ECE250: Signals and Systems
Practice sheet 5

September 25, 2024

1. (CO1,CO2,CO3) Let $x[n]$ be a real and odd periodic signal with period $N = 7$ and Fourier coefficients a_k. Given that

$$a_{15} = j,\ a_{16} = 2j,\ a_{17} = 3j$$

determine the values of a_0, a_{-1}, a_{-2} and a_{-3}.

2. (CO1,CO2,CO3) Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1 & 0 \leq n \leq 2 \\ -1 & -2 \leq n \leq -1 \\ 0 & otherwise \end{cases}$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k],$$

determine the Fourier series coefficients of the output y[n].

3. (CO1,CO2,CO3) Consider a causal continuous-time LTI system whose input $x(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{d}{dt}y(t) + 4y(t) = x(t),$$

Find the Fourier series representation of the output $y(t)$ for each of the following inputs:

(a) $x(t) = \cos 2\pi t$.

(b) $x(t) = \sin 4\pi t + \cos(6\pi t + \pi/4)$.

4. (CO1,CO2,CO3) In each of the following, we specify the Fourier series coefficients of a signal that is periodic with period 8. Determine the signal $x[n]$ in each case.
   (a) a_k in Figure1(a).
   (b) a_k in Figure1(b).

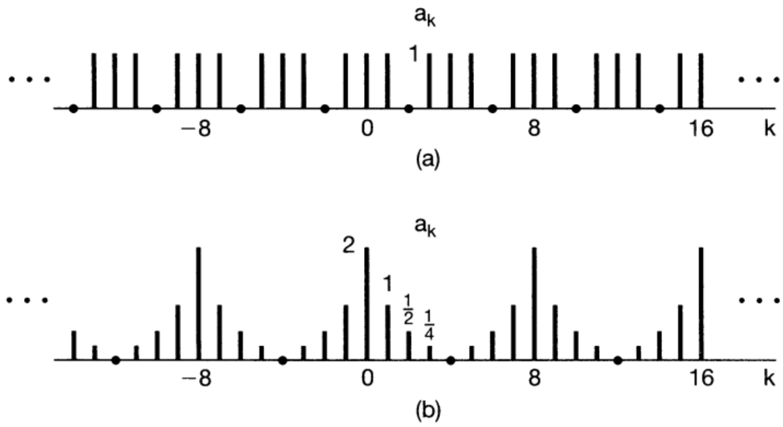

Figure 1: Figure for Q4.

5. (CO1,CO2,CO3) In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal $x(t)$ in following case.
   (a) $a_k = \begin{cases} jk, & |k| < 3 \\ 0 & otherwise \end{cases}$

6. (CO1,CO2,CO3) A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are specified as

$$a_1 = a_{-1}^* = j,\ a_5 = a_{-5} = 2.$$

Express $x(t)$ in the form

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$

7. (CO1,CO2,CO3) Considering a causal LTI system implemented as the RL circuit shown in Figure2. A current source produces an input current $x(t)$, and the system output is considered to be the current $y(t)$ flowing through the inductor. The differential equation relating $x(t)$ and $y(t)$ is given as

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

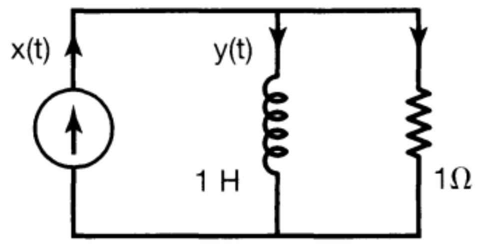

Figure 2: Figure for Q7.

(a) Determine the frequency response of this system by considering the output of the system to input of the form $x(t) = e^{j\omega t}$

8. (CO1,CO2,CO3) For the continuous-time periodic signal

$$x(t) = 2 + \cos\left(\tfrac{2\pi}{3}t\right) + 4\sin\left(\tfrac{5\pi}{3}t\right)$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

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Practice Sheet 6

Q1: (CO1, CO2, CO4) Verify the integration property; that is,

$$\int_{-\infty}^{t} x(\tau)\, d\tau \overset{\mathcal{F}}{\longleftrightarrow} \pi X(0)\delta(\omega) + \frac{1}{j\omega} X(\omega)$$

Q2: (CO1, CO2, CO4) Verify Parseval's relation; that is,

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(\omega)X_2(-\omega)\, d\omega$$

