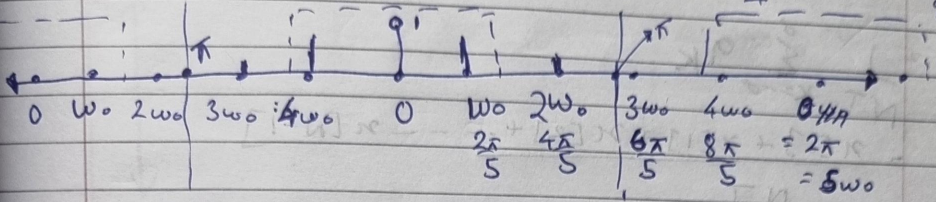
$$x[n] = \sum_{k=\langle 5\rangle} a_k e^{jk\frac{2\pi}{5}n} \qquad \boxed{a_0 = 1}$$

$$\boxed{N=5} \qquad \boxed{a_1 = a_{-1}} = \frac{1}{2} \qquad \boxed{a_2 = a_{-2}} = \frac{3}{4}$$

$$= 1 + \frac{1}{2} e^{j\frac{2\pi}{5}n} + \frac{1}{2} e^{-j\frac{2\pi}{5}n} + \frac{3}{4} e^{j4\pi/5\, n} + \frac{3}{4} e^{-j4\pi/5\, n}$$




$$x[n] = 1 + \cos(\omega_0 n) + \frac{3}{2}\cos(2\omega_0 n)$$

$$X(e^{-j\omega_0}) = 1$$

$$X(e^{+2j\omega_0}) = 0$$

$$X(e^{-2j\omega_0}) = 0$$

$x[n]$

$$g(t) = \sum_{k=-\infty}^{\infty} x[k]\,\delta(t + kT)$$

$$x[0] + x[1]\,\delta(t-T)$$

$$\cancel{x[2] + x[\ldots]}$$

$$x[2]\,\delta(t-2T)$$

$$+ x[-1]\,\delta(t+T)$$

g(t) is periodic with NT

$$a_0 = \frac{1}{NT}\int_{NT} \sum_{k=-\infty}^{\infty} e^{jn\frac{\omega_0 t}{NT}}$$

$$= \frac{a_k}{NT} \sum \int e^{jn}.$$

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$$a_0 = \frac{1}{NT}\int_{NT} g(t)\, dt$$

$$= \frac{1}{NT}\sum_{k=-\infty}^{\infty}\int_{NT} a_k\, e^{jk\frac{2\pi}{NT}t}$$

$$= \frac{1}{NT}\sum_{k=-\infty}^{\infty} a_k$$

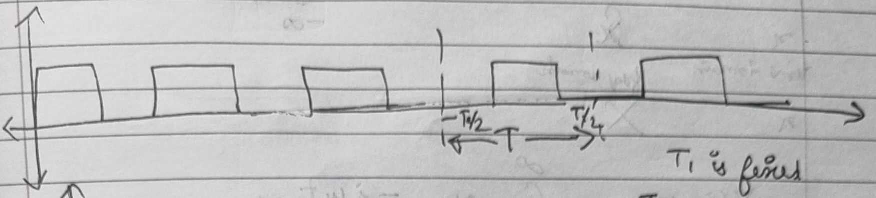
$$= \frac{x[0]+x[1]+x[2]+\cdots x[N-1]}{NT}$$

$$a_k = \frac{1}{NT}\int_{NT} g(t)\, e^{-jk\frac{2\pi}{NT}t}\, dt$$

$$= \frac{1}{NT}\int_{NT}\sum_{k=\infty}^{\infty} x[k]\,\delta(t-kT)\, e^{-jk\frac{2\pi}{NT}t}\, dt$$

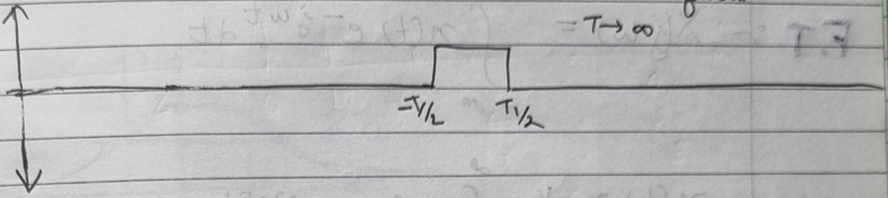
$$= \frac{1}{NT}\int_{0}^{NT}\sum_{k=0}^{N-1} x[k]\,\cdot$$

# Fourier Transform :-




T1 is fixed

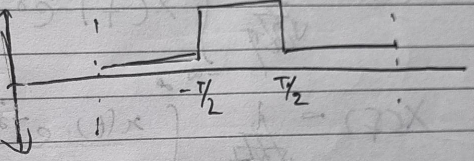
$T \to \infty$




$T_1$ is fixed

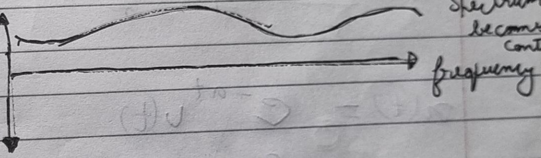
$T = 4T_1 \quad \omega_0 = 2\pi/8$

$T = 8T_1 \quad \omega_0 = 2\pi/16$



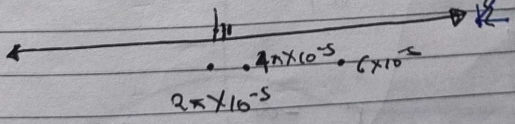

as $T \to \infty$

$\omega_0 \to 0$




$T = 1,00,000$

$$\omega_0 = \frac{2\pi}{100,000} = 2\pi \times 10^{-5}$$




K

Frequency

$$F.S. \; x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$k\omega_0 \rightarrow \omega$$

