Computational Quantum Physics

Week 6

Due on Week 8

Exercise 1: **Density Matrices**

Consider a quantum system formed by N subsystems (spins, atoms, particles etc..) each described by its wave function $\psi_i \in \mathcal{H}^D$ where $\mathcal{H}^D$ is a D-dimensional Hilbert space.

(a) How do you write the total wave function of the system $\Psi(\psi_1, \dots \psi_N)$? Write a Fortran code to
1) describe such a system (N-body non interacting, separable pure state) and
2) a general N-body pure wave function $\Psi \in \mathcal{H}^{D^N}$.
Comment on their efficiency.

(b) Given N=2, write the density matrix of a pure state Ψ, $\rho = |\Psi\rangle\langle\Psi|$.

(c) Given a generic density matrix in $\mathcal{H}^{D^2}$ compute the reduce density matrix of either the left or the right system, e.g. $\rho_1 = \mathrm{Tr}_2\, \rho$.

(d) Test the functions described before (and all others needed) on two-spin one-half (qubits) with different states.