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$\boxed{\text{C.T.}}$ Constant coefficient differential eqn. representation of Causal LTI System

$$\sum_{k=0}^{N} b_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} a_k \frac{d^k x(t)}{dt^k}$$

Condition of initial rest is satisfied
if $x(t)=0$ for $t<t_0$
$\frac{d^k}{dt^k} y(t)=0$ for $t<t_0$
$\forall k = 0, 1, \ldots N$

$N \equiv$ order of the system

Let $N=2$ & $M=3$

$$b_2 \frac{d^2}{dt^2} y(t) + b_1 \frac{d}{dt} y(t) + b_0 y(t) = a_0 x(t) + a_1 \frac{d}{dt} x(t) + a_2 \frac{d^2 x(t)}{dt^2} + a_3 \frac{d^3 x(t)}{dt^3}$$

Block Diagram representation

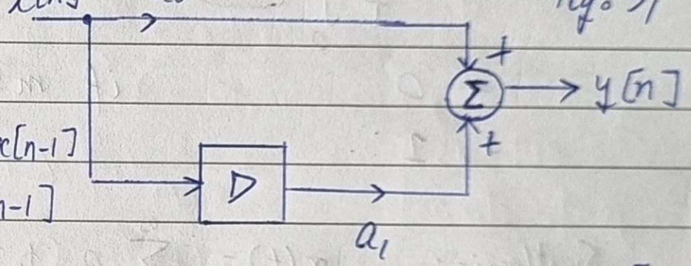
D.T.

$a_0 x[n] \equiv$ Scalar multiplier

$x[n-1] \equiv$ Delay element

$S_1: y[n] = a_0 x[n] + a_1 x[n-1]$

Fig: S_1



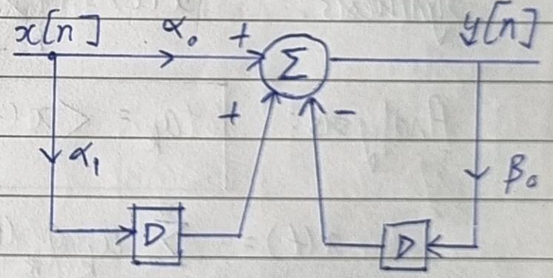
$S_2: b_1 y[n-1] + b_0 y[n] = a_0 x[n] + a_1 x[n-1]$

$$y[n] = \frac{a_0}{b_0} x[n] + \frac{a_1}{b_0} x[n-1] - \frac{b_1}{b_0} y[n-1]$$

where $= \alpha_0 x[n] + \alpha_1 x[n-1] - \beta_0 y[n-1]$

$\alpha_0 = a_0/b_0$

constants



Fourier Series

C.T.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \forall\, t \text{ for Periodic signal } x(t)$$

~~Synth~~ Synthesis equation $\Rightarrow$ $\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t)}$

Vector

basis fns.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$$

$$= \sum_{k=-\infty}^{\infty} a_k\, \delta[n-k]$$

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$\phi_k(t) = e^{jk\omega_0 t}$

$e^{jk\omega_0 t} = \cos k\omega_0 t + j\sin k\omega_0 t$

$\boxed{\omega_0 = \frac{2\pi}{T}}$ $T \equiv$ fundamental time period of $x(t)$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$\phi_1(t) = e^{j\omega_0 t}$ $\quad \phi_2(t) = e^{2j\omega_0 t}$

$\phi_{-1}(t) = e^{-j\omega_0 t}$ $\quad \dots \quad \phi_{-2}(t) = e^{-j2\omega_0 t}$

$\phi_0(t) = e^{j0} = \cos 0 + j\sin 0$
$= 1$

$$\boxed{\langle \phi_k(t), \phi_\ell(t)\rangle_T \triangleq \frac{1}{T}\int_T \phi_k(t)\,\phi_\ell^*(t)\,dt} \equiv \text{Defining inner product}$$

$$= \frac{1}{T}\int_T e^{jk\omega_0 t}\, e^{-j\ell\omega_0 t}\,dt \qquad = 1 \text{ if } m = 0$$

$$= \frac{1}{T}\int_T e^{j(k-\ell)\omega_0 t}\,dt$$

$$= \frac{1}{T}\int_0^T e^{jm\omega_0 t}\,dt = \left.\frac{e^{jm\omega_0 t}}{jm\omega_0 T}\right|_0^T = \frac{e^{jm2\pi} - e^{j0}}{jm\omega_0 T} = 0$$

$= 0$ if $m \neq 0$ if $k \neq \ell$

$= 1$ if $k = \ell$

Synthesis: $x(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t)$

$\{\phi_k(t)\}_{k\in\mathbb{Z}, T} \equiv$ span a vector space form an orthonormal basis (O.N.B)

Analysis: $\boxed{a_k = \langle x(t), \phi_k(t)\rangle_T}$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t)$$

$$\langle x(t), \phi_\ell(t)\rangle_T = \left\langle \sum_{k=-\infty}^{\infty} a_k\phi_k(t), \phi_\ell(t)\right\rangle_T = \langle \dots + a_{-10}\phi_{-10}(t) + \dots + a_0\phi_0(t) + \dots a_9\phi_9(t) + \dots, \phi_\ell(t)\rangle_T$$

$$= \sum_{k=-\infty}^{\infty}\left(a_k\langle\phi_k(t), \phi_\ell(t)\rangle_T\right)$$

$$= \dots\langle a_{-10}\phi_{-10}(t), \phi_\ell(t)\rangle_T + \dots + \langle a_0\phi_0(t), \phi_\ell(t)\rangle_T + \dots + \langle a_9\phi_9(t), \phi_\ell(t)\rangle_T + \dots$$

$$= \sum_{k=-\infty}^{\infty} a_k\,\delta[k-\ell]$$

$$\langle x(t), \phi_\ell(t)\rangle_T = a_\ell$$

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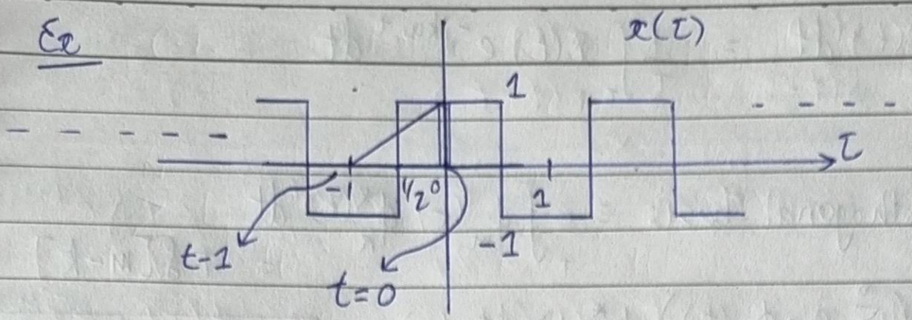
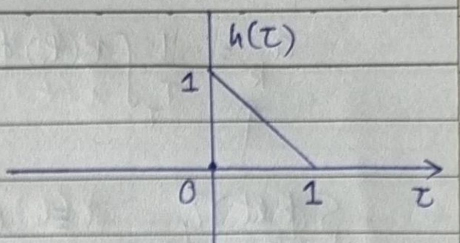
Ex





$h(-\tau) = 1+\tau$

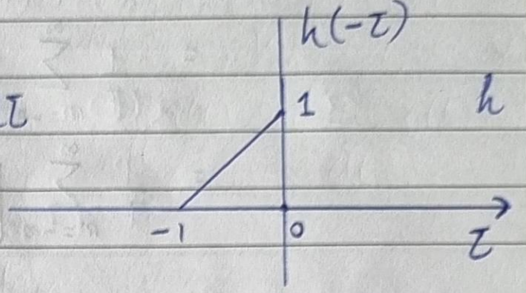
$h(-\tau+t) = 1+\tau-t$

Convolve $x(t)$ & $h(t)$

$$y(0) = \int_{-1}^{-1/2} x(\tau)h(t-\tau)\,d\tau + \int_{-1/2}^{0} x(\tau)(h(t-\tau))\,d\tau$$

$$= \int_{-1}^{-1/2} (-1)(1+\tau)\,d\tau + \int_{-1/2}^{0} (1)(1+\tau)\,d\tau$$

$$= -\left(\tau+\frac{\tau^2}{2}\right)\Big|_{-1}^{1/2} + \left(\tau+\frac{\tau^2}{2}\right)\Big|_{-1/2}^{0}$$



Case 1: $0 \le t \le 1/2$

$$y(t) = -\int_{t-1}^{-1/2} (1+\tau-t)\,d\tau + \int_{-1/2}^{t} (1+\tau-t)\,d\tau$$

C.T. Fourier Series

periodic signals

Synthesis

$$\rightarrow x(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t)$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$

