PHY 101

MECHANICS AND PROPERTIES OF MATTER I

Elements of Fluid Mechanics

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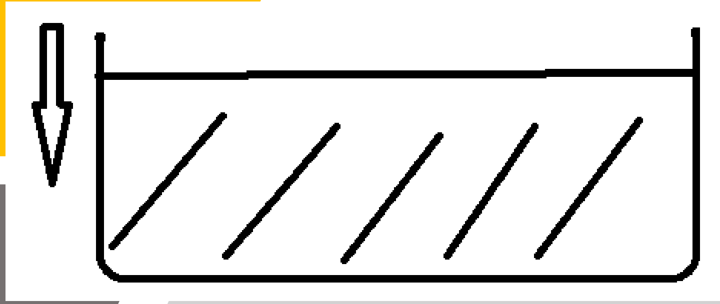
Introduction

- In a fluid, the pressure varies with depth.
- All points at the same height in a fluid have the same pressure.

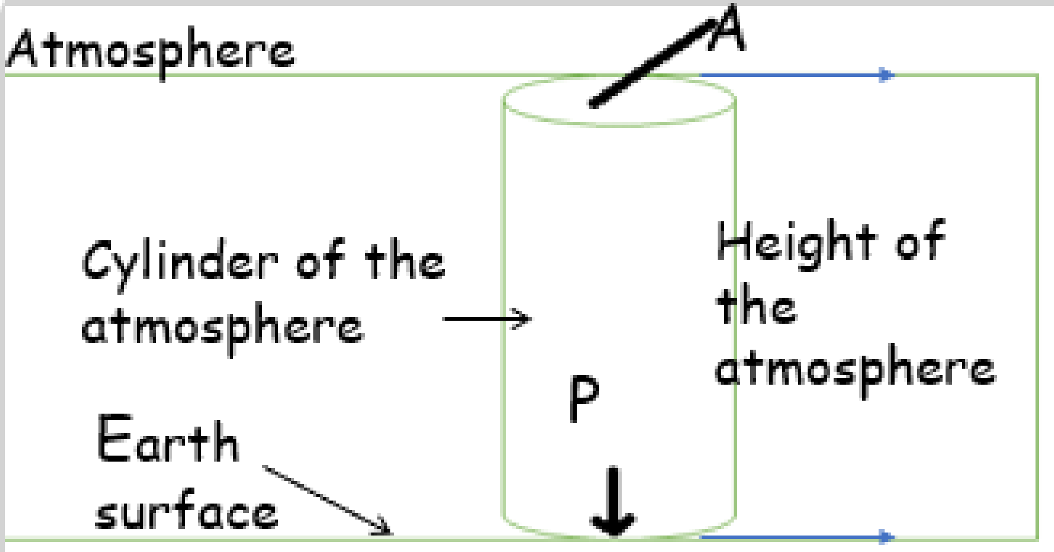


In this fluid, the pressure increases as the depth increases but all points at the same depth/height will have the same pressure.

Let's first consider atmospheric pressure:

Pressure = $\frac{Force}{Area}$

Considering the surface of the earth and as the atmosphere goes up at a certain point.



Consider a cylinder of the atmosphere where the cross-section area is A and the height h.

The pressure which is as result of all the air baring down on the surface of the earth is

$$p = \frac{F}{A}$$

Force is the weight of air; $W = mg$

Pressure, $\mathrm{P} = \frac{F}{A} = \frac{mg}{A}$

Density; $\rho = \frac{m}{Vol}$; mass, $\mathrm{m} = \rho \times vol$

$$Volume = Area \times height = A \times h = Ah$$

$$\mathrm{P} = \frac{F}{A} = \frac{mg}{A} = \frac{\rho volg}{A} = \frac{\rho Ahg}{A} = \rho hg$$

P is the pressure of this cylinder of air as it lands on the earth.

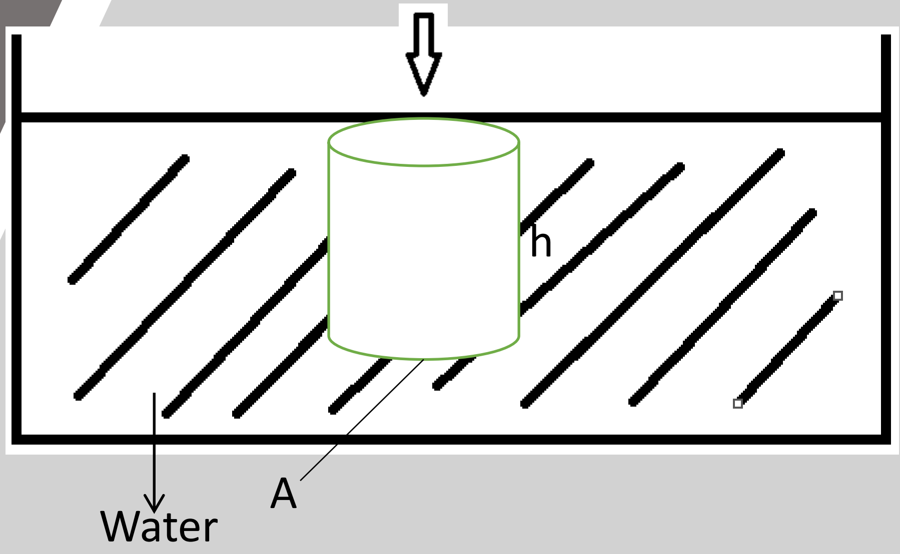
The density of the air vary as one goes up into the atmosphere, where it becomes less dense.

Secondly, g itself varies over the height of the atmosphere.

Atmospheric Pressure on the surface of the earth is 1.013×10^5Pas (N/m^2) which results from air having a height h.

The same pressure for mercury instead of air would result in $Hg_{,mecury} = 76\ cm$.

Pressure Under Water



On the surface of the water, atmospheric pressure P_A is applied.
What about a point under the water?
The pressure of the water at any point.

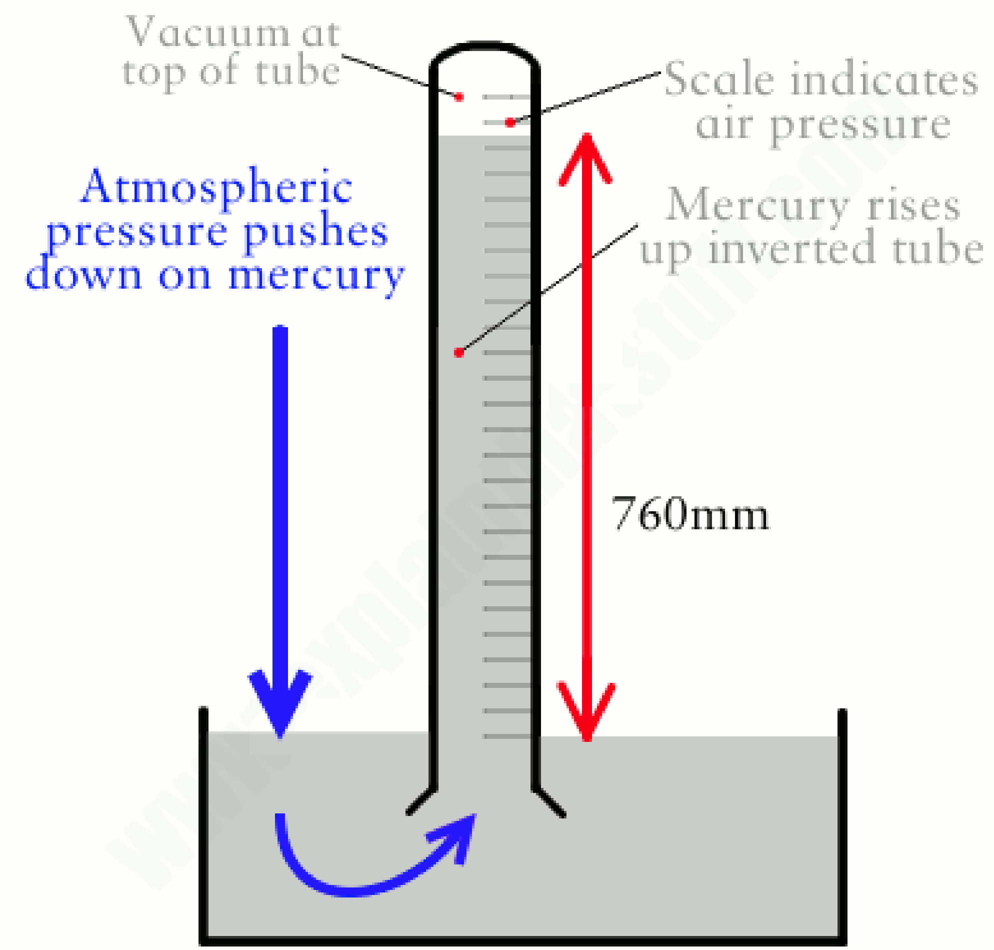
$$P_w = \frac{F}{A} = \frac{mg}{A} = \frac{\rho_w Volg}{A} = \frac{\rho_w Ahg}{A} = \rho_w hg$$

The total pressure at point A under the water = atmospheric pressure + pressure of the water.

$$P_r = P_A + P_w = P_A + \rho_w hg$$

This shows that pressure varies according to the depth/height of an object under the water.

