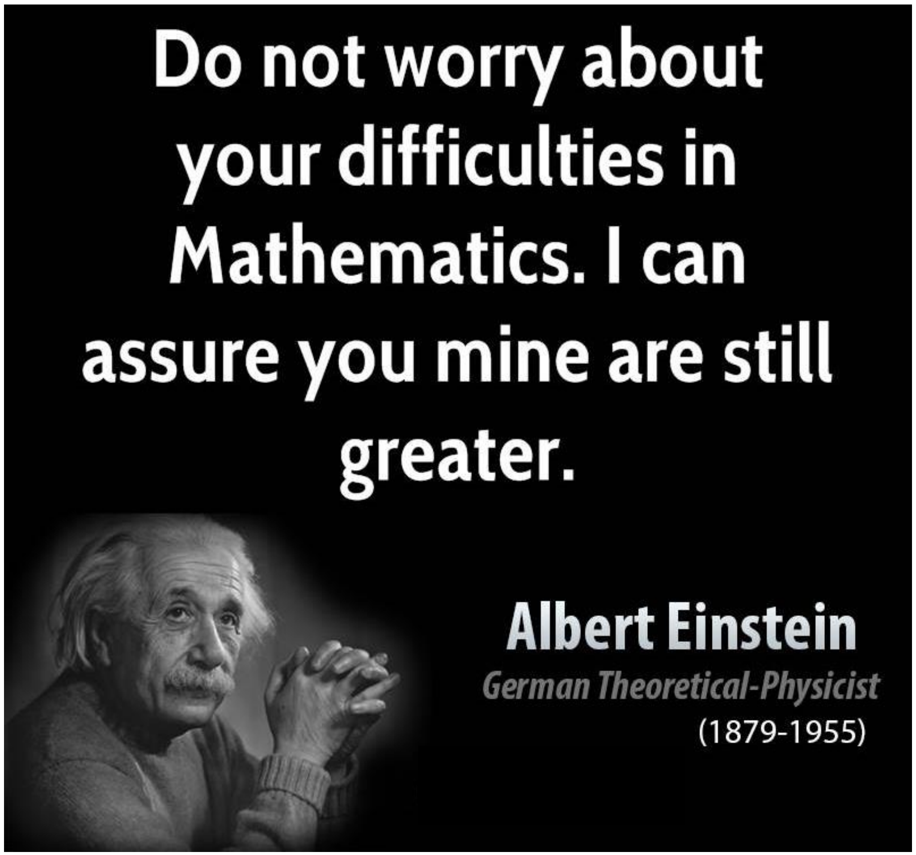
COM1002
Foundations of Computer Science



C7: Relations

WEEK	4	5	6	7
Mon				Lecture **Handout Ex5** (Assessed 5%)
Wed				Lecture
Thurs				Tut (ex 5)

WEEK	8	9	10	11
Mon	Lecture **Hand in Ex 5** **Handout Ex6** (Assessed 5%)	Lecture **Hand in Ex 6**	Lecture **Hand out ex 7** (Assessed 5%)	**Revision Lecture** **Hand in Ex 7**
Wed	Lecture	**Revision Lecture**	Lecture	**Revision Lecture**
Thurs	Tut (ex 6)	**Revision Tutorial** **QUIZ 2 (25%)** Diamond 101 4pm-5:30pm	Tut (ex 7)	**Revision Tutorial** **QUIZ 3 (25%)** Diamond 101 4pm-5:30pm

Quiz 1: 2013/2014/2015

Watton Lectures…

1BW
Before-Watton
(2013): (70 students)

Full marks:8
Failed: 16

2014:
101 students

Greater than 100	0
90 - 100	17
80 - 89	24
70 - 79	16
60 - 69	14
50 - 59	12
40 - 49	8
30 - 39	2
20 - 29	0
10 - 19	2
0 - 9	6