$\downarrow$ rad/s

$f_0 = 1/T$

$\equiv$ per sec or Hz

$e^{jk\omega_0 t} = \phi_k(t)$ = pure imaginary exponential (in power)

$T \equiv$ fundamental time period

$$x(t) = 3 + 2e^{j\omega_0 t} + e^{j2\omega_0 t}$$

$$= 3 + 2(\cos\omega_0 t + j\sin\omega_0 t) + \cos 2\omega_0 t + j\sin 2\omega_0 t$$

$$= \sum_{k=0}^{2} a_k e^{jk\omega_0 t}$$

$a_0 = 3$	all other
$a_1 = 2$	a_k's for $k \neq 0, 1, 2$
$a_2 = 1$	are zero

Analysis equation

$$a_k = \langle x(t), \phi_k(t) \rangle_T = \frac{1}{T}\int_T x(t)\phi_k^*(t)\,dt$$

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$$a_k = \frac{1}{T}\int_T x(t)\,\phi_k^*(t)\,dt = \frac{1}{T}\int_T x(t)\,e^{-jk\omega_0 t}\,dt$$

$$\{\phi_k(t)\}_{k\in\mathbb{Z}} \equiv \text{Orthogonal basis} \equiv \int_T \phi_m(t)\,\phi_k^*(t)\,dt = T\,\delta[m-k]$$

$$\text{R.H.S} = \frac{1}{T}\int_T \left(\sum_{m=-\infty}^{\infty} a_m \phi_m(t)\right)\phi_k^*(t)\,dt$$

$$= \frac{1}{T}\sum_{m=-\infty}^{\infty}\left[a_m\left(\int_T \phi_m(t)\,\phi_k^*(t)\right)dt\right]$$

$$= \frac{1}{T}\sum_{m=-\infty}^{\infty}\left[a_m \underbrace{\int_T e^{j(m-k)\omega_0 t}\,dt}\right]$$

$$= \frac{1}{\cancel{T}}\sum_{m=-\infty}^{\infty} a_m \cdot \cancel{T}\,\delta[m-k] = a_k = \text{L.H.S.}$$

$\phi_1(t),\ \phi_{-1}(t)$
$\downarrow$
I harmonic
$\phi_2(t),\ \phi_{-2}(t)$
$\downarrow$
2^{nd} harmonics & so on

Synthesis : $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$ | Analysis : $a_k = \frac{1}{T}\int_T x(t)\,e^{-jk\omega_0 t}\,dt$

—x—

e^{st} ≡ act as eigenfunctions of an LTI system where $H(s)$ ≡ corresponding eigenvalue

$$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

Let us assume $\boxed{x(t) = e^{st}}$ $\boxed{\text{where } s = j\omega_0}$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,e^{s(t-\tau)}\,d\tau = \int_{-\infty}^{\infty} h(\tau)\,e^{st}\cdot e^{-s\tau}\,d\tau$$

$$= e^{st}\underbrace{\int_{-\infty}^{\infty} h(\tau)\,e^{-s\tau}\,d\tau}_{H(s)}$$

$$e^{st} \longrightarrow \boxed{LTI} \longrightarrow e^{st}\cdot H(s) \qquad \boxed{y(t) = e^{st} H(s)}$$

$$A\underline{v} = \lambda\underline{v}$$

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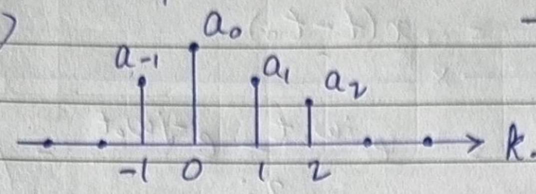
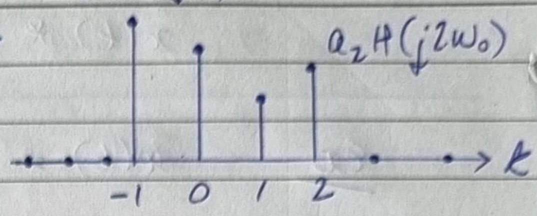
Given an line spectrum $x(t)$ $\xrightarrow[\text{System}]{LTI}$ line spectrum of $y(t)$

frequency domain representation



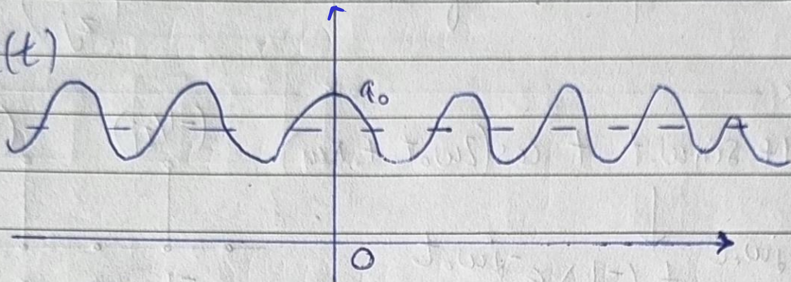


if $H(-j\omega_0) = 0$

$a_{-1} H(-j\omega_0) = 0$

$\downarrow$ this freq. will disappear in o/p

$x(t)$

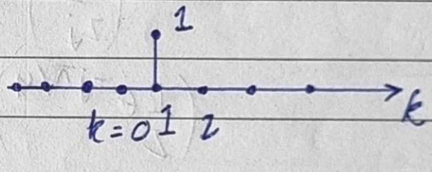


Given $\boxed{x(t) = a_{-1} e^{-j\omega_0 t} + a_0 + a_1 e^{j\omega_0 t} + a_2 e^{j2\omega_0 t}}$

Ex $\quad S: y(t) = x(t-2)$ $\quad$ Let $x(t) = e^{j2t}$ $\quad$ $\omega_0 = 2$ rad/s $= 2\pi/T$

$\boxed{T = \pi}$

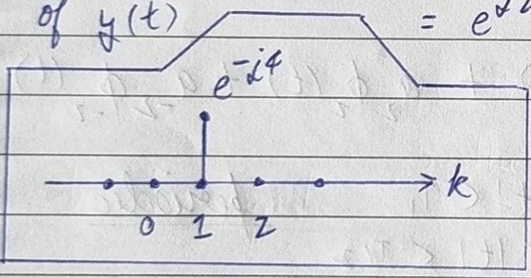
$y(t) = e^{j2(t-2)}$



$= e^{j2t} \cdot e^{-j4}$

$= e^{j2t}\, \underset{}{H(j2)}$

line spectrum of $y(t)$



$\boxed{H(j2) = e^{-j4}}$ $\quad$ $\boxed{s = j2}$

Ex $\quad$ what is $h(t)$? $\quad$ $h(t) = \delta(t-2)$

$$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau = \int_{-\infty}^{\infty} \delta(\tau-2) e^{-j2\tau} d\tau$$

$$= \int_{-\infty}^{\infty} \delta(\tau-2) \cdot e^{-j2\cdot 2} d\tau$$

$$= e^{-j4} \boxed{\int_{-\infty}^{\infty} \delta(\tau-2)\, d\tau}$$

$$= e^{-j4}$$

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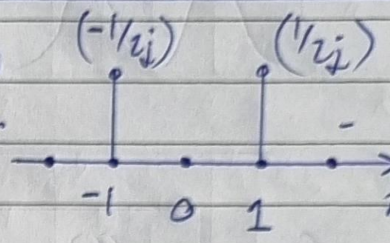
$$x(t) * \delta(t) = x(t)$$
$$x(t) * \delta(t-t_0) = x(t-t_0)$$

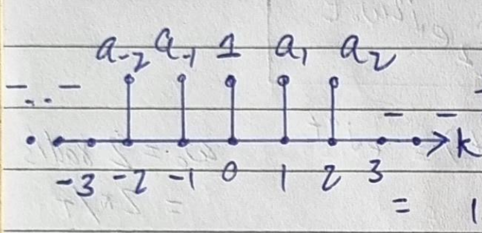
Ex $x(t) = \sin \omega_0 t = \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \left(\dfrac{1}{2j}\right) e^{j\omega_0 t} + \left(\dfrac{-1}{2j}\right) e^{-j\omega_0 t}$