Pressure In A Fluid

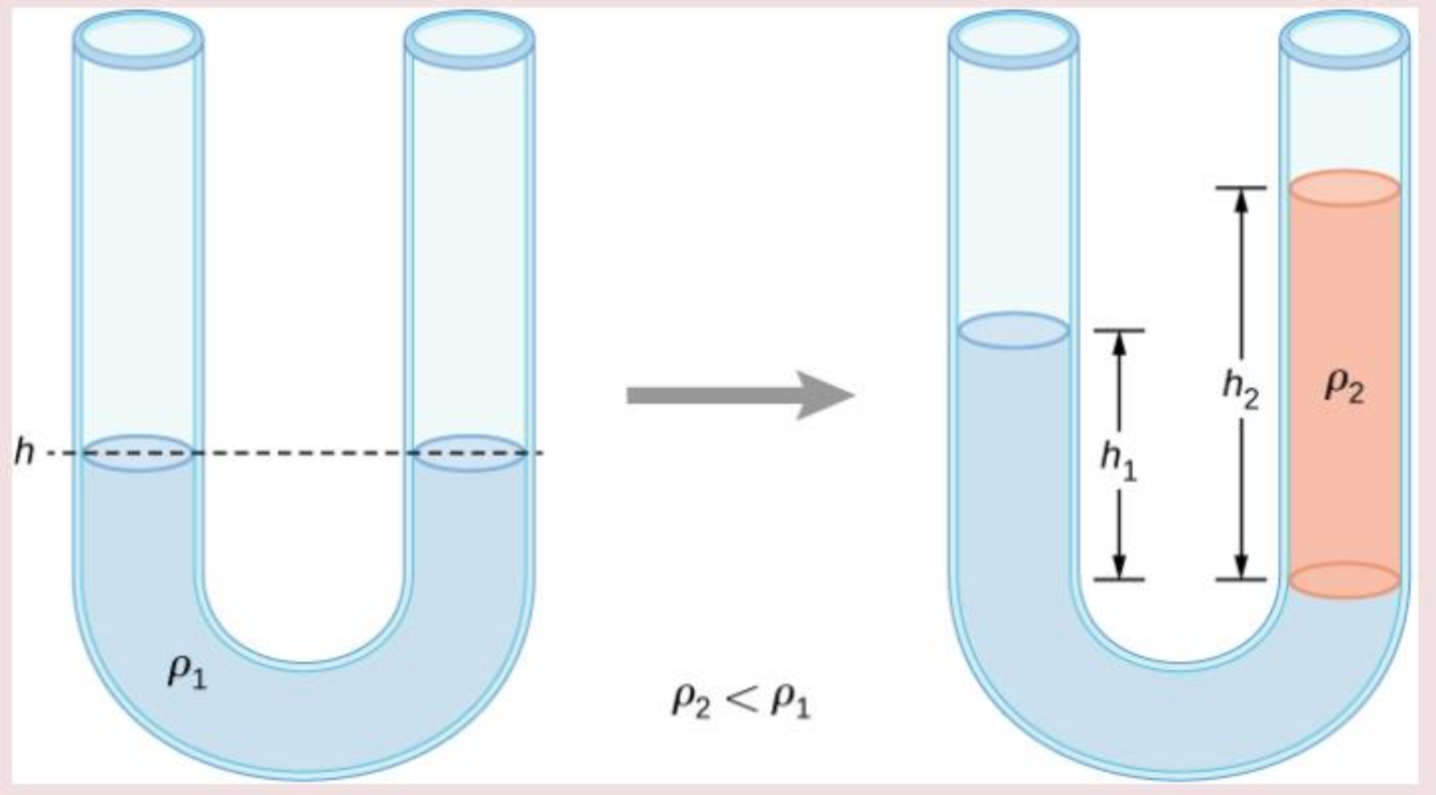


Atmospheric air pressure is measured using a barometer.

The simplest kind of barometer is a tall closed tube standing upside down in a bath of mercury (a dense liquid metal at room temperature) so the liquid rises partly up the tube a bit.

At sea level, the atmosphere will push down on a pool of mercury and make it rise up in a tube to a height of approximately $760\ mm$ (roughly 30in). This is called air pressure one atmosphere (1 atm).

Density of an Unknown Fluid



A U-tube with both ends open is filled with a liquid (e.g. water) of density ρ_1 to a height h on both sides (See Figure).

A liquid of density $\rho_2 < \rho_1$ (e.g. oil) is poured into one side and liquid 2 settles on top of liquid 1. The heights on the two sides are different. The height to the top of liquid 2 from the interface is h_2 and the height to the top of liquid 1 from the level of the interface is h_1.

Derive a formula for the height difference.

The pressure at points at the same height on the two sides of a U-tube must be the same as long as the two points are in the same liquid.

Therefore, we consider two points at the same level in the two arms of the tube:

One point is the interface on the side of the Liquid 2 and the other is a point in the arm with Liquid 1 that is at the same level as the interface in the other arm.

The pressure at each point is due to atmospheric pressure plus the weight of the liquid above it.

$$Pressure\ on\ the\ side\ of\ liquid\ 1 = P_A + \rho_1 g h_1$$
$$Pressure\ on\ the\ side\ of\ liquid\ 2 = P_A + \rho_2 g h_2$$

- Since the two points are in Liquid 1 and are at the same height, the pressure at the two points must be the same.
- Therefore, we have

$$P_A + \rho_1 g h_1 = P_A + \rho_2 g h_2$$

Hence,

$$\rho_1 g h_1 = \rho_2 g h_2$$

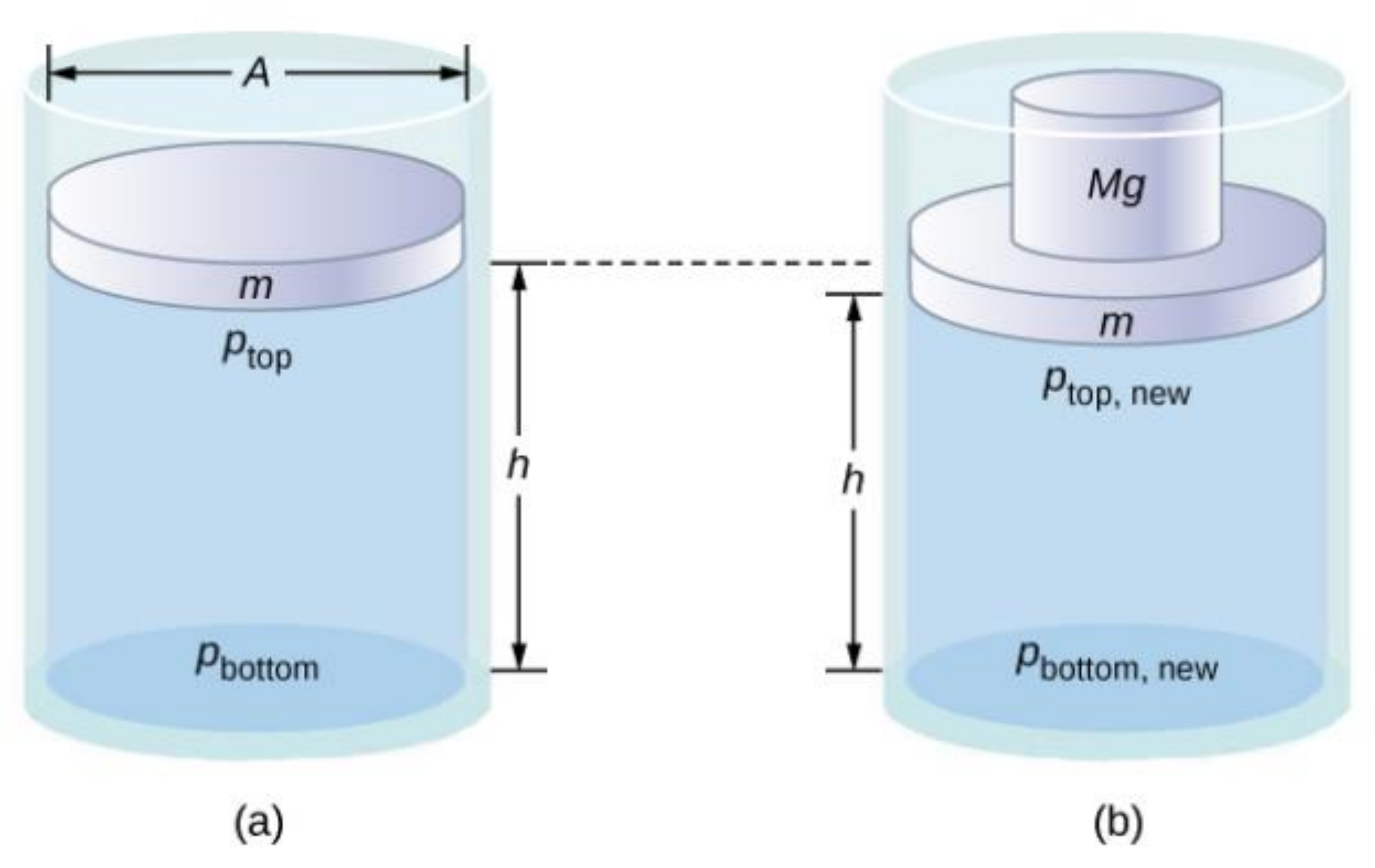
The difference in heights on the two sides of the U-tube is

$$h_2 - h_1 = \left(1 - \frac{\rho_2}{\rho_1}\right) h_2$$

If $\rho_2 = \rho_1$, then $h_2 = h_1$.
This implies that if the two sides have the same density, they have the same height.

