Tutorial 9

Problem 1)

4. For the circuit in Fig. 15.54, (a) derive an algebraic expression for the transfer function $\mathbf{H}(j\omega) = v_{out}/i_{in}$ terms of circuit components R_1, R_2, C_1, and C_2; and (b) evaluate the magnitude of $\mathbf{H}$ at frequencies of 100 Hz, 10 kHz, and 1 MHz for case where $R_1 = 20$ kΩ, $R_2 = 5$ kΩ, $C_1 = 10$ nF, and $C_2 = 40$ nF.

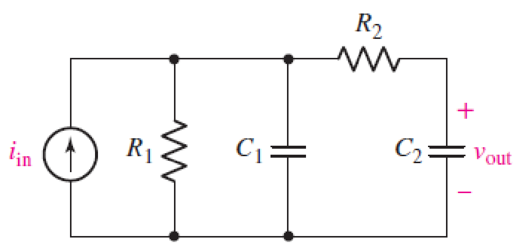


FIGURE 15.54

Problem 2)

6. For the circuit in Fig. 15.56, (a) determine the transfer function $\mathbf{H}(j\omega) = \mathbf{V}_{out}/\mathbf{V}_{in}$ in terms of circuit parameters R_1, R_2, and C; (b) determine the magnitude and phase of the transfer function at $\omega = 0$, 3×10^4 rad/s, and as $\omega \to \infty$ for the case where circuit values are $R_1 = 500$ Ω, $R_2 = 40$ kΩ, and $C = 10$ nF.

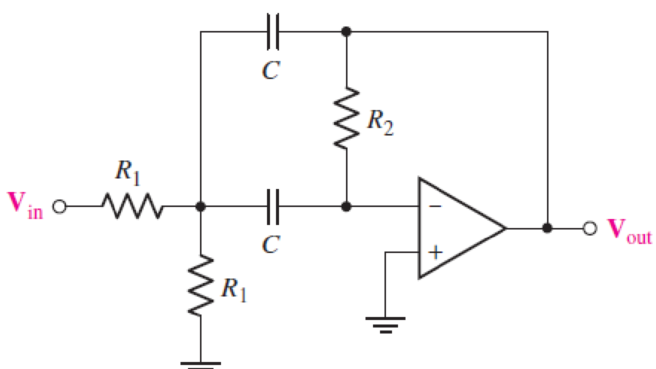


FIGURE 15.56

Problem 3)

8. Sketch the Bode magnitude and phase plots for the following functions:

(a) $3 + 4\mathbf{s}$; (b) $\dfrac{1}{3 + 4\mathbf{s}}$.

Problem 4)

9. For the following functions, sketch the Bode magnitude and phase plots:

$(a)\, 25\left(1+\frac{\mathbf{s}}{3}\right)(5+\mathbf{s})$; $(b)\, \frac{0.1}{(1+5\mathbf{s})(2+\mathbf{s})}$.

Problem 5)

10. Use the Bode approach to sketch the magnitude of each of the following response

$$\frac{4}{\mathbf{s}^3+7\mathbf{s}^2+12\mathbf{s}}.$$

Problem 6)

11. If a particular network is described by transfer function $\mathbf{H}(\mathbf{s})$, plot the magnitude and phase Bode plot

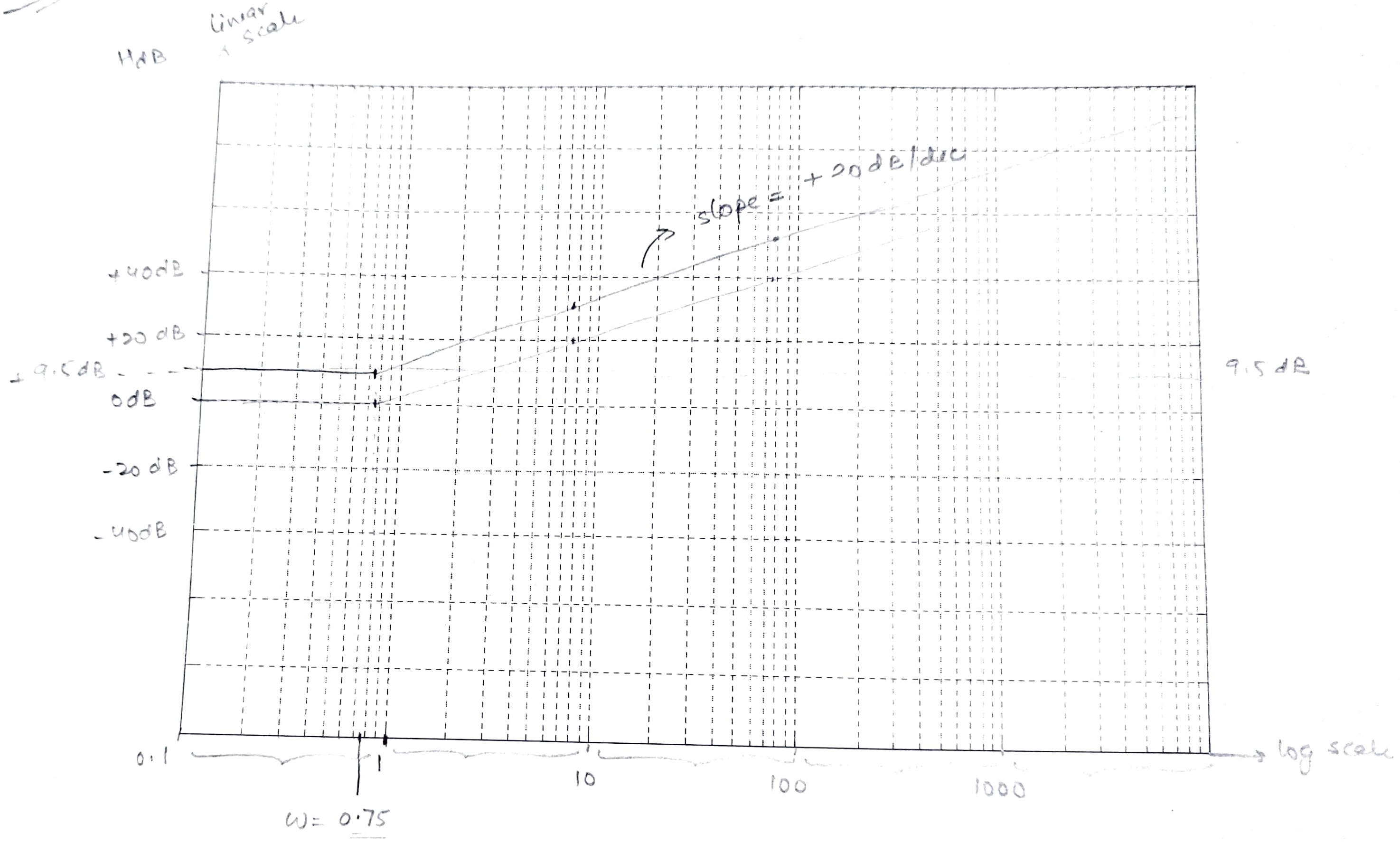
$(a)\, \frac{\mathbf{s}+300}{\mathbf{s}(5\mathbf{s}+8)}$

$H(s) = 3 + 4s = 3(1 + s/_{3/4}) = 3(1 + s/_{0.75})$

$20\log(3) = +9.5$ dB

Q8(a)

