Hello world! $2^2 = \sqrt{16}$

1.Translate the following system of 3 equations and 3 unknowns into an augmented matrix format and solve for the unknowns

$$\begin{cases} x+y+z &= 6 \\ 2x-y+z &= -3 \\ 3y-2z &= 0 \end{cases}$$

convert this linear algebra equation to a matrix: using Gaussian elimination

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 2 & -1 & 1 & | & -3 \\ 0 & 3 & -2 & | & 0 \end{bmatrix}$$

R2 = -2R1 + R2:

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 0 & -3 & -1 & | & -15 \\ 0 & 3 & -2 & | & 0 \end{bmatrix}$$

R3 = R3 + R2

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 0 & -3 & -1 & | & -15 \\ 0 & 0 & -3 & | & -15 \end{bmatrix}$$

Now, we have solve z parameter, which value is:

-3z = -15

z =5

Then, the linear algebra equation will be like:

$$\begin{cases} x+y+5 &= 6 \\ -3y-5 &= -15 \end{cases}$$

So, the y parameter value is:

-3y -5 = -15

-3y = -10

y = $\frac{10}{3}$

And then, we could use parameter y, z to solve x:

x+$\frac{10}{3}$ +5 = 6

x = -$\frac{7}{3}$

As a result, the solution to the system of equations is:

x = -$\frac{7}{3}$, y = $\frac{10}{3}$, z=5

2. Identify which, if any, of the following 2x2 matrices are singular

2.1:

$$\begin{pmatrix} 2 & 6 \\ 3 & -2 \end{pmatrix}$$

This determinant of this matrix is: **NOT** singular

$$\begin{aligned} \det(A) &= (2 \cdot (-2)) - (6 \cdot 3) \\ &= (-4) - (18) \\ &= -22 \\ -22 &\neq 0 \end{aligned}$$

2.2:

$$\begin{pmatrix} 1 & 11 \\ 3 & 40 \end{pmatrix}$$

This determinant of this matrix is: **NOT** singular

$$\begin{aligned} \det(A) &= (1 \cdot (40)) - (11 \cdot 3) \\ &= 40 - 33 \\ &= 7 \\ 7 &\neq 0 \end{aligned}$$

2.3:

$$\begin{pmatrix} 22 & 11 \\ 8 & 4 \end{pmatrix}$$

This determinant of this matrix is: **singular**

$$\begin{aligned} \det(A) &= (22 \cdot (4)) - (8 \cdot 11) \\ &= 88 - 88 \\ &= 0 \end{aligned}$$

2.4:

$$\begin{pmatrix} d^2 & d + c \\ d & c/d + 1 \end{pmatrix}$$

This determinant of this matrix is: **singular**

$$\begin{aligned} \det(A) &= (d^2 \cdot (c/d + 1)) - ((d + c) \cdot d) \\ &= cd + d^2 - d^2 - cd \\ &= 0 \end{aligned}$$

2.5:

$$\begin{pmatrix} 0.5 & 5 \\ 2 & 22 \end{pmatrix}$$

This determinant of this matrix is: **NOT** singular

$$\begin{aligned} \det(A) &= (0.5 \cdot 22) - (5 \cdot 2) \\ &= 11 - 10 \\ &= 1 \\ 1 &\neq 0 \end{aligned}$$

3. Find the determinant of each of the following 3x3 matrices.

3.1:

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 7 \\ 4 & 5 & 2 \end{bmatrix}$$

The determinant of this matrix is:

$$\begin{aligned} \det(A) &= (1 \cdot (5 \cdot 2 - 7 \cdot 5)) - (2 \cdot (-1 \cdot 2 - 7 \cdot 4)) + (3 \cdot (-1 \cdot 5 - 5 \cdot 4)) \\ &= (1 \cdot (-25)) - (2 \cdot (-30)) + (3 \cdot (-25)) \\ &= -25 + 60 - 75 \\ &= -40 \end{aligned}$$