I.F.T. :- $x(t) \longrightarrow x(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} X(j\omega) e^{j\omega t} d\omega$

$x$ time domain — $\cancel{x}$ freq domain

$x$ — $X$

F.T. :- $$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{\cancel{\sqrt{2\pi}}} \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

$$X(f) = \frac{1}{\cancel{\sqrt{2\pi}}} \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$x(t) = e^{-at} u(t)$$

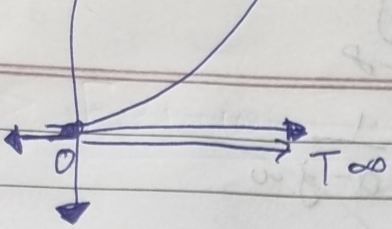
$$X(j\omega) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= -\frac{1}{(a+j\omega)} \left[ e^{-\infty} - e^{0} \right]$$

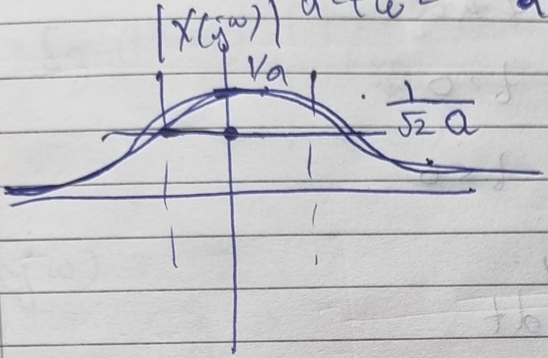
$$= \frac{1}{a+j\omega} \left[ 1 - \cancel{e^{-at}}^{0} e^{-j\omega t} \right.$$

$\cos(\infty) + j\sin(\infty)$




$$= \frac{a - j\omega}{a^2 + \omega^2}$$

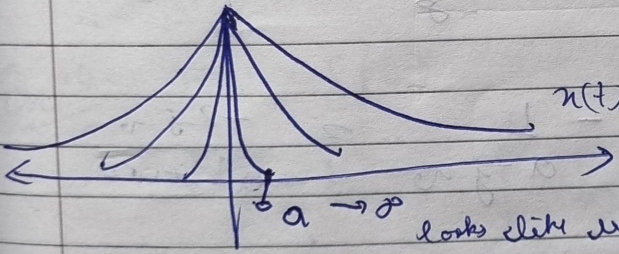
$$= \frac{a}{a^2+\omega^2} - \frac{j\omega}{a^2+\omega^2}$$



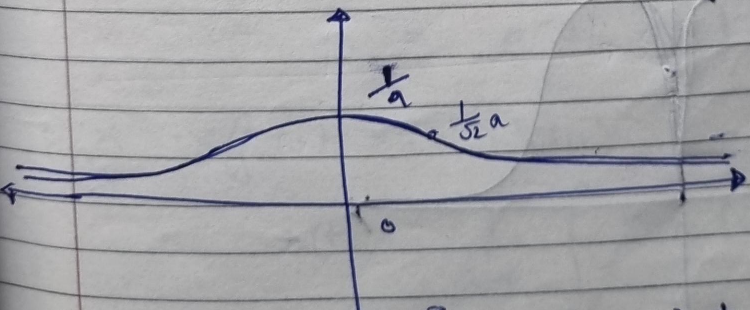

$10 \log_{10} A^2$

A is max amplitude

$$x(t) = e^{-a|t|} \quad a>0$$




looks like unit impulse




$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t} \, dt = \int_{0}^{\infty} e^{-(at+j\omega \ldots}$$

$$= \frac{1}{a+j\omega}) \int_{-\infty}^{0} e^{+at} e^{-j\omega t} \, dt = \int_{-\infty} \frac{1}{a+j \ldots}$$

$$\int_0^\infty + \int_{-\infty}^0$$

$$\frac{1}{a+j\omega} + \frac{1}{a-j\omega}$$

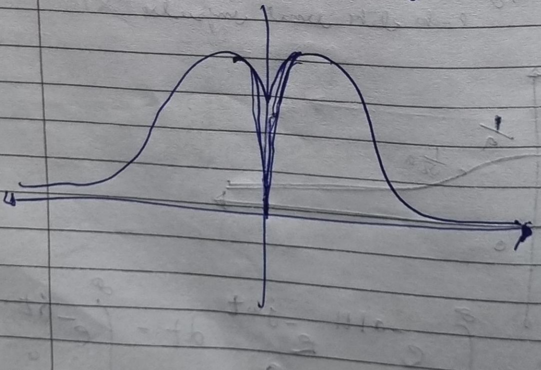
$$= \frac{2a}{(a+j\omega)(a-j\omega)} = \frac{2a}{a^2+\omega^2}$$

