COM1002
Foundations of Computer Science

Introduce key mathematical concepts that underpin:
computer science, software engineering, artificial intelligence.

Dr Paul Watton

Lewis Carroll (1832-98) was an English writer, mathematician, logician,…
His most famous writings are Alice's Adventures in Wonderland. Noted for his facility at word play, logic, and fantasy.

Logic Puzzle (Lewis Carroll)

1. All babies are illogical.

2. Nobody is despised who can manage a crocodile.

3. Illogical persons are despised.

What does this tell us about babies and crocodiles?

COM1002: Foundations of mathematics

Todays lecture will be about logic, propositions and deductions, e.g.

I am in a maths lecture IMPLIES $=>$ I am sleeping

Hopefully, this ***propositional statement*** is not true:

I am in a maths lecture IMPLIES $\Rightarrow$ I am sleeping

Note, if the above propositional statement is TRUE, this does not imply that the reverse is true, i.e.

I am sleeping IMPLIES $\Rightarrow$ I am in a maths lecture

…there are lots of other places I could be sleeping

Expressed in propositional logic $(A \Rightarrow B) \not\Rightarrow (B \Rightarrow A)$

this mathematical formulation is what we are learning about today

Motivation: Why is maths relevant for Computer Science ?

Computers do not work correctly (or as intended) all of the time.

The problem of systems failures becomes serious (costly, deadly) as automatic control systems find their way into our daily lives, e.g.

1968: the nuclear submarine USS Scorpion was destroyed killing all of its crew - own torpedoes designed to seek out nearest target… (**99 deaths**)

1985-87: Therac 25 Radiotherapy machine (faulty software – **3 deaths**)

1992: London Ambulance Service – new software system unable to cope with real-time data – **30 deaths**

1994: **Intel Pentium** –With certain inputs, the unit gave inaccurate results when performing division, rendering it useless of mathematical/scientific work.

1996: **Ariane 5 –** Satellite rocket launch – veered off course 40 seconds after lift off and self-destructed. Problem with conversion of 64 bit to 16 bit number.

Designing high-quality computing systems remains a challenge for **software engineers**.

Programs and software systems are mathematical structures.

Mathematics can help identify errors in systems.

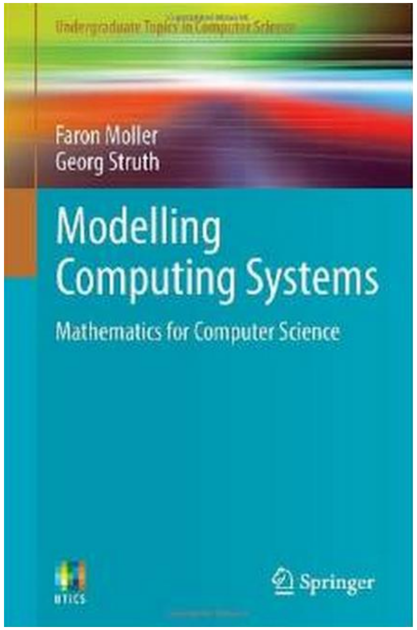
THIS COURSE: Introduces mathematical foundations to achieve this

Course Overview

Textbooks and lecture notes:

❑ A class textbook:

"**Modelling Computing Systems: Mathematics for Computer Science**", by Faron Moller & Georg Struth;



Follows chapters 1-10

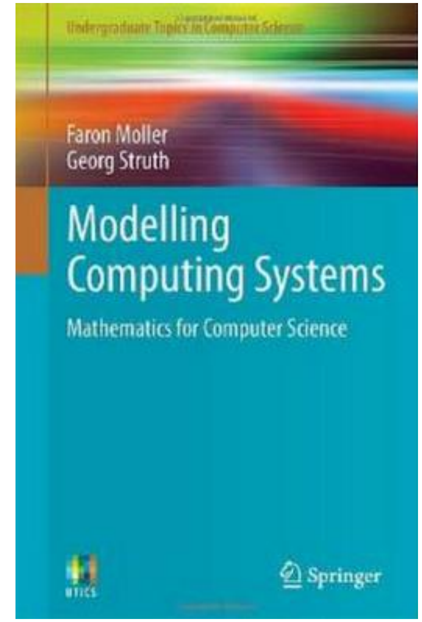
- **Propositional Logic**
- Set Theory
- Boolean Algebra
- Predicate Logic
- Proof strategies
- Functions and Relations
- Induction and Recursion

Other textbooks:

❖ see the module syllabus on the web,

❖ available from the library.

Obtaining a copy of book.
Detailed instructions are on MOLE.



