1

Que 1

Solve the equation $\log_4 (3x+4) = \log_2 (2x+1)$

Solution

$$\log_4 (3x+4) = \log_2 (2x+1)$$

$$\log_{2^2} (3x+4) = \log_2 2x+1$$

$$* \log_{a^x} b = \boxed{\frac{1}{x} \log_a b}$$

$$\frac{1}{2} \log_2 3x+4 = \log_2 2x+1$$

$$\cancel{\log_2} (3x+4)^{1/2} = \cancel{\log_2} (2x+1)$$

$$(3x+4)^{1/2} = (2x+1)$$

$$(3x+4)^{\frac{1}{2}\times 2} = (2x+1)^2$$

$$3x+4 = (2x+1)(2x+1)$$

$$3x+4 = 4x^2+2x+2x+1$$

$$3x+4 = 4x^2+4x+1$$

$$0 = 4x^2+4x-3x+1-4$$

$$4x^2+x-3 = 0$$

$$4x^2+4x-3x-3 = 0$$

$$(4x^2+4x)-(3x-3) = 0$$

$$4x(x+1)-3(x+1) = 0$$

$$(4x-3)(x+1) = 0$$

$$4x-3 = 0 \quad \text{or} \quad x+1 = 0$$

$$4x = 3 \quad \text{or} \quad x = 0+1$$

$$\frac{4x}{4} = \frac{3}{4} \quad \text{or} \quad x = -1$$

$$x = \frac{3}{4} \text{ or } -1$$

Solved
HoakLymatics

Que 2

Simplify $\frac{1}{1-x^2} - \frac{1}{1-x^{-2}}$

Solution

$$\frac{1}{1-x^2} - \frac{1}{1-x^{-2}}$$

$$\Rightarrow \frac{1}{1-x^2} - \frac{1}{1-\frac{1}{x^2}}$$

$$\Rightarrow \frac{1}{1-x^2} - \frac{1}{\frac{x^2-1}{x^2}}$$

$$\Rightarrow \frac{1}{1-x^2} - \frac{x^2}{x^2-1}$$

$$\Rightarrow \frac{\cancel{x^2}-1-\cancel{x^2}+x^4}{(1-x^2)(x^2-1)}$$

$$\Rightarrow \frac{x^4-1}{(1-x^2)(x^2-1)} = \frac{(x^2)^2-(1)^2}{(1-x^2)(x^2-1)} \longrightarrow \text{difference of two Squares}$$

$$\Rightarrow \frac{\cancel{(x^2-1)}(x^2+1)}{(1-x^2)\cancel{(x^2-1)}}$$

$$\Rightarrow \frac{1+x^2}{1-x^2}$$

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2

Que 3

Simplify $\left(\frac{a^x}{a^y}\right)^{x+y} \times \left(\frac{a^y}{a^z}\right)^{y+z} \times \left(\frac{a^z}{a^x}\right)^{z+x}$

Solution

$$\left(a^{x-y}\right)^{x+y} \times \left(a^{y-z}\right)^{y+z} \times \left(a^{z-x}\right)^{z+x}$$

$$a^{(x-y)(x+y)} \times a^{(y-z)(y+z)} \times a^{(z-x)(z+x)}$$

$$a^{x^2-y^2} \times a^{y^2-z^2} \times a^{z^2-x^2}$$

$$a^{(x^2-y^2)+(y^2-z^2)+(z^2-x^2)}$$

$$a^{\cancel{x^2}\cancel{-y^2}\cancel{+y^2}\cancel{-z^2}\cancel{+z^2}\cancel{-x^2}}$$

$$a^0$$

$$= 1$$

Solved

AoahLymatic

Que 4

Solve $\log_3(x+1) - \log_9(x-1) = 1$

Solution

$$\log_3(x+1) - \log_{3^2}(x-1) = 1$$

$$\log_3(x+1) - \frac{1}{2}\log_3(x-1) = 1$$

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$\log_3(x+1) - \frac{1}{2}\log_3(x-1) = 1$

$\log_3(x+1) - \log_3(x-1)^{1/2} = 1$

$\log_3(x+1) - \log_3\sqrt{(x-1)} = 1$

$\log_3\left[\frac{x+1}{\sqrt{x-1}}\right] = 1$

$\cancel{\log_3}\left[\frac{x+1}{\sqrt{x-1}}\right] = \cancel{\log_3} 3^1$

$\frac{x+1}{\sqrt{x-1}} = 3^1$

$\frac{x+1}{\sqrt{x-1}} \times \frac{3}{1}$

$x+1 = 3\sqrt{x-1}$

$\frac{3\sqrt{x-1}}{3} = \frac{x+1}{3}$

$\sqrt{x-1} = \frac{x+1}{3}$

On squaring both sides we have

$(\sqrt{x-1})^2 = \left(\frac{x+1}{3}\right)^2$

$x-1 = \frac{(x+1)^2}{3^2} \Rightarrow \frac{x-1}{1} \times \frac{x^2+2x+1}{9}$

$3(x-1) = x^2+2x+1$

$9x-9 = x^2+2x+1$

$x^2+2x-9x+1+9 = 0$

$x^2-7x+10 = 0$

$x^2-5x-2x+10 = 0$

$x(x-5)-2(x-5) = 0$

$(x-2)(x-5) = 0$

$x-2=0$ or $x-5=0$

$\therefore x = 2$ or 5

Solved ✓ Moan Lymatics

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Que 5 4

Solve the equation $\log_2 x + \log_x 4 = 3$

$\Rightarrow \log_2 x + \dfrac{1}{\log_4 x} = 3$

$\Rightarrow \log_2 x + \dfrac{1}{\log_{2^2} x} = 3$

$\Rightarrow \log_2 x + \dfrac{1}{\frac{1}{2}\log_2 x} = 3$

$\Rightarrow \log_2 x + \dfrac{2}{\log_2 x} = 3$

$\Rightarrow \log_2 x + \dfrac{2}{\log_2 x} = 3$

$* \dfrac{1}{\frac{1}{2}\log_2 x} \Rightarrow 1 \div \dfrac{\log_2 x}{2}$

$\Rightarrow 1 \times \dfrac{2}{\log_2 x}$

$= \dfrac{2}{\log_2 x}$

Let $p = \log_2 x$

$\Rightarrow p + \dfrac{2}{p} = 3$

$\Rightarrow \dfrac{p^2 + 2}{p} = 3 \Rightarrow p^2 + 2 = 3p$

$\Rightarrow p^2 - 3p + 2 = 0 \Rightarrow p^2 - 2p - p + 2 = 0$

$\Rightarrow p(p-2) - 1(p-2) = 0$

$\Rightarrow (p-1)(p-2) = 0$

$\therefore p = 1$ or 2

Recall that : $p = \log_2 x$

when $p = 1$

$p = \log_2 x$

$1 = \log_2 x$

$\cancel{\log_2} 2 = \cancel{\log_2} x$

$\therefore x = 2$

when $p = 2$

$p = \log_2 x$

$2 = \log_2 x$

$\cancel{\log_2} 2^2 = \cancel{\log_2} x$

$x = 2^2 = 4$

$\therefore x = 2$ or 4

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5

Que 6

$\log_3(x+2) = \log_9(6x+4)$

$\Rightarrow \log_3(x+2) = \log_{3^2}(6x+4)$

$\Rightarrow \log_3(x+2) = \frac{1}{2}\log_3(6x+4)$

$\Rightarrow \log_3(x+2) = \log_3(6x+4)^{1/2}$

$\Rightarrow (x+2) = (6x+4)^{1/2}$

on squaring both sides

$(x+2)^2 = \left[(6x+4)^{1/2}\right]^2$

$(x+2)^2 = 6x+4$

$x^2+2x+2x+4 = 6x+4$

$x^2+4x+4 = 6x+4$

$x^2+4x-6x+\cancel{4}-\cancel{4} = 0$

$x^2-2x = 0$

$x\left(\frac{x^2}{x} - \frac{2x}{x}\right) = 0$

$x(x-2) = 0$

$x = 0$ or $x-2 = 0$

$x = 0$ or 2

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Que 7

*

If α and β are the roots of the equation $2x^2 - x - 4 = 0$. Find the Value of $\alpha^4 + \beta^4$

Solution

$$2x^2 - x - 4 = 0$$

$$\alpha + \beta = \frac{-b}{a} = -\left(\frac{-1}{2}\right) = \frac{1}{2} \qquad \alpha + \beta = \frac{1}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{-4}{2} = -\frac{2}{2} \qquad \alpha\beta = -2$$

$$\Rightarrow \alpha^4 + \beta^4 = (\alpha^2)^2 + (\beta^2)^2 = \left[\alpha^2 + \beta^2\right]^2 - 2\alpha^2\beta^2$$

$$= \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2\alpha^2\beta^2$$

$$= \left[\left(\frac{1}{2}\right)^2 - 2(-2)\right]^2 - 2(\alpha\beta)^2$$

$$= \left(\tfrac{1}{4} + 4\right)^2 - 2(-2)^2$$

$$= \left(\frac{17}{4}\right)^2 - 2(4) = \frac{289}{16} - 8 = \frac{289 - 128}{16}$$

$$\therefore \alpha^4 + \beta^4 = \frac{161}{16}$$

solved

Que 8

If α and β are the root of the equation $2x^2 - x - 4 = 0$ find $\frac{1}{\alpha} + \frac{1}{\beta}$

Solution

$$2x^2 - x - 4 = 0$$

Recall from above $\Rightarrow \alpha + \beta = \tfrac{1}{2}$ and $\alpha\beta = -2$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{1/2}{-2} = \frac{1}{2} \div -2$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{2} \times \frac{-1}{2}$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-1}{4}$$

solved

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6

Que 9

Solve the equation $3^{2(x+1)} - 10 \cdot 3^{x} + 1 = 0$

Solution

$$3^{2x+2} - 10 \cdot 3^{x} + 1 = 0$$

$$3^{2x} \cdot 3^{2} - 10(3^{x}) + 1 = 0$$

$$(3^{x})^{2} \cdot 9 - 10(3^{x}) + 1 = 0$$

Let $3^{x} = P$

$$9P^{2} - 10P + 1 = 0$$

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$9p^2 - 10p + 1 = 0$

$9p^2 - 9p - p + 1 = 0$

$(9p^2 - 9p) - (p+1) = 0$

$9p(p-1) - 1(p-1) = 0$

$(9p-1)(p-1) = 0$

$9p - 1 = 0$ or $p - 1 = 0$

$p = 1/9$ or 1

Recall that $3^x = p$

when $p = 1/9$

$3^x = \frac{1}{9}$

$3^x = 9^{-1}$

$3^x = (3^2)^{-1}$

$\cancel{3}^x = \cancel{3}^{-2}$

$x = -2$

when $p = 1$

$3^x = 1$

$3^x = 3^0$

$\cancel{3}^x = \cancel{3}^0$

$x = 0$

$\therefore \quad x = 0$ or -2

Que 10

Solve the equation $9x^{2/3} + 4x^{-2/3} = 27$

$\Rightarrow 9x^{2/3} + 4(x^{2/3})^{-1} = 27$

$\Rightarrow$ Let $x^{2/3} = p$

$9p + 4p^{-1} = 27$

$9p + \frac{4}{p} = 27$ multiply through by p

$9p^2 + 4 = 27p$

$9p^2 - 27p + 4 = 0$

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$9p^2 - 27p + 4 = 0$

$a = 9, \; b = -27, \; c = 4$

By Formula

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-(-27) \pm \sqrt{(-27)^2 - (4 \times 9 \times 4)}}{2 \times 9}$$

$$p = \frac{27 \pm \sqrt{729 - 144}}{18} = \frac{27 \pm \sqrt{585}}{18} = \frac{27 \pm \sqrt{9 \times 65}}{18}$$

$$p = \frac{27 \pm 3\sqrt{65}}{18}; \; p = \frac{\cancel{3}(9 \pm \sqrt{65})}{\cancel{18}} = \frac{9 \pm \sqrt{65}}{6}$$

$$\therefore p = \frac{9 + \sqrt{65}}{6} \text{ or } \frac{9 - \sqrt{65}}{6}$$

Recall that $x^{2/3} = p$

$$x^{2/3} = \frac{9 + \sqrt{65}}{6}$$

$$\left(x^{\cancel{2/3}}\right)^{\cancel{3/2}} = \left(\frac{9 + \sqrt{65}}{6}\right)^{3/2}$$

$$x = \sqrt{\left(\frac{9 + \sqrt{65}}{6}\right)^3} \text{ or } \sqrt{\left(\frac{9 - \sqrt{65}}{6}\right)^3}$$