$$x(t) = \begin{cases} e^{-at} & t>0 \\ 0 & t=0 \\ e^{at} & t<0 \end{cases}$$

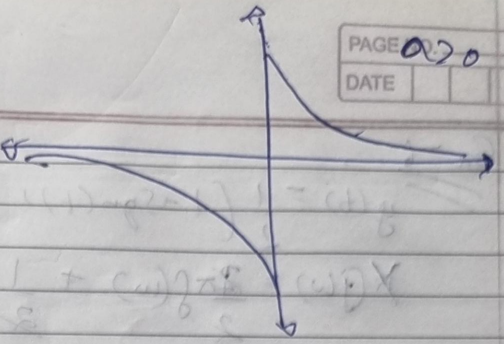
$$\int_0^\infty e^{-at-j\omega t}\,dt$$

$$\frac{1}{a+j\omega} + 0 + \int_{-\infty}^0 e^{at} e^{-j\omega t}\,dt$$

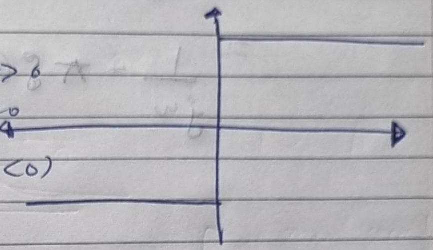
$$= \frac{1}{a+j\omega} - \frac{1}{a-j\omega} = \frac{-2j\omega}{a^2+\omega^2}$$



Let $a \to 0$




Let $a \to 0$



$$x(t) = Sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

$$X(j\omega) = \frac{2}{j\omega}$$

$$y(t) = \frac{1}{2}(1 + Sgn(t))$$

$\delta(t) \rightleftharpoons 1$

$1 \rightleftharpoons 2\pi\delta(\omega)$

Ex-4 $x(t) = \delta(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$$

$$= e^{-j\omega 0} \int 1 \, dt = 1$$

Ex-5 $X(j\omega) = 2\pi\delta(\omega)$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi \, \delta(\omega) \, e^{j\omega t} dt$$

$$= 1$$

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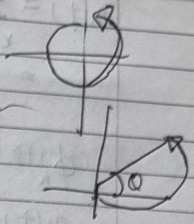
# Fourier Series



$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

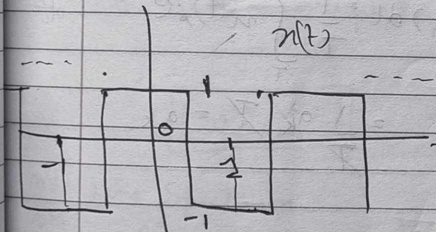
$$= \sum_{k=-\infty}^{\infty} a_k\,\delta[n-k]$$

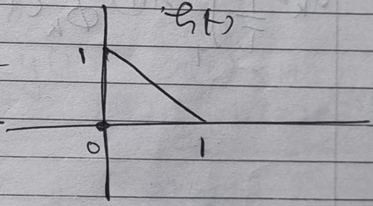
$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

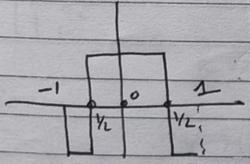
$$x(t) = \sum_{k=-\infty}^{\infty} a_n\,\phi_k(t)$$

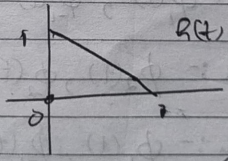
$$\phi_k(t) = e^{jk\omega_0 t}$$

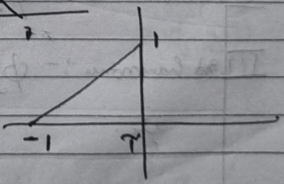
$\omega_0 = \frac{2\pi}{T}$ $\quad T$ = fundamental Time period of $x(t)$




$T(-1, -1/2)$ $\int_{-1}^{t} -1$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t) \qquad \omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{rad/sec} \qquad f_0 = \text{per sec or Hz}$$

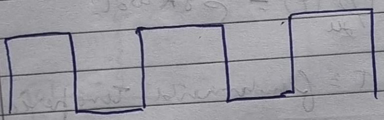
$\phi_k(t)$ = pure imaginary experimental (in power)

$$x(t) = 3 + 2e^{j\omega_0 t} + e^{j2\omega_0 t}$$

$$= 3 + 2(\cos\omega_0 t + j\sin\omega_0 t) + \cos 2\omega_0 t + j\sin 2\omega_0 t$$

Analysis equation:-

$$a_k = \langle x(t), \phi_k(t) \rangle_T = \frac{1}{T}\int_T x(t)\, \phi_k^*(t)\, dt$$



$$a_k = \frac{1}{T}\int_T x(t)\, \phi_k^*(t)\, dt = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{\cancel{T}} a_k \cancel{T} = a_k$$

Ist harmonic :- $\phi_1(t)$, $\phi_{-1}(t)$

IInd harmonic :- $\phi_2(t)$, $\phi_{-2}(t)$

IIIrd harmonic :- $\phi_3(t)$, $\phi_{-3}(t)$

etc.

Synthesis : $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

Analysis : $a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$

$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$

$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$

$y(t) = \int_{-\infty}^{\infty} h(\tau) e^{s(t-\tau)} d\tau = \int_{-\infty}^{\infty} h(\tau) e^{st} e^{-s\tau} d\tau$

$\boxed{x(t) = e^{st}}$ Let us assume

$= e^{st} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau}_{H(s)}$

$= e^{st} \times H(s)$

$e^{st} \equiv$ act as eigenfunction of an LTI system

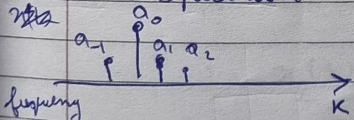
where $H(s)$ = corresponding eigenvalue

$\boxed{A v = \lambda v}$

$y(t) = x(t) \times H(s)$

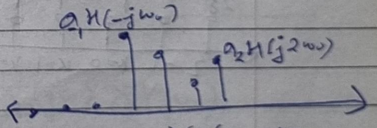
$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$

$x(t)$ Line Spectrum




frequency domain representation

$\xrightarrow{LTI}$ System




Line Spectrum of $y(t)$

if $H(-j\omega_0) = 0$

$a_{-1} \cdot H(-j\omega_0) = 0$

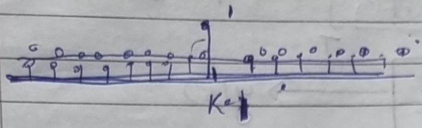
this freq. will also appear in O/P

It can only disappear not reappear.

