Robotics II Project

Short Preview Optimization Approach to Minimize The (Inverse Inertia Weighted) Norm of The Joint Torque

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Introduction: Torque Norm Minimization

We can achieve torque norm minimization, optimizing different targets (depending on the weighting)

$$H_1 = \frac{1}{2}\|\tau\|^2 \qquad \text{MTN}$$

$$H_2 = \frac{1}{2}\|\tau\|^2_{M^{-2}} = \frac{1}{2}\tau^T M^{-2}(q)\tau \qquad \text{MTSIWN}$$

$$H_3 = \frac{1}{2}\|\tau\|^2_{M^{-1}} = \frac{1}{2}\tau^T M^{-1}(q)\tau \qquad \text{MTIWN}$$

Introduction: Redundant Robots

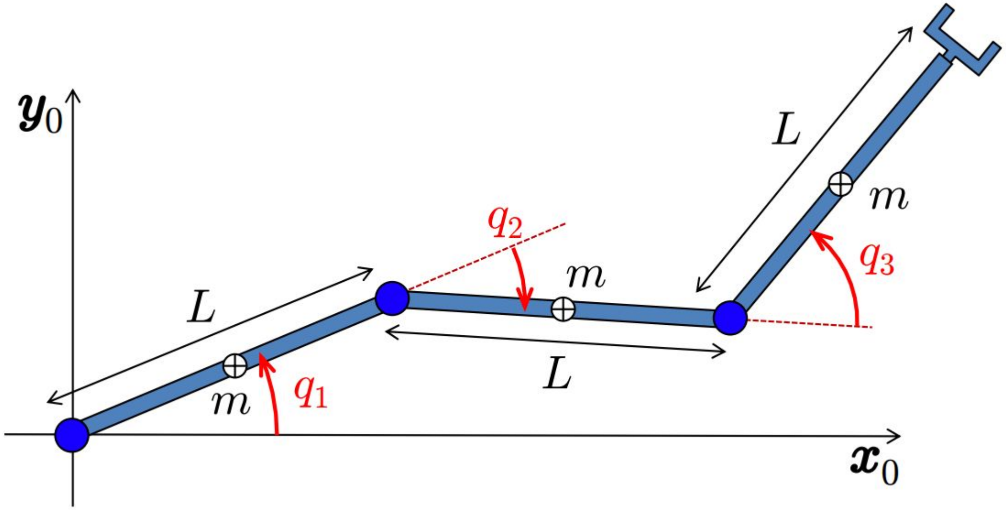
If the number of DOF of the robot (n) is bigger than the number of task constraints (m) the robot is **redundant** with respect to the task

The redundancy can be exploited to solve additional constraints like locally minimizing the **torque norm** projecting the desired behavior in the Null Space of the solution (to not affect the main task)

Introduction: Robot

We deal with a 3R planar robot placed on a horizontal plane, which allows us to neglect the effect of gravity.

- link mass (m = 10kg) uniformly distributed
- link length L = 1m
- moment of inertia $I_L = ml^2/12$
- relative joints positions (DH notation)



Introduction: Task Details

We want to perform a bang-bang profile on a linear path on a Cartesian plane (degree of redundancy n - m = 1)

Two versions of the task:

- long: 1.1738 m
- short: 0.2828 m

Given the task kinematics $x = f(q)$ we define the first- and second- order differential relations

$$\dot{x} = \frac{\partial f(q)}{\partial q}\dot{q} = J(q)\dot{q}$$

$$\ddot{x} = J(q)\ddot{q} + \dot{J}(q)\dot{q} = J(q)\ddot{q} + h$$

Robot Model

$$M(q)\ddot{q} + c(q,\dot{q}) = M(q)\ddot{q} + S(q,\dot{q})\dot{q} = M(q)\ddot{q} + n(q,\dot{q}) = \tau$$

