COM1002
Foundations of Computer Science

C7: Relations

Learning objective: To understand, attempt and correctly answer the following question…

Let the **EQUIVALENCE relation R** on the set A={1,2,3,…,29} of positive integers less than 30 be defined by

$$\boxed{(x,y)\in R \Leftrightarrow x \text{ and y have the same prime factors}}$$

How many equivalence classes does R partition A into ?

List each of these equivalence classes.

Let us use the eloquent, but still informal, account of equivalence relations given by Skemp (1977).He begins by introducing methods of sorting the elements of a parent set into sub-classes in which every object in the parent set belongs to one, and only one, subset (a partition of the parent set). He (ibid, p.174) considers two sorting methods: first, starting "with some characteristic properties, and form our sub-sets according to this"; and second, starting "with a particular matching procedure, and sort our set by putting all objects which match in this way into the same sub-set". The particularity of this matching procedure is in its "exactness", i.e. having an exact measure for the sameness; a necessity that if it is achieved, the matching procedure is called an equivalence relation. The exactness of the matching procedure also accounts for the transitive property. In addition to the transitive property, an equivalence relation has two further properties, reflexivity and symmetry (see below).

Skemp. R. R. (1977). The Psychology of Learning Mathematics. Penguin Books.

…he is talking about software verification and testing…

One of the best things about BioWare RPGs, going all the way back to the original **Baldur's Gate**, is the interaction between NPCs in your party. They flirt, they fight, and they talk, talk, talk, which is why some **Dragon Age: Inquisition** players found it odd that their companions were so very quiet, sometimes going hours at a time without speaking. Turns out, it's a bug.

Reports of the strange silence, as noted by **Reddit**, started popping up a couple of days ago. When it became clear that something was amiss, BioWare asked PC players afflicted by the bug who are 60-70 hours into the game to submit save files so it could analyze the problem. In an update posted today, however, it said that it has sufficient saves for now.

It would seem that one of the reasons it's taken so long for the issue to come to light is simply that players didn't realize it was a problem at all. It doesn't appear to affect the game in any other meaningful way—you can still play through from start to finish—but party members just don't talk, even though there should be some kind of interaction happening every 10-20 minutes. And if you're missing the party banter in a BioWare RPG, you're missing out on a lot.

At **last report**, BioWare said it's "getting somewhere" in correcting the problem, and asked anyone affected by it to post a report in this **EA forum thread**.

input combinations for any software system is impractical. Equivalence partitioning is a technique used in black box testing to develop test cases. As discussed in Sections V of Chapter 3 and of Chapter 6, black box testing is the testing approach used in validation for both conventional software and expert systems. The functional requirements of a software product serve as the basis for deriving test cases for black box testing. In input equivalence partitioning, the input domain of a software product is "partitioned into a finite number of equivalence classes such that a test of a representative of each class will, by induction, test the entire class, and, hence, the equivalent of exhaustive testing of the input domain can be performed." [Goodenough, 1975] Guidelines for defining equivalence classes are:

1. If an input condition specifies a range, one valid and two invalid equivalence classes are defined.
2. If an input condition requires a specific value, one valid and two invalid equivalence classes are defined.
3. If an input condition specifies a member of a set, one valid equivalence class and one invalid equivalence class are defined.
4. If an input condition is Boolean, one valid class and one invalid class are defined. [Pressman, 1987]

In addition to equivalence classes for the input of a software product, equivalence classes for the output are also beneficial in devising a comprehensive set of test cases for the validation process. [Sommerville, 1989] Output equivalence classes are defined similarly to input equivalence classes.

The concepts of input and output equivalence partitioning are utilized by the validation analyzer and enhancer component of SAVES. The input and output equivalence classes equate to the input and output values required of the expert system. Although such partitioning is simplistic, for the size of knowledge base used in the evaluation of SAVES, this level of partitioning is sufficient. For larger, more complicated knowledge bases, an extended method of equivalence partitioning needs to be adopted.

Terminology of Relations

Heterogeneous Homogeneous

Reflexive irreflexive

Symmetric antisymmetric asymmetric

transitive

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partial order total order equivalence

equivalence class

Let **Z** denote the set of all animals.