$y(t) = x(t-2)$ let $x(t) = e^{j2t}$ $\omega_0 = 2\,rad/s$

$= \frac{2\pi}{T}$

$\boxed{T = \pi}$



$y(t) = e^{j2(t-2)}$

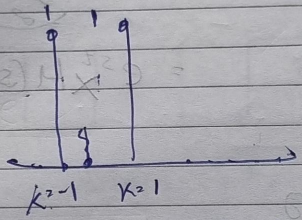
$= e^{j2t} \cdot e^{-j4}$

$= e^{j2t} H(j2)$

$H(s) = e^{-j4}$

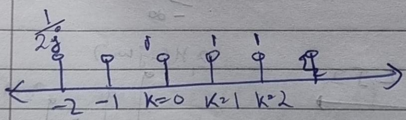
$x(t) = \sin \omega_0 t$

$= \cos(\omega_0 t - \frac{\pi}{2}) = \frac{1}{2j}(e^{j(\omega_0 t - \pi/2)} - e^{j(\omega_0 t + \pi/2)})$



$x(t) = 1 + \sin(\omega_0 t)$

$= 1 + \cos(\omega_0 t - \frac{\pi}{2}) + \cos(2\omega_0 t + \pi/4)$



b) $e^{-3|t|}\sin 2t$ $\qquad \frac{1}{2j}\left(e^{j2t} - e^{-j2t}\right)$

$$\int_0^\infty e^{-3t}\sin 2t \, e^{-j\omega t}\, dt$$

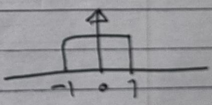
$$\frac{1}{2j}\int_0^\infty \left(e^{(-3+2j)t} - e^{(-3-2j)t}\right) e^{-j\omega t}\, dt$$

$$\frac{1}{2j}\left[\int_0^\infty e^{(-3+2j-j\omega)t} - \int_0^\infty e^{(-3-2j-j\omega)t}\right]$$

$$\frac{1}{2j}\left[\frac{-1}{-3+2j-j\omega} + \frac{-1}{+3+2j+j\omega}\right]$$

$$= \frac{1}{2j}\left[\frac{1}{3+2j-j\omega} - \frac{1}{3-2j-j\omega}\right]$$

$f(t) = t\left(\frac{\sin t}{\pi t}\right)^2$ $\qquad \frac{\sin t}{\pi t} \longleftrightarrow$



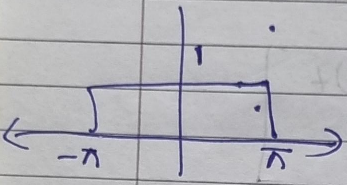

$x_1(t) * x_2(t) \longleftrightarrow X_1(j\omega)\, X_2(j\omega)$

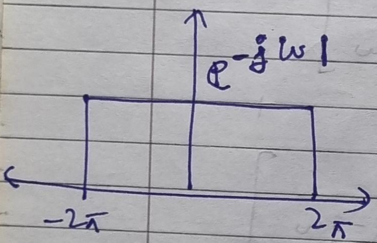
$x_1(t)\, x_2(t) \longleftrightarrow \frac{1}{2\pi}\cdot X_1(j\omega) * X_2(j\omega)$

$t\,X(j\omega) \longrightarrow j\frac{d}{d\omega} X(j\omega)$

$$\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$$

$$\frac{1}{2\pi}\left[\left(\quad\right) * \left(\quad\right)\right]$$



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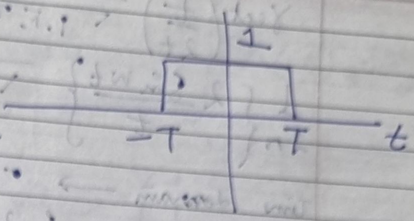
Fourier Transform:

$$x(t) = \begin{cases} 1 & -T \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

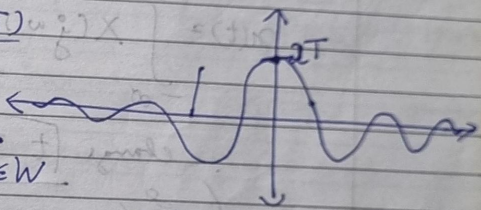



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

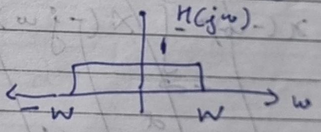
$$= \int_{-T}^{T} 1 \cdot e^{-j\omega t}\, dt = \frac{e^{-j\omega T} - e^{j\omega T}}{-(j\omega)}$$

$$= \frac{e^{j\omega T} - e^{-j\omega T}}{(j\omega)} = \frac{2j \sin(2\omega T)}{j\omega}$$

$$= \frac{2\sin(2\omega T)}{\omega}$$




$$X(j\omega) = \begin{cases} 1 & -W \le \omega \le W \\ 0 & \text{otherwise} \end{cases}$$