$$p_{c1} = \begin{pmatrix} \frac{l}{2}c_1 \\ \\ \frac{l}{2}s_1 \end{pmatrix} \quad p_{c2} = \begin{pmatrix} lc_1 + \frac{l}{2}c_{12} \\ \\ ls_1 + \frac{l}{2}s_{12} \end{pmatrix} \quad p_{c3} = \begin{pmatrix} lc_1 + lc_{12} + \frac{l}{2}c_{123} \\ \\ ls_1 + ls_{12} + \frac{l}{2}s_{123} \end{pmatrix}$$

$$v_{ci} = \frac{\partial p_{ci}}{\partial q_i} \qquad i = 1,2,3$$

$$T_i = \frac{1}{2}v_{ci}^T m_l v_{ci} + \frac{1}{2}I_l \left(\sum_{j=1}^{i} \dot{q}_i \right)^2 \qquad i = 1,2,3$$

Robot Model (contd.)

Given that $T = \sum_i T_i = \ddot{q}^T M \ddot{q}$

$$M = m_l l^2 \begin{pmatrix} 4 + 3c_2 + c_3 + c_{23} & \frac{5}{3} + \frac{3}{2}c_2 + c_3 + \frac{1}{2}c_{23} & \frac{1}{3} + \frac{1}{2}(c_3 + c_{23}) \\ & \frac{5}{3} + c_3 & \frac{1}{3} + \frac{1}{2}c_3 \\ \text{symm} & & \frac{1}{3} \end{pmatrix}$$

$$c_i = \dot{q}^T C_i(q)\dot{q}, \quad C_i = \frac{1}{2}\left(\left(\frac{\partial m_i}{\partial q}\right) + \left(\frac{\partial m_i}{\partial q}\right)^T - \frac{\partial M}{\partial q_i}\right)$$

$$c(q, \dot{q}) = S(q, \dot{q})\dot{q} = \text{col}\{\dot{q}^T C_i(q)\}\ \dot{q}$$

General Minimum Torque Norm Solution

The problem can be defined as a standard linear-quadratic optimization

$$\min_{\ddot{q}} H = \frac{1}{2}\|\tau\|^2 = \frac{1}{2}\ddot{q}^T Q \ddot{q} + r^T \ddot{q} + s$$

$$\text{s.t. } \tau = Q^{\frac{1}{2}}\ddot{q} + Q^{-\frac{1}{2}} r$$

$$b = A\ddot{q}$$

with the following unique solution

$$\ddot{q} = A_Q^{\#}(b + AQ^{-1}r) - Q^{-1}r$$

$$\text{where} \quad A_Q^{\#} = Q^{-1}A^T(AQ^{-1}A^T)^{-1}$$

Minimum Torque Norm Optimization (MTN)

We can perform the following substitutions to the general solution to minimize the torque norm

$$A = J \qquad s = \frac{1}{2} n^T n$$

$$Q = M^2 \qquad b = \ddot{x} - h$$

$$r = Mn$$

To reduce tracking error during motion, a stabilizing PD feedback control term is inserted at the task level in b

$$b = \ddot{x} + K_D(\dot{x_d} - \dot{x}) + K_P(x_d - x) - h$$

Minimum Inverse Inertia Torque Norm (MTIWN)

Use the same approach to minimize the Inverse Weighted Torque Norm:

$$\min_{\ddot{q}} H_3 = \frac{1}{2}\|\tau\|^2_{M^{-1}} = \frac{1}{2}\tau^T M^{-1}\tau$$
$$= \frac{1}{2}\ddot{q}^T M\ddot{q} + n^T\ddot{q} + \frac{1}{2}n^T M^{-1} n = \frac{1}{2}\ddot{q}^T Q\ddot{q} + r^T\ddot{q} + s$$

which has the form of the general solution, applying the following substitutions:

$$A = J \qquad r = n$$
$$Q = M \qquad s = \frac{1}{2}n^T M^{-1} n$$

Min. Squared Inv. Inertia Torque Norm (MTSIWN)

