

C.T.

## Constant coefficient differential eqn. representation of causal LTI System

$$\sum_{k=0}^N b_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^M a_k \frac{d^k x(t)}{dt^k}$$

Condition of initial rest  
is satisfied

$N \equiv$  Order of the system

Let  $N=2 \times M=3$

$$b_2 \frac{d^2 y(t)}{dt^2} + b_1 \frac{d y(t)}{dt} + b_0 y(t) = a_0 x(t) + a_1 \frac{d x(t)}{dt} + a_2 \frac{d^2 x(t)}{dt^2} + a_3 \frac{d^3 x(t)}{dt^3}$$

if  $x(t)=0$  for  $t < t_0$

$\frac{d^k y(t)}{dt^k} = 0$  for  $t < t_0$

$\forall k=0, 1, \dots, N$

## Block Diagram representation

D.T.

$a_0 x[n] =$  scalar multiplier

$x[n-1] =$  delay element

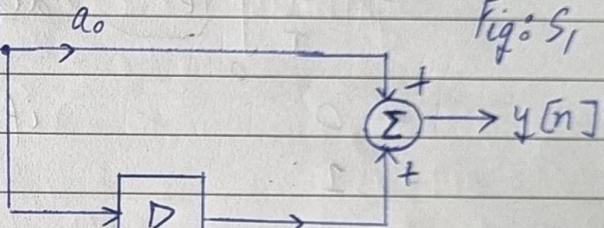
$$S_1: y[n] = a_0 x[n] + a_1 x[n-1]$$

$x[n]$

Fig: S<sub>1</sub>

$$S_2: b_1 y[n-1] + b_0 y[n] = a_0 x[n] + a_1 x[n-1]$$

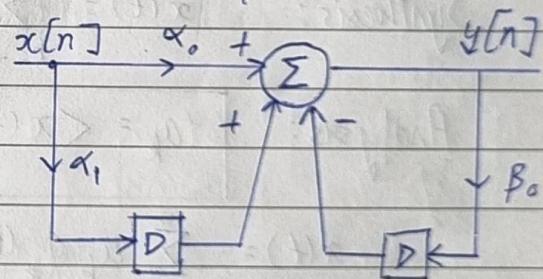
$$y[n] = \frac{a_0}{b_0} x[n] + \frac{a_1}{b_0} x[n-1] - \frac{b_1}{b_0} y[n-1]$$



where

$$\alpha_0 = a_0/b_0$$

constants



## Fourier Series

C.T.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j k \omega_0 t} \quad \forall t \text{ for Periodic signal } x(t)$$

Synthesis equation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t)$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] s[n-k]$$

$$= \sum_{k=-\infty}^{\infty} a_k s[n-k]$$

Vector

basis fns.

$$\phi_k(t) = e^{jk\omega_0 t}$$

$$e^{jk\omega_0 t} = \cos k\omega_0 t + j \sin k\omega_0 t$$

$\omega_0 = \frac{2\pi}{T}$   
 $T = \text{fundamental time}$   
 $\text{period of } x(t)$

$$x(t) = \int_{-\infty}^{\infty} x(\tau) s(t-\tau) d\tau$$

$$\phi_1(t) = e^{j\omega_0 t} \quad \phi_2(t) = e^{2j\omega_0 t}$$

$$\phi_{-1}(t) = e^{-j\omega_0 t} \quad \dots \quad \phi_{-2}(t) = e^{-2j\omega_0 t}$$

$$\phi_0(t) = e^{j0} = \cos 0 + j \sin 0$$

$$= 1$$

$$\langle \phi_k(t), \phi_l(t) \rangle_T \stackrel{\Delta}{=} \frac{1}{T} \int_T \phi_k(t) \phi_l^*(t) dt \equiv \text{Defining inner product}$$

$$= \frac{1}{T} \int_T e^{jk\omega_0 t} e^{-jl\omega_0 t} dt = 1 \text{ if } m=0$$

$$= \frac{1}{T} \int_T e^{j(m-l)\omega_0 t} dt$$

$$= \frac{1}{T} \int_0^T (e^{jm\omega_0 t}) dt = \frac{e^{jm\omega_0 T}}{jm\omega_0 T} \Big|_{m \neq 0} = \frac{e^{jm2\pi} - e^{j0}}{jm\omega_0 T} = 0$$

$$= 0 \quad \text{if } m \neq 0 \quad \text{if } k \neq l$$

$$= 1 \quad \text{if } k=l$$

Synthesis:  $x(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t)$   $\{\phi_k(t)\}_{k \in \mathbb{Z}}$   $\equiv$  Span of vector space

Analysis:  $a_k = \langle x(t), \phi_k(t) \rangle_T$   $T$  form an orthonormal basis (O.N.B.)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t)$$

$$\langle x(t), \phi_l(t) \rangle_T = \left\langle \sum_{k=-\infty}^{\infty} a_k \phi_k(t), \phi_l(t) \right\rangle_T = \langle \dots + a_{-10} \phi_{-10}(t) + \dots + a_0 \phi_0(t) + \dots + a_9 \phi_9(t) + \dots, \phi_l(t) \rangle_T$$

$$= \sum_{k=-\infty}^{\infty} (a_k \langle \phi_k(t), \phi_l(t) \rangle_T)$$

$$= \sum_{k=-\infty}^{\infty} a_k S[k-l]$$

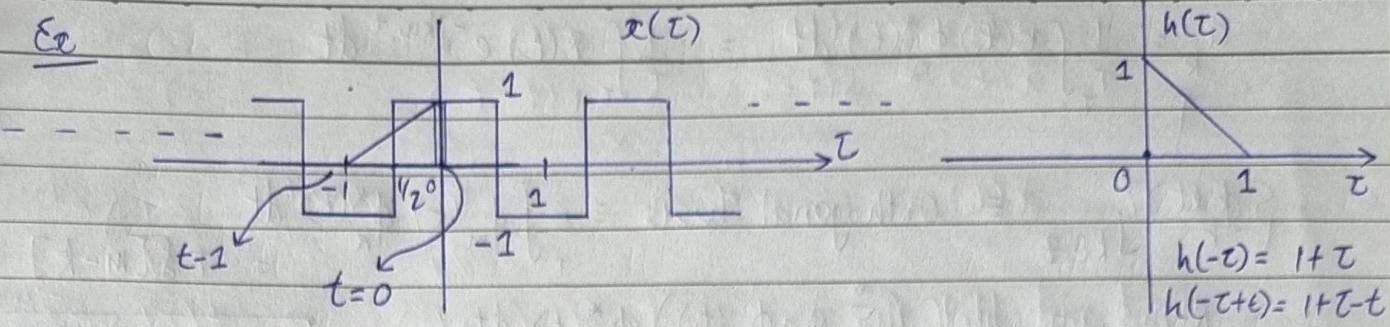
$$= \dots \langle a_{-10} \phi_{-10}(t), \phi_l(t) \rangle_T$$

$$+ \dots + \langle a_0 \phi_0(t), \phi_l(t) \rangle_T$$

$$+ \dots + \langle a_9 \phi_9(t), \phi_l(t) \rangle_T$$

=

Neelagagan  $\langle x(t), \phi_l(t) \rangle_T = a_l$

Ex(Convolve  $x(t) \times h(t)$ )

$$\begin{aligned} y(0) &= \int_{-1/2}^{1/2} x(\tau) h(t-\tau) d\tau + \int_{1/2}^0 x(\tau) (h(t-\tau)) d\tau \\ &= -\int_{-1/2}^{1/2} (-1)(1+\tau) d\tau + \int_{1/2}^0 (1)(1+\tau) d\tau \\ &= -\left(\tau + \frac{\tau^2}{2}\right) \Big|_{-1/2}^{1/2} + \left(\tau + \frac{\tau^2}{2}\right) \Big|_{-1/2}^0 \end{aligned}$$

Case 1:  $0 \leq t \leq 1/2$

$$y(t) = -\int_{-1/2}^{1/2} (1+\tau-t) d\tau + \int_{-1/2}^t (1+\tau-t) d\tau$$

C.T. Fourier Seriesperiodic signalsSynthesis

$$\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} a_k \phi_k(t) \\ &= \sum_{k=-\infty}^{\infty} a_k e^{j k \omega_0 t} \end{aligned}$$

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

$$\begin{array}{l} \downarrow \\ \text{rad/s} \\ \equiv \text{per sec} \\ \text{or Hz} \end{array}$$

