
toulbar2 User Manual

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WHAT IS TOULBAR2

toulbar2 is an exact black box discrete optimization solver targeted at solving cost function networks (CFN), thus solving the so-called “weighted Constraint Satisfaction Problem” or WCSP. Cost function networks can be simply described by a set of discrete variables each having a specific finite domain and a set of integer cost functions, each involving some of the variables. The WCSP is to find an assignment of all variables such that the sum of all cost functions is minimum and less than a given upper bound often denoted as k or \top . Functions can be typically specified by sparse or full tables but also more concisely as specific functions called “global cost functions” [Schiex2016a].

Using on the fly translation, toulbar2 can also directly solve optimization problems on other graphical models such as Maximum probability Explanation (MPE) on Bayesian networks [koller2009], and Maximum A Posteriori (MAP) on Markov random field [koller2009]. It can also read partial weighted MaxSAT problems, Quadratic Pseudo Boolean problems (MAXCUT) as well as Linkage **.pre** pedigree files for genotyping error detection and correction.

toulbar2 is exact. It will only report an optimal solution when it has both identified the solution and proved its optimality. Because it relies only on integer operations, addition and subtraction, it does not suffer from rounding errors. In the general case, the WCSP, MPE/BN, MAP/MRF, PWMaxSAT, QPBO or MAXCUT being all NP-hard problems and thus toulbar2 may take exponential time to prove optimality. This is however a worst-case behavior and toulbar2 has been shown to be able to solve to optimality problems with half a million non Boolean variables defining a search space as large as $2^{829,440}$. It may also fail to solve in reasonable time problems with a search space smaller than 2^{264} .

toulbar2 provides and uses by default an “anytime” algorithm [Katsirelos2015a] that tries to quickly provide good solutions together with an upper bound on the gap between the cost of each solution and the (unknown) optimal cost. Thus, even if it is unable to prove optimality, it will bound the quality of the solution provided. It can also apply a variable neighborhood search algorithm exploiting a problem decomposition [Ouali2017]. This algorithm is complete (if enough CPU-time is given) and it can be run in parallel using OpenMPI.

Beyond the service of providing optimal solutions, toulbar2 can also find a greedy sequence of diverse solutions [Ruffini2019a] or exhaustively enumerate solutions below a cost threshold and perform guaranteed approximate weighted counting of solutions. For stochastic graphical models, this means that toulbar2 will compute the partition function (or the normalizing constant Z). These problems being #P-complete, toulbar2 runtimes can quickly increase on such problems.

By exploiting the new toulbar2 python interface, with incremental solving capabilities, it is possible to learn a CFN from data and to combine it with mandatory constraints [Schiex2020b]. See examples at <https://forgemia.inra.fr/thomas.schiex/cfn-learn>.

HOW DO I INSTALL IT ?

toulbar2 is an open source solver distributed under the MIT license as a set of C++ sources managed with git at <http://github.com/toulbar2/toulbar2>. If you want to use a released version, then you can download there source archives of a specific release that should be easy to compile on most Linux systems.

If you want to compile the latest sources yourself, you will need a modern C++ compiler, CMake, Gnu MP Bignum library, a recent version of boost libraries and optionally the jemalloc memory management and OpenMPI libraries. You can then clone toulbar2 on your machine and compile it by executing:

```
git clone https://github.com/toulbar2/toulbar2.git
cd toulbar2
mkdir build
cd build
# cmake ..
cmake ..
make
```

Finally, toulbar2 is available in the debian-science section of the unstable/sid Debian version. It should therefore be directly installable using:

```
sudo apt-get install toulbar2
```

If you want to try toulbar2 on crafted, random, or real problems, please look for benchmarks in the [Cost Function benchmark Section](#). Other benchmarks coming from various discrete optimization languages are available at [Genotoul EvalGM \[Hurley2016b\]](#).

HOW DO I TEST IT ?

Some problem examples are available in the directory **toulbar2/validation**. After compilation with cmake, it is possible to run a series of tests using:

```
make test
```

For debugging toulbar2 (compile with flag `CMAKE_BUILD_TYPE="Debug"`), more test examples are available at [Cost Function Library](#). The following commands run toulbar2 (executable must be found on your system path) on every problems with a 1-hour time limit and compare their optimum with known optima (in .ub files).

```
cd toulbar2
git clone https://forgemia.inra.fr/thomas.schiex/cost-function-library.git
./misc/script/runall.sh ./cost-function-library/trunk/validation
```

Other tests on randomly generated problems can be done where optimal solutions are verified by using an older solver [toolbar](#) (executable must be found on your system path).

```
cd toolbar2
git clone https://forgemia.inra.fr/thomas.schiex/toolbar.git
cd toolbar/toolbar
make toolbar
cd ../..
./misc/script/rungenerate.sh
```


USING IT AS A BLACK BOX

Using `toulbar2` is just a matter of having a properly formatted input file describing the cost function network, graphical model, PWMaxSAT, PBO or Linkage **.pre** file and executing:

```
toulbar2 [option parameters] <file>
```

and `toulbar2` will start solving the optimization problem described in its file argument. By default, the extension of the file (either **.cfn**, **.cfn.gz**, **.cfn.xz**, **.wcsp**, **.wcsp.gz**, **.wcsp.xz**, **.wcnf**, **.wcnf.gz**, **.wcnf.xz**, **.cnf**, **.cnf.gz**, **.cnf.xz**, **.qpbo**, **.qpbo.gz**, **.qpbo.xz**, **.opb**, **.opb.gz**, **.opb.xz**, **.uai**, **.uai.gz**, **.uai.xz**, **.LG**, **.LG.gz**, **.LG.xz**, **.pre** or **.bep**) is used to determine the nature of the file (see *Input File formats*). There is no specific order for the options or problem file. `toulbar2` comes with decently optimized default option parameters. It is however often possible to set it up for different target than pure optimization or tune it for faster action using specific command line options.

QUICK START

- Download a binary weighted constraint satisfaction problem (WCSP) file `example.wcsp.xz`. Solve it with default options:

```
toulbar2 EXAMPLES/example.wcsp.xz
```

- Solve a WCSP using INCOP, a local search method [idwalk:cp04] applied just after preprocessing, in order to find a good upper bound before a complete search:

```
toulbar2 EXAMPLES/example.wcsp.xz -i
```

- Solve a WCSP with an initial upper bound and save its (first) optimal solution in filename “example.sol”:

```
toulbar2 EXAMPLES/example.wcsp.xz -ub=28 -w=example.sol
```

- ... and see this saved “example.sol” file:

```
cat example.sol  
# each value corresponds to one variable assignment in problem file order
```

- Download a larger WCSP file `scen06.wcsp.xz`. Solve it using a limited discrepancy search strategy [Ginsberg1995] with a VAC integrality-based variable ordering [Trosser2020a] in order to speed-up the search for finding good upper bounds first (by default, toulbar2 uses another diversification strategy based on hybrid best-first search [Katsirelos2015a]):

```
toulbar2 EXAMPLES/scen06.wcsp.xz -l -vacint
```

- Download a cluster decomposition file `scen06.dec` (each line corresponds to a cluster of variables, clusters may overlap). Solve the previous WCSP using a variable neighborhood search algorithm (UDGVNS) [Quali2017] during 10 seconds:

```
toulbar2 EXAMPLES/scen06.wcsp.xz EXAMPLES/scen06.dec -vns -time=10
```