And finally do the same for the Squared Inverse Weighted Torque Norm:

$$\min_{\ddot{q}} H_2 = \frac{1}{2}\|\tau\|^2_{M^{-2}} = \frac{1}{2}\tau^T M^{-2}\tau$$

$$= \frac{1}{2}\ddot{q}^T\ddot{q} + n^T M^{-1}\ddot{q} + \frac{1}{2}n_k^T M^{-2} n = \frac{1}{2}\ddot{q}^T Q\ddot{q} + r^T\ddot{q} + s$$

which has the form of the general solution, applying the following substitutions:

$$A = J \qquad Q = I$$

$$s = \frac{1}{2}n^T M^{-2} n \qquad r = (n^T M^{-1})^T = M^{-T} n = M^{-1} n$$

Short Preview Method

To prevent long term unstable behavior: include an estimation of the future robot state during the optimization process (Short Preview[1]). The method leads to the following optimization problem:

$$\min_{\ddot{q}_k, \ddot{q}_{k+1}} H = \frac{1}{2}\left(\omega_k \|\tau_k\|^2 + \omega_{k+1} \|\tau_{k+1}\|^2\right)$$

$$\begin{aligned} s.t. \quad \tau_k &= M_k \ddot{q}_k + n_k \\ \ddot{x}_k &= J_k \ddot{q}_k + h_k \\ \tau_{k+1} &= M_{k+1} \ddot{q}_{k+1} + n_{k+1} \\ \ddot{x}_{k+1} &= J_{k+1} \ddot{q}_{k+1} + h_{k+1} \end{aligned}$$

1. K. Al Khudir, G. Halvorsen, L. Lanari, and A. De Luca. Stable torque optimization for redundant robots using a short preview. IEEE Robotics and Automation Letters, 4(2):2046–2053, 2019.

Short Preview Method (contd.)

Some useful notation….

$$\dot{q}_{k+1} = \dot{q}(t_{k+1}) = \dot{q}_k + \ddot{q}_k T$$
$$q_{k+1} = q(t_{k+1}) = q_k + \dot{q}_k T + \frac{1}{2}\ddot{q}_k T^2$$

$$M_{k+1} = M(q_{k+1}) \qquad n_{k+1} = n(q_{k+1}, \dot{q}_{k+1})$$
$$J_{k+1} = J(q_{k+1}) \qquad h_{k+1} = h(q_{k+1}, \dot{q}_{k+1})$$

Short Preview Method (contd.)

The problem becomes nonlinear in the joint acceleration. This requires to apply the following approximations:

$$
\begin{aligned}
\ddot{x}_{k+1} &= J_{k+1}\ddot{q}_{k+1} + \dot{J}_{k+1}\dot{q}_{k+1} \\
&\approx J(q_k + \dot{q}_k T)\ddot{q}_{k+1} \\
&+ \frac{J(q_k + \dot{q}_k T) - J(q_k)}{T}(\dot{q}_k + \ddot{q}_k T) \\
&= J_{k+}\ddot{q}_{k+1} + (J_{k+} - J_k)\ddot{q}_k + h_{k+}
\end{aligned}
$$

where

$$
\begin{aligned}
J_{k+} &= J(q_k + \dot{q}_k T) \\
h_{k+} &= \frac{J_{k+} - J_k}{T}\dot{q}_k
\end{aligned}
$$

Short Preview Method (contd.)

