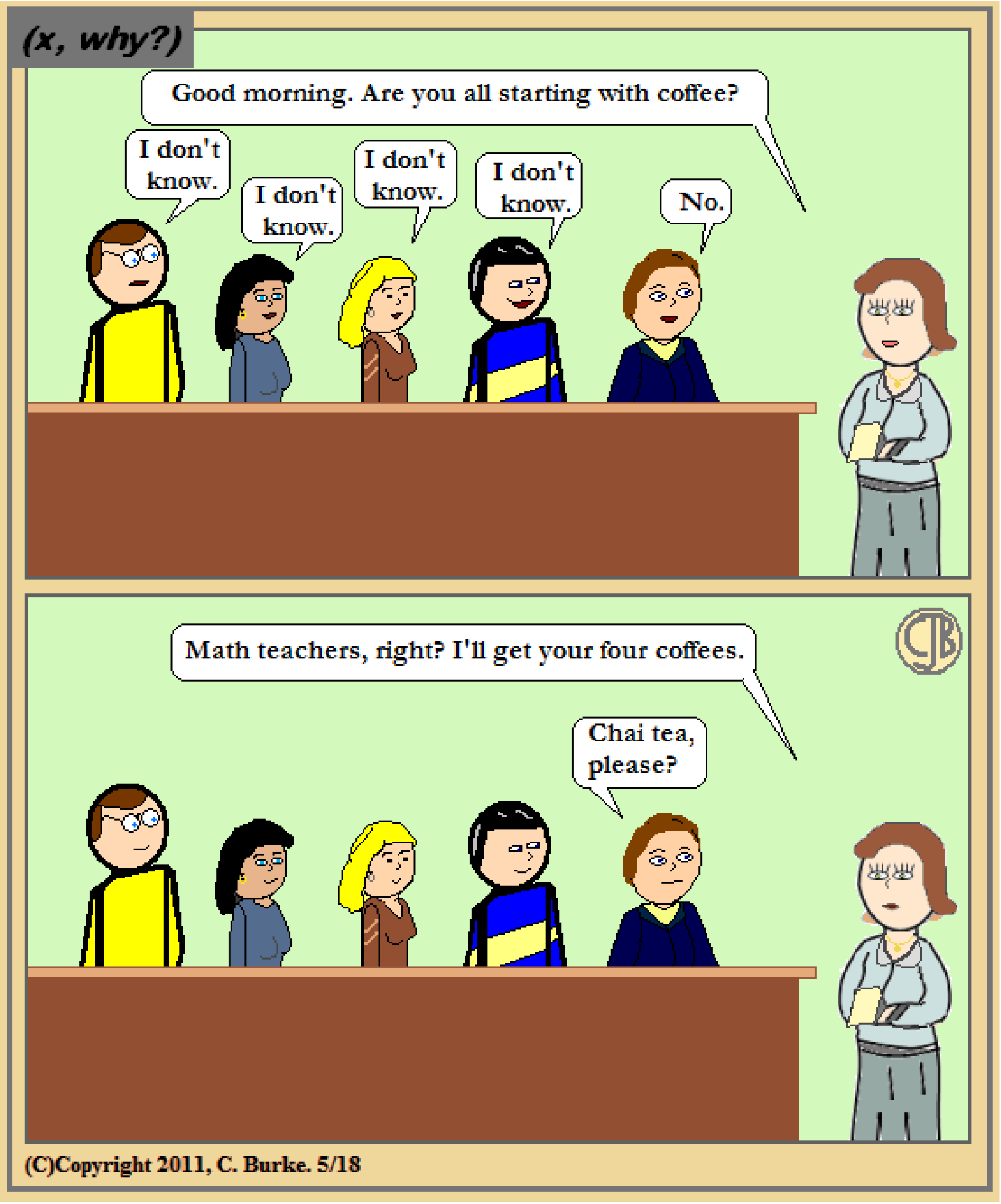
COM1002: Foundations of Computer Science



LEARNING OUTCOME

Universe of Discourse: Set of All Living Things **U** Set of All Night Clubs

Let S(x): "x is a Sheffield student,"
C(x): "x is a Sheffield cheesy nightclub night",
D(x,y): "x will dance in y."