$e^{jk\omega_0 t} = \phi_k(t) = \text{pure imaginary exponential (in power)}$

$T \equiv$  fundamental time period

$$x(t) = 3 + 2e^{j\omega_0 t} + e^{j2\omega_0 t}$$

$$= 3 + 2(\cos \omega_0 t + j \sin \omega_0 t) + \cos 2\omega_0 t + j \sin 2\omega_0 t$$

$$= \sum_{k=0}^2 a_k e^{j k \omega_0 t}$$

$$a_0 = 3$$

all other

$$a_1 = 2$$

$a_k$ 's for  $k \neq 0, 1, 2$

$$a_2 = 1$$

are zero

Analysis equation

$$a_k = \langle x(t), \phi_k(t) \rangle_T = \frac{1}{T} \int_T x(t) \phi_k^*(t) dt$$

$$a_k = \frac{1}{T} \int_T x(t) \phi_k^*(t) dt = \frac{1}{T} \int_T x(t) e^{-jkw_0 t} dt$$

$\{\phi_k(t)\}_{k \in \mathbb{Z}}$  = orthogonal basis =  $\int_T \phi_m(t) \phi_k^*(t) dt = T \delta[m-k]$

$$R.H.S = \frac{1}{T} \int_T \left( \sum_{m=-\infty}^{\infty} a_m \phi_m(t) \right) \phi_k^*(t) dt$$

$$= \frac{1}{T} \sum_{m=-\infty}^{\infty} \left[ a_m \left( \int_T \phi_m(t) \phi_k^*(t) dt \right) \right]$$

$$= \frac{1}{T} \sum_{m=-\infty}^{\infty} \left[ a_m \underbrace{\int_T e^{j(m-k)w_0 t} dt}_{\text{So}} \right]$$

$$= \frac{1}{T} \sum_{m=-\infty}^{\infty} a_m \cdot T \delta[m-k] = a_k = L.H.S.$$

$\phi_1(t), \phi_{-1}(t)$   
↓  
1st harmonic  
 $\phi_2(t), \phi_{-2}(t)$   
↓  
2nd harmonics &  
So on

Synthesis:  $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}$

Analysis:  $a_k = \frac{1}{T} \int_T x(t) e^{-jkw_0 t} dt$

$e^{st}$  act as eigenfunctions of an LTI system

where  $H(s) \equiv$  corresponding eigenvalue

$$x(t) \rightarrow \boxed{LTI} \rightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

Let us assume  $x(t) = e^{st}$

where  $s = j\omega_0$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) e^{s(t-\tau)} d\tau = \int_{-\infty}^{\infty} h(\tau) e^{st} e^{-s\tau} d\tau$$

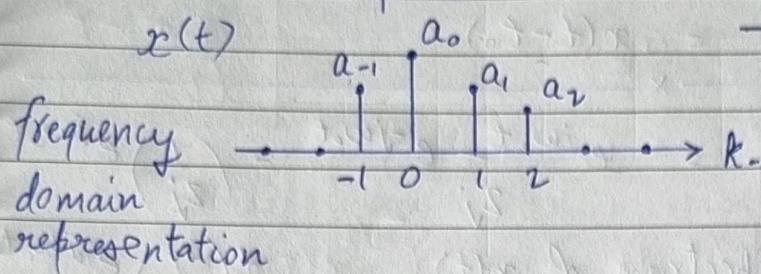
$$= e^{st} \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$$

$H(s)$

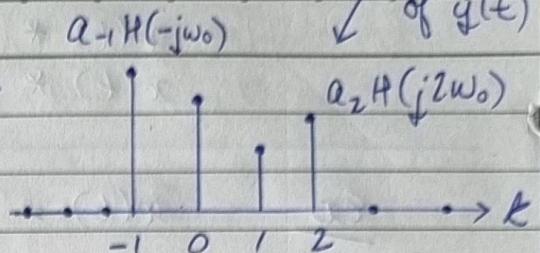
$$\frac{Av}{v} = \frac{1}{H(s)}$$

line  
spectrum  
of  $y(t)$

Given an line spectrum



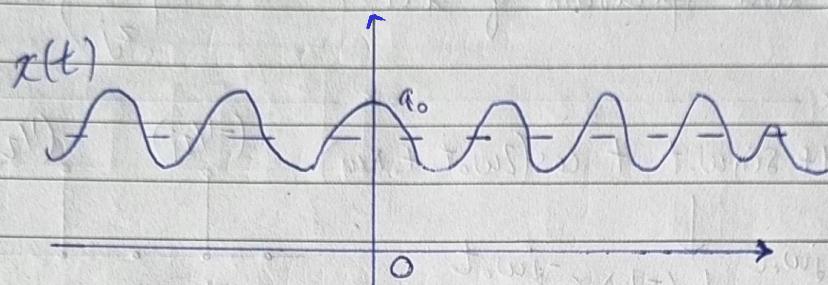
$LTI \rightarrow$   
System



$$\text{if } H(-j\omega_0) = 0$$

$$a_{-1}H(-j\omega_0) = 0$$

↓ this freq. will  
disappear in O/p



Given  $x(t) = a_{-1}e^{-j\omega_0 t} + a_0 + a_1e^{j\omega_0 t} + a_2e^{j2\omega_0 t}$

Ex

$$S: y(t) = x(t-2)$$

$$\text{let } x(t) = e^{j2t}$$

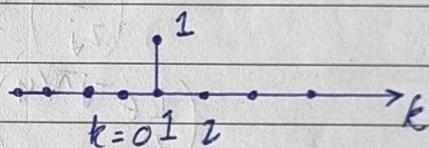
$$\omega_0 = 2 \text{ rad/s}$$

$$= 2\pi/T$$

$$T = \pi$$

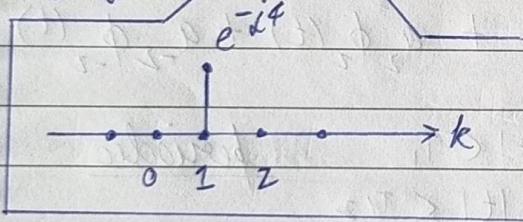
line spectrum  
of  $y(t)$

$$\begin{aligned} y(t) &= e^{j2(t-2)} \\ &= e^{j2t} \cdot e^{-j4} \\ &= e^{j2t} H(j2) \end{aligned}$$



$$H(j2) = e^{-j4}$$

$$S = j2$$



Ex

what is  $h(t)$ ?

$$h(t) = \delta(t-2)$$

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt = \int_{-\infty}^{\infty} s(t-2) e^{-st} dt$$

$$= \int_{-\infty}^{\infty} f(t-2) \cdot e^{-j2t} dt$$

$$\Rightarrow e^{-j4} \int_{-\infty}^{\infty} s(t-2) dt$$

$$= e^{-j4}$$

$$x(t) * \delta(t) = x(t)$$

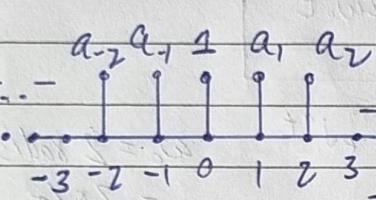
$$x(t) * \delta(t-t_0) = x(t-t_0)$$

Ex  $x(t) = \sin \omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \left(\frac{1}{z_j}\right) e^{j\omega_0 t} + \left(\frac{-1}{z_j}\right) e^{-j\omega_0 t}$

Find and plot the line spectrum of  $x(t)$

Ex  $x(t) = 1 + \sin \omega_0 t + \cos(2\omega_0 t + \pi/4)$

$$= 1 + \frac{e^{j\omega_0 t}}{z_j} + \left(\frac{-1}{z_j}\right) e^{-j\omega_0 t}$$



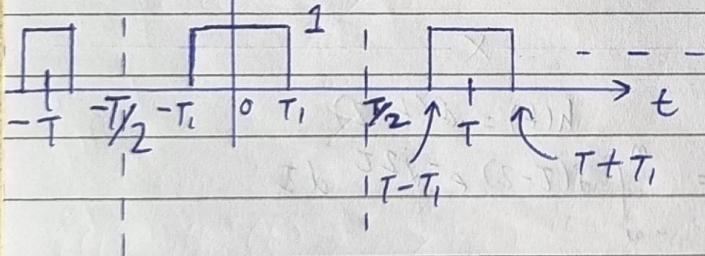
$$= 1 + \frac{1}{2} [e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}]$$

$$= 1 + \left(\frac{1}{z_j}\right) \phi_1(t) + \left(\frac{-1}{z_j}\right) \phi_{-1}(t) + \left(\frac{1}{2} e^{j\pi/4}\right) \phi_2(t) +$$

$$\left(\frac{1}{2} e^{-j\pi/4}\right) \phi_{-2}(t)$$

$$= a_0 + a_1 \phi_1(t) + a_{-1} \phi_{-1}(t) + a_2 \phi_2(t) + a_{-2} \phi_{-2}(t)$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T_1/2 \end{cases}$$



① if  $T \rightarrow \infty$ , signal will become aperiodic

② there should be a dc bias term, so  $a_0 \neq 0$

③

fundamental freq. . .