Que 11

Solve for x if $2\sqrt{2x-12} - \sqrt{2x-3} = 3$

Solution

$2\sqrt{2x-12} = 3 + \sqrt{2x-3}$ on squaring both sides we have;

$$\left(2\sqrt{2x-12}\right)^2 = \left(3 + \sqrt{2x-3}\right)^2$$

$$4(2x-12) = \left(3 + \sqrt{2x-3}\right)\left(3 + \sqrt{2x-3}\right)$$

$$8x - 48 = 9 + 6\sqrt{2x-3} + (2x-3)$$

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$$8x - 48 = 9 + 6\sqrt{(2x-3)} + 2x - 3$$

$$8x - 2x - 48 + 3 - 9 = 6\sqrt{2x-3}$$

$$6x - 54 = 6\sqrt{2x-3}$$

$$\cancel{6}(x-9) = \cancel{6}\sqrt{2x-3}$$

$$x - 9 = \sqrt{2x-3}$$ on squaring both sides

$$(x-9)^2 = \left(\sqrt{(2x-3)}\right)^2$$

$$(x-9)(x-9) = 2x-3$$

$$x^2 - 18x + 81 = 2x - 3.$$

$$x^2 - 18x - 2x + 81 + 3 = 0$$

$$x^2 - 20x + 84 = 0$$

$$x^2 - 14x - 6x + 84 = 0$$

$$(x^2 - 14x) - (6x + 84) = 0$$

$$x(x-14) - 6(x-14) = 0$$

$$(x-6)(x-14) = 0$$

$$x - 6 = 0 \text{ or } x - 14 = 0$$

$$x = 6 \text{ or } 14$$

Que 12

Find the only solution of the equation

$$\sqrt{(4x-2)} + \sqrt{x+1} - \sqrt{7-5x} = 0$$

$$\Rightarrow \sqrt{(4x-2)} + \sqrt{x+1} = \sqrt{7-5x}$$ on squaring we have

$$\Rightarrow \left[\left(\sqrt{4x-2}\right) + \left(\sqrt{x+1}\right)\right]^2 = \left(\sqrt{7-5x}\right)^2$$

$$\Rightarrow \left[\left(\sqrt{4x-2}\right) + \left(\sqrt{x+1}\right)\right].\left[\left(\sqrt{4x-2}\right) + \left(\sqrt{x+1}\right)\right] = 7 - 5x$$

$$\Rightarrow (4x-2) + 2\sqrt{(4x-2)(x+1)} + (x+1) = 7 - 5x$$

$$\Rightarrow 4x - 2 + 2\sqrt{(4x^2 + 4x - 2x - 2)} + x + 1 = 7 - 5x$$ Collect like terms

$$2\sqrt{4x^2 + 2x - 2} = 7 - 5x - x - 1 - 4x + 2$$

$$2\sqrt{4x^2 + 2x - 2} = -10x + 8$$

$$\cancel{2}\sqrt{4x^2 + 2x - 2} = \cancel{2}(-5x + 4)$$

$$\sqrt{4x^2 + 2x - 2} = 4 - 5x$$ on squaring both sides

$$\left(\sqrt{4x^2 + 2x - 2}\right)^2 = (4 - 5x)^2$$

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$$4x^2+2x-2 = (4-5x)(4-5x)$$ 10

$$4x^2+2x-2 = 16-20x-20x+25x^2$$
$$4x^2+2x-2 = 16-40x+25x^2$$
$$0 = 16-40x+25x^2-4x^2-2x+2$$
$$16-40x+25x^2-4x^2-2x+2 = 0$$
$$21x^2-42x+18 = 0$$

where $a = 21$ $b = -42$ $c = 18$

By formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-42) \pm \sqrt{(-42)^2-(4\times 21\times 18)}}{2\times 21} \Rightarrow \frac{42 \pm \sqrt{1764-1512}}{42}$$

$$x = \frac{42 \pm \sqrt{252}}{42} \Rightarrow \frac{42 \pm \sqrt{36\times 7}}{42}$$

$$x = \frac{42 \pm 6\sqrt{7}}{42} \Rightarrow \frac{42+6\sqrt{7}}{42} \text{ or } \frac{42-6\sqrt{7}}{42}$$

$$\therefore x = \frac{\cancel{6}(7+\sqrt{7})}{\cancel{42}} \text{ or } \frac{\cancel{6}(7-\sqrt{7})}{\cancel{42}}$$

$$x = \frac{7+\sqrt{7}}{7} \text{ or } \frac{7-\sqrt{7}}{7}$$

$$x = \frac{7 \pm \sqrt{7}}{7}$$ solved ✓ Noah Lynatics

Que 13

Solve; $3\sqrt{\frac{x}{x-1}} + 6\sqrt{\frac{x-1}{x}} = 11$

$$\Rightarrow 3\sqrt{\frac{x}{x-1}} = \left(11 - 6\sqrt{\frac{x-1}{x}}\right)$$ on squaring both sides

$$\left(3\sqrt{\frac{x}{x-1}}\right)^2 = \left(11-6\sqrt{\frac{x-1}{x}}\right)^2$$

$$9\left(\frac{x}{x-1}\right) = \left(11-6\sqrt{\frac{x-1}{x}}\right)\left(11-6\sqrt{\frac{x-1}{x}}\right)$$

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QUE 13 11

$$\sqrt[3]{\frac{x}{x-1}} + \sqrt[6]{\frac{x-1}{x}} = 11$$

$$3\left[\left(\frac{\sqrt{x}}{\sqrt{x-1}}\right) + \frac{2(\sqrt{x-1})}{\sqrt{x}}\right] = 11$$

$$\frac{\sqrt{x}}{\sqrt{x-1}} + \frac{2\sqrt{x-1}}{\sqrt{x}} = \frac{11}{3}$$

$$\frac{x + 2(x-1)}{\sqrt{x(x-1)}} = \frac{11}{3}$$

$$\frac{x + 2x - 2}{\sqrt{x}\sqrt{(x-1)}} = \frac{11}{3}$$

$$\frac{3x-2}{\sqrt{x^2-x}} = \frac{11}{3}$$

$$\frac{3(3x-2)}{11} = \frac{11\sqrt{x^2-x}}{11}$$

$$\sqrt{x^2-x} = \frac{9x-6}{11}$$ on squaring both sides we have

$$\left(\sqrt{x^2-x}\right)^2 = \left(\frac{9x-6}{11}\right)^2$$

$$x^2 - x = \frac{(9x-6)^2}{(11)^2} = \frac{(9x-6)(9x-6)}{121} = \frac{81x^2 - 108x + 36}{121}$$

$$(x^2 - x) \times 121 = 81x^2 - 108x + 36$$

$$121x^2 - 121x = 81x^2 - 108x + 36$$

$$121x^2 - 81x^2 - 121x + 108x = 36$$

$$40x^2 - 13x - 36 = 0$$

$$40x^2 - 45x + 32x - 36 = 0$$

$$(40x^2 - 45x) + (32x - 36) = 0$$

$$5x(8x-9) + 4(8x-9) = 0$$

$$(5x+4) = 0 \text{ or } 8x - 9 = 0$$ Solved

$$x = -\frac{4}{5} \text{ or } \frac{9}{8}$$ Haan Lymatics

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Que 14

Given that $\log_8 (p+2) + \log_8 q = r - \frac{1}{3}$ and $\log_2 (p-2) - \log_2 q = 2r+1$ Show that $p^2 = 4 + 32^r$. If $r = 1$, find the possible values of p and q

Solution

Proof: $\log_{2^3} (p+2) + \log_{2^3} q = r - \frac{1}{3}$

$\frac{1}{3}\log_2 (p+2) + \frac{1}{3}\log_2 q = r - \frac{1}{3}$

$\log_2 (p+2)^{1/3} + \log_2 (q)^{1/3} = r - \frac{1}{3}$

$\log_2 \left[(p+2)^{1/3} \cdot q^{1/3}\right] = r - \frac{1}{3}$

$\cancel{\log_2} \left[q^{1/3} (p+2)^{1/3}\right] = \cancel{\log_2} 2^{(r-1/3)}$

$q^{1/3} (p+2)^{1/3} = 2^{(r-1/3)}$ ----- (*)

* $\log_2 (p-2) - \log_2 q = 2r+1$

$\log_2 \left(\frac{p-2}{q}\right) = 2r+1$

$\cancel{\log_2} \left(\frac{p-2}{q}\right) = \cancel{\log_2} 2^{2r+1}$

$\frac{p-2}{q} = 2^{2r+1}$ ---- (*)(*)

$q^{1/3} = \frac{2^{(r-1/3)}}{(p+2)^{1/3}}$ -- from (*)

$(q^{1/3})^3 = \left(\frac{(2^{r-1/3})^3}{[(p+2)^{1/3}]^3}\right)$

$q = \frac{2^{3(r-1/3)}}{p+2}$ ---- (*)(*)(*)

Put ⊛⊛⊛ in ⊛⊛

$p-2 = q\left(2^{2r+1}\right)$

$p = 2 + q\left(2^{2r+1}\right)$

$p = 2 + \frac{2^{3r-1}}{p+2} \cdot 2^{2r+1}$

$p = 2 + \frac{2^{3r\cancel{-1}+2r\cancel{+1}}}{p+2}$

$p = 2 + \frac{2^{5r}}{p+2}$

$p = 2 + \frac{32^r}{p+2}$ multiply through by $p+2$

$p(p+2) = 2(p+2) + 32^r$

$p^2 + 2p = 2p + 4 + 32^r$

$p^2 \cancel{+2p} \cancel{-2p} = 4 + 32^r$

$\underline{p^2 = 4 + 32^r}$

Hence Proved

$*$ If $r = 1$

$p^2 = 4 + 32^r$

$p^2 = 4 + (32)^1$

$p^2 = 32 + 4$

$p^2 = 36$

$\sqrt{p^2} = \pm\sqrt{36}$

$p = \pm 6$

$\therefore p = \pm 6$

If $r = 1$

$q = \frac{2^{3r-1}}{p+2}$

$q = \frac{2^{3(1)-1}}{p+2}$

$q = \frac{2^{3-1}}{p+2}$

$q = \frac{2^2}{p+2}$

where $p = +6$ or -6

when $p = 6$

$q = \frac{4}{6+2} = \frac{4}{8}$

$q = \frac{1}{2}$

when $p = -6$

$q = \frac{2^2}{-6+2}$

$q = \frac{4}{-4}$

$q = -1$

$p = +6,\ q = \frac{1}{2}$

$p = -6,\ q = -1$

Solved

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Like i do → press

Que 15

14

(a) Solve the equations

$$(3x^2+2x)^2 - 9(3x^2+2x) + 8 = 0$$

$\Rightarrow$ Let $3x^2+2x = P$

$P^2 - 9P + 8 = 0$

$P^2 - 8P - P + 8 = 0$

$(P^2 - 8P) - (P+8) = 0$

$P(P-8) - 1(P-8) = 0$

$(P-1) = 0$ or $(P-8) = 0$

$P = 1$ or 8

Recall $3x^2+2x = P$

when $P = 1$

$3x^2+2x = 1$

$3x^2+2x-1 = 0$

$3x^2+3x-x-1 = 0$

$(3x^2+3x)-(x-1) = 0$

$3x(x+1) - 1(x+1) = 0$

$(3x-1)(x+1) = 0$

$3x-1 = 0$ or $x+1 = 0$

$\frac{3x}{3} = \frac{1}{3}$ $\quad x = 0-1$

$x = \frac{1}{3}$ or $x = 1$

when $P = 8$

$3x^2+2x = 8$

$3x^2+2x-8 = 0$

$3x^2+6x-4x-8 = 0$

$3x(x+2) - 2(x+2) = 0$

$(3x-2)(x+2) = 0$

$3x-2 = 0$ or $x+2 = 0$

$\frac{3x}{3} = \frac{2}{3}$ or $x = 0-2$

$x = \frac{2}{3}$ or $x = -2$

$\therefore x = \frac{1}{3}$ or 1 $\quad x = \frac{2}{3}$ or $x = -2$

Solved

Noah Lymatics

(ii) $x^2 + 2x - 2 = \frac{3}{x^2+2x}$

$x(x+2) - 2 = \frac{3}{x(x+2)}$

Let $x(x+2) = p$

$p - 2 = \frac{3}{p}$ multiply through by p

$p^2 - 2p = 3$

$p^2 - 2p - 3 = 0$

$p^2 - 3p + p - 3 = 0$

$p(p-3) + 1(p-3) = 0$

$(p+1)(p-3) = 0$

$p+1 = 0$ or $p-3 = 0$

$p = -1$ or 3

Recall that $x(x+2) = p$

when $p = -1$	when $p = 3$
$x(x+2) = -1$	$x(x+2) = 3$
$x^2+2x+1 = 0$	$x^2+2x-3 = 0$
$x^2+x+x+1 = 0$	$x^2+3x-x-3 = 0$
$x(x+1)+1(x+1) = 0$	$x(x+3)-1(x+3) = 0$
$(x+1)(x+1) = 0$	$(x-1)(x+3) = 0$
$x = -1$ twice	$x = +1$ or -3

$\therefore x = -1, -1, 1, -3$

Solved

~JoanLymatics

Que 16

Find a if $x^2 - 5x + 9 = 0$ has equal roots

Solution

By definition $\Rightarrow$ when $ax^2 + bx + c = 0$ has equal roots we have that $\Rightarrow$

$b^2 - 4ac = 0$ or $b^2 = 4ac$

from the Question given $x^2 - 5x + 9 = 0$

a b c

$a = 1,\ b = -5,\ c = 9$

* $b^2 - 4ac = 0$

$(-5)^2 - 4a(9) = 0$

$25 - \cancel{36}a = 0$

$25 - \cancel{36}a = 0$

$\frac{\cancel{36}a}{\cancel{36}} = \frac{\cancel{25}}{\cancel{36}}$ $\quad \frac{4a}{4} = \frac{25}{4}$

$\cancel{a = 0.69}$ $\quad a = 6.25$

Que 17

Q. Find the ranges of Value of K for which the equation $x^2+(k-3)x+k=0$ has roots of the same Sign.

Solution

Using $b^2-4ac \geq 0$

* $x^2+(k-3)x+k=0$

$\underset{a}{x^2} + \underset{b}{(k-3)x} + \underset{c}{k} = 0$

$a=1, \quad b=(k-3), \quad c=k$

$b^2-4ac \geq 0$

$(k-3)^2-(4.1.k) \geq 0$

$k^2-6k+9-4k \geq 0$

$k^2-10k+9 \geq 0$

$k^2-9k-k+9 \geq 0$

$k(k-9)-1(k-9) \geq 0$

$(k-1)(k-9) \geq 0$

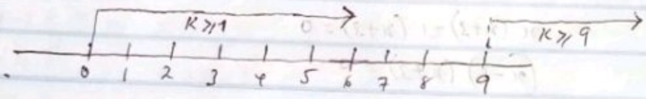
$(k-1) \geq 0$ or $k-9 \geq 0$

$k \geq 1$ or $k \geq 9$

$\therefore$ The Value of K has roots of the Same Sign

How to determine the range of Value of K $\Rightarrow$

$k \geq 1 \qquad k \geq 9$



* Note the Arrow must meet each other, which implies that i need to change the inequality Sign of $k > 9$ to $k \leq 9$

$\therefore k \geq 1, \quad k \leq 9$

$1 \leq k, \quad k \leq 9$

Range : $1 \leq K \leq 9$

$\therefore$ The range of Value for K $\Rightarrow$ $1 \leq K \leq 9$

Solved
YoanLymatics

Que 18

Find the Condition that must be Satisfied by P in order that the expression $2x^2+6x+1+K(x^2+2)=0$ may be Positive for all real Values of x.