BY THE OF TODAYS LECTURE YOU WILL BE ABLE TO EXPRESS IN PREDICATE LOGIC

"Every Sheffield student will dance in at least one cheesy Sheffield nightclub night."

The Leadmill

DJs spin cheese, pop and party tunes every week at the Leadmill.

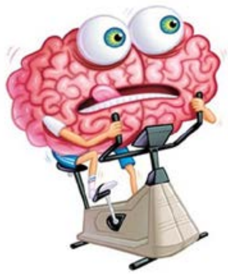
6-7 Leadmill Road,
SHEFFIELD,
S1 4SE

Music Venues

user reviews (2)
cinema listings
club listings

Our rating
User rating (2)

http://www.viewsheffield.co.uk/clubs/cheesy-nights-at-sheffield-clubs-recommended-Sheffield-4365.html



WARM-UP

Let $T(a,b)$ denote the propositional function "a trusts b" Let U be the set of all people in the world.

Everybody trusts Tony. $\forall x T(x,\text{Tony})$

Tony trusts somebody. $\exists x T(\text{Tony},x)$

Tony trusts George.  $T(\text{Tony},\text{George})$ 

Tony trusts everyone trusted by George. $\forall x(T(\text{George},x) \rightarrow T(\text{Tony},x))$

Nobody Trusts Tony. $\forall x \neg T(x,\text{Tony})$ $\neg \exists x\, T(x,\text{Tony})$

Kim trusts nobody. $\forall x \neg T(\text{Kim},x)$ 






P for the predicate "is a politician"
H for the predicate "is honest,"

Let U be the set of all people.

Express with Predicates:

Some politicians are honest.

$$\exists x\left[P(x) \wedge H(x)\right]$$

Some politicians are dishonest.

$$\exists x\left[P(x) \wedge \neg H(x)\right]$$

All politicians are dishonest.

$$\forall x\left[P(x) \Rightarrow \neg H(x)\right]$$

All dishonest people are politicians.

$$\forall x\left[\neg H(x) \Rightarrow P(x)\right]$$

The Frog Puzzle

1. All frogs are green.
2. Some frogs are green.
3. Not all frogs are green.
4. Some frogs are not green.
5. No frogs are green.
6. All frogs are not green.
7. Only frogs are green.
8. All and only frogs are green.
9. Kermit is a green frog.

$F(x)$: x is a frog

$G(x)$: x is green

$K(x)$: x is called Kermit

Universe of Discourse be the set which includes all living things and all green things.

All frogs are green.

$$\forall x\left[F(x) \Rightarrow G(x)\right]$$

Some frogs are green.

$$\exists x\left[F(x) \wedge G(x)\right]$$

Not all frogs are green.

$$\neg\forall x\left[F(x) \Rightarrow G(x)\right]$$

Some frogs are not green.

$$\exists x\left[F(x) \wedge \neg G(x)\right]$$

No frogs are green.

$$\neg\exists x\left[F(x) \wedge G(x)\right]$$

All frogs are not green.

$$\forall x\left[F(x) \Rightarrow \neg G(x)\right]$$

Only frogs are green.

$$\forall x\left[G(x) \Rightarrow F(x)\right]$$

All and only frogs are green.

$$\forall x\left[G(x) \Leftrightarrow F(x)\right]$$

Kermit is a green frog.

$$\exists x\left[F(x) \wedge G(x) \wedge K(x)\right]$$

Quantification 5

Bounded Quantification:

Restricts quantification to a subset of the universe of discourse;

For universal quantification:

- written $\forall x \in A\ P(x)$,
- equivalent to $\forall x\ (x \in A \Rightarrow P(x))$;

For existential quantification:

- written $\exists x \in A\ P(x)$,
- equivalent to $\exists x\ (x \in A \wedge P(x))$;

Quantification 6

Unique Quantification:

Where the predicate is true for exactly one element;

A form of existential quantification:

- written

$$\exists! x\ P(x),$$

- equivalent to

$$\exists x\ (P(x) \wedge \neg \exists y(P(y) \wedge y \neq x)).$$

Rules for Quantification 1

Negation:

- General rules:
  - $\neg \forall x\, P(x) \Leftrightarrow \exists x \neg P(x)$, and
  - $\neg \exists x\, P(x) \Leftrightarrow \forall x \neg P(x)$;
- These are related to De Morgan's Laws:
  - compare $\forall x\, P(x)$ with $P(a) \wedge P(b) \wedge \ldots$,
  - and $\exists x\, P(x)$ with $P(a) \vee P(b) \vee \ldots$.