Briefly,

- department will lend a book to students registered for COM1002.
- £20 deposit is needed to obtain the book from Reception.

- deposit refunded at the end of the academic year providing the book is returned in good condition.
- bar-code cover sheet is available for each student registered for COM1002 called “COM1002 Assignment 1 Textbook.” To access it log in at:

https://foe-coversheet.group.shef.ac.uk/

- To obtain book, print this sheet and bring it to DCS Reception with £20.
- You will be issued with a copy of the textbook with a unique copy number inside the back cover. The copy number will be recorded by your name and you will sign against it to confirm you have received the textbook.

Returning the book for a refund (see MOLE):

Teaching and Assessment 1

Lectures and tutorial sessions:

The lectures will:

- explain the concepts,
- show how to solve problems

However:

- You need to do some independent work!
- You need to read the book!

The class tutorials will start from Thursday 8th October:

- have three groups – each a third of the class,
- run by postgraduate students
- allow you to practice solving problems,
- allow you to get help if stuck.

Teaching and Assessment 2

Assessment:

For this semester:

- five examples sheets are assessed (5% each)
- three online exam-type “quizzes” (25% each),
  - Dates to be confirmed.
  - Most likely the Wednesday lecture slots in weeks 6, 9 and 11.

What is Logic ?

In science, logic means
"***the study of valid reasoning.***"

For centuries it has been a philosophical discipline. Nowadays, it is part of the scientific background assumed by researchers from a wide diversity of scientific backgrounds such as:

mathematics,
computer science,
linguistics,
cognitive science
economics.

Reason

Reason is the capacity for making sense of things, for establishing and verifying **facts**, and changing or justifying practices, institutions, and **beliefs** based on new or existing information. It is:

- associated with **thinking, cognition**, and **intellect.**

- one of the ways by which thinking comes from one idea to a related idea.

- the means by which rational beings understand themselves to think about **cause** and **effect, truth** and **falsehood**, and what is **good or bad**.

- closely associated with activities such as **philosophy, science, language, mathematics,** and **art.** It is normally considered to be a definitive characteristic of **human nature.**

- also closely identified with the ability to self-consciously change **beliefs, attitudes, traditions,** and **institutions,** and therefore with the capacity for freedom and **self-determination.**

Languages: Natural and Formal

A **natural language** is a language that is used for normal everyday communication in a human society, e.g. Chinese, English and French are all natural languages.

In mathematics, computer science, and linguistics, a **formal language** is a **set of strings of symbols** that may be constrained by **rules** that are specific to it.

A **programming language** is a **formal constructed language** designed to communicate instructions to a machine to control its behaviour or express algorithms

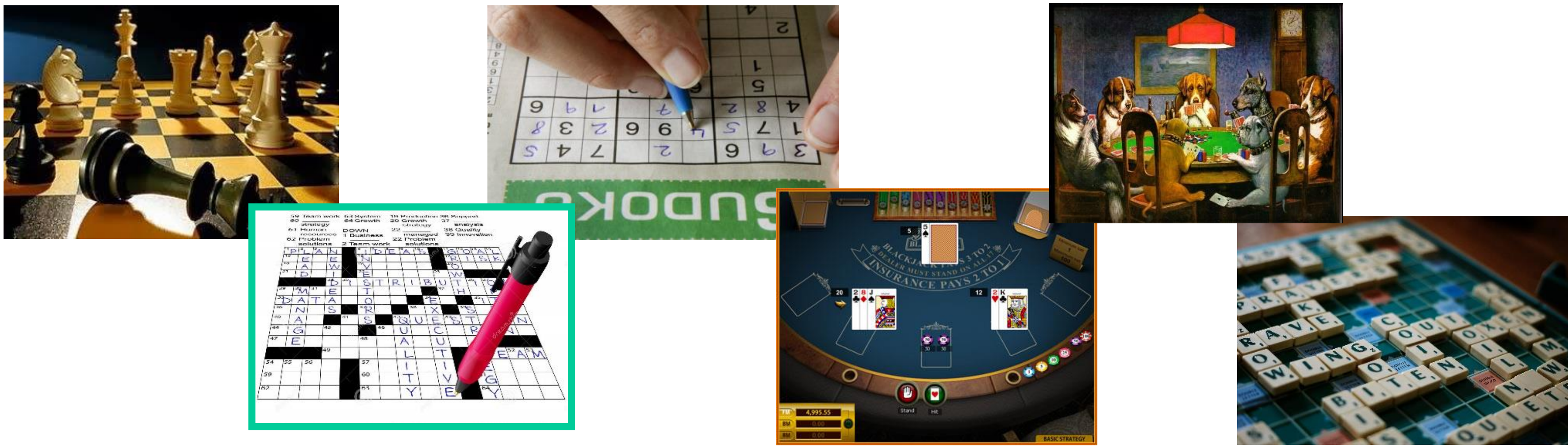
Logical Reasoning

Reasoning about situations involves complex sentences with the ‘logical connectives’ of **natural language**, such as ‘**not**’, ‘**and**’, ‘**or**’ and ‘**if** .. **then**’.