- Download another difficult instance `scen07.wcsp.xz`. Solve it using a variable neighborhood search algorithm (UDGVNS) with maximum cardinality search cluster decomposition and absorption [Ouali2017] during 5 seconds:

```
toulbar2 EXAMPLES/scen07.wcsp.xz -vns -O=-1 -E -time=5
```

- Download file `404.wcsp.xz`. Solve it using Depth-First Branch and Bound with Tree Decomposition and HBFS (BTD-HBFS) [Schiex2006a] based on a min-fill variable ordering:

```
toulbar2 EXAMPLES/404.wcsp.xz -O=-3 -B=1
```

- Solve the same problem using Russian Doll Search exploiting BTD [Sanchez2009a]:

```
toulbar2 EXAMPLES/404.wcsp.xz -O=-3 -B=2
```

- Solve another WCSP using the original Russian Doll Search method [Verfaillie1996] with static variable ordering (following problem file) and soft arc consistency:

```
toulbar2 EXAMPLES/505.wcsp.xz -B=3 -j=1 -svo -k=1
```

- Solve the same WCSP using a parallel variable neighborhood search algorithm (UPDGVNS) with min-fill cluster decomposition [Ouali2017] using 4 cores during 5 seconds:

```
mpirun -n 4 toulbar2 EXAMPLES/505.wcsp.xz -vns -O=-3 -time=5
```

- Download a cluster decomposition file `example.dec` (each line corresponds to a cluster of variables, clusters may overlap). Solve a WCSP using a variable neighborhood search algorithm (UDGVNS) with a given cluster decomposition:

```
toulbar2 EXAMPLES/example.wcsp.xz EXAMPLES/example.dec -vns
```

- Solve a WCSP using a parallel variable neighborhood search algorithm (UPDGVNS) with the same cluster decomposition:

```
mpirun -n 4 toulbar2 EXAMPLES/example.wcsp.xz EXAMPLES/example.dec -vns
```

- Download file `example.order`. Solve a WCSP using BTD-HBFS based on a given (min-fill) reverse variable elimination ordering:

```
toulbar2 EXAMPLES/example.wcsp.xz EXAMPLES/example.order -B=1
```

- Download file `example.cov`. Solve a WCSP using BTD-HBFS based on a given explicit (min-fill path-) tree-decomposition:

```
toulbar2 EXAMPLES/example.wcsp.xz EXAMPLES/example.cov -B=1
```

- Download a Markov Random Field (MRF) file `pedigree9.uai.xz` in UAI format. Solve it using bounded (of degree at most 8) variable elimination enhanced by cost function decomposition in preprocessing [Favier2011a] followed by BTD-HBFS exploiting only small-size (less than four variables) separators:

```
toulbar2 EXAMPLES/pedigree9.uai.xz -O=-3 -p=-8 -B=1 -r=4
```

- Download another MRF file `GeomSurf-7-gm256.uai.xz`. Solve it using Virtual Arc Consistency (VAC) in preprocessing [Cooper2008] and exploit a VAC-based value [Cooper2010a] and variable [Trosser2020a] ordering heuristics:

```
toulbar2 EXAMPLES/GeomSurf-7-gm256.uai.xz -A -V -vacint
```

- Download another MRF file `1CM1.uai.xz`. Solve it by applying first an initial upper bound probing, and secondly, use a modified variable ordering heuristic based on VAC-integrality during search [Trosser2020a]:

```
toulbar2 EXAMPLES/1CM1.uai.xz -A=1000 -vacint -rasps -vacthr
```

- Download a weighted Max-SAT file `brock200_4.clq.wcnf.xz` in wcnf format. Solve it using a modified variable ordering heuristic [Schiex2014a]:

```
toulbar2 EXAMPLES/brock200_4.clq.wcnf.xz -m=1
```

- Download another WCSP file `latin4.wcsp.xz`. Count the number of feasible solutions:

```
toulbar2 EXAMPLES/latin4.wcsp.xz -a
```

- Find a greedy sequence of at most 20 diverse solutions with Hamming distance greater than 12 between any pair of solutions:

```
toulbar2 EXAMPLES/latin4.wcsp.xz -a=20 -div=12
```

- Download a crisp CSP file `GEOM40_6.wcsp.xz` (initial upper bound equal to 1). Count the number of solutions using #BTD [[Favier2009a](#)] using a min-fill variable ordering (warning, cannot use BTD to find all solutions in optimization):

```
toulbar2 EXAMPLES/GEOM40_6.wcsp.xz -O=-3 -a -B=1 -ub=1 -hbfs:
```

- Get a quick approximation of the number of solutions of a CSP with Approx#BTD [[Favier2009a](#)]:

```
toulbar2 EXAMPLES/GEOM40_6.wcsp.xz -O=-3 -a -B=1 -D -ub=1 -hbfs:
```


COMMAND LINE OPTIONS

If you just execute:

```
toulbar2
```

toulbar2 will give you its (long) list of optional parameter which we now describe in more detail.

To deactivate a default command line option, just use the command-line option followed by `:`. For example:

```
toulbar2 -dee: <file>
```

will disable the default Dead End Elimination [Givry2013a] (aka Soft Neighborhood Substitutability) preprocessing.

6.1 General control

-agap=[decimal] stops search if the absolute optimality gap reduces below the given value (provides guaranteed approximation) (default value is 0)

-rgap=[double] stops search if the relative optimality gap reduces below the given value (provides guaranteed approximation) (default value is 0)

-a=[integer] finds at most a given number of solutions with a cost strictly lower than the initial upper bound and stops, or if no integer is given, finds all solutions (or counts the number of zero-cost satisfiable solutions in conjunction with BTD)

-D approximate satisfiable solution count with BTD

-logz computes log of probability of evidence (i.e. log partition function or log(Z) or PR task) for graphical models only (problem file extension .uai)

-timer=[integer] gives a CPU time limit in seconds. toulbar2 will stop after the specified CPU time has been consumed. The time limit is a CPU user time limit, not wall clock time limit.

-bt=[integer] gives a limit on the number of backtracks (9223372036854775807 by default)

-seed=[integer] random seed non-negative value or use current time if a negative value is given (default value is 1)

6.2 Preprocessing

- nopre** deactivates all preprocessing options (equivalent to -e: -p: -t: -f: -dec: -n: -mst: -dee: -trws:)
- p=[integer]** preprocessing only: general variable elimination of degree less than or equal to the given value (default value is -1)
- t=[integer]** preprocessing only: simulates restricted path consistency by adding ternary cost functions on triangles of binary cost functions within a given maximum space limit (in MB)
- f=[integer]** preprocessing only: variable elimination of functional (f=1) (resp. bijective (f=2)) variables (default value is 1)
- dec** preprocessing only: pairwise decomposition [Favier2011a] of cost functions with arity ≥ 3 into smaller arity cost functions (default option)
- n=[integer]** preprocessing only: projects n-ary cost functions on all binary cost functions if n is lower than the given value (default value is 10). See [Favier2011a].
- mst** find a maximum spanning tree ordering for DAC
- S** preprocessing only: performs singleton consistency (only in conjunction with option -A)
- M=[integer]** preprocessing only: apply the Min Sum Diffusion algorithm (default is inactivated, with a number of iterations of 0). See [Cooper2010a].
- trws=[float]** preprocessing only: enforces TRW-S until a given precision is reached (default value is 0.001). See Kolmogorov 2006.
- trws-order** replaces DAC order by Kolmogorov's TRW-S order.
- trws-n-iters=[integer]** enforce at most N iterations of TRW-S (default value is 1000).
- trws-n-iters-no-change=[integer]** stop TRW-S when N iterations did not change the lower bound up the given precision (default value is 5, -1=never).
- trws-n-iters-compute-ub=[integer]** compute a basic upper bound every N steps during TRW-S (default value is 100)