We can rewrite the torque norm at time step $t = t_{k+1}$

$$
\begin{aligned}
\|\tau_{k+1}\|^2 &= \ddot{q}_{k+1}^T M_{k+1}^2 \ddot{q}_{k+1} + 2n^T M_{k+1} \ddot{q}_{k+1} + n_{k+1}^T n_{k+1} \\
&= \ddot{q}_{k+1}^T M_{k+1}^2 \ddot{q}_{k+1} + 2(c_{k+1} + g_{k+1})^T M_{k+1} \ddot{q}_{k+1} \\
&\quad + (c_{k+1} + g_{k+1})^T (c_{k+1} + g_{k+1}) \\
&\approx \ddot{q}_{k+1}^T M_{k+}^2 \ddot{q}_{k+1} + 2(S_{k+} \dot{q}_{k+1} + g_{k+})^T M_{k+} \ddot{q}_{k+1} \\
&\quad + \dot{q}_{k+1}^T S_{k+}^T S_{k+} \dot{q}_{k+1} + 2g_{k+}^T S_{k+} \dot{q}_{k+1} + g_{k+}^T g_{k+} \\
&= \ddot{q}_{k+1}^T M_{k+}^2 \ddot{q}_{k+1} \\
&\quad + 2(S_{k+}(\dot{q}_k + T\ddot{q}_k) + g_{k+})^T M_{k+} \ddot{q}_{k+1} \\
&\quad + (\dot{q}_k + T\ddot{q}_k)^T S_{k+}^T S_{k+} (\dot{q}_k + T\ddot{q}_k) \\
&\quad + 2g_{k+}^T S_{k+} (\dot{q}_k + T\ddot{q}_k) + g_{k+}^T g_{k+}
\end{aligned}
$$

where

$$
M_{k+} = M(q_k + \dot{q}_k T), \qquad g_{k+} = g(q_k + \dot{q}_k T)
$$

$$
S_{k+} = S(q_k + \dot{q}_k T, \dot{q}_k) = \mathrm{col}\{\dot{q}_k^T C_i(q_k + \dot{q}_k T)\}
$$

Short Preview: Model Based Preview (MBP)

The problem can now be expressed in the general solution form with the following substitutions:

$$Q = \begin{pmatrix} \omega_k M_k^2 + \omega_{k+1} T^2 S_{k+}^T S_{k+} & \omega_{k+1} T S_{k+}^T M_{k+} \\ \text{symm} & \omega_{k+1} M_{k+}^2 \end{pmatrix}$$

$$r = \begin{pmatrix} \omega_k M_k (S_k \dot{q}_k + g_k) + \omega_{k+1} T S_{k+}^T (S_{k+} \dot{q}_k + g_{k+}) \\ \omega_{k+1} M_{k+} (S_{k+} \dot{q}_k + g_{k+}) \end{pmatrix}$$

$$A = \begin{pmatrix} J_k & \mathbf{0} \\ J_{k+} - J_k & J_{k+} \end{pmatrix}$$

$$b = \begin{pmatrix} \ddot{x}_k + K_D (\dot{x}_{dk} - \dot{x}_k) + K_P (x_{dk} - x_k) - h_k \\ \ddot{x}_{k+1} - h_{k+} \end{pmatrix}$$

Model Based Inv. Inertia Weighted Preview (MBIWP)

We use the same approach to minimize the Inverse Weighted Torque Norm with the Short Preview method:

$$\min_{\ddot{q}_k, \ddot{q}_{k+1}} H_3 = \frac{1}{2}(\omega_k \|\tau_k\|^2_{M^{-1}} + \omega_{k+1}\|\tau_{k+1}\|^2_{M^{-1}})$$

$$\|\tau_k\|^2_{M_k^{-1}} = \ddot{q}_k^T M_k \ddot{q}_k + 2n^T \ddot{q}_k + n_k^T M_k^{-1} n_k$$