Find and plot the line spectrum of $x(t)$



Ex $x(t) = 1 + \sin \omega_0 t + \cos(2\omega_0 t + \pi/4)$

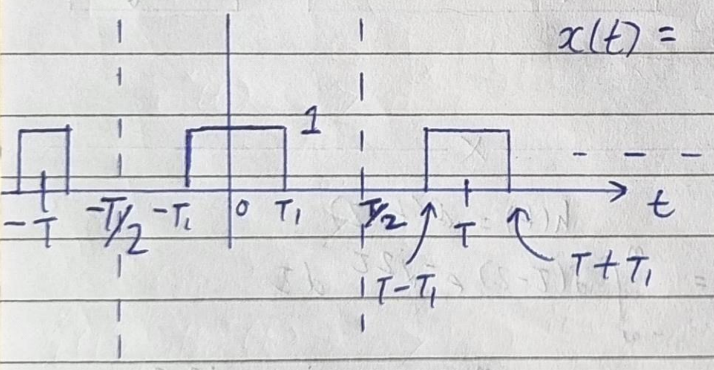
$$= 1 + \frac{e^{j\omega_0 t}}{2j} + \left(\frac{-1}{2j}\right) e^{-j\omega_0 t} + \frac{1}{2}\left[e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}\right]$$



$$= 1 + \left(\frac{1}{2j}\right)\phi_1(t) + \left(\frac{-1}{2j}\right)\phi_{-1}(t) + \left(\frac{1}{2} e^{j\pi/4}\right)\phi_2(t) + \left(\frac{1}{2} e^{-j\pi/4}\right)\phi_{-2}(t)$$

$$= a_0 + a_1\phi_1(t) + a_{-1}\phi_{-1}(t) + a_2\phi_2(t) + a_{-2}\phi_{-2}(t)$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T/2 \end{cases}$$ periodic



① if $T \to \infty$, signal will become aperiodic

② there should be a dc bias term, so $a_0 \neq 0$

③

$\omega_0 = \dfrac{2\pi}{T}$ → fundamental freq.

Solve it, plot the line spectrum

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① Linearity

$x(t) \xrightleftharpoons{F.S} a_k$

$y(t) \xrightleftharpoons{F.S.} b_k$

$x(t)$ & $y(t)$ are periodic with fundamental time period = T.

$z(t) = \boxed{\alpha x(t) + \beta y(t)} \rightleftharpoons \alpha a_k + \beta b_k$

& $z(t) \equiv$ Same fundamental time period T.

$$\boxed{z(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}} \qquad \omega_0 = \frac{2\pi}{T} \qquad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$$

$$c_k = \frac{1}{T}\int_T z(t) e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_T (\alpha x(t) + \beta y(t)) e^{-jk\omega_0 t}\, dt = \alpha \cdot \underbrace{\frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\, dt}_{a_k}$$

$$+ \beta \cdot \underbrace{\frac{1}{T}\int_T y(t) e^{-jk\omega_0 t}\, dt}_{b_k}$$

$$= \alpha a_k + \beta b_k \rightarrow \text{Proved.}$$

② Time Shifting

$x(t) \xrightarrow[T]{F.S} a_k$ $\qquad \omega_0 = \frac{2\pi}{T}$

Find F.S. coeff. of $x(t-t_0)$, where $t_0 \equiv$ Constant:

$$z(t) = x(t-t_0) \xrightarrow{T} ? = c_k = a_k e^{-jk\omega_0 t_0}$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} z(t) e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_{-T/2}^{T/2} x(t-t_0) e^{-jk\omega_0 t}\, dt$$

Let $t - t_0 = \tau$

$dt = d\tau$

$$= \frac{1}{T}\int_{-T/2-t_0}^{T/2-t_0} x(\tau) e^{-jk\omega_0(t_0+\tau)}\, d\tau$$

$$c_k = e^{-jk\omega_0 t_0} \underbrace{\left(\frac{1}{T}\int_T x(\tau) e^{-jk\omega_0 \tau}\, d\tau\right)}_{} = a_k e^{-jk\omega_0 t_0}$$

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③ Time-reversal

$$x(t) \xrightarrow[T]{F.S.} a_k$$

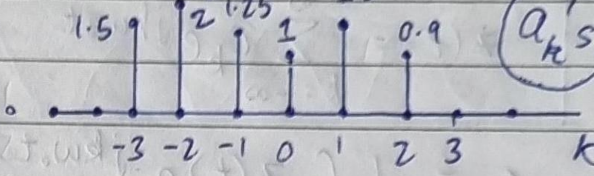
$$\tau = -t$$
$$d\tau = -dt$$

$$z(t) = x(-t) \xrightarrow[T]{F.S.} C_k = a_{-k}$$

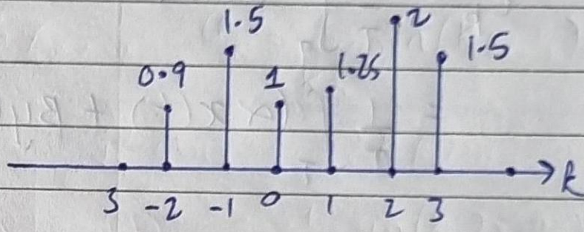
$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(-t)e^{-jk\omega_0 t}\,dt = \frac{-1}{T}\int_{T/2}^{-T/2} x(\tau)e^{jk\omega_0\tau}\,d\tau$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(\tau)e^{jk\omega_0\tau}\,d\tau$$

$$C_{-k} = \frac{1}{T}\int_T x(\tau)e^{-jk\omega_0\tau}\,d\tau = a_k$$



$$\boxed{C_k = a_{-k}}$$



$$C_0 = a_0$$
$$C_1 = a_{-1}$$

④ $$x(t) \xrightarrow[T]{F.S.} a_k$$ Time period T

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}}$$

$$z(t) = x^*(t) \longrightarrow a^*_{-k}$$

$$x^*(t) = \sum_{k=-\infty}^{\infty} a^*_k e^{-jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{\infty} a^*_{-k} e^{jk\omega_0 t}$$

$$z(t) = x^*(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

Case-1 Let $x(t)$ is real

$$x(t) = x^*(t)$$
$$\downarrow \qquad \downarrow$$
$$\boxed{a_k = a^*_{-k}}$$
$$\boxed{a^*_k = a_{-k}}$$

$x(t) = x(-t)$

Case-2 Let $x(t)$ is real & even.

$$\downarrow \qquad \downarrow$$
$$a^*_k = a_{-k} \qquad \boxed{a_k = a_{-k}}$$

Conclusion: a_k's are also real & even. $\longleftarrow$ $\boxed{a_k^* = a_k}$

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Case-3 $x(t)$ is real & odd

$$x(t) = x^*(t) \qquad\qquad x(-t) = -x(t)$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$a_k^* = a_{-k} \qquad\qquad -a_k = +a_{-k}$$

$$a_k^* = -a_k$$

Coeff. are purely imaginary.

---

⑤ Time Scaling

$$x(t) \xrightarrow[T]{F.S.} a_k \qquad \alpha > 0 \qquad \alpha \neq -1$$

$$z(t) = x(\alpha t) \xrightarrow[T/\alpha,\ \alpha\omega_0]{F.S.} C_k = ?$$

$$z(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk(\alpha\omega_0)t}$$

$$C_k = \frac{1}{T/\alpha} \int_{-T/2\alpha}^{T/2\alpha} x(\alpha t)\, e^{-jk(\alpha\omega_0)t}\, dt$$

let $\alpha t = \tau$

$\alpha\, dt = d\tau$

$dt = \frac{d\tau}{\alpha}$

$$= \frac{1}{T/\cancel{\alpha}} \int_{-T/2}^{T/2} x(\tau) e^{-jk\omega_0\tau} \frac{d\tau}{\cancel{\alpha}} = a_k$$

⑥ Multiplication

$$x(t) \xrightarrow[T]{F.S.} a_k \qquad x(t) = \sum_{l=-\infty}^{\infty} a_l e^{jl\omega_0 t}$$

$$y(t) \xrightarrow[T]{F.S.} b_k \qquad y(t) = \sum_{m=-\infty}^{\infty} b_m e^{jm\omega_0 t}$$

$$z(t) = x(t)y(t) \xrightarrow{F.S.} C_k = ?$$

$$C_k = \frac{1}{T}\int_T z(t) e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_T (x(t)\, y(t))\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_T \sum_l \sum_m a_l b_m e^{j(l+m)\omega_0 t} \cdot e^{-jk\omega_0 t}\, dt$$

$$= \sum_l \sum_m a_l b_m \left(\overbrace{\frac{1}{T}\int_T e^{-j(k-l-m)\omega_0 t}\, dt}^{1} \right) = 1 \qquad k-l-m = 0$$

$$l = k-m$$

$$= \boxed{\sum_m a_{k-m} b_m = \sum_l a_l b_{k-l}}$$

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multiplication in the time domain $\Longleftrightarrow$ Convolution in the frequency domain

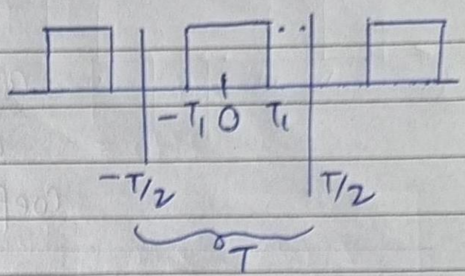
Fix T_1

---x---

① $T = 4T_1$

② $T = 8T_1$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T/2 \end{cases}$$



$$a_0 = \frac{1}{T}\int_T x(t)\,dt$$

$$= \frac{1}{T}\int_{-T_1}^{T_1} dt = \boxed{\frac{2T_1}{T} = a_0} = \frac{1}{2}$$

