COM1002
Foundations of Computer Science

2. Sets

LEARNING OUTCOME:

All babies are illogical

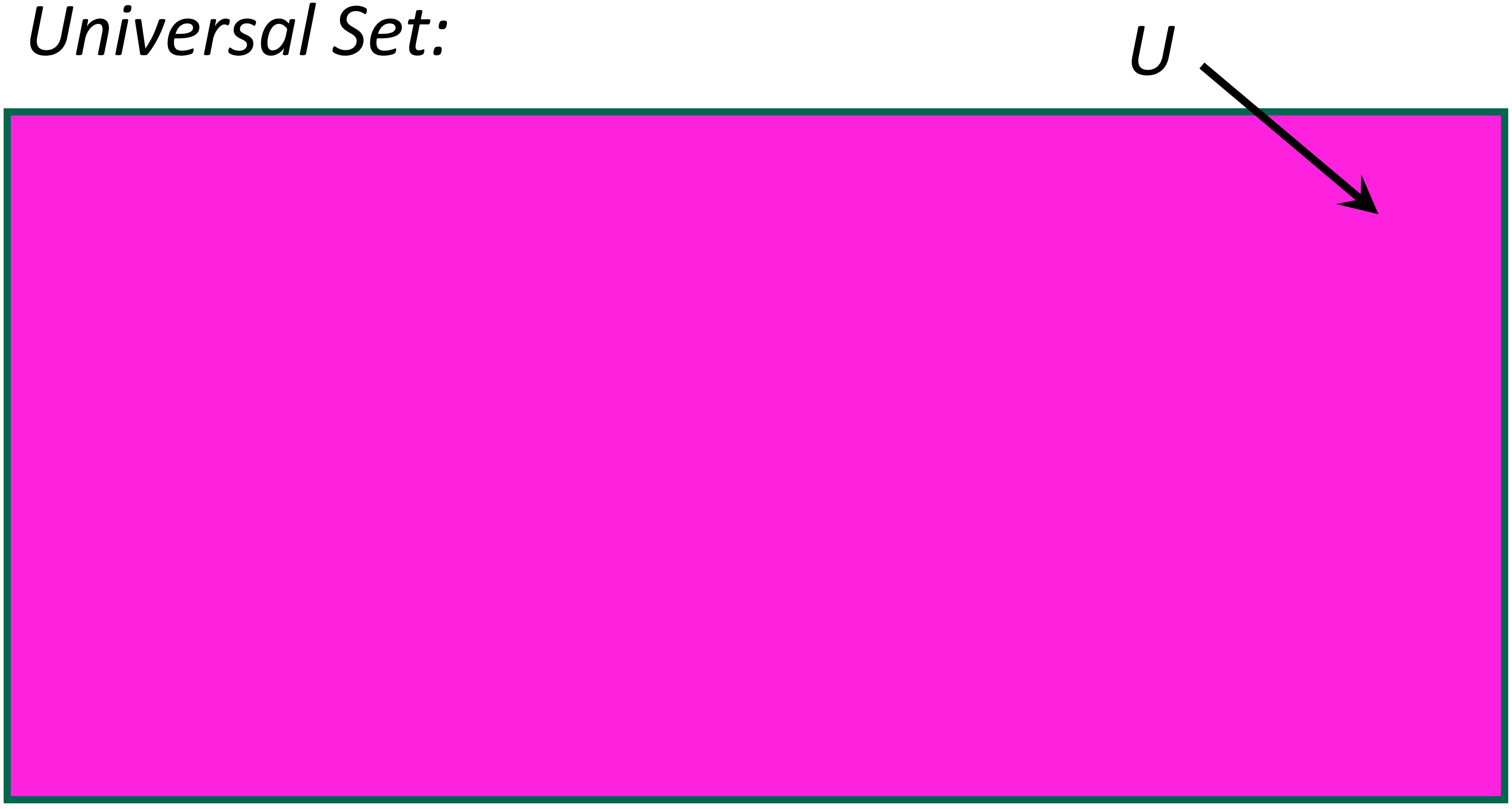
Nobody is despised who can manage a crocodile

Illogical persons are despised

We can solve with propositional Logic

We can also solve with Set Theory

Universal Set:



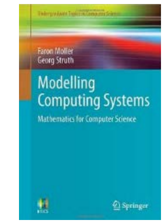
A **universal set** is the collection of all objects in a particular context or theory. All other **sets** in that framework constitute subsets of the **universal set**, which is denoted as an uppercase italic letter *U*. The objects themselves are known as elements or members of *U*.

