Multi-objective Optimization for Series-parallel Systems Using Genetic Algorithm

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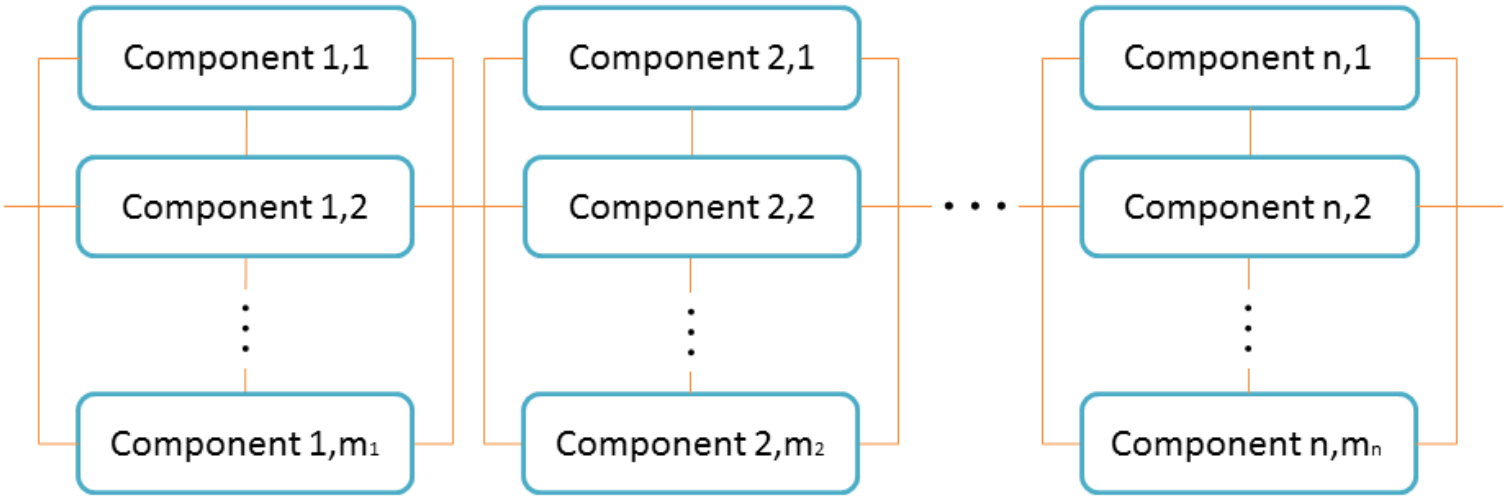
Project Description and Scope

- Implement genetic algorithm to optimize system reliability, weight, and cost of series-parallel systems using weighted sum.



Programming Challenge

- The algorithm must be able to increase population diversity of to keep exploring the problem space

Assumptions

- A component can only be functional or fail
- System reliability, cost, and weight of each component are fixed and provided
- A component can be selected only once for the whole system
- No repairing if a component fails.
- Failure of a component is independent from other components

Model Formulation

Z	Fitness of a system
X_{ij}	Architecture X for component j at subsystem i
n	Number of subsystem within the system (n = &o
m_i	Number of component choices available for subsystem I
R_i	Estimated reliability of the subsystem i
R_{ij}	Estimated reliability of using component j at subsystem i
C_i	Normalized cost of subsystem i
C_{ij}	Normalized cost of using component j at subsystem i
W_i	Normalized weight of subsystem i
W_{ij}	Normalized weight of using component j at subsystem i
α	Fitness weight of system reliability
β	Fitness weight of system cost
γ	Fitness weight of system weight

Objective Function

$$\boldsymbol{Max\ Z = \alpha \prod_{i=1}^{n} R_i + \beta \left(1 - \sum_{i=1}^{n} C_i\right) - \gamma \left(1 - \sum_{i=1}^{n} W_i\right)}$$