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\,dt \Longleftarrow k \neq 0$$

$$a_k = \frac{1}{k\pi}\sin(k\omega_0 T_1)$$

$k \neq 0$

$$= \frac{1}{k\pi}\sin\left(k\frac{2\pi\,\cancel{T_1}}{4\cancel{T_1}}\right)$$

$$= \frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)$$

⑦ Parseval's relation

$$x(t) \xrightarrow[T]{F.S} a_k$$

$$\frac{1}{T}\int_T |x(t)|^2\,dt = \sum_k |a_k|^2$$

---x---

D.T. F.S

Given a discrete-time periodic signal $x[n]$

Synthesis $\equiv$ $x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n}$ N = Fundamental time period

$$= \sum_{k=\langle N\rangle} a_k \phi_k[n]$$

$0, \omega_0, 2\omega_0, \ldots (N-1)\omega_0, N\omega_0$

$0, \frac{2\pi}{N}, 2\cdot\frac{2\pi}{N}, \ldots (N-1)\cdot\frac{2\pi}{N}, 2\pi = 0$

$$\phi_k[n] = e^{jk\omega_0 n}$$

$$\phi_{k+N}[n] = \phi_k[n] = e^{jk\omega_0 n}$$

$$\langle x[n], \phi_l[n] \rangle_N = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jl\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} \left(\sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \right) e^{-jl\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} \sum_{k=\langle N \rangle} a_k e^{j(k-l)\omega_0 n}$$

$$= \sum_{k=\langle N \rangle} a_k \underbrace{\left(\frac{1}{N} \sum_{n=\langle N \rangle} e^{j(k-l)\omega_0 n} \right)}_{0} = 1 \quad \text{when } k = l$$

$$= \boxed{a_l = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] \phi_l^*[n]}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n}$$

Ex $x[n] = \sin \omega_0 n$ $\qquad \omega_0 = \frac{2\pi}{N} \rightarrow$ periodic with period N.

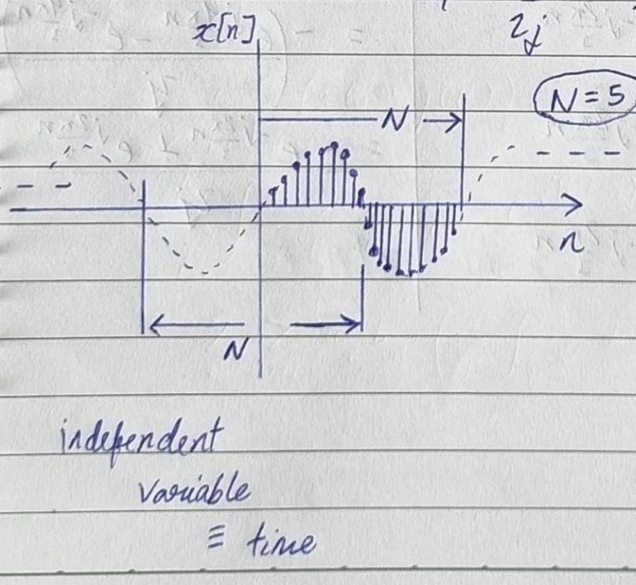
$a_l = ?$

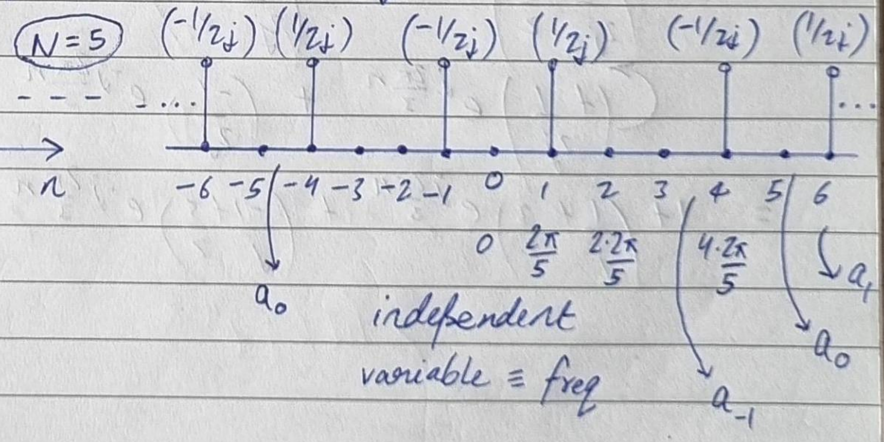
$$x[n] = \frac{1}{2j} \left[e^{j\omega_0 n} - e^{-j\omega_0 n} \right]$$

$$= \left(\frac{1}{2j} \right) e^{j\omega_0 n} + \left(\frac{-1}{2j} \right) e^{-j\omega_0 n}$$

a_1, a_{-1} are non-zero, rest all are zero in $k = \langle N \rangle$.

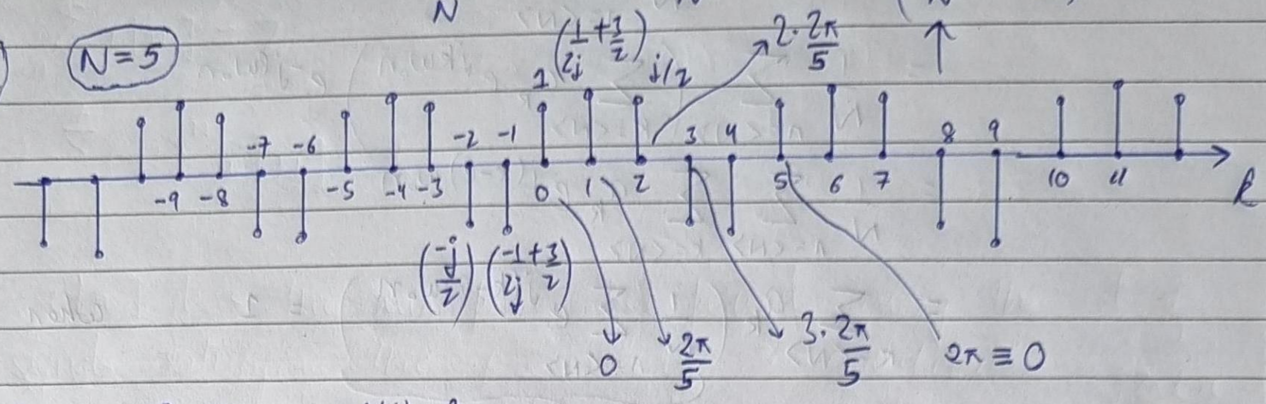
$a_1 = \frac{1}{2j}$; $a_{-1} = \left(\frac{-1}{2j} \right)$ → line spectrum





Ex $\quad x[n] = 1 + \sin\frac{2\pi n}{N} + 3\cos\frac{2\pi n}{N} + \cos\left(\frac{4\pi n}{N} + \frac{\pi}{2}\right)$

$k = \langle N \rangle$ $\qquad$ $N=5$



$N=7$ $\qquad$ 2 zero amplitude pts. in one N

$N=11$ $\qquad$ 6 zero-amplitude pts. in one period in the frequency domain

$$, \frac{-2\pi}{3}, 0, \frac{2\pi}{3}, 2 \cdot \frac{2\pi}{3}, 2\pi \equiv 0$$

$$e^{j2\cdot\frac{2\pi}{3}n} = e^{j\left(2\pi - \frac{2\pi}{3}\right)n} \quad N=3$$
$$= e^{j2\pi n} \cdot e^{-j\frac{2\pi}{3}n}$$
$$= (e^{j2\pi})^n \cdot e^{-j\frac{2\pi}{3}n}$$
$$= e^{-j\frac{2\pi}{3}n}$$

$$x[n] = 1 + \sin\frac{2\pi n}{3} + 3\cos\frac{2\pi n}{3} + \cos\left(\frac{4\pi n}{3} + \frac{\pi}{2}\right)$$

$$x[n] = 1 + \frac{1}{2j}\left(e^{j\frac{2\pi n}{3}} - e^{-j\frac{2\pi}{3}n}\right) + \frac{3}{2}\left(e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right) + \cancel{\cos\frac{4\pi n}{3}} \times 0$$

$$-\sin\frac{4\pi n}{3}$$

$$-\sin\frac{4\pi n}{3} = -\left(\frac{e^{j\frac{4\pi n}{3}} - e^{-j\frac{4\pi n}{3}}}{2j}\right) = \frac{-e^{-j\frac{2\pi}{3}n} + e^{j\frac{2\pi n}{3}}}{2j}$$

$$= 1 + \left(\frac{1}{2j} + \frac{3}{2}\right)e^{j\frac{2\pi n}{3}} + \left(\frac{-1}{2j} + \frac{3}{2}\right)e^{-j\frac{2\pi}{3}n}$$

$$\left(+\frac{1}{2j}\right)e^{j\frac{2\pi}{3}n} + \left(\frac{-1}{2j}\right)e^{-j\frac{2\pi}{3}n}$$

$$= 1 + \left(\frac{1}{j} + \frac{3}{2}\right)e^{j\frac{2\pi n}{3}} + \left(\frac{-1}{j} + \frac{3}{2}\right)e^{-j\frac{2\pi}{3}n}$$

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Ex $x[n] \equiv$ periodic with period N

Show that the F.S. coefficients of the periodic signal

$$g(t) = \sum_{k=-\infty}^{\infty} x[k]\,\delta(t-kT)$$

are periodic with period N.

