# COM1002
# Foundations of Computer Science


## Preparation for MOLE QUIZ 3

| WEEK | 1,2,3,4,5,6,7,8,9 | | 10 | 11 |
|---|---|---|---|---|
| **Mon** | | | Lecture | **REVISION LECTURE** |
| **Wed** | | | Lecture<br>**Hand out ex 7**<br><br>(Assessed 5%) | **NO LECTURE** |
| **Thurs** | | | Tut (ex 7) | **No Tutorial**<br><br>**QUIZ 3 (25%)**<br>Diamond 201<br>4pm-5pm<br>**Hand in ex 7**<br><br><br>5pm-6pm<br>(lecture clashes) |

Please don't enter room until 4pm if there are signs which say do not enter (tutorial class 3pm-4pm)

# MOLE QUIZ 3

**Thursday 10th December, 4pm-5pm**

- 8 multiple choice questions

    3 relations (Chapter 6)

    3 functions (Chapter 7)

    2 BNF grammars (Chapter 8)

# EXERCISES

Final exercise sheet uploaded onto MOLE.

Worth 5%

Hand in at MOLE QUIZ 3.

## Need to be able to understand and apply knowledge of terminology of Functions

domain co-domain graph image

Total function Partial function bijection

injection surjection pre-image

one-to-one composite functions

# Need to be able to understand and apply Knowledge of Terminology of Relations

Heterogeneous Homogeneous

Reflexive irreflexive

Symmetric antisymmetric asymmetric

transitive

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partial order total order equivalence

equivalence class

# BNF Grammars

A few subtleties and what we need to know…

# Basic Concepts 1

Defining Sets:

- **Finite sets** can be defined easily:
  - by listing their elements;
- This is not possible for infinite sets:
  - we use the inductive approach instead;
- An inductive definition has three parts:
  - **a basis clause**, for the “starting” elements,
  - **an inductive clause**, to build other elements,
  - **an extremal clause**, to eliminate unwanted elements.

# Basic Concepts 3

## **Syntactic Sets**:

- **Syntactic sets** can be defined in terms of:
  - an **alphabet** of symbols (characters),
  - **words** (strings) - sequences of symbols,
    - including the empty word, denoted $\varepsilon$,
  - and **sentences** - sequences of words.

# Basic Concepts 4

## Sets of Words:

- $A^*$ is the set of **words** over the **alphabet** $A$:
  - so $A^*$ is the smallest set such that
  - $\varepsilon \in A^*$, and if $w \in A^* \wedge a \in A$ then $aw \in A^*$,
    - this use of * is sometimes called the **Kleene star**;
- $\mathbf{A^+}$ is the set of **non-empty** words over $A$;
  - so $A^+$ is the smallest set such that
  - $a \in A \Rightarrow a \in \mathbf{A^+}$, and
  - if $w \in \mathbf{A^+} \wedge a \in A$ then $aw \in \mathbf{A^+}$.

# Basic Concepts 5

## Backus-Naur Form (BNF):

- a notation for writing inductive definitions:
  - as equations of the form:
    - **set_name :: = basis clauses | inductive clauses**
- e.g for $A^*$:
  - w ::= ε | aw
- e.g for $A^+$:
  - w ::= a | aw
- e.g for $\mathbf{N}$:
  - n ::= 0 | successor (n)

# BNF grammars

The set of propositional formulae can be defined inductively as the smallest set satisfying the following:

1. True and false are propositional formulae, as is every propositional variable P
2. If p and q are propositional formulae then so are

$$\neg p,\ p \vee q,\ p \wedge q,\ p \Rightarrow q \quad \text{and} \quad p \Leftrightarrow q$$

$$\phi ::= \underbrace{true \mid\ false \mid\ P}_{\text{Basis clauses}} \mid\ \underbrace{\neg\phi \mid\ \phi \wedge \phi \mid\ \phi \vee \phi \mid\ \phi \Rightarrow \phi \mid\ \phi \Leftrightarrow \phi}_{\text{Inductive clauses}}$$

Give an inductive definition of the set of formulae of predicate logic.

$$\phi ::= true \mid\ false \mid\ P(x_1,\ldots,x_n) \mid\ \neg\phi \mid\ \phi \wedge \phi \mid\ \phi \vee \phi \mid\ \phi \Rightarrow \phi \mid\ \phi \Leftrightarrow \phi \mid\ \forall x\phi \mid\ \exists x\phi$$

(*x* is taken to range over all variables)

Define all the words (ie strings) of up to 3 character in length constructed from the letters a, b, c.  These words need to have a particular form, which is defined by the following BNF grammar, where S denotes the permitted word structures.

$$S ::= \underbrace{a \mid b \mid c}_{\text{Basis clauses}} \mid \underbrace{aS \mid Sb \mid cS \mid}_{\text{Inductive clauses}}$$