# Random Number Characterization via Quantum Inspired Machine Learning


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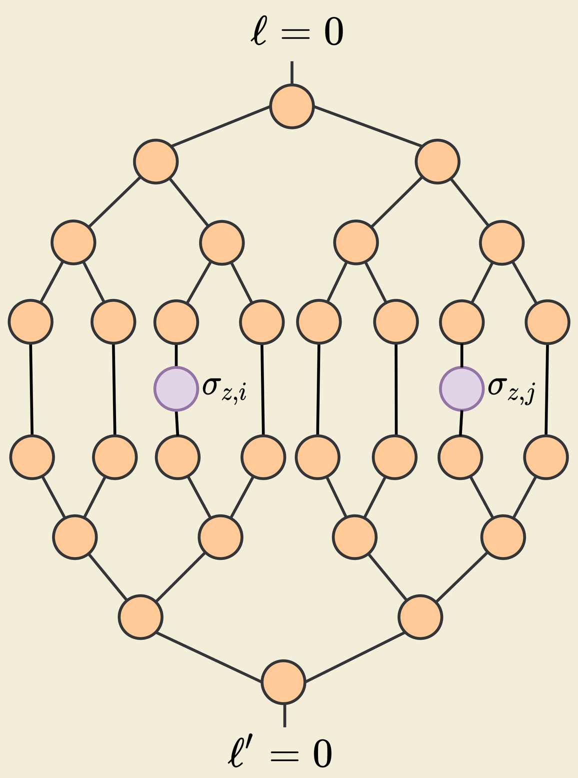
## 1. Introduction

We introduce an innovative approach that exploits **Tensor Networks** (TNs) to solve the problem of discriminating pseudo- from quantum random number sequences.

## 4. Analytical tools

**Site-site correlations** give an index of pseudo-randomness: in **truly** random sequences $C_{ij}^{\ell\ell'} \approx 0\,\forall\, i,j$.

$$C_{ij}^{\ell\ell'} = \frac{\langle \Psi_{\mathrm{TTN}}^{\ell} | \sigma_z^i \sigma_z^j | \Psi_{\mathrm{TTN}}^{\ell'} \rangle}{\sqrt{\langle \Psi_{\mathrm{TTN}}^{\ell} | \Psi_{\mathrm{TTN}}^{\ell} \rangle \langle \Psi_{\mathrm{TTN}}^{\ell'} | \Psi_{\mathrm{TTN}}^{\ell'} \rangle}}$$




The **entanglement entropy** serves as index of the information distribution among nodes,

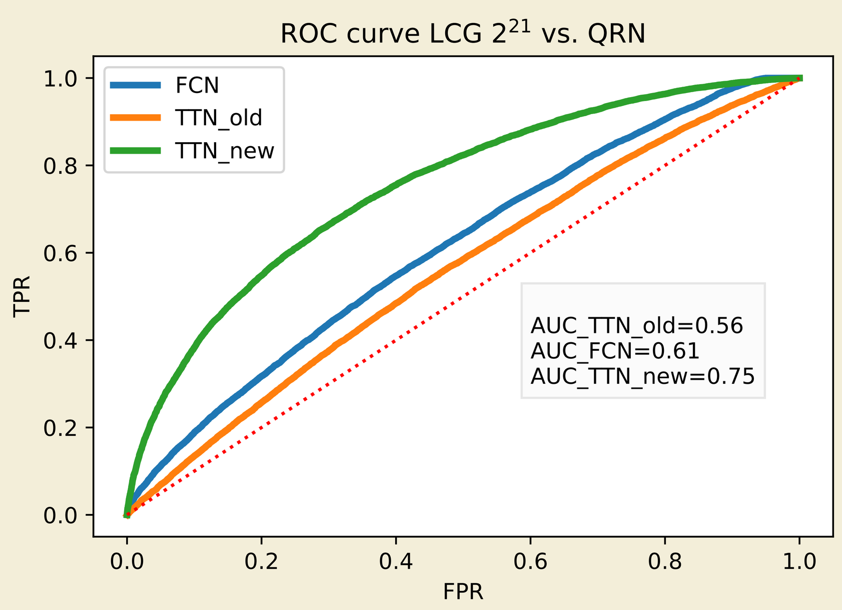
$$S(X) = -\operatorname{Tr}\left(\rho_X \log \rho_X\right),\ \rho_X = \operatorname{Tr}_{\bar{X}} |\Psi_{\mathrm{TN}}\rangle \langle \Psi_{\mathrm{TN}}| \,.$$