Where

$$R_i = 1 - \prod_{j=1}^{m_i} \left(1 - X_{ij} R_{ij}\right)$$

$$C_i = \sum_{j=1}^{m_i} X_{ij} C_{ij}$$

$$W_i = \sum_{j=1}^{m_i} X_{ij} W_{ij}$$

Constrains

- System must be functional

$$\prod_{i=1}^{n} R_i > 0$$

- Sum of the fitness weight must equal to 1

$$\alpha + \beta + \gamma = 1$$

- The number of selected components must be in between 1 and m_i

$$\sum_{j=1}^{m_i} X_{ij} = 1,2, \dots, m_i$$

$$X_{ij} = 0,1$$

$$i = 1,2, \dots, n$$

Input Data

Configurable Parameters

- Fitness weight of system reliability (α)
- Fitness weight of system cost (β)
- Fitness weight of system weight (γ)

Sub-system	Design Alternative											
	Comp Choice 1			Comp Choice 2			Comp Choice 3			Comp Choice 4		
	R	C	W	R	C	W	R	C	W	R	C	W
1	0.90	1	3	0.93	1	4	0.91	2	2	0.95	4	5
2	0.95	4	8	0.94	2	10	0.93	1	9	-	-	-
3	0.85	2	7	0.95	3	5	0.87	1	6	0.92	4	4
4	0.83	3	5	0.87	4	6	0.85	5	4	-	-	-
5	0.94	2	4	0.93	2	3	0.95	5	5	0.94	2	4
6	0.99	6	5	0.98	4	4	0.97	2	5	0.96	2	4
7	0.91	4	7	0.92	4	8	0.94	5	0	-	-	-

Problem Size

$$Problem\ size = \prod_{i=1}^{7} \sum_{j=1}^{m_i} \binom{m_i}{j}$$

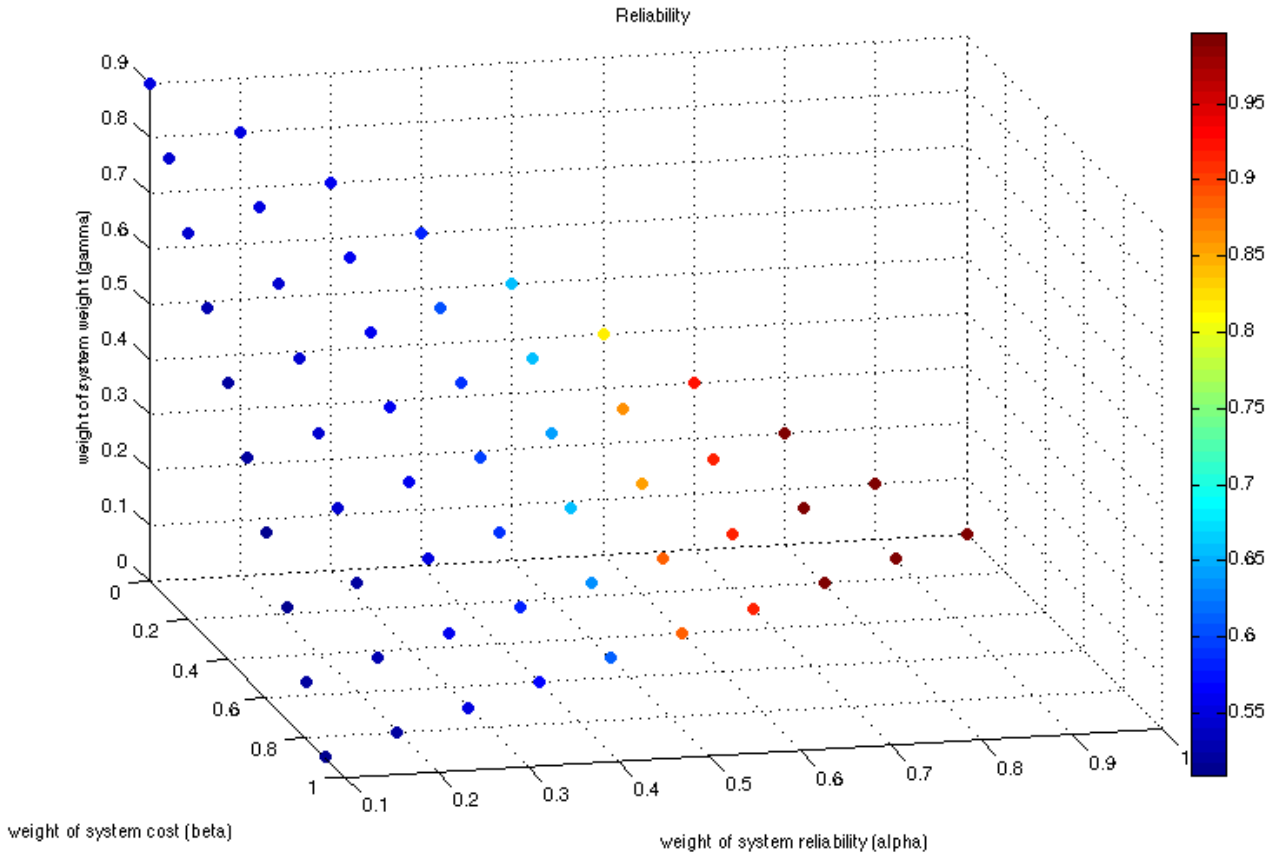
$$Problem\ size = \left[\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}\right]^4 \times \left[\binom{3}{1} + \binom{3}{2} + \binom{3}{3}\right]^3$$

