(ML-)VAMP state evolution

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1 Gaussian likelihood

$$A^+ = \frac{1}{V^+(A^-)} - A^-, \qquad A^- = \frac{1}{V^-(A^+)} - A^+, \tag{1}$$

where

$$V^+(A) = \mathbb{E}_{x,z}\partial_B^2 \log Z_x(A, Ax + \sqrt{A}z), \tag{2}$$

$$V^-(A) = \sigma^2 \lim_{N\to\infty} \frac{1}{N} \operatorname{Tr}\left[(\Phi^T\Phi + \frac{A}{\sigma^2})^{-1}\right] = \sigma^2\,\mathcal{S}(-A\sigma^2). \tag{3}$$

2 Non-Gaussian likelihoods

Following He at al. (2017), we can track the variances using

$$A_x^+ = \frac{1}{V_x^+(A_x^-)} - A_x^-, \qquad A_z^+ = \frac{1}{V_z^+(A_x^+, 1/A_z^-)} - A_z^-, \tag{4}$$

$$A_x^- = \frac{1}{V_x^-(A_x^+, 1/A_z^-)} - A_x^+, \qquad A_z^- = \frac{1}{V_z^-(A_z^+)} - A_z^+, \tag{5}$$

where

$$V_x^+(A) = \mathbb{E}_{x,z}\partial_B^2 \log Z_x(A, Ax + \sqrt{A}z), \tag{6}$$

$$V_z^+(A, \sigma^2) = \sigma^2 \lim_{M\to\infty} \frac{1}{M} \operatorname{Tr}\left[\Phi(\Phi^T\Phi + \frac{A}{\sigma^2})^{-1}\Phi^T\right] = \alpha^{-1}\sigma^2\big(1 - A\sigma^2\,\mathcal{S}(-A\sigma^2)\big), \tag{7}$$

$$V_x^-(A, \sigma^2) = \sigma^2 \lim_{N\to\infty} \frac{1}{N} \operatorname{Tr}\left[(\Phi^T\Phi + \frac{A}{\sigma^2})^{-1}\right] = \sigma^2\,\mathcal{S}(-A\sigma^2), \tag{8}$$

$$V_z^-(A) = \frac{1}{A} + \frac{1}{A^2}\underbrace{\mathbb{E}_{y,w,z}\partial_w^2 \log Z_z(y, w, 1/A)}_{-\mathbb{g}(A)}. \tag{9}$$

Combining (5) and (9) yields

$$1/A_z^- = \frac{1}{\mathbb{g}(A_z^+)} - \frac{1}{A_z^+}. \tag{10}$$

At the fixed points

$$V_x \equiv V_x^+ = V_x^- = \frac{1}{A_x^+ + A_x^-}, \qquad V_z \equiv V_z^+ = V_z^- = \frac{1}{A_z^+ + A_z^-}. \tag{11}$$

as well as

$$V_z = \frac{1 - A_x^+ V_x}{\alpha A_z^-} = \frac{A_x^- V_x}{\alpha A_z^-} \Rightarrow \alpha\frac{A_z^-}{A_z^- + A_z^+} = \frac{A_x^-}{A_x^- + A_x^+} \tag{12}$$

These variables relate to ours according to

$$V = V_x, \qquad \tilde{A} = A_x^-, \qquad \tilde{V} = 1/A_z^+, \qquad A = A_z^- A_z^+ V_z = \mathbb{g}(A_z^+), \qquad \theta = A_z^- V_z = \frac{1}{1 + A_z^+/A_z^-}. \tag{13}$$