Pascal's Principle and Hydraulics

- A static fluid is a fluid that is not in motion.
- When a fluid is not flowing, the fluid is said to be in static equilibrium. If the fluid is water, it is in **hydrostatic equilibrium**.
- For a fluid in static equilibrium, the net force on any part of the fluid must be zero; otherwise the fluid will start to flow.
- **Pascal's principle** (also known as Pascal's law) states that when a **change in pressure** is applied to an enclosed fluid, it is transmitted undiminished to all portions of the fluid and to the walls of its container.



- This principle applies to the *change* in pressure.
- Suppose some water is place in a cylindrical container of height h and cross-sectional area A that has a movable piston of mass m (See Figure).
- Adding weight Mg at the top of the piston increases the pressure at the top by Mg/A, since the additional weight also acts over area A of the lid:

$$\Delta p_{top} = \frac{Mg}{A}$$

- According to Pascal's principle, the change in pressure at all points in the water changes by the same amount, Mg/A.

- Thus, the pressure at the bottom also increases by Mg/A.

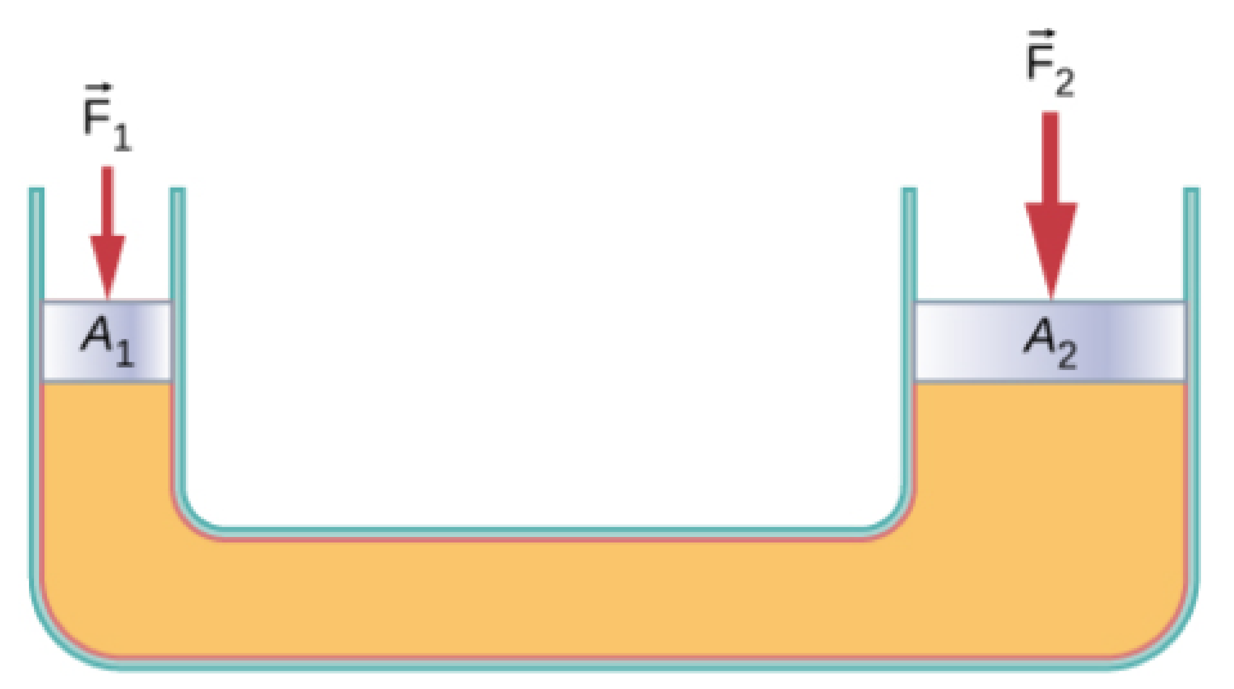
- The pressure at the bottom of the container is equal to the sum of the atmospheric pressure, the pressure due the fluid, and the pressure supplied by the mass.
- The change in pressure at the bottom of the container due to the mass is

$$\Delta p_{bottom} = \frac{Mg}{A}$$

Hence,

$$\Delta p = \Delta p_{top} = p_{bottom} = p_{everywhere}$$

Hydraulics Systems



- Hydraulic systems operate by applying forces (F_1, F_2) to an incompressible fluid in a U-tube, using a movable piston (A_1, A_2) on each side of the tube

- Pascal's principle is used to derive a relationship between the forces in this simple hydraulic system.

- Note:
- First, that the two pistons in the system are at the same height, so there is no difference in pressure due to a difference in depth.
- The pressure due to F_1 acting on area A_1 is simply

$$p_1 = \frac{F_1}{A_1}$$

According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container.

Thus, a pressure p_2 is felt at the other piston that is equal to p_1 i.e., $p_1 = p_2$.

However, since $p_2 = F_2/A_2$, then,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

- This equation relates the ratios of force to area in any hydraulic system, provided that the pistons are at the same vertical height and that friction in the system is negligible.

- Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area.

- For example, if a $100\ N$ force is applied to the left cylinder in the Figure and the right cylinder has an area five times greater, then the output force is $500\ N$.

- Hydraulic systems are used to operate automotive brakes, hydraulic jacks, and numerous other mechanical systems.

- A **hydraulic jack** is used to lift heavy loads, such as the ones used by auto mechanics to raise an automobile. It consists of an incompressible fluid in a U-tube fitted with a movable piston on each side.

- One side of the U-tube is narrower than the other. A small force applied over a small area can balance a much larger force on the other side over a larger area (see Fig.)

- From Pascal's principle, it can be shown that the force needed to lift the car is less than the weight of the car:

$$F_1 = \frac{A_1}{A_2} F_2$$

- where F_1 is the force applied to lift the car, A_1 is the cross-sectional area of the smaller piston, A_2 is the cross sectional area of the larger piston, and F_2 is the weight of the car.