6.3 Initial upper bounding

- l=[integer]** limited discrepancy search [Ginsberg1995], use a negative value to stop the search after the given absolute number of discrepancies has been explored (discrepancy bound = 4 by default)
- L=[integer]** randomized (quasi-random variable ordering) search with restart (maximum number of nodes/VNS restarts = 10000 by default)
- i=["string"]** initial upper bound found by INCOP local search solver [idwalk:cp04]. The string parameter is optional, using "0 1 3 idwa 100000 cv v 0 200 1 0 0" by default with the following meaning: *stoppinglowerbound randomseed nbiterations method nbmoves neighborhoodchoice neighborhoodchoice2 minnbneighbors maxnbneighbors neighborhoodchoice3 autotuning tracemode*.
- x=[(,i[= # <>]a)*]** performs an elementary operation ('=:assign, '#':remove, '<':decrease, '>':increase) with value a on variable of index i (multiple operations are separated by a comma and no space) (without any argument, a complete assignment – used as initial upper bound and as a value heuristic – read from default file "sol" taken as a certificate or given directly as an additional input filename with ".sol" extension and without -x)
- ub=[decimal]** gives an initial upper bound

- rasps=[integer]** VAC-based upper bound probing heuristic (0: disable, >0: max. nb. of backtracks, 1000 if no integer given) (default value is 0)
- raspslds=[integer]** VAC-based upper bound probing heuristic using LDS instead of DFS (0: DFS, >0: max. discrepancy) (default value is 0)
- raspsdeg=[integer]** automatic threshold cost value selection for probing heuristic (default value is 10 degrees)
- raspsini** reset weighted degree variable ordering heuristic after doing upper bound probing

6.4 Tree search algorithms and tree decomposition selection

- hbfs=[integer]** hybrid best-first search [Katsirelos2015a], restarting from the root after a given number of backtracks (default value is 10000)
 - open=[integer]** hybrid best-first search limit on the number of stored open nodes (default value is -1, i.e., no limit)
 - B=[integer]** (0) HBFS, (1) BT-D-HBFS [Schiex2006a] [Katsirelos2015a], (2) RDS-BTD [Sanchez2009a], (3) RDS-BTD with path decomposition instead of tree decomposition [Sanchez2009a] (default value is 0)
 - O=[filename]** reads either a reverse variable elimination order (given by a list of variable indexes) from a file in order to build a tree decomposition (if BT-D-like and/or variable elimination methods are used) or reads a valid tree decomposition directly (given by a list of clusters in topological order of a rooted forest, each line contains a cluster number, followed by a cluster parent number with -1 for the first/root(s) cluster(s), followed by a list of variable indexes). It is also used as a DAC ordering.
 - O=[negative integer]** build a tree decomposition (if BT-D-like and/or variable elimination methods are used) and also a compatible DAC ordering using
 - (-1) maximum cardinality search ordering,
 - (-2) minimum degree ordering,
 - (-3) minimum fill-in ordering,
 - (-4) maximum spanning tree ordering (see -mst),
 - (-5) reverse Cuthill-McKee ordering,
 - (-6) approximate minimum degree ordering,
 - (-7) default file ordering
- If not specified, then use the variable order in which variables appear in the problem file.
- j=[integer]** splits large clusters into a chain of smaller embedded clusters with a number of proper variables less than this number (use options “-B=3 -j=1 -svo -k=1” for pure RDS, use value 0 for no splitting) (default value is 0).
 - r=[integer]** limit on the maximum cluster separator size (merge cluster with its father otherwise, use a negative value for no limit) (default value is -1)
 - X=[integer]** limit on the minimum number of proper variables in a cluster (merge cluster with its father otherwise, use a zero for no limit) (default value is 0)
 - E=[float]** merges leaf clusters with their fathers if small local treewidth (in conjunction with option “-e” and positive threshold value) or ratio of number of separator variables by number of cluster variables above a given threshold (in conjunction with option -vns) (default value is 0)
 - R=[integer]** choice for a specific root cluster number
 - I=[integer]** choice for solving only a particular rooted cluster subtree (with RDS-BTD only)

6.5 Variable neighborhood search algorithms

- vns** unified decomposition guided variable neighborhood search [Ouali2017] (UDGVNS). A problem decomposition into clusters can be given as *.dec, *.cov, or *.order input files or using tree decomposition options such as -O. For a parallel version (UPDGVNS), use “mpirun -n [NbOfProcess] toulbar2 -vns problem.wcsp”.
- vnsini=[integer]** initial solution for VNS-like methods found: (-1) at random, (-2) min domain values, (-3) max domain values, (-4) first solution found by a complete method, (k=0 or more) tree search with k discrepancy max (-4 by default)
- ldsmin=[integer]** minimum discrepancy for VNS-like methods (1 by default)
- ldsmax=[integer]** maximum discrepancy for VNS-like methods (number of problem variables multiplied by maximum domain size -1 by default)
- ldsinc=[integer]** discrepancy increment strategy for VNS-like methods using (1) Add1, (2) Mult2, (3) Luby operator (2 by default)
- kmin=[integer]** minimum neighborhood size for VNS-like methods (4 by default)
- kmax=[integer]** maximum neighborhood size for VNS-like methods (number of problem variables by default)
- kinc=[integer]** neighborhood size increment strategy for VNS-like methods using: (1) Add1, (2) Mult2, (3) Luby operator (4) Add1/Jump (4 by default)
- best=[integer]** stop VNS-like methods if a better solution is found (default value is 0)

6.6 Node processing & bounding options

- e=[integer]** performs “on the fly” variable elimination of variable with small degree (less than or equal to a specified value, default is 3 creating a maximum of ternary cost functions). See [Larrosa2000].
- k=[integer]** soft local consistency level (NC [Larrosa2002] with Strong NIC for global cost functions=0 [LL2009], (G)AC=1 [Schiex2000b] [Larrosa2002], D(G)AC=2 [CooperFCSP], FD(G)AC=3 [Larrosa2003], (weak) ED(G)AC=4 [Heras2005] [LL2010]) (default value is 4). See also [Cooper2010a] [LL2012asa].
- A=[integer]** enforces VAC [Cooper2008] at each search node with a search depth less than a given value (default value is 0)
 - V** VAC-based value ordering heuristic (default option)
- T=[decimal]** threshold cost value for VAC (default value is 1)
- P=[decimal]** threshold cost value for VAC during the preprocessing phase only (default value is 1)
- C=[float]** multiplies all costs internally by this number when loading the problem (cannot be done with cfn format and probabilistic graphical models in uai/LG formats) (default value is 1)
 - vacthr** automatic threshold cost value selection for VAC during search (must be combined with option -A)
- dee=[integer]** restricted dead-end elimination [Givry2013a] (value pruning by dominance rule from EAC value (dee >= 1 and dee <= 3)) and soft neighborhood substitutability (in preprocessing (dee=2 or dee=4) or during search (dee=3)) (default value is 1)
 - o** ensures an optimal worst-case time complexity of DAC and EAC (can be slower in practice)