Solve it, plot the line spectrum

C.T. F.S① Linearity

$$\begin{aligned} x(t) &\xrightarrow[\text{F.S.}]{\text{F.S.}} a_k \\ y(t) &\xrightarrow[\text{F.S.}]{\text{F.S.}} b_k \end{aligned}$$

$x(t)$  &  $y(t)$  are periodic  
with fundamental time  
period =  $T$ .

$$z(t) = [\alpha x(t) + \beta y(t)] \xrightarrow{\quad} \alpha a_k + \beta b_k$$

&  $z(t) \equiv$  same fundamental time period  $T$ .

$$z(t) = \sum_{k=-\infty}^{\infty} c_k e^{j k \omega_0 t}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j k \omega_0 t}$$

$$c_k = \frac{1}{T} \int_T z(t) e^{-j k \omega_0 t} dt$$

$$= \frac{1}{T} \int_T (\alpha x(t) + \beta y(t)) e^{-j k \omega_0 t} dt = \underbrace{\alpha \cdot \frac{1}{T} \int_T x(t) e^{-j k \omega_0 t} dt}_{a_k}$$

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j k \omega_0 t}$$

$$+ \underbrace{\beta \cdot \frac{1}{T} \int_T y(t) e^{-j k \omega_0 t} dt}_{b_k}$$

$$= \alpha a_k + \beta b_k \rightarrow \text{Proved.}$$

② Time Shifting

$$x(t) \xrightarrow[\text{T}]{\text{F.S.}} a_k$$

$$\omega_0 = \frac{2\pi}{T}$$

Find F.S. coeff. of  $x(t-t_0)$ , where  $t_0 \equiv$  constant:

$$\begin{aligned} z(t) &= x(t-t_0) \xrightarrow{T} z = c_k = a_k e^{-j k \omega_0 t_0} \\ c_k &= \frac{1}{T} \int_{-T/2}^{T/2} z(t) e^{-j k \omega_0 t} dt = \frac{1}{T} \int_{-T/2}^{T/2} x(t-t_0) e^{-j k \omega_0 t} dt \\ &= \frac{1}{T} \int_{-T/2-t_0}^{T/2-t_0} x(\tau) e^{-j k \omega_0 (t_0+\tau)} d\tau \end{aligned}$$

let  $t-t_0 = \tau$

$dt = d\tau$

$$c_k = e^{-j k \omega_0 t_0} \left( \frac{1}{T} \int_T x(\tau) e^{-j k \omega_0 \tau} d\tau \right) = a_k e^{j k \omega_0 t_0}$$

③ Time-reversal

$$x(t) \xrightarrow[T]{F.S.} a_k$$

$$z(t) = x(-t) \xrightarrow[T]{F.S.} c_k = a_{-k}$$

$$\begin{aligned} T &= -t \\ dt &= -dt \end{aligned}$$

$$c_k = \frac{1}{T} \int_{-T/2}^{T/2} x(-t) e^{-jkw_0 t} dt = \frac{-1}{T} \int_{-T/2}^{T/2} x(t) e^{jkw_0 t} dt$$

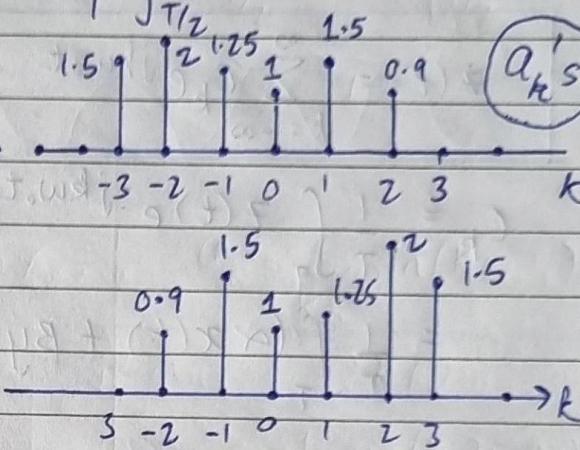
$$= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{jkw_0 t} dt$$

$$c_{-k} = \frac{1}{T} \int_T x(t) e^{-jkw_0 t} dt = a_k$$

$$c_k = a_{-k}$$

$$c_0 = a_0$$

$$c_1 = a_{-1}$$



④

$$x(t) \xrightarrow[T]{F.S.} a_k$$

Time period T

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}$$

$$z(t) = x^*(t) \longrightarrow a_{-k}^*$$

Case-1

(let  $x(t)$  is real)

$$x(t) = x^*(t)$$

$$\downarrow \qquad \downarrow$$

$$a_k = a_{-k}^*$$

$$a_k^* = a_{-k}$$

$$x(t) = x(t)$$

Case-2 (let  $x(t)$  is real & even.)

Conclusion:  $a_k'$ 's are also real & even.

$$a_k^* = a_{-k}$$

$$a_k = a_{-k}$$

$$\leftarrow a_k^* = a_k$$

Case-3  $x(t)$  is real & odd



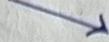
$$x(t) = x^*(t)$$



$$a_k^* = a_{-k}$$

$$a_k^* = -a_k$$

Coeff. are purely imaginary.



$$x(-t) = -x(t)$$



$$-a_k = +a_{-k}$$

### (5) Time Scaling

$$z(t) = \sum_{k=-\infty}^{\infty} c_k e^{j k(\omega_0 t)}$$

$$x(t) \xrightarrow[T]{F.S.} a_k \quad \alpha > 0 \quad \alpha \neq -1$$

$$z(t) = x(\alpha t) \xrightarrow[T/\alpha]{F.S.} c_k = ?$$

$$\begin{aligned} c_k &= \frac{1}{T/\alpha} \int_{-T/2\alpha}^{T/2\alpha} x(\alpha t) e^{-jk(\alpha \omega_0 t)} dt \quad \text{let } \alpha t = \tau \\ &= \frac{1}{T/\alpha} \int_{-T/2}^{T/2} x(\tau) e^{-jk\omega_0 \tau} \frac{d\tau}{\alpha} = a_k \end{aligned}$$

### (6) Multiplication

$$x(t) \xrightarrow[T]{F.S.} a_k$$

$$x(t) = \sum_{l=-\infty}^{\infty} a_l e^{j l \omega_0 t}$$

$$y(t) \xrightarrow[T]{F.S.} b_k$$

$$y(t) = \sum_{m=-\infty}^{\infty} b_m e^{j m \omega_0 t}$$

$$z(t) = x(t) y(t) \xrightarrow[F.S.]{T} c_k = ?$$

$$c_k = \frac{1}{T} \int_T z(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T} \int_T (x(t) y(t)) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T} \int_T \sum_l \sum_m a_l b_m e^{j(l+m)\omega_0 t} e^{-jk\omega_0 t} dt$$

$$= \sum_l \sum_m a_l b_m \left( \frac{1}{T} \int_T e^{-j(k-l-m)\omega_0 t} dt \right) = 1 \quad k-l-m=0 \\ \quad l=k-m$$

$$= \boxed{\sum_m a_{k-m} b_m = \sum_l a_l b_{k-l}}$$

multiplication in the time domain  $\longleftrightarrow$  convolution in the frequency domain

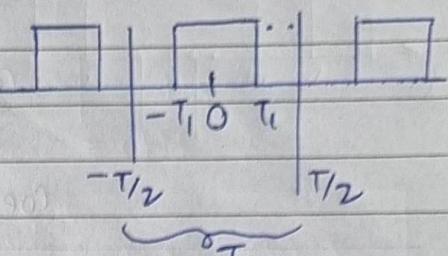
Fix  $T_1$

$\times$

$\times$

$$\textcircled{1} \quad T = 4T_1$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T/2 \end{cases}$$



$$\textcircled{2} \quad T = 8T_1$$

$$a_0 = \frac{1}{T} \int_T x(t) dt$$

$$= \frac{1}{T} \int_{-T_1}^{T_1} dt = \boxed{\frac{2T_1}{T} = a_0 = \frac{1}{2}}$$

$$a_k = \frac{1}{T} \int_{-T_1}^{T_1} e^{-jkw_0 t} dt \iff k \neq 0$$

$$a_k = \frac{1}{k\pi} \sin(Kw_0 T_1)$$

$$k \neq 0$$

$$= \frac{1}{k\pi} \sin\left(\frac{k2\pi}{4T_1}\right)$$

$$= \frac{1}{k\pi} \sin\left(\frac{k\pi}{2}\right)$$

### ⑦ Parseval's relation

$$x(t) \xrightarrow[T]{\text{F.S}} a_k$$

$$\frac{1}{T} \int_T |x(t)|^2 dt = \sum_k |a_k|^2$$

D.T. F.S

Given a discrete-time periodic signal  $x[n]$

$$\text{Synthesis} \equiv x[n] = \sum_{k=-N}^N a_k e^{jkw_0 n} \quad N = \text{Fundamental time period}$$