Ex ① $x(t)$ is a real signal $\Rightarrow \boxed{a_k^* = a_{-k}} \Rightarrow a_{-1} = a_1^*$

② $x(t)$ is periodic with $T = 4$.

③ $a_k = 0$ for $|k| > 1$ — only a_{-1}, a_0, a_1 have non-zero amplitude

④ A signal with F.S. coeff. $b_k = e^{-j\frac{\pi k}{2}} a_{-k}$ is odd

⑤ $\boxed{\frac{1}{4}\int_4 |x(t)|^2 dt = \frac{1}{2}}$ $\equiv$ average power in one time period

$$= |a_{-1}|^2 + |a_0|^2 + |a_1|^2 = \frac{1}{2}$$

$$x(t) \xrightarrow[T]{F.S} a_k$$

$x(t)$ is real

$$z(t) = x(-t+1) \xrightarrow[T]{F.S} z$$

$z(t) = x(-t+1)$

real & odd

odd & purely imaginary

$b_0 = 0$

$|b_1|^2 + |b_{-1}|^2 = \frac{1}{2}$

$b_1 = \frac{j}{2}$, $b_{-1} = \frac{-j}{2}$

$b_1 = ja$

$b_{-1} = -ja$

$|ja|^2 + |-ja|^2 = \frac{1}{2}$

$a^2 + a^2 = \frac{1}{2}$

$a^2 = \frac{1}{4}$

$a = \pm \frac{1}{2}$

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① $x[n]$ is periodic with period $N=6$

② $\sum_{n=0}^{5} x[n] = 2$

③ $\sum_{n=2}^{7} (-1)^n x[n] = 1$

④ $x[n]$ has the min. power per period among the set of signals satisfying the preceeding 3 conditions

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j\frac{2\pi}{N}kn}$$

C.T. $x(t) = e^{st} \longrightarrow \boxed{LTI} \longrightarrow y(t) = e^{st} H(s)$

C.T. F.S. $s_k = jk\omega_0$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{s_k t} \longrightarrow y(t) = \sum_{k=-\infty}^{\infty} a_k H(s_k) e^{s_k t}$$

$$= \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$$

$$\boxed{H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} \, dt}$$

D.T. $x[n] = z^n \longrightarrow \boxed{LTI} \longrightarrow y[n] = ?$

$h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k] z^{n-k}$$

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$$= z^n \boxed{\sum_{k=-\infty}^{\infty} h[k]\, z^{-k}}$$

$y[n] = z^n H(z)$

Let $z = e^{j\omega_0}$

$x[n] = e^{j\omega_0 n} \longrightarrow e^{j\omega_0 n} H(e^{j\omega_0})$

D.T.F.S. $\quad x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n} \xrightarrow[\text{system } h[n]]{\text{LTI}} y[n] = \sum_{k=\langle N\rangle} a_k H(e^{jk\omega_0}) e^{jk\omega_0 n}$

———————— x ———————— x ————————

$x[n] = \sum_{k=\langle 5\rangle} a_k e^{jk\frac{2\pi}{5}n}$ $\quad N = 5$

$= 1 + \cos\omega_0 n + \frac{3}{2}\cos 2\omega_0 n$

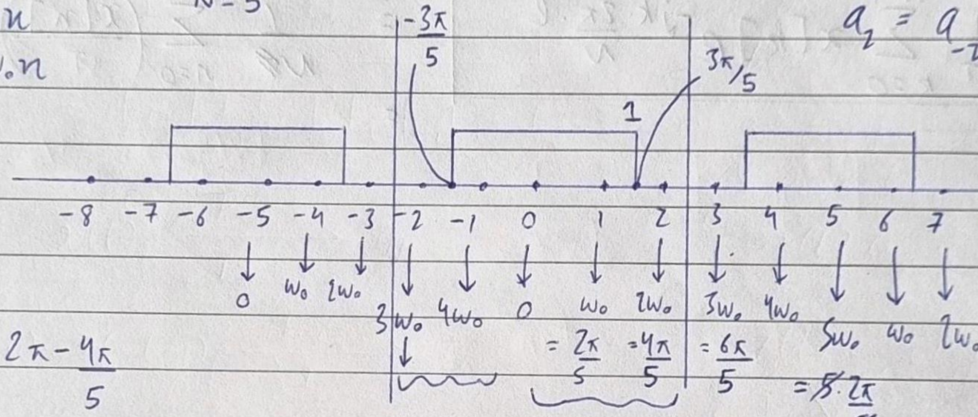
$a_0 = 1$

$a_1 = a_{-1} = 1/2$

$a_2 = a_{-2} = 3/4$

$H(0) = 1$

$H(e^{j\omega_0}) = 1$



$3\omega_0 = \frac{6\pi}{5} = 2\pi - \frac{4\pi}{5}$

$= -\frac{4\pi}{5}$

$H(e^{-j\omega_0}) = 1$

$H(e^{+2j\omega_0}) = 0$

$H(e^{-2j\omega_0}) = 0$

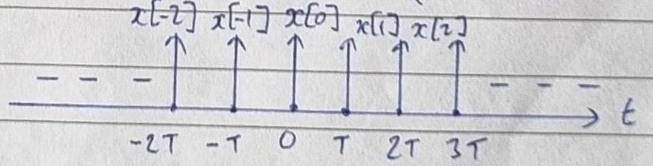
Ex $\quad x[n]$ is a periodic signal, N. Show that F.S. coeff. of a periodic signal $g(t) = \sum_{k=-\infty}^{\infty} x[k]\,\delta(t-kT)$ are also periodic with period N.

N=3

———————— x ———————— x ————————

$g(t)$

$= x[0]\,\delta(t) + x[1]\,\delta(t-T) + x[2]\,\delta(t-2T) + \ldots + \ldots + x[-1]\,\delta(t+T)$



$g(t)$ is periodic with NT.

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$$g(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{NT}t}$$

$$a_0 = \frac{1}{NT}\int_{NT} g(t)\,dt = \frac{x[0] + x[1] + \cdots + x[N-1]}{NT}$$

$$a_l = \frac{1}{NT}\int_{NT} g(t) e^{-jl\frac{2\pi}{NT}t}\,dt$$

$$a_l = \frac{1}{NT}\int_{NT} \sum_{k=-\infty}^{\infty} x[k]\,\delta(t-kT)\, e^{-jl\frac{2\pi}{NT}t}\,dt$$

$$= \frac{1}{NT}\int_0^{NT} \sum_{k=\cancel{-\infty}\,0}^{N-1} x[k] e^{-jl\frac{2\pi}{N\cancel{T}}\cdot k\cancel{T}}\,\delta(t-kT)\,dt$$

$$= \frac{1}{NT}\sum_{k=0}^{N-1} x[k] \int_0^{NT} e^{-jlk\cdot\frac{2\pi}{N}}\,\delta(t-kT)\,dt$$

$$\boxed{a_l = \frac{1}{NT}\sum_{k=0}^{N-1} x[k] e^{-jk\cdot\frac{2\pi}{N}\cdot l}} = \frac{1}{N\cancel{T}}\sum_{n=0}^{N-1}\left(\frac{x[n]}{T}\right) e^{-jl\cdot\frac{2\pi n}{N}}$$

$$=$$

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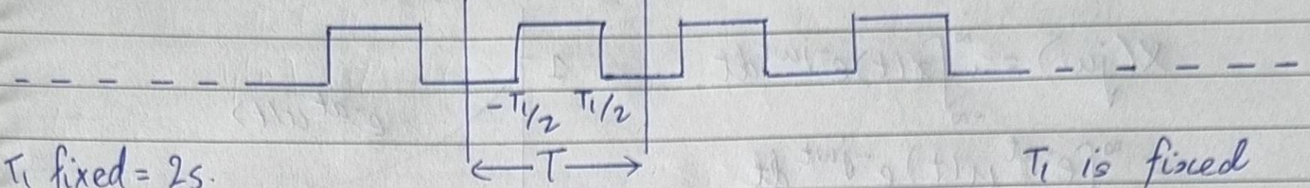
Fourier Transform



T_1 fixed = 2s.