Rules for Quantification 2

Conjunction and Disjunction:

Equivalences:

- $\forall x\,(P(x) \wedge Q(x)) \Leftrightarrow (\forall x\, P(x)) \wedge (\forall x\, Q(x))$,
- $\exists x\,(P(x) \vee Q(x)) \Leftrightarrow (\exists x\, P(x)) \vee (\exists x\, Q(x))$,
- from the same comparisons;

Examples of non-equivalences:

- assume x is an arbitrary integer: we have
- $(\exists x\, \mathrm{Prime}(x)) \wedge (\exists x\, \mathrm{Square}(x))$,
- but $\exists x\,(\mathrm{Prime}(x) \wedge \mathrm{Square}(x))$ is false;

Rules for Quantification 3

Examples of non-equivalences (cont.):

- similarly, we have $\forall x\ (\mathrm{Even}(x) \vee \mathrm{Odd}(x))$,
- but $(\forall x\ \mathrm{Even}(x)) \vee (\forall x\ \mathrm{Odd}(x))$ is false;

Properties for these cases:

- $\exists x\ (P(x) \wedge Q(x)) \Rightarrow (\exists x\ P(x)) \wedge (\exists x\ Q(x))$;
- $(\forall x\ P(x)) \vee (\forall x\ Q(x)) \Rightarrow \forall x\ (P(x) \vee Q(x))$.

Rules for Quantification 4

Re-ordering Quantifiers:

Equivalences:

- $\forall x\ \forall y\ P(x, y) \Leftrightarrow \forall y\ \forall x\ P(x, y)$, and
- $\exists x\ \exists y\ P(x, y) \Leftrightarrow \exists y\ \exists x\ P(x, y)$;

A non-equivalence:

- $\forall x\ \exists y\ P(x, y)$ is not equivalent to $\exists y\ \forall x\ P(x, y)$.

- For example, let K(x,y) be x has kissed y

And interpret the following predicates…

$\forall x\ \exists y\ K(x, y)$;　　　　$\exists y\ \forall x\ K(x, y)$

Express:

1. All bees like all flowers

2. Bees only like flowers

3. Only bees like flowers

Using the **predicates:**

B(x) = "x is a bee"

F(x) = "x is a flower"

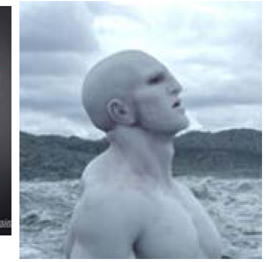
L(x,y) = "x likes y"

For solution see book…

E(x): "x is an earth-like planet."

L(x): "x supports life."

Universe of Discourse be the set of all planets

E(x): "x is an earth-like planet."

L(x): "x supports life."

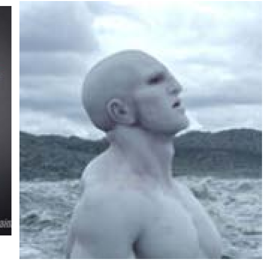
What do the following statements say ?

$$(a) \quad \forall x\left[L(x) \Rightarrow E(x)\right]$$

"Every earth-like planet supports life."
"Every life-supporting planet is earth-like."
"Some life-supporting planets are not earth-like."
"Some earth-like planets do not support life."
"All planets support life or are earth-like."

 $\Longrightarrow$ 



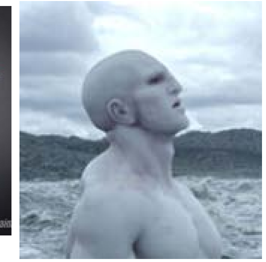



$$\forall x E(x) \vee \forall x L(x)$$

"All planets are either earth-like or support life."
"Either all planets are earth-like, or all of them support life."
"All planets are earth-like and support life."
"Some planets are earth-like; others support life."
"A planet is either earth-like or it is not."