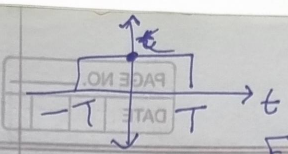



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, dt = \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{jWt} - e^{-jWt}}{jt}\right]$$

$$= \frac{1}{2\pi j t} \times 2j\sin(Wt) = \frac{\sin(Wt)}{\pi t}$$

Causal, Stable, Memoryless




$$\text{rect}\left(\frac{t}{2t}\right) \underset{\text{I.F.T.}}{\overset{\text{F.T.}}{\rightleftharpoons}} \frac{2\sin\omega T}{\omega}$$

$$\frac{1}{2\pi}\left\{2\frac{\sin Wt}{t}\right\} \rightleftharpoons \text{rect}\left(\frac{\omega}{2W}\right)$$

Time domain $\longrightarrow \omega$

& multiply by $2\pi$

freq domain to time domain

$\omega \longrightarrow t$

& divide by $2\pi$

Done using Integral properties

$$x(t) = \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

change $\boxed{t \to -\omega}$

$$\cancel{x(-\omega) = \int_{-\infty}^{\infty} X(-j\omega)\, e^{j\omega t}\, d\omega}$$

$$\cancel{x(\omega) = \int_{-\infty}^{\infty} X(j\omega)\, e^{-j\omega t}\, d\omega}$$

$$2\, x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(t)\, e^{-j\omega t}\, dt$$

$t \to -\omega$
$\omega \to t$

$t = t$

$$-\int_{\infty}^{-\infty} X(-t)\, e^{j\omega t}\, dt = \int_{-\infty}^{\infty} X(-t)\, e^{j\omega t}\, dt$$

$$\cancel{\int_{-\infty}^{\infty} X(t)\, e^{j\omega t}\, dt} = \boxed{\int_{-\infty}^{\infty} X(t)\, e^{-j\omega t}\, dt}$$

$t \to -t$

$x(t) = e^{-|t|}$

$X(j\omega) = \dfrac{2}{1+\omega^2}$

$x(t) = \dfrac{2}{1+t^2}$

$X(j\omega) = \dfrac{1}{2\pi} \quad \dfrac{2\pi}{1+\omega}$

$2\pi e$

$x(t) \rightleftharpoons X(j\omega)$

$t\,x(t) \rightleftharpoons j \dfrac{d}{d\omega} X(j\omega)$

Scaling :- $x(t) \rightleftharpoons X(j\omega)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt \qquad t = a\tau, \quad dt = a\, d\tau$$

$$= \int_{-\infty}^{\infty} x(a\tau)\, e^{-j\omega a\tau}\, a \cdot dt$$

$$x(at) \rightleftharpoons \frac{1}{a} X\left(j\frac{\omega}{a}\right)$$

(5) Parseval's relation

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

(6) Multiplication in time domain convolution in frequency :-

(7) Convolution in time domain :-

$$y(j\omega) = \left(\frac{1}{a+j\omega}\right)\left(\frac{1}{b+j\omega}\right)$$

$$= \frac{A}{a+j\omega} + \frac{B}{b+j\omega}$$

$$A(b+j\omega) + B(a+j\omega) = 1$$

$\boxed{\omega = jb}$

$$B\cdot(a-b) = 1$$

$$B = \frac{1}{a-b}$$

$$A = \frac{1}{b-a}$$

---

FS

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$a_k = \int_T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$\frac{2\sin(\omega T)}{\omega} = rect\left(\frac{t}{2T}\right)$$

$$\frac{1}{2\pi}\left\{\frac{2\sin(Wt)}{t}\right\} \rightleftharpoons rect\left(\frac{\omega}{2W}\right)$$

time domain to freq
$t \to -\omega$
& multiply by $2\pi$
freq domain to time
$\omega \to -t$
& divide by $2\pi$

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$$y[n] = \frac{1}{2}\{x[n] - x[n-1]\}$$

$$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{F} \pi X(0)\delta(\omega) + \frac{1}{j\omega}X(\omega)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{t} x(\tau)\,d\tau\, e^{-j\omega t}\,dt$$

$$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

$$\int_{-\infty}^{t} x(\tau)\,d\tau = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{t} X(j\omega)e^{j\omega\tau}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{X(j\omega)}{j\omega}\left(e^{j\omega t} - e^{-j\omega\infty}\right)d\omega.$$

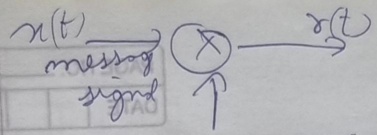
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{X(j\omega)}{j\omega}\left(e^{j\omega t} - \cancel{x}\right.$$

$$\frac{1}{2\pi}\delta(\omega)$$

$$\boxed{U(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)}$$

$$1 \rightleftharpoons 2\pi\delta(\omega).$$

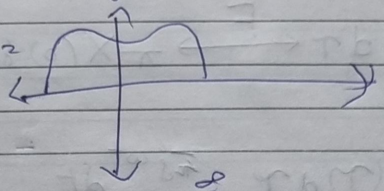
$$\delta(t) \rightleftharpoons 1$$




$r(t) = x(t)\cos(\omega_0 t)$

$x(t)$ = message

$X(j\omega) =$



$$F(\cos\omega_0 t) = \int_{-\infty}^{\infty} \cos(\omega_0 t)\, e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \qquad e^{j\omega t}$$

