Design of a wheel with dynamically variable curvature.

Rough draft*

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***Abstract*—This paper presents the design and control of a wheel capable of varying its radius over a range of values using a novel transformation mechanism. The transformation occurs without any additional actuators. Interestingly the wheel can vary its radius ranging from a large diameter to small diameter which enhances its maneuverability like small turning radius and fast/efficient locomotion. Mechanical design of the wheel has been explained in a detailed manner in this paper and also the transforming mechanism and motion study has been shown clearly.**
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***Index Terms*—variable radius, maneuverability, locomotion**

I. Introduction

Automobiles are complex machines, in part due to the transmission, suspension,and steering mechanism. By replacing the traditionally static tire with a re-invented wheel that dynamically changes radius, the mechanical complexity of these three systems can be reduced. Reducing the mechanical complexity lowers the cost and required maintenance of a vehicle, all of which are desirable traits for all terrain vehicles and rovers. The design of the wheel consists of three sub-assemblies each of which has dedicated functions to accomplish. The mathematics and working behind each sub-assembly is described in great detail below.
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II. Nomenclature

d	diameter of sun gear
D	diameter of ring gear
$\frac{D-d}{2}$	diameter of planet gear
k	$\frac{d}{D}$
ϕ	gear mesh angle
n	number of teeth in sun gear
γ	angle of oscillation
θ_{max}	maximum angular displacement of disc
a	radius of inner circle
b	radius of peripheral circle
m	$\frac{a}{b}$
A	area of triangle formed by intersection of two circles
L	perimeter of triangle formed by intersection of two circles
r	radial displacement of spokes on wheel
ω	relative angular velocity of disc
t	any instant of time

III. Design Methodology:

The control of the transformation of wheel is developed in a way to avoid requirement of any additional actuators in order to bring down the hardware complexity. The input shaft when fed with anti-clockwise rotation tends to actuate the transformation mechanism, whereas when fed with clockwise rotation drives the wheel forward. This is type of control over the transformation mechanism is implemented with a trapped one-way bearing,

A. Trapped roller one-way bearing:

A trapped roller one-way bearings is designed to transmit torque between the shaft and housing in one direction and allow free motion in the opposite direction. Proper mounting is easily accomplished with a simple press fit in the casing.

Fig. 1. Trapped roller one-way bearing.

Relative motion between the casing and the input shaft is permitted only when the rotation fed is in clockwise direction. This eliminates any additional actuators required to control the curvature of the wheel.

IV. Oscillatory mechanism:

The first sub-assembly converts continuous rotation fed in by the input shaft into rotary oscillation of desired angular displacement along the same axis,

A. Gear system that changes direction:

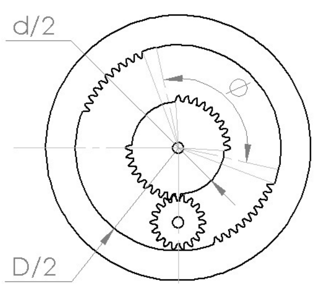

Fig. 2. Planetary gear assembly.

$$\phi\left(\frac{2d}{D-d}\right) = (\pi-\phi)\left(\frac{2D}{D-d}\right)$$

$$\phi = \frac{\pi}{(1+k)} \qquad \text{(IV-A.1)}$$

B. Angle of oscillation:

The angle of oscillation can be determined as,

$$\gamma_0 = \left(\frac{2D}{D-d}\right)\phi$$

substituting Eq (IV-A.1),

$$\gamma_0 = \frac{2\pi}{(1-k^2)}$$

the above mentioned angle of oscillation is for an ideal case. While implementation considerations for gear clearance must be made,

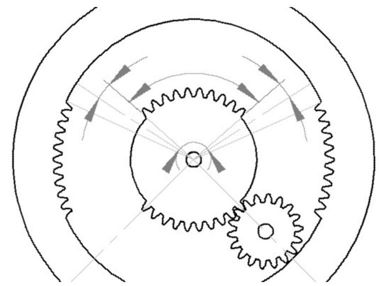

Fig. 3. Clearance between gears.

therefore, $\phi = \phi - 2\left(\frac{2\pi}{n}\right)$,

$$\gamma = \left(\frac{2}{1-k}\right)\gamma_0$$

$$\gamma = \frac{2\pi}{n}\left(\frac{n-4(1+k)}{1-k^2}\right) \qquad \text{(IV-B.1)}$$

C. Obtaining the desired angle of oscillation:

The value of θ computed by Eq: (IV-B.1) is completely determined by the ratio of diameters of the sun and ring gears. It is crucial to reduce this value of θ with a gear train in order to reduce the complexity in design of the gear system that changes direction.

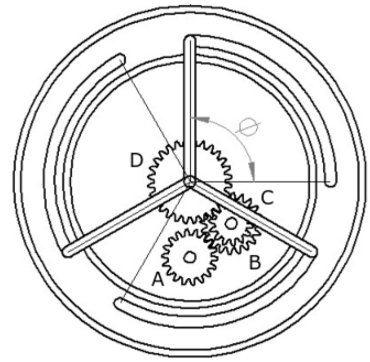

Fig. 4. Oscillating shaft.