$$= \sum_{k=-N}^N a_k \phi_k[n]$$

$$0, w_0, 2w_0, \dots, (N-1)w_0, Nw_0, 0, \frac{2\pi}{N}, 2 \cdot \frac{2\pi}{N}, \dots, (N-1) \cdot \frac{2\pi}{N}, \frac{2\pi}{N} = 0$$

$$\phi_k[n] = e^{jkw_0 n}$$

$$\phi_{k+N}[n] = \phi_k[n] = e^{jkw_0 n}$$

$$\begin{aligned}
 \langle x[n], \phi_e[n] \rangle_N &= \frac{1}{N} \sum_{n \in \{N\}} x[n] e^{-j\omega_0 n} \\
 &= \frac{1}{N} \sum_{n \in \{N\}} \left( \sum_{k \in \{N\}} a_k e^{jk\omega_0 n} \right) e^{-j\omega_0 n} \\
 &= \frac{1}{N} \sum_{n \in \{N\}} \sum_{k \in \{N\}} a_k e^{j(k-l)\omega_0 n} \\
 &= \sum_{k \in \{N\}} a_k \underbrace{\left( \frac{1}{N} \sum_{n \in \{N\}} e^{j(k-l)\omega_0 n} \right)}_0 = 1 \quad \text{when } k=l
 \end{aligned}$$

$$= \boxed{a_l = \frac{1}{N} \sum_{n \in \{N\}} x[n] \phi_e^*[n]}$$

$$= \frac{1}{N} \sum_{n \in \{N\}} x[n] e^{-jk\omega_0 n}$$

Ex  $x[n] = \sin \omega_0 n$   $\omega_0 = \frac{2\pi}{N} \rightarrow$  periodic with period  $N$ .

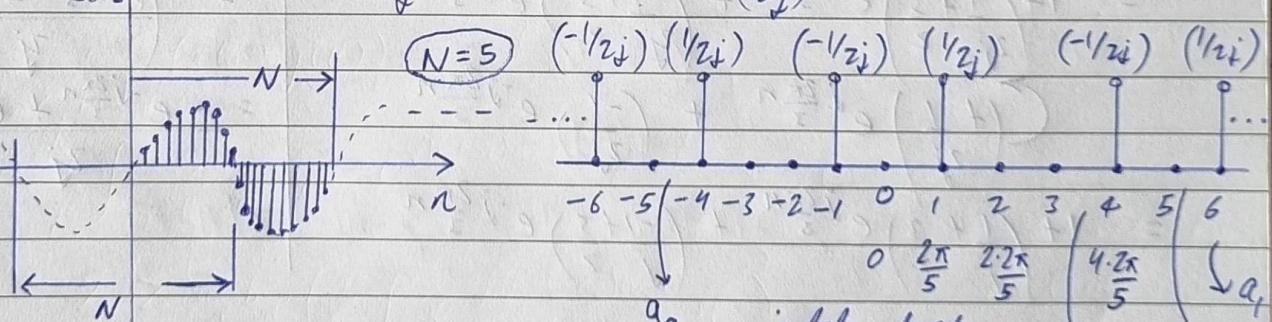
$$x[n] = \frac{1}{2j} [e^{j\omega_0 n} - e^{-j\omega_0 n}]$$

$$= \left(\frac{1}{2j}\right) e^{j\omega_0 n} + \left(\frac{-1}{2j}\right) e^{-j\omega_0 n}$$

line spectrum

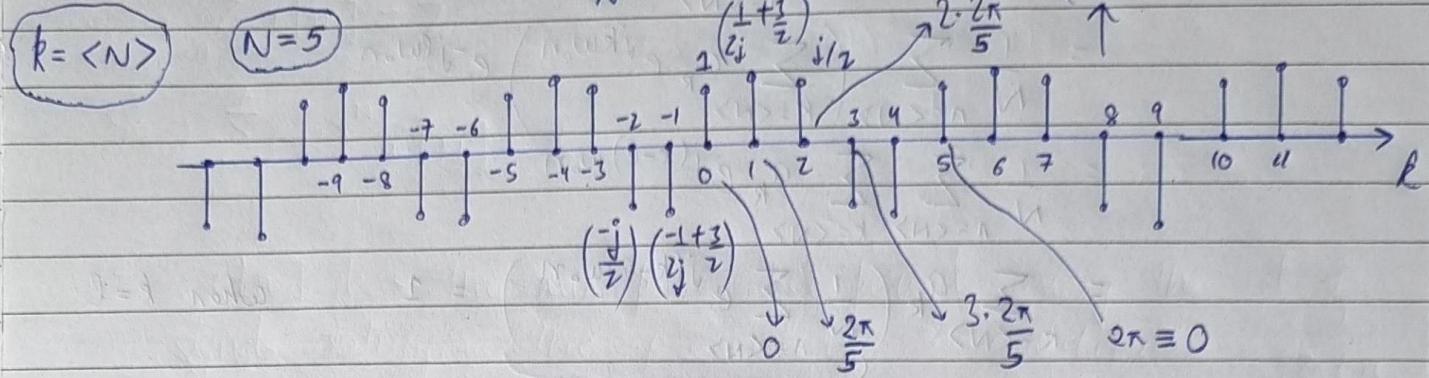
$a_1, a_{-1}$  are non-zero, rest all are zero in  $k \in \{N\}$ .

$$a_1 = \frac{1}{2j}; \quad a_{-1} = \left(\frac{-1}{2j}\right)$$



independent  
variable  
 $\equiv$  time

$$\text{Ex} \quad x[n] = 1 + \sin \frac{2\pi n}{N} + 3 \cos \frac{2\pi n}{N} + \cos \left( \frac{4\pi n + \pi}{2} \right)$$



N=7 2 zero amplitude pts. in one N

N=11 6 zero-amplitude pts. in one period in the frequency domain

$$\rightarrow -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, \frac{2 \cdot 2\pi}{3}, 2\pi \equiv 0$$

$$e^{j2 \cdot \frac{2\pi n}{3}} = e^{j(2\pi - \frac{2\pi}{3})n} \quad N=3$$

$$= (e^{j2\pi n}) \cdot e^{-j\frac{2\pi n}{3}}$$

$$= (e^{j2\pi n}) \cdot e^{-j\frac{2\pi n}{3}}$$

$$= e^{-j\frac{2\pi n}{3}}$$

$$x[n] = 1 + \sin \frac{2\pi n}{3} + 3 \cos \frac{2\pi n}{3} + \cos \left( \frac{4\pi n + \pi}{2} \right)$$

$$x[n] = 1 + \frac{1}{2} \left( e^{j\frac{2\pi n}{3}} - e^{-j\frac{2\pi n}{3}} \right) + \frac{3}{2} \left( e^{j\frac{2\pi n}{3}} + e^{-j\frac{2\pi n}{3}} \right) + \cancel{\cos \frac{4\pi n}{3}} \times 0$$

$$- \sin \frac{4\pi n}{3}$$

$$= 1 + \left( \frac{1+3}{2} \right) e^{j\frac{2\pi n}{3}} + \left( \frac{-1+3}{2} \right) e^{-j\frac{2\pi n}{3}}$$

$$\left( \frac{1+3}{2} \right) e^{j\frac{2\pi n}{3}} + \left( \frac{-1}{2} \right) e^{-j\frac{2\pi n}{3}}$$

$$= 1 + \left( \frac{1+3}{2} \right) e^{j\frac{2\pi n}{3}} + \left( \frac{-1+3}{2} \right) e^{-j\frac{2\pi n}{3}}$$

$$-\sin \frac{4\pi n}{3}$$

$$= - \frac{\left( e^{j\frac{4\pi n}{3}} - e^{-j\frac{4\pi n}{3}} \right)}{2j}$$

$$= - \frac{e^{-j\frac{2\pi n}{3}} + e^{j\frac{2\pi n}{3}}}{2j}$$

Ex  $x[n] \equiv$  periodic with period  $N$

Show that the F.S. coefficients of the periodic signal

$$g(t) = \sum_{k=-\infty}^{\infty} x[k] S(t - kT)$$

are periodic with period  $N$ .

Ex

①  $x(t)$  is a real signal

②  $x(t)$  is periodic with  $T = 4$ .