$T \longrightarrow \infty$

T_1 is fixed
$T \longrightarrow \infty$

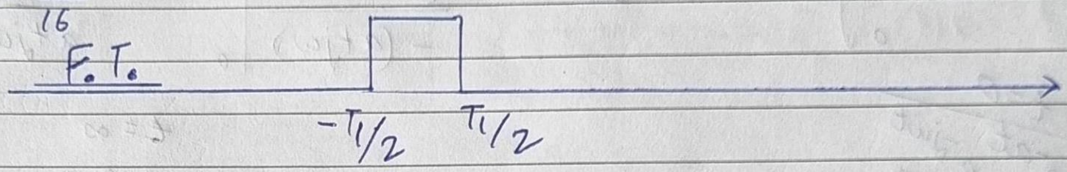
$T = 4T_1 \Longrightarrow \omega_0 = \frac{2\pi}{8}$

$\Downarrow$

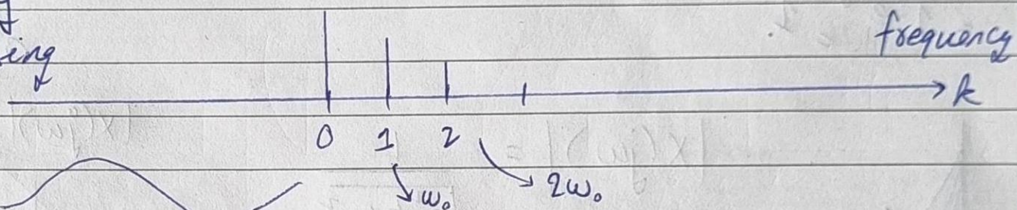
$T = 8T_1 \Longrightarrow \omega_0 = \frac{2\pi}{16}$

∘
∘
∘

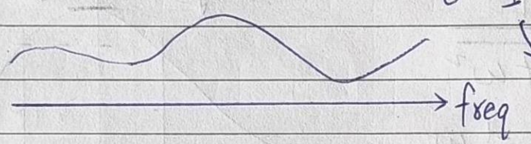
F.T.



as $T \longrightarrow$ increasing
$\omega_0 \longrightarrow$ decreasing



$T \longrightarrow \infty$
$\omega \longrightarrow 0$



C.T. F.T. Synthesis eqn $\equiv x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$

Inverse F.T.

We will use analysis eqn. $\Longleftarrow \equiv X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$

F.T.

F.S.

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$k\omega_0 \rightarrow \bar{\omega}$

$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt$

$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi f t}\,\cancel{dt}\,df$ $\qquad \omega = 2\pi f$

$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}\,dt$

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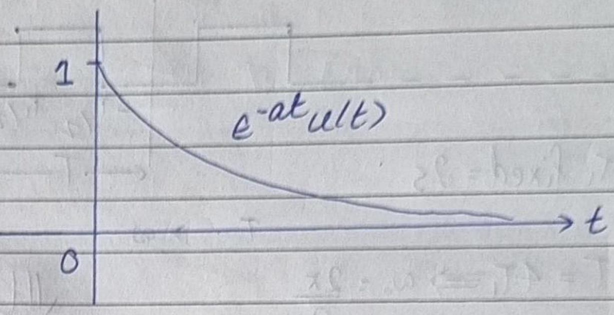
Ex $x(t) = e^{-at}u(t)$ $\quad a>0$



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$= \int_{-\infty}^{\infty} e^{-at}u(t)e^{-j\omega t}dt$$

$$= \int_{0}^{\infty} e^{-at}\cdot e^{-j\omega t}dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}dt = \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_0^{\infty} = \frac{1}{a+j\omega}$$

$t=0$

$e^{-at}\cdot e^{-j\omega t}$

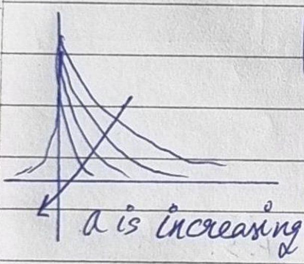
$= 1\cdot e^{-j0}$

$= 1$

$t=\infty$

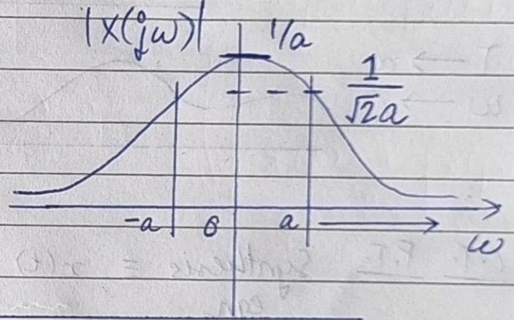
$e^{-a\cdot\infty}\cdot e^{-j\omega\cdot\infty}$

$0 \times ? = 0$



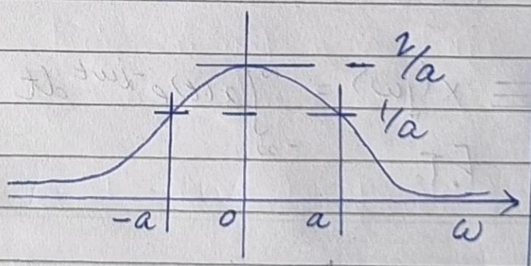
$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\angle X(j\omega) = -\tan^{-1}\frac{\omega}{a}$$



Ex-2 $\quad x(t) = e^{-a|t|}$ $\quad a>0$

$$X(j\omega) = \frac{2a}{a^2+\omega^2}$$

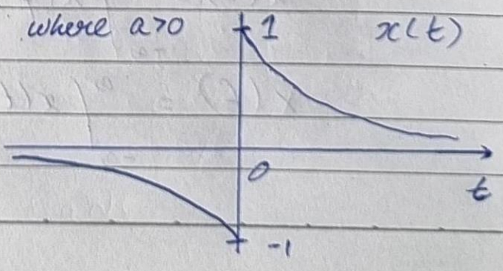


$10\log_{10} A^2$

Max ampli A

$A^2/2$

Ex-3 $\quad x(t) = \begin{cases} e^{-at} & t>0 \\ 0 & t=0 \\ -e^{at} & t<0 \end{cases}$ $\quad$ where $a>0$



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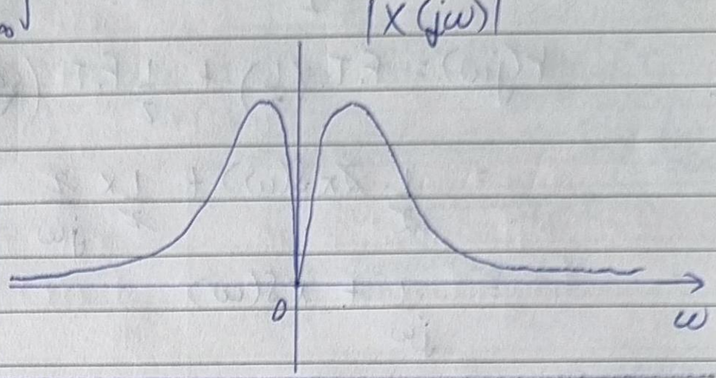
$$X(j\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t}\, dt - \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt$$

$$= \frac{-2j\omega}{a^2+\omega^2}$$

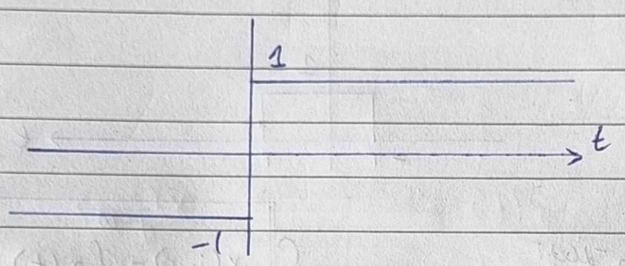


Case: let $a \to 0$

$$x(t) = sgn(t) = \begin{cases} 1 & t>0 \\ 0 & t=0 \\ -1 & t<0 \end{cases}$$

$$y(t) = \frac{1}{2}(1+ sgn(t))$$

$$X(j\omega) = \frac{2}{j\omega}$$



Ex: 4

$$x(t) = \delta(t)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt$$

$$= 1$$

$$x(t) \underset{I.F.T.}{\overset{F.T.}{\rightleftharpoons}} X(j\omega)$$

$$\boxed{\begin{aligned} \delta(t) &\rightleftharpoons 1 \\ 1 &\rightleftharpoons 2\pi\delta(\omega) \end{aligned}}$$

