# ECE250: Signals & Systems
# Monsoon 2024
## Mid-Semester Examination

Date: **4/10/2024** Duration: **1.30 Hours** Total Marks: **36+6 Marks**

**Note:**

**(1)** Please provide proper mathematical justifications with your answers. No marks will be awarded without a valid justification.

**(2) Do not use any property without proving it mathematically in the paper. No shortcuts or statements are allowed. This will fetch you zero marks.**

**(3)** Institute Plagiarism policy are strictly applicable.

**[CO1, CO2] Q1: [6+3 Marks]** Given that

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & otherwise \end{cases}$$

and $h(t) = \beta x(t/\alpha)$, where $0 < \alpha \le 1$

**(a)** Calculate and sketch $y(t) = x(t) * h(t)$.

**(b)** If $\frac{d}{dt}y(t)$ contains only three discontinuities, then what is the value of $\alpha$ and $\beta$.

**[CO1, CO2] Q2: [4+6 Marks]** The cascade of the following two systems ***S₁*** and ***S₂*** is depicted in Figure-1.

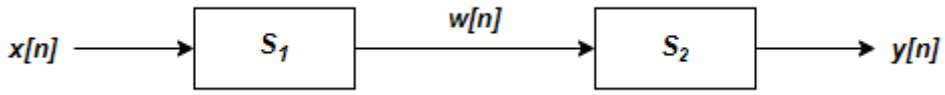


**Figure 1**

$S_1$: Causal LTI; $w[n] = \frac{1}{2}w[n-1] + x[n]$

$S_2$: Causal LTI; $y[n] = \alpha y[n-1] + \beta w[n]$

The systems are initially at rest and the difference equation relating $x[n]$ and $y[n]$ is:

$$y[n] = -\frac{1}{8}y[n-2] + \frac{3}{4}y[n-1] + x[n]$$

**(a)** Determine $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

**(b)** Show the impulse response of the cascade connection of ***S₁*** and ***S₂***.

**[CO1, CO2, CO3] Q3: [6 Marks]** Determine the Fourier series representation for the periodic signal $x(t)$ with time period **4**

$$x(t) = \begin{cases} \sin \pi t, & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}$$

**[CO1, CO2, CO3] Q4: [2+6+2+1 Marks]** Given an impulse train $x[n]$,

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$$

**(a)** Find the Fourier Series of $x[n]$ and plot the line spectrum.

**(b)** This signal is applied as an input to a particular LTI system with frequency response $H(e^{jw})$, the output of the system is found to be

$$y[n] = \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right)$$

Determine the values of $H\left(e^{jk\pi/2}\right)$ for $k = 0, 1, 2, and\ 3$.

**(c)** Plot the line spectrum of $y[n]$.

**(d)** Write your inference comparing the line spectrum of $x[n]$ and $y[n]$.

**[CO1, CO2] Q5[Bonus Question]: [6 Marks]** Consider the signal

$$x[n] = \alpha^n u[n]$$

**(a)** Sketch the signal $g[n] = x[n] - \alpha x[n-1]$

**(b)** Use the result of part (a) to determine a sequence $h[n]$ such that

$$x[n] * h[n] = \left(\frac{1}{2}\right)^n \{u[n+2] - u[n-2]\}$$