6.7 Branching, variable and value ordering

- svo** searches using a static variable ordering heuristic. The variable order value used will be the same order as the DAC order.
- b** searches using binary branching (by default) instead of n-ary branching. Uses binary branching for interval domains and small domains and dichotomic branching for large enumerated domains (see option -d).
- c** searches using binary branching with last conflict backjumping variable ordering heuristic [Lecoutre2009].
- q=[integer]** use weighted degree variable ordering heuristic [boussemart2004] if the number of cost functions is less than the given value (default value is 1000000).
- var=[integer]** searches by branching only on the first [given value] decision variables, assuming the remaining variables are intermediate variables that will be completely assigned by the decision variables (use a zero if all variables are decision variables, default value is 0)
- m=[integer]** use a variable ordering heuristic that selects first variables such that the sum of the mean (m=1) or median (m=2) cost of all incident cost functions is maximum [Schiex2014a] (in conjunction with weighted degree heuristic -q) (default value is 0: unused).
- d=[integer]** searches using dichotomic branching. The default d=1 splits domains in the middle of domain range while d=2 splits domains in the middle of the sorted domain based on unary costs.
- sortd** sorts domains in preprocessing based on increasing unary costs (works only for binary WCSPs).
- sortc** sorts constraints in preprocessing based on lexicographic ordering (1), decreasing DAC ordering (2 - default option), decreasing constraint tightness (3), DAC then tightness (4), tightness then DAC (5), randomly (6) or the opposite order if using a negative value.
- solr** solution-based phase saving (reuse last found solution as preferred value assignment in the value ordering heuristic) (default option).
- vacint** VAC-integrality/Full-EAC variable ordering heuristic (can be combined with option -A)

6.8 Diverse solutions

toulbar2 can search for a greedy sequence of diverse solutions with guaranteed local optimality and minimum pairwise Hamming distance [Ruffini2019a].

- div=[integer]** minimum Hamming distance between diverse solutions (use in conjunction with -a=integer with a limit of 1000 solutions) (default value is 0)
- divm=[integer]** diversity encoding method: 0: Dual 1: Hidden 2: Ternary (default value is 0)
- mdd=[integer]** maximum relaxed MDD width for diverse solution global constraint (default value is 0)
- mddh=[integer]** MDD relaxation heuristic: 0: random, 1: high div, 2: small div, 3: high unary costs (default value is 0)

6.9 Console output

- help** shows the default help message that toulbar2 prints when it gets no argument.
- v=[integer]** sets the verbosity level (default 0).
- Z=[integer]** debug mode (save problem at each node if verbosity option -v=num >= 1 and -Z=num >= 3)
- s=[integer]** shows each solution found during search. The solution is printed on one line, giving by default (-s=1) the value (integer) of each variable successively in increasing file order. For -s=2, the value name is used instead, and for -s=3, variable name=value name is printed instead.

6.10 File output

- w=[filename]** writes last/all solutions found in the specified filename (or “sol” if no parameter is given). The current directory is used as a relative path.
- w=[integer]** 1: writes value numbers, 2: writes value names, 3: writes also variable names (default value is 1, this option can be used in combination with -w=filename).
- z=[filename]** saves problem in wcsv or cfn format in filename (or “problem.wcsv”/”problem.cfn” if no parameter is given) writes also the graphviz dot file and the degree distribution of the input problem
- z=[integer]** 1 or 3: saves original instance in 1-wcsv or 3-cfn format (1 by default), 2 or 4: saves after preprocessing in 2-wcsv or 4-cfn format (this option can be used in combination with -z=filename)
- x=[(,i[= # <>]a)*]** performs an elementary operation (‘=’:assign, ‘#’:remove, ‘<’:decrease, ‘>’:increase) with value a on variable of index i (multiple operations are separated by a comma and no space) (without any argument, a complete assignment – used as initial upper bound and as value heuristic – read from default file “sol” or given as input filename with “.sol” extension)

6.11 Probability representation and numerical control

- precision=[integer]** probability/real precision is a conversion factor (a power of ten) for representing fixed point numbers (default value is 7)
- epsilon=[float]** approximation factor for computing the partition function (greater than 1, default value is infinity)

6.12 Random problem generation

-random=[bench profile] bench profile must be specified as follows.

- n and d are respectively the number of variable and the maximum domain size of the random problem.

bin-{n}-{d}-{t1}-{p2}-{seed}

- t1 is the tightness in percentage % of random binary cost functions
- p2 is the number of binary cost functions to include
- the seed parameter is optional

binsub-{n}-{d}-{t1}-{p2}-{p3}-{seed} binary random & submodular cost functions

- t1 is the tightness in percentage % of random cost functions

- p2 is the number of binary cost functions to include
- p3 is the percentage % of submodular cost functions among p2 cost functions (plus 10 permutations of two randomly-chosen values for each domain)

tern-{n}-{d}-{t1}-{p2}-{p3}-{seed}

- p3 is the number of ternary cost functions

nary-{n}-{d}-{t1}-{p2}-{p3}...-{pn}-{seed}

- pn is the number of n-ary cost functions

salldiff-{n}-{d}-{t1}-{p2}-{p3}...-{pn}-{seed}

- pn is the number of salldiff global cost functions (p2 and p3 still being used for the number of random binary and ternary cost functions). *salldiff* can be replaced by *gcc* or *regular* keywords with three possible forms (e.g., *sgcc*, *sgccdp*, *wgcc*) and by *knapsack*.

INPUT FILE FORMATS

Notice that by default `toulbar2` distinguishes file formats based on their extension. It is possible to read a file from a unix pipe using option `-stdin=[format]`; e.g., `cat example.wcsp | toulbar2 --stdin=wcsp`

It is also possible to read and combine multiple problem files (warning, they must be all in the same format, either `wcsp`, `cfn`, or `xml`). Variables with the same name are merged (domains must be identical), otherwise the merge is based on variable indexes (`wcsp` format).

7.1 cfn format (.cfn, .cfn.gz, and .cfn.xz file extension)

With this JSON compatible format, it is possible:

- to give a name to variables and functions.
- to associate a local label to every value that is accessible inside `toulbar2` (among others for heuristics design purposes).
- to use decimal and possibly negative costs.
- to solve both minimization and maximization problems.
- to debug your `.cfn` files: the parser gives a cause and line number when it fails.
- to use gzip'd or xz compressed files directly as input (`.cfn.gz` and `.cfn.xz`).
- to use dense descriptions for dense cost tables.

See a full description here (`CFN_format.pdf` file).

7.2 wcsp format (.wcsp file extension)

It is a text format composed of a list of numerical and string terms separated by spaces. Instead of using names for making reference to variables, variable indexes are employed. The same for domain values. All indexes start at zero.

Cost functions can be defined in intention (see below) or in extension, by their list of tuples. A default cost value is defined per function in order to reduce the size of the list. Only tuples with a different cost value should be given (not mandatory). All the cost values must be positive. The arity of a cost function in extension may be equal to zero. In this case, there is no tuples and the default cost value is added to the cost of any solution. This can be used to represent a global lower bound constant of the problem.