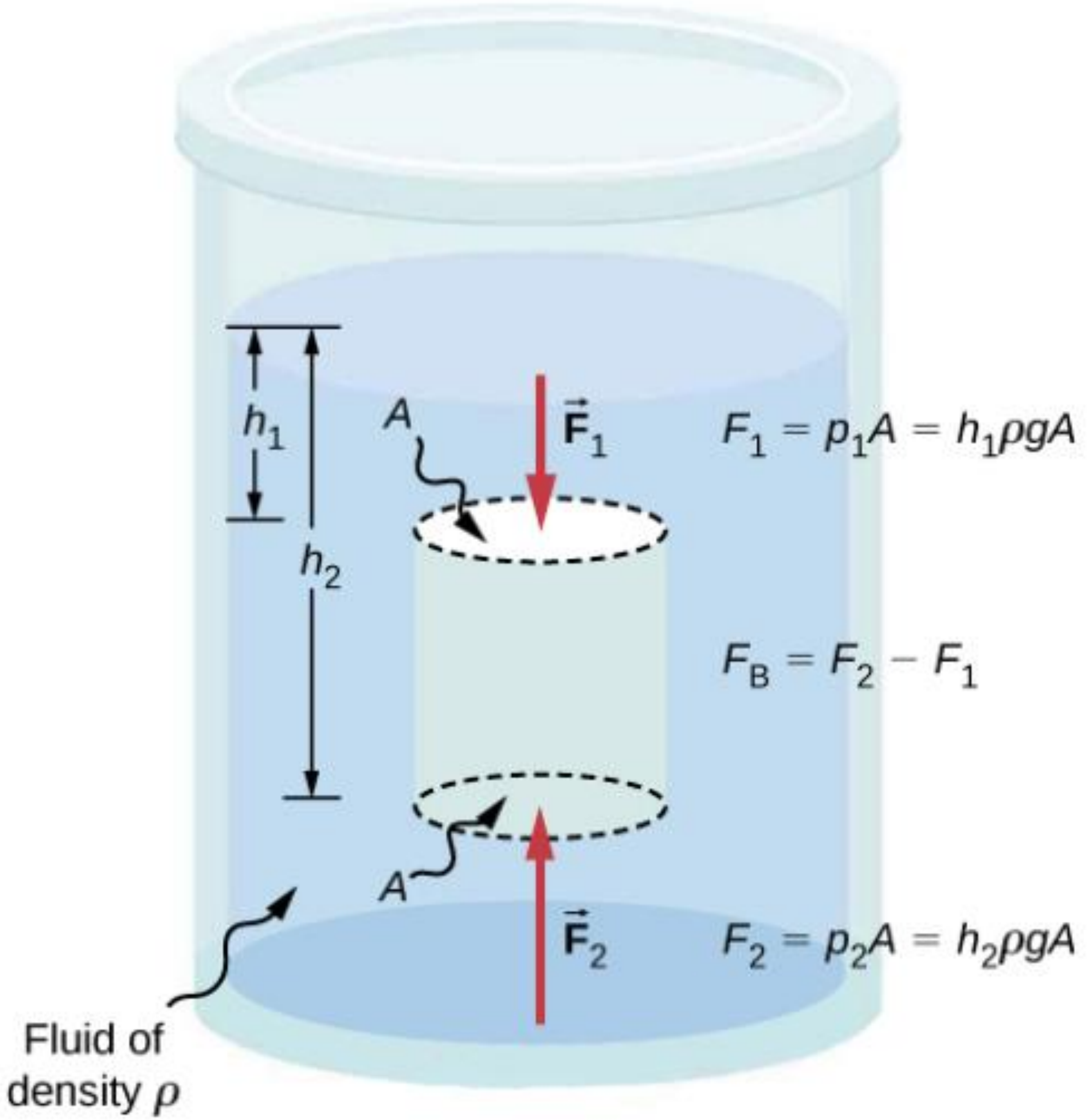
Question

What force must be exerted on the master cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on a second cylinder? The master cylinder has a 2.00-cm diameter and the second cylinder has a 24.0-cm diameter.

Buoyancy

- When placed in a fluid, some objects float due to a buoyant force.
- Where does this buoyant force come from?
- Why is it that some things float and others do not?
- Do objects that sink get any support at all from the fluid?
- Is your body buoyed by the atmosphere, or are only helium balloons affected?
- (See Figures)

- The fact is that pressure increases with depth in a fluid.
- This means that the **upward force** on the bottom of an object in a fluid is greater than the **downward force** on top of the object.
- There is an upward force, or **buoyant force**, on any object in any fluid (Figure).
- If the buoyant force is greater than the object's weight, the object rises to the surface and floats.
- If the buoyant force is less than the object's weight, the object sinks.
- If the buoyant force equals the object's weight, the object can remain suspended at its present depth.
- The buoyant force is always present, whether the object floats, sinks, or is suspended in a fluid.



- **Therefore, buoyant force is the upward force on any object in any fluid.**

- Pressure due to the weight of a fluid increases with depth because $p = h\rho g$

- This change in pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder.

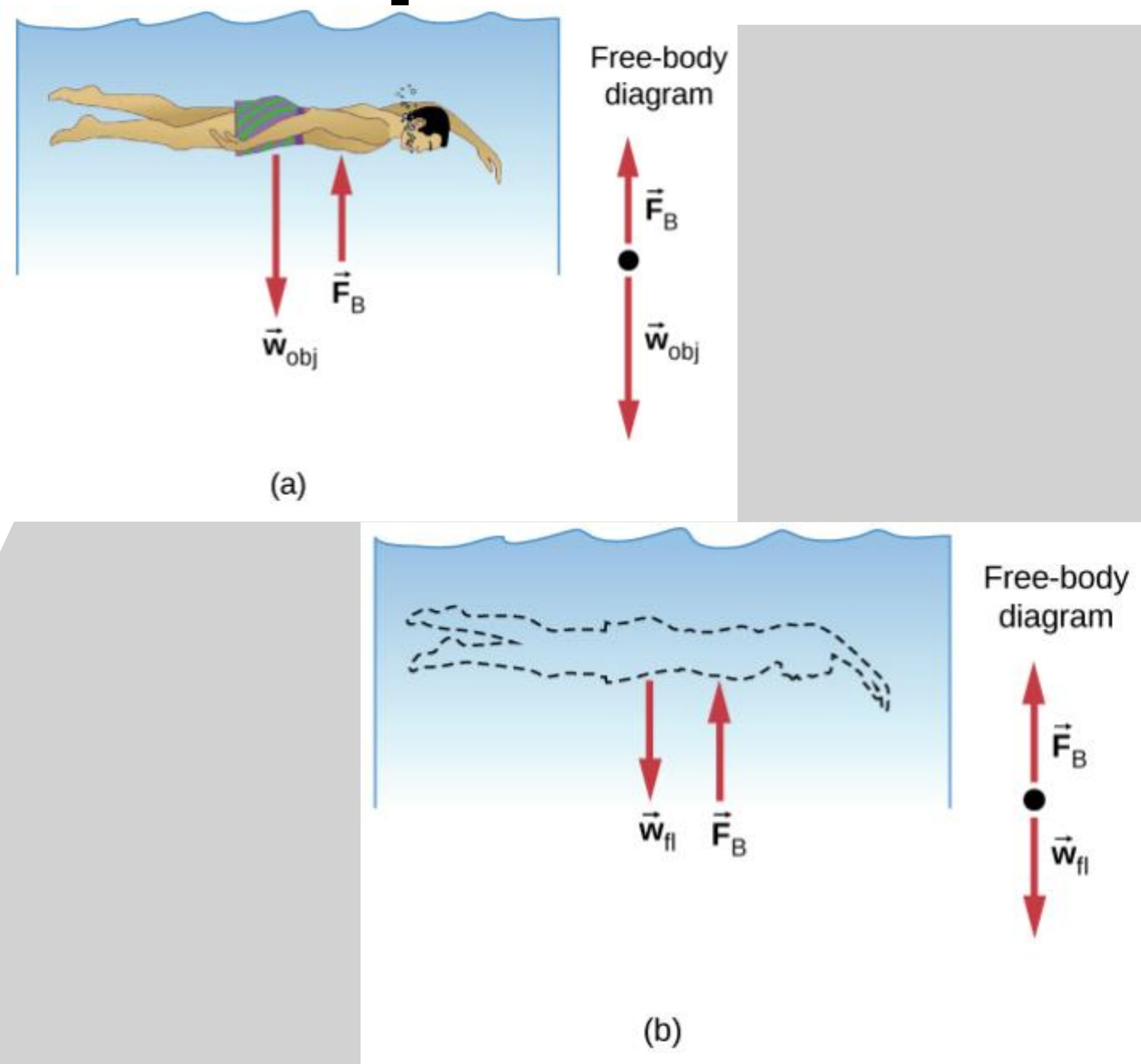
- The differences in the force results in the buoyant force F_B.

Archimedes' Principle

- **Archimedes' principle** states that the buoyant force on an object equals the weight of the fluid it displaces.
- In equation form,

$$F_B = w_f$$

where F_B is the buoyant force and w_f is the weight of the fluid displaced by the object.



- Archimedes' principle refers to the force of buoyancy that results when a body is submerged in a fluid, whether partially or wholly.
- The force that provides the pressure of a fluid acts on a body perpendicular to the surface of the body.
- In other words, the force due to the pressure at the bottom is pointed up, while at the top, the force due to the pressure is pointed down; the forces due to the pressures at the sides are pointing into the body.
- Since the bottom of the body is at a greater depth than the top of the body, the pressure at the lower part of the body is higher than the pressure at the upper part (See Figure).
- Therefore a net upward force acts on the body which is the force of buoyancy, or simply buoyancy.

Density and Archimedes' Principle

(a)

(b)

- The extent to which a floating object is submerged depends on how the object's density compares to the density of the fluid.
- In the Figure, for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship when loaded.
- We can derive a quantitative expression for the fraction submerged by considering density.

- The fraction submerged is the ratio of the volume submerged to the volume of the object, or

$$fraction\ submerged = \frac{V_{sub}}{V_{obj}} = \frac{V_{fl}}{V_{obj}}$$

- The volume submerged equals the volume of fluid displaced.
- Now we can obtain the relationship between the densities:

$$\frac{V_{fl}}{V_{obj}} = \frac{m_{fl}/\rho_{fl}}{m_{obj}/\rho_{obj}}$$

where ρ_{obj} is the average density of the object and ρ_{fl} is the density of the fluid.

