End-Sem Examination

Date: 27.12.2021 Time: 02:00PM - 04:00PM

Instruction:

- *Uploading of your answer sheet needs to be completed on or before 04:05PM in your google classroom section page. Any upload after 04:05PM will result in severe penalty including the possibility of assignment of 0 as your midterm score.*
- *In case you face difficulty with google classroom upload, you may email your section supervisor with your answer-sheet attached. The time of receipt of the email will be considered to be your time of submission.*
- *You may refer to the text book or class-notes. If you need to write verbatim from the book or class notes, you need to provide clear citation. You are not allowed to discuss with your peers or anyone else. Plagiarism policies are strictly enforced.*
- *Your answer sheet needs to contain your name, roll number and tutorial section in the header.*
- *Your answers needs to be correct and properly explained in order to obtain full score.*
- *This question paper has **2 pages**, containing **11 questions**. You can score a maximum of 40 points.*

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1. Determine if the surface $x^2 + y^2 + z^2 = 1$ is tangent to the surface $x^2 + y^2 + z^4 = 1$ at $(0, 0, 1)$. *[2 point]*
2. Determine if the curve $(\ln t, t \ln t, 1)$ is tangent to the surface $xz - yz^2 + \cos xy = 1$ at $(0, 0, 1)$. *[4 point]*
3. Let $a_1, a_2, \ldots, a_n$ be n non-zero real numbers for some integer $n \geq 2$. Find the maximum value of $\sum_{i=1}^{n} a_i^2 x_i$ subject to the constraints $\sum_{i=1}^{n} x_i^2 = 16$. *[4 point]*
4. Find the work done by the force $F(x, y, z) = (x, -z, 2y)$ in moving straight from $(0, 0, 0)$ to $(1, 1, 0)$, then straight to $(1, 1, 1)$, and finally straight back to $(0, 0, 0)$. *[4 point]*
5. Find an example of a vector field $F(x, y, z) = (F_1(x, y, z), F_2(x, y, z), F_3(x, y, z))$ defined and smooth on all of $\mathbb{R}^3$ but not conservative, but satisfies
$$\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z} \quad \text{and} \quad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}$$
[4 point]
6. Find the circulation of $F(x, y, z) = (-y \sin z, x \sin z, xy \cos z + 12)$ around the circle cut from the sphere $x^2 + y^2 + z^2 = 5$ by the plane $z = 1$ clockwise as viewed from above. *[4 point]*

7. Show that if $u = x - y$ and $v = x + y$, then for any continuous f,

$$2\int_0^\infty \int_0^\infty e^{-2sx} f(x-y, x+y)dydx = \int_0^\infty \int_0^\infty e^{-s(u+v)} f(u,v)dudv$$

[4 point]

8. What domain D in space minimizes the value of the integral

$$\iiint_D (4x^2 + y^4 + z^2 - 4)dV$$

[2 point]

9. Solve for a:

$$\int_0^1 \int_0^{2-a-x^2} \int_a^{2-x^2-y} dzdydx = \frac{4}{15}$$

[4 point]

10. Suppose $f(x,y)$ is continuous and non-negative over a non-empty open region R in the plane with a defined area $A(R)$. If $\iint_R f(x,y)dA = 0$, then prove that $f(x,y) = 0$ at every point $(x,y) \in R$. [*Hint: You may first try to deduce that for a continuous f, if $f(a,b) = 2\varepsilon$ for some $\varepsilon > 0$, then there exists $\delta > 0$ such that $f(x,y) > \varepsilon$ for any $(x,y) \in B_\delta(a,b) = \{(x,y) : \|(x,y) - (a,b)\| < \delta\}$.*] *[4 point]*

11. Among all smooth, simple closed curves in the plane, oriented counterclockwise, find the one along which the work done by $F(x,y) = (x^2y + \frac{1}{3}y^3, 4x)$ is the largest? *[4 point]*