$$Problem\ size = 50{,}625 \times 343$$

$$\boldsymbol{Problem\ size = 17{,}364{,}375}$$

Weighted Sum

Sum of the fitness weight must equal to 1: $\alpha + \beta + \gamma = 1$



Genetic Algorithm

- Max Generation Run: 5,000
- Population size: 50
- Elitism size: 3
- Chromosome size: 25 (1 gene for 1 component)
- Crossover Method: Tournament Selection
- Tournament Size: 10 (randomly pick from population)
- Crossover Rate: 80%
- Mutation Method: Single point mutation
- Mutation Rate: 20%

Chromosome Repairing Algorithm

$$\prod_{i=1}^{n} R_i > 0$$

- A subsystem with no component will be randomly inserted a component
- The repairing function will execute after each evolution phase

Population Elimination

- Prevent algorithm from local optimum
- Improve population diversity
- Select 10 top worst individuals, and regenerate them

Image from: http://www.independent.co.uk/life-style/health-and-families/health-news/birdflu-victim-had-no-direct-contact-with-poultry-6283803.html

Execution Result

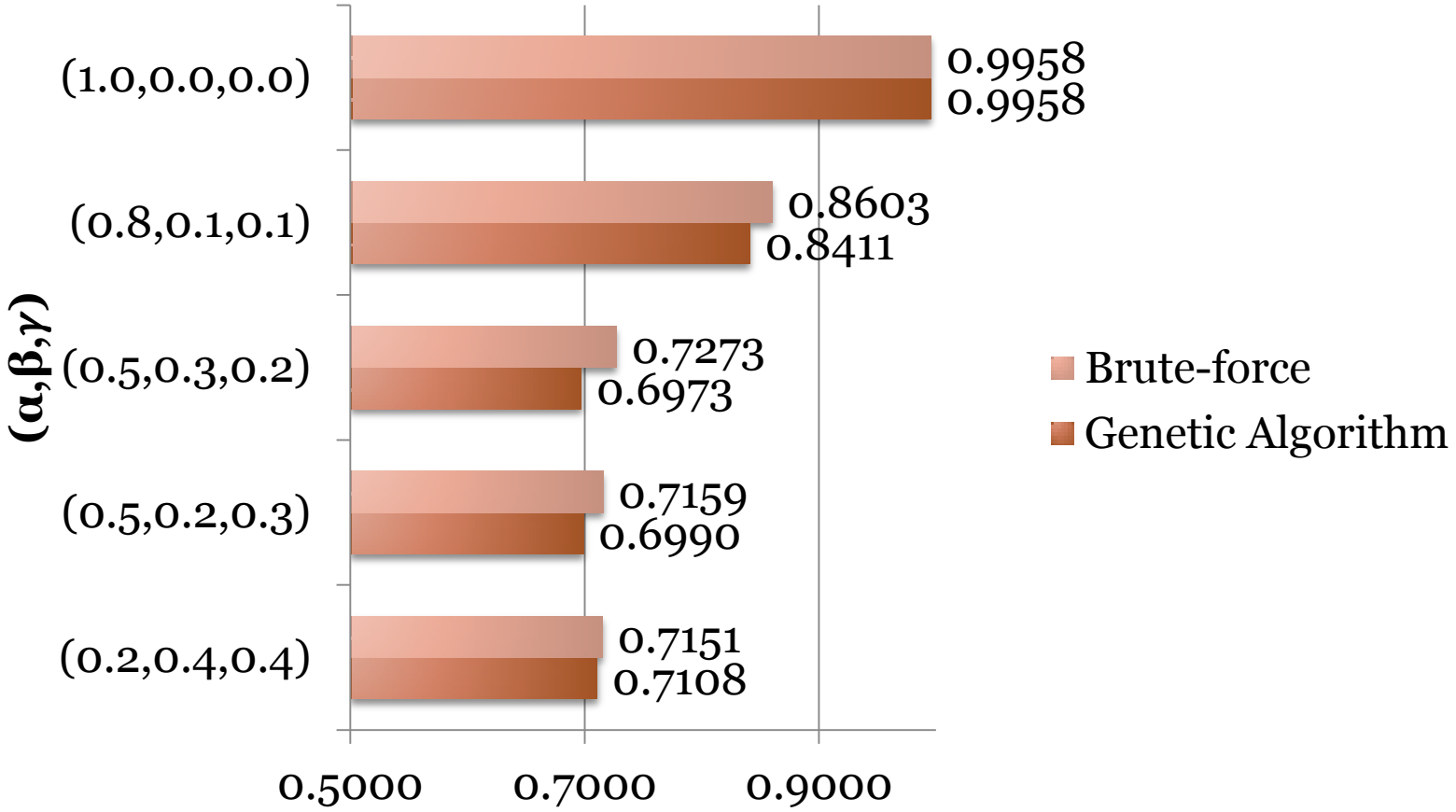
- Genetic Algorithm

α	β	γ	Fitness	Reliability	Cost	Weight	CPU Time (sec)
0.2	0.4	0.4	0.7108	0.5249	16	37	0.5555
0.5	0.3	0.2	0.6990	0.8359	32	63	0.8449
0.5	0.2	0.3	0.6973	0.7821	29	52	0.4965
0.8	0.1	0.1	0.8411	0.9868	55	102	0.1185
1.0	0.0	0.0	0.9958	0.9958	75	136	0.0457

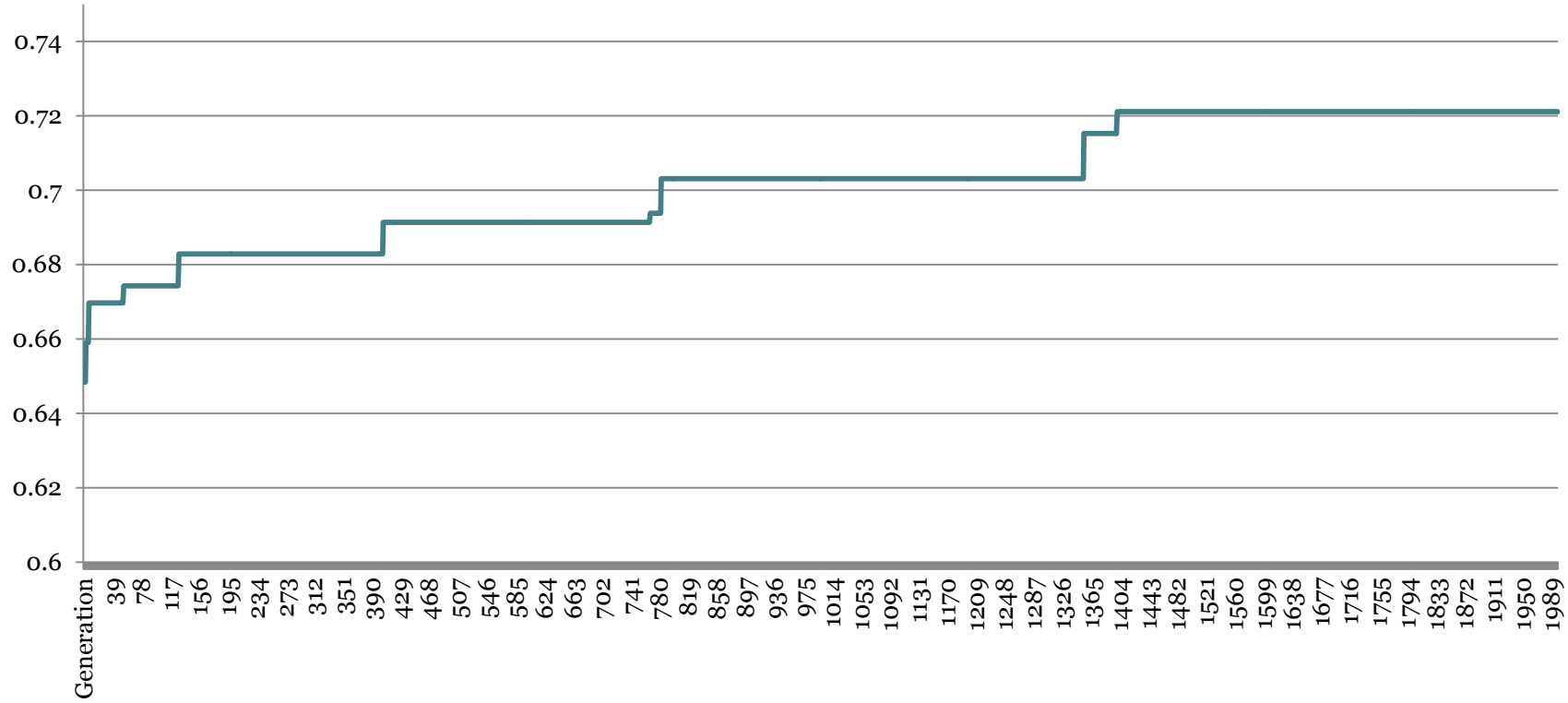
- Brute Force

α	β	γ	Fitness	Reliability	Cost	Weight	CPU Time (sec)
0.2	0.4	0.4	0.7151	0.507	14	38	89.1709
0.5	0.3	0.2	0.7159	0.9152	30	75	87.8117
0.5	0.2	0.3	0.7273	0.8796	36	58	86.5529
0.8	0.1	0.1	0.8603	0.9692	40	84	84.3733
1.0	0.0	0.0	0.9958	0.9958	75	136	83.2641

Result Comparison



GA Convergence Graph



Conclusion

- GA can return acceptable results
- GA uses very small amount of CPU time
- Due to the enormous size of problem space, GA stuck in local optimum very often