$$\begin{aligned}
\|\tau_{k+1}\|^2_{M_{k+1}^{-1}} &= \ddot{q}_{k+1}^T M_{k+1}\ddot{q}_{k+1} + 2n^T\ddot{q}_{k+1} + n_{k+1}^T M_{k+1}^{-1} n_{k+1} \\
&= \ddot{q}_{k+1}^T M_{k+1}\ddot{q}_{k+1} + 2(c_{k+1} + g_{k+1})^T \ddot{q}_{k+1} \\
&\quad + (c_{k+1} + g_{k+1})^T M_{k+1}^{-1}(c_{k+1} + g_{k+1}) \\
&\approx \ddot{q}_{k+1}^T M_{k+}\ddot{q}_{k+1} + 2(S_{k+}\dot{q}_{k+1} + g_{k+})^T \ddot{q}_{k+1} \\
&\quad + \dot{q}_{k+1}^T S_{k+}^T M_{k+}^{-1} S_{k+}\dot{q}_{k+1} + \dot{q}_{k+1}^T S_{k+}^T M_{k+}^{-1} g_{k+} \\
&\quad + g_{k+}^T S_{k+} M_{k+}^{-1}\dot{q}_{k+1} + g_{k+}^T M_{k+}^{-1} g_{k+} \\
&= \ddot{q}_{k+1}^T M_{k+}\ddot{q}_{k+1} + 2(S_{k+}(\dot{q}_k + T\ddot{q}_k) + g_{k+})^T \ddot{q}_{k+1} \\
&\quad + (\dot{q}_k + T\ddot{q}_k)^T S_{k+}^T M_{k+}^{-1} S_{k+}(\dot{q}_k + T\ddot{q}_k) \\
&\quad + (\dot{q}_k + T\ddot{q}_k)^T S_{k+}^T M_{k+}^{-1} g_{k+} \\
&\quad + g_{k+}^T S_{k+} M_{k+}^{-1}(\dot{q}_k + T\ddot{q}_k) + g_{k+}^T M_{k+}^{-1} g_{k+}
\end{aligned}$$

Model Based Inv. Inertia Weighted Preview (MBIWP)

where the substitutions in the general solution now become:

$$Q = \begin{pmatrix} \omega_k M_k + \omega_{k+1} T^2 S_{k+}^T M_{k+}^{-1} S_{k+} & \omega_{k+1} T S_{k+}^T \\ \text{symm} & \omega_{k+1} M_{k+} \end{pmatrix}$$

$$r = \begin{pmatrix} \omega_k (S_k \dot{q}_k + g_k) + \omega_{k+1} T S_{k+}^T M_{k+}^{-1} (S_{k+} \dot{q}_k + g_{k+}) \\ \omega_{k+1} (S_{k+} \dot{q}_k + g_{k+}) \end{pmatrix}$$

$$A = \begin{pmatrix} J_k & \mathbf{0} \\ J_{k+} - J_k & J_{k+} \end{pmatrix}$$

$$b = \begin{pmatrix} \ddot{x}_k + K_D (\dot{x_{dk}} - \dot{x_k}) + K_P (x_{dk} - x_k) - h_k \\ \ddot{x}_{k+1} - h_{k+} \end{pmatrix}$$

MB Squared Inv. Inertia Weighted Preview (MBSIWP)

Finally we do the same for the Squared Inv. Inertia Weighted Torque Norm for which the problem can be expressed as:

$$\min_{\ddot{q}_k, \ddot{q}_{k+1}} H_2 = \frac{1}{2}(\omega_k \|\tau_k\|^2_{M^{-2}} + \omega_{k+1} \|\tau_{k+1}\|^2_{M^{-2}})$$

$$\|\tau_k\|^2_{M_k^{-2}} = \ddot{q}_k^T \ddot{q}_k + 2n^T M_k^{-1} \ddot{q}_k + n_k^T M_k^{-2} n_k$$