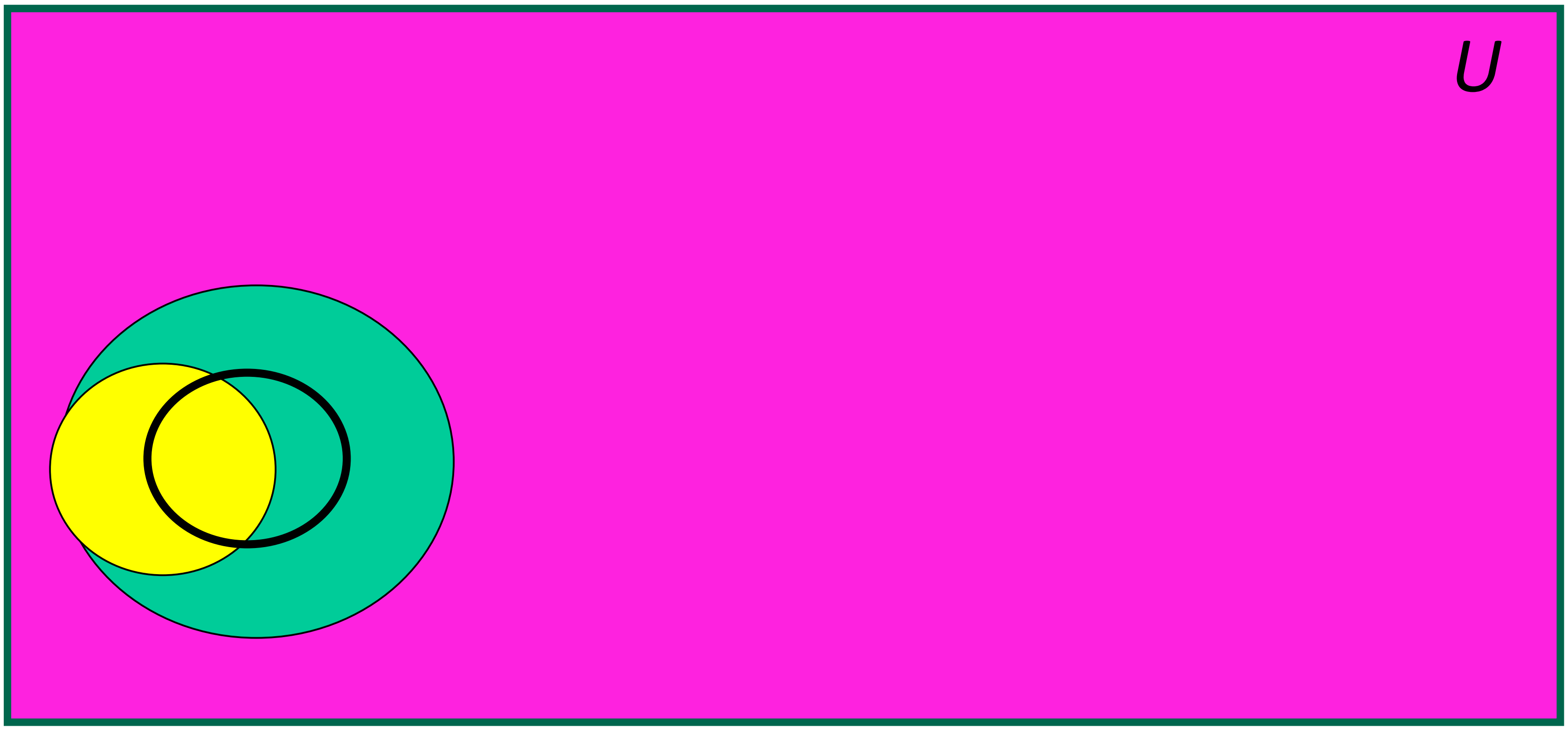
Universal Set: All students at University of Sheffield