These are not the only expressions that drive **logical reasoning**, but they do form the most basic level.

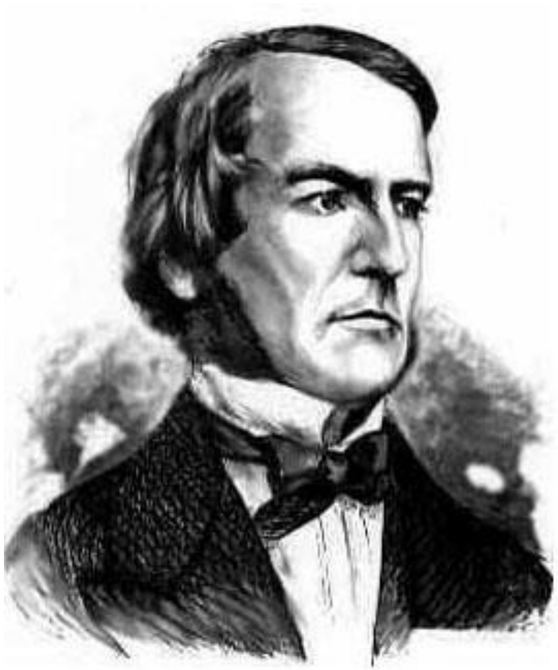
We could stay close to **natural language** itself to define our system, but it has become clear over time that working with well-chosen notation makes things much clearer, and easier to manipulate. We use formal notations to improve understanding and facilitate computation.

Reasoning is fun! ?

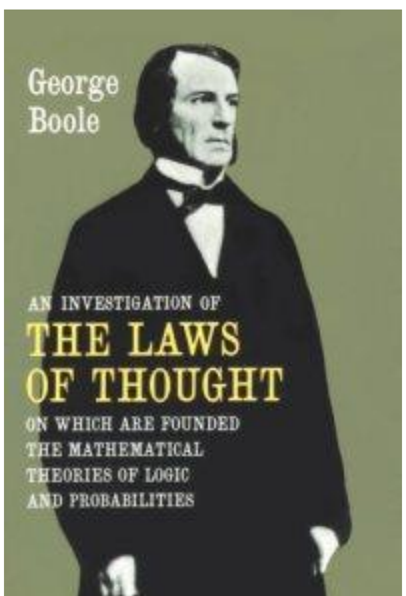


COM1002: challenging material
develops mathematical reasoning skills
Is enjoyable – work hard and stay on top of material!
Must do all exercise sheets

Propositional Logic



George Boole



The first completely formal version of propositional logic was presented around 1850 by George Boole, and published in his famous '***The Laws of Thought***'.

Although the language of propositional logic is simple, the logic itself is very expressive and powerful.

All digital computation can be modelled as propositional reasoning (low-level representation).

George Boole (1815-1864)
English mathematician, philosopher and logician

Propositions and Deductions 1

Atomic propositions:

Statements that can only be either ‘true’ or ‘false’

my watch is ticking,

my parrot is dead;

Compound propositions:

State relationships between propositions, e.g

‘my parrot is dead **or** my watch has stopped’

‘my parrot is dead **and** my watch has stopped’

‘this lecture is stimulating **and** I am awake’

Exercise 1.1 (Book)

Which of the following are propositional statements ?

- 2+3 = 5
- 2+3 = 6
- Do your homework, Joel!
- Joel didn't do his homework
- Is there life on Mars ?
- What Felix says is false.

Propositions and Deductions 2

- Deduction is a process with three steps:
  - Start from premises – true propositions;
  - Construct an inference from these;
  - This gives a conclusion, as a truth value.

Deduction

Are the deductions are valid ?

If you take my medication, you will get better.
But you are not taking my medication.
So, you will not get better.