$\vee$



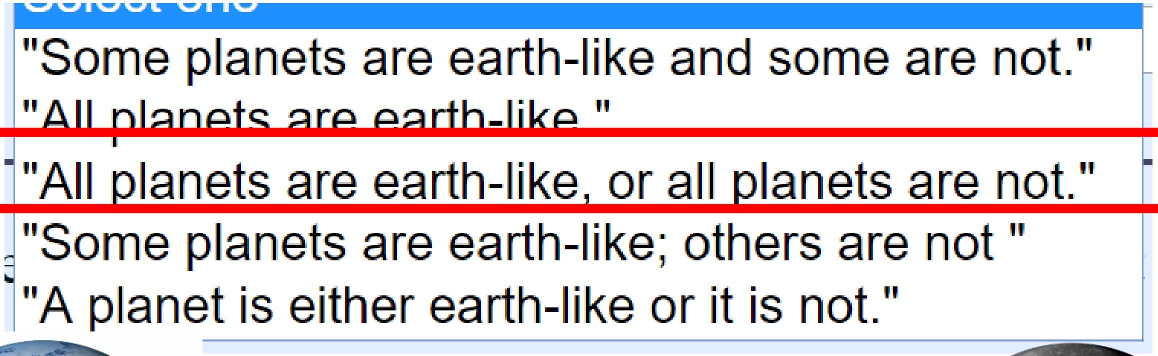



$$\forall x\left[E(x) \vee \neg E(x)\right]$$

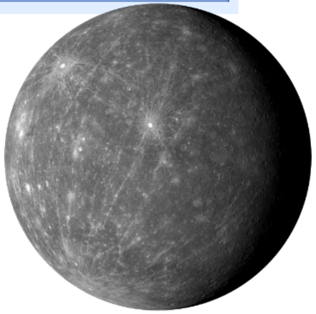
"Some planets are earth-like."
"All planets are earth-like."
"All planets are earth-like, or all planets are not."
"Some planets are earth-like; others are not "
"A planet is either earth-like or it is not."



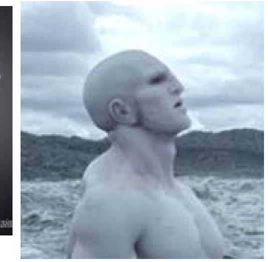

$$\forall x E(x) \vee \forall x \neg E(x)$$



$\vee$

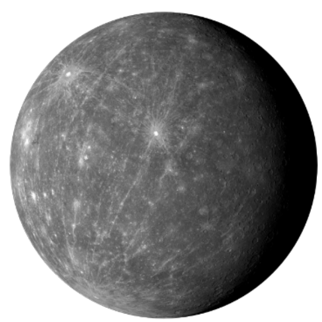




$$\neg\forall x\left[E(x)\vee L(x)\right]$$

"Some planets are neither earth-like nor support life."
"Not all planets are both earth-like and support life"
"All planets are neither earth-like nor support life."
"All planets are either not earth-like or do not support life."
"A planet is either earth-like or supports life."



Universe of Discourse: Set of All Living Things U Set of All Night Clubs

Let S(x): "x is a student,"
C(x): "x is a cheesy nightclub",
D(x,y): "x will dance in y."

"Every student will dance in at least one cheesey nightclub."

$$\forall x\left[S(x) \wedge \left(\exists y\left[C(y) \wedge D(x,y)\right]\right)\right]$$

$$\forall x\left[\left(\exists y\left[C(y) \wedge D(x,y)\right]\right)\right]$$

$$\forall x\left[S(x) \vee \left(\exists y\left[C(y) \wedge D(x,y)\right]\right)\right]$$

$$\boxed{\forall x\left[S(x) \Rightarrow \left(\exists y\left[C(y) \wedge D(x,y)\right]\right)\right]}$$

Exercise

EXPRESS AS PREDICATES.

Some Sheffield students will dance in every Sheffield cheesy nightclub night.

Some Sheffield students will dance in no Sheffield cheesy nightclub nights