Semi-log Paper for Bode Plots

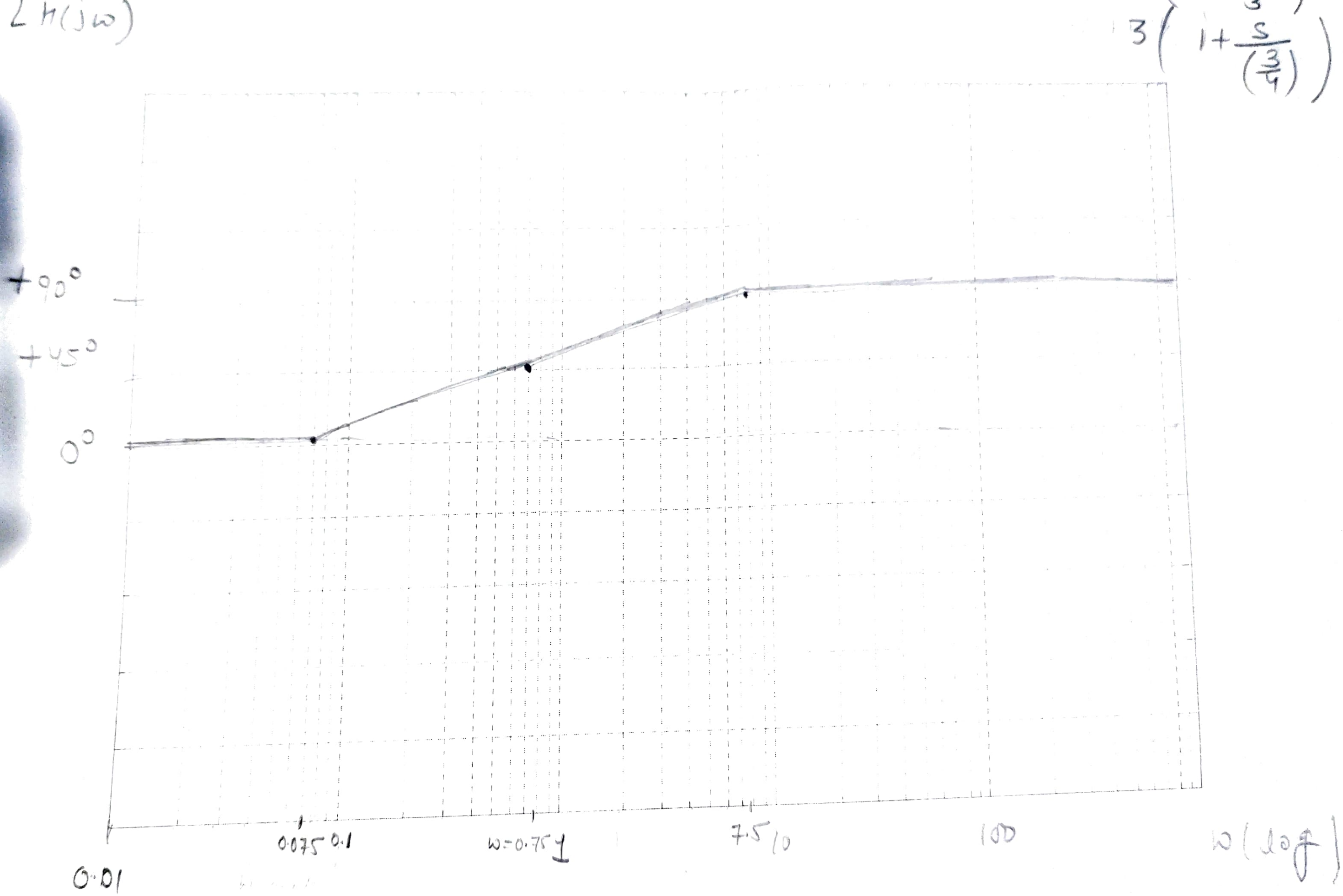


Semi-log Paper for Bode Plots

P-3 (Q-8) (9)

$$H(s) = 3\left(1 + \frac{4}{3}s\right)$$

$$3\left(1 + \frac{s}{\left(\frac{3}{4}\right)}\right)$$

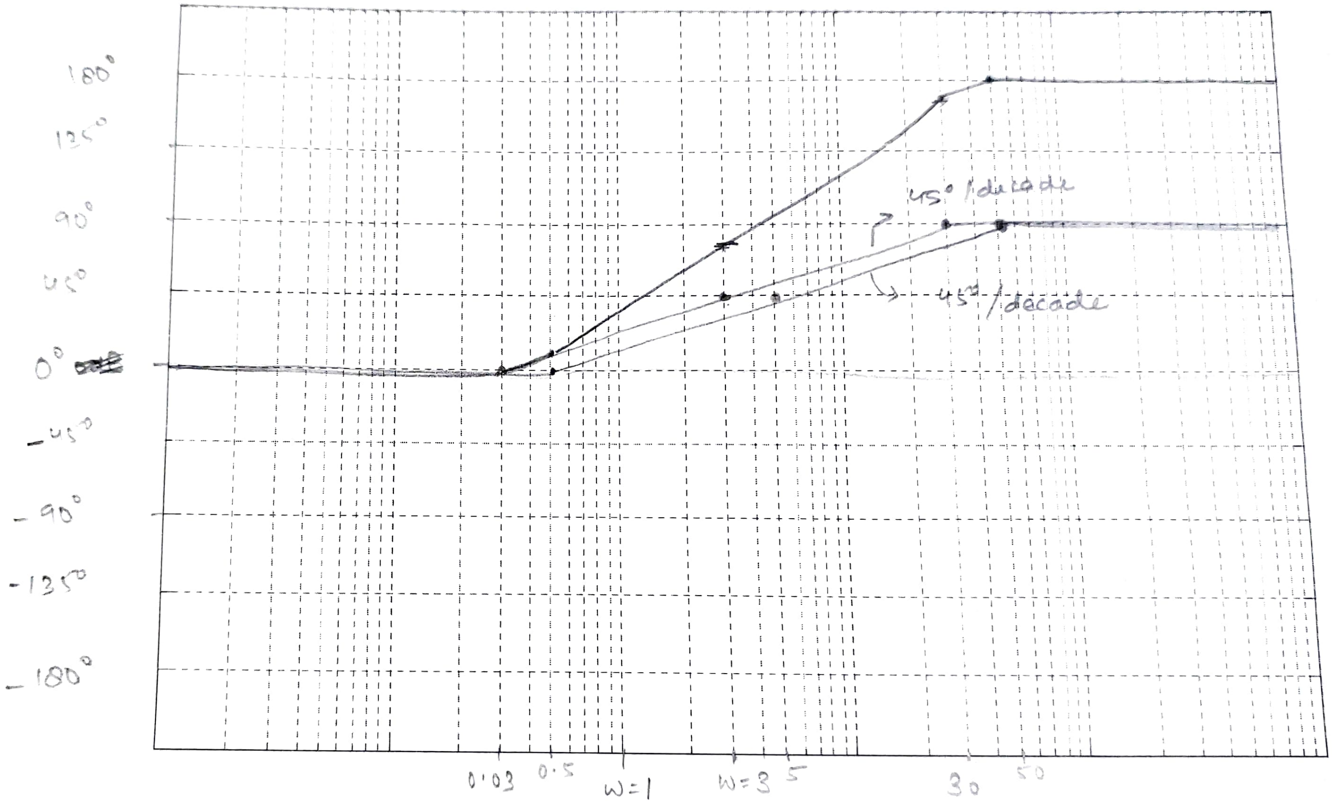


Q9(a)