Angle of oscillation of the driver gear (A) is θ which is to be converted into θ_{max} with a compound gear (BC) train with gear ratios,

$$ratio_{(AB)} = \frac{\pi}{\gamma}, \quad ratio_{(CD)} = \frac{\theta_{max}}{\pi}, \quad ratio_{(AD)} = \frac{\theta_{max}}{\gamma}$$

the shaft has three arms, each of which are slotted to prevent overrunning when the driven gear is under clearance angle.

V. Expansion mechanism:

The expansion mechanism is derived from a self centering mechanism that easily and quickly adjusts to the size of a closed segment ring. This is done by using a pair of disks which rotate against each other, drawing in and out a hex-head nut on a series of opposing arcs. But instead of opposing arcs, the derived system has radial slots in one of the disk opposing the other disc with an arc.

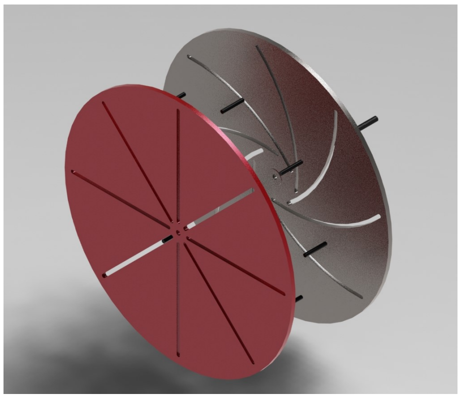

Fig. 5. Discs and spokes.

A. Locus of the arc:

The arcs on the rotating disc influences the radial expansion of wheel with much impact as they are the driving force behind it. It is important to find their optimal radius of curvature and their corresponding locus assuming center of the shaft as origin.

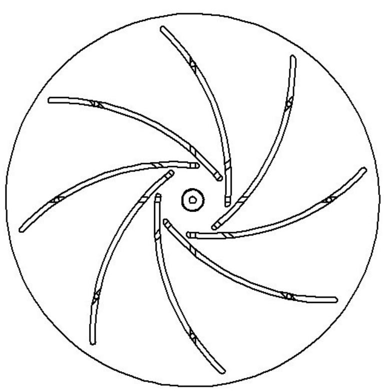

Fig. 6. arcs.

Locus of the arc is nothing but the intersection of two circles with radii $(a+b)$ and (t) at coordinates $(-b,0)$ and $(0,0)$ respectively. Where (t) is an arbitrary variable.

$$\begin{aligned}(x-b)^2+y^2 &= (a+b)^2\\ x^2+y^2 &= t^2\end{aligned}$$

It is straightforward to show that the intersection of these two circles consists of two points, with the coordinates

$$\begin{aligned}x_0 &= \frac{(-b+0)}{2}+\left(\frac{(a+b)^2-t^2}{2L^2}\right)(0-(-b))+2A\left(\frac{0}{L^2}\right)\\ y_0 &= \frac{(0+0)}{2}+\left(\frac{(a+b)^2-t^2}{2L^2}\right)(0)+2A\left(\frac{0-(-b)}{L^2}\right)\end{aligned}$$

where L is the distance between centers of the two intersecting circles and A is the area of the triangle with edge lengths $(a+b)$, (t), L as given by Herons formula

$$A=\frac{\sqrt{(L+b+a+t)(L+b+a-t)(L+t-b-a)(b+a+t-L)}}{4}$$

since $L=b$,

$$\begin{aligned}A &= \frac{\sqrt{(2b+a+t)(2b+a-t)(t-a)(t+a)}}{4}\\ A &= \frac{\sqrt{((2b+a)^2-t^2)(t^2-a^2)}}{4}\end{aligned}$$

Substituting the values of A and L,

$$\begin{aligned}x_0 &= \frac{-b}{2}+\left(\frac{(a+b)^2-t^2}{2b}\right)\\ y_0 &= \left(\frac{\sqrt{((2b+a)^2-t^2)(t^2-a^2)}}{2b}\right)\end{aligned}$$

Eliminating t, we arrive at locus of the arc,

$$y^2 = (a-x)(2b+a+x) \qquad \text{(V-A.1)}$$

where x lies in the domain,

$$\left(\frac{a^2+2ab-b^2}{2}\right) <= x <= (a)$$

B. Polar coordinates:

The locus of arc (Eq:V-A.1) when shifted to polar plane reduces numerous computations during rotation,

$$x=r\cos\theta, \quad y=r\sin\theta$$

$$\begin{aligned}r^2\sin^2\theta &= (a-r\cos\theta)(2b+a+r\cos\theta)\\ r^2 &= 2ab+a^2-2br\cos\theta\end{aligned}$$

$$r = \sqrt{(a+b)^2-b^2\sin^2\theta}-b\cos\theta$$

where θ lies in the domain,

$$0 <= \theta <= \cosh\left(\frac{m^2+2m-1}{2}\right)$$

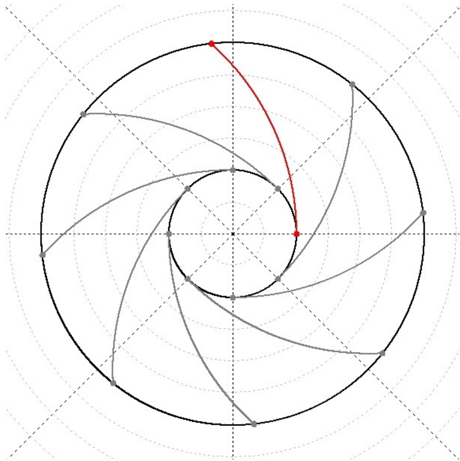

Fig. 7. Locus of arc.