Ex: 5

$$X(j\omega) = 2\pi\delta(\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t}\, d\omega = \frac{1}{\cancel{2\pi}}\int_{-\infty}^{\infty} \cancel{2\pi}\delta(\omega) e^{j\omega t}\, d\omega$$

$$= 1$$

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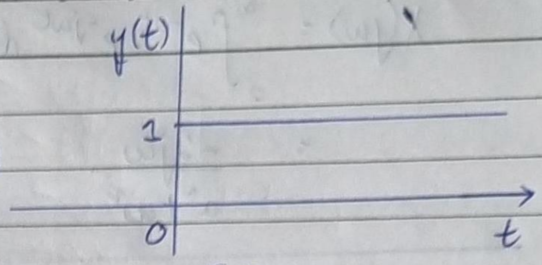
Ex-6 $\quad y(t) = \frac{1}{2}\left(1 + \text{sgn}(t)\right) = u(t)$



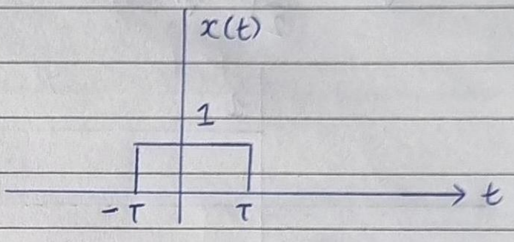
$Y(j\omega) = F.T.\left(\frac{1}{2}\right) + \frac{1}{2} F.T.\left(\text{sgn}(t)\right)$

$= \frac{1}{\cancel{2}} \cdot \cancel{2}\pi \delta(\omega) + \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{j\omega}$

$= \frac{1}{j\omega} + \pi\delta(\omega)$

$u(t) \underset{I.F.T.}{\overset{F.T.}{\rightleftharpoons}} \frac{1}{j\omega} + \pi\delta(\omega)$

$x(t) = \begin{cases} 1 & -T \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$



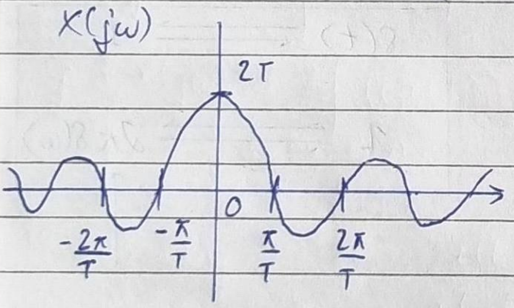
$X(j\omega) = \int_{-T}^{T} e^{-j\omega t}\, dt$

$= \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-T}^{T} = \frac{e^{j\omega T} - e^{-j\omega T}}{j\omega}$

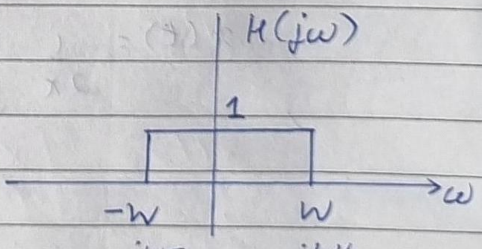
$= \frac{2}{\omega}\left(\frac{e^{j\omega T} - e^{-j\omega T}}{2j}\right) = \left(\frac{2}{\omega}\right)\sin\omega T$

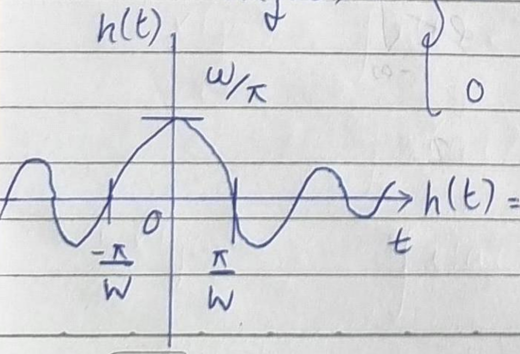
$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$



$H(j\omega) = \begin{cases} 1 & -W \leq \omega \leq W \\ 0 & \text{otherwise} \end{cases}$





$h(t) = \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t}\, d\omega = \frac{e^{jWt} - e^{-jWt}}{j2\pi t}$

$= \frac{1}{\pi t}\sin Wt$

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$\text{rect}\left(\frac{t}{2T}\right) \underset{\text{I.F.T.}}{\overset{\text{F.T.}}{\rightleftharpoons}} \frac{2\sin\omega T}{\omega}$

$\frac{1}{2\pi}\left\{\frac{2\cdot\sin Wt}{t}\right\} \underset{\text{I.F.T.}}{\overset{\text{F.T.}}{\rightleftharpoons}} \text{rect}\left(\frac{\omega}{2W}\right)$

time domain to freq.
$t \longrightarrow -\omega$
& multiply by 2π

freq. domain to Time domain
$\omega \longrightarrow -t$
& divide by 2π

$$\text{rect}\left(\frac{t}{2T}\right) = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\underbrace{\left(\frac{2\sin\omega T}{\omega}\right)}_{X(j\omega)} e^{j\omega t}\,d\omega$$

Change $t \longrightarrow -\omega$ by multiply by 2π

$$x(-\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(t)\,e^{-j\omega t}\,dt$$

$$2\pi x(-\omega) = \int_{-\infty}^{\infty} X(t)\,e^{-j\omega t}\,dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt$$

$\downarrow$
Change ω to $-t$

$$X(-t) = \int_{-\infty}^{\infty} x(-\omega)\,e^{j\omega t}\,d\omega$$

divide by 2π

$$\frac{X(-t)}{2\pi} = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(-\omega)\,e^{j\omega t}\,d\omega$$

Ex $\quad g(t) = \frac{2}{1+t^2}$ | Find $G(j\omega)$.

$$G(j\omega) = \int_{-\infty}^{\infty}\left(\frac{2}{1+t^2}\right)e^{-j\omega t}\,dt$$

Ex $\quad x(t) = e^{-|t|}$

Prove it $\downarrow$

$$X(j\omega) = \frac{2}{1+\omega^2}$$

$$e^{-|t|} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{2}{1+\omega^2}\right)e^{+j\omega t}\,d\omega$$

Change t with $-\omega$

$$2\pi e^{-|\omega|} = \int_{-\infty}^{\infty}\left(\frac{2}{1+t^2}\right)e^{-j\omega t}\,dt$$

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Properties — ① Linearity

$$x(t) \rightleftharpoons X(j\omega)$$
$$g(t) \rightleftharpoons G(j\omega)$$
$$ax(t) + bg(t) \rightleftharpoons aX(j\omega) + bG(j\omega)$$

—X—

② Differentiation in time domain

$$x(t) \rightleftharpoons X(j\omega)$$
$$\frac{d}{dt}x(t) \rightleftharpoons j\omega X(j\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

$$\frac{d}{dt}x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(X(j\omega)\,j\omega\right)e^{j\omega t}\,d\omega$$

③ Differentiation in the freq. domain

$$x(t) \rightleftharpoons X(j\omega)$$
$$tx(t) \rightleftharpoons j\frac{d}{d\omega}X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$\frac{d}{d\omega}X(j\omega) = \int_{-\infty}^{\infty} x(t)(-jt)\,e^{-j\omega t}\,dt$$

$$j\frac{d}{d\omega}X(j\omega) = \int_{-\infty}^{\infty}\left(tx(t)\right)e^{-j\omega t}\,dt$$

④ Scaling

$$x(t) \rightleftharpoons X(j\omega)$$
$$x(at) \rightleftharpoons \frac{1}{|a|}X\left(j\frac{\omega}{a}\right)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} x(a\tau)\,e^{-j\omega . a\tau}\,a . d\tau$$

$$\frac{1}{a}X(j\omega) = \int_{-\infty}^{\infty} x(a\tau)\,e^{-j(\omega . a)\tau}\,d\tau$$

$a > 0$

$t = a\tau$

$dt = a\,d\tau$

$\omega' = \omega a$

$\omega = \frac{\omega'}{a}$

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$$\frac{1}{a}X\left(j\frac{\omega'}{a}\right) = \int_{-\infty}^{\infty} x(a\tau)e^{-j\omega'\tau}\,d\tau \Rightarrow$$

⑤ Parseval's relation

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$\int_{-\infty}^{\infty} x(t)x^*(t)\,dt = \int_{-\infty}^{\infty} x(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega)e^{-j\omega t}d\omega\right]dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega)\left(\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt\right)d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega)X(j\omega)d\omega$$

Proved

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$|X(j\omega)|^2 \equiv$ Energy spectrum

⑥ Convolution in time domain $\Rightarrow$ Multiplication in the freq. domain

$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \rightleftharpoons X(j\omega)H(j\omega)$$

$$x(t) \rightleftharpoons X(j\omega)$$
$$h(t) \rightleftharpoons H(j\omega)$$

---

⑦ Multiplication in time domain $\Rightarrow$ Convolution in the freq. domain

$$r(t)g(t) \rightleftharpoons \frac{1}{2\pi}\left(R(j\omega) \underset{\text{Convolution}}{*} G(j\omega)\right)$$

