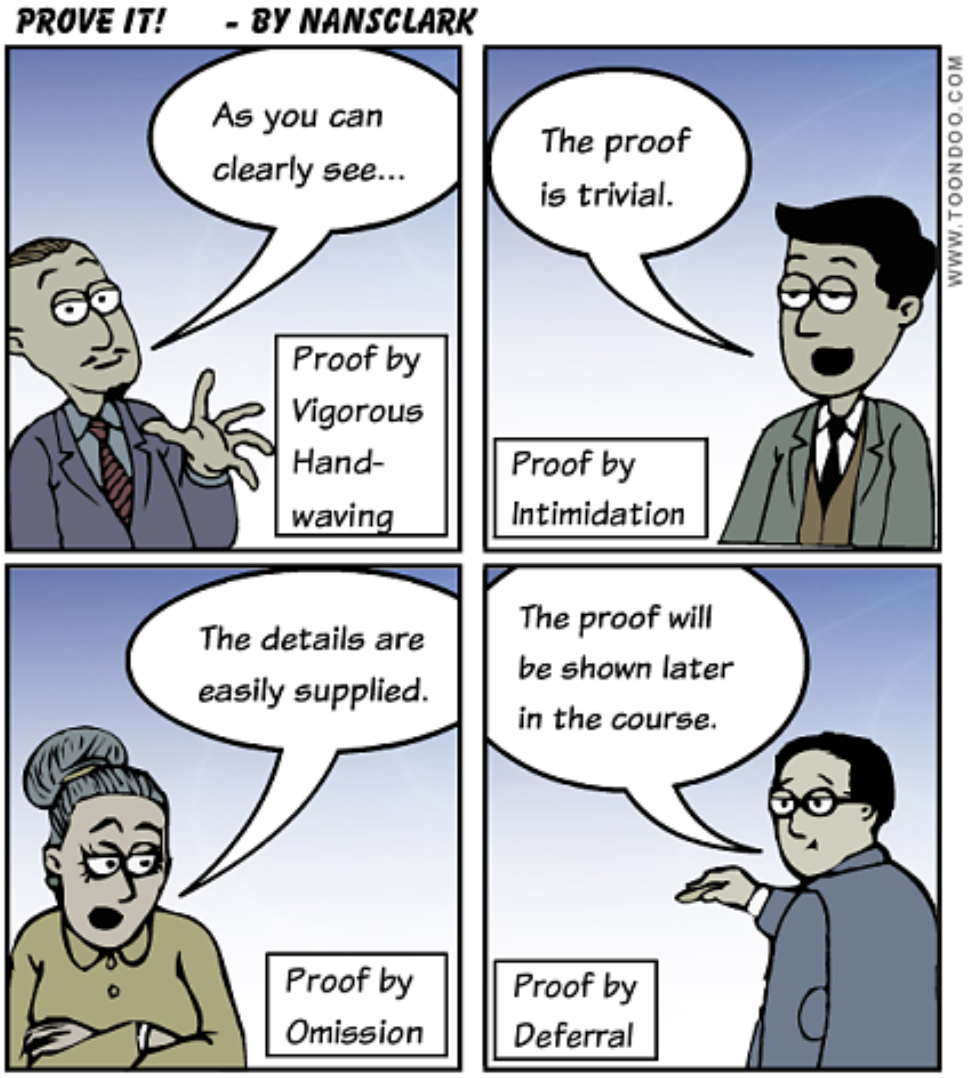
COM1002:
Foundations of Computer Science



Proof Strategies

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WEEK	8	9	10	11
Mon		Lecture **Hand in Ex 6**	Lecture **Hand out ex 7** (Assessed 5%)	Lecture **Hand in Ex 7**
Wed	Lecture	**Revision Lecture**	Lecture	**Revision Lecture**
Thurs	Tut (ex 6)	**Revision Tutorial** **QUIZ 2 (25%)** Diamond 101 4pm-5:30pm	Tut (ex 7)	**Revision Tutorial** **QUIZ 3 (25%)** Diamond 101 4pm-5:30pm

We are nearly there…

QUIZ 2

6 Questions:

Predicate Logic (x2),
Composition of Functions,
Proof in propositional logic,
Proof in predicate logic over sets
Simplication of a predicate statement

$p \Rightarrow (p \vee q)$	*Disjunction introduction*
$((p \Rightarrow q) \wedge (r \Rightarrow q) \wedge (p \vee r)) \Rightarrow q$	*Disjunction elimination*
$(p \wedge q) \Rightarrow p$	*Conjunction elimination*
$(p \wedge q) \Rightarrow q$	*Conjunction elimination*
$(p) \wedge (q) \Rightarrow (p \wedge q)$	*Conjunction introduction*
$((p \Rightarrow q) \wedge p) \Rightarrow q$	*Modus Ponens*
$((p \Rightarrow q) \wedge \neg q) \Rightarrow \neg p$	*Modus Tollens*
$(\neg p \wedge (p \vee q)) \Rightarrow q$	*Disjunctive syllogism*
$((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$	*Hypothetical syllogism*

Simplification of a predicate

Negate each quantified statement, simplifying so that only the simple statements are negated. Show each step of your work.

(a) $\forall x(\sim P(x) \wedge \sim Q(x))$

(b) $\exists x(Q(x) \rightarrow \sim P(x))$

Negate $\forall x\left(\neg P(x) \wedge \neg Q(x)\right)$

$$\begin{aligned}\neg\forall x\left(\neg P(x) \wedge \neg Q(x)\right) &\Leftrightarrow \exists x\neg\left(\neg P(x) \wedge \neg Q(x)\right)\\ &\Leftrightarrow \exists x\left(\neg\neg P(x) \vee \neg\neg Q(x)\right)\\ &\Leftrightarrow \exists x\left(P(x) \vee Q(x)\right)\end{aligned}$$

Negate $\exists x\big(Q(x) \Rightarrow \neg P(x)\big)$

$$\begin{aligned}
\neg\exists x\big(Q(x) \Rightarrow \neg P(x)\big) &\Leftrightarrow \forall x \neg\big(Q(x) \Rightarrow \neg P(x)\big) \\
&\Leftrightarrow \forall x \neg\big(\neg Q(x) \vee \neg P(x)\big) \\
&\Leftrightarrow \forall x\big(\neg\neg Q(x) \wedge \neg\neg P(x)\big) \\
&\Leftrightarrow \forall x\big(Q(x) \wedge P(x)\big)
\end{aligned}$$

Proof in Propositional Logic

Show that the hypotheses:

- It is not sunny this afternoon and it is colder than yesterday.
- We will go swimming only if it is sunny.
- If we do not go swimming, then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

lead to the conclusion:

We will be home by the sunset.

Main steps:

Translate the statements into proposional logic.

Write a formal proof, a sequence of steps that state hypotheses or apply inference rules to previous steps.

Show that the hypotheses:

- It is not sunny this afternoon and it is colder than yesterday.
- We will go swimming only if it is sunny.
- If we do not go swimming, then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

lead to the conclusion:

- We will be home by the sunset.

Step	Reason

Where:
s: "it is sunny this afternoon"
c: "it is colder than yesterday"
w: "we will go swimming"
t: "we will take a canoe trip.
h: "we will be home by the sunset."