We propose a **new spin encoding**, invariant under modulo operations:

$$\phi^{s_j}(x_j) = \left[\cos\left(2\pi x_j\right), \sin\left(2\pi x_j\right)\right].$$

## 6. Fully connected network

We compare TTN performances with a fully connected network composed of 3 layers, 32 input nodes and binary output, performing a random grid search over **500 models**.




The dataset is composed of sequences of 32 PRNs via linear congruential generator with $p = 2^{21}$. Interestingly, the TTN with **our map** proposal achieves the **best** performance for long-range correlations detection: this is highly significant, since the TN sees the sequence only once.

**Pre-processing** the data by spectral analysis via **discrete cosine transform** equates TTN and FCN scores.

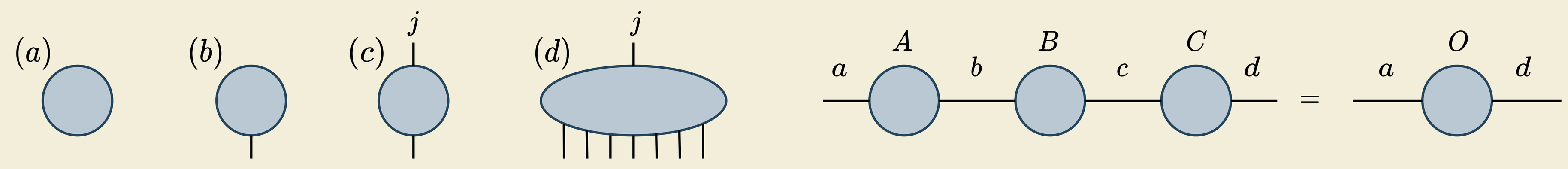
## 7. Conclusion and outlook

- TTNs show **promising** results in distinguishing QR sequences from periodic PR ones up to periods well beyond the input size;
- TTNs **fail** to discriminate QRNs from PRNs generated from e.g. the Marsenne-Twister algorithm.

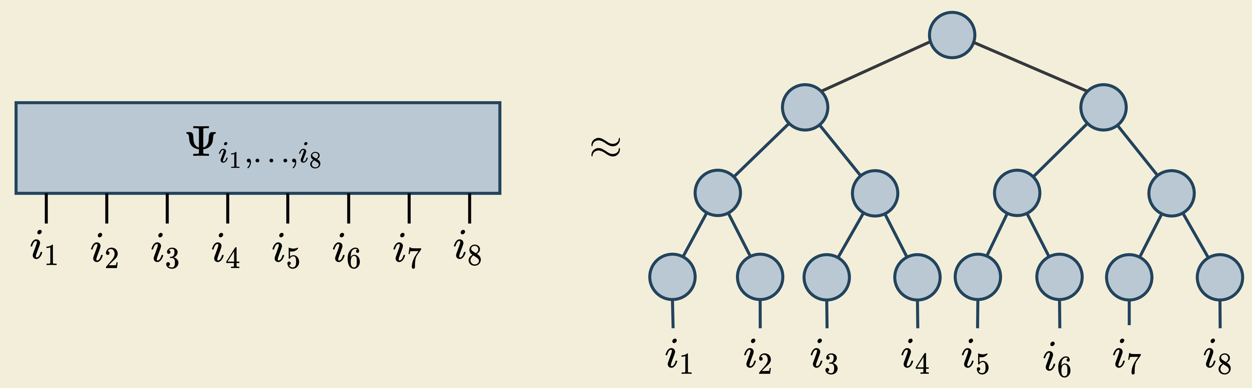
For pseudo-random sequences, increasing the bond dimension saturates the accuracy: there is a **limit** to the amount of information the network can extract, independently of its representative power.
Such methods can be thus used to certify cases of **impossible information compression**.