Since the object floats, its mass and that of the displaced fluid are equal, so they cancel from the equation, leaving

$$fraction\ submerged = \frac{\rho}{\rho_{fl}}$$

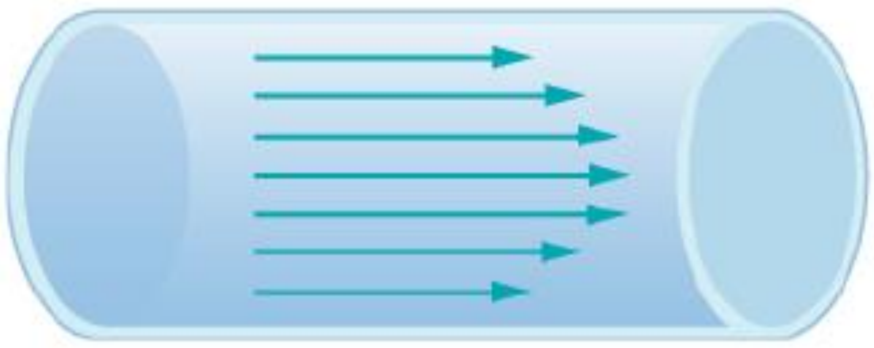
This relationship is used to measure densities.

- **EXAMPLE**

Suppose a 60.0-kg woman floats in fresh water with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

- Ans: $970\ kg/m^3$

Characteristics of Flow



(a) Laminar Flow

(b) Turbulent Flow

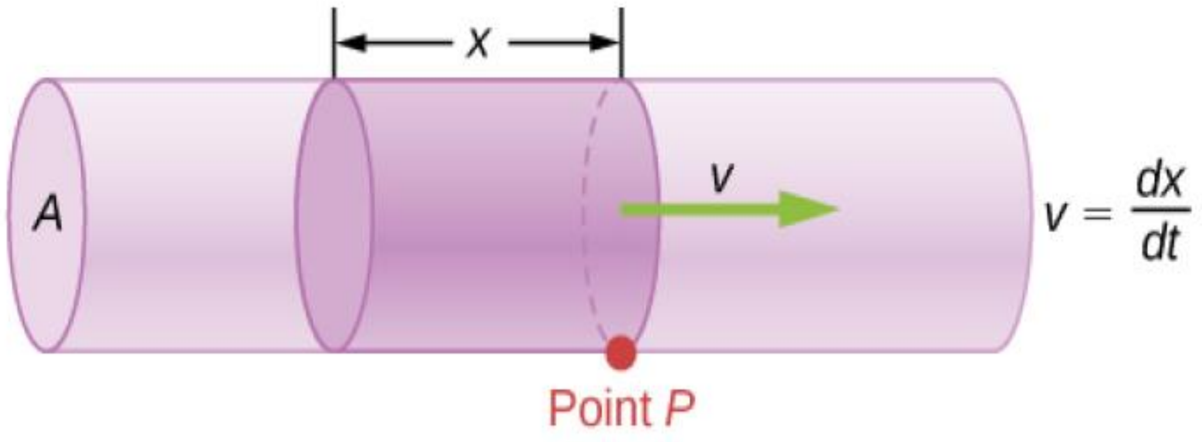
- A method for representing fluid motion is called a streamline.
- A streamline represents the path of a small volume of fluid as it flows.
- The velocity is always tangential to the streamline. (See Figure)
- The first fluid exhibits a laminar flow (sometimes described as a steady flow), represented by smooth, parallel streamlines.

- The second diagram represents turbulent flow, in which streamlines are irregular and change over time.
- In turbulent flow, the paths of the fluid flow are irregular as different parts of the fluid mix together or form small circular regions that resemble whirlpools.
- This can occur when the speed of the fluid reaches a certain critical speed.

Fluid Dynamics

Flow Rate and its Relation to Velocity

- The volume of fluid passing by a given location through an area during a period of time is called flow rate Q, or more precisely, volume flow rate.



- In symbols, this is written as

$$Q = \frac{dV}{dt}$$

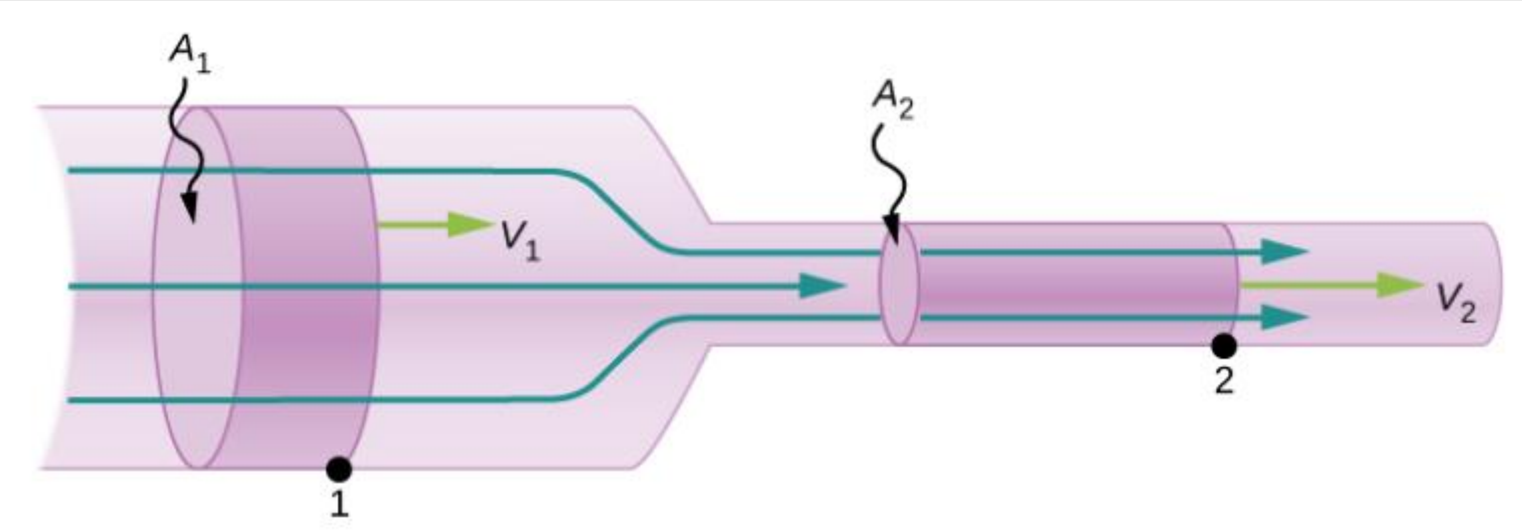
where V is the volume and t is the elapsed time.

In (Figure), the volume of the cylinder is Ax, so the flow rate is

$$Q = \frac{dV}{dt} = \frac{d}{dt}(Ax) = A\frac{dx}{dt} = Av\ (m^3/s)$$

- where A is the cross-sectional area of the pipe and v is the magnitude of the velocity.
- Flow rate and velocity are related, but quite different, physical quantities.
- To make the distinction clear, consider the flow rate of a river. The greater the velocity of the water, the greater the flow rate of the river.
- Flow rate also depends on the size and shape of the river.

- The relationship tells us that flow rate is directly proportional to both the average speed of the fluid and the cross-sectional area of a river, pipe, or other conduit.
- The larger the conduit, the greater its cross-sectional area.



- The figure shows an incompressible fluid flowing along a pipe of decreasing radius.

- If the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow.
- The flow is continuous because there are no sources or sinks that add or remove mass, so the mass flowing into the pipe must be equal the mass flowing out of the pipe.
- In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase.
- Hence, considering arbitrary points 1 and 2, then

$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

This is called the *equation of continuity* and is valid for any incompressible fluid (with constant density).

- The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: It emerges with a large speed—that is the purpose of the nozzle.

- In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.

- Since liquids are essentially incompressible, the equation of continuity is valid for all liquids.

- However, gases are compressible, so the equation must be applied with caution to gases if they are subjected to compression or expansion.

Example

A nozzle with a diameter of $0.500\ cm$ is attached to a garden hose with a radius of $0.900\ cm$. The flow rate through hose and nozzle is $0.500\ L/s$. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

Use the relationship between flow rate and speed to find both speeds. We use the subscript 1 for the hose and 2 for the nozzle.