S: First year Students registered for COM1002

C: Students in this lecture

B: Students who have lecture who obtained class textbook





$S \subset U$ $\quad C \subseteq S$ $\quad B \subseteq S$

Course Overview

Textbooks and lecture notes:

❑ *A class textbook:*

*"**Modelling Computing Systems: Mathematics for Computer Science**", by Faron Moller & Georg Struth;*

Follows chapters 1-10

Propositional Logic

Set Theory

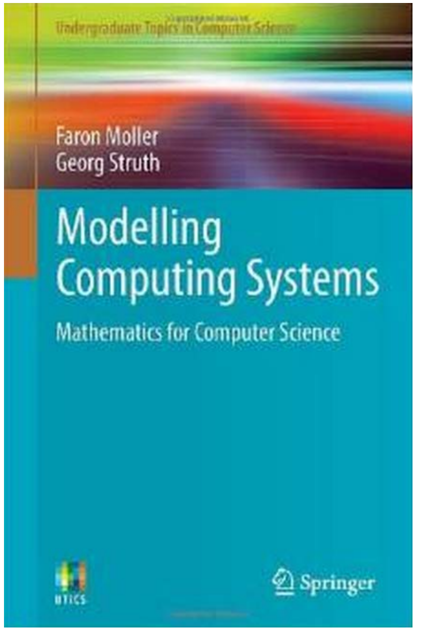
Boolean Algebra

Predicate Logic

Proof strategies

Functions and Relations

Induction and Recursion



COM 1002: Foundations of Computer Science

Exercise Sheet 1

This sheet will be marked and account for 5% of your mark for this Semester. You need to hand it in in the tutorial on Thursday 15th October.

Exercise 1. Huey, Dewey and Louie have signs with their names on their bedroom doors. One day, when their uncle Donald comes to visit, they mix up the signs and their mother tells Donald that no door has its correct sign. Huey, Dewey and Louie are all sitting quietly in their own rooms. Donald is challenged to put the signs into their proper places, but is only allowed to open one door to look who is inside.

1. Give a propositional formula which models this situation. Use the propositional variable XY to express that Name X is at Y's door. For instance, HD states that the sign "Huey" is on Dewey's door. [10%]
2. Suppose Donald opens the door with the sign "Huey" and finds Louie. Can you deduce the correct door for each sign? [10%]

HINT: See exercise 1.16 in the course text book 'Modelling Computer Systems'

Exercise 2. Use truth tables to test whether the following formulas are valid.

1. $P \Rightarrow (Q \Rightarrow P)$. [10%]
2. $((P \Rightarrow (Q \Rightarrow R)) \Rightarrow ((P \Rightarrow Q) \Rightarrow (P \Rightarrow R)))$. [10%]
3. $(\neg P \Rightarrow \neg Q) \Rightarrow (Q \Rightarrow P)$. [10%]
4. $P \vee (P \wedge Q) \Leftrightarrow P$. [10%]
5. $P \vee Q \Leftrightarrow \neg(\neg P \wedge \neg Q)$. [10%]
6. $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$. [10%]

NOTE: A propositional logic formula is *valid* if, and only if, it is *TRUE* for all possible interpretations of the propositional variables (each proposition variable can be *TRUE* or *FALSE*).

Exercise 3. 1. Formalise each of the following statements in propositional logic. [10%]

(1) The only animals in this house are cats.
(2) Every animal that loves to gaze at the moon is suitable for a pet.
(3) When I detest an animal, I avoid it.
(4) No animals are carnivorous, unless they prowl at night.
(5) No cat fails to kill mice.
(6) No animal ever takes to me, except those that are in this house.
(7) Kangaroos are not suitable for pets.
(8) None but carnivora kill mice.
(9) I detest animals that do not take to me.
(10) Animals that prowl at night always love to gaze at the moon.

2. Argue that they imply *I always avoid a kangaroo*. [10%]
(Hint: Using a truth table might be very time consuming...)

1

Exercise Sheet 1 – Now on MOLE

Assessed – 5% of Mark for this Semester

Hand in on Thursday in tutorial.

