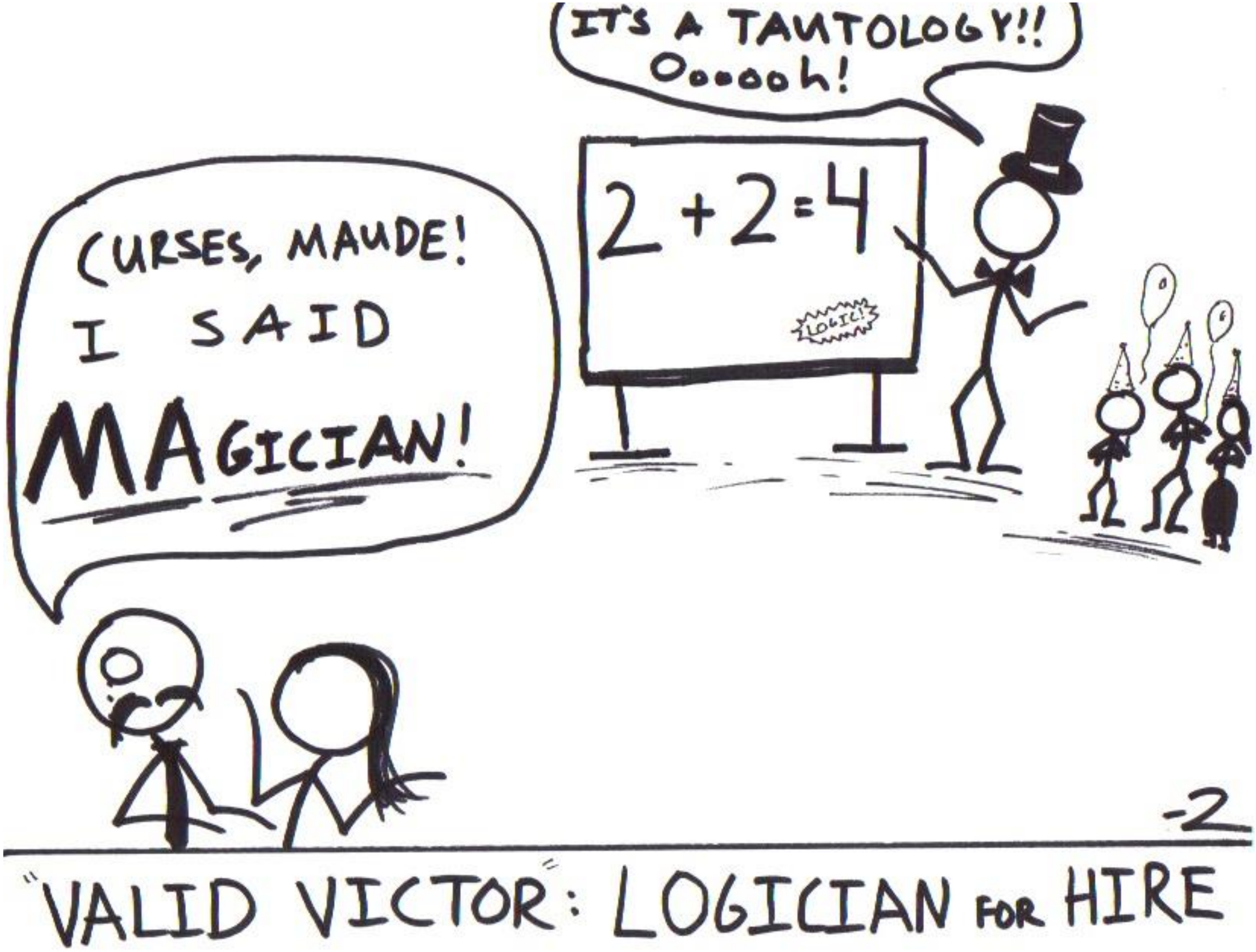
COM1002: Foundations of Computer Science



Assessment

Hand in exercise sheet in tomorrows tutorial (worth 5% of this Semester COM1002)

A new exercise sheet will be uploaded onto MOLE today (not assessed)

Learning Objectives

- Finish off Set Theory (C2)
- Introduction to Predicate Logic. Express…

1. All bees like all flowers
2. Bees only like flowers
3. Only bees like flowers

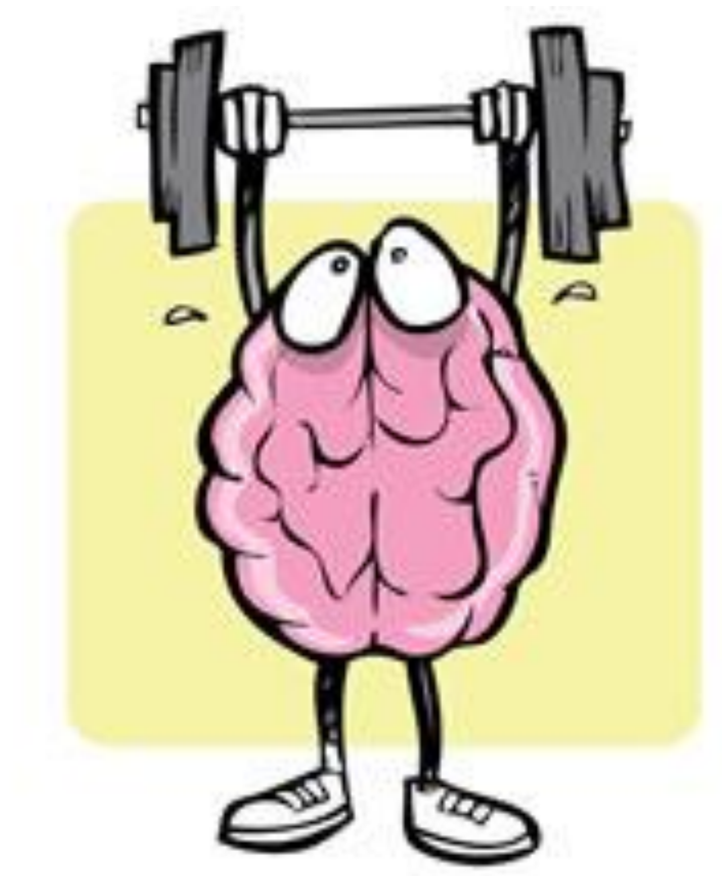
Using the **predicates:**

B(x) = "x is a bee"

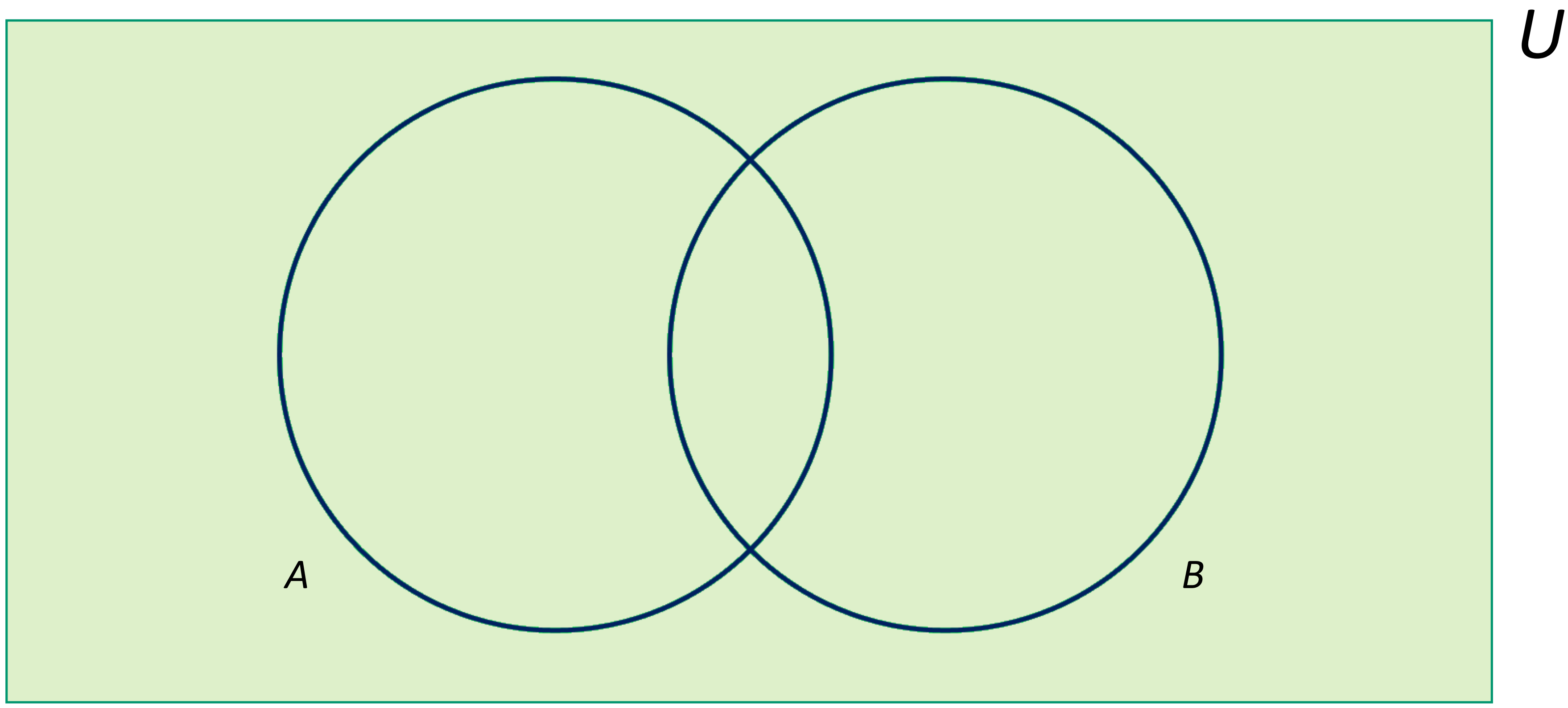
F(x) = "x is a flower"

L(x,y) = "x likes y"

Warm up!



$A \cup B'$



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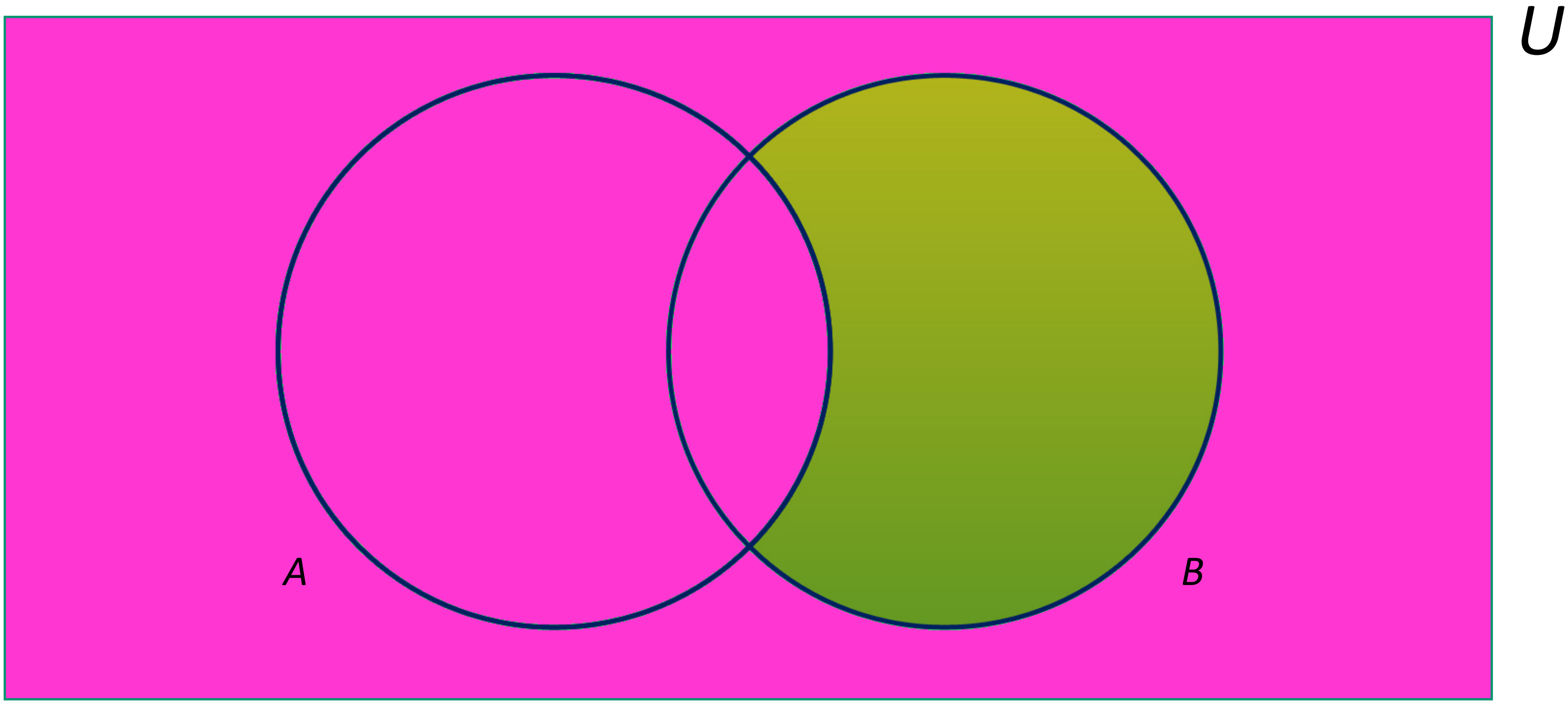
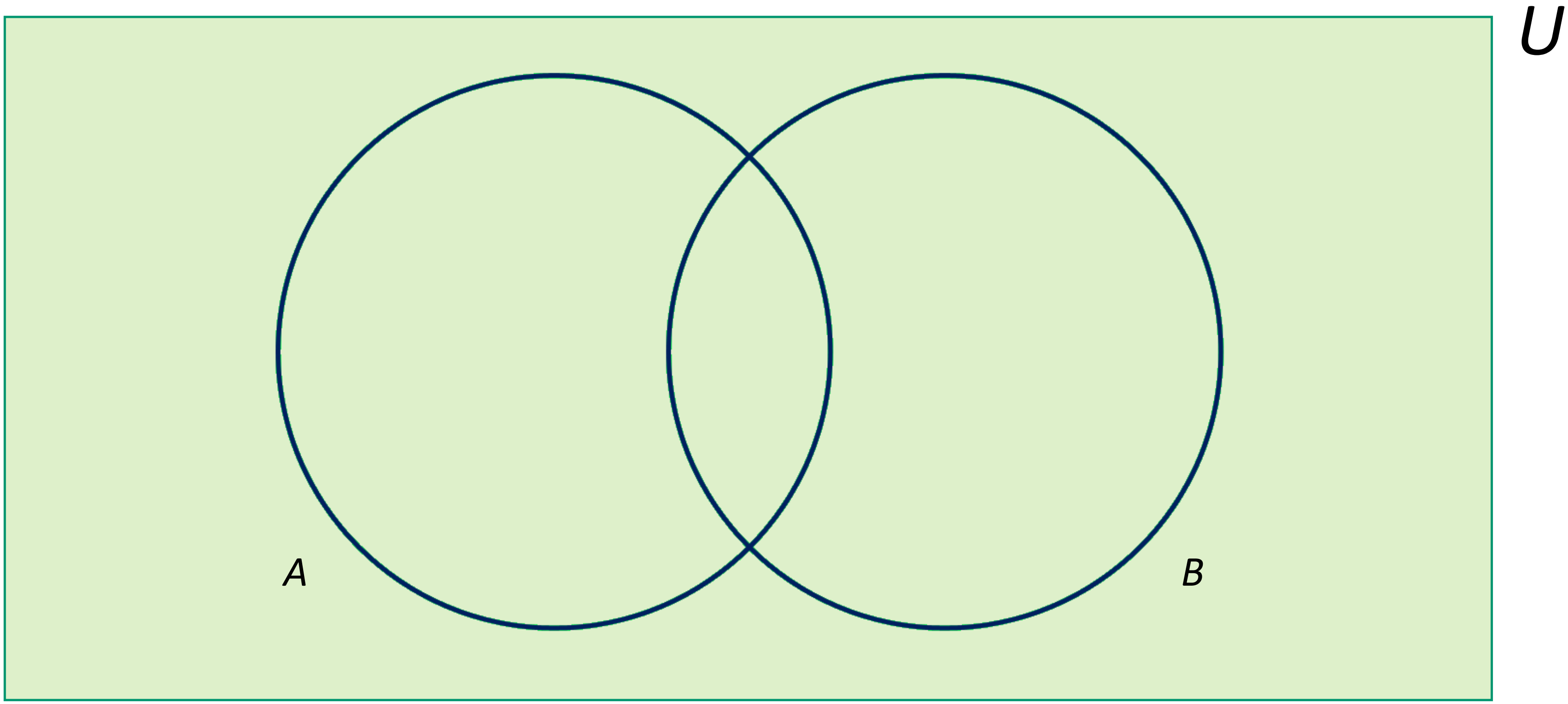
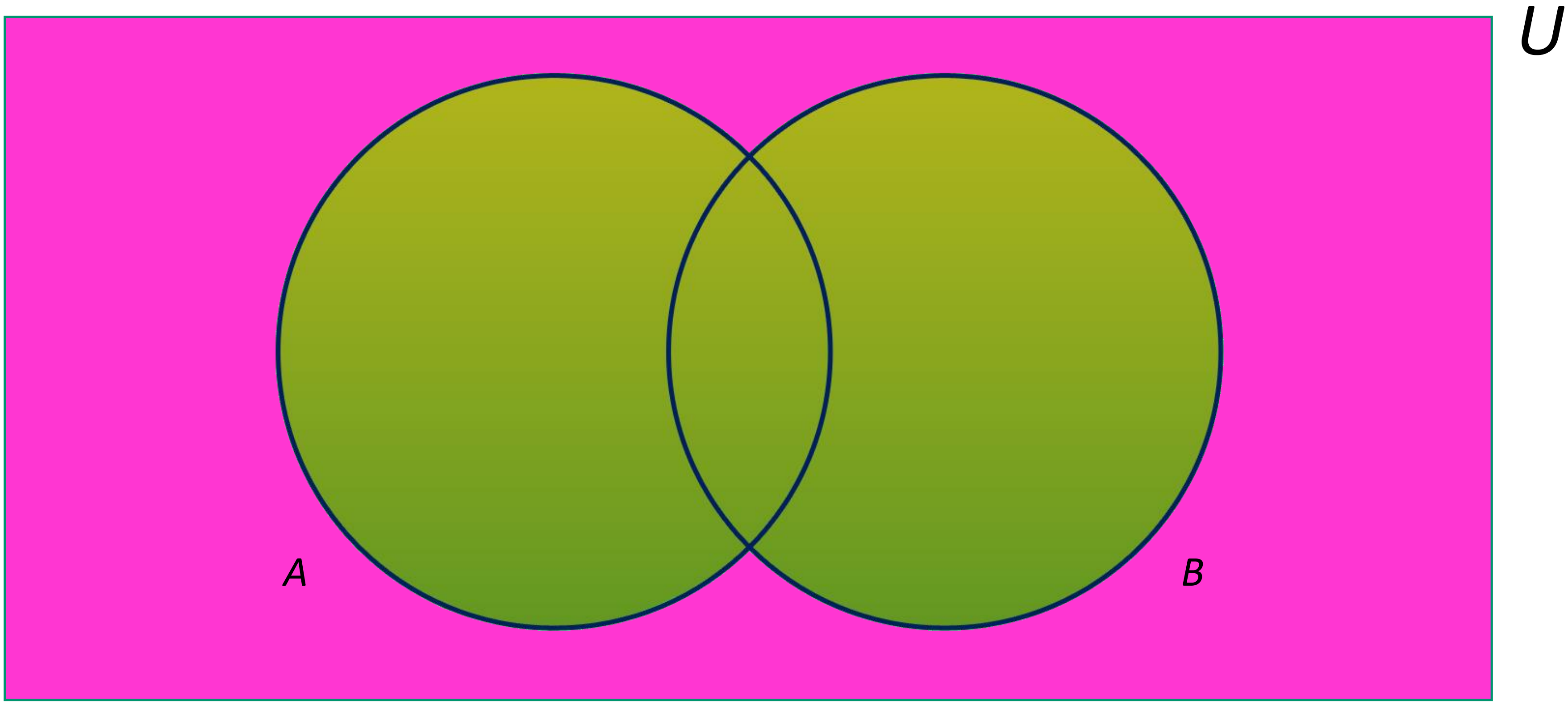


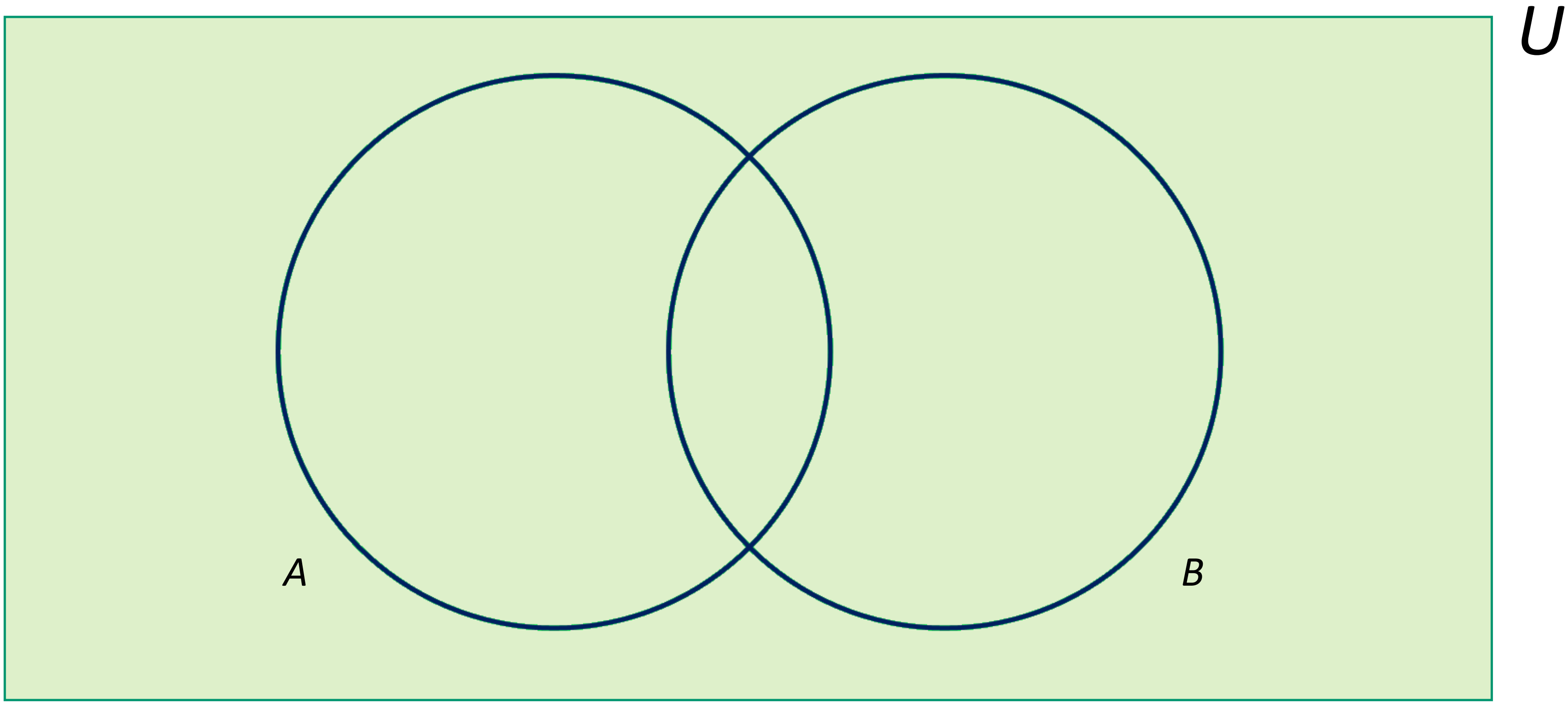
$A' \cap B'$



$A' \cap B'$



$A \cap B'$



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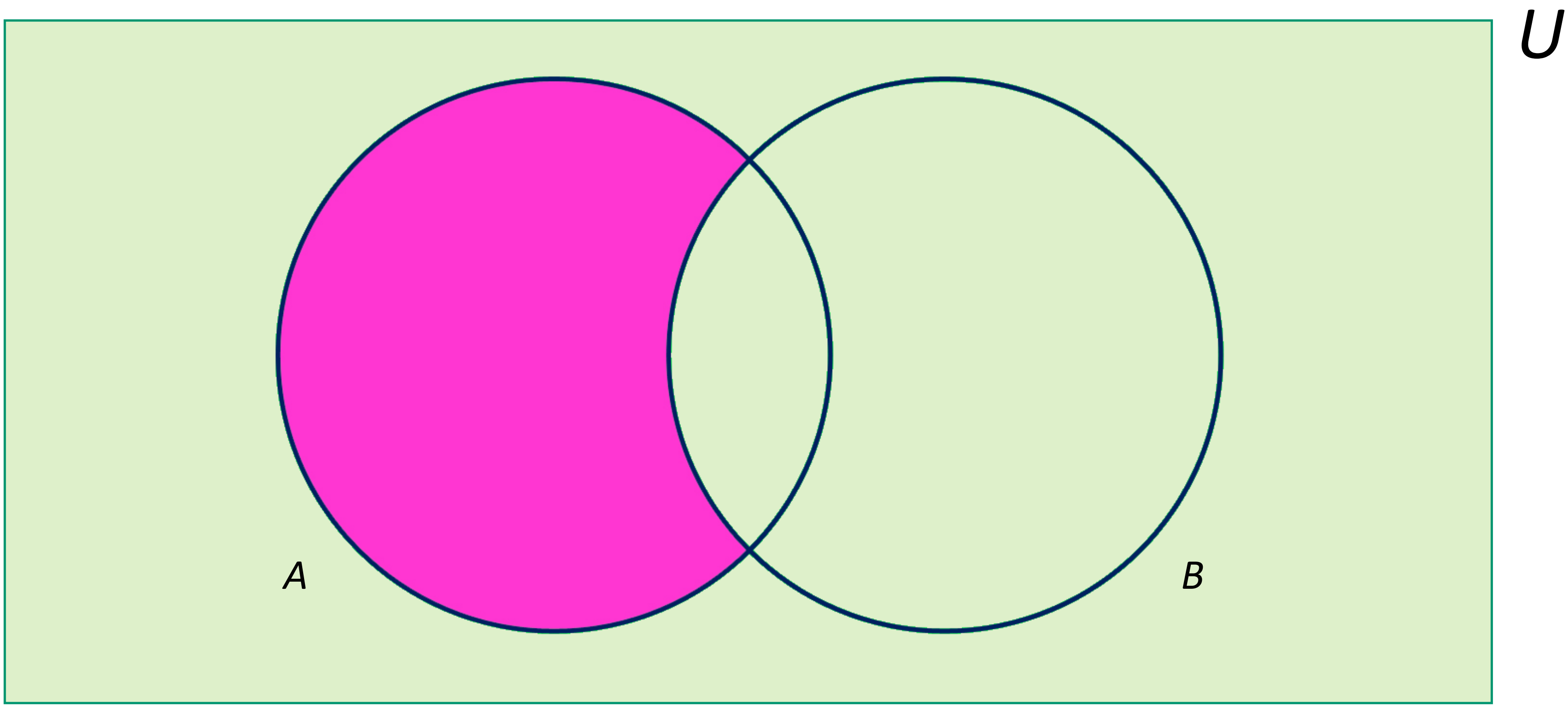
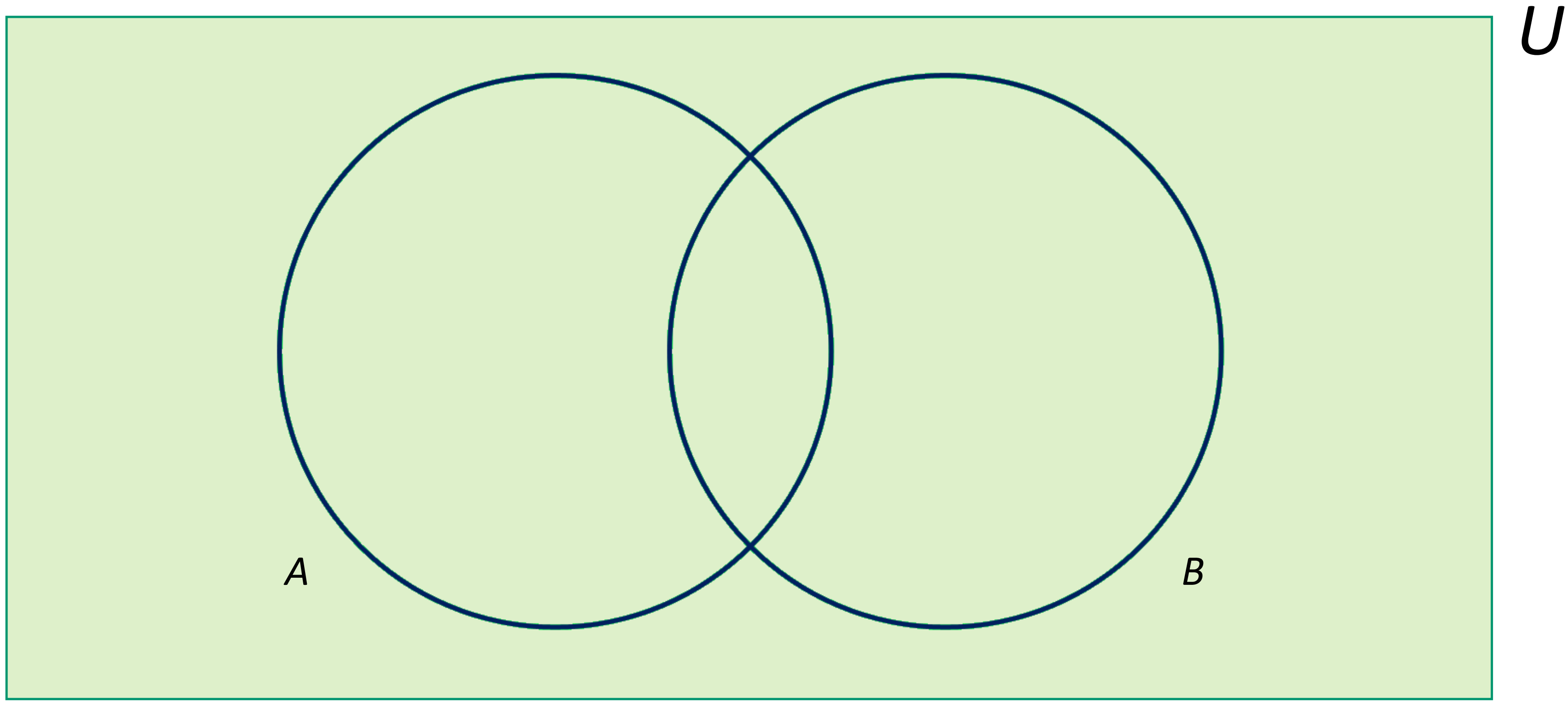
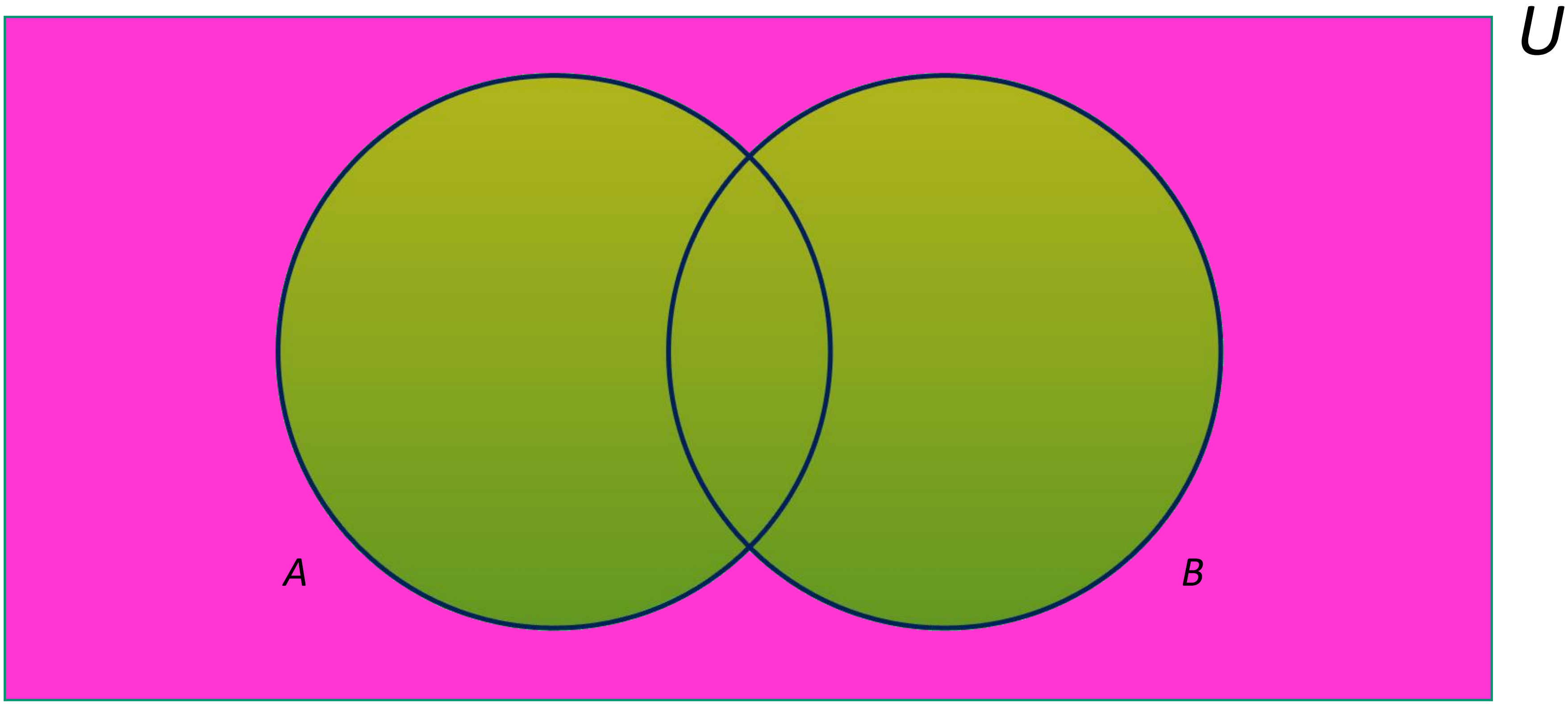


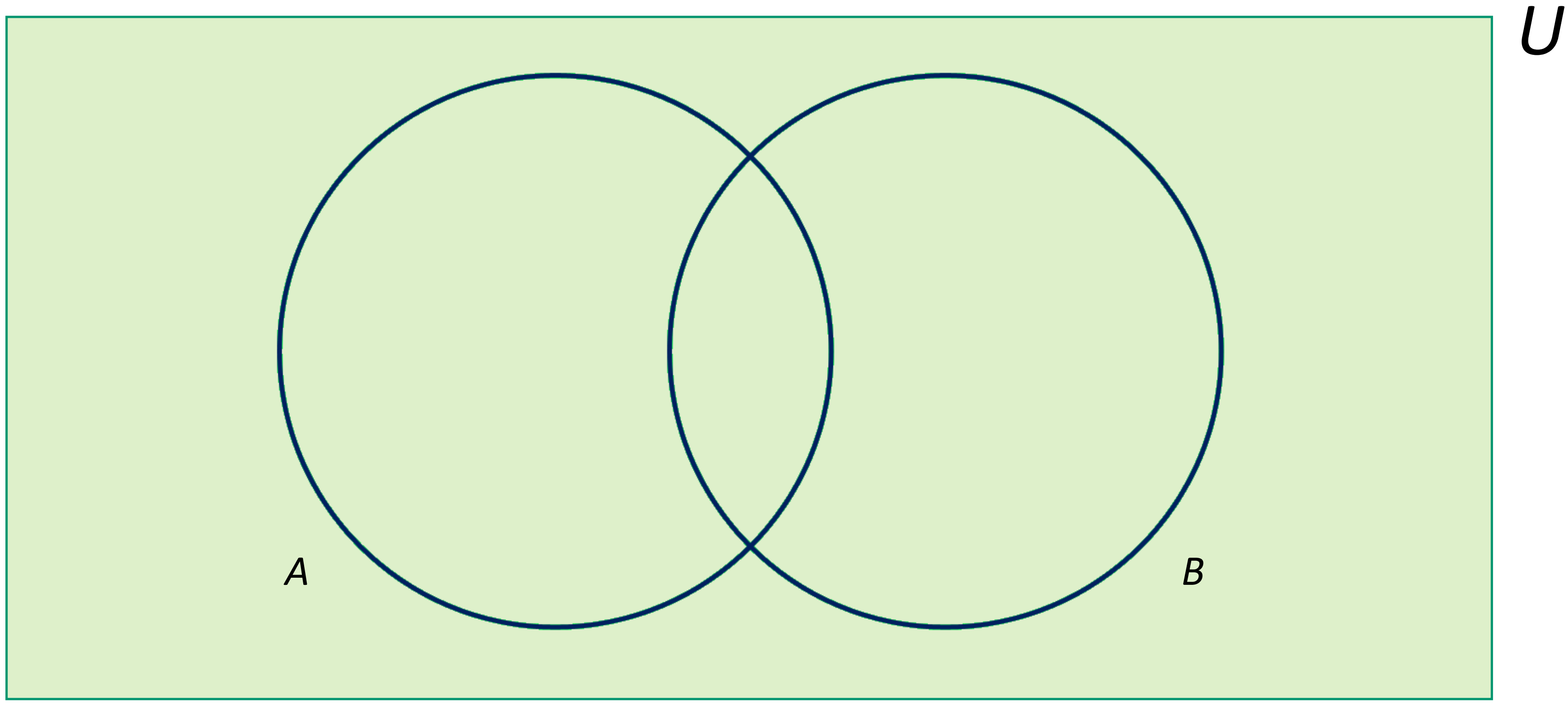
$(A \cup B)'$



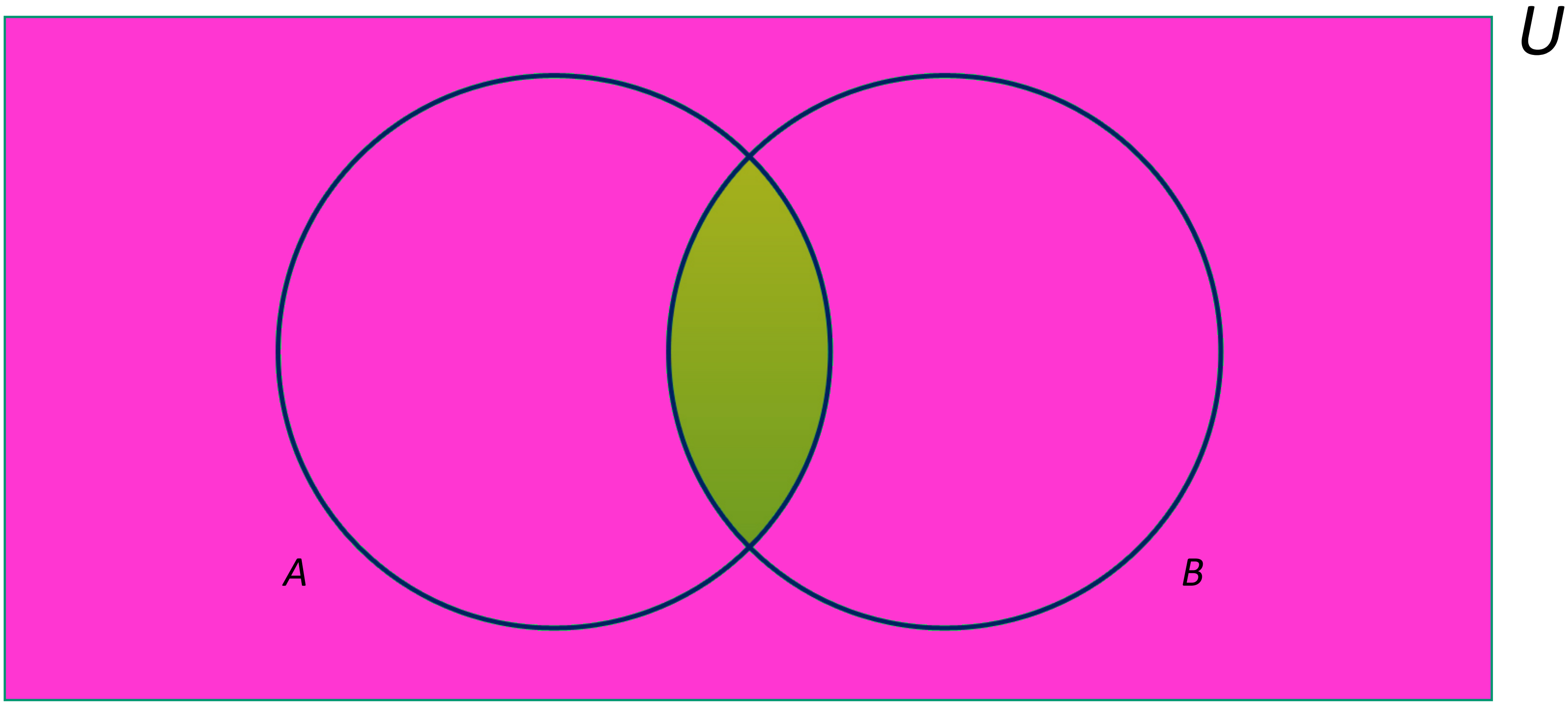
$(A \cup B)'$



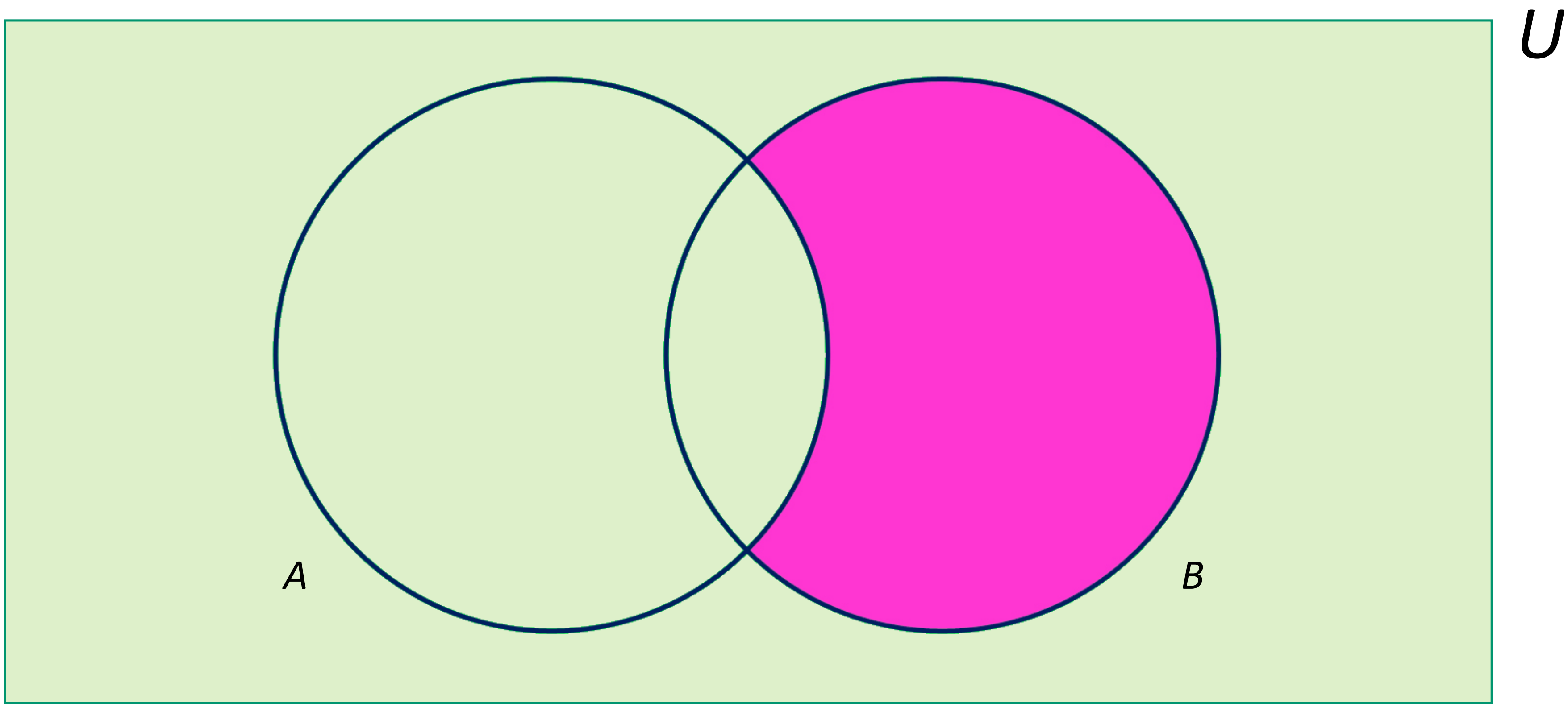
$A' \cup B'$



$A' \cup B'$

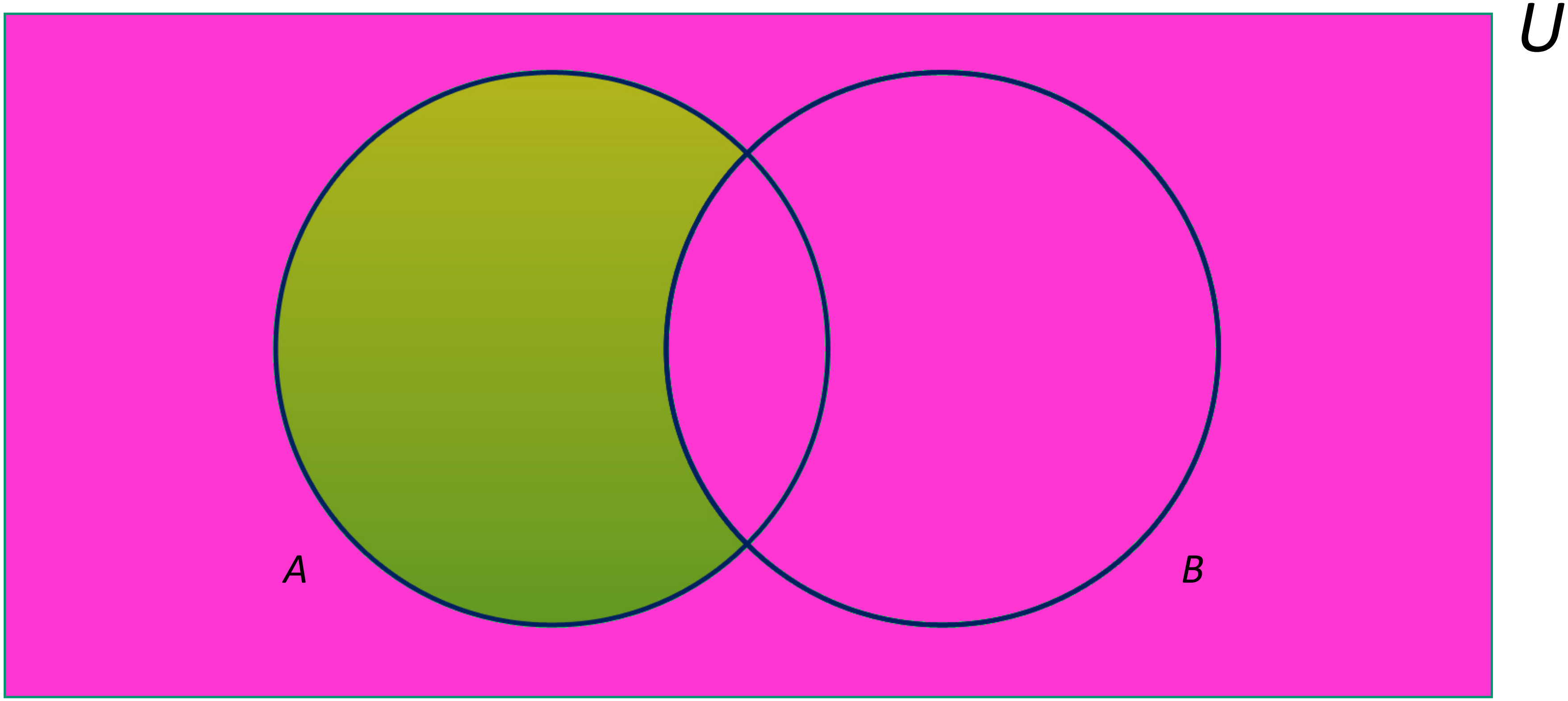


What is the shaded region?



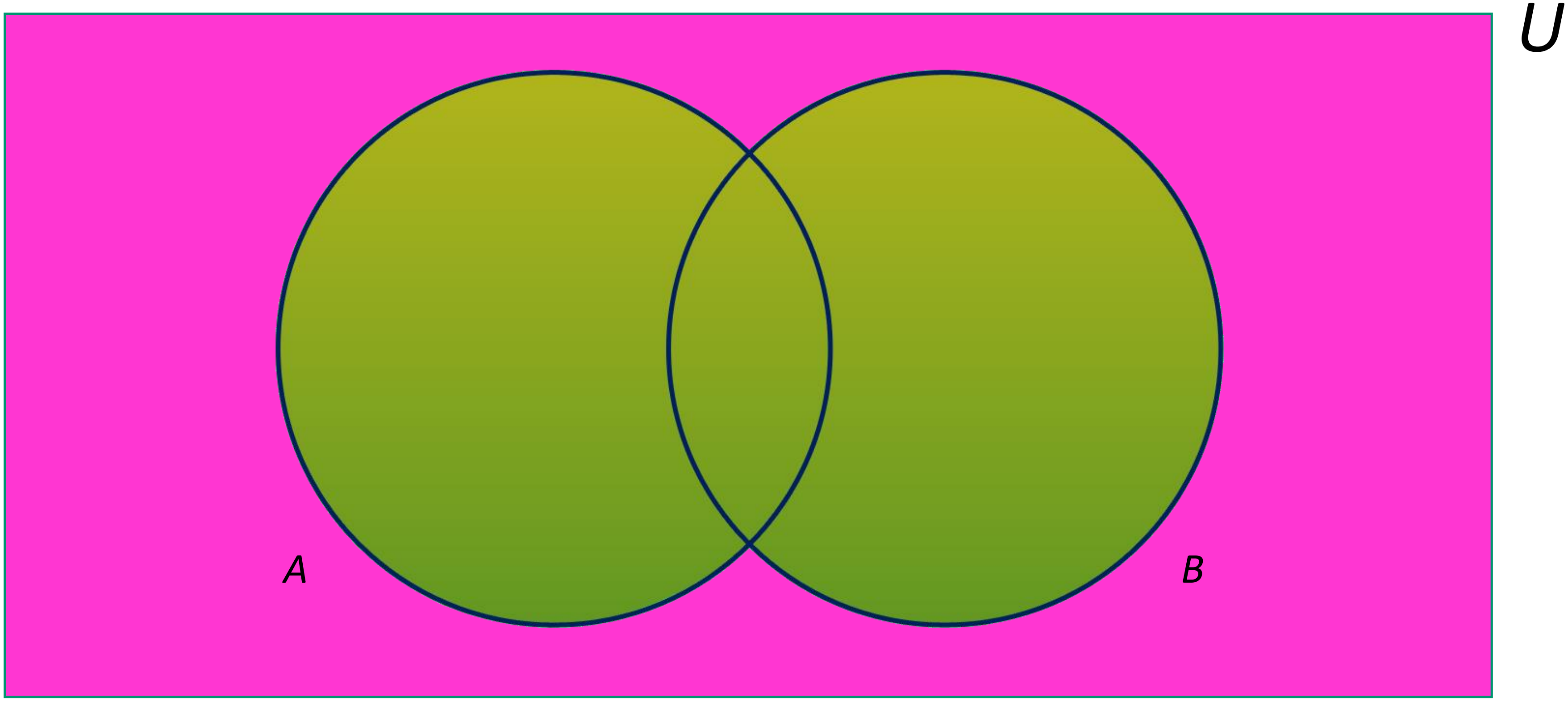
$(B \cap A')$

What is the shaded region?



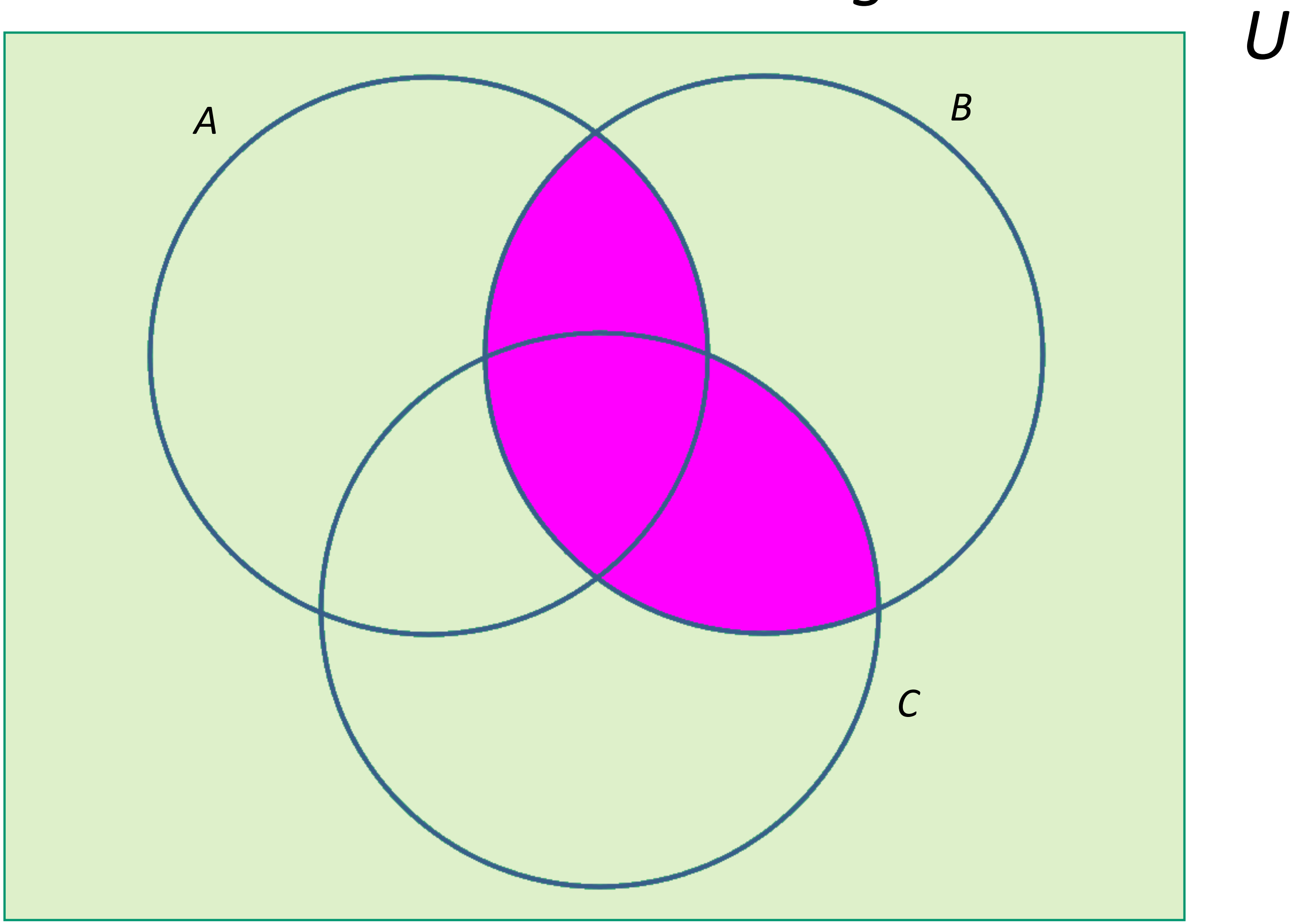
$(A' \cup B)$

What is the shaded region?



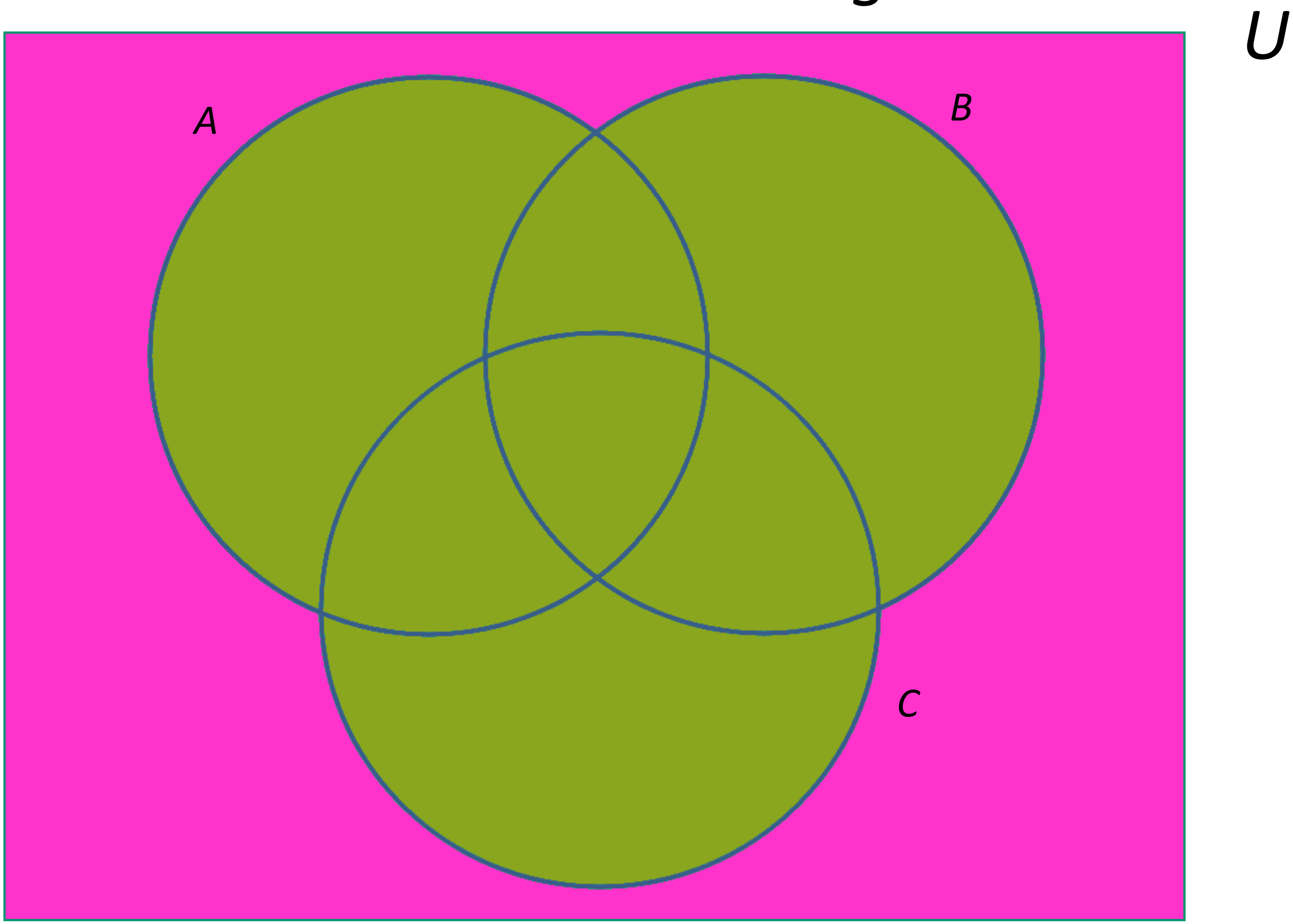
$(A \cup B)'$

What is the shaded region?



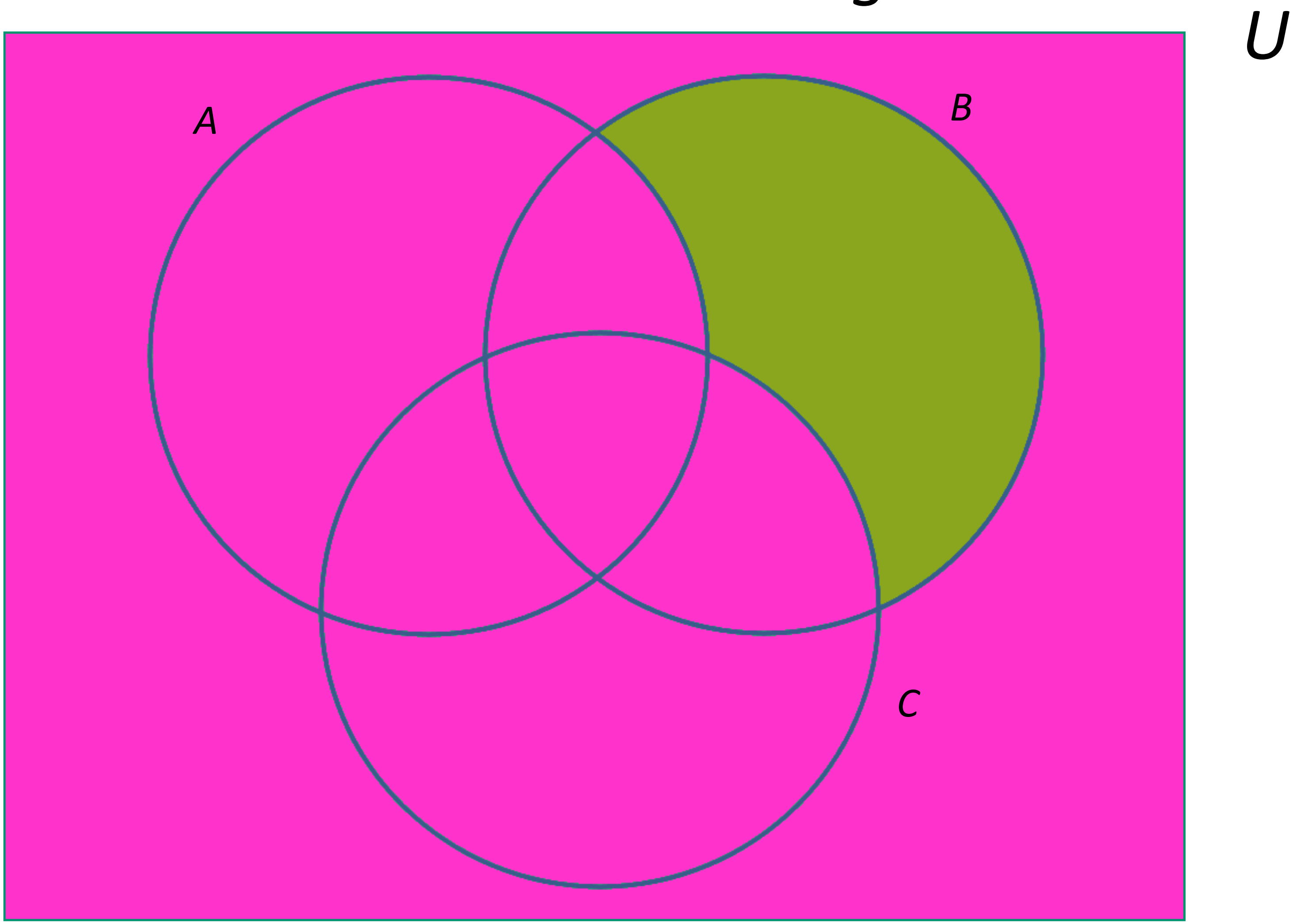
$B \cap (A \cup C)$

What is the shaded region?



$(A \cup B \cup C)'$

What is the shaded region?



$(A \cup C \cup B')$

Simplify, with the aid of Venn Diagrams,

$$A \cap (\bar{A} \cup B)$$

$$A \cap (\bar{A} \cup B) = A \cap B$$

We can also prove this using algebraic laws

Algebraic Laws for Sets 1

These define equality of expressions:

Commutativity:

$A \cup B = B \cup A$ and $A \cap B = B \cap A$;

Associativity:

$A \cup (B \cup C) = (A \cup B) \cup C$ and

$A \cap (B \cap C) = (A \cap B) \cap C$;

Idempotence:

$A \cup A = A$ and $A \cap A = A$;

Algebraic Laws for Sets 2

Distributivity:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C);$$

De Morgan's Laws:

$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ and $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

Double Complement Law:

$\overline{\overline{A}} = A$,

this assumes a suitable universe;

Algebraic Laws for Sets 3

Universe Laws:

$A \cup \mathcal{U} = \mathcal{U}$ and $A \cap \mathcal{U} = A$;

Empty Set Laws:

$A \cup \varnothing = A$ and $A \cap \varnothing = \varnothing$;

Complement Laws:

$A \cup \overline{A} = \mathcal{U}$ and $A \cap \overline{A} = \varnothing$;

Absorption Laws:

$A \cup (A \cap B) = A$ and $A \cap (A \cup B) = A$.

Try exercises 2.28

Exercise 2.28

Give a derivation of $A \cap (\overline{A} \cup B) = A \cap B$

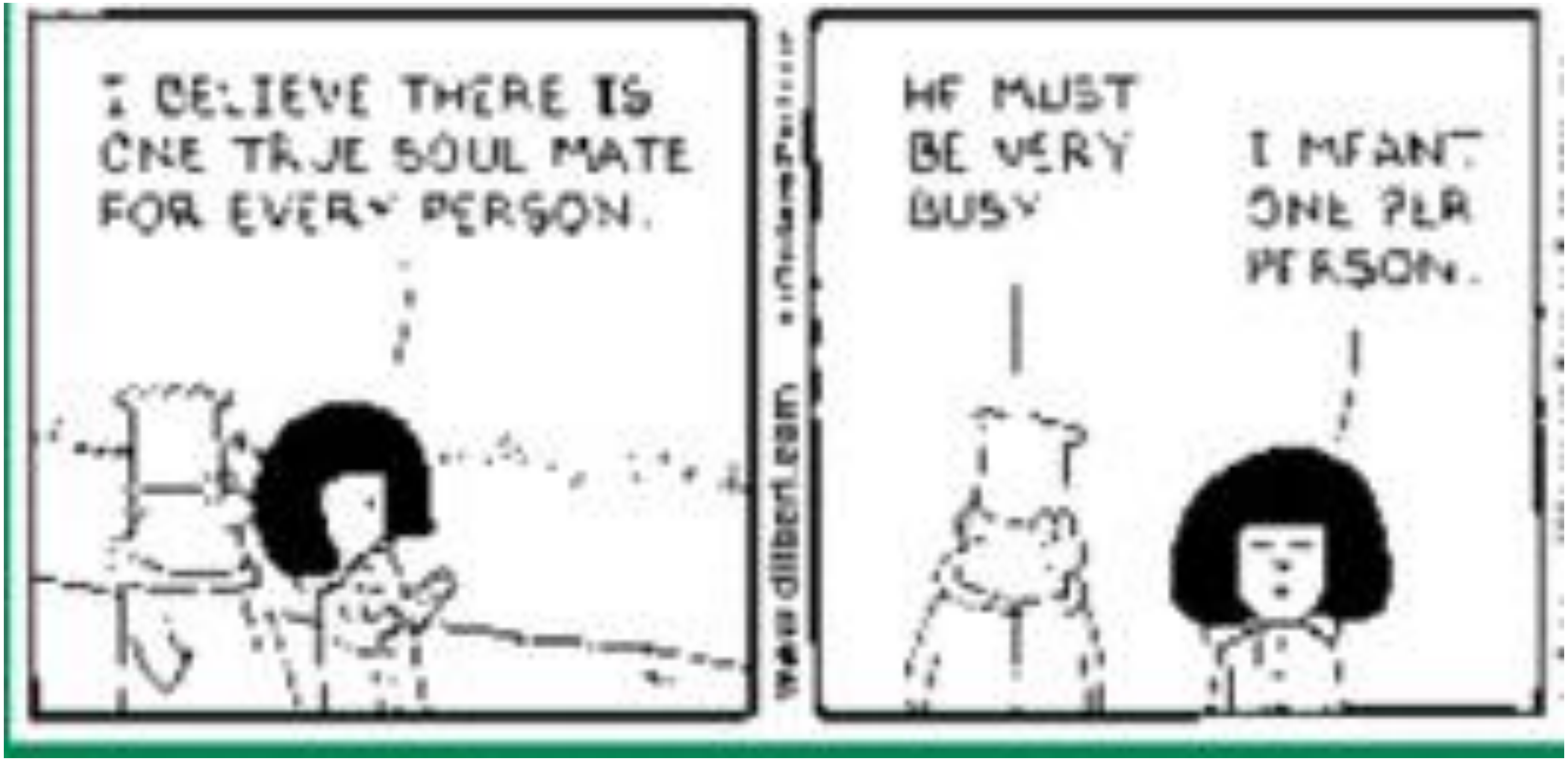
$$A \cap (\overline{A} \cup B) = \qquad \textit{Distributive law}$$

$$= \qquad \textit{Complement law}$$

$$= \qquad \textit{Commutative law}$$

$$= \qquad \textit{Empty set law}$$

Predicate Logic



Learning Objective

We can write each of the following statements in predicate logic.

1. All bees like all flowers

2. Bees only like flowers

3. Only bees like flowers

Using the predicates

B(x) = "x is a bee"

F(x) = "x is a flower"

L(x,y) = "x likes y"

Motivation

To generalise from propositional logic:

In order to avoid its limitations:

it can not handle some concepts properly, i.e.

- *the distinctions between none, one, some and all;*

These are important in specifications:

E.g. must a property hold for some or all values;

And in verification:

E.g. does a system match some part of its specification for some cases, or all.

Predicates 1

Propositions defined over variables:

- Eg the set builder notation:
  - { x : x has property P },
- Such a property is a **predicate**:
  - denoted P (x),
  - its **truth value** depends on the value of x,
  - its **truth set** is the set of x for which it is true;
- The variable is called a free variable:
  - giving it a value produces a proposition.

Predicates 2

Universe of Discourse:

- The set of possible values for a free variable:
  - the **truth set** is a subset of this;
- Predicates may have multiple free variables;
  - Their universe of discourse is a Cartesian product
    - they have **tuples** of values as their elements.

Predicates 3

Pairs of free variables are very common:

- Eg **equality, set inclusion**:
  - infix notation is commonly used for these:
    - eg ‘x = y’ rather than prefix ‘= (x, y)’
- Universe of discourse is a set of pairs;
- So is the truth set.

Try some of exercise 4.2.

Exercise 4.2

What are the truth sets of the following predicates ?

1. Even (x) = “x is an even integer”
2. EvenPrime(x) “x is an even prime”

Quantification 1

Universal Quantification:

If the predicate is true for all values:

- The truth set must be the **universe of discourse**;
- It is said to be universally quantified;
- This is denoted $\forall x\ P(x)$:
  - which is a proposition,
  - where x is now bound by the quantifier $\forall x$,
  - which is read “for all x”.

Quantification 2

Negation of Universal Quantification:

- If the predicate is true for no values:
  - the **truth set** must be the **empty set**;
- This is still universal quantification:
  - *the predicate is not true for all values*;
- It is denoted $\forall x \neg P(x)$:
  - which is again a proposition,
  - where x is bound by the quantifier $\forall x$.

Try exercise 4.5.

Exercise 4.5

Using the predicates

B(x) = "x is a bee"

F(x) = "x is a flower"

L(x,y) = "x likes y"

Write each of the following statements in predicate logic.

1. All bees like all flowers
2. Bees only like flowers
3. Only bees like flowers

All bees like all flowers

UNIVERSE OF DISCOURSE = All living things

$$\forall x, y\left[\left(B(x) \wedge F(y)\right) \Rightarrow L(x, y)\right]$$

Bees only like flowers

$$\forall x, y\left[(B(x) \wedge L(x, y)) \Rightarrow F(y)\right]$$

Only bees like flowers

$$\forall x, y\left[(F(y) \wedge L(x, y)) \Rightarrow B(x) \right]$$