③  $a_k = 0$  for  $|k| > 1$

④ A signal with F.S. coeff.  $b_k = e^{-j\frac{2\pi k}{4}} a_k$  is odd

⑤  $\frac{1}{4} \int_0^4 |x(t)|^2 dt = \frac{1}{2}$     $\equiv$  average power in one time period

$$= |a_{-1}|^2 + |a_0|^2 + |a_1|^2 = \frac{1}{2}$$

$$x(t) \xrightarrow[T]{F.S.} a_k$$

$x(t)$  is real

$$z(t) = x(-t+1) \xrightarrow[T]{F.S.} b_k$$

$z(t) = x(-t+1)$

real & odd

$$b_0 = 0$$

$$|b_1|^2 + |b_{-1}|^2 = \frac{1}{2}$$

$$b_1 = \frac{j}{2}, b_{-1} = -\frac{j}{2}$$

$$|ja|^2 + |-ja|^2 = \frac{1}{2}$$

add & purely imaginary

$$b_1 = ja$$

$$b_{-1} = -ja$$

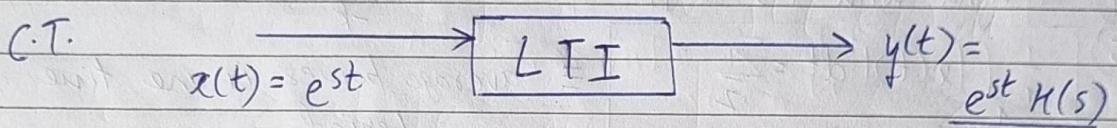
$$a^2 + a^2 = \frac{1}{2}$$

$$a^2 = \frac{1}{4}$$

$$a = \pm \frac{1}{2}$$

- ①  $x[n]$  is periodic with period  $N=6$
- ②  $\sum_{n=0}^5 x[n] = 2$
- ③  $\sum_{n=2}^5 (-1)^n x[n] = 1$
- ④  $x[n]$  has the min. power per period among the set of signals satisfying the preceding 3 conditions

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N} kn}$$

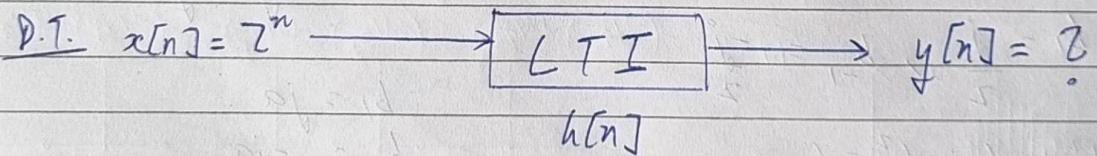


C.T. F.S.  $s_k = jkw_0$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{s_k t} \xrightarrow{} y(t) = \sum_{k=-\infty}^{\infty} a_k H(s_k) e^{s_k t}$$

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$



$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k] 2^{n-k}$$

$$= z^n \left| \sum_{k=-\infty}^{\infty} h[k] z^{-k} \right|$$

$$y[n] = z^n H(z) \uparrow$$

$$\text{let } z = e^{j\omega_0}$$

$$x[n] = e^{j\omega_0 n} \rightarrow e^{j\omega_0 n} H(e^{j\omega_0})$$

$$\text{D.T.F.S. } x[n] = \sum_{k=-N}^{\infty} a_k e^{j k \omega_0 n} \xrightarrow[\text{system } h[n]]{LT} y[n] = \sum_{k=-N}^{\infty} a_k H(e^{j k \omega_0}) e^{j k \omega_0 n}$$

$$x[n] = \sum_{k=-5}^{\infty} a_k e^{j k \frac{2\pi}{5} n}$$

$$a_0 = 1$$

$$= 1 + \cos \omega_0 n + \frac{3}{2} \cos 2\omega_0 n$$

$$N=5$$

$$a_1 = a_{-1} = \frac{1}{2}$$

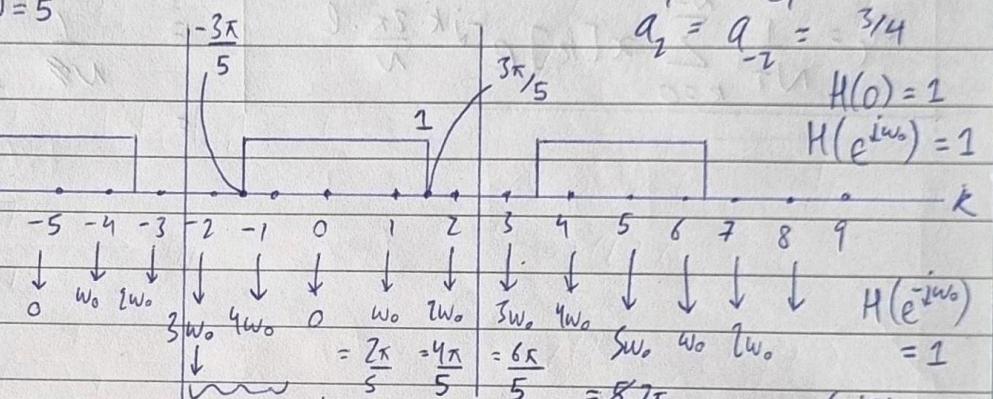
$$a_2 = a_{-2} = \frac{3}{4}$$

$$H(0) = 1$$

$$H(e^{j\omega_0}) = 1$$

$$3\omega_0 = \frac{6\pi}{5} = 2\pi - \frac{4\pi}{5}$$

$$= -\frac{4\pi}{5}$$

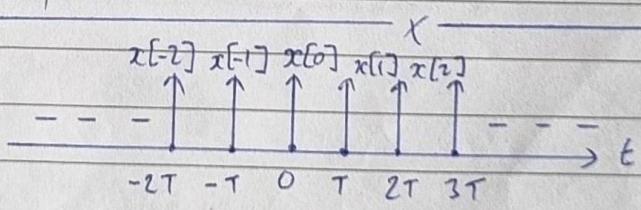


Ex  $x[n]$  is a periodic signal,  $N$ . Show that F.S. coeff. of a periodic signal  $g(t) = \sum_{k=-\infty}^{\infty} x[k] s(t-kT)$

( $N=3$ )

are also periodic with period  $N$ .

$$g(t) = x[0] s(t) + x[1] s(t-T) + x[2] s(t-2T) + \dots + x[-1] s(t+T)$$



$g(t)$  is periodic with  $NT$ .

$$g(t) = \sum_{k=-\infty}^{\infty} a_k e^{j k \frac{2\pi}{NT} t}$$

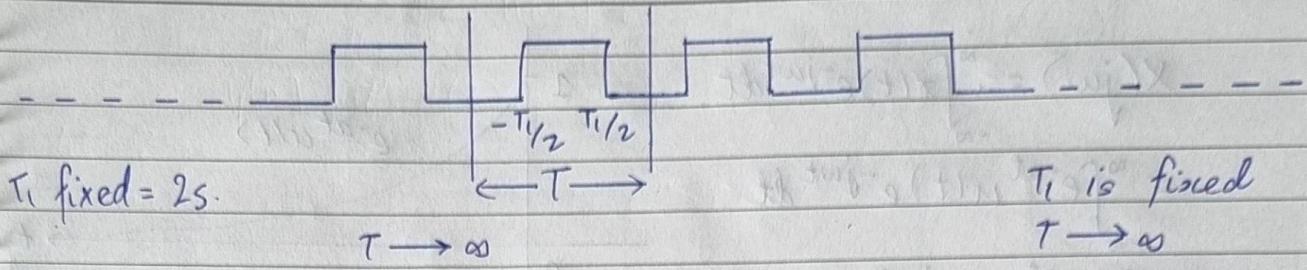
$$a_0 = \frac{1}{NT} \int_{NT} g(t) dt = \frac{x[0] + x[1] + \dots + x[N-1]}{NT}$$

$$a_l = \frac{1}{NT} \int_{NT} g(t) e^{-j l \frac{2\pi}{NT} t} dt$$

$$\begin{aligned} a_l &= \frac{1}{NT} \int_{NT} \sum_{k=-\infty}^{\infty} x[k] s(t-kT) e^{-j l \frac{2\pi}{NT} t} dt \\ &= \frac{1}{NT} \int_0^{NT} \sum_{k=0}^{N-1} x[k] e^{-j l \frac{2\pi}{NT} \cdot kT} s(t-kT) dt \\ &= \frac{1}{NT} \sum_{k=0}^{N-1} x[k] \int_0^{NT} e^{-j lk \cdot \frac{2\pi}{N}} s(t-kT) dt \end{aligned}$$

$$a_l = \frac{1}{NT} \sum_{k=0}^{N-1} x[k] e^{-j k \cdot \frac{2\pi}{N} \cdot l} = \frac{1}{N} \sum_{n=0}^{N-1} \left( \frac{x[n]}{T} \right) e^{-j l \cdot \frac{2\pi n}{N}}$$

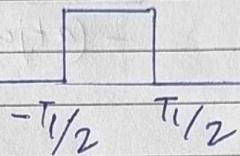
## Fourier Transform



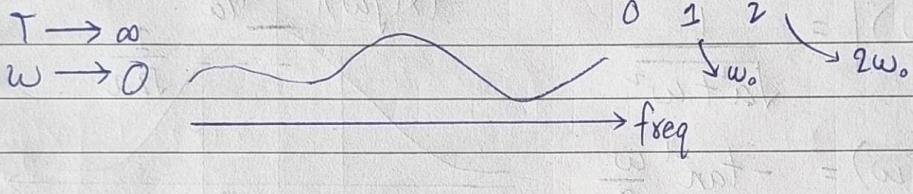
$$T = 4T_1 \Rightarrow \omega_0 = \frac{2\pi}{8}$$

$$T = 8T_1 \Rightarrow \omega_0 = \frac{2\pi}{16}$$

F.T.



as  $T \rightarrow$  increasing  
 $\omega_0 \rightarrow$  decreasing



frequency  $\rightarrow k$

C.T. F.T.  $\text{Synthesis} \equiv x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$

F.S.  $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

Inverse F.T.