Operations on Sets 1

Union:

- For sets A and B:
  - written $A \cup B$,
  - elements that are in A, or B, or both,
  - $A \cup B = \{ x : x \in A \text{ or } x \in B \}$.
- So that:
  - $(x \in A \cup B) \Leftrightarrow (x \in A) \vee (x \in B)$.

Operations on Sets 2

Intersection:

- For sets A and B:
  - written $A \cap B$,
  - elements that are in both A and B,
  - $A \cap B = \{ x : x \in A \text{ and } x \in B \}$.
- So that:
  - $(x \in A \cap B) \Leftrightarrow (x \in A) \wedge (x \in B)$;
- Two sets A and B are disjoint if they have no elements in common:
  - ie $A \cap B = \emptyset$.

Operations on Sets 3

Difference:

- For sets A and B:
  - written $A \setminus B$,
  - elements that are in A but not in B,
  - $A \setminus B = \{ x \in A : x \notin B \}$.
- So that:
  - $(x \in A \setminus B) \Leftrightarrow (x \in A) \wedge (x \notin B)$.

Operations on Sets 4

Complement:

- For some set A and a universe set:
  - written $\overline{A}$,
  - elements that are not in A (but in the universe),
  - $\overline{A} = \{ x : x \notin A \}$, or
  - $\overline{A} = \{ x \in \text{universe} : x \notin A \}$.
- So that:
  - $(x \in \overline{A}) \Leftrightarrow (x \notin A)$.

Exercise 2.16

Let A, B, C be sets

1. If $A \subseteq B$ what can you say about $(A \cup B)$ and $(A \cap B)$?
2. What is $\overline{\overline{A}}$, the double-complement of A?
3. If $C \subseteq A$ and $C \subseteq B$, how is C related to $(A \cap B)$?
4. If $A \subseteq C$ and $B \subseteq C$, how is C related to $(A \cup B)$?

Operations on Sets 5

Powerset $\mathcal{P}$ (A)**:**

For a set A:

- written $\mathcal{P}$ (A),
- elements are subsets of A,
- $\mathcal{P}(A) = \{ X : X \subseteq A \}$.

- So that:
  - $(X \in \mathcal{P}(A)) \Leftrightarrow (X \subseteq A)$, and
  - $\varnothing \in \mathcal{P}(A)$, and
  - $A \in \mathcal{P}(A)$.

- Try exercise 2.17.

Exercise 2.17

Simon has invited the following five friends to his birthday party: *Louis, Cheryl, Mel, Danni, Tulisa*. However, some of them might not show up. If we let

$$\text{Friends} = \{\text{Louis}, \text{Cheryl}, \text{Mel}, \text{Danni}, \text{Tulisa}\}$$

then the collection of combinations of friends that might come to Simon's party is given by the Powerset: $P(\text{Friends})$

However, perhaps *Louis* is busy getting his hair done and *Cheryl* is visiting Ashley, but the others will come; then the set of friends that will come to Simons' party is

$$\{\text{Mel}, \text{Danni}, \text{Tulisa}\} \in P(\text{friends})$$

Or everyone decides Simon's party is boring and they don't want to go, then the set of friends that come to the party is the **empty set**.

List the elements of $P(\text{Friends})$

How many sets of each size are there ?

Modelling with Sets

Approach:

- Define an appropriate universe set;
- Represent the problem's statements:
  - either by simple sets,
  - or by set operations;
- Construct Venn diagrams:
  - to represent relationships between the sets.

Illustrate with example 2.24.

