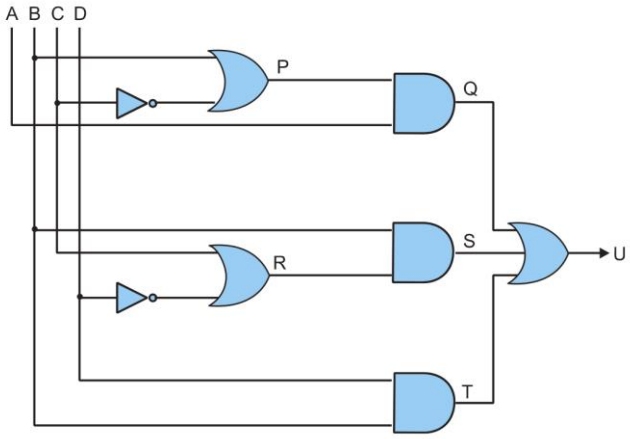
COM1006/COM1090 Devices and Networks (Autumn)
Tutorial Sheet #5: Simplifying and Implementing Circuits

1. In the previous tutorial sheet, you derived a sum-of-products expression for the output U of the following circuit. Now simplify this expression using a Karnaugh map.

   Afterwards, check your answer by opening your circuit in Logisim, clicking Project → Analyze Circuit and selecting the "Minimized" tab. Note that rectangles might be mirrored or shifted if variables are assigned differently to rows and columns. This does not affect the minimised formula, though.



2. Derive simplified Boolean formulas in sum-of-product form (e.g. F = AC + BCD) from the following Karnaugh maps. The expressions should be simplified as much as possible, i.e. have a minimum number of products, all of which cannot be further shortened.

(i)

CD \ AB	00	01	11	10
00				
01	1	1	1	
11	1		1	
10				

(ii)

CD \ AB	00	01	11	10
00	1			1
01		X		
11			X	1
10	1	X	1	X

3. In this exercise you shall prove that every Boolean function can be implemented using only NAND gates (using only NOR works as well).

   (i) Show that AND, OR, and NOT can be expressed using NAND with one or two operands, i. e. use Boolean algebra to show that

   $$\overline{A}, \qquad A+B, \quad \text{and} \quad A \cdot B$$

   can be expressed using only NAND operations. You might need to use nested NAND operations for $A+B$ and/or $A \cdot B$.

   (ii) Implement your solutions for (i) in Logisim using only NAND gates.

4. What is tri-state logic and why is it used in digital systems? In Logisim, connect two input pins with a line. Make sure that the "Three-state" property of each input pin is "No". Set the inputs to different values, and you'll see that the line glows red, indicating that an error has occurred. Now change the "Three-state" property of both pins to "Yes". By clicking on a pin with the poke tool, you'll now find that there are three values (0, 1 and x). You'll find that if one pin is set to 1, you can avoid an error by disconnecting the other pin (by clicking until it shows x).

5. A device accepts natural binary numbers in the range 0000 to 1111 that represent the integers 0 to 15. The output of the circuit is true if the input to the circuit represents a prime number and is false otherwise. Write the truth table for this device. A prime number is an integer that is greater than 1 and is divisible only by itself and 1. Zero and one are not prime numbers. How would you use a 4-line to 16-line demultiplexer to implement this system?

   Build your solution in Logisim. Demultiplexers are found in the "Plexers" folder. Add an east-facing demultiplexer and set "Select Bits" to 4. You'll need an input signal of 1 attached to the "data" input (west side). For the "select" input, the right one on the demultiplexer's south side, you need a 4-bit input pin: add an input pin and set its "data bits" to 4. You will see a bit string of 4 bits; you can "poke" bits individually. Complete your circuit and use a LED that lights red when the select input is a prime number.