$$y_1[n] = x[nL]$$

$$x_p[n] = x[n]\,p[n]$$

where $$p[n] = \sum_{n=-\infty}^{\infty} \delta[n-kL]$$

F.S. representation

$$= \sum_{k=\langle L\rangle} a_k e^{jk\frac{2\pi}{L}n}$$

if $L=2$

$$p(n) = \sum_{k=0}^{1} a_k e^{jk\frac{2\pi}{2}n}$$

$$= \frac{1}{L}\sum_{k=0}^{1} e^{jk\pi n}$$

$$a_k = \frac{1}{L} \quad \forall k$$

$$Y_1(e^{j\omega})$$

$$X_p(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})\,P(e^{j(\omega-\theta)})\,d\theta$$

In 2π interval

$X(e^{j\omega})$ = Bandlimited between $-\frac{\pi}{L}$ to $\frac{\pi}{L}$

$$X(e^{j\omega}) = \begin{cases} \text{nonzero} & -\frac{\pi}{L} \text{ to } \pi/L \\ 0 & \text{otherwise} \end{cases}$$

where $-\pi \leq \omega < \pi$

$$P(e^{j\omega}) = \frac{2\pi}{L}\sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{L}\right)$$

$$X_p(e^{j\omega}) = \frac{1}{L}\sum_{k=0}^{L-1} X\left(e^{j\left(\omega - k\frac{2\pi}{L}\right)}\right)$$

$L=2$ $\quad X(e^{j\omega})$, $\quad X\left(e^{j\left(\omega-\frac{2\pi}{L}\right)}\right)$ & $X\left(e^{j(\omega-\pi)}\right)$

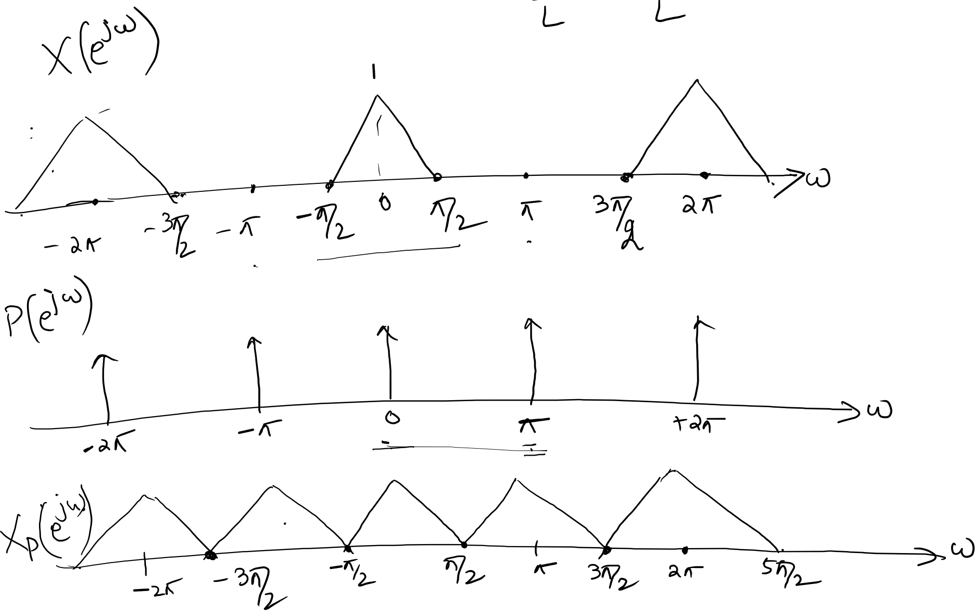
located at $\omega=0$ corresponding to $\delta(\omega)$

Shifted by $\omega=\frac{2\pi}{L}$ corresponding to $\delta\left(\omega-\frac{2\pi}{L}\right) \equiv \delta(\omega-\pi)$

$L=3$ $\quad X(e^{j\omega})$ $\quad X\left(e^{j\left(\omega-\frac{2\pi}{3}\right)}\right)$ $\quad X\left(e^{j\left(\omega-\frac{4\pi}{3}\right)}\right)$