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$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin\left(k\frac{\pi}{4}\right)}{\left(k\frac{\pi}{4}\right)} \delta\left(t - k\frac{\pi}{4}\right)$$

$$x(j\omega) = \frac{1}{2\pi}\left(\frac{\sin t}{\pi t}\right) * g(t)$$

$$\frac{d}{dt} U(t) = \frac{d\,u(t)}{dt}\ \delta(t)$$

$$\frac{d\,x(t)}{dt} \Rightarrow j\omega\, X(j\omega)$$

$$\frac{d\,\delta(t)}{dt} = j\omega(1)$$

$$\frac{d\,x(t)}{d(t)} = j\omega X(j\omega)$$

$$\frac{dx(t)}{dt} = g(t)$$

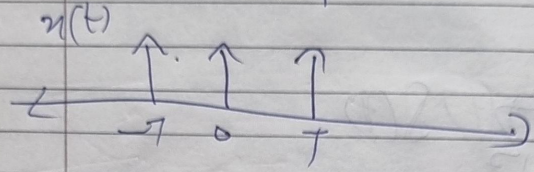
$$x(t) = \int_{-\infty}^{t} g(\tau)\,d\tau.$$

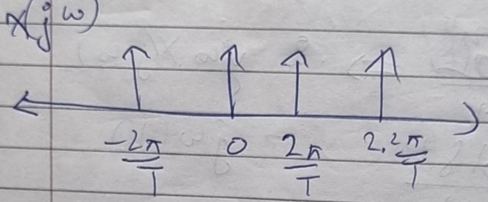
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

$$\omega_0 = \frac{2\pi}{T}$$

Fourier Series of

$x(t)$ is $= \frac{1}{T}$




$$\int_{-\infty}^{\infty} X(j\omega)\, d\omega = \int_{-\infty}^{\infty} \frac{2a}{a^2+\omega^2}\, d\omega$$

$$= 2a\left[\tan^{-1}\left(\frac{\omega}{a}\right)\right]_{-\infty}^{\infty}$$

$$= 2a\left[\cancel{\frac{\pi}{2}}\right] = \frac{2}{a}\frac{1}{a}\tan^{-1}\left(\frac{\omega}{a}\right)$$

$$= \frac{2}{a}\,\frac{1}{\frac{1}{a}}\left[\tan^{-1}\left(\frac{\infty}{a}\right) - \tan^{-1}(-\infty)\right]$$

$$= 2\pi$$

$a \to 0$

$X(j\omega) \to 2\pi\delta(\omega)$

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# Discrete Time Fourier Transform :-

CTFT

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

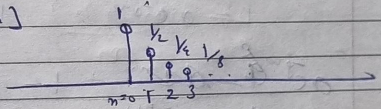
Analysis eqn

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

Ex $x[n] = a^n u[n] \quad |a|<1$

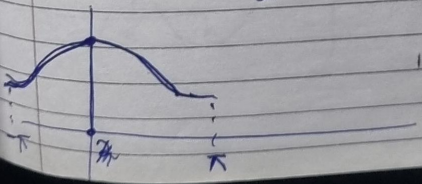
$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$




$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n]\, e^{-j\omega n}$$

$$\sum_{-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n]\, e^{-j\omega n} = \sum_{0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n}$$

$$= \frac{1}{1-\frac{1}{2}e^{-j\omega}} = \frac{2}{2-e^{-j\omega}}$$



$x[n] = a^{|n|} \qquad |a|<1$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} a^{|n|} e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} e^{-j\omega n} a^{n} + \sum_{n=-\infty}^{0} a^{-n} e^{-j\omega n} - 1$$

$$= \cancel{\frac{1}{1-ae^{-j\omega}} + \frac{1}{1-a^{+}j\omega}}$$

$$= \sum_{n=0}^{\infty} (a e^{-j\omega})^{n} + \sum_{n=0}^{\infty} (a e^{j\omega})^{n} - 1$$

$$= \frac{1}{1-ae^{-j\omega}} + \frac{1}{1-ae^{j\omega}} - 1$$

$$= \frac{\cancel{1-1}+ae^{-j\omega}}{1-ae^{-j\omega}} + \frac{1}{1-ae^{j\omega}}$$

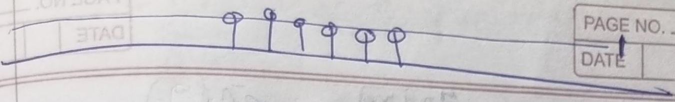
$$= \frac{ae^{-j\omega}\cancel{-a^{2}} + 1 - ae^{-j\omega}}{1-ae^{-j\omega}-ae^{j\omega}+a^{2}}$$

$$= \frac{1-a^{2}}{1-2a\cos\omega + a^{2}}$$

$x[n] = \delta[n]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \delta[n] e^{-j\omega n}$$

$x[n] = 1$



$$X(e^{j\omega}) = \sum_{n=-2}^{2} e^{-j\omega n}$$

$$= e^{+j\omega \cdot 2} + e^{+j\omega} + 1 + e^{-j\omega} + e^{-2j\omega}$$

$$\cancel{= 1 + 2\cos(2\omega) + 2\cos(\omega)}$$

$\cancel{x[n] =}$

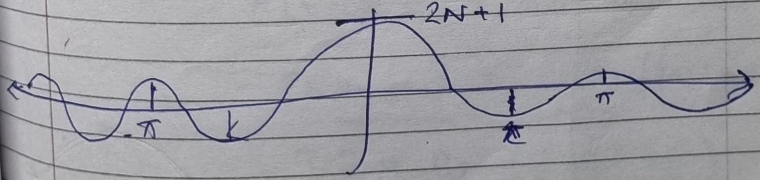
$$X(e^{j\omega}) = \sum_{n=-N}^{N} e^{-j\omega n} = e^{j\omega N}\left(1 + \cdots + e^{-j\omega 2N}\right)$$

