ECE250: Signals & Systems

Monsoon 2023

End Semester Examination

Date: **1/12/2023** Duration: **2.00 Hours** Total Points: **34 Points**

Instructions

- **Please do not plagiarize. Any act of plagiarism will be dealt with strictly as per the institute's policy.**
- Please provide proper mathematical justifications with your answers. No marks will be awarded without a valid justification.

[CO1, CO2, CO5] Q1 [4 marks]: The Z-transform of the signal $x[n]$ is $X(z)$, where

$$X(z) = log(1 + az^{-1}) \qquad |z| > a$$

Determine the value of x[n].

[CO1, CO2, CO4] Q2 [6 marks]: A causal LTI system with impulse response $h(t)$ has the following properties:

(1) When the input to the system is $x(t) = e^{2t}$ for all "t", the output is $y(t) = (1/6)e^{2t}$ for all "t".

(2) The impulse response $h(t)$ satisfies the differential equation

$$\frac{dh(t)}{dt} + 2h(t) = (e^{-4t})u(t) + bu(t),$$

where "b" is an unknown constant.

Determine the system function $H(s)$ of the system, consistent with the information above. There should be no unknown constants in your answer; that is, the constant "b" should not appear in the answer.

[CO1, CO2, CO3] Q3 [5 marks]: Consider a causal discrete-time LTI system whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n] - (1/4)y[n-1] = x[n]$$

Find the Fourier series representation of the output $y[n]$ for the following input:

$$x[n] = \cos\left(\frac{\pi}{4}n\right) + 2\sin\left(\frac{\pi}{3}n\right)$$

[CO1, CO2, CO4] Q4 [2+4 marks]: A causal LTI system is described by the difference equation

$$y[n] - ay[n-1] = bx[n] + x[n-1],$$

where "a" is real and is less than "1" in magnitude.

(a) Find the value of "b" such that the frequency response of the system satisfies

$$|H(e^{jw})| = 1, \qquad \text{for all "}w\text{"}$$

(b) Find and plot the output of this system with $a = -\left(\frac{1}{2}\right)$, when the input is

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

[CO1, CO2, CO3] Q5 [3+3+3 marks]: A signal $x(t) = 10 * \cos(2\pi f_m t)$ that undergoes impulse-train sampling to generate

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT_s).\delta(t - nT_s)$$

This is passed through a LPF (given in Figure-1). Consider the following cases:

(a) $f_m = 2\ KHz,\ f_s = 3\ KHz\ and\ f_c = f_s/2\ KHz$
(b) $f_m = 0.5\ KHz,\ f_s = 3\ KHz\ and\ f_c = \frac{f_s}{2} KHz$
(c) $f_m = 4.8\ KHz,\ f_s = 3\ KHz\ and\ f_c = f_s/2\ KHz$

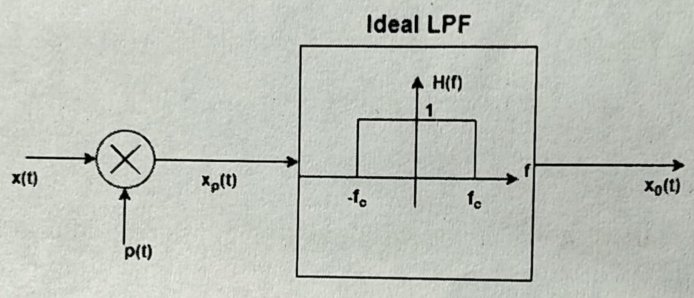

Figure 1

For each of the above cases (a) to (c):

(i) Draw $X_p(f)$
(ii) Draw $X_0(f)$
(iii) Find the frequency of the reconstructed signal $x_0(t)$.

[CO1, CO2, CO4] Q6 [1+1+1+1 marks]: A message signal x(t) is transmitted over ideal noiseless channel using the below block diagram (Figure-2) and it is reconstructed at receiver side as the output of ideal LPF (Figure-2).

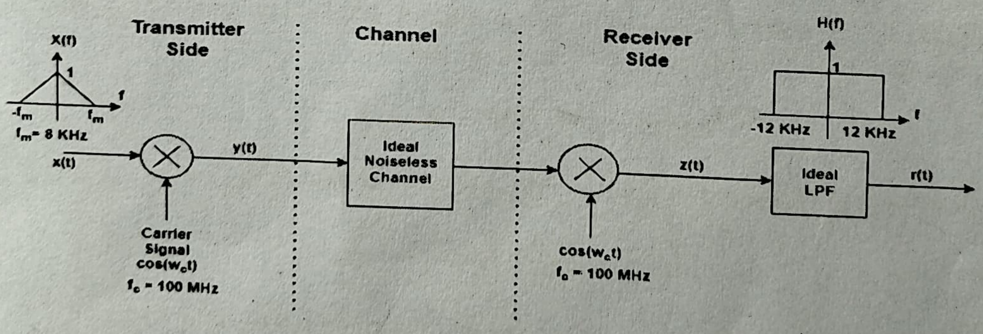

Figure 2

(a) Draw $Y(f)$
(b) Draw $Z(f)$
(c) Draw $R(f)$
(d) Also, write your inference on the final output.