We will use

analysis  $\equiv x(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

F.T.

$$a_k = \frac{1}{T} \int_T x(t) e^{-j\omega_0 t} dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

$$\omega = 2\pi f$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

Ex  $x(t) = e^{-at} u(t)$   $a > 0$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt = \left[ \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty} = \frac{1}{a+j\omega}$$

~~$t=0$~~

$$\begin{aligned} & -at - j\omega t \\ & e^{-at} e^{-j\omega t} \\ & = 1 \cdot e^{-j0} \\ & = 1 \end{aligned}$$

$e^{-at} u(t)$

0

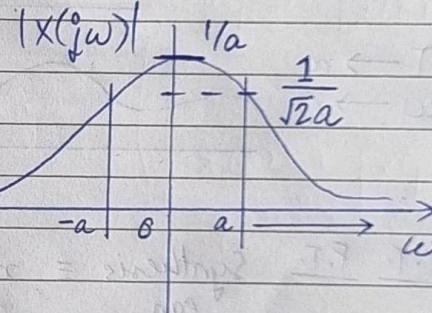
T

$$\begin{aligned} & t=\infty \\ & e^{-a \cdot \infty} \cdot e^{-j\omega \cdot \infty} \\ & 0 \times ? = 0 \end{aligned}$$

$$|X(j\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}$$

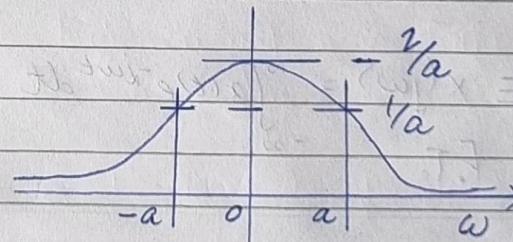
$$X(j\omega) = -\tan^{-1} \frac{\omega}{a}$$

$a$  is increasing



Ex-2  $x(t) = e^{-alt}$   $a > 0$

$$X(j\omega) = \frac{2a}{a^2 + \omega^2}$$



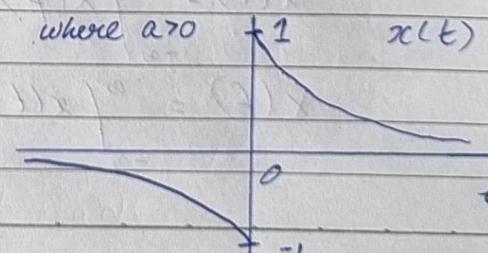
$\log A^2 / \log 10$

Max  
ampli A  
 $A^2/2$

Ex-3

$$x(t) = \begin{cases} e^{-at} & t > 0 \\ 0 & t = 0 \\ -e^{-at} & t < 0 \end{cases}$$

where  $a > 0$



$$X(j\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt - \int_{-\infty}^0 e^{at} e^{-j\omega t} dt$$

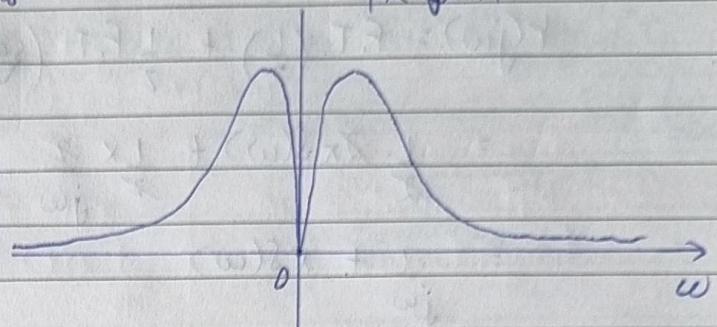
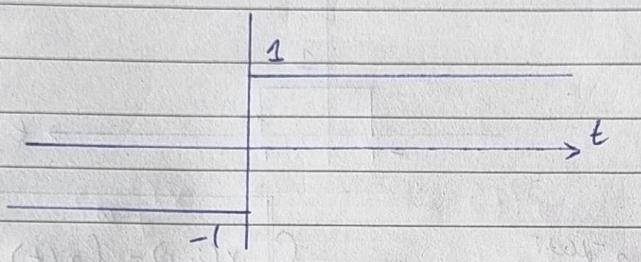
$$= \frac{-2j\omega}{a^2 + \omega^2}$$

$$|X(j\omega)|$$

Case: let  $a \rightarrow 0$

$$x(t) = \text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

$$\psi(t) = \frac{1}{2}(1 + \text{sgn}(t)) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



$$X(j\omega) = \frac{2}{j\omega}$$

Ex: 4

$$x(t) = \delta(t)$$

$$x(t) \xrightarrow{\frac{F.O.T.}{I.O.F.O.T.}} X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$$

$$= 1$$

$$\delta(t) \xrightarrow{\quad} 1$$

$$1 \xrightarrow{\quad} 2\pi \delta(\omega)$$

Ex: 5

$$X(j\omega) = 2\pi \delta(\omega)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi \delta(\omega) e^{j\omega t} d\omega$$

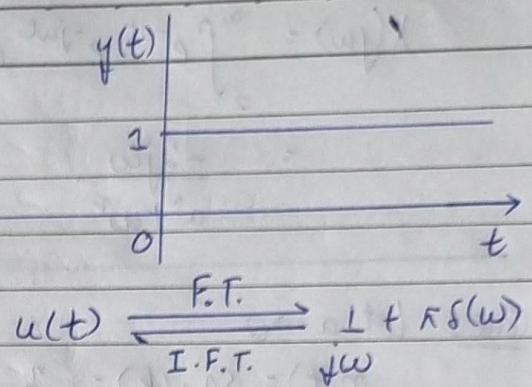
$$= 1$$

$$\text{Ex-6} \quad y(t) = \frac{1}{2} (1 + \text{sgn}(t)) = u(t)$$

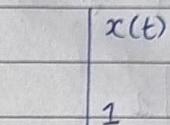
$$Y(j\omega) = \text{F.T.}\left(\frac{1}{2}\right) + \frac{1}{2} \text{F.T.}(\text{sgn}(t))$$

$$= \frac{1}{2} \cdot \frac{1}{j\omega} \pi S(\omega) + \frac{1}{j\omega} \times \frac{\pi}{j\omega}$$

$$= \frac{1}{j\omega} + \pi S(\omega)$$



$$x(t) = \begin{cases} 1 & -T \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

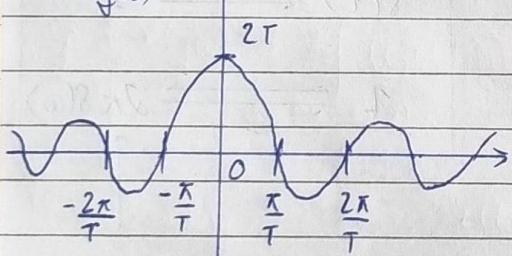


$$X(j\omega) = \int_{-T}^T e^{-j\omega t} dt$$

$$= \frac{e^{-j\omega T}}{-j\omega} \Big|_{-T}^T = \frac{e^{j\omega T} - e^{-j\omega T}}{j\omega}$$

$$= \frac{2}{\omega} \left( \frac{e^{j\omega T} - e^{-j\omega T}}{2j} \right) = \left( \frac{2}{\omega} \sin \omega T \right)$$

$$X(j\omega)$$

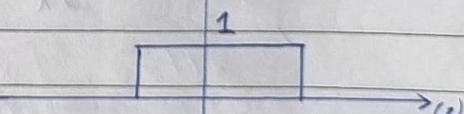
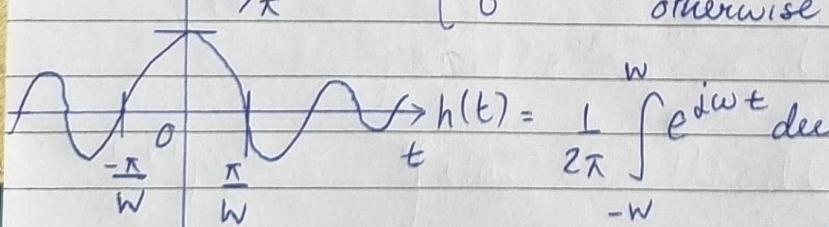