If $L=2$, consider $X(e^{j\omega})$ to be non-zero from

$-\frac{\pi}{L}$ to $\frac{\pi}{L}$ or $-\frac{\pi}{2}$ to $\pi/2$



$$y_1[n] = x[nL]$$
$$= x[2n]$$

$$Y_1(e^{j\omega}) = X_p\left(e^{j\frac{\omega}{L}}\right)$$

$$= X_p\left(e^{j\frac{\omega}{2}}\right)$$

$$= \frac{1}{L}\sum_{k=0}^{L-1} X\left(e^{j\left(\frac{\omega}{2} - k.\frac{2\pi}{2}\right)}\right)$$

$$= \frac{1}{2}\sum_{k=0}^{1} X\left(e^{j\left(\frac{\omega - k2\pi}{2}\right)}\right)$$

$$= \frac{1}{2}X\left(e^{j\omega/2}\right) + \frac{1}{2}X\left(e^{j\frac{\omega}{2} - \pi}\right)$$

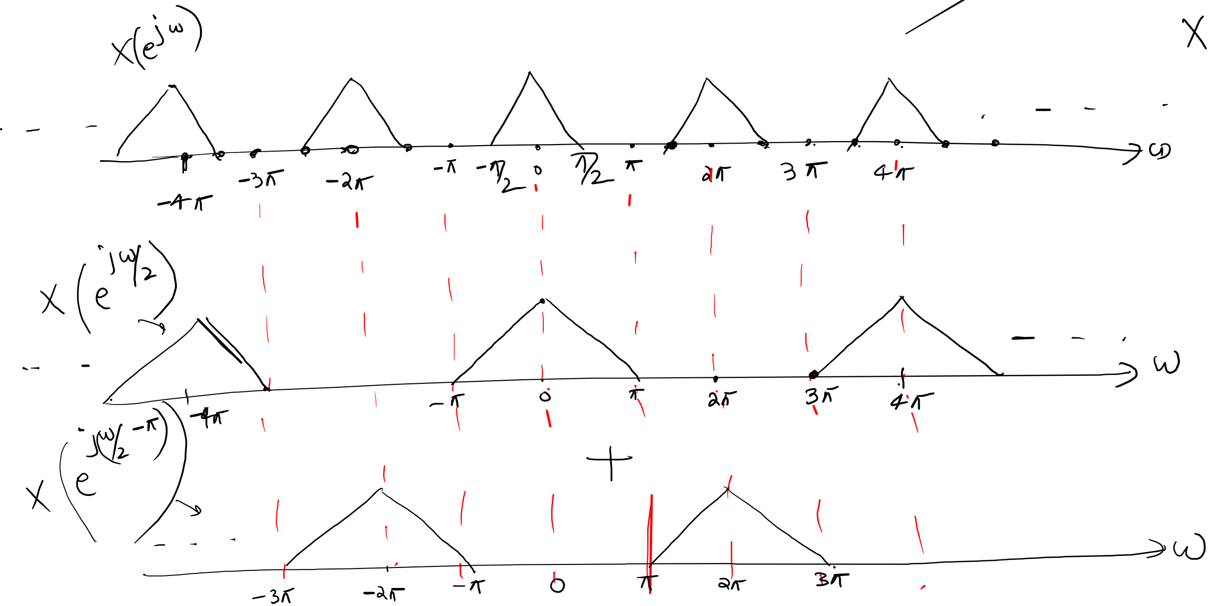
$$Y_1(e^{j\omega}) = \frac{1}{2} X\left(e^{j\omega/2}\right) + \frac{1}{2} X\left(e^{j(\omega/2 - \pi)}\right)$$