The `wcsp` file format is composed of three parts: a problem header, the list of variable domain sizes, and the list of cost functions.

- Header definition for a given problem:

```
<Problem name>
<Number of variables (N)>
<Maximum domain size>
<Number of cost functions>
<Initial global upper bound of the problem (UB)>
```

The goal is to find an assignment of all the variables with minimum total cost, strictly lower than UB. Tuples with a cost greater than or equal to UB are forbidden (hard constraint).

- Definition of domain sizes

```
<Domain size of variable with index 0>
...
<Domain size of variable with index N - 1>
```

Note domain values range from zero to *size-1* a negative domain size is interpreted as a variable with an interval domain in $[0, -size - 1]$

Warning variables with interval domains are restricted to arithmetic and disjunctive cost functions in intention (see below)

- General definition of cost functions

- Definition of a cost function in extension

```
<Arity of the cost function>
<Index of the first variable in the scope of the cost function>
...
<Index of the last variable in the scope of the cost function>
<Default cost value>
<Number of tuples with a cost different than the default cost>
```

followed by for every tuple with a cost different than the default cost:

```
<Index of the value assigned to the first variable in the scope>
...
<Index of the value assigned to the last variable in the scope>
<Cost of the tuple>
```

Note Shared cost function : A cost function in extension can be shared by several cost functions with the same arity (and same domain sizes) but different scopes. In order to do that, the cost function to be shared must start by a negative scope size. Each shared cost function implicitly receives an occurrence number starting from 1 and incremented at each new shared definition. New cost functions in extension can reuse some previously defined shared cost functions in extension by using a negative number of tuples representing the occurrence number of the desired shared cost function. Note that default costs should be the same in the shared and new cost functions. Here is an example of 4 variables with domain size 4 and one AllDifferent hard constraint decomposed into 6 binary constraints.

- Shared CF used inside a small example in wcsp format:

```
AllDifferentDecomposedIntoBinaryConstraints 4 4 6 1
4 4 4 4
-2 0 1 0 4
0 0 1
1 1 1
2 2 1
```

(continues on next page)

(continued from previous page)

```

3 3 1
2 0 2 0 -1
2 0 3 0 -1
2 1 2 0 -1
2 1 3 0 -1
2 2 3 0 -1

```

- Definition of a cost function in intension by replacing the default cost value by -1 and by giving its keyword name and its K parameters

```

<Arity of the cost function>
<Index of the first variable in the scope of the cost function>
...
<Index of the last variable in the scope of the cost function>
-1
<keyword>
<parameter1>
...
<parameterK>

```

Possible keywords of cost functions defined in intension followed by their specific parameters:

- $\geq cst\ delta$ to express soft binary constraint $x \geq y + cst$ with associated cost function $max((y + cst - x \leq delta)?(y + cst - x) : UB, 0)$
- $> cst\ delta$ to express soft binary constraint $x > y + cst$ with associated cost function $max((y + cst + 1 - x \leq delta)?(y + cst + 1 - x) : UB, 0)$
- $\leq cst\ delta$ to express soft binary constraint $x \leq y + cst$ with associated cost function $max((x - cst - y \leq delta)?(x - cst - y) : UB, 0)$
- $< cst\ delta$ to express soft binary constraint $x < y + cst$ with associated cost function $max((x - cst + 1 - y \leq delta)?(x - cst + 1 - y) : UB, 0)$
- $= cst\ delta$ to express soft binary constraint $x = y + cst$ with associated cost function $(|y + cst - x| \leq delta)?|y + cst - x| : UB$
- $disj\ cstx\ csty\ penalty$ to express soft binary disjunctive constraint $x \geq y + cstx \vee y \geq x + cstx$ with associated cost function $(x \geq y + cstx \vee y \geq x + cstx)?0 : penalty$
- $sdisj\ cstx\ csty\ xinfy\ yinfy\ costx\ costy$ to express a special disjunctive constraint with three implicit hard constraints $x \leq xinfy$ and $y \leq yinfy$ and $x < xinfy \wedge y < yinfy \Rightarrow (x \geq y + cstx \vee y \geq x + costx)$ and an additional cost function $((x = xinfy)?costx : 0) + ((y = yinfy)?costy : 0)$
- Global cost functions using a dedicated propagator:
 - $clique\ l\ (nb_values\ (value)^*)^*$ to express a hard clique cut to restrict the number of variables taking their value into a given set of values (per variable) to at most l occurrence for all the variables (warning! it assumes also a clique of binary constraints already exists to forbid any two variables using both the restricted values)
 - $knapsack\ capacity\ (weight)^*$ to express a reverse knapsack hard constraint with minimum capacity and a list of weights associated to the variables in the scope of the constraint (warning! it assumes Boolean 0/1 variables and is equivalent to a linear constraint with \geq operator, use negative numbers to express the \leq operator)
- Global cost functions using a flow-based propagator:

- `salldiff var|dec|decbi cost` to express a soft alldifferent constraint with either variable-based (*var* keyword) or decomposition-based (*dec* and *decbi* keywords) cost semantic with a given *cost* per violation (*decbi* decomposes into a binary cost function complete network)
- `sgcc var|dec|wdec cost nb_values (value lower_bound upper_bound (shortage_weight excess_weight)?)*` to express a soft global cardinality constraint with either variable-based (*var* keyword) or decomposition-based (*dec* keyword) cost semantic with a given *cost* per violation and for each value its lower and upper bound (if *wdec* then violation cost depends on each value shortage or excess weights)
- `ssame cost list_size1 list_size2 (variable_index)* (variable_index)*` to express a permutation constraint on two lists of variables of equal size (implicit variable-based cost semantic)
- `sregular var|edit cost nb_states nb_initial_states (state)* nb_final_states (state)* nb_transitions (start_state symbol_value end_state)*` to express a soft regular constraint with either variable-based (*var* keyword) or edit distance-based (*edit* keyword) cost semantic with a given *cost* per violation followed by the definition of a deterministic finite automaton with number of states, list of initial and final states, and list of state transitions where symbols are domain values
- Global cost functions using a dynamic programming DAG-based propagator:
 - `sregulardp var cost nb_states nb_initial_states (state)* nb_final_states (state)* nb_transitions (start_state symbol_value end_state)*` to express a soft regular constraint with a variable-based (*var* keyword) cost semantic with a given *cost* per violation followed by the definition of a deterministic finite automaton with number of states, list of initial and final states, and list of state transitions where symbols are domain values
 - `sgrammar|sgrammardp var|weight cost nb_symbols nb_values start_symbol nb_rules ((0 terminal_symbol value)|(1 nonterminal_in nonterminal_out_left nonterminal_out_right)|(2 terminal_symbol value weight)|(3 nonterminal_in nonterminal_out_left nonterminal_out_right weight))*` to express a soft/weighted grammar in Chomsky normal form
 - `samong|samongdp var cost lower_bound upper_bound nb_values (value)*` to express a soft among constraint to restrict the number of variables taking their value into a given set of values
 - `salldifdp var cost` to express a soft alldifferent constraint with variable-based (*var* keyword) cost semantic with a given *cost* per violation (decomposes into *samongdp* cost functions)
 - `sgccdp var cost nb_values (value lower_bound upper_bound)*` to express a soft global cardinality constraint with variable-based (*var* keyword) cost semantic with a given *cost* per violation and for each value its lower and upper bound (decomposes into *samongdp* cost functions)
 - `max|smaxdp defCost nbtuples (variable value cost)*` to express a weighted max cost function to find the maximum cost over a set of unary cost functions associated to a set of variables (by default, *defCost* if unspecified)
 - `MST|smstdp` to express a spanning tree hard constraint where each variable is assigned to its parent variable index in order to build a spanning tree (the root being assigned to itself)
- Global cost functions using a cost function network-based propagator [Ficolof2012]:
 - `wregular nb_states nb_initial_states (state and cost)* nb_final_states (state and cost)* nb_transitions (start_state symbol_value end_state cost)*` to express a weighted regular constraint with weights on initial states, final states, and transitions, followed by the definition of a deterministic finite automaton with number of states, list of initial and final states with their costs, and list of weighted state transitions where symbols are domain values
 - `walldiff hard|lin|quad cost` to express a soft alldifferent constraint as a set of *wamong* hard constraint (*hard* keyword) or decomposition-based (*lin* and *quad* keywords) cost semantic with a given *cost* per violation
 - `wgcc hard|lin|quad cost nb_values (value lower_bound upper_bound)*` to express a soft global cardinality constraint as either a hard constraint (*hard* keyword) or with decomposition-based (*lin* and *quad* keyword) cost semantic with a given *cost* per violation and for each value its lower and upper bound