Solution

- 1. To solve the flow rate equation for speed, use $\pi {r_1}^2$ for the cross-sectional area of the hose, obtaining

$$v = \frac{Q}{A} = \frac{Q}{\pi {r_1}^2}$$

$$v = \frac{(0.500\ \ L/s)(10^{-3}\ \ m/L)}{3.142(9.00 \times 10^{-3}\ m)^2}$$

$$= 1.96\ \ m/s.$$

- We could repeat this calculation to find the speed in the nozzle v2, but we can use the equation of continuity to give a somewhat different insight.
- The equation states;

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi {r_1}^2}{\pi {r_2}^2} v_1 = \frac{{r_1}^2}{{r_2}^2} v_1$$

$$v_2 = \frac{(0.900\ cm)^2}{(0.250\ cm)^2} 1.96\ m/s = 25.5\ m/s$$

A speed of 1.96 m/s is about right for water emerging from a hose with no nozzle. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

Mass Conservation

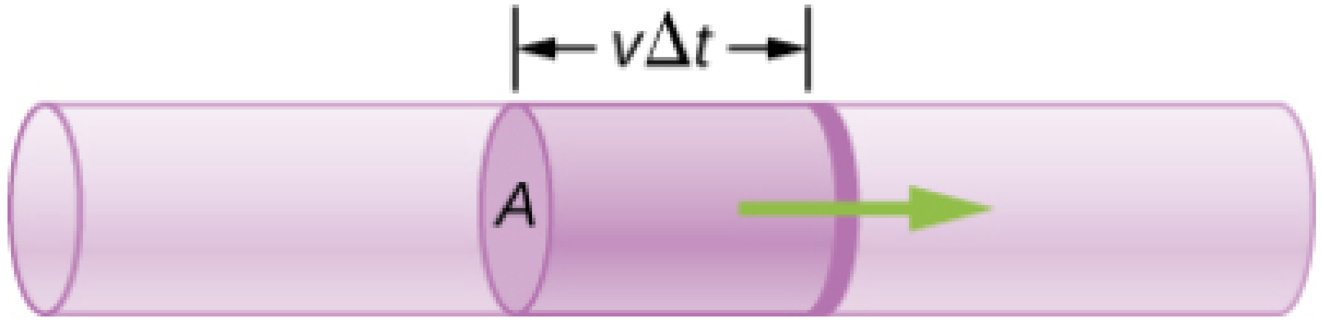
- The rate of flow of a fluid can also be described by the *mass flow rate* or mass rate of flow.
- This is the rate at which a mass of the fluid moves past a point.
- The mass can be determined from the density and the volume:

$$m = \rho V = \rho A x$$

The mass flow rate is then

$$\frac{dm}{dt} = \frac{d}{dt}(\rho A x) = \rho A \frac{dx}{dt} = \rho A v$$

where ρ is the density, A is the cross-sectional area, and v is the magnitude of the velocity.



Consider the pipe in the figure starts at the inlet with a cross sectional area of A_1 and constricts to an outlet with a smaller cross-sectional area of A_2.

The mass of fluid entering the pipe has to be equal to the mass of fluid leaving the pipe.

For this reason, the velocity at the outlet (v_2) is greater than the velocity of the inlet (v_1).

- Using the fact that the mass of fluid entering the pipe must be equal to the mass of fluid exiting the pipe, we can find a relationship between the velocity and the cross-sectional area by taking the rate of change of the mass in and the mass out:

$$\left(\frac{dm}{dt}\right)_2 = \left(\frac{dm}{dt}\right)_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

- This is also known as the continuity equation in general form.
- If the density of the fluid remains constant through the constriction—that is, the fluid is incompressible—then the density cancels from the continuity equation,

$$A_1 v_1 = A_2 v_2$$

- The equation reduces to show that the volume flow rate into the pipe equals the volume flow rate out of the pipe.

Question

Water is moving at a velocity of $2.00\ m/s$ through a hose with an internal diameter of $1.60\ cm$.

(a) What is the flow rate in litres per second?

(b) The fluid velocity in this hose's nozzle is $15.0\ m/s$. What is the nozzle's inside diameter?

- What is the fluid speed in a fire hose with a $9.00\ cm$ diameter carrying $80.0\ L$ of water per second?
- (b) What is the flow rate in cubic meters per second?
- (c) Would your answers be different if salt water replaced the fresh water in the fire hose?

Bernoulli's Equation

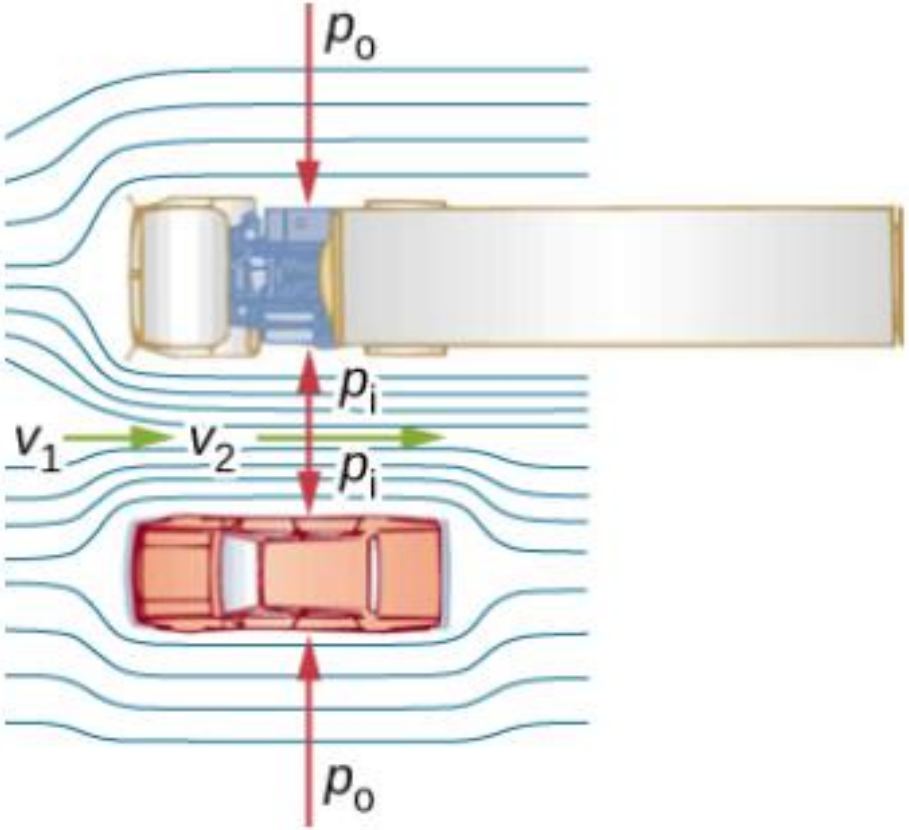
- Recall that when a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases.
- The increased kinetic energy comes from the net work done on the fluid to push it into the channel.
- Also, if the fluid changes vertical position, work is done on the fluid by the gravitational force.
- A pressure difference occurs when the channel narrows.
- This pressure difference results in a net force on the fluid because the pressure times the area equals the force, and this net force does work.

- Recall the work-energy theorem,

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2$$

- The net work done increases the fluid's kinetic energy.
- As a result, the pressure drops in a rapidly moving fluid whether or not the fluid is confined to a tube.
- There are many common examples of pressure dropping in rapidly moving fluids.
- For instance, shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on.

- The reason is that the high-velocity stream of water and air creates a region of lower pressure inside the shower, whereas the pressure on the other side remains at the standard atmospheric pressure. This pressure difference results in a net force, pushing the curtain inward.



- Similarly, when a car passes a truck on the highway, (See Figure) the two vehicles seem to pull toward each other.
- The reason is the same: The high velocity of the air between the car and the truck creates a region of lower pressure between the vehicles, and they

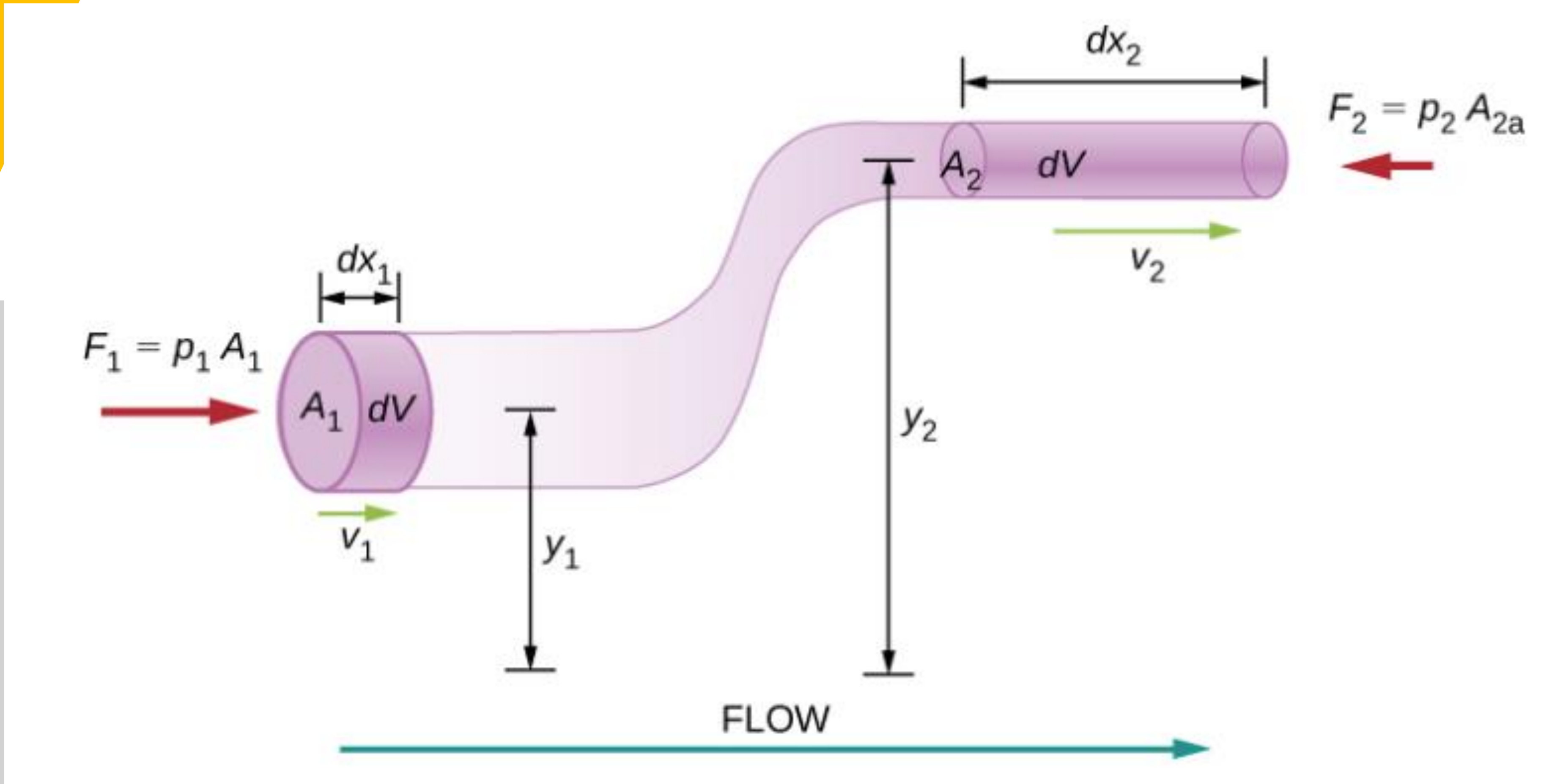
- are pushed together by greater pressure on the outside.
- This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.