$$X\left(e^{j\omega/2}\right)\Big|_{\omega=\pi} = X\left(e^{j\pi/2}\right)$$

$$X\left(e^{j0/2}\right) = X\left(e^{j0}\right)$$

$$X\left(e^{j\omega/2}\right)\Big|_{\omega=2\pi} = X\left(e^{j\pi}\right)$$

$$X\left(e^{j3\omega/2}\right)\Big|_{\omega=3\pi}$$



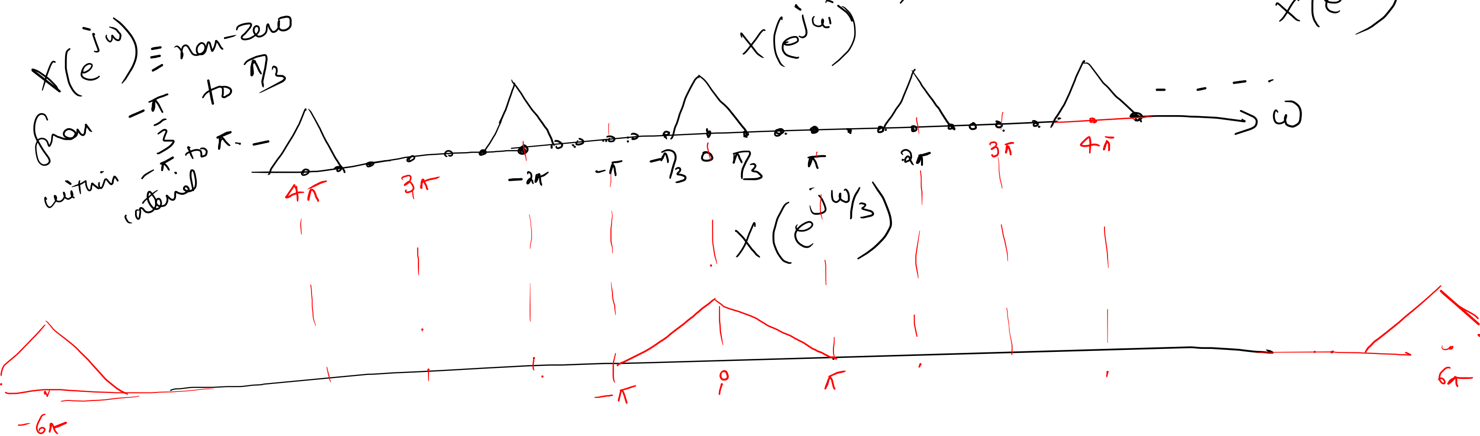
$L = 3$

$$Y_1(e^{j\omega}) = \frac{1}{3}\sum_{k=0}^{2} X\left(e^{j\left(\frac{\omega-2\pi k}{3}\right)}\right)$$

$$Y_1(e^{j\omega}) = \frac{1}{3} X\left(e^{j\omega/3}\right) + \frac{1}{3} X\left(e^{j\left(\frac{\omega-2\pi}{3}\right)}\right) + \frac{1}{3} \cancel{X}\left(e^{j\left(\frac{\omega-4\pi}{3}\right)}\right)$$

$X(e^{j\omega})$

$X(e^{j\omega}) \equiv$ non-zero from $-\frac{\pi}{3}$ to $\pi/3$ within $-\pi$ to π. centered



$h[n]$ = interpolator in time domain
= anti-imaging & anti-aliasing filter in freq. domain

$y_1[n] = x[2n]$ downsample / decimation by 2

$y_2[n] = x\left[\frac{3}{2}n\right]$

$y_2[0] = v[0]$

$y_2[1] = v[3]$

$w[0] = x[0]$

$w[1] = 0$

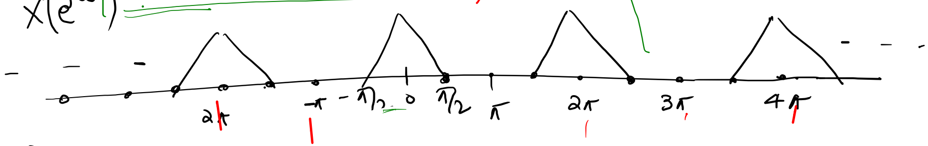
$w[2] = x[1]$

$w[3] = 0$

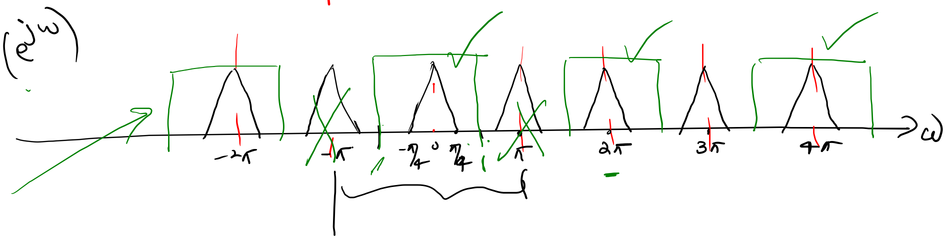
$w[4] = x[2]$

$$x[n] \to \boxed{\uparrow 2} \xrightarrow{w[n]} \boxed{h[n]} \xrightarrow{v[n]} \boxed{\downarrow 3} \to y_2[n]$$

$X(e^{j\omega})$



$W(e^{j\omega})$



$$w[n] = \begin{cases} x[n/2] & n = \text{multiple of } 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{W(e^{j\omega}) = X\left(e^{j\omega \cdot 2}\right)}$$