$$= e^{j\omega N} \sum_{n=0}^{2N} e^{-j\omega n}$$

$$= e^{j\omega N}\left(\frac{1 - e^{-j\omega(2N+1)}}{1 - e^{-j\omega}}\right)$$

$$= \frac{e^{j\omega N} - e^{-j\omega(N+1)}}{1 - e^{-j\omega}}$$

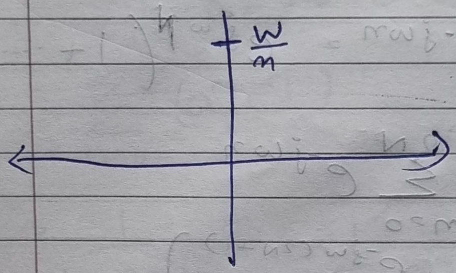
$$X(e^{j\omega}) = \frac{\sin \omega\left(N+\frac{1}{2}\right)}{\sin \omega/2}$$



$$= \frac{1}{2\pi j n}\left(e^{\pi j w n} - e^{-\pi j w}\right)$$

$$= \frac{1}{2\pi j n}\left(e^{\pi j n} - e^{-j\pi n}\right)$$

sin w=π $= \frac{1}{2\pi j n}\left(2j \sin(wn)\right)$




$$= \frac{1}{2\pi}\,\frac{\sin(\pi n)}{\pi n}$$

$$\boxed{\text{At } n = 0 \quad 1}$$

D.T.F.T. of a periodic Signal

$x[n] = e^{j\omega_0 n}$ $\qquad \omega_0 = \frac{2\pi}{N}$

$= \frac{\cos(\omega_0 n)}{2}$

$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \cdot \delta(\omega - k\omega_0)$

$X(j\omega) = \sum_{n=-\infty}^{\infty} e^{j\omega_0 n} e^{-j\omega n}$

$= \sum_{n=-\infty}^{\infty} e^{-jn(-\omega_0+\omega)}$

$= \cancel{2\pi\,\delta(\omega-\omega_0)}$

$= \frac{1}{1-}$

$\boxed{2\pi\delta}$

Let $x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n}$ $\qquad \omega_0 = \frac{2\pi}{N}$

Let $N = 3$

$x[n] = a_0 + a_1 e^{j\frac{2\pi}{3}n} + a_2 e^{j\frac{4\pi}{3}n}$

$= 2\pi a_0 \delta(\omega) + 2\pi a_1 \delta\left(\omega - \frac{2\pi}{3}\right)$

$+ 2\pi a_2 \delta\left(\omega - \frac{4\pi}{3}\right)$

# DTFT

~~$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}$~~

~~$x[n]$~~

$x(t) =$ C.T./aperiodic signal $\xrightarrow{\text{CTFT}}$ $X(j\omega)$ (aperiodic)

$x(t)$ (C.T./aperiodic signal) $\downarrow$ sampling → $x[n]$ D.T./aperiodic

$X(j\omega)$ (aperiodic) $\downarrow$ periodicity → $X(e^{j\omega})$ (periodic)

$x[n]$ D.T./aperiodic $\xrightarrow{\text{DTFT}}$ $X(e^{j\omega})$ (periodic)

$x[n]$ D.T./aperiodic $\downarrow$ periodicity → $x[n]$ D.T./periodic

$X(e^{j\omega})$ (periodic) $\downarrow$ Sampling → $X(e^{j\omega})$

$x[n]$ D.T./periodic $\xrightarrow{\text{D.T.F.S}}$ $X(e^{j\omega})$

~~DF.~~ D.T.F.S

Discrete Fourier Transform D.F.T $[0, N-1]$

CT/aperiodic $x(t)$ $\xrightarrow{\text{CTFT}}$ $X(j\omega)$ aperiodic

$x(t)$ $\downarrow$ periodic → $x(t)$

$X(j\omega)$ $\downarrow$ sample → $X(e^{j\omega})$

$x(t)$ $\xrightarrow{\text{C.T.F.S}}$ $X(e^{j\omega})$ periodic

DF

$x(t)$ $\downarrow$ Sampling → $x[n]$

$X(e^{j\omega})$ periodic $\downarrow$ $X(e^{j\omega})$ periodic

$x[n]$ DT/periodic $\xrightarrow{\text{D.T.F.S}}$ $X(e^{j\omega})$ periodic

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Ex $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-kN]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta[n-kN]\, e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta[n-kN]\, e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} e^{-j\omega kN}$$

$$= \sum_{k=-\infty}^{\infty}$$

$$x_0[n] = a_0$$

$$X_0(e^{j\omega}) = 2\pi a_0 \delta(\omega)$$

$0 \leq \omega < 2\pi$

$$x_1[n] = a_1 e^{j\omega_0 n}$$

$0 \leq \omega \leq 2\pi$ $\quad X_1(e^{j\omega}) = 2\pi a_1 \delta(\omega - \omega_0)$

$$\Rightarrow X_1(e^{j\omega}) = 2\pi \sum a_1$$

$$X_0(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} a_0 \delta(\omega - k2\pi)$$

$\boxed{a_k = \frac{1}{N}}$

for $x[n] = \sum_{n=-\infty}^{\infty} \delta[n-kN]$

$$X(e^{j\omega}) = \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$

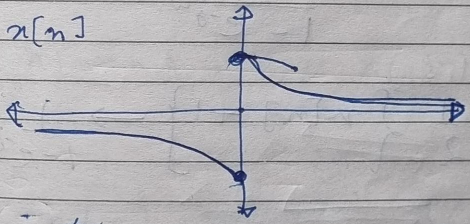
a generic D.T. periodic signal

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}$$

P.T.S.