Semi-log Paper for Bode Plots

$\angle H(j\omega)$

$125\left(1+\frac{s}{3}\right)\left(1+\frac{s}{5}\right)$



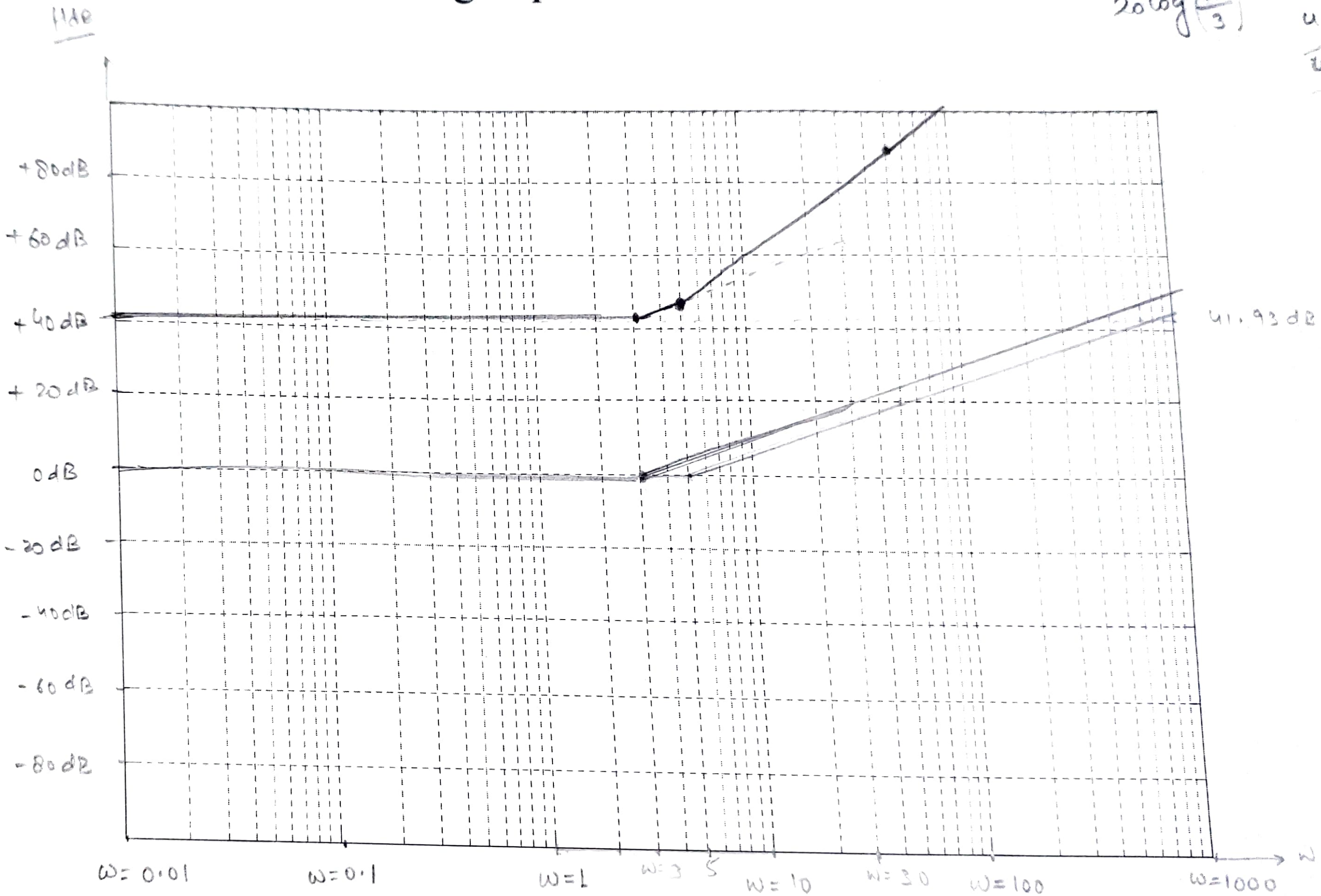
Q9(a)