## 2. Tensor Network states

An $N$-rank tensor is a mathematical object with $N$ indices, $\mathcal{T}_{\alpha_1 \dots \alpha_N}$.




A tensor network state is a tailored quantum many-body wave function *ansatz*, $|\psi_{\mathrm{QMB}}\rangle = \sum_{i_1,\dots,i_N} \mathcal{T}_{i_1,\dots,i_N} |i_1,\dots,i_N\rangle$. The TN prescription expresses the amplitudes tensor as the contraction of a set of smaller tensors $\mathcal{T}^{[q]}$ over auxiliary indices $q \in \{1, Q\}$.




We make use of **binary tree tensor networks** (TTNs), hierarchical TNs architectures suitable in physics applications for both open and closed boundary conditions.

## 3. Tensor Network Machine Learning

TTNs turn out to also be a very natural way to parameterize ML models. We consider

$$f^{\ell}(\mathbf{x}) = W^{\ell} \cdot \phi(\mathbf{x}) = \langle W^{\ell} | \phi(\mathbf{x}) \rangle \qquad f^{\ell} : [0,1]^{\times N} \to [0,1]^{\times 2}\ ;$$

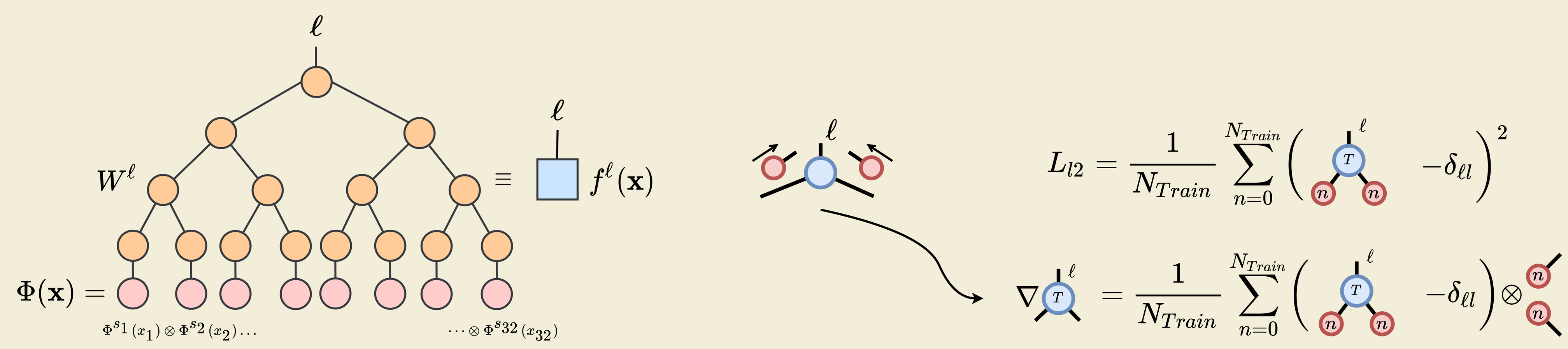
$W$ is the **optimized weight tensor** and $\phi(\cdot)$ is a multi-dimensional **feature map**. The most physically relevant feature map is the **spin-map**:

$$\phi^{s_j}(x_j) = \left[\cos\left(\frac{\pi}{2}x_j\right), \sin\left(\frac{\pi}{2}x_j\right)\right].$$