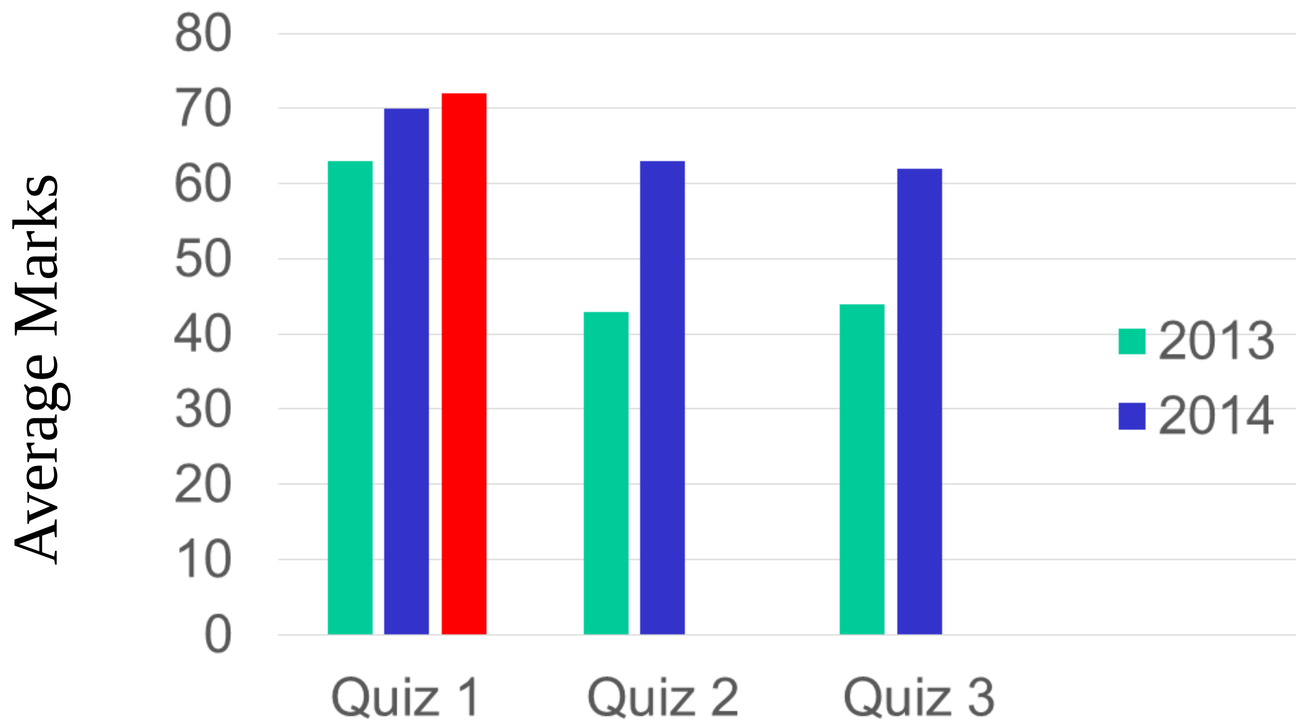
Full marks:17
Failed: 2
No show: 6

2015:
119 students

90 - 100	33
80 - 89	27
70 - 79	16
60 - 69	18
50 - 59	13
40 - 49	3
30 - 39	1
20 - 29	0
10 - 19	0
0 - 9	7

Full marks: 25
Failed: 1 (32/90)
No show: 6
Left Course: 1

Quiz Results 2013, 2014, 2015



COM1002: Difficult material. Potential uninteresting for some…

However, important to attend lectures and attend MOLE quizzes…

Question 1: Multiple Choice

In the truth table below, each row and column has been numbered. Exactly one of the entries in the truth table is incorrect, on the basis that all other entries have been calculate accurately describes the incorrect entry.

	1	2	3	4	5	6	7	8
	P	Q	R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$	$P \vee \neg R$	$(P \vee \neg R) \Rightarrow Q$	$((P \wedge Q) \Rightarrow R) \wedge ((P \vee \neg R) \Rightarrow Q)$
1	F	F	F	F	T	T	F	F
2	F	F	T	F	T	F	T	T
3	F	T	F	F	F	T	T	T
4	F	T	T	F	T	F	T	T
5	T	F	F	F	T	T	F	F
6	T	F	T	F	T	T	F	F
7	T	T	F	T	F	T	T	F
8	T	T	T	T	T	T	T	T

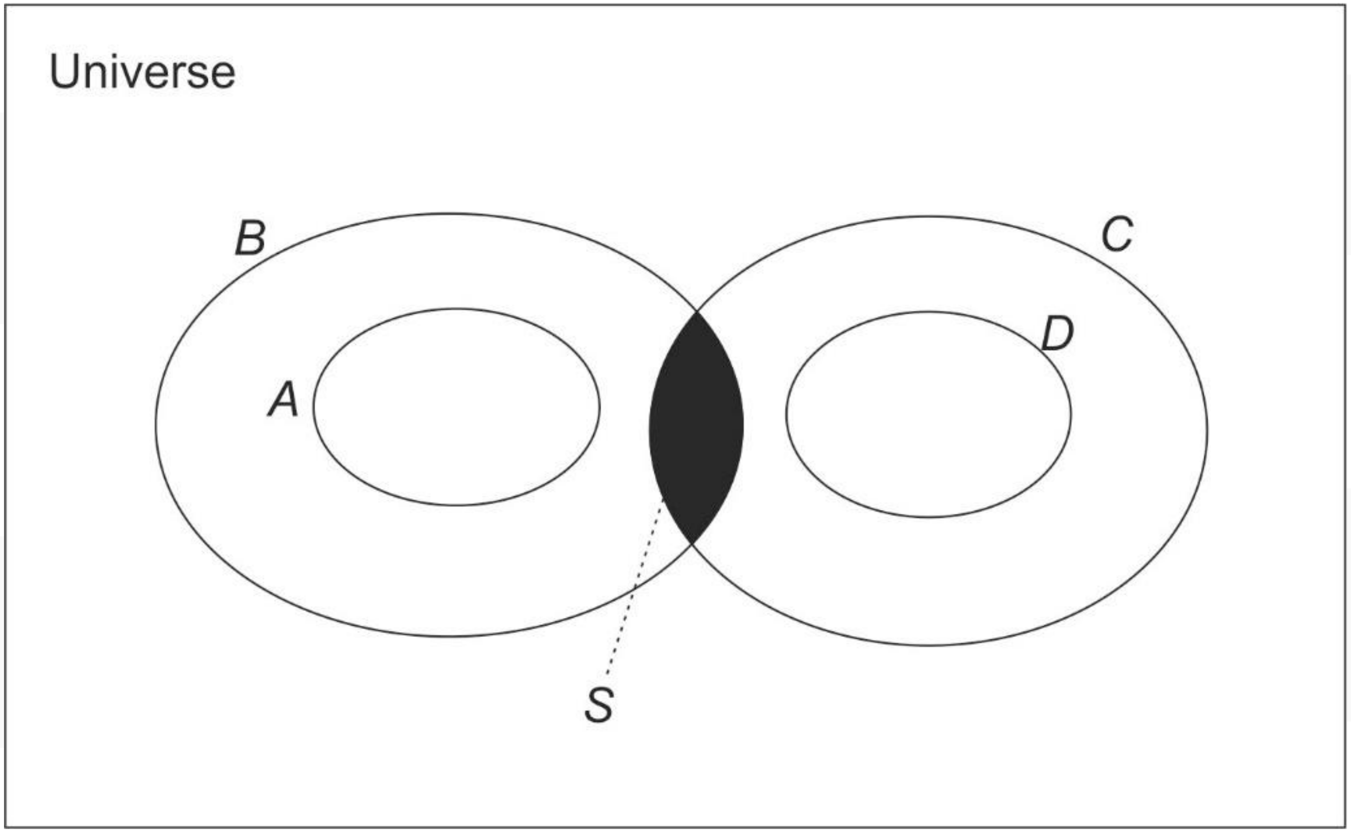
Correct		Per cent Answered
✓	The entry in column 5 of row 3 is the only incorrect one.	84.685%
	The entry in column 7 of row 3 is the only incorrect one.	0%
	The entry in column 7 of row 5 is the only incorrect one.	0%
	The entry in column 8 of row 7 is the only incorrect one.	0%
	The entry in column 8 of row 3 is the only incorrect one.	13.514%
	The entry in column 8 of row 5 is the only incorrect one.	0.901%

accurately describes the incorrect entry.

	1	2	3	4	5	6	7	8
	P	Q	R	$P \oplus Q$	$(P \oplus Q) \wedge R$	$P \oplus \neg R$	$(P \oplus \neg R) \vee Q$	$((P \oplus Q) \wedge R) \vee ((P \oplus \neg R) \vee Q)$
1	F	F	F	F	F	T	T	T
2	F	F	T	F	F	F	F	F
3	F	T	F	T	F	T	T	T
4	F	T	T	T	T	F	T	T
5	T	F	F	T	F	T	F	F
6	T	F	T	T	T	T	T	T
7	T	T	F	F	F	F	T	T
8	T	T	T	F	F	T	T	T

- ✅ The entry in column 6 of row 5 is the only incorrect one. 89.189%
- The entry in column 7 of row 5 is the only incorrect one. 8.108%
- The entry in column 8 of row 5 is the only incorrect one. 1.802%
- The entry in column 8 of row 7 is the only incorrect one. 0%
- The entry in column 4 of row 8 is the only incorrect one. 0.901%
- The entry in column 5 of row 8 is the only incorrect one. 0%

Correct



$A \subseteq B$
$D \subseteq C$
$S = B \cap C$
$A \cap C = \varnothing$
$B \cap D = \varnothing$

73.874%

$A \subseteq B$
$C \subseteq D$
$S = B \cap C$
$A \cap C = \varnothing$
$B \cap D = \varnothing$

1.802%

$A \subseteq B$
$D \subseteq C$
$S = B \cup C$
$A \cap C = \varnothing$
$B \cap D = \varnothing$

0.901%

$A \subseteq B$
$D \subseteq C$
$S = B \cap C$
$A \cap D = \varnothing$

19.82%

$B \subseteq A$
$D \subseteq C$
$S = B \cap C$
$A \cap C = \varnothing$
$B \cap D = \varnothing$

0%

$A \subseteq B$
$D \subseteq C$
$S = B \cap C$
$A \cap B = \varnothing$
$B \cap D = \varnothing$

3.604%

For the set model that is given below of some system, which one of the following Venn di

$A \subseteq B$

$B \subseteq C$

$D \subseteq C$

$A \cap D = \varnothing$

$B \cap D \neq \varnothing$

Diagram 1

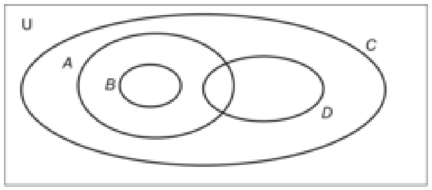


Diagram 2

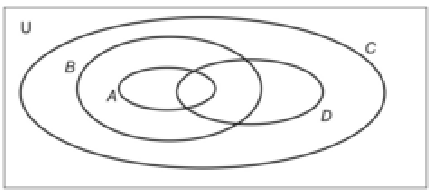


Diagram 3

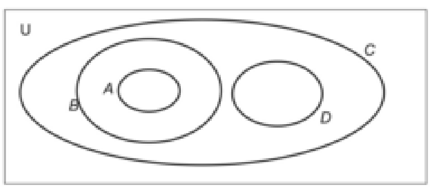


Diagram 4

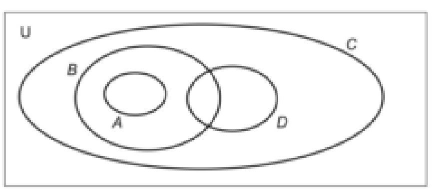


Diagram 5

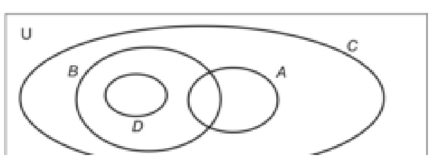


Diagram 4

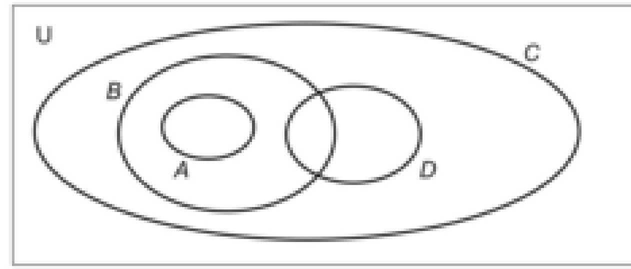
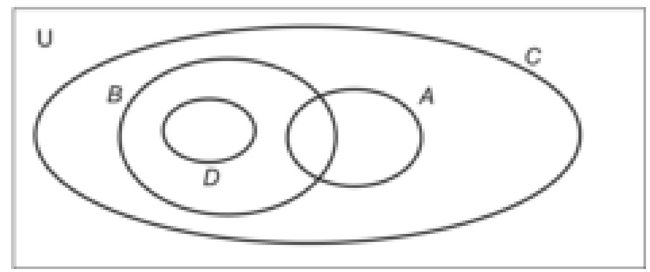


Diagram 5



Correct

✔	Diagram 4	83.784%
	Diagram 1	0%
	Diagram 2	0%
	Diagram 3	16.216%
	Diagram 5	0%

Terminology of Relations

Heterogeneous Homogeneous

Reflexive irreflexive

Symmetric antisymmetric asymmetric

transitive

---

partial order total order equivalence

equivalence class

Motivation

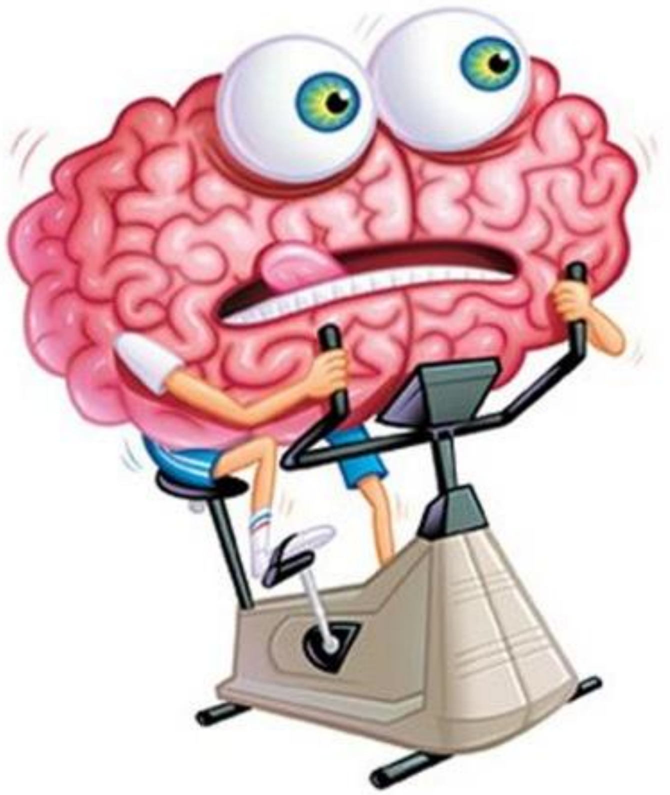
2012 Summer Olympics

Gold	Silver	Bronze	Total
46	29	29	104
38	27	23	88
29	17	19	65
24	26	32	82
13	8	7	28
11	19	14	44
11	11	12	34

How do we order the data in the table?

We need to define a relation

Warm-up: Ex 7.14



Consider the relationships:

- ***Is-an-ancestor-of***
- ***Is-married-to***

defined over people.

Indicate whether these are:

- **reflexive**
- **Irreflexive**
- **Symmetric**
- **anti-symmetric**
- **transitive**.

try exercise 7.14.

for any relation $R \subseteq A \times A$. R is:

- **reflexive** iff $\forall x \in A\ (xRx)$,
- **irreflexive** iff $\forall x \in A\ \neg(xRx)$:
- **symmetric** iff $\forall x,y \in A\ (xRy \Rightarrow yRx)$
- **antisymmetric** iff $\forall x,y \in A\ (xRy \wedge yRx \Rightarrow x = y)$:

or EQUIVALENTLYif $R(a,b)$ with $a \neq b$, then $R(b,a)$ must not hold.

- **transitive** iff $\forall x, y, z \in A\ ((x\ R\ y) \wedge (y\ R\ z) \Rightarrow x\ R\ z)$

	Is-an-ancestor-of	*Is-married-to*
Reflexive ?	**NO**	**NO**
Irreflexive ?	**YES**	**YES**
Symmetric ?	**NO**	**YES**
Antisymmetric ?	**YES**	**NO**
Transitive ?	**YES**	**NO**

Binary Relations

Operations on Relations:

- Boolean operations:
  - viz **union, intersection, set difference**:
    - – apply to the set of pairs, eg:
    - – Parent = Father $\cup$ Mother, Father = Parent \ Mother;
- Inversion:
  - Given $R \subseteq A \times B$, we have
  - $R^{-1} \subseteq B \times A = \{ (b, a) : (a, b) \in R \}$.

Binary Relations

Composing Relations:

- For $R \subseteq A \times B$ and $S \subseteq B \times C$:
  - the composition of S and R is $S \circ R \subseteq A \times C$,
  - $S \circ R = \{ (a, c) : \exists\, b \in B\ ((a, b) \in R \wedge (b, c) \in S) \}$:
    - eg grandfather = Father ° Parent;
- Domain and Range:
  - $\mathrm{dom}(R) = \{ a \in A : \exists\, b \in B\ ((a, b) \in R) \}$:
    - so implicitly relations are always **total**;
  - $\mathrm{ran}(R) = \{ b \in B : \exists\, a \in A\ ((a, b) \in R) \}$.

Composite Relations
(may be useful for MOLE quizzes!)

S={a,b,c,d}

R1={(a,b),(b,c),(c,d),(d,a)}
R2 = {(a,d),(b,a),(c,b),(d,c)}
R3 = {(c,d)}

What are the composite relations:

i) R1oR2 ?
ii) R2oR2 ?
iii) R1\R3 ?

Classes of Binary Relations 1

Similarities with Classes of Functions:

- for $R \subseteq A \times B$, $(a1, b1) \in R$, $(a2, b2) \in R$:
- $a1 = a2 \Rightarrow b1 = b2$ means R is a **function**,
  - if we also have $a1 = a2 \Leftrightarrow b1 = b2$ it is one-to-one, i.e. an **injective** function,
  - otherwise it is **many-to-one**;
- If $\neg\,(a1 = a2 \Rightarrow b1 = b2)$,
  - $b1 = b2 \Rightarrow a1 = a2$ means R is **one-to-many**,
  - otherwise it is **many-to-many**.

Classes of Binary Relations 2

Order Relations:

- For any relation $R \subseteq A \times A$:
  - R is a **partial order** iff it is ***reflexive***, ***antisymmetric*** and ***transitive***;
  -
  - R is a **total order** iff it is a ***partial order***, **and** $\forall x, y \in A\ ((x\ R\ y) \vee (y\ R\ x))$:

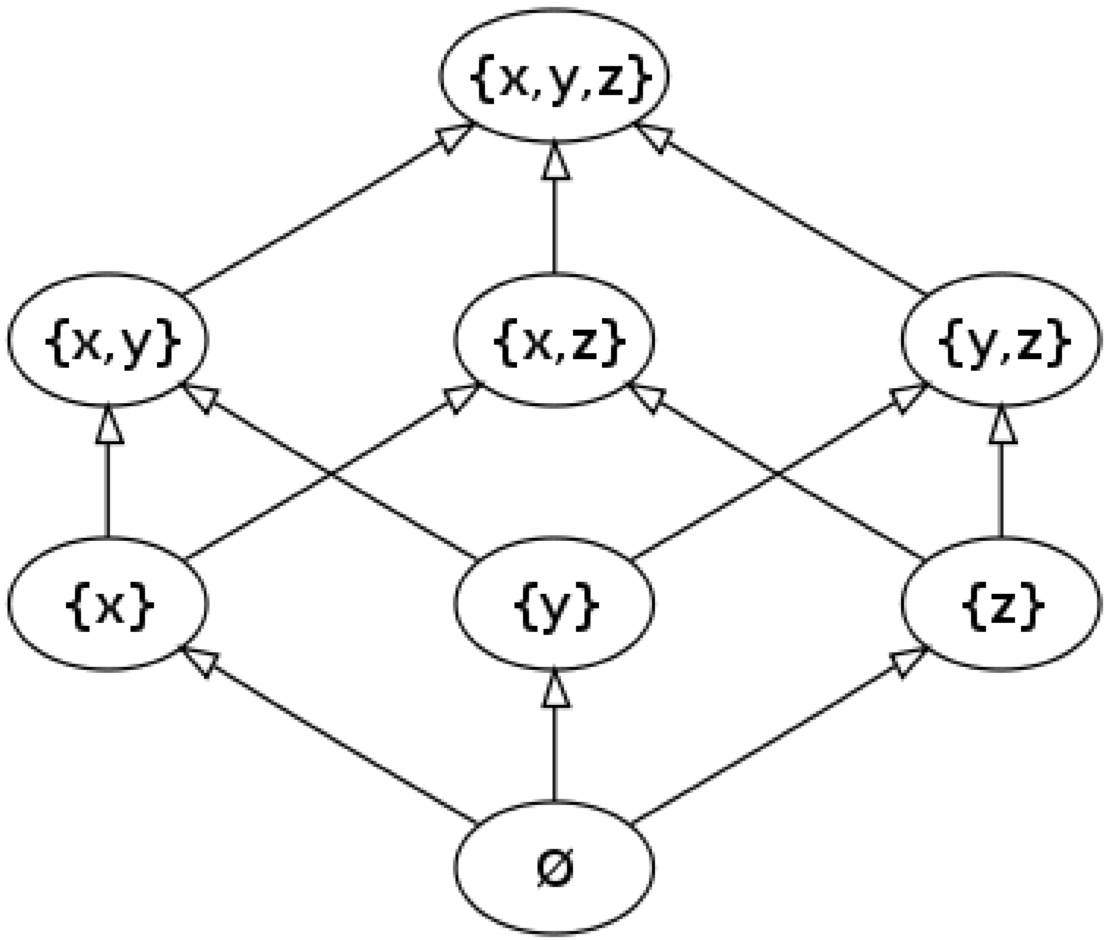
e.g.:

= for integers is a **partial order**,

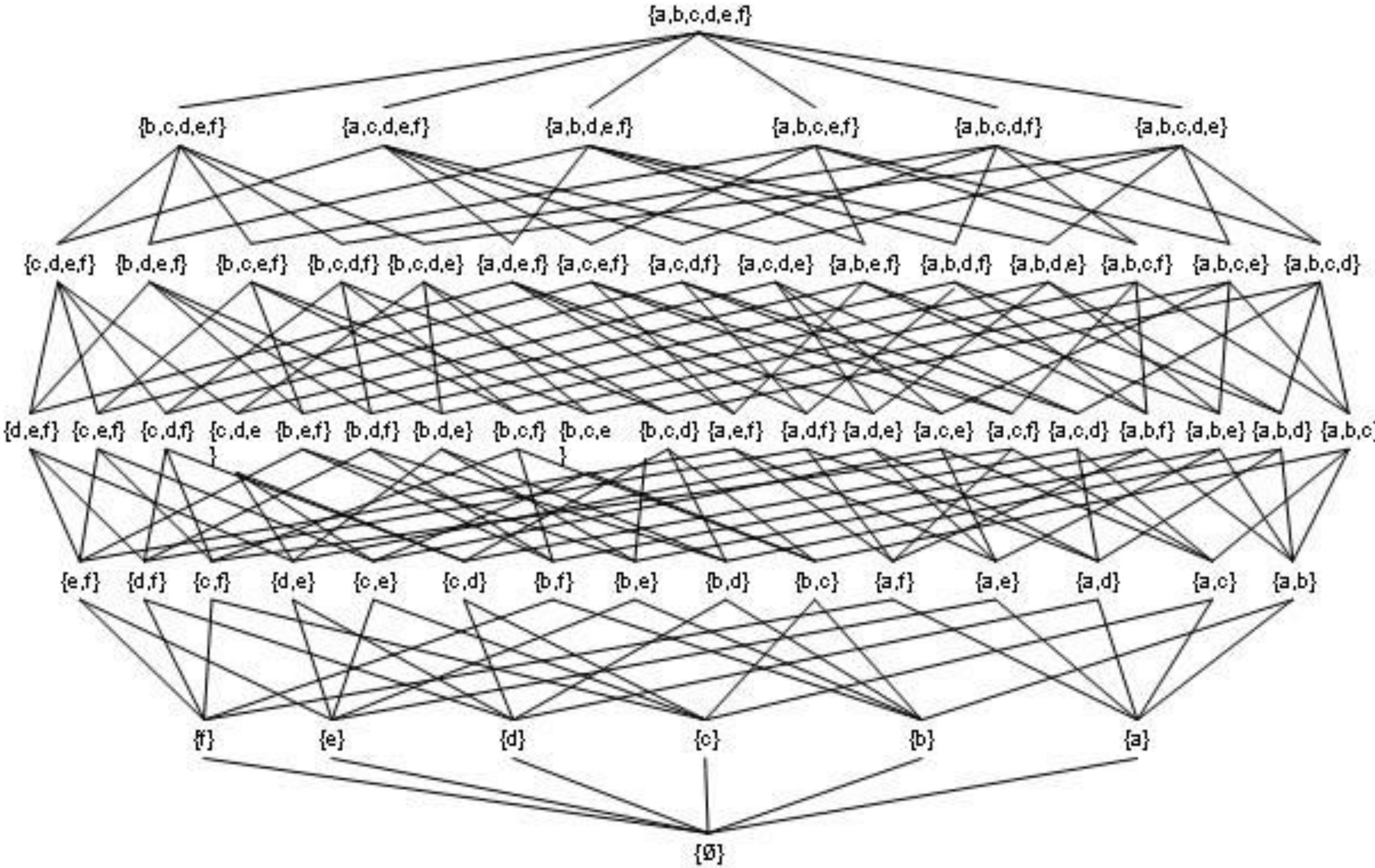
$\leq$ for integers is a **total order**, but **<** is not an **order**,

$\subseteq$ for sets is a **partial order** (cf { 1 } and { 2 }).

EXAMPLE of a Partial Order: The **set of all subsets** of a three-element set {x, y, z}, ordered by inclusion. Sets on the same horizontal level don't share a precedence relationship. Other pairs, such as {x} and {y,z}, do not either.



Partially ordered set of **set of all subsets** of a six-element set {a, b, c, d, e, f}, ordered by the subset relation.



Classes of Binary Relations 3

Equivalence Relations:

- for any relation $R \subseteq A \times A$:
  - R is an **equivalence** iff it is *reflexive*, *symmetric* and *transitive*:
    - eg = for integers is an equivalence,
    - $\leq$ for integers and < are not equivalences,
    - $\subseteq$ for sets is not an equivalence.

- **reflexive** iff $\forall x \in A\ (xRx)$,
- **symmetric** iff $\forall x, y \in A\ (xRy \Rightarrow yRx)$
- **transitive** iff $\forall x, y, z \in A\ ((x\ R\ y) \wedge (y\ R\ z) \Rightarrow x\ R\ z)$

Exercises 7.18

- which of the following relations on N are **partial orders**? Which are **total orders** ? Which are **equivalences** ? Explain your answers.

The Identity relation I = {(n,n) : n $\in$ N}

The Universal relation U= {(m,n): m,n $\in$ N}

The parity relation P = {m=n(mod 2): m,n $\in$ N}

For a positive integer *n*, two integers *a* and *b* are said to be *congruent modulo n*, and written as

$$a \equiv b \pmod{n},$$

if their difference $a - b$ is an integer multiple of *n* (or *n* divides $a - b$).

PARTIAL ORDER

reflexive iff $\forall x \in A\ (xRx)$,

antisymmetric iff $\forall x,y \in A\ (xRy \wedge yRx \Rightarrow x = y)$:

or EQUIVALENTLYif *R(a,b)* with $a \neq b$, then *R(b,a)* must not hold.

transitive iff $\forall x, y, z \in A\ ((x\ R\ y) \wedge (y\ R\ z) \Rightarrow x\ R\ z)$

TOTAL ORDER iff PARTIAL ORDER and

$\forall x, y \in A\ ((x\ R\ y) \vee (y\ R\ x))$:

EQUIVALENCE

- **reflexive** iff $\forall x \in A\ (xRx)$,
- **symmetric** iff $\forall x,y \in A\ (xRy \Rightarrow yRx)$
- **transitive** iff $\forall x, y, z \in A\ ((x\ R\ y) \wedge (y\ R\ z) \Rightarrow x\ R\ z)$

i) The Identity relation $I = \{(n,n) : n \in N\}$

ii) The Universal relation $U = \{(m,n): m,n \in N\}$

iii) The parity relation $P = \{m=n (\text{mod } 2): m,n \in N\}$

Exercises 7.19

Consider a set S of students who are each taking some number of courses chosen from a set C of courses. Define the following binary relation on S:

$$R_1 = \{(s_1, s_2): \quad s_1 \text{ and } s_2 \quad \text{take all the same courses}\}$$
$$R_2 = \{(s_1, s_2): \quad s_1 \text{ and } s_2 \quad \text{take some courses together}\}$$

Are either of these an equivalence relation ? Justify your answer.

EQUIVALENCE

- **reflexive** iff $\forall x \in A\ (xRx)$,
- **symmetric** iff $\forall x,y \in A\ (xRy \Rightarrow yRx)$
- **transitive** iff $\forall x, y, z \in A\ ((x\ R\ y) \wedge (y\ R\ z) \Rightarrow x\ R\ z)$

Classes of Binary Relations 4

Equivalence Partitions:

- A **partition** of a set A is:
  - a set of subsets A_i for $i \in I$ of A:
    - where A_i are the **blocks** of the partition,
  - which are all disjoint (and non-empty),
  - and together contain all of A, so that:
    - $(i \neq j \Rightarrow A_i \cap A_j = \emptyset) \wedge \cup_{i \in I} A_i = A$;
- A **refinement** of a partition is:
  - a partition where every block is a subset of a block of the one being refined.