Semi-log Paper for Bode Plots

$20\log\left(\frac{\omega}{3}\right)^5 = 6.66\,dB$

41.9

48.5



Q11

$$\frac{37.5\,(1 + s/300)}{s\,(1 + s/(8/5))}$$

1.6

Semi-log Paper for Bode Plots

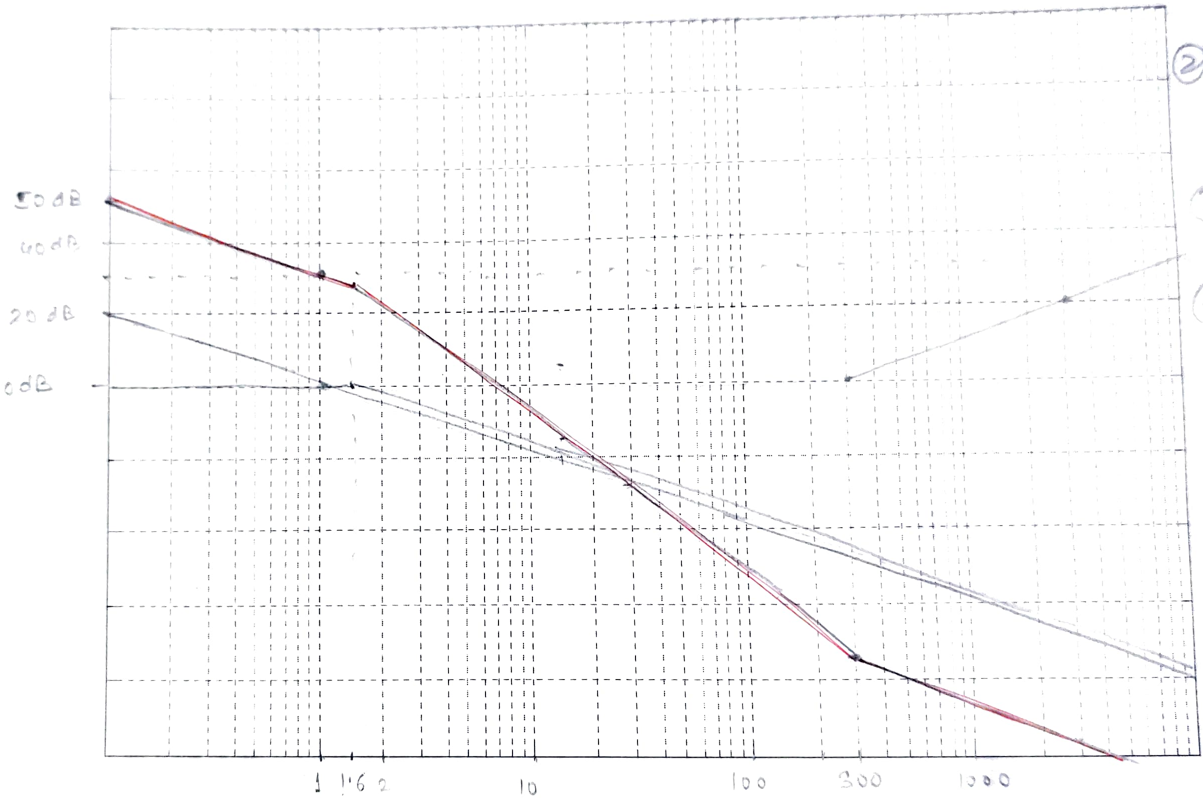
① $20\log(37.5) = 31.5$ dB

② $\left(\frac{1}{s}\right)^n \Rightarrow -20n\log\omega$

at $\omega = 1$ 0dB

③ $\frac{1}{(1 + s/1.6)}$

④ $(1 + s/300)$

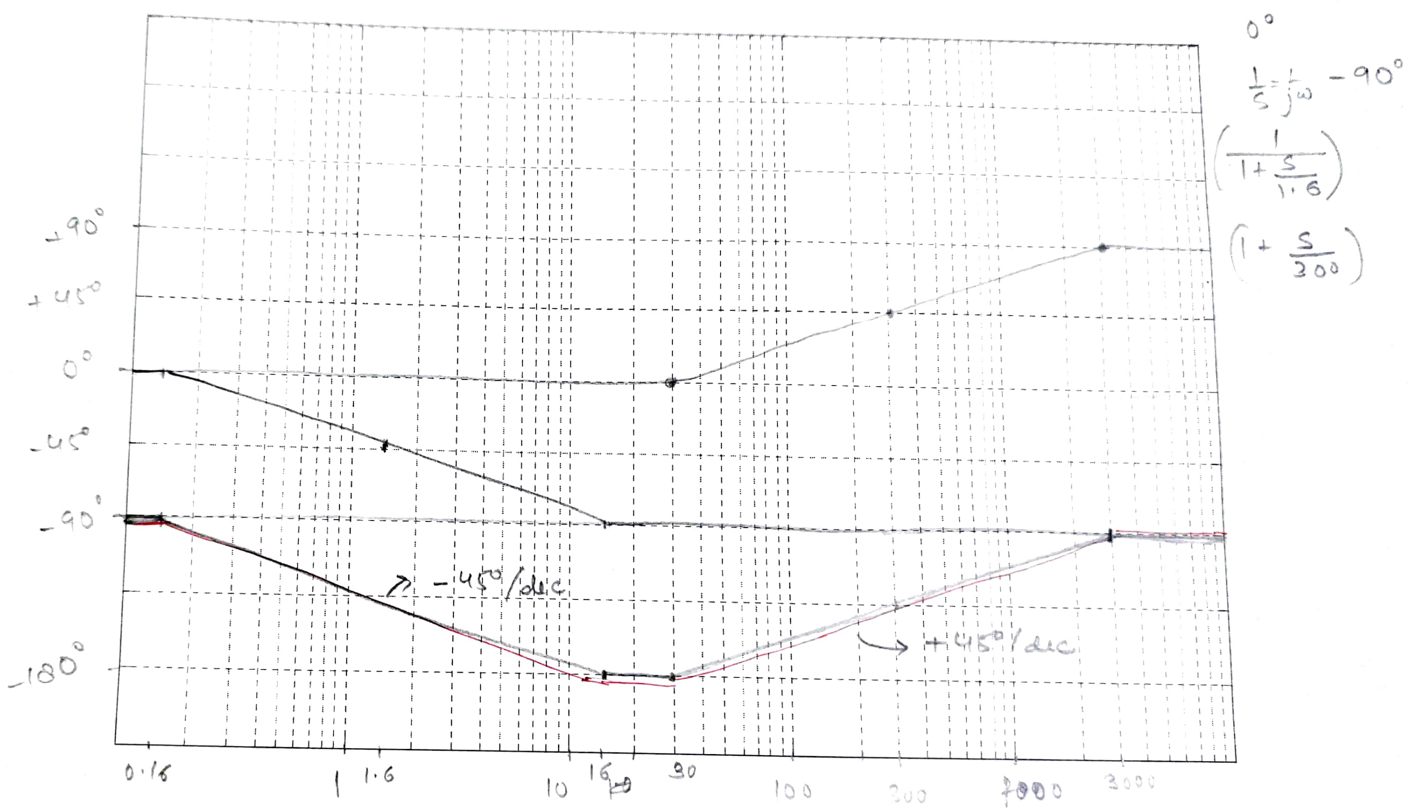


Q.11

$\angle H(j\omega)$

Semi-log Paper for Bode Plots

$$\frac{37.5(1+s/300)}{s(1+s/1.6)}$$

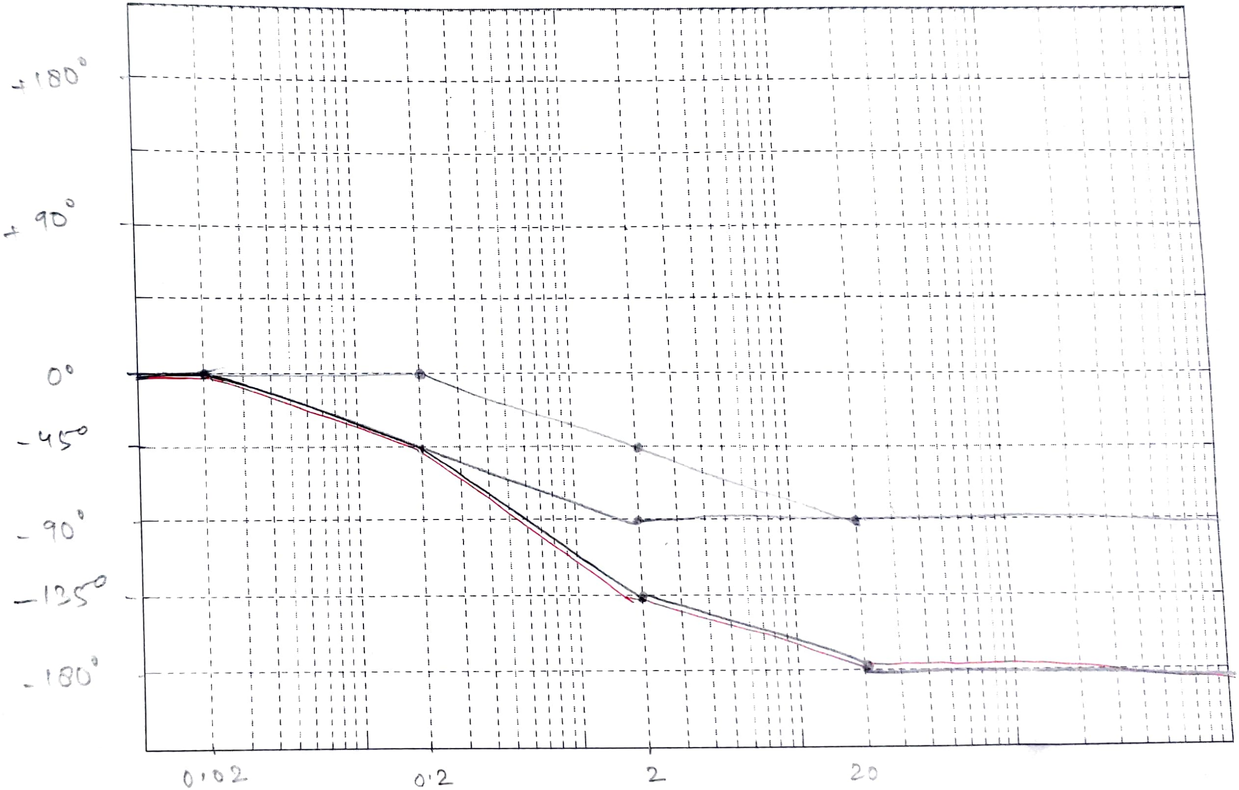


Q9(b)