- Air passing between the vehicles flows in a narrower channel and must increase its speed (v_2 is greater than v_1), causing the pressure between them to drop (p_1 is less than p_0).

- Greater pressure on the outside pushes the car and truck together.

Energy Conservation and Bernoulli's Equation

- The application of the principle of conservation of energy to frictionless laminar flow leads to a very useful relation between pressure and flow speed in a fluid.
- This relation is called **Bernoulli's equation**, named after Daniel Bernoulli (1700–1782), who published his studies on fluid motion in his book *Hydrodynamica* (1738).
- Consider an incompressible fluid flowing through a pipe that has a varying diameter and height, as shown in the Figure.
- Subscripts 1 and 2 in the figure denote two locations along the pipe and illustrate the relationships between the areas of the cross sections A, the speed of flow v, the height



from ground y, and the pressure p at each point.

We assume here that the density at the two points is the same—therefore, density is denoted by ρ without any subscripts—and since the fluid in incompressible, the shaded volumes must be equal.

- We also assume that there are no viscous forces in the fluid, so the energy of any part of the fluid will be conserved.
- To derive Bernoulli's equation, first calculate the work that was done on the fluid as:

$$dW = F_1 dx_1 - F_2 dx_2$$

$$dW = p_1 A_1 dx_1 - p_2 A_2 dx_2 = p_1 dV - p_2 dV = (p_1 - p_2) dV$$

The work done was due to the conservative force of gravity and the change in the kinetic energy of the fluid.

The change in the kinetic energy of the fluid is equal to

$$dK = \frac{1}{2} m_2 {v_2}^2 - \frac{1}{2} m_1 {v_1}^2 = \frac{1}{2} \rho dV ({v_2}^2 - {v_1}^2)$$

- The change in potential energy is

$$dU = mgy_2 - mgy_1 = \rho dV g(y_2 - y_1)$$

- The energy equation then becomes

$$dW = dK + dU$$

$$(p_1 - p_2)dV = \frac{1}{2}\rho dV({v_2}^2 - {v_1}^2) + \rho dV g(y_2 - y_1)$$

$$(p_1 - p_2) = \frac{1}{2}\rho({v_2}^2 - {v_1}^2) + \rho g(y_2 - y_1)$$

Rearranging the equation gives Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g y_2$$

- This relation states that the mechanical energy of any part of the fluid changes as a result of the work done by the fluid external to that part, due to varying pressure along the way.
- Since the two points were chosen arbitrarily, we can write Bernoulli's equation more generally as a conservation principle along the flow.
- For an incompressible, frictionless fluid, the combination of pressure and the sum of kinetic and potential energy densities is constant not only over time, but also along a streamline:

$$p + \frac{1}{2}\rho v^2 + \rho g y = constant$$

- A special note must be made here of the fact that in a dynamic situation, the pressures at the same height in different parts of the fluid may be different if they have different speeds of flow.
- According to Bernoulli's equation, if a small volume of fluid flow along a path, various quantities in the sum may change, but the total remains constant.
- **Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction.**
- The general form of Bernoulli's equation has three terms in it, and it is broadly applicable.
- To understand it better, let us consider some specific situations that simplify and illustrate its use and meaning.

Bernoulli's Equation for Static Fluids

- First consider the very simple situation where the fluid is static—that is, $v_1 = v_2 = 0$.
- Bernoulli's equation in that case is

$$p_1 + \rho g h_1 = p_2 + \rho g h_2$$

- We can further simplify the equation by setting h2=0. (Any height can be chosen for a reference height of zero, as is often done for other situations involving gravitational force, making all other heights relative.) In this case, we get

$$p_2 = p_1 + \rho g h_1$$

- This equation tells us that, in static fluids, pressure increases with depth.
- As we go from point 1 to point 2 in the fluid, the depth increases by h_1, and consequently, p_2 is greater than p_1 by an amount $\rho g h_1$.
- In the very simplest case, p_1 is zero at the top of the fluid, and we get the familiar relationship $p = \rho g h$. (Recall that $p = \rho g h$ and $\Delta U g = -mgh$.)
- Thus, Bernoulli's equation confirms the fact that the pressure change due to the weight of a fluid is $\rho g h$.
- Although we introduce Bernoulli's equation for fluid motion, it includes much of what we studied for static fluids earlier.

Bernoulli's Principle

- Suppose a fluid is moving but its depth is constant—that is, $h_1 = h_2$.
- Under this condition, Bernoulli's equation becomes

$$p_1 + \frac{1}{2}\rho {v_1}^2 = p_2 + \frac{1}{2}\rho {v_2}^2$$

- Situations in which fluid flows at a constant depth are so common that this equation is often also called **Bernoulli's principle**, which is simply Bernoulli's equation for fluids at constant depth.
- Note that this principle applies to a small volume of fluid as it flows along its path.

- Bernoulli's principle reinforces the fact that pressure drops as speed increases in a moving fluid:
- If v_2 is greater than v_1 in the equation, then p_2 must be less than p_1 for the equality to hold.

Example:

- If the speed of water in a hose increased from $1.96\, m/s$ to $25.5\, m/s$ going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is $1.01 \times 10^5\, N/m^2$ (atmospheric, as it must be) and assuming level, frictionless flow.

Solution:

- Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find p_1.
- Solving Bernoulli's principle for p1p1 yields

$$p_1 = p_2 + \frac{1}{2}\rho {v_2}^2 - \frac{1}{2}\rho {v_1}^2 = p_2 + \frac{1}{2}\rho({v_2}^2 - {v_1}^2)$$

- Substituting known values,

$$p_1 = 1.01 \times 10^5 \ N/m^2 + \frac{1}{2}(10^3 \ kg/m^3)[(25.5 \ m/s)^2 - (1.96 \ m/s)^2]$$