Q3: (CO1, CO2, CO4) Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t} \cos \omega_0 t]u(t), \quad \alpha > 0$

(b) $e^{-3|t|} \sin 2t$

Q4: (CO1, CO2, CO4)

(a) Determine the Fourier transform of the following signal

$$(x(t) = t \left(\frac{\sin(t)}{\pi t} \right)^2$$

(b) Use Parseval's relation and the result of the previous part to determine the numerical value of

$$A = \int_{-\infty}^{\infty} t^2 \left(\frac{\sin(t)}{\pi t} \right)^4 dt$$

Q5: (CO1, CO2, CO4) Find the impulse response of a system with the frequency response

$$H(j\omega) = \frac{\sin^2(3\omega)\cos\omega}{\omega^2}.$$

Q6: (CO1, CO2, CO4) Consider the signal $x(t)$ in Figure 1.

(a) Find the Fourier transform $X(j\omega)$ of $x(t)$.

(b) Sketch the signal

$$\bar{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$

(c) Find another signal $g(t)$ such that $g(t)$ is not the same as $x(t)$ and

$$\bar{x}(t) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$

(d) Argue that, although $G(j\omega)$ is different from $X(j\omega)$, $G\left(j\frac{\pi k}{2}\right) = X\left(j\frac{\pi k}{2}\right)$ for all integers k. You should not explicitly evaluate $G(j\omega)$ to answer this question.

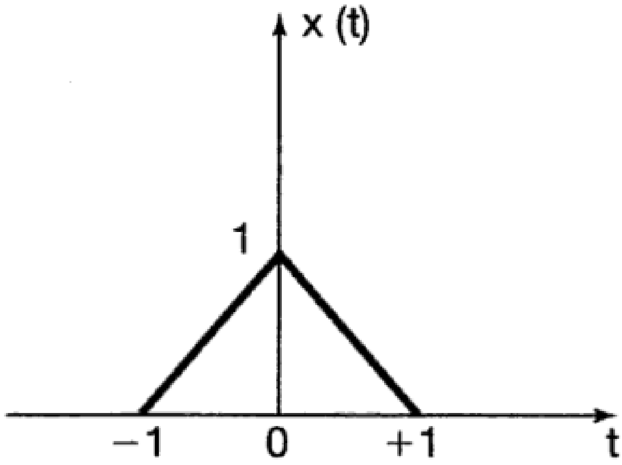

Figure 1: $x(t)$.

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Practice sheet 7

October 23, 2024

1. Suppose that a signal $x(t)$ has Fourier transform $X(j\omega)$. Now consider another signal $g(t)$ whose shape is the same as the shape of $X(j\omega)$; that is,

$$g(t) = X(jt),$$

   (a) Show that the Fourier transform $G(j\omega)$ of $g(t)$ has the same shape as $2\pi x(-t)$; that is, show that
   $$G(j\omega) = 2\pi x(-\omega).$$
   (b) Using the fact that
   $$\mathscr{F}\{\delta(t+B)\} = e^{jB\omega}$$
   in conjunction with the result from part (a). show that
   $$\mathscr{F}\{e^{jBt}\} = 2\pi\delta(\omega - B)$$

2. Suppose $g(t) = x(t)\cos t$ and the Fourier transform of the $g(t)$ is,

$$G(j\omega) = \begin{cases} 1, & |\omega| \leq 2 \\ 0, & otherwise \end{cases}$$

   (a) Determine $x(t)$.
   (b) Plot the Fourier transform $X_1(j\omega)$ of a signal $x_1(t)$ such that
   $$g(t) = x_1(t)\cos\left(\tfrac{2}{3}t\right)$$

3. The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation,

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)\,d\tau - x(t)$$

where $z(t) = e^{-1}u(t) + 3\delta(t)$.

(a) Find the frequency response $H(j\omega) = Y(j\omega)/X(j\omega)$ of this system.

(b) Determine the impulse response of the system.

4. A causal and stable LTI system S has the frequency response,

$$H(j\omega) = \frac{j\omega+4}{6-\omega^2+5j\omega}.$$

(a) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of S.

(b) Determine the impulse response $h(t)$ of S.

(c) what is the output of S when the input is

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t)?$$

5. Consider the signal,

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & otherwise \end{cases}$$

Determine the Fourier transform of each of the signals shown in Figure1. You should be able to do this by explicitly evaluating only the transform of $x_0(t)$ and then using properties of the Fourier transform.