$$\frac{0.05}{(1+s/0.2)(1+s/2)}$$

Semi-log Paper for Bode Plots

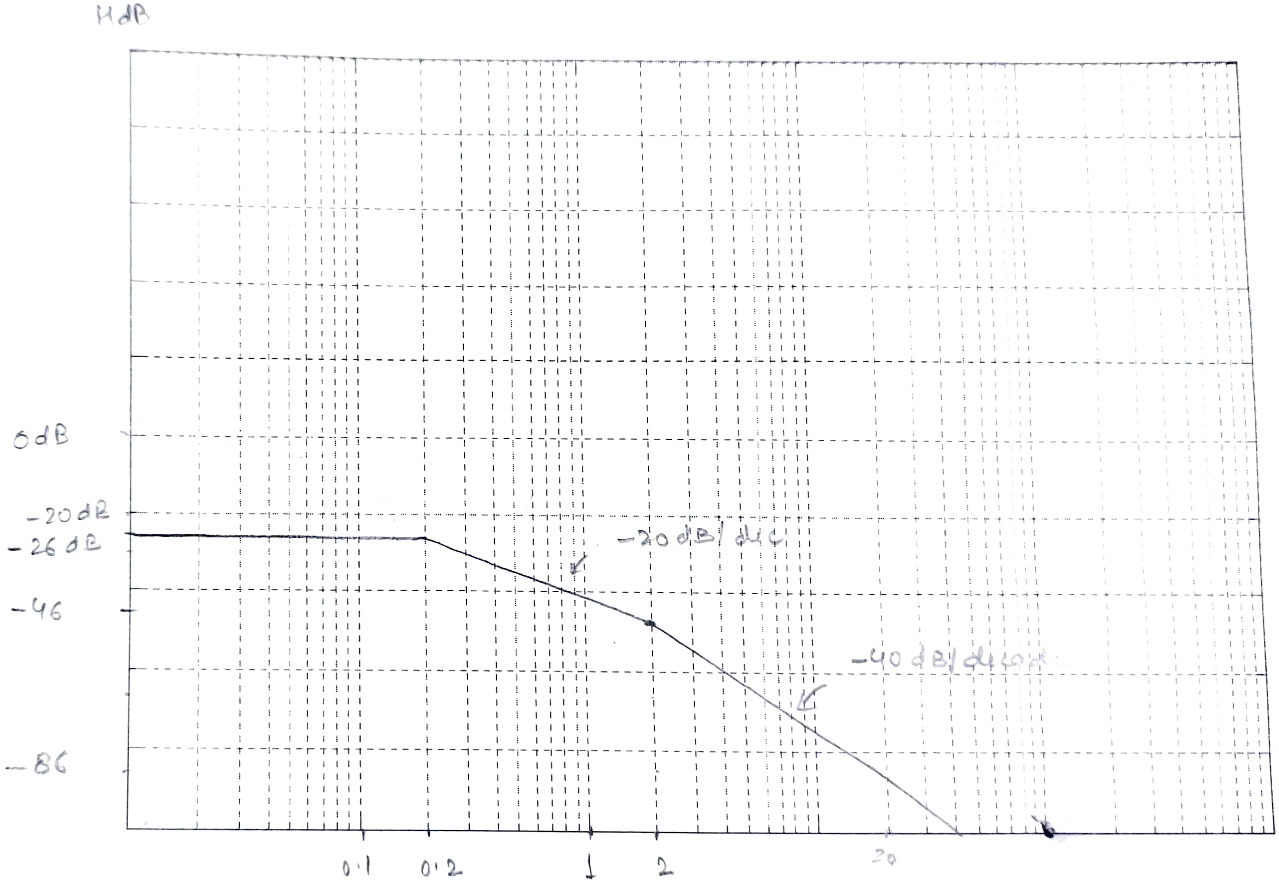
$\angle H(j\omega)$



Q 9(b)

$$\frac{0.05}{(1+s/0.2)(1+s/2)}$$

Semi-log Paper for Bode Plots



Q-10 (P-5)

$$H(s) = \frac{4}{s^3+7s^2+12s}$$

$$= \frac{4}{s(s^2+7s+12)} = \frac{4}{s(s+3)(s+4)}$$

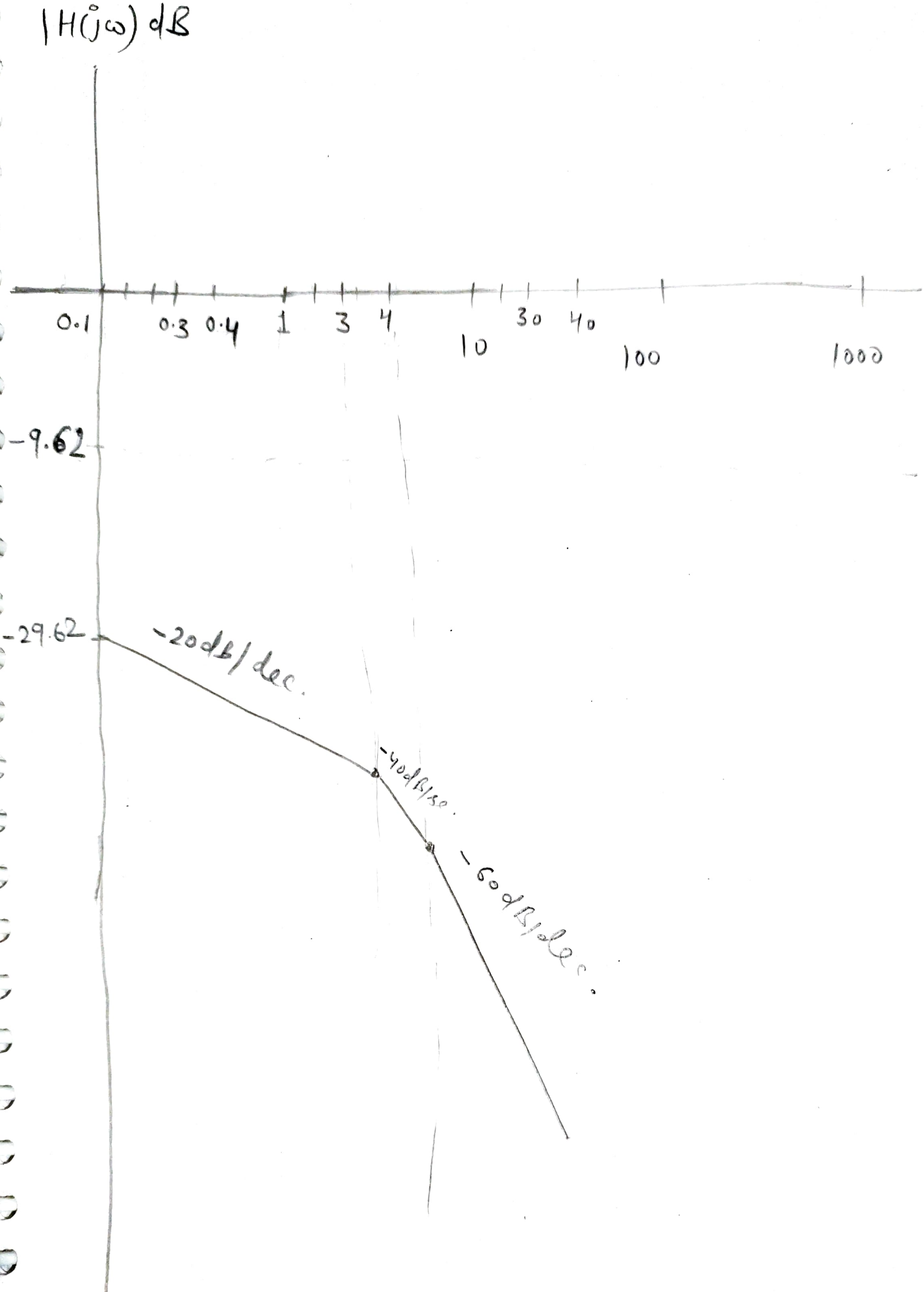
$$= \frac{4}{12 \times s\left(1+\frac{s}{3}\right)\left(1+\frac{s}{4}\right)} = \frac{1}{3s\left(1+\frac{s}{3}\right)\left(1+\frac{s}{4}\right)}$$