The elements of $f^{\ell}(\mathbf{x})$ are the **amplitude probabilities** for $\phi(\mathbf{x})$ to belong to each class,

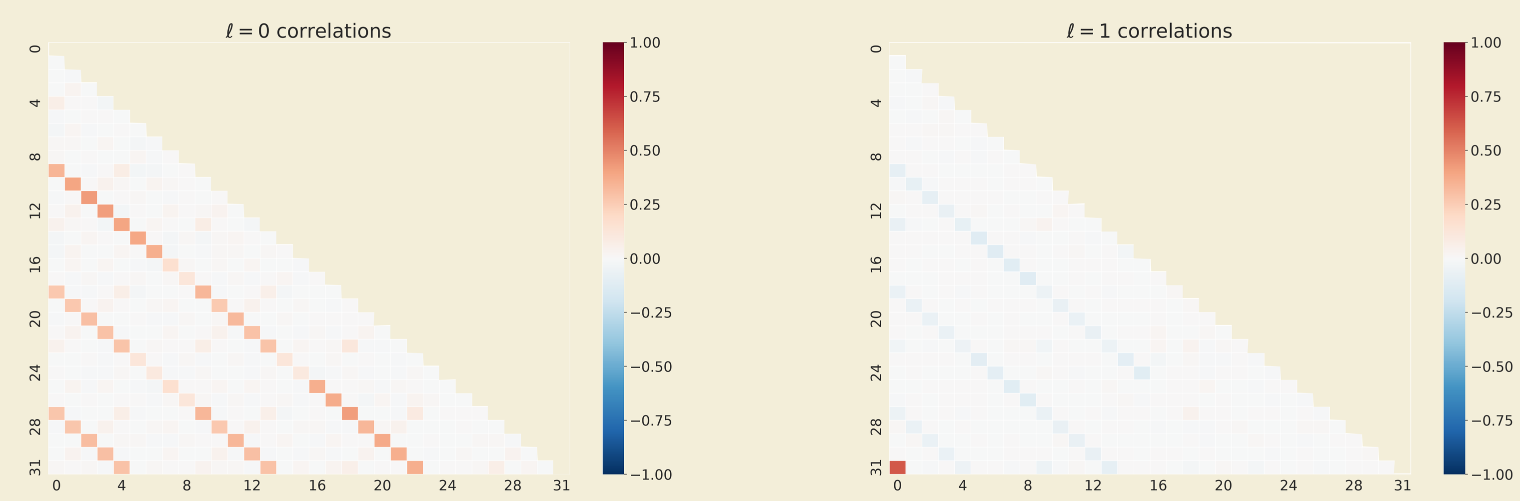
$$\left|f^{\ell}(\mathbf{x})\right|^2 \equiv \mathcal{P}(\ell, \mathbf{x}) \qquad \|f^{\ell}(\mathbf{x})\|_2 = 1\,.$$

For the optimization, we exploit **gauge transformations** via QR decomposition: using the MSE loss $L_{l2}$ the value of the tensors are updated following the gradient.




## 5. Results examples

We study the model against increasingly complex generated pseudo-random sequences by progressively increasing their **period** $p$, using also the new spin map proposal.




The images are for $p = 9$ vs. $p = 31$. The network classifies $\ell = 1$ (0) by exploiting positive (negative) correlations as shown in the red (blu) squares. By swapping the labels the results are mirrored: the network is trained **symmetrically**.

## References and acknowledgements


[1] Edwin M. Stoudenmire and David J. Schwab: *Supervised learning with tensor networks*, Advances in Neural Information Processing Systems, 29 (2016)

[2] Simone Montangero: *Introduction to tensor network methods numerical simulations of low-dimensional many-body quantum systems* (2018)



The TTN code has been developed by Marco Trenti & Timo Felser from "Tensor Solutions: focus is transparent AI", a spin-off of Ulm university.