3.2:

$$\begin{bmatrix} 6 & 2 & -1 \\ -4 & 1/5 & 0 \\ 6 & 10 & -5 \end{bmatrix}$$

The determinant of this matrix is:

$$\begin{aligned} \det(A) &= (6 \cdot (1/5 \cdot (-5) - 0 \cdot 10)) - (2 \cdot (-4 \cdot (-5) - 0 \cdot 6)) + (-1 \cdot (-4 \cdot 10 - 1/5 \cdot 6)) \\ &= (6 \cdot (-1)) - (2 \cdot 20) + (-1 \cdot (-40)) \\ &= -6 - 40 + 40 \\ &= -6 \end{aligned}$$

3.3:

$$\begin{bmatrix} 3 & -1 & 2 \\ 5 & 1 & 0 \\ -2 & 3 & 4 \end{bmatrix}$$

The determinant of this matrix is:

$$\begin{aligned}\det(A) &= (3\cdot(1\cdot 4-0\cdot 3)) - (-1\cdot(5\cdot 4-0\cdot(-2))) + (2\cdot(5\cdot 3-1\cdot(-2)))\\ &= (3\cdot 4)-(-1\cdot 22)+(2\cdot 17)\\ &= 12+22+34\\ &= 68\end{aligned}$$

4. Find the inverse of the following 3x3 matrices. You can use any method you like, but it would be good practice to try 2 different methods.

4.1:

$$\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}$$

The process of finding the inverse of this matrix is to find the determinant of this matrix first:

$$\begin{aligned}\det(A) &= (2\cdot(1\cdot 3-0\cdot 1)) - (0\cdot(5\cdot 3-0\cdot 0)) + (-1\cdot(5\cdot 1-1\cdot 0))\\ &= (2\cdot 3)-(0\cdot 15)+(-1\cdot 5)\\ &= 6-5\\ &= 1\end{aligned}$$

Transpose of this matrix is:

$$\begin{bmatrix}2 & 5 & 0\\ 0 & 1 & 1\\ -1 & 0 & 3\end{bmatrix}$$

The Matrix R1C1 is:

$$\begin{pmatrix}1 & 1\\ 0 & 3\end{pmatrix}$$

And the value of this matrix is: 3

The Matrix R1C2 is:

$$\begin{pmatrix}0 & 1\\ -1 & 3\end{pmatrix}$$

And the value of this matrix is: 1

The Matrix R1C3 is:

$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

And the value of this matrix is: -1

The Matrix R2C1 is:

$$\begin{pmatrix}5 & 0\\ 0 & 3\end{pmatrix}$$

And the value of this matrix is: 15

The Matrix R2C2 is:

$$\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$$

And the value of this matrix is: 6

The Matrix R2C3 is:

$$\begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix}$$

And the value of this matrix is: -5

The Matrix R3C1 is:

$$\begin{pmatrix} 5 & 0 \\ 1 & 1 \end{pmatrix}$$

And the value of this matrix is: 5

The Matrix R3C2 is:

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

And the value of this matrix is: 2

The Matrix R3C3 is:

$$\begin{pmatrix} 2 & 5 \\ 0 & 1 \end{pmatrix}$$

And the value of this matrix is: 2

Collect all of these value together, we could get the temporary matrix:

$$\begin{bmatrix} 3 & 1 & -1 \\ 15 & 6 & -5 \\ 5 & 2 & 2 \end{bmatrix}$$

adjoint matrix: change the temporary matrix each value's symbol:

$$\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & 5 \\ 5 & -2 & 2 \end{bmatrix}$$

Using adjoint matrix divide the determinant of this matrix:
And the inverse of this matrix is:

$$\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & 5 \\ 5 & -2 & 2 \end{bmatrix}$$

Using Gaussian elimination to find the inverse of this matrix:

$$\begin{bmatrix} 2 & 0 & -1 & | & 1 & 0 & 0 \\ 5 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & 1 & 3 & | & 0 & 0 & 1 \end{bmatrix}$$

R1 = R1/2

$$\begin{bmatrix} 1 & 0 & -1/2 & | & 1/2 & 0 & 0 \\ 5 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & 1 & 3 & | & 0 & 0 & 1 \end{bmatrix}$$