Solution

$$2x^2+6x+1+K(x^2+2)=0$$
$$2x^2+6x+1+Kx^2+2K=0$$
$$2x^2+Kx^2+6x+1+2K=0$$
$$\underset{a}{(2+K)}x^2+\underset{b}{6}x+\underset{c}{(1+2K)}=0$$

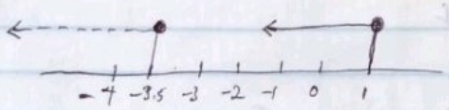
$a = 2+K$, $b=6$, $c=1+2K$

* The expression given may be Positive $\forall$ Real Values of x iff $b^2-4ac \geq 0$

$$b^2-4ac \geq 0$$
$$6^2-(4\times(2+K)\times(1+2K)) \geq 0$$
$$36-(4(2+4K+K+2K^2)) \geq 0$$
$$36-(4(2+5K+2K^2)) \geq 0$$
$$36-(8+20K+8K^2) \geq 0$$
$$36-8-20K-8K^2 \geq 0$$
$$0 \geq 8K^2+20K+8-36$$
$$0 \geq 8K^2+20K-28$$

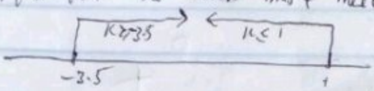
(By turning this Inequality Sideways then the sign will Change)

$$8K^2+20K-28 \leq 0$$
$$4(2K^2+5K-7) \leq 0$$
$$2K^2+5K-7 \leq \frac{0}{4}$$
$$2K^2+5K-7 \leq 0$$
$$2K^2+7K-2K-7 \leq 0$$
$$K(2K+7)-1(2K+7) \leq 0$$
$$(K-1)(2K+7) \leq 0$$
$$(K-1) \leq 0 \text{ or } 2K+7 \leq 0$$
$$K \leq 1 \text{ or } 2K \leq -7$$
$$K \leq 1 \text{ or } K \leq -7/2$$
$$K \leq 1 \text{ or } -3.5$$



Solved
Afoah Lymatics

* Note that the arrow must meet each other which makes $K \leq -3.5$ becomes $K \geq -3.5$



$K \geq -3.5$ or $K \leq 1$

$-3.5 \leq K$ a $K \leq 1$

$-3.5 \leq K \leq 1$ $\forall$ (+ve) Positive Values of x → $\boxed{K \Rightarrow -3.5, -3, -2, -1}$ $\boxed{K \Rightarrow 0, 1}$

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Que 19

The roots of the Quadratic equation $x^2 - px + q = 0$ are α and β form in terms of p and q, the quadratic equation whose roots are: $\alpha^3 - p\alpha^2$ and $\beta^3 - p\beta^2$

Solution

To Construct the quadratic equation we use $\Rightarrow$

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots})$$

Since the roots given are $\Rightarrow$ $\alpha^3 - p\alpha^2$, $\beta^3 - p\beta^2$ then the Sum & Product are shown below:

Sum of roots $\Rightarrow (\alpha^3 - p\alpha^2) + (\beta^3 - p\beta^2)$

$\Rightarrow \alpha^3 + \beta^3 - p\alpha^2 - p\beta^2$

$\Rightarrow \alpha^3 + \beta^3 - p(\alpha^2 + \beta^2)$

Note $\Rightarrow \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

& $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^3 + \beta^3 - p(\alpha^2 + \beta^2)$

$\Rightarrow (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) - p\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$ --- (*)

from the equation given above; $x^2 - px + q = 0$

$\downarrow$ a, $\downarrow$ b, $\downarrow$ c

$a = 1$, $b = -p$, $c = q$

But we believed that: $\alpha + \beta = \frac{-b}{a}$ $\quad \alpha\beta = c/a$

$\therefore \alpha + \beta = -\frac{(-p)}{1}$ $\quad \alpha\beta = \frac{q}{1}$

$\therefore$ Finally $\alpha + \beta = p$, $\alpha\beta = q$

On Substituting the value of $\alpha+\beta$ & $\alpha\beta$ in expression (*) above we have

Sum of roots $\Rightarrow (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) - p\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$

Sum of roots $\Rightarrow p^3 - 3q(p) - p\left[p^2 - 2q\right]$

$\Rightarrow \cancel{p^3} - 3pq - \cancel{p^3} + 2pq$

$\Rightarrow -3pq + 2pq = \underline{\underline{-pq}}$

$\therefore$ Sum of roots $\Rightarrow -(pq)$ --- (**)

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How to get the Product of d roots $\Rightarrow$

Product of roots $\Rightarrow (\alpha^3 - p\alpha^2) \cdot (\beta^3 - p\beta^2)$

$\Rightarrow \alpha^3\beta^3 - p\alpha^3\beta^2 - p\alpha^2\beta^3 + p^2\alpha^2\beta^2$

$\Rightarrow (\alpha\beta)^3 - p\alpha^2\beta^2(\alpha+\beta) + p^2(\alpha\beta)^2$

$\Rightarrow (\alpha\beta)^3 - p(\alpha\beta)^2(\alpha+\beta) + p^2(\alpha\beta)^2$

Recall that $\alpha+\beta = p$ & $\alpha\beta = q$

$\Rightarrow (\alpha\beta)^3 - p(\alpha\beta)^2(\alpha+\beta) + p^2(\alpha\beta)^2$

$\Rightarrow q^3 - p(q)^2(p) + p^2(q)^2$

$\Rightarrow q^3 - \cancel{p^2q^2} + \cancel{p^2q^2}$

$\Rightarrow q^3$

$\therefore$ Product of d roots $= q^3$

Recall that : $x^2 - (\text{Sum of roots})x + (\text{Product of roots})$

$\Rightarrow x^2 - (-(pq))x + (q^3)$

$x^2 + pqx + q^3$

Solved.

Afoan Lymatrics

Que 20 20

Find the quadratic equation with roots which exceed by 2 the roots of the quadratic equation $3x^2 - (p-4)x - (2p+1) = 0$

Solution

we believed that:

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots})$$

Since the roots is not given, now let the roots be α, β but the question says the roots exceed by 2

i.e the roots will become, $\alpha + 2$, $\beta + 2$

Sum of roots $\Rightarrow \alpha + 2 + \beta + 2$

$\Rightarrow (\alpha + \beta) + 4$

Let's consider the equation given: $\underset{a}{3x^2} - \underset{b}{(p-4)x} - \underset{c}{(2p+1)} = 0$

$a = 3x^2$, $b = -(p-4)$, $c = -(2p+1)$

but we believe that $\alpha + \beta = \frac{-b}{a}$ & $\alpha\beta = \frac{c}{a}$

$$\alpha + \beta = -\frac{[-(p-4)]}{3\cancel{x}} \qquad \alpha\beta = \frac{-(2p+1)}{3\cancel{x}}$$

$$\alpha + \beta = \frac{p-4}{3\cancel{x}}, \quad \alpha\beta = \frac{-(2p+1)}{3\cancel{x}}$$

On substituting into the expression we get in sum of roots

$$\text{Sum of roots} = (\alpha+\beta) + 4$$

$$= \frac{p-4}{3} + 4 \quad \text{-----} \quad (*)$$

$\odot$ Sum of roots $= \boxed{\frac{p-4}{3} + 4}$

Now to get the product of roots

Product of roots $\Rightarrow (\alpha + 2)(\beta + 2)$

$\Rightarrow \alpha\beta + 2\alpha + 2\beta + 4$

$\Rightarrow \alpha\beta + 2(\alpha+\beta) + 4$

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On substituting the value of $\alpha+\beta$, $\alpha\beta$ respectively.

Product of roots $\Rightarrow$ $\cancel{\alpha\beta+2\alpha}\ \alpha\beta + 2(\alpha+\beta) + 4$

$$\Rightarrow \frac{-(2p+1)}{3} + 2\left(\frac{p-4}{3}\right) + 4$$

$$\Rightarrow \frac{-(2p+1)}{3} + \frac{2(p-4)}{3} + 4$$

$$\Rightarrow \frac{-2p-1+(2(p-4))}{3} + 4$$

$$\Rightarrow \frac{-2p-1+(2p-8)}{3} + 4$$

$$\Rightarrow \frac{\cancel{-2p}-1\cancel{+2p}-8}{3} + 4$$

Product of roots $\Rightarrow$ $\frac{-7}{3} + 4 = \frac{-7+12}{3} \Rightarrow \frac{5}{3}$

Recall

$$x^2 - (\text{Sum of root})\,x + (\text{Product of roots})$$

$$x^2 - \left(\frac{p-4}{3} + 4\right)x + \left(\cancel{\frac{4p-25}{3x^2}}\ \frac{5}{3}\right)$$

$$\cancel{x^2 - \left(\frac{p-4+4x^2}{x^2}\right)x + \left(\frac{4p-25+12x^2}{3x^2}\right)}$$

$$\cancel{x^2 - \frac{4x^2+p-4}{x} + \frac{12x^2+4p-25}{3x^2} = 0}$$

$$x^2 - \left(\frac{p-4+12}{3}\right)x + \frac{5}{3} = 0 \Rightarrow \text{multiply through by } 3$$

$$\boxed{3x^2 - (p+8)x + 5 = 0}$$

Solved
Hoan Lymatics

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Que 21

If α and β are the roots of the equation $x^2 = \frac{x}{2} + 2$, find the equation where roots are $\Rightarrow \alpha - \frac{\beta}{\alpha}$ and $\beta - \frac{\alpha}{\beta}$

Solution

To find the equation for the roots;

$$x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$$

$$x^2 - \left[\left(\alpha - \frac{\beta}{\alpha}\right) + \left(\beta - \frac{\alpha}{\beta}\right)\right]x + \left[\left(\alpha - \frac{\beta}{\alpha}\right) \times \left(\beta - \frac{\alpha}{\beta}\right)\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{\alpha^2\beta - \beta^2 + \alpha\beta^2 - \alpha^2}{\alpha\beta}\right]x + \left[\alpha\beta - \frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha} + 1\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{\alpha^2\beta + \alpha\beta^2 - \alpha^2 - \beta^2}{\alpha\beta}\right]x + \left[\frac{1 - \alpha^3 - \beta^3 + \alpha\beta}{\alpha\beta}\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{\alpha\beta[\alpha+\beta] - [\alpha^2+\beta^2]}{\alpha\beta}\right]x + \left[\frac{1 - (\alpha^3+\beta^3) + \alpha\beta}{\alpha\beta}\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{\alpha\beta(\alpha+\beta) - ((\alpha+\beta)^2 - 2\alpha\beta)}{\alpha\beta}\right]x + \left[\frac{1 + \alpha\beta - ((\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta))}{\alpha\beta}\right] = 0$$

$\Rightarrow$ From the equation given $x^2 = \frac{x}{2} + 2$ let's multiply through by 2

$$2x^2 = x + 4$$

$$2x^2 - x - 4 = 0$$

$\therefore \alpha + \beta = \frac{-b}{a}$ $\qquad \alpha\beta = \frac{c}{a}$

$\alpha + \beta = -\frac{(-1)}{2}$ $\qquad \alpha\beta = -\frac{4}{2}$

$\alpha + \beta = \frac{1}{2}$ $\qquad \alpha\beta = -2$

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$$\Rightarrow x^2 - \left[\frac{\alpha\beta(\alpha+\beta) - [(\alpha+\beta)^2 - 2\alpha\beta]}{\alpha\beta}\right]x + \left[\frac{1+\alpha\beta - [(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)]}{\alpha\beta}\right] = 0$$

On substituting $\alpha+\beta = 1/2$ & $\alpha\beta = -2$ we get ;

$$\Rightarrow x^2 - \left[\frac{-2(1/2) - [(1/2)^2 - 2(-2)]}{-2}\right]x + \left[\frac{1+(-2) - [(1/2)^3 - 3(-2)(1/2)]}{-2}\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{-1 - [1/4 + 4]}{-2}\right]x + \left[\frac{1-2-[1/8+3]}{-2}\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{-1 - 17/4}{-2}\right]x + \left[\frac{-1 - 25/8}{-2}\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{-21/4}{-2}\right]x + \left[\frac{-33/8}{-2}\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{21}{4} \div 2\right]x + \left[\frac{33}{8} \div 2\right] = 0$$

$$\Rightarrow x^2 - \left[\frac{21}{4} \times \frac{1}{2}\right]x + \left[\frac{33}{8} \times \frac{1}{2}\right] = 0$$

$$\Rightarrow x^2 - \left(\frac{21}{8}\right)x + \left(\frac{33}{16}\right) = 0$$

$$\Rightarrow x^2 - \frac{21}{8}x + \frac{33}{16} = 0$$

multiplying through by 16

$$\Rightarrow 16x^2 - (21x \times 2) + 33 = 0$$

$$\Rightarrow 16x^2 - 42x + 33 = 0$$

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Que 22

Factorize $2x^4+7x^3-17x^2-7x+15$ Completely.