6. Given the relationships

$$y(t) = x(t) * h(t).$$

and

$$g(t) = x(3t) * h(3t).$$

and given that $x(t)$ has Fourier transform $X(j\omega)$ and $h(t)$ has Fourier transform $H(j\omega)$, use Fourier transform properties to show that $g(t)$ has the form

$$g(t) = Ay(Bt).$$

Determine the values of A and B.

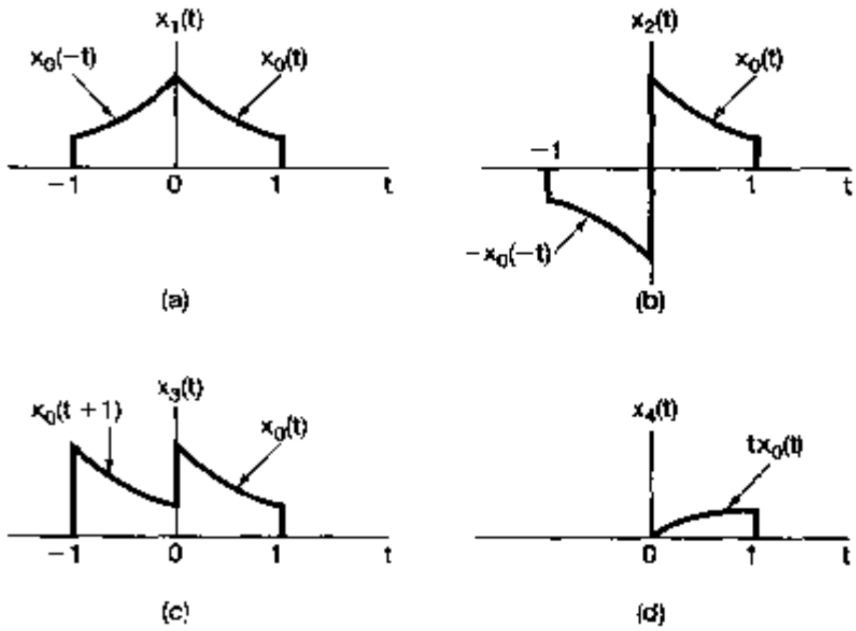

Figure 1: Figure for Q5.

7. The Fourier transform of a particular signal is,

$$X(e^{j\omega}) = \sum_{k=0}^{3} \frac{(\frac{1}{2})^k}{1-\frac{1}{4}e^{-j(\omega-\frac{\pi}{2}k)}}.$$

It can be shown that

$$x[n] = g[n]q[n],$$

where $g[n]$ is of the form $\alpha^n u[n]$ and $q[n]$ is a periodic signal with period N.

(a) Determine the value of α.

(b) Determine the value of N.

(c) Is $x[n]$ real?

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Practice Sheet 8

Q1: (CO1, CO2, CO4, CO5) An LTI system with impulse response $h_1[n] = \left(\frac{1}{2}\right)^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response:

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}$$

Determine $h_2[n]$.

Q2: (CO1, CO2, CO4, CO5) Compute the Discrete-time Fourier transform of the given discrete-time signals:

$$x[n] = (n-1)\left(\frac{1}{3}\right)^{|n|}$$

Q3: (CO1, CO2, CO4, CO5) A discrete-time LTI system which has the property that if the input is

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} u[n-1],$$

then the output is

$$y[n] = \left(\frac{1}{3}\right)^n u[n].$$

Find a difference equation relating $x[n]$ and $y[n]$ that characterizes the system.

Q4: (CO1, CO2, CO4, CO5) Consider a system consisting of the cascade of two LTI systems with frequency responses

$$H_1(e^{j\omega}) = \frac{2 - e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}$$

and

$$H_2(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega} + \frac{1}{4}e^{-j2\omega}}.$$

Find the difference equation describing the overall system.

Q5: (CO1, CO2, CO4, CO5) Consider a causal LTI system described by the difference equation

$$y[n] + \frac{1}{2}y[n-1] = x[n] \tag{1}$$

1. Determine the frequency response $H(e^{j\omega})$ of this system
2. What is the response of the system to the following inputs?
   (a) $x[n] = (1/2)^n u[n]$
   (b) $x[n] = \delta[n] + 1/2\delta[n-1]$
3. Find the response to the inputs with the following Fourier transforms:
   (a) $X(e^{j\omega}) = \frac{1-1/4e^{-j\omega}}{1+1/2e^{-j\omega}}$
   (b) $X(e^{j\omega}) = 1 + 2e^{-3j\omega}$

Q6: (CO1, CO2, CO4) Shown in Figure 1 is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For each of the input signals $x(t)$ below, determine the filtered output signal $y(t)$.