$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$H(j\omega) = \begin{cases} 1 & -W \leq \omega \leq W \\ 0 & \text{otherwise} \end{cases}$$

$$H(j\omega)$$



$$h(t) = \frac{1}{2\pi} \int_{-W}^W e^{j\omega t} d\omega = \frac{e^{j\omega t} - e^{-j\omega t}}{j2\pi t}$$

$$= \frac{1}{\pi t} \sin \omega t$$

Neelgagan

$\text{rect}\left(\frac{t}{2T}\right)$	$\xrightarrow{\text{F.T.}}$	$\frac{2 \sin \omega T}{\omega}$	time domain to freq
$\xleftarrow{\text{I.F.T.}}$			$t \rightarrow -\omega$
$\frac{1}{2\pi} \left\{ 2 \cdot \frac{\sin \omega t}{t} \right\}$	$\xrightarrow{\text{F.T.}}$	$\text{rect}\left(\frac{\omega}{2\omega}\right)$	$\times \text{ multiply by } 2\pi$
$\xleftarrow{\text{I.F.T.}}$			freq. domain to time dom
			$\omega \rightarrow -t$
			$\times \text{ divide by } 2\pi$

$$\text{rect}\left(\frac{t}{2T}\right) = x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \frac{2 \sin \omega T}{\omega} \right) e^{j\omega t} d\omega \xrightarrow{\quad} x(j\omega)$$

Change  $t \rightarrow -\omega$  by multiply by  $2\pi$

$$x(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(t) e^{-j\omega t} dt$$

$$2\pi x(-\omega) = \int_{-\infty}^{\infty} X(t) e^{-j\omega t} dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

↓  
Change  $\omega$  to  $-t$

$$x(-t) = \int_{-\infty}^{\infty} x(-\omega) e^{j\omega t} d\omega$$

divide by  $2\pi$

$$x(-t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(-\omega) e^{j\omega t} d\omega$$

Ex

$$g(t) = \frac{2}{1+t^2} \quad \text{Find } G(j\omega).$$

$$G(j\omega) = \int_{-\infty}^{\infty} \left( \frac{2}{1+t^2} \right) e^{-j\omega t} dt$$

Ex

$$x(t) = e^{-|t|}$$

Prove it ↓

$$X(j\omega) = \frac{2}{1+\omega^2}$$

Neelgagan

$$e^{-|t|} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \frac{2}{1+\omega^2} \right) e^{j\omega t} d\omega$$

Charge  $t$  with  $-\omega$

$$2\pi e^{-|t|} = \int_{-\infty}^{\infty} \left( \frac{2}{1+\omega^2} \right) e^{-j\omega t} d\omega$$

Properties — ① Linearity

$$\begin{array}{ccc} x(t) & \xleftrightarrow{} & X(j\omega) \\ g(t) & \xleftrightarrow{} & G(j\omega) \\ ax(t) + bg(t) & \xleftrightarrow{} & aX(j\omega) + bG(j\omega) \end{array}$$

② Differentiation in time domain

$$\begin{array}{ccc} x(t) & \xleftrightarrow{} & X(j\omega) \\ \frac{d}{dt}x(t) & \xleftrightarrow{} & j\omega X(j\omega) \end{array}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$\frac{d}{dt}x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (X(j\omega) j\omega) e^{j\omega t} d\omega$$

③ Differentiation in the freq. domain

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$\begin{array}{ccc} x(t) & \xleftrightarrow{} & X(j\omega) \\ tx(t) & \xleftrightarrow{} & j \frac{d}{d\omega} X(j\omega) \end{array}$$

$$\frac{d}{d\omega} X(j\omega) = \int_{-\infty}^{\infty} x(t) (-jt) e^{-j\omega t} dt$$

$$\int \frac{d}{d\omega} X(j\omega) = \int_{-\infty}^{\infty} (tx(t)) e^{-j\omega t} dt$$

④ Scaling

$$\begin{array}{ccc} x(t) & \xleftrightarrow{} & X(j\omega) \\ x(at) & \xleftrightarrow{} & \frac{1}{|a|} X(j\frac{\omega}{a}) \end{array}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} x(at) e^{-j\frac{\omega}{a}at} a \cdot dt$$

$$\frac{1}{a} X(j\omega) = \int_{-\infty}^{\infty} x(at) e^{-j(\omega a)t} dt$$

$$a > 0$$

$$t = at$$

$$dt = adt$$

$$\omega' = \omega a$$

$$\omega = \frac{\omega}{a}$$

$$\frac{1}{a} X(j\omega) = \int_{-\infty}^{\infty} x(at) e^{-j\omega t} dt \Rightarrow$$

⑤ Parseval's relation

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$\begin{aligned} \int_{-\infty}^{\infty} x(t)x^*(t) dt &= \int_{-\infty}^{\infty} x(t) \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(j\omega) e^{-j\omega t} d\omega \right] dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(j\omega) \left( \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \right) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(j\omega) X(j\omega) d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega \quad \text{proved} \end{aligned}$$

$|X(j\omega)|^2 \equiv \text{Energy spectrum}$

⑥ Convolution in time domain  $\Rightarrow$  Multiplication in the freq. domain

$$\int_{-\infty}^{\infty} x(t) h(t-\tau) dt \longleftrightarrow X(j\omega) H(j\omega)$$

$$\begin{aligned} x(t) &\longleftrightarrow X(j\omega) \\ h(t) &\longleftrightarrow H(j\omega) \end{aligned}$$

⑦ Multiplication in time domain  $\Rightarrow$  Convolution in the freq. domain

$$r(t) g(t) \longleftrightarrow \frac{1}{2\pi} (R(j\omega))^* G(j\omega)$$

Convolution

Ex. impulse response  $h(t) = e^{-at} u(t) \quad a > 0$

$$\begin{aligned} &\int \frac{d}{d\omega} \left( \frac{1}{at+j\omega} \right) \\ &= \frac{1}{(at+j\omega)^2} \end{aligned}$$

i/p  $x(t) = e^{-bt} u(t) \quad b > 0$

$y(t) = x(t) * h(t)$

Case I:  $a = b$

$$X(j\omega) = \frac{1}{b+j\omega}$$

$$H(j\omega) = \frac{1}{a+j\omega}$$

$$\therefore a=b \Rightarrow X(j\omega) H(j\omega) = \left(\frac{1}{a+j\omega}\right)^2 = P(j\omega) = \frac{d}{d\omega} \left(\frac{1}{a+j\omega}\right)$$

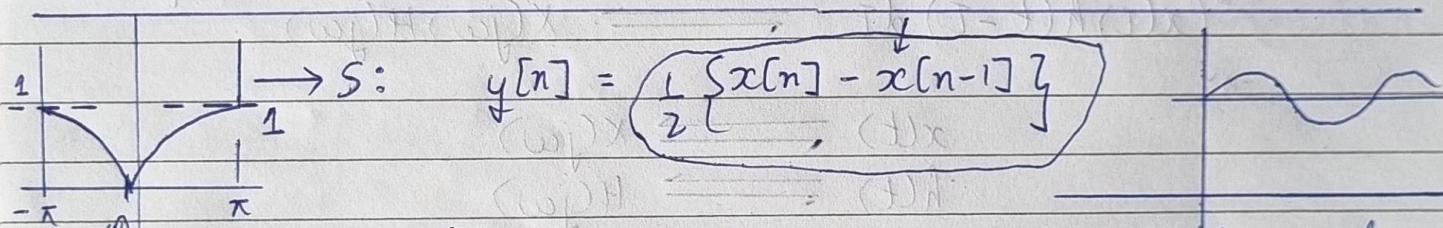
$$y(t) = te^{-at} u(t)$$

Case II  $b \neq a$

$$Y(j\omega) = \left(\frac{1}{a+j\omega}\right) \left(\frac{1}{b+j\omega}\right)$$

$$Y(j\omega) = \frac{1}{b-a} \left[ \frac{1}{(a+j\omega)} - \frac{1}{(b+j\omega)} \right]$$

$$y(t) = \frac{1}{b-a} e^{-at} u(t) - \left(\frac{1}{b-a}\right) e^{-bt} u(t)$$



Find the frequency response of the corresponding system -

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{2} \left\{ x[n] - x[n-1] \right\} \\ \text{Let } x[n] &= e^{j\omega n} \Rightarrow e^{j\omega n} H(e^{j\omega}) \\ y[n] &= \frac{1}{2} e^{j\omega n} - \frac{1}{2} e^{j\omega(n-1)} \\ &= e^{j\omega n} \left[ \frac{1 - e^{-j\omega}}{2} \right] \\ x[n] \rightarrow & \frac{1}{2} \left\{ x[n] - x[n-1] \right\} = e^{j\omega n} \left[ \frac{1 - e^{-j\omega}}{2} \right] \quad H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{2} \\ & \text{OR } H(e^{j\omega}) \cdot e^{j\omega n} = j e^{-j\omega/2} \left[ e^{j\omega/2} - e^{-j\omega/2} \right] \end{aligned}$$

$$\begin{aligned} z^n &\rightarrow H(z) z^n \\ e^{j\omega n} &\rightarrow H(e^{j\omega}) e^{j\omega n} \\ \text{let } z &= e^{j\omega} \\ &= j e^{-j\omega/2} \sin \frac{\omega}{2} \end{aligned}$$

$$x(t) = \cos \omega_0 t \rightarrow \cos \omega_0 t = \frac{1}{2} (e^{j\omega_0 t} + e^{-j\omega_0 t})$$