$$p_1 = 4.24 \times 10^5 \ N/m^2$$

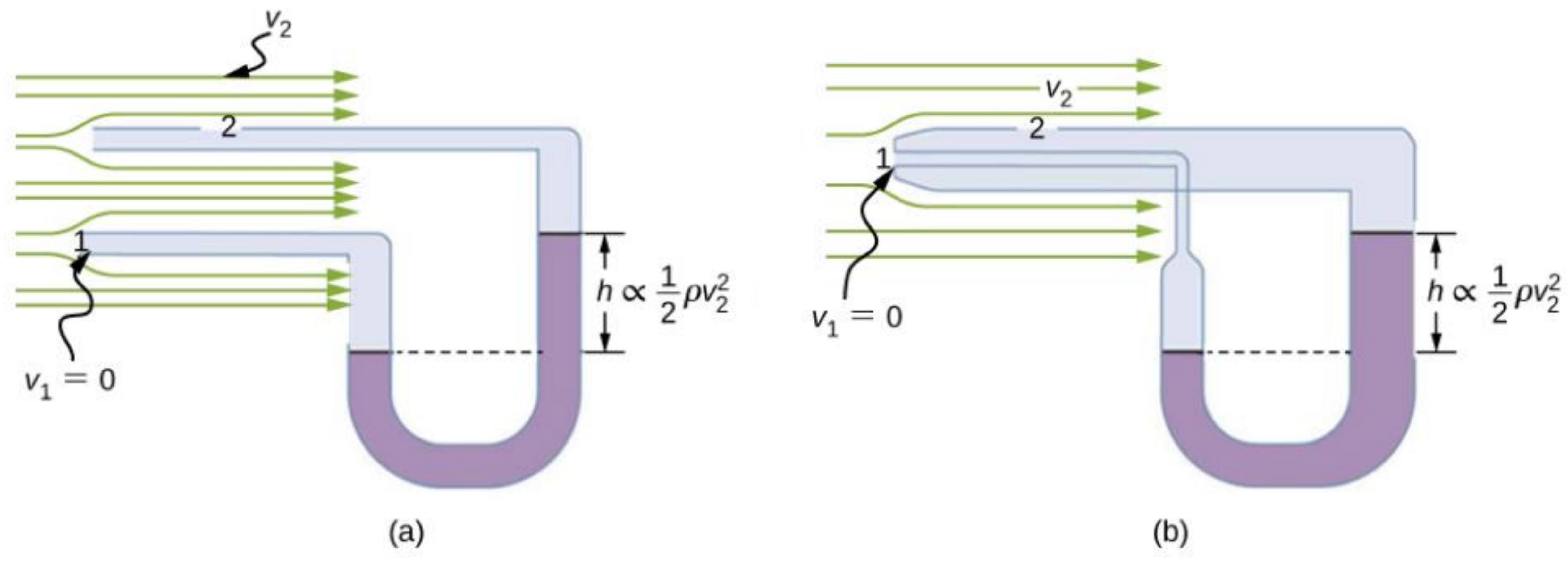
- This absolute pressure in the hose is greater than in the nozzle, as expected, since v is greater in the nozzle.
- The pressure p_2 in the nozzle must be atmospheric, because the water emerges into the atmosphere without other changes in conditions.

Application of Bernoulli's Principle

- Many devices and situations occur in which fluid flows at a constant height and thus can be analysed using Bernoulli's principle.
- Example of such devices are:
- Entrainment
- Velocity Measurement
- Fire Hose

Velocity measurement

- The Figure below shows two devices that apply Bernoulli's principle to measure fluid velocity.



- The manometer in part (a) is connected to two tubes that are small enough not to appreciably disturb the flow.
- The tube facing the oncoming fluid creates a dead spot having zero velocity $(v_1 = 0)$ in front of it, while fluid passing the other tube has velocity v_2.
- This means that Bernoulli's principle as stated in

$$p_1 + \frac{1}{2}\rho {v_1}^2 = p_2 + \frac{1}{2}\rho {v_2}^2$$

- becomes

$$p_1 = p_2 + \frac{1}{2}\rho {v_2}^2$$

- Thus, pressure p_2 over the second opening is reduced by $\frac{1}{2}\rho {v_2}^2$, so the fluid in the manometer rises by h on the side connected to the second opening, where,

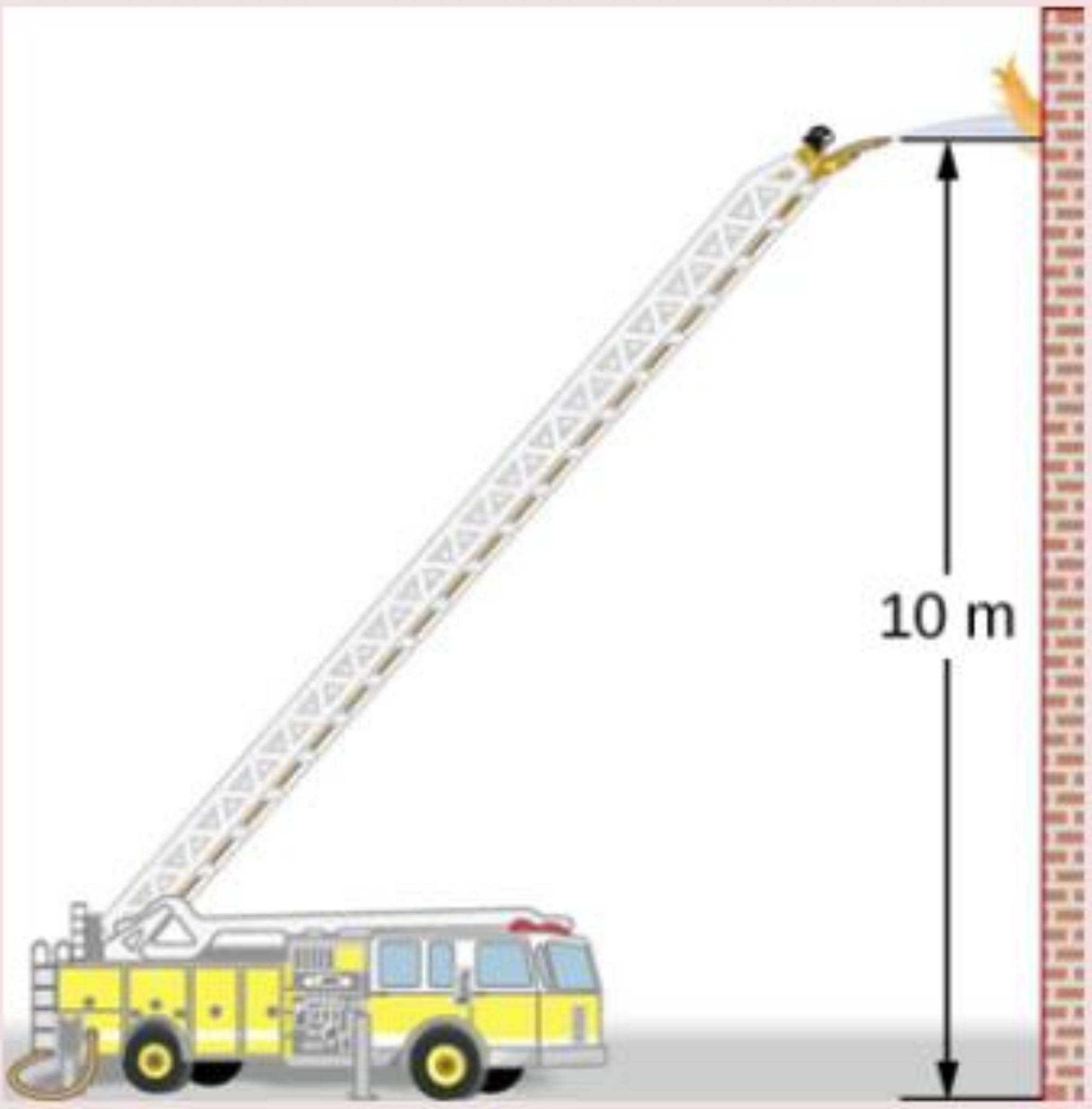
$$h \propto \frac{1}{2}\rho {v_2}^2$$

Solving for v_2, we see that

$$v_2 \propto \sqrt{h}$$

Part (b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air-speed indicators in aircraft.

Calculating Pressure: A Fire Hose



- Fire hoses used in major structural fires have an inside diameter of $6.40\ cm$ (See Figure).
- Suppose such a hose carries a flow of $40.0\ L/s$, starting at a gauge pressure of $1.62 \times 10^6\ N/m^2$. The hose rises up $10.0\ m$ along a ladder to a nozzle having an inside diameter of $3.00\ cm$. What is the pressure in the nozzle?

- We must use Bernoulli's equation to solve for the pressure, since depth is not constant.
- Bernoulli's equation is

$$p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_2$$

where subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively.

- We must first find the speeds $v_1$and v_2. Since $Q = A_1 v_1$, we get

$$v_1 = \frac{Q}{A_1} = \frac{40.0 \times 10^{-3}\ m^3/s}{\pi(3.20 \times 10^{-2}\ m)^2} = 12.4\ m/s$$

- Similarly, we find

$$v_2 = 56.6\ m/s$$

- This rather large speed is helpful in reaching the fire.
- Now, taking h_1 to be zero, we solve Bernoulli's equation for p_2:

$$p_2 = p_1 + \frac{1}{2}\rho({v_1}^2 - {v_2}^2) - \rho g h_2$$

- Substituting known values yields

$$p_2 \\ = 1.62 \times 10^6\ N/m^2 + \frac{1}{2}(1000\ kg/m^3)[(12.4\ m/s)^2 - (56.6\ m/s)^2] \\ - (1000\ kg/m^3)(9.8\ m/s^2)(10.0\ m) = 0$$

- **Significance**
- This value is a gauge pressure, since the initial pressure was given as a gauge pressure.
- Thus, the nozzle pressure equals atmospheric pressure as it must, because the water exits into the atmosphere without changes in its conditions.

Questions

1. Based on Bernoulli's equation, what are three forms of energy in a fluid? (Note that these forms are conservative, unlike heat transfer and other dissipative forms not included in Bernoulli's equation.)

2. A fluid of a constant density flows through a reduction in a pipe.

Find an equation for the change in pressure, in terms of v_1, A_1 , A_2, and the density.