(a) $x(t) = \cos(2\pi t + \theta)$

(b) $x(t) = \cos(4\pi t + \theta)$

(c) $x(t)$ is a half-wave rectified sine wave of period, as sketched in Figure 2.

$$x(t) = \begin{cases} \sin(2\pi t), & m \le t \le \left(m+\frac{1}{2}\right) \\ 0, & \left(m+\frac{1}{2}\right) \le t \le m \quad \text{for any integer } m \end{cases}$$

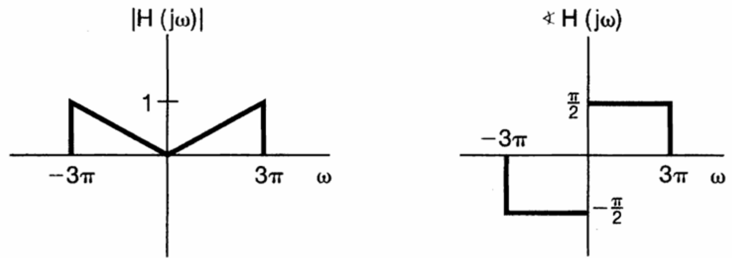

Figure 1:

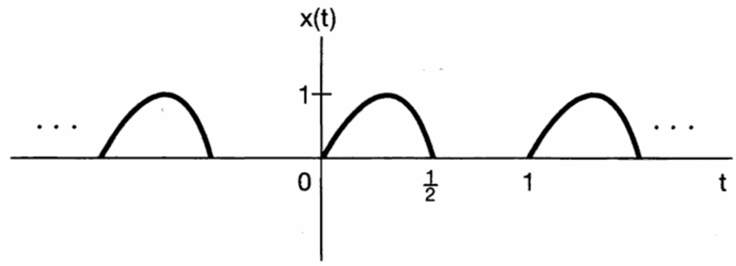

Figure 2:

Q7: (CO1, CO2, CO4, CO5) Consider an ideal discrete-time lowpass filter with impulse response $h[n]$ and for which the frequency response $H(e^{j\omega})$ is that shown in Figure 3. Let us consider obtaining a new filter with impulse response $h_1[n]$ and frequency response $H_1(e^{j\omega})$ as follows:

$$h_1[n] = \begin{cases} h[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

This corresponds to inserting a sequence value of zero between each sequence value of $h[n]$. Determine and sketch $H_1(e^{j\omega})$ and state the class of ideal filters to which it belongs (e.g., lowpass, highpass, bandpass, multiband, etc.).

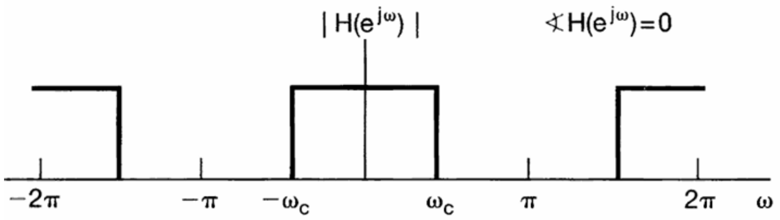

Figure 3:

Q8: (CO1, CO2, CO4, CO5) A discrete-time system is implemented as shown in Figure 4. The system S shown in the figure is an LTI system with impulse response $h_{lp}[n]$.

(a) Show that the overall system is time invariant.

(b) If $h_{lp}[n]$ is a lowpass filter, what type of filter does the system of the figure implement?

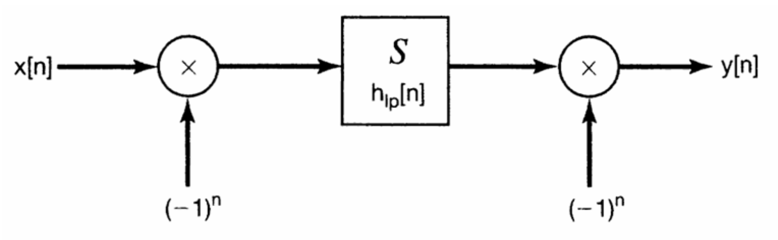

Figure 4: