Solving ZDT1

Constraints

```matlab
N = 3; % Decision Variables
Functions = {@f1, @f2}; % Objective functions
M = length(Functions);
Maximum = 1;

RefPoints = 25;
f1_vals = zeros(1, RefPoints);
f2_vals = zeros(1, RefPoints);

wvals = linspace(1e-5, Maximum-1e-5, RefPoints);
zvals = linspace(1e-5, Maximum-1e-5, RefPoints);

rng(128);

for i = 1:RefPoints
    % z = [zvals(i) zvals(i)];  % Initial decision vector
    z = rand(1, M);
    % w = [wvals(i), 1-wvals(i)];
    w = rand(1, M);
    w = w / norm(w); % Normalize `w'

    assert (length(z) == M);
    assert (length(w) == M);

    % Non-Linear Conditions for ASF
    C1 = @(x) ASFCondition(x(1:N), Functions{1}, z(1), w(1));
    C2 = @(x) ASFCondition(x(1:N), Functions{2}, z(2), w(2));

    % Final Objective function
    Objective = @(x) ASF(x, Functions, M, z, w);

    % Bounds
    L = zeros(1, N+1);
    U =  ones(1, N+1) * Maximum;

    fprintf("Iter [%2d] Getting feasible solution ...\n", i);
    % x0 = ones(1, N+1);
    % while C1(x0) > 0 || C2(x0) > 0
    %     x0 = [rand(1, N+1) * Maximum];
    % end

    x0 = [rand(1, N) * Maximum 0.9999]

    fprintf("Iter [%2d] Solving ...\n", i);
```

```
    options = optimoptions('fmincon', 'Algorithm', 'sqp', 'TolFun', 1e-9, 'TolX',
1e-9, 'MaxFunctionEvaluations', 1e5, 'Display', 'final');
    [x, fval, exitflag, output] = fmincon(Objective, x0, [], [], [], [], L, U,
@(x)Constraint(x, C1, C2), options);

    f1_vals(i) = f1(x(1:end-1));
    f2_vals(i) = f2(x(1:end-1));
end
```

Iter [1] Getting feasible solution ...
x0 = 1×4
    0.2389   0.6455   0.7906   0.9999
Iter [1] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [2] Getting feasible solution ...
x0 = 1×4
    0.1097   0.6021   0.2513   0.9999
Iter [2] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [3] Getting feasible solution ...
x0 = 1×4
    0.1021   0.7065   0.9858   0.9999
Iter [3] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [4] Getting feasible solution ...
x0 = 1×4
    0.1257   0.9076   0.5732   0.9999
Iter [4] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [5] Getting feasible solution ...
x0 = 1×4
    0.1338   0.6559   0.4235   0.9999
Iter [5] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

```
<stopping criteria details>
Iter [ 6] Getting feasible solution ...
x0 = 1×4
    0.8439    0.6908    0.2029    0.9999
Iter [ 6] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [ 7] Getting feasible solution ...
x0 = 1×4
    0.2249    0.2416    0.8776    0.9999
Iter [ 7] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [ 8] Getting feasible solution ...
x0 = 1×4
    0.4363    0.5678    0.7552    0.9999
Iter [ 8] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [ 9] Getting feasible solution ...
x0 = 1×4
    0.8213    0.2015    0.1738    0.9999
Iter [ 9] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [10] Getting feasible solution ...
x0 = 1×4
    0.6336    0.1886    0.4499    0.9999
Iter [10] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [11] Getting feasible solution ...
x0 = 1×4
    0.8863    0.5316    0.7586    0.9999
Iter [11] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
```

the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [12] Getting feasible solution ...
x0 = 1×4
    0.3058   0.5624   0.4999   0.9999
Iter [12] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [13] Getting feasible solution ...
x0 = 1×4
    0.1968   0.4190   0.9930   0.9999
Iter [13] Solving ...
Feasible point with lower objective function value found.

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [14] Getting feasible solution ...
x0 = 1×4
    0.8519   0.9816   0.8622   0.9999
Iter [14] Solving ...
Feasible point with lower objective function value found.