Solution

Putting $x = 1$; the expression gives zero $= 0$

$\therefore$ $x-1=0$ $\Rightarrow$ ie $x-1$ is a factor of the expression

Let's use the $2x^4+7x^3-17x^2-7x+15$ to divide $x-1$ Such that the remainder must give zero since the $x-1$ is a factor of the expression.

$$
\begin{array}{rl}
 & 2x^3+9x^2-8x-15 \\
x-1 & \overline{)\,2x^4+7x^3-17x^2-7x+15} \\
 & -2x^4+2x^3 \\
\hline
 & 9x^3-17x^2-7x+15 \\
 & -9x^3+9x^2 \\
\hline
 & -8x^2-7x+15 \\
 & +8x^2 \boxed{-} 8x \\
\hline
 & -15x+15 \\
 & +15x \boxed{-} 15 \\
\hline
 & 0
\end{array}
$$

The Quotient $\Rightarrow$ $2x^3+9x^2-8x-15$

Putting $x = -1$ the quotient becomes zero $= 0$

$\therefore$ $x+1 = 0$ $\Rightarrow$ ie $x+1$ is a factor of the Quotient above

Let's use $x+1$ to divide the Quotient again;

$$
\begin{array}{rl}
 & 2x^2-11x+3 \\
x+1 & \overline{)\,2x^3+9x^2-8x-15} \\
 & -2x^3 \boxed{+} 2x^2 \\
\hline
 & -11x^2-8x-15 \\
 & +11x^2+11x \\
\hline
 & 3x-15 \\
 & -3x \boxed{+} 3 \\
\hline
 & -18
\end{array}
\qquad
\begin{array}{rl}
 & 2x^2+7x-15 \\
x+1 & \overline{)\,2x^3+9x^2-8x-15} \\
 & -2x^3 \boxed{+} 2x^2 \\
\hline
 & 7x^2-8x-15 \\
 & -7x^2 \boxed{+} 7x \\
\hline
 & -15x-15 \\
 & +15x+15 \\
\hline
 & 0
\end{array}
$$

$\Rightarrow$ Quotient = Remainder × Divisor

$2x^2-11x+3 = (x+1) \times -18$

$2x^2-11x+3 = -18x-18$

$2x^2-11x+18x$

$\cancel{(\text{Quotient}) + (\text{Remainder} \times \text{divisor}) = 0}$

$\cancel{(2x^2-11x+3)+(-18\times(x+1)) = 0}$

$\cancel{2x^2-11x+3+(-18x-18) = 0}$

$\cancel{2x^2-11x+3-18x-18 = 0}$

$\cancel{2x^2 -}$

$\therefore$ Quotient $\Rightarrow 2x^2+7x-15$

$\Rightarrow 2x^2+10x-3x-15$

$\Rightarrow 2x(x+5)-3(x+5)$

$\Rightarrow (2x-3)(x+5)$

$\therefore (x-1)(x+1)(2x-3)(x+5) = 2x^4+7x^3-17x^2-7x+15$

L.H.S = R.H.S

$\Rightarrow (x^2-1)(2x-3)(x+5)$

Solved

ProbLyMaths

Que 23

The Polynomial $P(x) = x^3+ax^2+bx+c$ leaves remainder -36, -20, 0 on division by $(x+1)$, $(x+2)$, $(x+3)$ respectively. Solve the equation $P(x) = 0$

Solution

Put $(x+1) = 0$
$x = -1$

Since $P(x) = 0$ then

$x^3+ax^2+bx+c = 0$

but $x = -1$

$(-1)^3+a(-1)^2+b(-1)+c = 0$

$-1+a(1)-b+c = 0$

$\boxed{a-b+c = 1}$

Put $(x+2) = 0$
$x = -2$

$x^3+ax^2+bx+c = 0$

but $x = -2$

$(-2)^3+a(-2)^2+b(-2)+c = 0$

$-8+a(4)-2b+c = 0$

$\boxed{4a-2b+c = 8}$

and Put $(x+3) = 0$
$x = -3$

$x^3+ax^2+bx+c = 0$

but $x = -3$

$(-3)^3+a(-3)^2+b(-3)+c = 0$

$-27+a(9)-3b+c = 0$

$\boxed{9a-3b+c = 27}$

Solving it simultaneously we have

$\Rightarrow$ $a - b + c = 1$ --- (i)

$4a - 2b + c = 8$ --- (ii)

$9a - 3b + c = 27$ --- (iii)

eqn (ii) − eqn (i) we have and also eqn (iii) − eqn (ii)

$$\begin{array}{r} 4a - 2b + c = 8 \\ - \quad a - b + c = 1 \\ \hline 3a - b + 0 = 7 \end{array}$$

$\boxed{3a - b = 7}$ --- (iv)

$$\begin{array}{r} 9a - 3b + c = 27 \\ - \quad 4a - 2b + c = 8 \\ \hline 5a - b + 0 = 19 \end{array}$$

$\boxed{5a - b = 19}$ → (v)

$\Rightarrow$ eqn (v) − eqn (iv)

$$\begin{array}{r} 5a - b = 19 \quad \text{--- (v)} \\ - \quad 3a - b = 7 \quad \text{--- (iv)} \\ \hline 2a + 0 = 12 \end{array}$$

$2a = 12$

$\boxed{a = 6}$ ✓

Since $a = 6$

from eqn (iv)

$3a - b = 7$

$3(6) - b = 7$

$18 - b = 7$

$b = 18 - 7 = 11$

$\therefore \boxed{b = 11}$

$\Rightarrow$ Since $a = 6$, $b = 11$

from eqn (i)

$a - b + c = 1$

$6 - 11 + c = 1$

$c = 1 + 11 - 6$

$c = 12 - 6$

$\therefore c = 6$

$\therefore a = 6$, $b = 11$, $c = 6$

$P(x) = x^3 + ax^2 + bx + c$

$P(x) = x^3 + 6x^2 + 11x + 6$ Using factorization

$P(x) = x^3 + 6x^2 + 11x + 6$

Recall from the question $P(x) = 0$

$0 = x^3 + 6x^2 + 11x + 6$

$x^3 + 6x^2 + 11x + 6 = 0$

If $x = -1$ then $0 = 0$ i.e L.H.S = R.H.S

$x + 1 = 0$

Use $x+1$ to divide $x^3 + 6x^2 + 11x - 6$ Using long division

$$
\begin{array}{r l}
 & x^2 + 5x + 6 \longrightarrow \text{Quotient} \\
\text{divisor} \longleftarrow x+1 & \overline{)\,x^3 + 6x^2 + 11x + 6} \longrightarrow \text{dividend} \\
 & -x^3 \boxed{\mp} x^2 \\
 & \quad 5x^2 + 11x + 6 \\
 & \quad -5x^2 \boxed{\mp} 5x \\
 & \qquad 6x + 6 \\
 & \qquad -6x \boxed{\mp} 6 \\
 & \qquad\quad 0 \longrightarrow \text{Remainder.}
\end{array}
$$

Quotient $= x^2 + 5x + 6 \Rightarrow x^2 + 3x + 2x + 6$

$x(x+3) + 2(x+3)$

$(x+2)(x+3)$

$\therefore x^2 + 5x + 6 = (x+2)(x+3) =$ Quotient

But $\Rightarrow$ divisor $\times$ Quotient $=$ Dividend

i.e $(x+1)(x+2)(x+3) = x^3 + 6x^2 + 11x + 6$

Recall $x^3 + 6x^2 + 11x + 6 = 0$

$(x+1)(x+2)(x+3) = 0$

$(x+1) = 0$ or $x+2 = 0$ or $x+3 = 0$

$x = -1$ or $x = -2$ or $x = -3$

$\therefore x = -1$ or -2 or -3

Solved
Ican Lymstries

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Que 24 28

Find the Values of a and b which make the quadratic expression $x^4-2x^3+3x^2+ax+b$ a Perfect Square.

Solution

Since $x^4-2x^3+3x^2+ax+b$ is a Perfect Square then;
Let the Polynomial be the Square of px^2+qx+r Such that

$$(px^2+qx+r)^2 = x^4-2x^3+3x^2+ax+b$$

$(px^2+qx+r)(px^2+qx+r) \Rightarrow$

$p^2x^4+pqx^3+prx^2+pqx^3+q^2x^2+qrx+prx^2+qrx+r^2 \Rightarrow$

$p^2x^4+q^2x^2+r^2+2pqx^3+2prx^2+2qrx \Rightarrow$

$p^2x^4+q^2x^2+2prx^2+2pqx^3+2qrx+r^2 \Rightarrow$

$p^2x^4+2pqx^3+(q^2+2pr)x^2+(2qr)x+(r^2) \Rightarrow x^4-2x^3+3x^2+ax+b$

on relating

$p^2=1$, $2pq=-2$, $q^2+2pr=3$, $2qr=a$, $r^2=b$

$\sqrt{p^2}=\pm\sqrt{1}$ $\quad pq=-2/2$ $\quad (\pm1)^2+2(\pm1)(\pm1)=3$ $\quad 2(\pm1)(\pm1)=a$ $\quad (\pm1)^2=b$

$p=\pm1$ $\quad pq=-1$ $\quad (1)\pm 2r=3$ $\quad a=2$ $\quad b=1$

$(\pm1)q=-1$ $\quad \pm2r=3-1$

$q=\pm1$ $\quad \pm2r=2$

$r=\pm1$

$\therefore\ a=2$ and $b=1$

Que 25

Express $\dfrac{x^4-6x^2+3}{x(x+1)^2}$ in Partial Fractions

Solution

$$\frac{x^4-6x^2+3}{x(x+1)^2} = \frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

multiplying through by $x(x+1)^2$

$x^4-6x^2+3 = A(x+1)^2+Bx(x+1)+Cx$

Put $x=-1$

$(-1)^4-6(-1)^2+3 = 0+0-C$

$1-6+3=-C$

$C=6-3-1$

$C=2$

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$* \quad x^4 - 6x^2 + 3 = A(x+1)^2 + Bx(x+1) + Cx$

Put $x = 0$

$x^4 - 6x^2 + 3 = A(0+1)^2 + 0 + 0$

$0^4 - 6(0)^2 + 3 = A(1)^2$

$0 - 0 + 3 = A$

$\therefore A = \underline{3}$

$* \quad x^4 - 6x^2 + 3 = A(x+1)^2 + Bx(x+1) + Cx$

Put $x =$ Any Value except -1 so let's put $x = 1$

$1^4 - 6(1)^2 + 3 = A(1+1)^2 + B(1)(1+1) + C(1)$

$1 - 6 + 3 = A(2)^2 + B(2) + C$ but $A = 3$ and $C = 2$

$-2 = 3(2)^2 + B(2) + 2$

$-2 = 3(4) + 2B + 2$

$-2 = 12 + 2B + 2$

$2B = -12 - 2 - 2$

$2B = -16$

$B = -16/2$

$\therefore B = -8$

$\therefore A = 3, \ B = -8, \ C = 2$

Recall that

$$\frac{x^4 - 6x^2 + 3}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\therefore \frac{x^4 - 6x^2 + 3}{x(x+1)^2} = \frac{3}{x} + \frac{-8}{x+1} + \frac{2}{(x+1)^2}$$

$$\frac{x^4 - 6x^2 + 3}{x(x+1)^2} = \boxed{\frac{3}{x} - \frac{8}{x+1} + \frac{2}{(x+1)^2}}$$

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Que 26 30

Express $\frac{2x+1}{x^3-1}$ in Partial fraction.

Solution

Put $x^3-1 = 0$

Such that if $x = 1$ then the expression $x^3-1 = 0$

Since $x^3 = 1$

$x-1 = 0$

Now use $x-1$ to divide x^3-1 (using division of Polynomial

$$x^2+x+1 \longrightarrow \text{Quotient}$$

$$x-1 \,\big)\, x^3-1 \longrightarrow \text{dividend}$$

divisor

$$-x^3+x^2$$

$$x^2-1$$

$$-x^2+x$$

$$x-1$$

$$-x+1$$

$$0$$

$\therefore$ divisor $\times$ Quotient $=$ Dividend

$\therefore$ $(x-1)(x^2+x+1) = x^3-1$

Recall: $\frac{2x+1}{x^3-1} = \frac{2x+1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

multiply through by $(x-1)(x^2+x+1)$

$\rightarrow$ $2x+1 = A(x^2+x+1) + (Bx+C)(x-1)$

put $x = 1$

$2(1)+1 = A(1^2+1+1) + 0$

$2+1 = A(3)$

$3A = 3$

$A = 3/3 = 1$

$\therefore$ $A = 1$

* $2x+1 = A(x^2+x+1) + (Bx+C)(x-1)$

Put $x = 0$

$2(0)+1 = A(0^2+0+1) + (0+C)(-1)$

$1 = A - C$ Recall that $A = 1$

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$1 = A - C$

$C = A - 1$

$C = 1 - 1$

$C = \underline{0}$

$* \quad 2x + 1 = A(x^2+x+1) + (Bx+C)(x-1)$

put $x =$ Any Value except 0 and 1

$\therefore$ put $x = 2$

$2(2) + 1 = A(2^2+2+1) + (B(2)+C)(2-1)$

$4 + 1 = A(4+3) + (2B+C)(1)$

$5 = 7A + 2B + C$ but $C = 0$ and $A = 1$

$5 = 7(1) + 2B + 0$

$5 = 7 + 2B$

$2B = 5 - 7$

$2B = -2$

$B = -2/2$

$\therefore B = \underline{\underline{-1}}$

$\therefore \quad A = 1, \; B = -1, \; C = 0$

Recall $\dfrac{2x+1}{x^3-1} = \dfrac{2x+1}{(x-1)(x^2+x+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$

$= \dfrac{1}{x-1} + \dfrac{(-1)x+0}{x^2+x+1}$

$$\therefore \quad \frac{2x+1}{x^3-1} = \boxed{\frac{1}{x-1} - \frac{x}{x^2+x+1}}.$$

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Que 27

Express $\frac{1}{x^4+5x^2+6}$ in Partial Fraction.