Let **S** be the relation animal x is the same species as animal y

Is **S** an equivalence relation ?

Let S denote the set of Sheffield University Students.

Let F be the relation student x is friends with student y

F = {(x,y): x is friends with y where x,y $\in$ S}

Is F an Equivalence relation ?

Let S denote the set of Sheffield University Students.

Let A be the relation student x is the same age (in years) as student y

A = {(x,y): x is the same age as y where x,y $\in$ S}

Is A an Equivalence relation ?

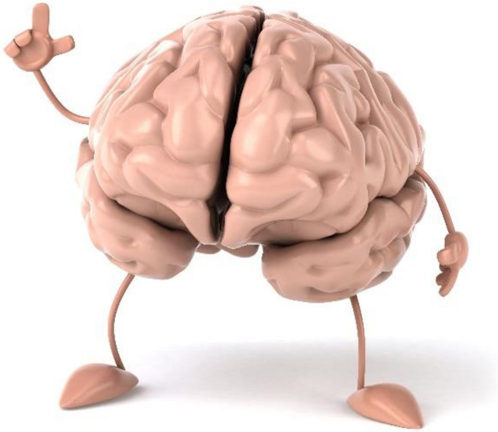
Let S denote the set of Sheffield University Students.

Let E be the relation student x has the same colour eyes as student y

$E = \{(x,y):\ x \text{ has the same colour eyes as } y \text{ where } x,y \in S\}$

Is E an Equivalence relation ?

What are the Equivalence classes of E ?



Let S denote the set of Sheffield University Students.

Let B be the relation student x has the same brain as student y

B = {(x,y): x has the same brain as student y where x,y $\in$ S}

Is B an Equivalence relation ?

Let S denote the set of Sheffield University Students.

Let M be the relation student x shares the same viewpoint on Marmite as student y

M = {(x,y): x and y agree on y where x,y $\in$ S}

Is M an Equivalence relation ?

What are the equivalence classes?

PARTIAL ORDER

reflexive iff $\forall x \in A\ (xRx)$,

antisymmetric iff $\forall x,y \in A\ (xRy \wedge yRx \Rightarrow x = y)$:

or EQUIVALENTLYif *R(a,b)* with $a \neq b$, then *R(b,a)* must not hold.

transitive iff $\forall x, y, z \in A\ ((x\ R\ y) \wedge (y\ R\ z) \Rightarrow x\ R\ z)$

TOTAL ORDER iff PARTIAL ORDER and

$$\forall x, y \in A\ ((x\ R\ y) \vee (y\ R\ x)):$$

EQUIVALENCE

- **reflexive** iff $\forall x \in A\ (xRx)$,
- **symmetric** iff $\forall x,y \in A\ (xRy \Rightarrow yRx)$
- **transitive** iff $\forall x, y, z \in A\ ((x\ R\ y) \wedge (y\ R\ z) \Rightarrow x\ R\ z)$

i) The Identity relation $I = \{(n,n) : n \in N\}$

ii) The Universal relation $U= \{(m,n): m,n \in N\}$

iii) The parity relation $P = \{m=n (\text{mod } 2): m,n \in N\}$

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Equivalence Partitions:

- A **partition** of a set A is:
  - a set of subsets A_i for $i \in I$ of A:
    - where A_i are the **blocks** of the partition,
  - which are all disjoint (and non-empty),
  - and together contain all of A, so that:
    - $(i \neq j \Rightarrow A_i \cap A_j = \emptyset) \;\; \wedge \bigcup_{i \in I} A_i = A$;
- A **refinement** of a partition is:
  - a partition where every block is a subset of a block of the one being refined.

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Equivalence Classes:

for an equivalence relation $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{A}$:

the **equivalence class** of $\mathbf{a} \in \mathbf{A}$ with respect to **R** is the set of elements related to **a** by **R**:

denoted $[\mathbf{a}]_{\mathbf{R}} = \{ \mathbf{x} \in \mathbf{A} : \mathbf{a}\ \mathbf{R}\ \mathbf{x} \}$;

sets of equivalence classes:

for an equivalence relation **R**, the set of **equivalence classes** of **A** with respect to **R** is a **partition** of **A**.

Try Exercise 7.24.

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(Answer: – see page 440 in Class textbook)