If I resist, the enemy will kill me.
But I am not resisting.
So, the enemy will not kill me

If you take my medication, you will get better.
But you are not getting better.
So, you have not taken my medication

Logical Language 1

Defining Languages:

- Involves at least two languages:
  - the object language – the one being defined,
  - a meta-language – the one used for defining it;
- An object language has two aspects:
  - its syntax – the rules for writing it
  - its semantics – deciding what constructions written in it mean.

Logical Language 2

Defining Syntax:

Involves two steps:

- define the symbols, and then
- define larger constructions:
  - eg words, sentences, etc for natural languages,
  - the formulae for propositional logic.

Propositional Variables:

- used to stand for propositions,
- names often abbreviate the propositions, e.g.

D for the proposition "**this man is dead**".

'DEAD' for the statement "**this man is dead**".

Logical Language 3

Propositional Connectives:

- The “operations” of propositional logic;
- Similar role to +, -, /,* etc in arithmetic;
- Form compound propositions:
  - from one or two simpler ones;
- …needed for any kind of deduction.

Each connective has a specific meaning, i.e its ‘semantics’

The Propositional Connectives 1

Negation:

For a proposition p:

- negation is written **$\neg$ p**,
- usually pronounced "**not p**",
- meaning of $\neg$ p is "proposition p is false".

Laws for negation:

- $\neg$ *true* = *false*, and $\neg$ *false* = *true*;
- for any p: $\neg\neg$ p = p (law of double negation.

Exercise 1.4

Rewrite the following sentences without negations at the start

1. ¬“The earth revolves around the sun”

2. ¬“All of my children are boys”

3. ¬“$(2+2\leq 4)$”

The Propositional Connectives 2

Disjunction:

For propositions p, q:

- written $p \vee q$,
- usually pronounced "p or q",
- **is true if '***either p is true, or q is true, or both*,
- p and q are often called disjuncts.

Law for disjunction:

$p \vee \neg p = \text{true}$ '*the law of the excluded middle.'*

Exercise 1.5

Are the following disjunctions true or false ?

1. $(3 < 2) \lor (3 < 5)$
2. $(5 < 4) \lor (7 < 5)$
3. $(5 < 6) \lor (6 < 8)$

The Propositional Connectives 3

Conjunction:

- For propositions p and q:
  - written $p \wedge q$,
  - usually pronounced "p and q",
  - **is true if** '*both p is true and q is true*'
  - p and q are often called conjuncts.

Law for conjunction: $p \wedge \neg p = \text{false}$.

Exercise 1.7

Are the following conjunctions true or false ?

1. $(3 < 2) \wedge (3 < 5)$

2. $(5 < 4) \wedge (7 < 5)$

3. $(5 < 6) \wedge (6 < 8)$

The Propositional Connectives 4

Implication:

For propositions **p** and **q**:

- written **p** $\Rightarrow$ **q**,
- usually pronounced "**p implies q**", or "**if p then q**",
- p is the *premise* and q is the *conclusion*.

Law for implication:

p $\Rightarrow$ q is equivalent to $\neg$ q $\Rightarrow$ $\neg$ p.

Implication (to think about…)

$p \rightarrow q$ is true

if and only if, ‘**if *p* is true, then *q* must be true**’.

EQUIVALENTLY

if, and only if, ‘**p is false or q is true**’,

EQUIVALENTLY

$p \rightarrow q$ is false, if, and only if, ‘**p is true and q is false**’.

$p \rightarrow q$ may be expressed in English in several ways:

p implies **q** | if **p** then **q**

q if p | **p** only if **q**

q whenever **p**

p is a sufficient condition for **q**

q is a necessary condition for **p**

Useful for some of the exercises…

Subtleties of language: translating arguments or sentences in natural languages into the notations of formal logic – can be tricky even for native English speakers…

Alice in Wonderland
Lewis Carroll (1832-98)
writer, logician,
mathematician,
Christ Church College
University of Oxford

March Hare: "Then you should ***say what you mean***."

Alice: "*I do, at least -- at least* ***I mean what I say --***
that's the same thing, you know."

Mad Hatter: "**Not the same thing a bit!** Why, you might just as well say that
'**I see what I eat**' is the same thing as '**I eat what I see**'!"

March Hare: "You might just as well say that
'**I like what I get**' is the same thing as '**I get what I like**!"

Dormouse: "You might just as well say that '**I breathe when I sleep**'
is the same thing as '**I sleep when I breathe**!"

Mad Hatter: "It's the same thing with you"

Logic Puzzle

1. All babies are illogical.

2. Nobody is despised who can manage a crocodile.

3. Illogical persons are despised.

What does this tell us about babies and crocodiles?

HOMEWORK…

Can we express the above statements using propositional logic and draw a deduction