Solution

$\frac{1}{x^4+5x^2+6} \longrightarrow$ Factorize ;

Let's use $3x^2+2x^2 = 5x^2$

$$\frac{1}{x^4+5x^2+6} = \frac{1}{x^4+3x^2+2x^2+6} = \frac{1}{x^2(x^2+3)+2(x^2+3)}$$

$$= \frac{1}{(x^2+2)(x^2+3)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{x^2+3}$$

multiply through by $(x^2+2)(x^2+3)$ -

$$\Rightarrow 1 = (Ax+B)(x^2+3) + (Cx+D)(x^2+2)$$

$$1 = Ax^3+3Ax+Bx^2+3B + Cx^3+2Cx+Dx^2+2D$$

$$1 = Ax^3+Cx^3+Bx^2+Dx^2+3Ax+2Cx+3B+2D$$

$$1 = (A+C)x^3+(B+D)x^2+(3A+2C)x+(3B+2D)$$

$$0x^3+0x^2+0x+1 = (A+C)x^3+(B+D)x^2+(3A+2C)x+(3B+2D)$$

On relating

$A+C=0$

$B+D=0$

$3A+2C=0$

$3B+2D=1$

$\rightarrow A = -C$

$\rightarrow 3(-C)+2C = 0$

$-3C+2C = 0$

$-C = 0$

$C = \underline{0}$

Since $C=0$

$A = -C$

$A = -(0) = 0$

$\therefore A = \underline{0}$

$*$ $B+D=0$, $B = -\underline{D}$

from $3B+2D=1$

$3(-D)+2D=1$

$-3D+2D=1$

$-D=1$

$D = \underline{-1}$

Since $D=-1$

$*$ $B = -D$

$B = -(-1)$

$B = \underline{1}$

$\therefore$ $A=0$, $B=1$, $C=0$, $D=-1$

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Recall that

$$\frac{1}{x^4+5x^2+6} = \frac{1}{(x^2+2)(x^2+3)} = \frac{Ax+B}{(x^2+2)} + \frac{Cx+D}{(x^2+3)}$$

= where $A=0$, $B=1$, $C=0$; $D=-1$

$$\Rightarrow \frac{(0)x+1}{x^2+2} + \frac{0(x)+(-1)}{x^2+3}$$

$$\Rightarrow \frac{1}{x^2+2} - \frac{1}{x^2+3}$$

$$\therefore \frac{1}{x^4+5x^2+6} = \boxed{\frac{1}{x^2+2} - \frac{1}{x^2+3}}$$

Que. 28

Prove by Induction that

$$1^3+2^3+3^3+\cdots+n^3 = \frac{1}{4}n^2(n+1)^2$$

* Put $n=1$

$$\Rightarrow 1^3+2^3+3^3+\cdots+1^3 = \frac{1}{4}(1)^2(1+1)^2$$

$$1^3 = \frac{1}{4}(1)(2)^2$$

$$1 = \frac{1}{\cancel{4}}\cdot\cancel{4}$$

$$1 = 1$$

L.H.S = R.H.S

* Put $n=k$

$$1^3+2^3+3^3+\cdots+k^3 = \frac{1}{4}k^2(k+1)^2 \quad \ldots (1)$$

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* Put $n = k+1$

$$1^3+2^3+3^3+\dots+n^3 = \frac{1}{4}n^2(n+1)^2$$

$$1^3+2^3+3^3+\dots+(k+1)^3 = \frac{1}{4}(k+1)^2((k+1)+1)^2$$

$$1^3+2^3+3^3+\dots+(k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2 \quad \text{---} \quad (\text{II})$$

Now let's Consider eqn (I) and (II)

$$1^3+2^3+3^3+\dots+k^3 = \frac{1}{4}k^2(k+1)^2 \quad \text{---} \quad (\text{I})$$

$$1^3+2^3+3^3+\dots+(k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2 \quad \text{---} \quad (\text{II})$$

On adding the two equations together, we need to show that

$$1^3+2^3+3^3+\dots+k^3+(k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2 = \text{when } n=k$$

$$1^3+2^3+3^3+\dots+k^3 = \frac{1}{4}(k+1)^2(k+2)^2-(k+1)^3$$

$$= \frac{1}{4}(k+1)^2\left[(k+2)^2-4(k+1)\right]$$

$$= \frac{1}{4}(k+1)^2\left[k^2+\cancel{4k}+\cancel{4}-\cancel{4k}-\cancel{4}\right]$$

$$1^3+2^3+3^3+\dots+k^3 = \frac{1}{4}(k+1)^2\left[k^2\right] = \text{when } n=k$$

$\therefore$ It is true for $n=k$ and $n=k+1$

Que 29

Prove that $7^{2n} + 16n - 1$ is divisible by 64 and by Induction.

Proof

Base Case, Show when $n = 1$

$7^{2n} + 16n - 1 = 7^{2(1)} + 16(1) - 1 \Rightarrow 7^2 + 16 - 1 \Rightarrow 49 + 15 \Rightarrow 64$

which is true for $n = 1$

Inductive Step

Show when $n = k$ and show when $n = k+1$ are both true.

For $n = k$

$7^{2n} + 16n - 1 \Rightarrow 7^{2k} + 16k - 1 = 64q$

$\frac{7^{2k} + 16k - 1}{64} = q$

also show for $n = k+1$ is also true

$7^{2n} + 16n - 1 \Rightarrow 7^{2(k+1)} + 16(k+1) - 1$

$\Rightarrow 7^{2k+2} + 16k + 16 - 1$

$\Rightarrow 7^{2k} \cdot 7^2 + 16k + 15$

$\Rightarrow 7^{2k} \cdot 49 + (784k - 768k) + 64 - 49$

$\Rightarrow 49(7^{2k}) + 784k - 49 - 768k + 64$

$\Rightarrow 49[7^{2k} + 16k - 1] - 768k + 64$

$\Rightarrow 49[64q] - 768k + 64$ Recall

$\Rightarrow 64q(49) - 768k + 64$

$\Rightarrow 64[49q - 12k + 1]$

Let $49q - 12k + 1 = r$

$\Rightarrow 64r$

This means that if $n = k+1$

$7^{2(k+1)} + 16(k+1) - 1 = 64r$

$\frac{7^{2(k+1)} + 16(k+1) - 1}{64} = r$ which is true for $n = k+1$

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Que 30

The Coefficient of x^5 in the expansion of $(1+5x)^8$ is equal to the coefficient of x^4 in the expansion of $(a+5x)^7$. find the value of a

Solution

By Definition we have that ;

$$(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + {}^nC_2 x^2 + {}^nC_3 x^3 + \cdots + {}^nC_{n-1} x^{n-1} + x^n$$

$$\Rightarrow (1+5x)^8 = {}^8C_0 (5x)^0 + {}^8C_1 (5x) + {}^8C_2 (5x)^2 + {}^8C_3 (5x)^3 + {}^8C_4 (5x)^4 + {}^8C_5 (5x)^5 + \cdots$$

The Coefficient of x^5 $\Rightarrow$ Using the 6th term of the sequence above

ie $${}^8C_5 (5x)^5 \Rightarrow \frac{8!}{(8-5)!\,5!} 5^5 \times x^5 \Rightarrow \frac{8!}{3!\,5!} (3125).\,x^5$$

$$\Rightarrow \frac{8 \times 7 \times 6 \times \cancel{5!}}{3!\,\cancel{5!}} (3125)\, x^5$$

$$\Rightarrow \frac{336 \times 3125}{3 \times 2 \times 1} x^5$$

$$\Rightarrow \frac{1050000}{6} x^5 \Rightarrow 175000\, x^5$$

The coefficient of x^4 [5th term].

$$^7C_4\, a^3 (5x)^4 = \frac{7!}{3!4!} \times a^3 \times 5^4 \times x^4$$

$$= \frac{7 \times 6 \times 5 \times \cancel{4!}}{3 \times 2 \times \cancel{4!}} \times a^3 \times 625 \times x^4$$

$$= 7 \times 5 \times 625 \times a^3 \times x^4 = \underline{21875\, a^3}\, x^4$$

The coefficients : $\frac{21875\, a^3}{21875} = \frac{175000}{21875}$

$$a^3 = 8$$

$$a^3 = 2^3$$

$$a = \underline{\underline{2}}$$

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Que 31.

Find the term Independent of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^6$.

Solution

We know that the General term of the expansion $(a+b)^n$ is

$$T_{r+1} = {}^nC_r\, a^{n-r} b^r$$

$$\Rightarrow (a+b)^n = \left(x^2 - \frac{2}{x}\right)^6$$ where $n=6$, $a = x^2$, $b = -\frac{2}{x}$

$$(a+b)^n = \left(x^2 + \left(-\frac{2}{x}\right)\right)^6$$

$$T_{r+1} = {}^nC_r\, a^{n-r} b^r$$

$$= {}^6C_r \left(x^2\right)^{6-r} \left(\frac{-2}{x}\right)^r$$

$$= {}^6C_r\, x^{12-2r} \cdot \left(-2 \cdot \frac{1}{x}\right)^r$$

$$= {}^6C_r\, x^{12-2r} \cdot (-2)^r \cdot \left(\frac{1}{x}\right)^r$$

$T_{r+1} = {}^6C_r\, x^{12-2r} \cdot (-2)^r \cdot x^{-r}$

$T_{r+1} = {}^6C_r\, x^{12-2r} \cdot x^{-r} \cdot (-2)^r$

$T_{r+1} = {}^6C_r\, x^{12-2r-r} \cdot (-2)^r$

$T_{r+1} = {}^6C_r\, x^{12-3r} \cdot (-2)^r$

$T_{r+1} = {}^6C_r\, (-2)^r\, x^{12-3r}$ ---- ($*$)

equate the Power of x to Zero

i.e $x^{12-3r} = x^0$

$\Rightarrow 12-3r = 0$

$3r = 12$

$r = 12/3 = 4$

$\therefore r = 4$

Since $r = 4$ we need to find the term independent of x

Recall from eqn ($*$)

$T_{r+1} = {}^6C_r\, (-2)^r\, x^{12-3r}$ since $r = 4$

$T_{4+1} = {}^6C_4\, (-2)^4\, x^{12-3(4)}$

$T_5 = {}^6C_4\, (-2)^4 \cdot x^{12-12}$

$T_5 = {}^6C_4\, (16)\, x^0$ where ${}^6C_4 = 15$

$T_5 = 15 \times 16 \times 1$

$T_5 = 240$

Hence the term which is independent of x is the 5th term i.e

$\underline{\underline{T_5 = 240}}$

Que 32

Find the remainder when $x^3 - 5x^2 + 6x - 2$ is divided by $(x-2)$

$$\begin{array}{r} x^2 - 3x \\ x-2 \,\overline{)\, x^3 - 5x^2 + 6x - 2} \\ -x^3 + 2x^2 \\ \hline -3x^2 + 6x - 2 \\ +3x^2 - 6x \\ \hline -2 \end{array}$$

Remainder $= -2$

Solved

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Que 33

Determine the Coefficient of x^3 in the binomial expansion of $(1-2x)^7$

Solution

We know that :-

$$(a+b)^n = {}^nC_r \, a^{n-r} b^r \; ; r \geq 0$$

$$(1-2x)^7 = (1+(-2x))^7 \Rightarrow \text{where } a=1 ; \; b=-2x , \; n=7$$

$$[1+(-2x)]^7 = {}^7C_0 \, 1^7(-2x)^0 + {}^7C_1 \, 1^6(-2x)^1 + {}^7C_2 \, 1^5(-2x)^2 + {}^7C_3 \, 1^4(-2x)^3$$

$$= 1 + {}^7C_1(-2x) + {}^7C_2(-2x)^2 + {}^7C_3(-2x)^3$$

$$= 1 + 7(-2x) + 21(-2x)^2 + 35(-2x)^3$$

$$= 1 - 14x + 21(4x^2) + 35(-8x^3)$$

$$= 1 - 14x + 84x^2 - 280x^3$$

$\therefore$ Coefficient of x^3 in the binomial expansion of $(1-2x)^7 = -280$

Que 34

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Que 34

How many terms of the Series $-9, -6, -3, \ldots$ may be taken so that their sum is 66?

Solution

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

where $d = -6-(-9) = -3-(-6)$
$= -6+9 = -3+6$
$d = 3 = 3$
$\therefore d = 3$

Sum is 66 $\Rightarrow \frac{n}{2}\left(2 \times (-9) + (n-1)(3)\right)$

$$66 = \frac{n}{2}\left(-18 + 3n - 3\right)$$

$$2 \times 66 = \frac{n}{2}\left(-18 + 3n - 3\right) \times 2$$

$$132 = -18n + 3n^2 - 3n$$

$$3n^2 - 21n - 132 = 0$$

$$3(n^2 - 7n - 44) = 0$$

$$n^2 - 7n - 44 = 0/3$$

$$\therefore n^2 - 7n - 44 = 0$$

$$n^2 - 11n + 4n - 44 = 0$$

$$n(n-11) + 4(n-11) = 0$$

$$(n+4) = 0 \text{ or } (n-11) = 0$$

$\therefore n = -4$ or 11 $\quad\therefore$ 11 terms of the Series will be taken

(C)

Que 35

The expansion of $(x+y)^0$ is

Answer $\Rightarrow (x+y)^0$

$\Rightarrow 1$

(C)

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Que 36

Convert $y = \frac{1}{2}\left[(x+4)^2 + (x-2)^2\right]$ to the form $y = (x+p)^2 + q$ and hence find the minimum Value of y and the Value of x where this occur

Solution

$$y = \frac{1}{2}\left[(x+4)^2 + (x-2)^2\right]$$

$$y = \frac{1}{2}\left[x^2+8x+16 + x^2-4x+4\right]$$

$$y = \frac{1}{2}\left[2x^2+4x+20\right]$$

$$y = \frac{1}{\cancel{2}}\cdot\cancel{2}\left(x^2+2x+10\right)$$

$$y = x^2+2x+10$$

* take half of the Coefficient of x i.e $\frac{1}{2}\cdot 2 = \frac{1}{\cancel{2}}\times\cancel{2} = 1$
* Square the result $\Rightarrow (1)^2 \Rightarrow 1$
* Add and Subtract the result on the R.H.S of the equation above

$$y = x^2+2x+10+1-1$$

$$y = x^2+2x+1+10-1$$

$$y = (x^2+2x+1)+9$$

$$y = (x+1)^2+9 \quad \text{In form of} \quad y = (x+p)^2+q$$

where $p = \boxed{1}$ and $q = \boxed{9}$

Hence To get the minimum Value of y:

$$\frac{dy}{dx} = 0$$

$$y = (x+1)^2+9$$

$$\frac{dy}{dx} = \frac{d}{dx}(x+1)^2 + \frac{d}{dx}(9)$$

$$\frac{dy}{dx} = 2(x+1)+0$$

$$\frac{dy}{dx} = 2x+2 \quad \text{but} \quad \frac{dy}{dx} = 0 \text{ at minimum}$$

$$2x+2 = 0$$

$$2(x+1) = 0$$

$$x+1 = 0 \quad \therefore x = -1$$ {Value of x where it occur}

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Recall that $y = (x+1)^2 + 9$

minimum Value of $y = (-1+1)^2 + 9$

$y = 0^2 + 9$

$\therefore y = 9$ (minimum Value)