$$H(s) = \frac{0.33}{s\left(1+\frac{s}{3}\right)\left(1+\frac{s}{4}\right)}$$

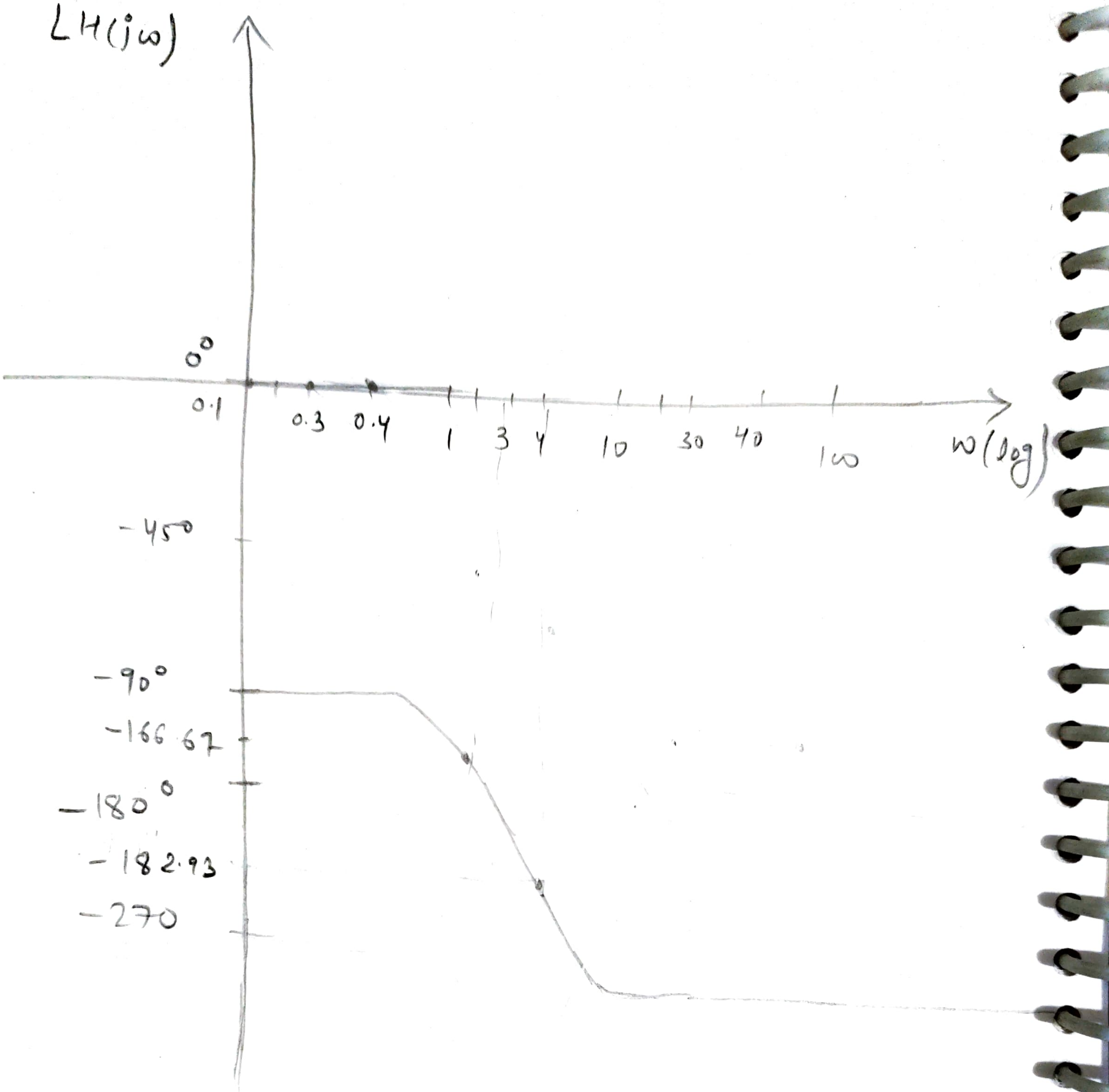
0.33 : $20\log(0.33) = -9.62\ dB$

$\left(1+\frac{s}{3}\right)$: Corner freq. $w_c = 3$ = $-20\ dB$

$\left(1+\frac{s}{4}\right)$: Corner freq. $w_c = 4$ = $-20\ dB$.



|H(jω) dB
0.1
0.3
0.4
1
3
4
10
30
40
100
1000
-9.62
-29.62
-20dB/dec.
-40dB/se.
-60dB/dec.


∠H(jω)
0°
0.1
0.3
0.4
1
3
4
10
30
40
100
ω(log)
-45°
-90°
-166.67
-180°
-182.93
-270

Q-8 (b)

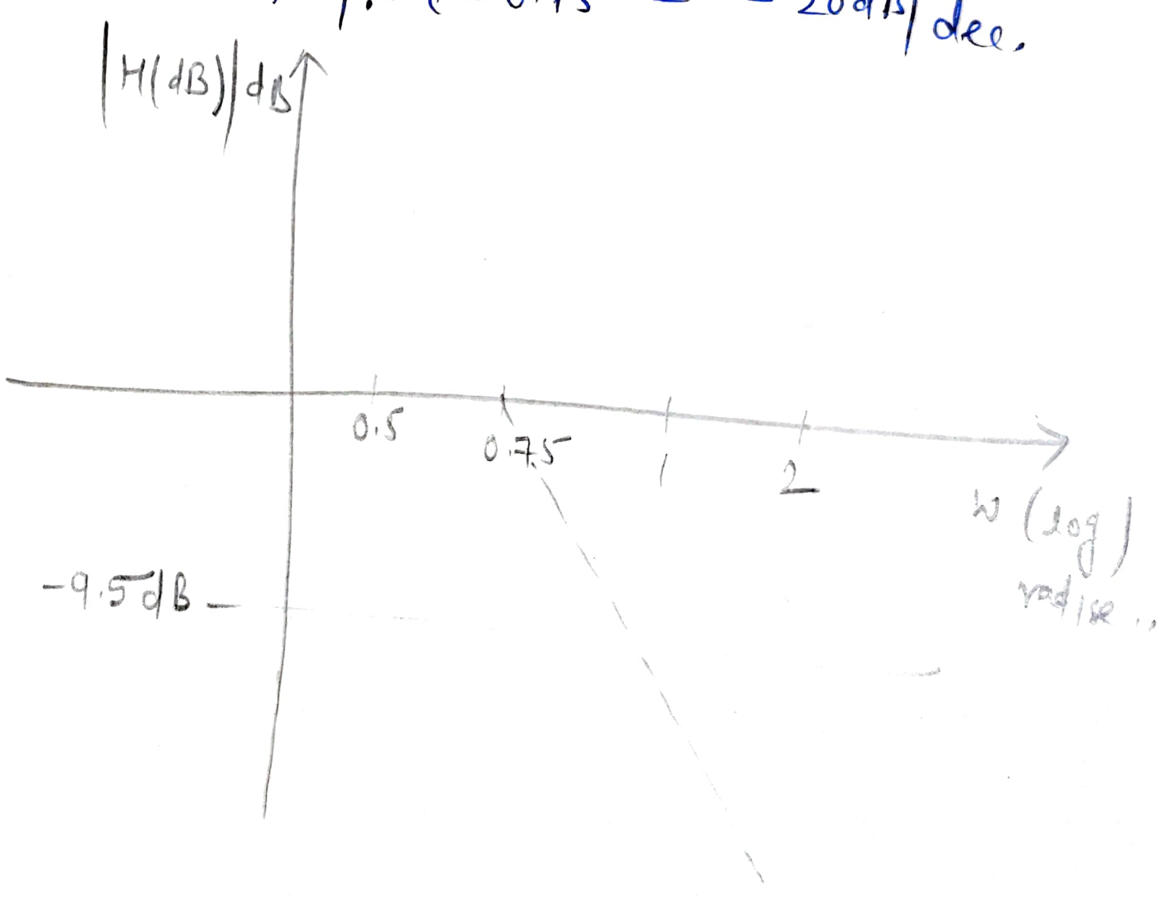
$$H(s) = \frac{1}{3+4s}$$

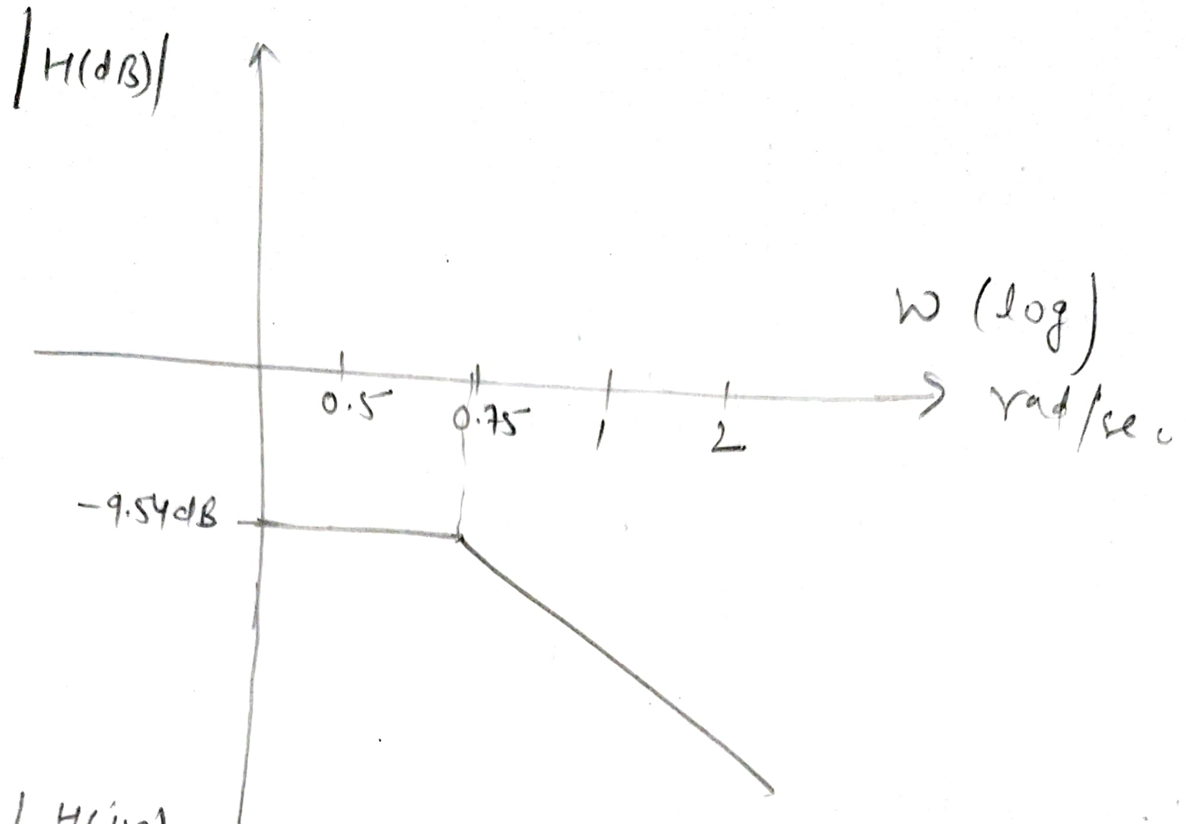
$$H(s) = \frac{1}{3}\left(\frac{1}{\left(1+\frac{4s}{3}\right)}\right)$$