- `wsame hard|lin|quad cost` to express a permutation constraint on two lists of variables of equal size (implicitly concatenated in the scope) using implicit decomposition-based cost semantic
- `wsamegcc hard|lin|quad cost nb_values (value lower_bound upper_bound)*` to express the combination of a soft global cardinality constraint and a permutation constraint
- `wamong hard|lin|quad cost nb_values (value)* lower_bound upper_bound` to express a soft among constraint to restrict the number of variables taking their value into a given set of values
- `wvaramong hard cost nb_values (value)*` to express a hard among constraint to restrict the number of variables taking their value into a given set of values to be equal to the last variable in the scope
- `woverlap hard|lin|quad cost comparator righthandside` overlaps between two sequences of variables X, Y (i.e. set the fact that X_i and Y_i take the same value (not equal to zero))
- `wsum hard|lin|quad cost comparator righthandside` to express a soft sum constraint with unit coefficients to test if the sum of a set of variables matches with a given comparator and right-hand-side value
- `wvarsum hard cost comparator` to express a hard sum constraint to restrict the sum to be *comparator* to the value of the last variable in the scope

Let us note $\langle \rangle$ the comparator, K the right-hand-side value associated to the comparator, and Sum the result of the sum over the variables. For each comparator, the gap is defined according to the distance as follows:

- * if $\langle \rangle$ is `==` : $gap = abs(K - Sum)$
- * if $\langle \rangle$ is `<=` : $gap = max(0, Sum - K)$
- * if $\langle \rangle$ is `<` : $gap = max(0, Sum - K - 1)$
- * if $\langle \rangle$ is `!=` : $gap = 1$ if $Sum \neq K$ and $gap = 0$ otherwise
- * if $\langle \rangle$ is `>` : $gap = max(0, K - Sum + 1)$;
- * if $\langle \rangle$ is `>=` : $gap = max(0, K - Sum)$;

Warning The decomposition of `wsum` and `wvarsum` may use an exponential size (sum of domain sizes).

list_size1 and *list_size2* must be equal in *ssame*.

Cost functions defined in intention cannot be shared.

Note More about network-based global cost functions can be found here <https://metivier.users.greyc.fr/decomposable/>

Examples:

- quadratic cost function $x_0 * x_1$ in extension with variable domains $\{0, 1\}$ (equivalent to a soft clause $\neg x_0 \vee \neg x_1$):

```
2 0 1 0 1 1 1 1
```

- simple arithmetic hard constraint $x_1 < x_2$:

```
2 1 2 -1 < 0 0
```

- hard temporal disjunction $x_1 \geq x_2 + 2 \vee x_2 \geq x_1 + 1$:

```
2 1 2 -1 disj 1 2 1000
```

- clique cut ($\{x_0, x_1, x_2, x_3\}$) on Boolean variables such that value 1 is used at most once (warning! must add a clique of binary constraints too) :

```
4 0 1 2 3 -1 clique 1 1 1 1 1 1 1 1
2 0 1 0 1 1 1 1000
2 0 2 0 1 1 1 1000
2 0 3 0 1 1 1 1000
2 1 2 0 1 1 1 1000
2 1 3 0 1 1 1 1000
2 2 3 0 1 1 1 1000
```

- knapsack constraint ($\{x_0, x_1, x_2, x_3\}$) on Boolean variables such that $2 * x_0 + 3 * x_1 + 4 * x_2 + 5 * x_3 \geq 10$:

```
4 0 1 2 3 -1 knapsack 10 2 3 4 5
```

- soft_alldifferent($\{x_0, x_1, x_2, x_3\}$) :

```
4 0 1 2 3 -1 salldiff var 1
```

- soft_gcc($\{x_1, x_2, x_3, x_4\}$) with each value v from 1 to 4 only appearing at least $v-1$ and at most $v+1$ times :

```
4 1 2 3 4 -1 sgcc var 1 4 1 0 2 2 1 3 3 2 4 4 3 5
```

- soft_same($\{x_0, x_1, x_2, x_3\}, \{x_4, x_5, x_6, x_7\}$) :

```
8 0 1 2 3 4 5 6 7 -1 ssame 1 4 4 0 1 2 3 4 5 6 7
```

- soft_regular($\{x_1, x_2, x_3, x_4\}$) with DFA $(3^*)+(4^*)$:

```
4 1 2 3 4 -1 sregular var 1 2 1 0 2 0 1 3 0 3 0 0 4 1 1 4 1
```

- soft_grammar($\{x_0, x_1, x_2, x_3\}$) with hard cost (1000) producing well-formed parenthesis expressions :

```
4 0 1 2 3 -1 sgrammardp var 1000 4 2 0 6 1 0 0 0 1 0 1 2 1 0 1 3 1 2 0 3 0 1 0 0 3 1
```

- soft_among($\{x_1, x_2, x_3, x_4\}$) with hard cost (1000) if $\sum_{i=1}^4 (x_i \in \{1, 2\}) < 1$ or $\sum_{i=1}^4 (x_i \in \{1, 2\}) > 3$:

```
4 1 2 3 4 -1 samongdp var 1000 1 3 2 1 2
```

- soft_max($\{x_0, x_1, x_2, x_3\}$) with cost equal to $\max_{i=0}^3 ((x_i \neq i) ? 1000 : (4 - i))$:

```
4 0 1 2 3 -1 smaxdp 1000 4 0 0 4 1 1 3 2 2 2 3 3 1
```

- wregular($\{x_0, x_1, x_2, x_3\}$) with DFA $(0(10)^*2^*)$:

```
4 0 1 2 3 -1 wregular 3 1 0 0 1 2 0 9 0 0 1 0 0 1 1 1 0 2 1 1 1 0 0 1 0 0 1 1 2 0
→ 1 1 2 2 0 1 0 2 1 1 1 2
1
```