Que 37

Express the function $8+2x-x^2$ as a Sum or difference of two Squares one of which is independent of x. Deduce the maximum or minimum Value of the function

Soln

$\Rightarrow 8+2x-x^2$

$\Rightarrow -(-8+x^2-2x)$

$-(x^2-2x-8)$

$\Rightarrow -(x^2-4x+2x-8)$

$\Rightarrow -[x(x-4)+2(x-4)]$

$\Rightarrow -[(x+2)(x-4)]$

$\Rightarrow -(x+2)(x-4)$ where $(x-4) \Rightarrow (\sqrt{x}-2)(\sqrt{x}+2)$

$\Rightarrow -(x+2)[(\sqrt{x}-2)(\sqrt{x}+2)]$

Let $y = -(x+2)[(\sqrt{x}-2)(\sqrt{x}+2)]$

To determine the minimum or maximum Value of the function then $dy/dx = 0$

$y = -(x+2)(x-4) = -8+2x-x^2$

$y = -8+2x-x^2$

$\frac{dy}{dx} = \frac{d}{dx}(-8) + \frac{d}{dx}(2x) - \frac{d}{dx}(x^2)$

$\frac{dy}{dx} = 0 + 2 - 2x$

$\frac{dy}{dx} = 2(1-x)$ but $\frac{dy}{dx} = 0$

$2(1-x) = 0$

$(1-x) = 0/2$

$1-x = 0$

$x = 1$

Since $x = 1$

$y = -8+2x-x^2$

$y = -8+2(1)-1^2$

$y = -8+2-1$

$y = -7$ (maximum Value Function)

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Question 37

$8+2x-x^2$

$= -x^2+2x+8$

$= -(x^2-2x-8)$

$= -(x^2-2x+(-1)^2-8+(-1)^2)$

$= -(x^2-2x+(-1)^2-8-1)$

$= -((x-1)^2-9)$

$= 9-(x-1)^2 = 3^2-(x-1)^2$

To determine of minimum or maximum Value

$\frac{dy}{dx} = 0$

$y = 8+2x-x^2$

$\frac{dy}{dx} = 2-2x$

when $\frac{dy}{dx} = 0$, $2-2x = 0$

$\frac{2x}{2} = \frac{2}{2}$

$x = \underline{\underline{1}}$

when $x=1$

$y = 8+2(1)-(1)^2$

$= 8+2-1$

$= 9$

Maximum value $y = 9$

$x = 1$

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Que 38

Given that $p = \sqrt{2}$ and $q = \sqrt{3}$, express $\dfrac{\sqrt{50} - \sqrt{12}}{\sqrt{8} + \sqrt{75}}$ in terms of p and q as simple as possible

Solution

$$\frac{\sqrt{50} - \sqrt{12}}{\sqrt{8} + \sqrt{75}}$$

$$\Rightarrow \frac{\sqrt{25\times2} - \sqrt{4\times3}}{\sqrt{4\times2} + \sqrt{25\times3}} = \frac{5\sqrt{2} - 2\sqrt{3}}{2\sqrt{2} + 5\sqrt{3}} \text{ where}$$

$$\Rightarrow \frac{(5\sqrt{2} - 2\sqrt{3})}{2\sqrt{2} + 5\sqrt{3}} \times \frac{(2\sqrt{2} - 5\sqrt{3})}{2\sqrt{2} - 5\sqrt{3}}$$

$$\Rightarrow \frac{10\sqrt{4} - 25\sqrt{6} - 4\sqrt{6} + 10\sqrt{9}}{4\sqrt{4} - 25\sqrt{9}} = \frac{10(2) - 29\sqrt{6} + 10(3)}{4(2) - 25(3)}$$

$$\Rightarrow \frac{20 - 29(\sqrt{3}\times\sqrt{2}) + 30}{8 - 75} \quad \text{where } p = \sqrt{2} \text{ and } q = \sqrt{3}$$

$$\Rightarrow \frac{20 - 29(q\times p) + 30}{-67} = \frac{20 + 30 - 29(pq)}{-67}$$

$$\Rightarrow \frac{50 - 29pq}{-67}$$

$$\Rightarrow -\frac{(50 - 29pq)}{67}$$

$$\Rightarrow \frac{29pq - 50}{67}$$

Que 39

Que 39

If $A = \{1,2,3,4,5\}$, $B = \{3,4,5,6\}$ and $C = \{4,5,6,7\}$ then $(A \cup B) - C$ is ?

Solution

$A = \{1,2,3,4,5\}$ $\quad C = \{4,5,6,7\}$

$B = \{3,4,5,6\}$

$\Rightarrow A \cup B = \{1,2,3,4,5,6,7\}$

$\therefore (A \cup B) - C \Rightarrow \{1,2,3,4,5,6,7\} - \{\overbrace{4,5,6,7}^{\text{deleting is left}}\}$

$\therefore (A \cup B) - C = \{1,2,3\} \Rightarrow \{1,2,3\}$

(C)

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~~On solving $a^2+b^2=2$, $2a=-3$, $b=1$~~

Que 40

Simplify (i) $\dfrac{27^{n+2} - 6\cdot 3^{3n+3}}{3^n \cdot 9^{n+2}}$ (ii) $\dfrac{\left(x^{3/2}+x^{1/2}\right)\left(x^{1/2}-x^{-1/2}\right)}{\left(x^{2/3}-x^{1/2}\right)^2}$

Solution

(i) $\dfrac{27^{n+2} - 6\cdot 3^{3n+3}}{3^n \cdot 9^{n+2}} \Rightarrow \dfrac{3^{3(n+2)} - 6\cdot 3^{3n+3}}{3^n \cdot 3^{2(n+2)}}$

$\Rightarrow \dfrac{3^{3n+6} - 6\cdot 3^{3n+3}}{3^n \cdot 3^{2n+2}} \Rightarrow \dfrac{3^{3n}\cdot 3^6 - 6\cdot 3^{3n}\cdot 3^3}{3^{n+2n+2}}$

$\Rightarrow \dfrac{3^{3n}\cdot 3^6 - 6\cdot 3^{3n}\cdot 3^3}{3^{3n+2}} \Rightarrow \dfrac{\cancel{3^{3n}}\left[3^6 - 6\cdot 3^3\right]}{\cancel{3^{3n}}\cdot 3^2}$

$\Rightarrow \dfrac{3^6 - 6\cdot 3^3}{3^2} \Rightarrow \dfrac{\cancel{3^2}\left[3^4 - 6\cdot(1)\right]}{\cancel{3^2}}$

$\Rightarrow 3^4 - 6$

$\Rightarrow 81 - 6$

$= \underline{75}$

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Que 40 (ii)

Simplify

(ii) $\dfrac{\left(x^{3/2}+x^{1/2}\right)\left(x^{1/2}-x^{-1/2}\right)}{\left(x^{3/2}-x^{1/2}\right)^2}$

$\Rightarrow \left(x^{1/2}\left[\dfrac{x^{3/2}}{x^{1/2}}+\dfrac{x^{1/2}}{x^{1/2}}\right]\cdot x^{\frac{1}{2}}\left[\dfrac{x^{1/2}}{x^{1/2}}-\dfrac{x^{-1/2}}{x^{1/2}}\right]\right)\div\left(x^{\frac{3}{2}}-x^{\frac{1}{2}}\right)^2$

$\Rightarrow \dfrac{x^{\frac{1}{2}}\left[x^{(3/2-1/2)}+1\right]\cdot x^{\frac{1}{2}}\left[1-x^{-1/2-\frac{1}{2}}\right]}{\left(x^{\frac{1}{2}}\left[\dfrac{x^{3/2}}{x^{1/2}}-\dfrac{x^{1/2}}{x^{1/2}}\right]\right)^2}$

$= \dfrac{x^{1/2}\left[x+1\right]\cdot x^{\frac{1}{2}}\left[1-x^{-1}\right]}{\left(x^{1/2}\right)^2\left[x-1\right]^2}$

$\Rightarrow \dfrac{x^{\frac{1}{2}}\cdot x^{1/2}(x+1)\left(1-\frac{1}{x}\right)}{x(x-1)^2} \Rightarrow \dfrac{x^{\left(\frac{1}{2}+\frac{1}{2}\right)}(x+1)\left(\frac{x-1}{x}\right)}{x(x-1)^2}$

$\Rightarrow \dfrac{\cancel{x}(x+1)\left(\frac{x-1}{x}\right)}{\cancel{x}(x-1)^2} \Rightarrow \dfrac{\left[\dfrac{(x+1)(x-1)}{x}\right]}{(x-1)^2}$

$\Rightarrow \dfrac{(x+1)(x-1)}{x}\div(x-1)^2 \Rightarrow \dfrac{(x+1)\cancel{(x-1)}}{x}\times\dfrac{1}{\cancel{(x-1)^2}}$

$\Rightarrow \dfrac{(x+1)}{x(x-1)}$

$\Rightarrow \dfrac{x+1}{x(x-1)}$

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Que 41

Given that $\log_x U + \log_x V = P$ and

$\log_x U - \log_x V = q$ Prove that $U = x^{(p+q)}$

and find a similar expression for V

Solution

$\log_x U + \log_x V = P$ ---- (i)

$\log_x U - \log_x V = q$ ---- (ii)

On adding the two equations

$\log_x U + \log_x U + \log_x V + (-\log_x V) = P + q$

$\log_x (u \times u) + \cancel{\log_x V} - \cancel{\log_x V} = P + q$

$\log_x U^2 = P + q$ $P+q$ is equivalent to $\log_x x^{P+q}$

or

$\log_x U^2 = (P+q) \log_x x$ Since $\log_x x = 1$

$\cancel{\log_x} U^2 = \cancel{\log_x} x^{P+q}$

$U^2 = x^{P+q}$

$U^{2 \times 1/2} = x^{(P+q) \cdot 1/2}$

$U = x^{\left(\frac{P+q}{2}\right)}$ or $U = \sqrt{x^{(P+q)}}$

Now to find a similar expression for V

from eqn (i)

$\log_x U + \log_x V = P$

Since $U = x^{\frac{(P+q)}{2}}$

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$$\log_x U + \log_x V = P$$

$$\log_x x^{\left(\frac{p+q}{2}\right)} + \log_x V = P$$

$$\log_x \left[x^{\left(\frac{p+q}{2}\right)} \cdot V \right] = P$$

$$\log_x \left[V x^{\left(\frac{p+q}{2}\right)} \right] = P \cdot \log_x x$$ Since $\log_x x = 1$

$$\cancel{\log_x} \left[V x^{\left(\frac{p+q}{2}\right)} \right] = \cancel{\log_x} x^p$$

$$V x^{\frac{p+q}{2}} = x^p$$

$$\frac{V \cancel{x^{\left(\frac{p+q}{2}\right)}}}{\cancel{x^{\left(\frac{p+q}{2}\right)}}} = \frac{x^p}{x^{\left(\frac{p+q}{2}\right)}}$$

$$V = x^p \div x^{\frac{p+q}{2}}$$

$$V = x^{p - \frac{p+q}{2}}$$

$$V = x^{\frac{2p-(p+q)}{2}}$$

$$V = x^{\frac{2p-p-q}{2}}$$

$$\therefore V = x^{\frac{p-q}{2}}$$

Solved
FoanLymathics

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Que 42

The Value Of the Product of $\log_2 3 \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8$

Solution

$\log_2 3 \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8$

* Using $\log_b a = \dfrac{\log_{10} a}{\log_{10} b}$

$$\frac{\cancel{\log_{10} 3}}{\log_{10} 2} \times \frac{\cancel{\log_{10} 4}}{\cancel{\log_{10} 3}} \times \frac{\cancel{\log_{10} 5}}{\cancel{\log_{10} 4}} \times \frac{\cancel{\log_{10} 6}}{\cancel{\log_{10} 5}} \times \frac{\cancel{\log_{10} 7}}{\cancel{\log_{10} 6}} \times \frac{\log_{10} 8}{\cancel{\log_{10} 7}}$$

$$\Rightarrow \frac{\log_{10} 8}{\log_{10} 2} = \frac{\log_{10} 2^3}{\log_{10} 2^1} \Rightarrow \frac{3 \log_{10} 2}{1 \log_{10} 2} \Rightarrow \frac{3\,\cancel{(\log_{10} 2)}}{1\,\cancel{(\log_{10} 2)}}$$

$$\Rightarrow \frac{3}{1} = \underline{\underline{3}}$$

Que 43

The Value of $\begin{vmatrix} a & a-c & a-b \\ b & c+a & b-a \\ c & c-a & a+b \end{vmatrix}$ is

Solution

$$\begin{vmatrix} + & - & + \\ a & a-c & a-b \\ b & c+a & b-a \\ c & c-a & a+b \end{vmatrix}$$

$$= +a \begin{vmatrix} c+a & b-a \\ c-a & a+b \end{vmatrix} - (a-c) \begin{vmatrix} b & b-a \\ c & a+b \end{vmatrix} + (a-b) \begin{vmatrix} b & c+a \\ c & c-a \end{vmatrix}$$

$$= a\Big((c+a)(a+b) - (b-a)(c-a)\Big) - (a-c)\Big(b(a+b) - (b-a)c\Big)$$

$$+ (a-b)\Big(b(c-a) - (c+a)c\Big)$$

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$= a\left[ac+\cancel{bc}+\cancel{a^2}+ab-\cancel{bc}+ab+ac-\cancel{a^2}\right]-(a-c)\left[ab+b^2-bc+ac\right]$
$+(a-b)\left[bc-ab-c^2+ac\right]$

$= a\left[2ac+2ab\right]-(a-c)\left[b^2+ab+ac-bc\right]+(a-b)\left[-c^2-ab+bc+ac\right]$

$\Rightarrow 2a^2c+2a^2b-ab^2-a^2b-a^2c+abc+b^2c+abc+ac^2-bc^2$
$-ac^2-a^2b+abc+a^2c+bc^2+ab^2-b^2c+abc$

$$\boxed{\Rightarrow(2a^2c-a^2c-a^2c)+(2a^2b-a^2b-a^2b)-ab^2+ab^2-a^2b-a^2c+a^2c+abc+abc+abc+abc+b^2c-b^2c+ac^2-ac^2-bc^2+bc^2}$$