$$\begin{aligned}
\|\tau_{k+1}\|^2_{M_{k+1}^{-2}} &= \ddot{q}_{k+1}^T \ddot{q}_{k+1} + 2n^T M_{k+1}^{-1} \ddot{q}_{k+1} + n_{k+1}^T M_{k+1}^{-2} n_{k+1} \\
&= \ddot{q}_{k+1}^T \ddot{q}_{k+1} + 2(c_{k+1} + g_{k+1})^T M_{k+1}^{-1} \ddot{q}_{k+1} \\
&\quad + (c_{k+1} + g_{k+1})^T M_{k+1}^{-2} (c_{k+1} + g_{k+1}) \\
&\approx \ddot{q}_{k+1}^T \ddot{q}_{k+1} + 2(S_{k+} \dot{q}_{k+1} + g_{k+})^T M_{k+}^{-1} \ddot{q}_{k+1} \\
&\quad + \dot{q}_{k+1}^T S_{k+}^T M_{k+}^{-2} S_{k+} \dot{q}_{k+1} + \dot{q}_{k+1}^T S_{k+}^T M_{k+}^{-2} g_{k+} \\
&\quad + g_{k+}^T S_{k+} M_{k+}^{-2} \dot{q}_{k+1} + g_{k+}^T M_{k+}^{-2} g_{k+} \\
&= \ddot{q}_{k+1}^T \ddot{q}_{k+1} + 2(S_{k+}(\dot{q}_k + T\ddot{q}_k) + g_{k+})^T M_{k+}^{-1} \ddot{q}_{k+1} \\
&\quad + (\dot{q}_k + T\ddot{q}_k)^T S_{k+}^T M_{k+}^{-2} S_{k+} (\dot{q}_k + T\ddot{q}_k) \\
&\quad + (\dot{q}_k + T\ddot{q}_k)^T S_{k+}^T M_{k+}^{-2} g_{k+} \\
&\quad + g_{k+}^T S_{k+} M_{k+}^{-2} (\dot{q}_k + T\ddot{q}_k) + g_{k+}^T M_{k+}^{-2} g_{k+}
\end{aligned}$$

MB Squared Inv. Inertia Weighted Preview (MBSIWP)

We adapt the general solution with these substitutions:

$$Q = \begin{pmatrix} \omega_k I + \omega_{k+1} T^2 S_{k+}^T M_{k+}^{-2} S_{k+} & \omega_{k+1} T S_{k+}^T M_{k+}^{-1} \\ \text{symm} & \omega_{k+1} I \end{pmatrix}$$

$$r = \begin{pmatrix} \omega_k M_k^{-1}(S_k \dot{q}_k + g_k) + \omega_{k+1} T S_{k+}^T M_{k+}^{-2}(S_{k+}\dot{q}_k + g_{k+}) \\ \omega_{k+1} M_{k+}^{-1}(S_{k+}\dot{q}_k + g_{k+}) \end{pmatrix}$$

$$A = \begin{pmatrix} J_k & \mathbf{0} \\ J_{k+} - J_k & J_{k+} \end{pmatrix}$$

$$b = \begin{pmatrix} \ddot{x}_k + K_D(\dot{x_{dk}} - \dot{x_k}) + K_P(x_{dk} - x_k) - h_k \\ \ddot{x}_{k+1} - h_{k+} \end{pmatrix}$$

Damping In The Null-Space

To reduce joint-motion oscillation we minimize the difference between the torque and a given reference:

$$\tau_r = \tau_{D_k} = -D_k M_k \dot{q}_k$$

Target Torque

$$\ddot{q}_k = -M_k^{-1}(S_k \dot{q}_k + g_k) - d\dot{q}_k$$

Resulting Joint acceleration
(assuming $D_k = dI > 0$)

$$H = \frac{1}{2}\|\tau_k - \tau_{D_k}\|^2$$
$$= \|M_k \ddot{q}_k + (S_k + D_k M_k)\dot{q}_k + g_k\|^2$$

Minimization Problem

$$H = \frac{1}{2}(\omega_k \|\tau_k - \tau_{D_k}\|^2 + \omega_{k+1}\|\tau_{k+1} - \tau_{D_k}\|^2)$$

$$\text{where} \quad \tau_{D_k} = -D_{k+1} M_{k+}(\dot{q}_k + T\ddot{q}_k)$$

Minimization Problem with Preview Window

Final substitutions in **r** and **Q** needed for the damping.