$$X(e^{j\omega}) = \sum_{l=-\infty}^{\infty} \sum_{k=\langle N \rangle} a_k \delta(\omega - k\omega_0 - 2\pi l)$$

$$x[n] = \begin{cases} a^n & n \geq 0 \quad 0 < a < 1 \\ -a^{-n} & n < 0 \end{cases}$$




$$X(e^{j\omega}) \Rightarrow \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$\Rightarrow \sum_{n=0}^{\infty} a^n e^{-j\omega n} + \sum_{n=-\infty}^{-1} (-a^{-n}) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (ae^{-j\omega})^n - \sum_{n=0}^{\infty} (ae^{j\omega})^n + 1 + \sum_{n=1}^{n=\infty} -e^{j\omega n} a^n + 1 - 1$$

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$$\frac{1}{1-ae^{-j\omega}} - \frac{1}{1-ae^{j\omega}} + 1$$

$$\frac{\cancel{1} - \cancel{ae^{j\omega}} - \cancel{1} + \cancel{ae^{j\omega}} \quad ae^{-j\omega} - ae^{j\omega}}{1 + a^2 - ae^{-j\omega} - ae^{j\omega}}$$

$$= \frac{1 + a^2 - 2ae^{j\omega}}{1 + 2a\cos\omega + a^2}$$

If $a = 1$

$$X(e^{j\omega}) = \frac{2 - 2e^{j\omega}}{2 - 2\cos\omega}$$

$$= \frac{(1 - e^{j\omega})}{(1 - \cos\omega)}$$

$$= \frac{2}{1 - e^{-j\omega}}$$

$$u[n] = \frac{1}{2}\left\{x[n] + 1\right\} \longrightarrow \frac{1}{1-e^{-j\omega}} + \pi\delta(\omega)$$

# Properties

① Periodicity

② Linearity

$$x_1[n] \rightleftharpoons X_1(e^{j\omega})$$
$$x_2[n] \rightleftharpoons X_2(e^{j\omega})$$

③ $\alpha x_1[n] + \beta x_2[n] \rightleftharpoons \alpha X_1(e^{j\omega}) + \beta X_2(e^{j\omega})$

④ n shifting

$$x[n] \rightleftharpoons X(e^{j\omega})$$
$$x[n-n_0] \rightleftharpoons e^{-j\omega n_0} X(e^{j\omega})$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} \, d\omega$$

$$x[n-n_0] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega(n-n_0)} \, d\omega$$

$$x[n] \rightleftharpoons X(e^{j\omega})$$
$$? \rightleftharpoons X(e^{j(\omega-\omega_0)})$$

$e^{j\omega_0 n}$

$$X(e^{j\omega}) \rightleftharpoons \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$X(e^{j(\omega-\omega_0)}) \rightleftharpoons \sum_{n=-\infty}^{\infty} x[n] e^{-j(\omega-\omega_0)n}$$
$$= \sum_{n=-\infty}^{\infty} (x[n] e^{j\omega_0 n}) e^{-j\omega n}$$

(5) Differencing in time domain:

$$x[n] \rightleftharpoons X(e^{j\omega})$$

$$x[n] - x[n-1] \rightleftharpoons X(e^{j\omega})(1-e^{-j\omega})$$

(6) Accumulation

$$x[n] \rightleftharpoons X(e^{j\omega})$$

$$\sum_{k=-\infty}^{n} x[k] = x[n] * u[n] \rightleftharpoons X(e^{j\omega}) \cdot U(e^{j\omega})$$

(7) Convolution in time domain

$$x[n] \rightleftharpoons X(e^{j\omega}) \; ; \; h[n] \rightleftharpoons H(e^{j\omega})$$

$$x[n] * h[n] \rightleftharpoons X(e^{j\omega}) H(e^{j\omega})$$

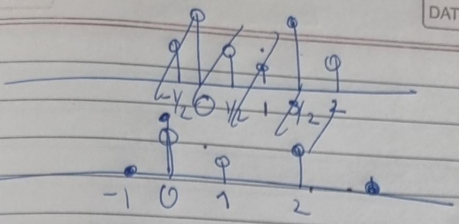
(8) Differentiation in the frequency domain

$$x[n] \rightleftharpoons X(e^{j\omega})$$

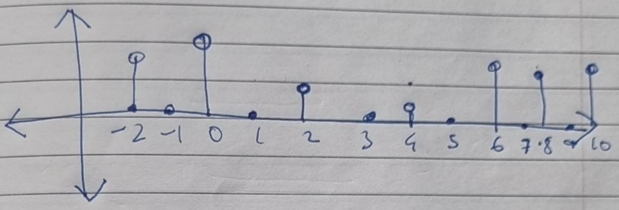
$$\rightleftharpoons \frac{d}{d\omega} X(e^{j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$j \frac{d}{d\omega} X(e^{j\omega}) \rightleftharpoons \sum_{n=-\infty}^{\infty} \left(n\, x[n]\right) e^{-j\omega n}$$



$$y_2[n] = \begin{cases} x[n/2] & ; \text{ when } n \text{ is a multiple of } 2 \\ 0 & ; \text{ otherwise} \end{cases}$$



$$y[n] = x[nL] \rightleftharpoons ?$$

where $L$ is an integer