$= abc+abc+abc+abc$

$= \underline{\underline{4abc}}$ (D)

Que 44

If $\log(x^3y^3)=a$ and $\log\left(\frac{x}{y}\right)=b$, then $\log x$ is

Solution

$\log x^3y^3=a$, $\log\left(\frac{x}{y}\right)=b$

$\log x^3+\log y^3=a$ $\qquad \log x-\log y=b$

$\log x^3=a-\log y^3$ $\qquad \log y=\log x-b$

$3\log x=a-\log y^3$

$\boxed{\log x=\frac{a-\log y^3}{3}}$

$\log x=\frac{a-3\log y}{3}$

$\log x=\frac{a-3(\log x-b)}{3}$

$\log x=\frac{a-3\log x+3b}{3} \Rightarrow 3\log x=a-3\log x+3b$

$3\log x+3\log x=a+3b$

$\log x(3+3)=a+3b$

$6\log x=a+3b$

$\log x=\frac{a+3b}{6}$ — None

$\therefore \log x=\underline{\underline{\frac{1}{6}(a+3b)}}$

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Que 45

If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}$ find a matrix C such that $(A+B) + C = 0$

Soln

Since $(A+B) + C = 0$

then $C = -(A+B)$

where $A + B = \overset{A}{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}} + \overset{B}{\begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}}$

$A+B = \begin{pmatrix} 5 & 7 \\ 9 & 11 \end{pmatrix}$

Recall $C = -(A+B) = -\begin{pmatrix} 5 & 7 \\ 9 & 11 \end{pmatrix}$

$\therefore$ (M)ATRIX $C = \begin{pmatrix} -5 & -7 \\ -9 & -11 \end{pmatrix}$ (A)

Que 46

If $A = \begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}$ find $A^2 + 3A + 5I$. where I is the Unit Vector of order 2

Solution

Since I is the Unit Vector of Order 2 then $I = \begin{pmatrix} 1/5 & 0 \\ 0 & 1/5 \end{pmatrix}$

$A^2 + 3A + 5I = \overset{A^2}{\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}} + \overset{3A}{3\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}} + \overset{5I}{5\begin{pmatrix} 1/5 & 0 \\ 0 & 1/5 \end{pmatrix}}$

$= \begin{pmatrix} (1-6) & (2+0) \\ (-3+0) & (-6+0) \end{pmatrix} + \begin{pmatrix} 3 & 6 \\ -9 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} -5 & 2 \\ -3 & -6 \end{pmatrix} + \begin{pmatrix} 3 & 6 \\ -9 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} -2 & 8 \\ -12 & -6 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -2+1 & 8+0 \\ -12+0 & -6+1 \end{pmatrix}$

$= \begin{pmatrix} -1 & 8 \\ -12 & -5 \end{pmatrix}$ (D)

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Question 46

$A = \begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^2 + 3A + 5I$

$$= \begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix} + 3\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-6 & 2+0 \\ -3+0 & -6+0 \end{pmatrix} + \begin{pmatrix} 3 & 6 \\ -9 & 0 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 2 \\ -3 & -6 \end{pmatrix} + \begin{pmatrix} 3 & 6 \\ -9 & 0 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 8 \\ -12 & -6 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 8 \\ -12 & -1 \end{pmatrix}$$

Question 49

$$\left(\frac{1-i}{1+i}\right)^2 = \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^2 = \left(\frac{1-2i+i^2}{1-\cancel{i}+\cancel{i}-i^2}\right)^2$$

But $i^2 = -1$

$$\left(\frac{\cancel{1}-2i-\cancel{1}}{1+1}\right)^2 = \left(\frac{-2i}{2}\right)^2 = (-i)^2 = (-i)(-i)$$

$$= i^2 = 1.$$

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Que 47

The Inverse of the matrix $\begin{pmatrix} -1 & 2 \\ 4 & 6 \end{pmatrix}$ is

Solution

Let $A = \begin{pmatrix} -1 & 2 \\ 4 & 6 \end{pmatrix}$ Such that $A^{-1} = \frac{1}{\det A}(\text{Adjoint } A)$

$A^{-1} = \frac{1}{\begin{vmatrix} -1 & 2 \\ 4 & 6 \end{vmatrix}} \times \begin{bmatrix} 6 & -2 \\ -4 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{(-6-8)}\begin{pmatrix} 6 & -2 \\ -4 & -1 \end{pmatrix} = \frac{1}{-14}\begin{pmatrix} 6 & -2 \\ -4 & -1 \end{pmatrix}$

$A^{-1} = -\frac{1}{14}\begin{pmatrix} 6 & -2 \\ -4 & -1 \end{pmatrix}$ OR $\begin{pmatrix} -6/14 & -2/14 \\ +4/14 & +1/14 \end{pmatrix}$

OR

$A^{-1} = \begin{pmatrix} -3/7 & -1/7 \\ 2/7 & 1/14 \end{pmatrix}$ — None (D)

Que 48

If α and β are the roots of $2x^2+3x+7=0$, find the Value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

Solution

$\Rightarrow \frac{\alpha}{\beta}+\frac{\beta}{\alpha} \Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta} \Rightarrow$ where $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$; from $2x^2+3x+7=0$

$\alpha+\beta = -b/a \quad \alpha\beta = c/a$

$\alpha+\beta = -3/2 \quad \alpha\beta = 7/2$

$\Rightarrow \frac{\left(\frac{-3}{2}\right)^2 - \cancel{2}\left(\frac{7}{\cancel{2}}\right)}{7/2} \Rightarrow \frac{9/4 - 7}{7/2} \Rightarrow \frac{\frac{9-28}{4}}{7/2} \Rightarrow \frac{\frac{-19}{4}}{7/2}$

$\Rightarrow \frac{-19}{4} \div \frac{7}{2} \Rightarrow \frac{-19}{\cancel{4}} \times \frac{\cancel{2}}{7} \Rightarrow \frac{-19}{14}$

$\Rightarrow \frac{-19}{14}$ — (D)

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$\frac{}{(1+i)(1+i)}$ $1+2i+i^2$

Que 49

In the form of $a+ib$ $\left(\frac{1-i}{1+i}\right)^2$ is

Solution

$$\left(\frac{1-i}{1+i}\right)^2 \Rightarrow \left[\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right]^2 \Rightarrow \left[\frac{1-2i+i^2}{1-\cancel{i}+\cancel{i}-i^2}\right]^2$$

$$\Rightarrow \left[\frac{1-2i+(-1)}{1-i^2}\right]^2 \quad \text{where } i^2 = \left(\sqrt{(-1)}\right)^2 \Rightarrow \left((-1)^{1/\cancel{2}}\right)^{\cancel{2}} \Rightarrow -1$$

$$\Rightarrow \left[\frac{\cancel{1}-2i\cancel{-1}}{1-(-1)}\right]^2 = \left[\frac{-2i}{1+1}\right]^2 = \left[\frac{-\cancel{2}i}{\cancel{2}}\right]^2$$

$$\Rightarrow (-i)^2 \Rightarrow i$$

$\therefore$ $a+ib \Rightarrow 0+(i)(1)$ $\therefore$ $a=0,\ b=1$ (B)

Que 50

Simplify $(\sqrt{3}+\sqrt{2})^3+(\sqrt{3}-\sqrt{2})^3$

Solution

$$(\sqrt{3}+\sqrt{2})^3+(\sqrt{3}-\sqrt{2})^3$$

let $(\alpha)^3$ $(\beta)^3$ $\Rightarrow \alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$$= \left(\sqrt{3}\cancel{+\sqrt{2}}+\sqrt{3}\cancel{-\sqrt{2}}\right)^3 - 3(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})\left(\sqrt{3}\cancel{+\sqrt{2}}+\sqrt{3}\cancel{-\sqrt{2}}\right)$$

$$\Rightarrow (2\sqrt{3})^3 - 3\left(3-\cancel{\sqrt{6}}+\cancel{\sqrt{6}}-2\right)(2\sqrt{3})$$

$\Rightarrow (2\sqrt{3})^3 - 3(3-2)(2\sqrt{3})$

$\Rightarrow (2\sqrt{3})^3 - 3(1)(2\sqrt{3})$

$\Rightarrow 2^3(\sqrt{3})^3 - 3(2\sqrt{3})$

$\Rightarrow 8(\sqrt{3})^3 - 6\sqrt{3}$

$\Rightarrow 2\sqrt{3}\left(4(\sqrt{3})^2 - 3\right)$

$\Rightarrow 2\sqrt{3}\left(4(3) - 3\right)$

$\Rightarrow 2\sqrt{3}(12-3)$

$\Rightarrow 2\sqrt{3}\,(9)$

$\Rightarrow \underline{18\sqrt{3}}$ ✓ B

Que 51

The Coefficient of x^6 in the expansion of $(x+2)^9$ is

Solution

$(x+2)^9 \Rightarrow (a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$ where $r = 0, 1, 2, \ldots n$

but $a = x$, $b = 2$

$= {}^9C_0\, x^9 (2)^0 + {}^9C_1\, x^{9-1}(2)^1 + {}^9C_2\, x^{9-2}(2)^2 + {}^9C_3\, x^{9-3}(2)^3 + \cdots + {}^9C_9\, x^0 (2)^9$

$\Rightarrow {}^9C_0\, x^9 + {}^9C_1\, x^8(2) + {}^9C_2\, x^7(2)^2 + {}^9C_3\, x^6(2)^3$

Using this

$${}^9C_3\, x^6 (8)$$

$$= \frac{9!}{(9-3)!\,3!} \cdot 8x^6$$

$$= \frac{9!}{6!\,3!} \cdot 8x^6 = \frac{9\times8\times7\times6!}{6!\,3!} \cdot 8x^6$$

$$= \frac{9\times8\times7}{3\times2\times1} \cdot 8x^6 \Rightarrow 3\times8\times7\times4\, x^6$$

$$\Rightarrow 672\, x^6$$

Coefficient of $x^6 \Rightarrow \underline{672}$ ✓ done

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Que 52

If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$ and $B = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

then the Product BA is ?

Solution

$$BA = \overset{B}{\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}} \times \overset{A}{\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}}$$

$$= \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & \cancel{\sin\alpha\cos\alpha} - \sin\alpha\cos\alpha \\ \cancel{\sin\alpha\cos\alpha - \sin\alpha\cos\alpha} & \sin^2\alpha + \cos^2\alpha \end{pmatrix}$$

From MAT 111 (TRIG) $\sin^2\alpha + \cos^2\alpha = 1$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (C)$$

Que 53

Solve the equation $2^{3n+1} = 5^{n+1}$

Solution

Take Log of both sides

$$\log 2^{3n+1} = \log 5^{n+1}$$

$$(3n+1)\log 2 = (n+1)\log 5$$

$$\frac{(3n+1)\cancel{\log 2}}{\cancel{\log 2}} = \frac{(n+1)\log 5}{\log 2}$$

$$(3n+1) = (n+1)\cdot\left(\frac{\log 5}{\log 2}\right)$$ multiply both sides by $\frac{1}{(n+1)}$

$$(3n+1)\cdot\frac{1}{n+1} = \cancel{(n+1)}\left(\frac{\log 5}{\log 2}\right)\cdot\frac{1}{\cancel{(n+1)}}$$

$$\frac{3n+1}{n+1} = \frac{\log 5}{\log 2} \Rightarrow \frac{3n+1}{n+1} = \frac{0.69897}{0.30103}$$

$$\frac{3n+1}{n+1} = 2.3219$$

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$$\frac{3n+1}{n+1} \times \frac{2.3219}{1}$$

$3n+1 = (n+1)(2.3219)$

$3n+1 = 2.3219n + 2.3219$

$3n - 2.3219n = 2.3219 - 1$

$0.6781 n = 1.3219$

$$n = \frac{1.3219}{0.6781}$$

$n = 1.949$

$n \simeq 1.95$ ✓ (B)

Que 54

The third and thirteenth term of an A.P are -40, and 0 respectively, the A.P is

Solution

T_3 = Third term = -40

T_{13} = Thirteenth term = 0.