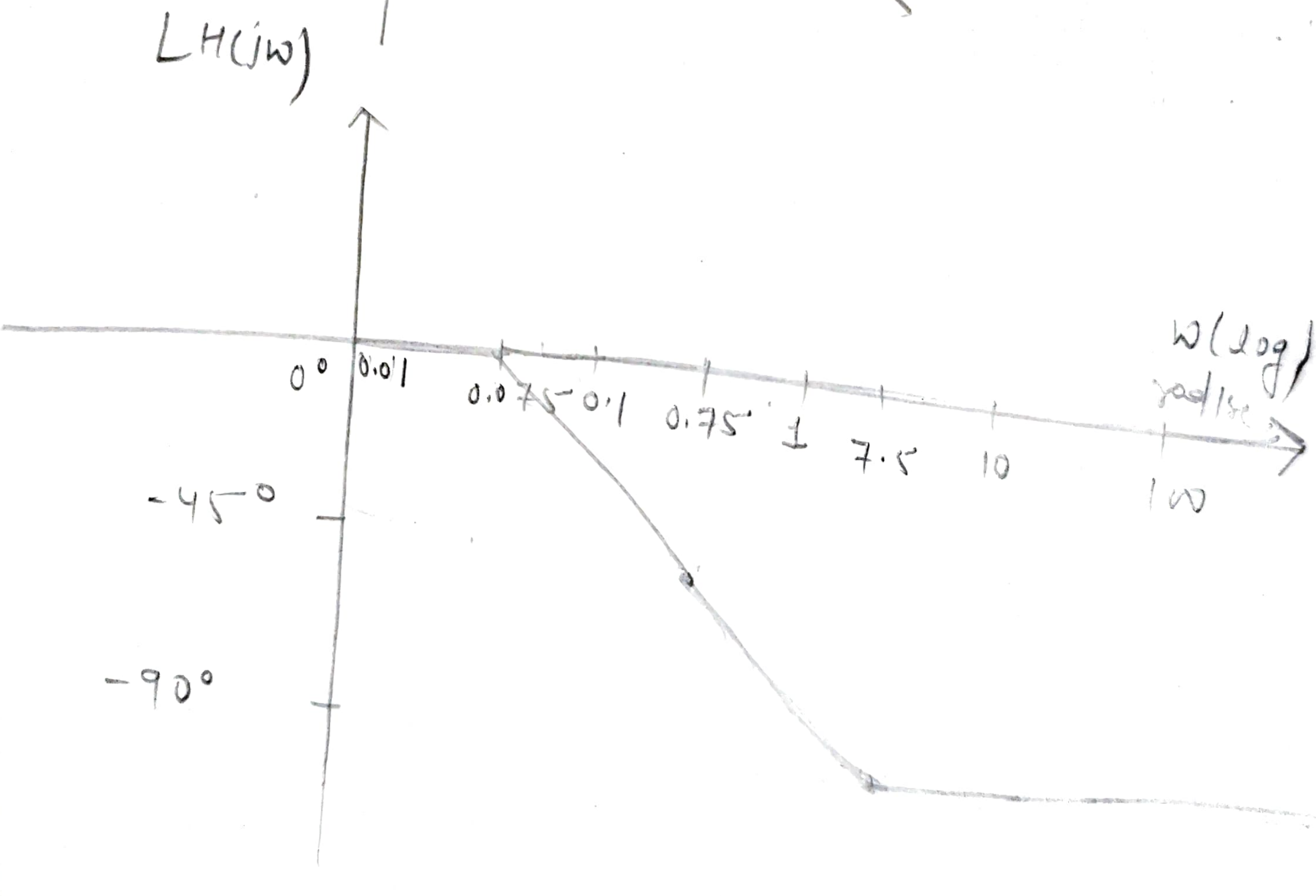
$$= \frac{1}{3}\left(\frac{1}{1+\frac{s}{\left(\frac{3}{4}\right)}}\right)$$

$\frac{1}{3}$: $20\log(1/3) = -9.54\,dB$

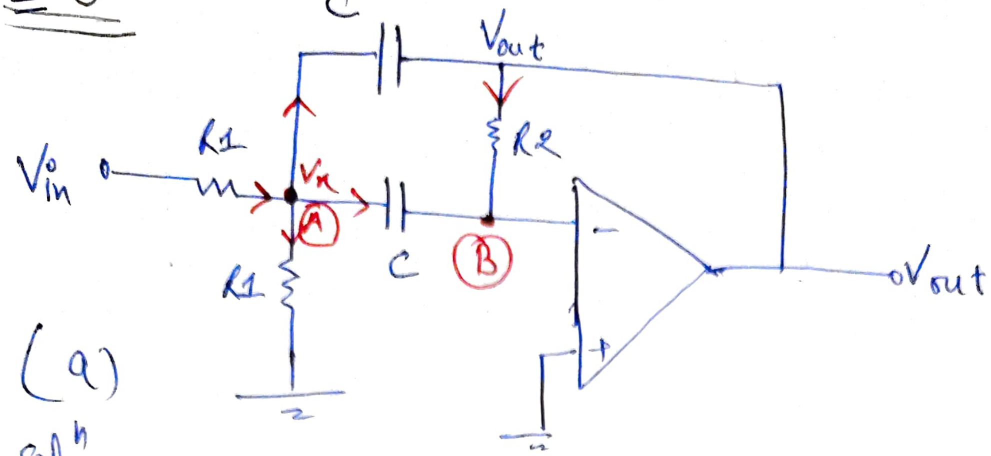
$\left(\frac{1}{1+\frac{s}{\left(\frac{3}{4}\right)}}\right)$: corner freq. $w_c = 0.75$ = $-20\,dB/dec.$




|H(dB)|
w (log) rad/sec
0.5
0.75
1
2
-9.54dB


∠H(jw)
w(log) rad/sec
0°
0.01
0.075
0.1
0.75
1
7.5
10
100
-45°
-90°

Q 6



(a)