- wamong ($\{x_1, x_2, x_3, x_4\}$) with hard cost (1000) if $\sum_{i=1}^4 (x_i \in \{1, 2\}) < 1$ or $\sum_{i=1}^4 (x_i \in \{1, 2\}) > 3$:

```
4 1 2 3 4 -1 wamong hard 1000 2 1 2 1 3
```

- wvamong ($\{x_1, x_2, x_3, x_4\}$) with hard cost (1000) if $\sum_{i=1}^3 (x_i \in \{1, 2\}) \neq x_4$:

```
4 1 2 3 4 -1 wvamong hard 1000 2 1 2
```

- woverlap($\{x_1, x_2, x_3, x_4\}$) with hard cost (1000) if $\sum_{i=1}^2 (x_i = x_{i+2}) \geq 1$:

```
4 1 2 3 4 -1 woverlap hard 1000 < 1
```

- wsum ({x1,x2,x3,x4}) with hard cost (1000) if $\sum_{i=1}^4 (x_i) \neq 4$:

```
4 1 2 3 4 -1 wsum hard 1000 == 4
```

- wvarsum ({x1,x2,x3,x4}) with hard cost (1000) if $\sum_{i=1}^3 (x_i) \neq x_4$:

```
4 1 2 3 4 -1 wvarsum hard 1000 ==
```

Latin Square 4 x 4 crisp CSP example in wcsp format :

```
latin4 16 4 8 1
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4 0 1 2 3 -1 salldiff var 1
4 4 5 6 7 -1 salldiff var 1
4 8 9 10 11 -1 salldiff var 1
4 12 13 14 15 -1 salldiff var 1
4 0 4 8 12 -1 salldiff var 1
4 1 5 9 13 -1 salldiff var 1
4 2 6 10 14 -1 salldiff var 1
4 3 7 11 15 -1 salldiff var 1
```

4-queens binary weighted CSP example with random unary costs in wcsp format :

```
4-WQUEENS 4 4 10 5
4 4 4 4
2 0 1 0 10
0 0 5
0 1 5
1 0 5
1 1 5
1 2 5
2 1 5
2 2 5
2 3 5
3 2 5
3 3 5
2 0 2 0 8
0 0 5
0 2 5
1 1 5
1 3 5
2 0 5
2 2 5
3 1 5
3 3 5
2 0 3 0 6
0 0 5
0 3 5
1 1 5
2 2 5
3 0 5
```

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```

3 3 5
2 1 2 0 10
0 0 5
0 1 5
1 0 5
1 1 5
1 2 5
2 1 5
2 2 5
2 3 5
3 2 5
3 3 5
2 1 3 0 8
0 0 5
0 2 5
1 1 5
1 3 5
2 0 5
2 2 5
3 1 5
3 3 5
2 2 3 0 10
0 0 5
0 1 5
1 0 5
1 1 5
1 2 5
2 1 5
2 2 5
2 3 5
3 2 5
3 3 5
1 0 0 2
1 1
3 1
1 1 0 2
1 1
2 1
1 2 0 2
1 1
2 1
1 3 0 2
0 1
2 1

```


7.3 UAI and LG formats (.uai, .LG)

It is a simple text file format specified below to describe probabilistic graphical model instances. The format is a generalization of the Ergo file format initially developed by Noetic Systems Inc. for their Ergo software.

- **Structure**

A file in the UAI format consists of the following two parts, in that order:

```
<Preamble>

<Function tables>
```

The contents of each section (denoted < ... > above) are described in the following:

- **Preamble**

The preamble starts with one line denoting the type of network. This will be either BAYES (if the network is a Bayesian network) or MARKOV (in case of a Markov network). This is followed by a line containing the number of variables. The next line specifies each variable's domain size, one at a time, separated by whitespace (note that this implies an order on the variables which will be used throughout the file).

The fourth line contains only one integer, denoting the number of functions in the problem (conditional probability tables for Bayesian networks, general factors for Markov networks). Then, one function per line, the scope of each function is given as follows: The first integer in each line specifies the size of the function's scope, followed by the actual indexes of the variables in the scope. The order of this list is not restricted, except when specifying a conditional probability table (CPT) in a Bayesian network, where the child variable has to come last. Also note that variables are indexed starting with 0.

For instance, a general function over variables 0, 5 and 11 would have this entry:

```
3 0 5 11
```

A simple Markov network preamble with three variables and two functions might for instance look like this:

```
MARKOV
3
2 2 3
2
2 0 1
3 0 1 2
```

The first line denotes the Markov network, the second line tells us the problem consists of three variables, let's refer to them as X, Y, and Z. Their domain size is 2, 2, and 3 respectively (from the third line). Line four specifies that there are 2 functions. The scope of the first function is X,Y, while the second function is defined over X,Y,Z.

An example preamble for a Belief network over three variables (and therefore with three functions) might be:

```
BAYES
3
2 2 3
3
1 0
2 0 1
2 1 2
```

The first line signals a Bayesian network. This example has three variables, let's call them X, Y, and Z, with domain size 2, 2, and 3, respectively (from lines two and three). Line four says that there are 3 functions (CPTs

in this case). The scope of the first function is given in line five as just X (the probability $P(X)$), the second one is defined over X and Y (this is $(Y | X)$). The third function, from line seven, is the CPT $P(Z | Y)$. We can therefore deduce that the joint probability for this problem factors as $P(X, Y, Z) = P(X).P(Y | X).P(Z | Y)$.

- **Function tables**

In this section each function is specified by giving its full table (i.e, specifying the function value for each tuple). The order of the functions is identical to the one in which they were introduced in the preamble.

For each function table, first the number of entries is given (this should be equal to the product of the domain sizes of the variables in the scope). Then, one by one, separated by whitespace, the values for each assignment to the variables in the function's scope are enumerated. Tuples are implicitly assumed in ascending order, with the last variable in the scope as the 'least significant'.

To illustrate, we continue with our Bayesian network example from above, let's assume the following conditional probability tables:

X	P(X)	
0	0.436	
1	0.564	

X	Y	P(Y X)
0	0	0.128
0	1	0.872
1	0	0.920
1	1	0.080

Y	Z	P(Z Y)
0	0	0.210
0	1	0.333
0	2	0.457
1	0	0.811
1	1	0.000
1	2	0.189

The corresponding function tables in the file would then look like this:

```

2
0.436 0.564

4
0.128 0.872
0.920 0.080

6
0.210 0.333 0.457
0.811 0.000 0.189

```

(Note that line breaks and empty lines are effectively just whitespace, exactly like plain spaces " ". They are used here to improve readability.)

In the LG format, probabilities are replaced by their logarithm.

- **Summary**

To sum up, a problem file consists of 2 sections: the preamble and the full the function tables, the names and the labels.

For our Markov network example above, the full file could be:

```
MARKOV
3
2 2 3
2
2 0 1
3 0 1 2

4
4.000 2.400
1.000 0.000

12
2.2500 3.2500 3.7500
0.0000 0.0000 10.0000
1.8750 4.0000 3.3330
2.0000 2.0000 3.4000
```

Here is the full Bayesian network example from above:

```
BAYES
3
2 2 3
3
1 0
2 0 1
2 1 2

2
0.436 0.564

4
0.128 0.872
0.920 0.080

6
0.210 0.333 0.457
0.811 0.000 0.189
```

- **Expressing evidence**

Evidence is specified in a separate file. This file has the same name as the original problems file but an added .evid extension at the end. For instance, problem.uai will have evidence in problem.uai.evid.

The file simply starts with a line specifying the number of evidence variables. This is followed by the pairs of variable and value indexes for each observed variable, one pair per line. The indexes correspond to the ones implied by the original problem file.

If, for our above example, we want to specify that variable Y has been observed as having its first value and Z with its second value, the file example.uai.evid would contain the following:

```
2
1 0
2 1
```

7.4 Partial Weighted MaxSAT format

Max-SAT input format (.cnf)

The input file format for Max-SAT will be in DIMACS format:

```
c
c comments Max-SAT
c
p cnf 3 4
1 -2 0
-1 2 -3 0
-3 2 0
1 3 0
```

- The file can start with comments, that is lines beginning with the character ‘c’.
- Right after the comments, there is the line “p cnf nbvar nbclauses” indicating that the instance is in CNF format; nbvar is the number of variables appearing in the file; nbclauses is the exact number of clauses contained in the file.
- Then the clauses follow. Each clause is a sequence of distinct non-null numbers between -nbvar and nbvar ending with 0 on the same line. Positive numbers denote the corresponding variables. Negative numbers denote the negations of the corresponding variables.

Weighted Max-SAT input format (.wcnf)

In Weighted Max-SAT, the parameters line is “p wcnf nbvar nbclauses”. The weights of each clause will be identified by the first integer in each clause line. The weight of each clause is an integer greater than or equal to 1.

Example of Weighted Max-SAT formula:

```
c
c comments Weighted Max-SAT
c
p wcnf 3 4
10 1 -2 0
3 -1 2 -3 0
8 -3 2 0
5 1 3 0
```

Partial Max-SAT input format (.wcnf)

In Partial Max-SAT, the parameters line is “p wcnf nbvar nbclauses top”. We associate a weight with each clause, which is the first integer in the clause. Weights must be greater than or equal to 1. Hard clauses have weight top and soft clauses have weight 1. We assume that top is a weight always greater than the sum of the weights of violated soft clauses.

Example of Partial Max-SAT formula:

```
c
c comments Partial Max-SAT
c
p wcnf 4 5 15
15 1 -2 4 0
15 -1 -2 3 0
```

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```

1 -2 -4 0
1 -3 2 0
1 1 3 0

```

Weighted Partial Max-SAT input format (.wcnf)

In Weighted Partial Max-SAT, the parameters line is “p wcnf nbvar nbclauses top”. We associate a weight with each clause, which is the first integer in the clause. Weights must be greater than or equal to 1. Hard clauses have weight top and soft clauses have a weight smaller than top. We assume that top is a weight always greater than the sum of the weights of violated soft clauses.

Example of Weighted Partial Max-SAT formula:

```

c
c comments Weighted Partial Max-SAT
c
p wcnf 4 5 16
16 1 -2 4 0
16 -1 -2 3 0
8 -2 -4 0
4 -3 2 0
3 1 3 0

```

7.5 QPBO format (.qpbo)

In the quadratic pseudo-Boolean optimization (unconstrained quadratic programming) format, the goal is to minimize or maximize the quadratic function:

$$X' * W * X = \sum_{i=1}^N \sum_{j=1}^N W_{ij} * X_i * X_j$$

where W is a symmetric squared $N \times N$ matrix expressed by all its non-zero half ($i \leq j$) squared matrix coefficients, X is a vector of N binary variables with domain values in $\{0, 1\}$ or $\{1, -1\}$, and X' is the transposed vector of X .

Note that for two indices $i \neq j$, coefficient $W_{ij} = W_{ji}$ (symmetric matrix) and it appears twice in the previous sum. It can be controlled by the option {tt -qpmult=[double]} which defines a coefficient multiplier for quadratic terms (default value is 2).

Note also that coefficients can be positive or negative and are real float numbers. They are converted to fixed-point real numbers by multiplying them by $10^{\text{precision}}$ (see option {em -precision} to modify it, default value is 7). Infinite coefficients are forbidden.

Notice that depending on the sign of the number of variables in the first text line, the domain of all variables is either $\{0, 1\}$ or $\{1, -1\}$.

Warning! The encoding in Weighted CSP of variable domain $\{1, -1\}$ associates for each variable value the following index: value 1 has index 0 and value -1 has index 1 in the solutions found by toulbar2. The encoding of variable domain $\{0, 1\}$ is direct.

Qpbo is a file text format:

- First line contains the number of variables N and the number of non-zero coefficients M .

If N is negative then domain values are in $\{1, -1\}$, otherwise $\{0, 1\}$. If M is negative then it will maximize the quadratic function, otherwise it will minimize it.

- Followed by $|M|$ lines where each text line contains three values separated by spaces: position index i (integer belonging to $[1, |N|]$), position index j (integer belonging to $[1, |N|]$), coefficient W_{ij} (float number) such that $i \leq j$ and $W_{ij} \neq 0$.

7.6 OPB format (.opb)

The OPB file format is used to express pseudo-Boolean satisfaction and optimization models. These models may only contain 0/1 Boolean variables. The format is defined by an optional objective function followed by a set of linear constraints. Variables may be multiplied together in the objective function, but currently not in the constraints due to some restriction in the reader. The objective function must start with the **min:** or **max:** keyword followed by **coef_1 varname_1_1 varname_1_2 ... coef2 varname_2_1 ...** and end with a **;**. Linear constraints are composed in the same way, ended by a comparison operator (**<=**, **>=**, or **!=**) followed by the right-hand side coefficient and **;**. Each coefficient must be an integer beginning with its sign (+ or - with no extra space). Comment lines start with a *****.

An example with a quadratic objective and 7 linear constraints is:

```
max: +1 x1 x2 +2 x3 x4;  
+1 x2 +1 x1 >= 1;  
+1 x3 +1 x1 >= 1;  
+1 x4 +1 x1 >= 1;  
+1 x3 +1 x2 >= 1;  
+1 x4 +1 x2 >= 1;  
+1 x4 +1 x3 >= 1;  
+2 x1 +2 x2 +2 x3 +2 x4 <= 7;
```

Internally, all integer costs are multiplied by a power of ten depending on the -precision option. For problems with big integers, try to reduce the precision (*e.g.*, use option -precision 0).

7.7 XCSP2.1 format (.xml)

CSP and weighted CSP in XML format XCSP 2.1, with constraints in extension only, can be read. See a description of this deprecated format here http://www.cril.univ-artois.fr/CPAI08/XCSP2_1.pdf.

Warning, toulbar2 must be compiled with a specific option XML in the cmake.

7.8 Linkage format (.pre)

See **mendelsoft** companion software at <http://miat.inrae.fr/MendelSoft> for pedigree correction. See also <https://carlit.toulouse.inra.fr/cgi-bin/awki.cgi/HaplotypeInference> for haplotype inference in half-sib families.

USING IT AS A LIBRARY

See `toulbar2` reference manual which describes the `libtb2.so` C++ library API.

USING IT FROM PYTHON

A Python interface is now available. Compile `toulbar2` with cmake option `PYTB2` (and without MPI options) to generate a Python module **pytoulbar2** (in `lib` directory). See examples in `src/pytoulbar2.cpp` and `web/TUTORIALS` directory.

An older version of `toulbar2` was integrated inside `Numberjack`. See <https://github.com/eomahony/Numberjack>.

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