Ex impulse response $h(t) = e^{-at}u(t)$ $a>0$

i/p $x(t) = e^{-bt}u(t)$ $b>0$

$y(t) = ?$ $= x(t) * h(t)$

Case I: $a = b$

$$\boxed{j\frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right) = \frac{1}{(a+j\omega)^2}}$$

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$$X(j\omega) = \frac{1}{b+j\omega}$$

$$H(j\omega) = \frac{1}{a+j\omega}$$

$$\because \; a=b \Rightarrow X(j\omega)H(j\omega) = \left(\frac{1}{a+j\omega}\right)^2 = Y(j\omega) = j\frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right)$$

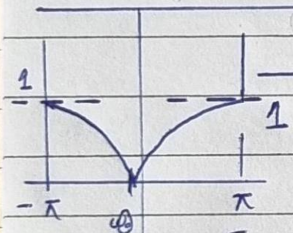
$$y(t) = te^{-at}u(t)$$

Case II $\quad b \neq a$

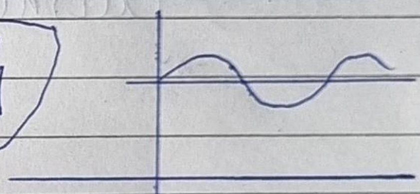
$$Y(j\omega) = \left(\frac{1}{a+j\omega}\right)\left(\frac{1}{b+j\omega}\right)$$

$$= \frac{1}{b-a}\left[\frac{1}{(a+j\omega)} - \frac{1}{(b+j\omega)}\right]$$

$$y(t) = \frac{1}{b-a}e^{-at}u(t) - \left(\frac{1}{b-a}\right)e^{-bt}u(t)$$



$\rightarrow S: \quad y[n] = \frac{1}{2}\{x[n] - x[n-1]\}$



Find the frequency response of the corresponding system —

$|H(e^{j\omega})| = \left|\sin\frac{\omega}{2}\right|$

Let $x[n] = e^{j\omega n}$

$$y[n] = \frac{1}{2}e^{j\omega n} - \frac{1}{2}e^{j\omega(n-1)}$$

$$= e^{j\omega n}\left\{\frac{1-e^{-j\omega}}{2}\right\}$$

$$= H(e^{j\omega}) \cdot e^{j\omega n}$$

$x[n] \rightarrow \boxed{} \rightarrow \frac{1}{2}\{x[n]-x[n-1]\}$

$$z^n \longrightarrow H(z)\,z^n$$

$$e^{j\omega n} \longrightarrow H(e^{j\omega})\,e^{j\omega n}$$

Let $z = e^{j\omega}$

$\Rightarrow \cancel{e^{j\omega n}}\,H(e^{j\omega}) = \cancel{e^{j\omega n}}\left\{\frac{1-e^{-j\omega}}{2}\right\}$

$$H(e^{j\omega}) = \frac{1-e^{-j\omega}}{2}$$

$$= je^{-j\omega/2}\left\{\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right\}$$

$$= je^{-j\omega/2}\sin\frac{\omega}{2}$$

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$x(t) = \cos\omega_0 t \rightarrow \quad \cos\omega_0 t = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$

M1 $\quad X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$

$$= \frac{1}{2}\left[e^{j(\omega_0 - \omega)t} - - - \right] \quad * \text{ can't do.}$$

M2 we know $\rightarrow \quad 1 \rightleftharpoons 2\pi\delta(\omega)$ ——— (1)

&

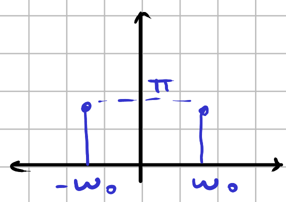
$X(j(\omega - \omega_0)) \rightleftharpoons e^{j\omega_0 t}\, x(t)$ — (2)

using (1) in (2)

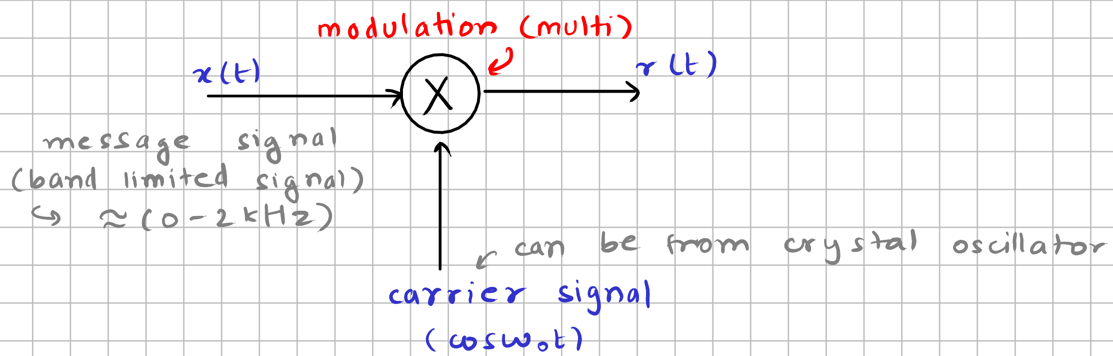
$$2\pi\delta(\omega - \omega_0) \rightleftharpoons e^{j\omega_0 t}.$$

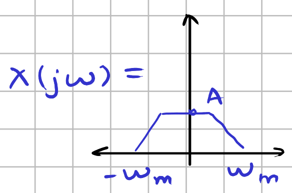
so $\quad \cos\omega_0 t = \frac{1}{2}\left[2\pi\delta(\omega - \omega_0) + 2\pi\delta(\omega + \omega_0)\right]$

$\cos\omega_0 t = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$



* modulation & carrier signal





calc $\quad R(j\omega) = ?$

so $\quad r(t) = x(t) \cdot \cos\omega_0 t$

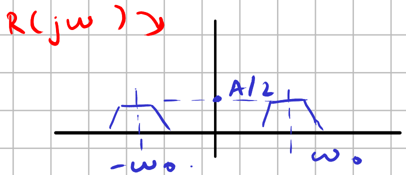
$\cos(\omega_0 t) = g(t)$

multiplication in time domain $\rightarrow$ convulution in freq. domain.

so $\rightarrow \quad R(j\omega) = \frac{1}{2\pi}\left\{ X(j\omega) * G(j\omega) \right\}$

$$= \frac{1}{2\cancel{\pi}}\left[X(j\omega) \cdot \cancel{\pi}\left(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right)\right]$$

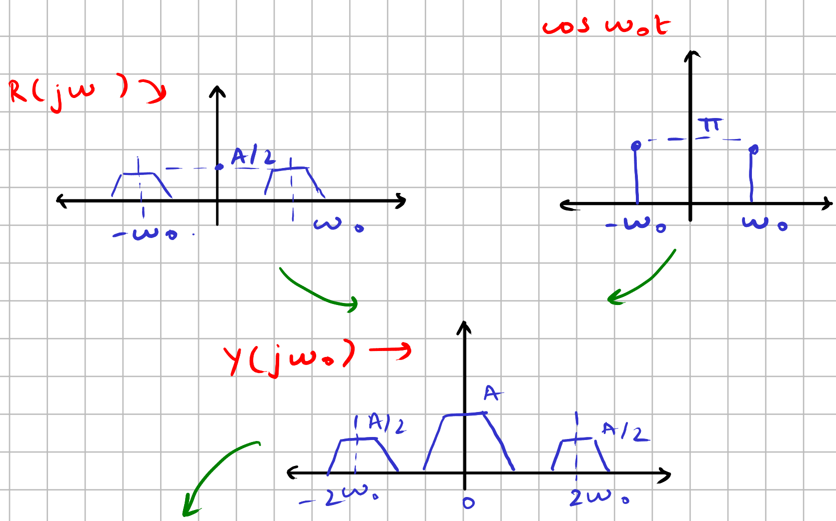
$$= \frac{1}{2}\left[X(j\omega) \cdot \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]\right]$$

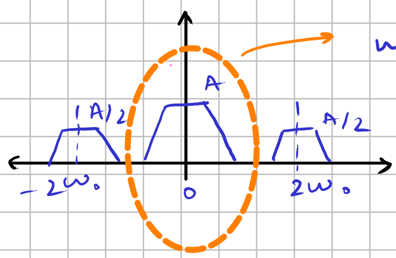


$$\left\{ x(\omega) \cdot \delta(\omega - \omega_0) = x(\omega_0) \right\}$$

now lets convulate $r(t)$ with $g(t)$

$\hookrightarrow$ $y(t) = r(t) \cdot \cos \omega_0 t$





we got our orignal signal back.

$X(j\omega) =$