R1 = 5R1

$$\begin{bmatrix} 5 & 0 & -5/2 & | & 5/2 & 0 & 0 \\ 5 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & 1 & 3 & | & 0 & 0 & 1 \end{bmatrix}$$

R2 = R2 - R1

$$\begin{bmatrix} 5 & 0 & -5/2 & | & 5/2 & 0 & 0 \\ 0 & 1 & 5/2 & | & -5/2 & 1 & 0 \\ 0 & 1 & 3 & | & 0 & 0 & 1 \end{bmatrix}$$

R1 = R1/5

$$\begin{bmatrix} 1 & 0 & -1/2 & | & 1/2 & 0 & 0 \\ 0 & 1 & 5/2 & | & -5/2 & 1 & 0 \\ 0 & 1 & 3 & | & 0 & 0 & 1 \end{bmatrix}$$

R3 = R3 - R2

$$\begin{bmatrix} 1 & 0 & -1/2 & | & 1/2 & 0 & 0 \\ 0 & 1 & 5/2 & | & -5/2 & 1 & 0 \\ 0 & 0 & 1/2 & | & 5/2 & -1 & 1 \end{bmatrix}$$

R3 = 2R3

$$\begin{bmatrix} 1 & 0 & -1/2 & | & 1/2 & 0 & 0 \\ 0 & 1 & 5/2 & | & -5/2 & 1 & 0 \\ 0 & 0 & 1 & | & 5 & -2 & 2 \end{bmatrix}$$

R3 = R3 * -1/2

$$\begin{bmatrix} 1 & 0 & -1/2 & | & 1/2 & 0 & 0 \\ 0 & 1 & 5/2 & | & -5/2 & 1 & 0 \\ 0 & 0 & -1/2 & | & -5/2 & 1 & -1 \end{bmatrix}$$

R1 = R1 - R3

$$\begin{bmatrix} 1 & 0 & 0 & | & 3 & -1 & 1 \\ 0 & 1 & 5/2 & | & -5/2 & 1 & 0 \\ 0 & 0 & -1/2 & | & -5/2 & 1 & -1 \end{bmatrix}$$

R3 = R3 * -5

$$\begin{bmatrix} 1 & 0 & 0 & | & 3 & -1 & 1 \\ 0 & 1 & 5/2 & | & -5/2 & 1 & 0 \\ 0 & 0 & 5/2 & | & 25/2 & -5 & 5 \end{bmatrix}$$

R2 = R2 - R3

$$\begin{bmatrix} 1 & 0 & 0 & | & 3 & -1 & 1 \\ 0 & 1 & 0 & | & -15 & 6 & -5 \\ 0 & 0 & 5/2 & | & 25/2 & -5 & 5 \end{bmatrix}$$

R3 = R3 * 2/5

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 1 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right]$$

As a result, the inverse of this matrix is:

$$\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

4.2:

$$\begin{bmatrix} 3 & -1 & 2 \\ 5 & 1 & 0 \\ -2 & 3 & 4 \end{bmatrix}$$

The inverse of this matrix is:

$$\begin{bmatrix} 2/33 & 5/33 & -1/33 \\ -10/33 & 8/33 & 5/33 \\ 17/66 & -7/66 & 4/33 \end{bmatrix}$$

5. Write down the transpose of each of the following matrices.

5.1:

$$\begin{bmatrix} 5 & -55 & 6 & 5 \\ 12 & 1/2 & -7 & 22 \end{bmatrix}$$

The transpose of this matrix is:

$$\begin{bmatrix} 5 & 12 \\ -55 & 1/2 \\ 6 & -7 \\ 5 & 22 \end{bmatrix}$$

5.2:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 9 \\ 12 & 20 & -1 & 11 \\ -2 & -15 & 0 & 13 \end{bmatrix}$$

The transpose of this matrix is:

$$\begin{bmatrix} 1 & 0 & 12 & -2 \\ 2 & 2 & 20 & -15 \\ 3 & 4 & -1 & 0 \\ 4 & 9 & 11 & 13 \end{bmatrix}$$