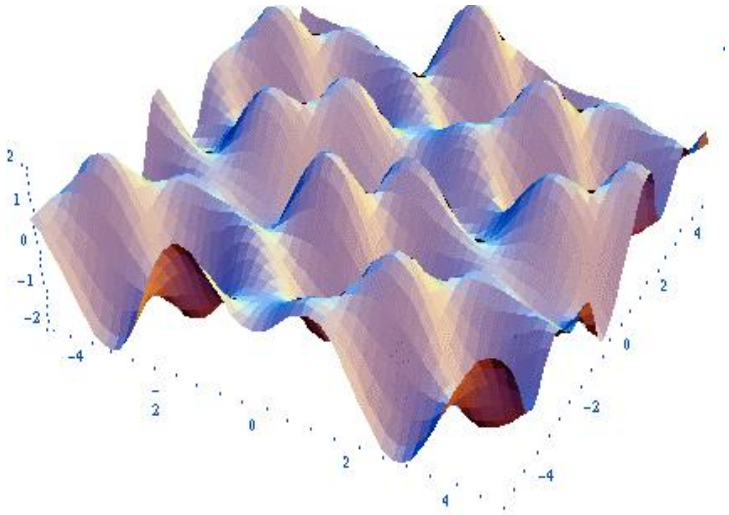


Image from: http://plato.asu.edu/gom.html

Further Improvement

- Advance GA mechanism to increase exploration

Image from: https://secure.flickr.com/photos/lakelandlocal/404688085/

Reference

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- Jacobson, L. (2012, February 12). Creating a genetic algorithm for beginners. Retrieved from http://www.theprojectspot.com/tutorial-post/creating-a-genetic-algorithm-for-beginners/3

- Input data from Dr Naruemon Wattanapongsakorn

Thank you

Q&A