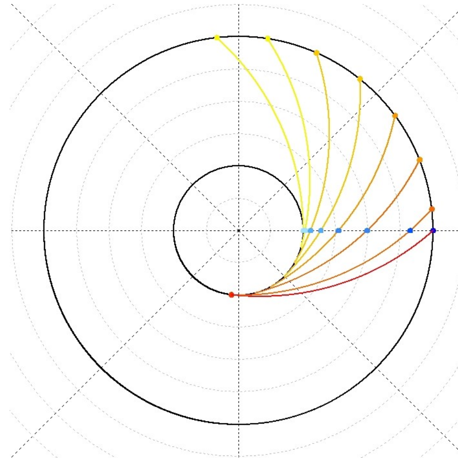

Fig. 8. Trajectory.

C. Trajectory of the spoke:

The arcs on the rotating discs determine the curvature of wheel therefore it is crucial to device an equation relating the angular displacement of the disc to the radial expansion of the spokes. Let us consider the trajectory of the spoke which is parallel to x-axis, assuming the spokes are initially at their minimum distance from the origin,

we know that, $y = 0$ at any point on the trajectory of the above considered spoke and also,

$$a <= x <= b$$

Now let us say that the two discs have a relative angular velocity of ω during expansion, at any time t the locus of the arc is given by,

$$r = \sqrt{(a+b)^2 - b^2\sin^2(\theta-\omega t)} - b\cos(\theta-\omega t) \qquad \text{(V-C.1)}$$

Trajectory of the spoke in polar coordinates,

$$r\sin\theta = 0, \quad a <= r <= b \qquad \text{(V-C.2)}$$

Computing the intersection of the equations (V-C.1) and (V-C.2) will present us the relationship between the relative angular velocity and the radial expansion of wheel,

$$\begin{aligned} r\sin(\theta-\omega t) &= 0 \\ \implies \omega t &= \theta - n\pi \end{aligned}$$

where $\theta <= \cosh\left(\dfrac{m^2+2m-1}{2}\right)$,

$$\omega t = \theta$$

Therefore the angular velocity of the disc and the radial expansion of wheel can be related as,

$$r = \sqrt{(a+b)^2 - b^2\sin^2\omega t} - b\cos\omega t \qquad \text{(V-C.3)}$$

VI. Motion Study:

A. Rate of expansion:

Now that we have the relationship between angular velocity of the disc and the radial displacement, it is much easier to derive an equation for the rate of expansion. Let us assume at some time t the angular displacement of the disc is θ,

$$\begin{aligned} r &= \sqrt{(a+b)^2 - b^2\sin^2\theta} - b\cos\theta \\ \frac{dr}{dt} &= \left(b\sin\theta - \frac{b^2\cos\theta\sin\theta}{\sqrt{(a+b)^2 - b^2\sin^2\theta}}\right)\frac{d\theta}{dt} \end{aligned}$$

It is clear that the rate of expansion of the wheel is dependent on the previous state and the angular velocity of the disc.

B. Dependence of 'θ_{max}' on 'm':

The dependence of maximum angular displacement of the disc on the ratio of inner and outer radii determines the range over which the curvature of wheel could be varied.
We know that,

$$\theta_{max} = \cosh\left(\frac{m^2+2m-1}{2}\right)$$

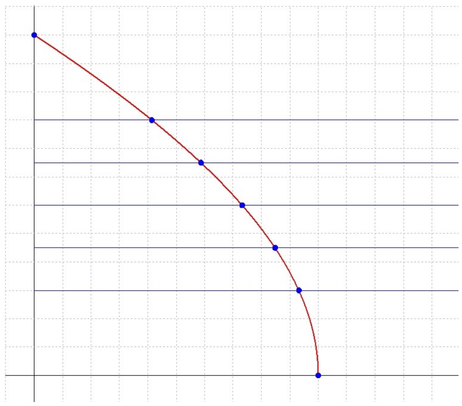

Fig. 9. θ_{max} vs m.

From the above graph, it is evident that θ_{max} increases with decreasing m, where

$$0 < m <= 1$$

also,

$$\frac{-1}{2} < \frac{m^2+2m-1}{2} <= 1$$

$$0 <= \cosh\left(\frac{m^2+2m-1}{2}\right) < \frac{2\pi}{3}$$

$$\implies 0 <= \theta_{max} < \frac{2\pi}{3}$$

Therefore θ_{max} ε $\left[0, \frac{2\pi}{3}\right)$, though the value of θ_{max} is subjected to change with m it is important to note that it lies well within the constraints posed by the oscillatory mechanism.