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [15] Getting feasible solution ...
x0 = 1×4
    0.8040   0.2373   0.1901   0.9999
Iter [15] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [16] Getting feasible solution ...
x0 = 1×4
    0.8144   0.8395   0.0367   0.9999
Iter [16] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

```
<stopping criteria details>
Iter [17] Getting feasible solution ...
x0 = 1×4
    0.3627    0.0417    0.6135    0.9999
Iter [17] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [18] Getting feasible solution ...
x0 = 1×4
    0.5622    0.2837    0.6748    0.9999
Iter [18] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [19] Getting feasible solution ...
x0 = 1×4
    0.1768    0.6510    0.6534    0.9999
Iter [19] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [20] Getting feasible solution ...
x0 = 1×4
    0.6778    0.9382    0.0478    0.9999
Iter [20] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [21] Getting feasible solution ...
x0 = 1×4
    0.9887    0.6819    0.0105    0.9999
Iter [21] Solving ...
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [22] Getting feasible solution ...
x0 = 1×4
    0.9152    0.6188    0.5749    0.9999
Iter [22] Solving ...
Feasible point with lower objective function value found.
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [23] Getting feasible solution ...
x0 = 1×4
    0.3929    0.1748    0.4207    0.9999
Iter [23] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [24] Getting feasible solution ...
x0 = 1×4
    0.9488    0.2274    0.8223    0.9999
Iter [24] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Iter [25] Getting feasible solution ...
x0 = 1×4
    0.2715    0.8514    0.8543    0.9999
Iter [25] Solving ...
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```
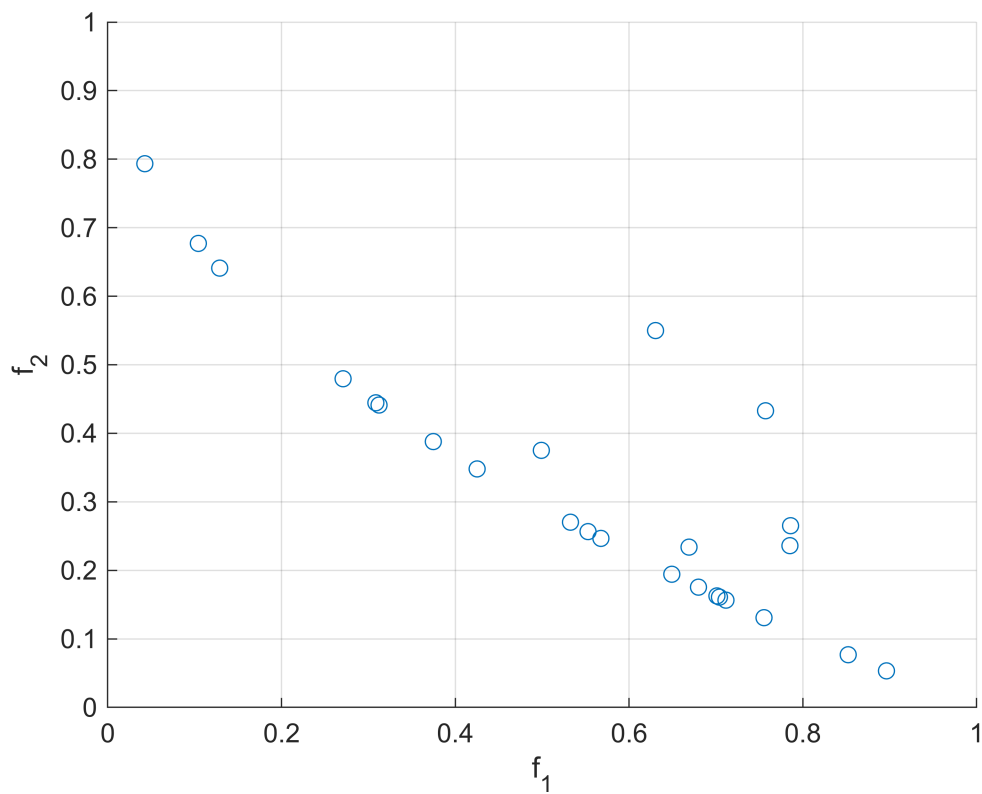
```
f1_vals
```

```
f1_vals = 1×25
    0.6491    0.5676    0.2708    0.3747    0.6798    0.7011    0.3122    0.5325 ···
```

```
f2_vals
```

```
f2_vals = 1×25
    0.1943    0.2466    0.4796    0.3879    0.1755    0.1627    0.4412    0.2702 ···
```

```matlab
figure;
scatter(f1_vals, f2_vals);
xlim([0 1]);
xlabel("f_1")
ylim([0 1]);
ylabel("f_2");
grid on;
```

Functions

f1, f2, ASFCondition don't handle for alpha

```matlab
function ret = f1(x)
    ret = x(1);
end

function ret = f2(x)
    G = 1 + sum(x(2:end));
    ret = G * (1 - sqrt(x(1) ./ G));
end

function ret = ASFCondition(x, Fn, z, w)
    ret = (Fn(x) - z) ./ w - x(end);
end

function [c, ceq] = Constraint(x, C1, C2)
    c = [C1(x); C2(x)];
    ceq = [];
end

% Handles alpha
function ret = ASF(x, Functions, M, z, w)
    C = zeros([1 M]);
```

```matlab
    x = x(1:end-1);  % alpha is not required
    for i = 1:M
        C(i) = (Functions{i}(x) - z(i)) ./ w(i);
    end
    ret = max(C);
end
```