Try exercises 2.25

Modelling with Sets (Ex 2.24)

All candy has sugar

John eats only healthy foods

No healthy foods contain sugar

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(C)

(H)

(S)

(J)

$$C \subseteq S$$

$$J \subseteq H$$

$$\left(S \cap H\right) = \varnothing$$

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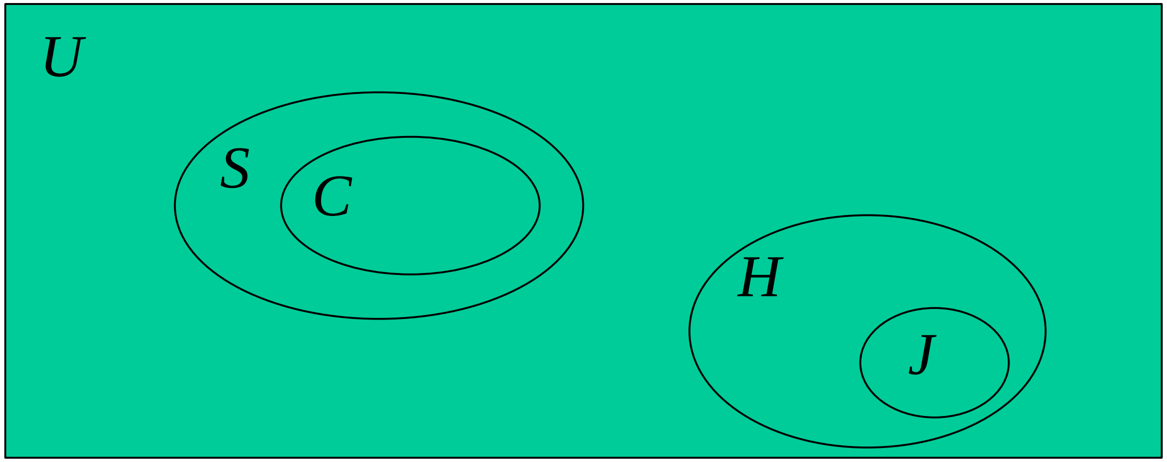

Solving puzzles with Set Theory

All babies are illogical

Nobody is despised who can manage a crocodile

Illogical persons are despised

We can solve with Venn Diagrams!

Solving puzzles with Set Theory

All babies are illogical

Nobody is despised who can manage a crocodile

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What is the Universe?

What are the subsets ?

What are the relationships between the subsets?

U = Set of people

Set of babies (B)

Set of all illogical (I) persons

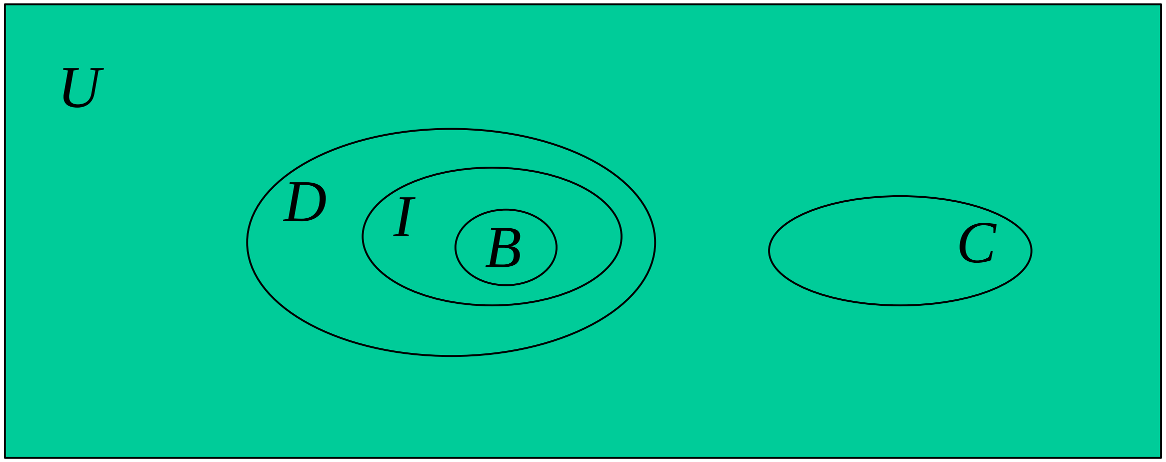
Set of all despised (D) persons

Set of all persons who can manage a crocodile (C)

$$B \subseteq I$$

$$I \subseteq D$$

$$\left(D \cap C\right) = \varnothing$$



It is clear from this that no baby can manage a crocodile, as a baby would be illogical and hence despised; and no despised person, such as this baby, can manage a crocodile.

LEARNING OUTCOME:

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We can now do this!?