Since $T_3 = -40$, $T_{13} = 0$

$a+2d = -40$ $\quad$ $a+12d = 0$

⇒ Solving Simultaneously we have

$a+2d = -40$ --(i)
$a+12d = 0$ --(ii) } on Subtracting

$\cancel{a-a} + 2d - 12d = -40 - 0$

$-10d = -40$

$d = \cancel{-}40/\cancel{-}10 = 4$

∴ $d = 4$

Since $d = 4$, from eqn (1) $a+2d = -40$

$a+2(4) = -40$

$a+8 = -40$

$a = -40 - 8 = -48$

∴ $a = -48$, $d = 4$ But Common difference $d = 4$

A.P ⇒ $-48, (-48+4), (-48+4+4), (-48+4+4+4), \ldots$

$-48, -44, -40, \ldots$ ✓ (A)

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Que 55

The modulus of $z = a+ib$ is

ANSWER ⇒ $|z| = \sqrt{a^2+b^2}$ ✓ (A)

Que 56

Solve the equation $3^{x^2} = 9^{x+4}$

Solution

$3^{x^2} = 3^{2(x+4)}$

$x^2 = 2(x+4)$

$x^2 = 2x+8$

$x^2 - 2x - 8 = 0$

$(x^2 - 4x) + (2x - 8) = 0$

$x(x-4) + 2(x-4) = 0$

$(x+2)(x+4) = 0$

$x+2 = 0$ or $x+4 = 0$

$x = -2$ or $x = -4$

$\therefore x = -2$ or -4 ✓ (C)

Que 57

The 16th term of the series 3.75, 3.5, 3.25, ... is

Solution

a = 1st term = 3.75

d = Common diff = $3.5 - 3.75 = 3.25 - 3.5 = -0.25$

$\therefore d = -0.25$

but $T_{16} = a + 15d$

$= 3.75 + 15(-0.25)$

$= 3.75 - 3.75$

$= 0$

$\therefore T_{16}$ = 16th term is 0 ✓ (B)

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Que 58

The Variable $Z = \frac{2+3i}{3-3i}$ in the form $Z = a+ib$ is

Solution

$Z = \frac{2+3i}{3-3i} \times \frac{3+3i}{3+3i}$

$= \frac{6+6i+9i+9i^2}{9+9i-9i-9i^2}$. where $i^2 = [\sqrt{(-1)}]^2 = (-1)$

$= \frac{6+15i+9(-1)}{9-9(-1)} = \frac{6+15i-9}{9+9}$

$= \frac{15i-3}{18}$

$= \frac{-3+15i}{18}$ (C)

Que 59

If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ then $BA =$

Solution

$BA = \overset{B}{\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}} \times \overset{A}{\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}}$

$= \begin{bmatrix} (2-1) & (4+3) \\ (0-1) & (0+3) \end{bmatrix}$

$= \begin{bmatrix} 1 & 7 \\ -1 & 3 \end{bmatrix}$

$\therefore BA = \begin{bmatrix} 1 & 7 \\ -1 & 3 \end{bmatrix}$ (C)

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Que 60

60

The Series representing e^x is

Solution

$$e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$$

Since the series has a Summation Σ with range of $n = 0, 1, 2, \ldots \infty$

$$\therefore e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{(A)}$$

Que 61

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Que 61

Find the Coefficient of x^8 in $\left(x^2+\frac{2y}{x}\right)^{10}$

Sol

By Formula $\Rightarrow$ $(a+b)^n = {}^nC_0 a^{n-0} b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_r a^{n-r} b^r$

where $r = 3,4,5,6,\dots$

$\left(x^2+\frac{2y}{x}\right)^{10}$ where $a = x^2$, $b = \frac{2y}{x} = 2y \times \frac{1}{x} \Rightarrow 2y \times (x^{-1}) \Rightarrow 2yx^{-1}$

$$\left(x^2+\frac{2y}{x}\right)^{10} = \left(x^2+2yx^{-1}\right)^{10}$$

$$\left(x^2+2yx^{-1}\right)^{10} = {}^{10}C_0 (x^2)^{10} (2yx^{-1})^0 + {}^{10}C_1 (x^2)^9 (2yx^{-1})^1 + \dots + {}^{10}C_4 (x^2)^6 (2yx^{-1})^4$$

Using the 5th term $\Rightarrow$ ${}^{10}C_4 (x^2)^6 (2yx^{-1})^4$

$\Rightarrow \frac{10!}{(10-4)!\,4!} \times x^{12} \cdot (2)^4 (y)^4 (x^{-1})^4$

$\Rightarrow \frac{10!}{6!\,4!} \times x^{12} \cdot 16 \cdot y^4 \cdot x^{-4}$

$\Rightarrow \frac{10 \times 9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!} \times 4!} \cdot x^{12} \times x^{-4} \times 16 \times y^4$

$\Rightarrow \frac{10 \times \cancel{9}^3 \times 8 \times 7}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \cdot x^8 \times \cancel{16}^2 y^4$

$\Rightarrow 10 \times 3 \times 8 \times 7 \times x^8 \times 2y^4$

$\Rightarrow 10 \times 3 \times 8 \times 7 \times 2 \times x^8 \times y^4$

$\Rightarrow 3360\, x^8 y^4$

$\Rightarrow 3360\, y^4 x^8$

$\therefore$ The Coefficient of $x^8 = 3360\, y^4$

(c)

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Que 62

Find the Inverse of the function $f(x) = 3x+4$

Solution

$f(x) = 3x+4$

Let $f(x) = y$

Such that $y = 3x+4$

Now make x the subject of the formula

$y = 3x+4$

$y-4 = 3x$

$3x = y-4$

$x = \frac{y-4}{3}$

Put $x = f^{-1}(x)$ and Put $y = x$

$x = \frac{y-4}{3}$

$f^{-1}(x) = \frac{x-4}{3}$ Done

Que 63

If $\sqrt{x^2+9} = x+1$ Solve for x

Solution

$\left(\sqrt{x^2+9}\right)^2 = (x+1)^2$ on squaring both sides we have

$x^2+9 = (x+1)^2 = x^2+2x+1$

$x^2+9 = x^2+2x+1$

$\cancel{x^2}-\cancel{x^2} = 2x+1-9$

$0 = 2x-8$

$2x = 8$

$x = 8/2$

$\therefore x = 4$ (B)

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Que 64

Simplify $\frac{\log_{10} 8}{\log_{10} 4}$

Solution

$$\frac{\log_{10} 2^3}{\log_{10} 2^2} = \frac{3\log_{10} 2}{2\log_{10} 2} = \frac{3\,\cancel{(\log_{10} 2)}}{2\,\cancel{(\log_{10} 2)}}$$

$$\therefore \frac{\log_{10} 8}{\log_{10} 4} = \frac{3}{2}$$ (C)

Que 65

If $f(x-4) = x^2+2x+3$, find $f(2)$

Solution

Since $f(x-4) = x^2+2x+3$

then $f(2) = ?$

$x-4 = 2$
$x = 2+4$
$x = 6$

$$f(2) = x^2+2x+3 = 6^2+2(6)+3$$
$$= 36+12+3$$
$$= 51$$

$\therefore f(2) = 51$ D

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Que 66

Find the eleventh term of the Progression 4, 8, 16, - - -

Solution

$a = 4$, Common ratio $= \frac{8}{4} = \frac{16}{8} = 2 \therefore r = 2$

T_{11} = eleventh term = ?

but $T_n = ar^{n-1}$ where $n = 1$

$T_{11} = ar^{11-1}$

$T_{11} = ar^{10}$ $a = 4, r = 2$

$T_{11} = 4(2)^{10} = 2^2(2^{10}) = 2^{2+10} = 2^{12}$

$\therefore T_{11}$ = Eleventh term $= 2^{12}$ (B)

Que 67

An A.P has the first term 11 and fourth term 32. The sum of the first nine term is

Solution

Since 1st term = 11, Fourth term = 32

$a = 11$ $T_4 = 32$

where $T_4 = 32$

$a + 3d = 32$

$11 + 3d = 32$

$3d = 32 - 11 = 21$

$3d = 21$

$d = 21/3 = 7$

Since $d = 7$ then The sum of the 1st to nine term is

$S_{(1^{st}-9^{th})} = S_9 = \frac{n}{2}(2a + (n-1)d)$

$= \frac{9}{2}((2 \times 11) + (9-1)(7)) = \frac{9}{2}(22 + (8)(7)) = \frac{9}{2}(22 + 56$

$S_9 = \frac{9}{2} \times 78 \Rightarrow 9 \times 39 = 351$

$\therefore$ The Sum of the first nine term is 351 (A)

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Que 68

If α and β are the roots of the equation $2x^2 - x - 4 = 0$. Find the Value of $\alpha^3 + \beta^3$.

Solution

Given $2x^2 - x - 4 = 0$

then $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = c/a$

$\alpha + \beta = -\left(\frac{-1}{2}\right)$, $\alpha\beta = \frac{-4}{2}$

$\alpha + \beta = \boxed{1/2}$, $\alpha\beta = \boxed{-2}$

But $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(\frac{1}{2}\right)^3 - 3(-2)(1/2)$

$= \frac{1}{8} + 3$

$= \frac{1+24}{8}$

$= \frac{25}{8}$

(c)

$\therefore \alpha^3 + \beta^3 = \frac{25}{8}$ ✓

Que 69

If $\log_4 U - \log_8 V = 1$, find U in terms of V

Solution

$\log_{2^2} U - \log_{2^3} V = 1$ (Take the reciprocal of the power of Base)

$\frac{1}{2}\log_2 U - \frac{1}{3}\log_2 V = 1$

$\log_2 U^{1/2} - \log_2 V^{1/3} = 1$

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$$\log_2 U^{1/2} - \log_2 V^{1/3} = 1$$

$$\log_2 \left[\frac{U^{1/2}}{V^{1/3}} \right] = 1 \qquad \text{Let } 1 = \log_2 2$$

$$\cancel{\log_2} \left[\frac{U^{1/2}}{V^{1/3}} \right] = \cancel{\log_2} 2$$

$$\frac{U^{1/2}}{V^{1/3}} = 2$$

$$U^{1/2} = 2V^{1/3} \qquad \text{multiply both by Power of 2}$$

$$U^{1/2 \times 2} = \left(2V^{1/3}\right)^2$$

$$U = \left(2V^{1/3}\right)^2 = 2V^{1/3} \times 2V^{1/3} = 4V^{1/3 + 1/3} = 4V^{2/3}$$

$$\therefore U = \underline{\underline{4V^{2/3}}} \quad (\text{A})$$

QUE 70

Find the Inverse of matrix $A = \begin{pmatrix} 2 & 4 & 3 \\ 1 & -2 & -2 \\ -3 & 3 & 2 \end{pmatrix}$

Solution

$$\text{Inverse of matrix } A = \frac{1}{\det A} \times \left(\text{Cofactor of MATRIX } A\right)^T$$

(i.e Transpose of Cofactor of A)

$$\therefore \det A = |A| = \begin{vmatrix} \overset{+}{2} & \overset{-}{4} & \overset{+}{3} \\ 1 & -2 & -2 \\ -3 & 3 & 2 \end{vmatrix}$$

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$$|A| = \det A \Rightarrow +2\begin{vmatrix} -2 & -2 \\ 3 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & -2 \\ -3 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -2 \\ -3 & 3 \end{vmatrix}$$

$$\det A \Rightarrow 2(-4+6) - 4(2-6) + 3(3-6)$$

$$\Rightarrow 2(2) - 4(-4) + 3(-3)$$

$$\Rightarrow +4 + 16 - 9$$

$$\Rightarrow 20 - 9$$

$$\det A \Rightarrow 11$$

$$\therefore \boxed{\det A = 11}$$

Now get the Cofactor of matrix $A \Rightarrow$

$$\text{Matrix } A = \begin{pmatrix} 2 & 4 & 3 \\ 1 & -2 & -2 \\ -3 & 3 & 2 \end{pmatrix}$$

Cofactor of $2 = +\begin{vmatrix} -2 & -2 \\ 3 & 2 \end{vmatrix} = +\big((-4) - (-6)\big) = +(-4+6) = 2$

Cofactor of $4 = -\begin{vmatrix} 1 & -2 \\ -3 & 2 \end{vmatrix} = -\big((2) - (6)\big) = -(2-6) = 4$

Cofactor of $3 = +\begin{vmatrix} 1 & -2 \\ -3 & 3 \end{vmatrix} = +\big((3) + (6)\big) = +(3-6) = -3$

Cofactor of $1 = -\begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} = -\big((8) - (9)\big) = -(8-9) = 1$

Cofactor of $-2 = +\begin{vmatrix} 2 & 3 \\ -3 & 2 \end{vmatrix} = +\big((4) - (-9)\big) = +(4+9) = 13$

Cofactor of $-2 = -\begin{vmatrix} 2 & 4 \\ -3 & 3 \end{vmatrix} = -\big((6) - (-12)\big) = -(6+12) = -18$

Cofactor of $-3 = +\begin{vmatrix} 4 & 3 \\ -2 & -2 \end{vmatrix} = +\big((-8) - (-6)\big) = +(-8+6) = -2$

Cofactor of $3 = -\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -\big((-4) - (3)\big) = -(-4-3) = 7$

Cofactor of $2 = +\begin{vmatrix} 2 & 4 \\ 1 & -2 \end{vmatrix} = +\big((-4) - (4)\big) = +(-4-4) = -8$

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$\therefore$ Since Matrix $A = \begin{pmatrix} 2 & 4 & 3 \\ 1 & -2 & 2 \\ -3 & 3 & 2 \end{pmatrix}$ then the Cofactor of matrix $A \Rightarrow$

Cofactor of matrix $A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 13 & -18 \\ -2 & 7 & -8 \end{pmatrix}$

The Transpose of the Cofactor of matrix $A = \begin{pmatrix} 2 & 1 & -2 \\ 4 & 13 & 7 \\ -3 & -18 & -8 \end{pmatrix}$

Recall that Inverse of matrix $A = A^{-1} = \frac{1}{\det A}$ (Transpose of Cofactor of A)

Since $\det A = 11$

then $A^{-1} = \frac{1}{11}\begin{bmatrix} 2 & 1 & -2 \\ 4 & 13 & 7 \\ -3 & -18 & -8 \end{bmatrix}$ (C)

Solved

Noalymatics

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$a - b + c = -35 \quad -①$

$4a - 2b + c = -12 \quad -②$

$9a - 3b + c = 27 \quad -③$

③ - ② : $5a - b = 39 \quad -④$

② - ① : $3a - b = 23 \quad -⑤$

④ - ⑤ $\frac{2a}{\cancel{2}} = \frac{16}{2}$

$a = \underline{8}$

Substitute 8 for a in ④

$5(8) - b = 39$

$40 - b = 39$

$-b = 39 - 40$

$-b = -1$

$b = \underline{1}$

Substitute 1 for b and 8 for a in ①

$8 - 1 + c = -35$

$7 + c = -35$

$c = -35 - 7 = -42.$

$\Rightarrow P(x) = x^3 + 8x^2 + x - 42$

$x = -3$ is a root, because $P(-3) = 0$.

$\Rightarrow (x+3)$ is a factor

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$$\begin{array}{r|l} & x^2+5x-14 \\ \hline x+3 & x^3+8x^2+x-42 \\ & -\underline{(x^3+3x^2)} \\ & \quad 5x^2+x-42 \\ & \quad -\underline{(5x^2+15x)} \\ & \qquad -14x-42 \\ & \qquad \underline{-14x-42} \\ & \qquad\quad - \quad - \end{array}$$

$(x+3)(x^2+5x-14)=0$

$(x+3)(x^2+7x-2x-14)=0$

$(x+3)(x(x+7)-2(x+7))=0$

$(x+3)(x-2)(x+7)=0$

$x+3=0 \qquad x-2=0 \qquad x+7=0$

$x=\underline{\underline{-3}} \qquad x=\underline{\underline{2}} \qquad x=\underline{\underline{-7}}$

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