(M1)

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt.$$

$$\hookrightarrow = \frac{1}{2} [ e^{j(\omega_0 - \omega)t} - \dots ] \quad * \text{ can't do.}$$

(M2) we know  $\stackrel{!}{\rightarrow} 1 \rightleftharpoons 2\pi \delta(\omega) \quad \text{--- (1)}$

$$x(j(\omega - \omega_0)) \rightleftharpoons e^{j\omega_0 t} x(t) \quad \text{--- (2)}$$

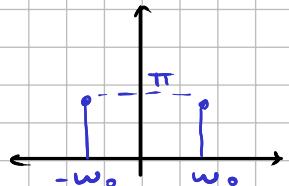
using (1) in (2)

$$\swarrow \hookrightarrow 2\pi \delta(\omega - \omega_0) \rightleftharpoons e^{j\omega_0 t}.$$

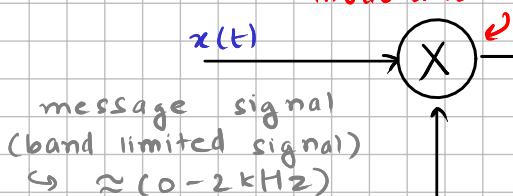
so  $\cos \omega_0 t = \frac{1}{2} [ 2\pi \delta(\omega - \omega_0) + 2\pi \delta(\omega + \omega_0) ]$

$$\cos \omega_0 t = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$$

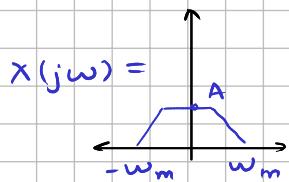
\* modulation & carrier signal



modulation (multi)



carrier signal  
( $\cos \omega_0 t$ )  
 $\hookrightarrow$  can be from crystal oscillator



$$\text{calc } R(j\omega) = ?$$

$$\text{so } r(t) = x(t) \cdot \cos \omega_0 t. \quad \text{if } \cos(\omega_0 t) = g(t)$$

multiplication in time domain  $\rightarrow$  convolution in freq. domain.

$$\begin{aligned} \text{so } \rightarrow R(j\omega) &= \frac{1}{2\pi} \int x(j\omega) * g(j\omega) d\omega \\ &= \frac{1}{2\pi} [ x(j\omega) * \pi [ \delta(\omega - \omega_0) + \delta(\omega + \omega_0) ] ] \\ &= \frac{1}{2} [ x(j\omega) * [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] ] \end{aligned}$$

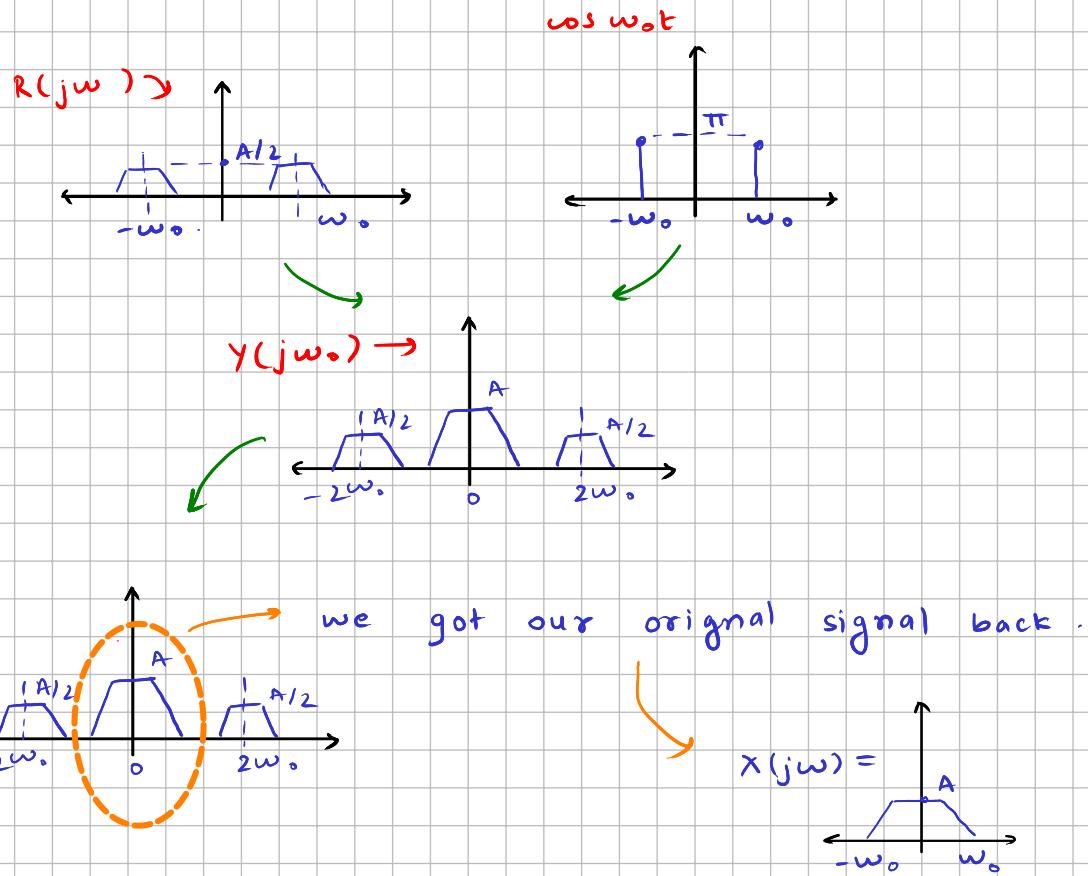
$R(j\omega) \propto$



$$\left[ x(\omega) * \delta(\omega - \omega_0) \right] = x(\omega_0)$$

now lets convolute  $r(t)$  with  $g(t)$

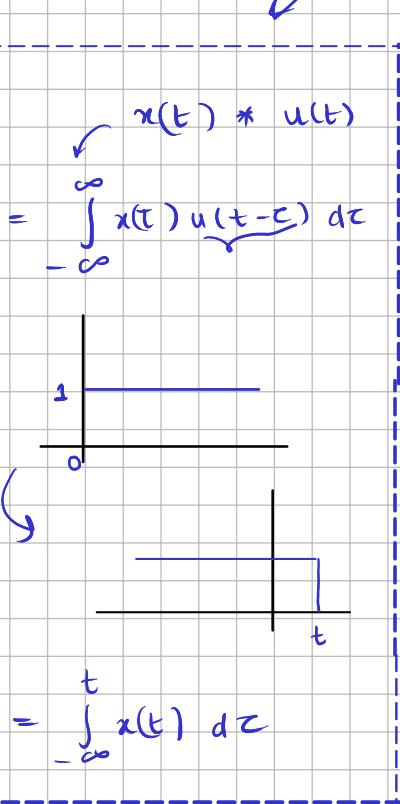
$$\hookrightarrow y(t) = r(t) \cdot \cos \omega_0 t$$



### Integration Property ↴

$$x(t) \iff X(j\omega)$$

$$\int_{-\infty}^t x(\tau) d\tau \iff ?$$



$$x(t) \cdot u(t) \iff X(j\omega) \cdot U(j\omega)$$

$$= X(j\omega) \cdot \left[ \frac{1}{j\omega} + \pi \delta(\omega) \right]$$

$$\int_{-\infty}^t x(\tau) d\tau \iff \frac{x(j\omega)}{(j\omega)} + \pi x(0) \delta(\omega)$$

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \rightarrow \text{periodic}$$

[we use F.T for aperiodic  
but it works for  
periodic as well]