+a^2\,c^2\,l^2\,Sin[b]^2}}{\sqrt{-1.\,\,a^2\,l^2+q^2-2.\,\,q\,s+s^2+a^2\,l^2\,Sin[b]^2}}<\sqrt{c^2}\,\&\& \\ q>s\,\&\&\,l>0\,\&\&\,a=\frac{q-s}{l}\,\&\&\,Sin[b]=0\,\&\&\,c>0 \\ = \left(-\sqrt{c^2}<\frac{\sqrt{-a^2\,c^2\,l^2+c^2\,q^2-2\,c^2\,q\,s+c^2\,s^2+a^2\,c^2\,l^2\,Sin[b]^2}}{\sqrt{-1.\,\,a^2\,l^2+q^2-2.\,\,q\,s+s^2+a^2\,l^2\,Sin[b]^2}}<\sqrt{c^2}\,\&\&\,q>s\,\&\& \\ l>0\,\&\&\,a>\frac{q-s}{l}\,\&\&\,Sin[b]=\sqrt{\frac{a^2\,l^2-q^2+2\,q\,s-s^2}{a^2\,l^2}}\,\&\&\,c<0 \\ = \left(-\sqrt{c^2}<\frac{\sqrt{-a^2\,c^2\,l^2+c^2\,q^2-2\,c^2\,q\,s+c^2\,s^2+a^2\,c^2\,l^2\,Sin[b]^2}}{\sqrt{-1.\,\,a^2\,l^2+q^2-2.\,\,q\,s+s^2+a^2\,l^2\,Sin[b]^2}}<\sqrt{c^2}\,\&\&\,c>0 \\ = \sqrt{c^2}<\frac{\sqrt{-a^2\,c^2\,l^2+c^2\,q^2-2\,c^2\,q\,s+c^2\,s^2+a^2\,c^2\,l^2\,Sin[b]^2}}{\sqrt{-1.\,\,a^2\,l^2+q^2-2.\,\,q\,s+s^2+a^2\,l^2\,Sin[b]^2}}<\sqrt{c^2}\,\&\&\,c>0 \\ = \sqrt{c^2}<\frac{\sqrt{-a^2\,c^2\,l^2+c^2\,q^2-2\,c^2\,q\,s+c^2\,s^2+a^2\,l^2\,Sin[b]^2}}{\sqrt{-1.\,\,a^2\,l^2+q^2-2.\,\,q\,s+s^2+a^2\,l^2\,Sin[b]^2}}$$

In[*]:= q := c

In[*]:= S := 5

In[•]:= **a :=** π

In[*]:= b := 1.2468502254630345`

$$q > s \&\& l > 0 \&\& a > \frac{q-s}{l} \&\& Sin[b] = \sqrt{\frac{a^2 l^2 - q^2 + 2 q s - s^2}{a^2 l^2}} \&\& c > 0$$

Out[*]= True