Soln

KCL at node A

$$\frac{V_{in}-V_x}{R_1} = (V_x - V_{out})SC + \frac{V_x}{R_1} + V_x SC$$

$$\frac{V_{in}}{R_1} + V_{out}SC = 2V_x SC + \frac{V_x}{R_1} + \frac{V_x}{R_1}$$

$$\frac{V_{in} + V_{out}SCR_1}{\cancel{R_1}} = \frac{(2SCR_1 + 2)}{\cancel{R_1}} V_x$$

$$V_x = \frac{V_{in} + V_{out}SCR_1}{2 + 2SCR_1}$$

KCL at node B

$$(V_x - 0)SC + \frac{V_{out} - 0}{R_2} = 0$$

$$\frac{V_{in} \times SC + V_{out}S^2C^2R_1}{2(1+SCR_1)} + \frac{V_{out}}{R_2} = 0$$

$$\frac{V_{out}}{R_2} = \frac{-V_{in} sC}{2(1+sCR_1)} - \frac{V_{out}\, s^2C^2R_1}{2(1+sCR_1)}$$

$$V_{out} = \frac{-V_{in}\, sCR_2}{2(1+sCR_1)} - \frac{V_{out}\, s^2C^2R_1R_2}{2(1+sCR_1)}$$

$$V_{out}\left[1+\frac{s^2C^2R_1R_2}{2(1+sCR_1}\right] = \frac{-V_{in}\, sCR_2}{2(1+sCR_1)}$$

$$V_{out}\left[2(1+sCR_1)+s^2C^2R_1R_2\right] = -V_{in}sCR_2$$

$$\boxed{H(s) = \frac{V_{out}}{V_{in}} = \frac{-sCR_2}{2(1+sCR_1)+s^2C^2R_1R_2}}$$

$$H(j\omega) = \frac{-j\omega CR_2}{2(1+j\omega CR_1)+(-\omega^2C^2R_1R_2)}$$

$$H(j\omega) = \frac{-j\omega CR_2}{2(1+j\omega CR_1)-\omega^2C^2R_1R_2}$$

$$|H(j\omega)| = \frac{\omega CR_2}{\sqrt{(2-\omega^2C^2R_1R_2)^2+(2\omega CR_1)^2}}$$

At $\omega = 0$

$$\boxed{|H(j\omega)| = \frac{0}{\sqrt{(2-0)^2+(0)^2}} = 0}$$

$$\angle |H(j\omega)| = 0 - \tan^{-1}\left(\frac{2\omega C R_1}{2-\omega^2 C^2 R_1 R_L}\right)$$

at $\omega = 0$

$$\boxed{\angle |H(j\omega)| = 0^{\circ}}$$

(Phase can not be determined)

$$|H(j\omega)| = \frac{3\times10^4 \times 10\times10^{-9} \times 40\times10^3}{\sqrt{\left(2-(3\times10^4)^2 \times (10\times10^{-9})^2 \times 40\times10^3 \times 500\right)^2 + \left(2\times3\times10^4 \times 10\times10^{-9} \times 500\right)^2}}$$

$$= \frac{12}{\sqrt{(2-0.18)^2 + (0.3)^2}}$$

$$|H(j\omega)| = 6.507$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{2\times3\times10^4\times10^{-8}\times500}{(2-0.18)^2}\right)$$

$$= -\tan^{-1}\left(\frac{3000\times10^{-4}}{3.3}\right)$$

$$= -\tan^{-1}\left(\frac{0.3}{3.3}\right)$$

$$\boxed{\angle H(j\omega) = -5.199^{\circ}}$$

$\omega = 3\times10^4$ rad/s.

$$|H(j\omega)|\Big|_{\omega=\infty} = \frac{\omega C R_2}{\omega^2\sqrt{\left(\frac{2}{\omega^2} - C^2R_1R_2\right)^2 + \left(\frac{2CR_1}{\omega^2}\right)^2}}$$

$$\boxed{|H(j\omega)| = \frac{1}{\infty} = 0}$$

$$\angle H(j\omega)| = \text{not defined}$$

TUTORIAL - 9

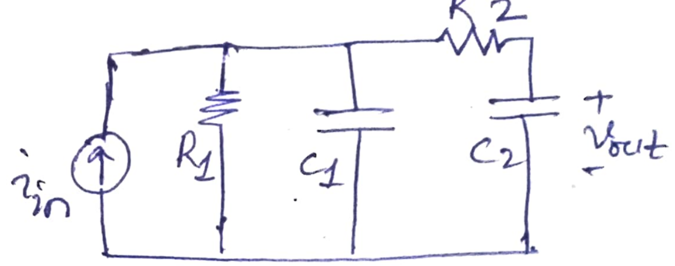
1) Given,

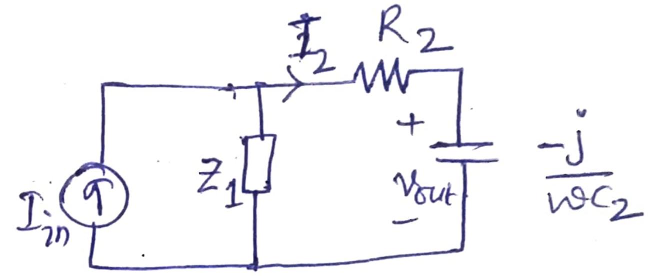
$R_1 = 20\,K\Omega$

$R_2 = 5\,K\Omega$

$C_1 = 10\,nF$

$C_2 = 40\,nF$





$$Z_1 = R_1 \parallel \frac{1}{j\omega C_1}$$

$$\therefore Z_1 = \frac{R_1}{1 + j\omega R_1 C_1}$$

By using current division rule

$$I_2 = \frac{Z_1}{Z_1 + R_2 + \frac{1}{j\omega C_2}} \times I_{in}$$

$$\Rightarrow I_2 = \frac{Z_1 j\omega C_2}{j\omega C_2 Z_1 + j\omega C_2 R_2 + 1} I_{in}$$

By using ohm's law,

$$V_{out} = \frac{1}{j\omega C_2} I_2$$

$$\therefore V_{out} = \frac{1}{j\omega C_2} \times \frac{Z_1 j\omega C_2}{j\omega C_2 Z_1 + j\omega C_2 R_2 + 1} I_{in}.$$

$$\Rightarrow \frac{V_{out}}{I_{in}} = \frac{\left(R_1 / 1 + j\omega R_1 C_1\right)}{j\omega C_2\left(\frac{R_1}{1 + j\omega R_1 C_1} + R_2\right) + 1}$$

$$\Rightarrow \frac{V_{out}}{I_{in}} = \frac{R_1}{j\omega C_2 (R_1 + R_2 + jR_1R_2\omega C_1) + 1 + j\omega R_1 C_1}$$

$$\Rightarrow \frac{V_{out}}{I_{in}} = \frac{R_1}{j\omega C_2 R_1 + j\omega C_2 R_2 - R_1R_2C_1C_2\omega^2 + j\omega R_1 C_1 + 1}$$

$$\Rightarrow \frac{V_{out}}{I_{in}} = \frac{R_1}{(1 - R_1R_2C_1C_2\omega^2) + j(\omega C_1 R_1 + \omega C_2 R_2 + \omega C_2 R_1)}$$

$$\Rightarrow H(j\omega) = \frac{R_1}{(1 - \omega^2 R_1R_2C_1C_2) + j(R_1C_1 + R_2C_2 + R_1C_1)\omega}$$

$$\Rightarrow H(j\omega) = \frac{20K}{(1 - \omega^2 \times 4 \times 10^{-8}) + j\omega(1.2 \times 10^{-3})}$$

(b) If $f = 100\,Hz$

$\omega = 2\pi f \Rightarrow \omega = 628.318$ rad/sec

$$\therefore H(j\omega) = \frac{20K}{0.989 + j0.754} = 16131.36\angle -37.45^\circ$$

$$\boxed{|H(j\omega)|_{f=100Hz} = 16131.3623}$$

If $f = 10\,KHz \Rightarrow \omega = 62831.853$ rad/sec

$$\therefore H(j\omega) = \frac{20K}{-156.914 + j75.398}$$

If $f = 1 MHz \Rightarrow \omega = 6283185.307$ rad/sec

$\therefore H(j\omega) = \dfrac{20k}{-1579135.704 + j2539.822}$

$H(j\omega) = 0.01266 \angle -179.726°$

$\therefore \boxed{|H(j\omega)|_{f=1MHz} = 0.01266}$

---

2) Given, $H(s) = 3+4s$

$= 3\left[1 + \dfrac{s}{\frac{3}{4}}\right]$

gain, $K = 3$ & cutt-off frequency $\omega_c = \frac{3}{4}$ rad/

→ One zero at $s = -\frac{3}{4}$

→ no pole or zero is present at the origin. So the starting slope will be 0 dB/dec.

starting point, $20\log(3) \pm 20\times0 = 9.54$ dB

$H(\omega) = 3 + 4j\omega$

$|H(\omega)| = \sqrt{(3^2)+(4\omega)^2}$

$|H(j\omega)|_{dB} = 20\log(|H(\omega)|)$