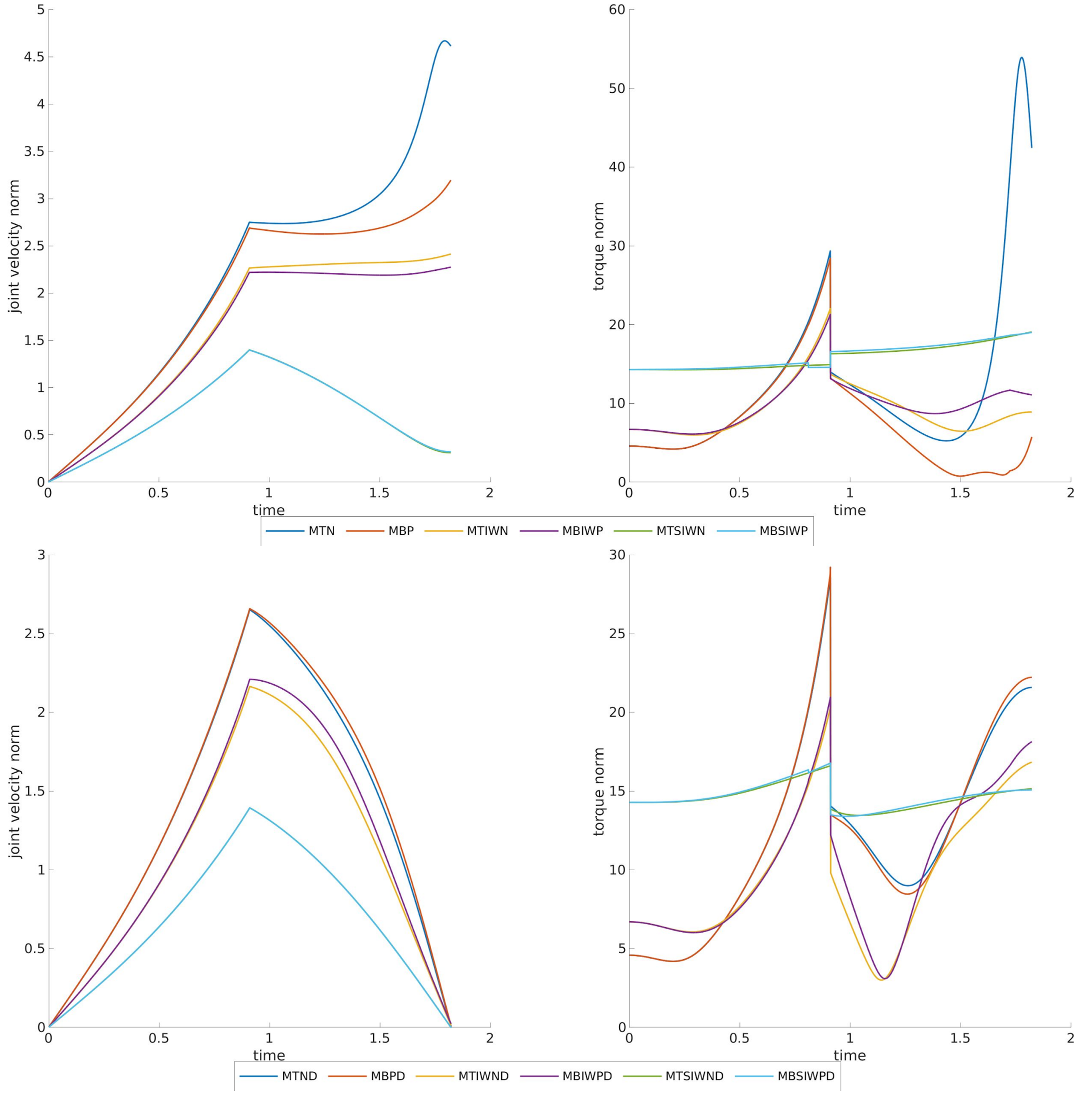
$$S_k \rightarrow (S_k + D_k M_k)$$
$$S_{k+} \rightarrow (S_{k+} + D_{k+1} M_{k+})$$

Results: Experimental Setup

Results are obtained with the same settings of the original paper:

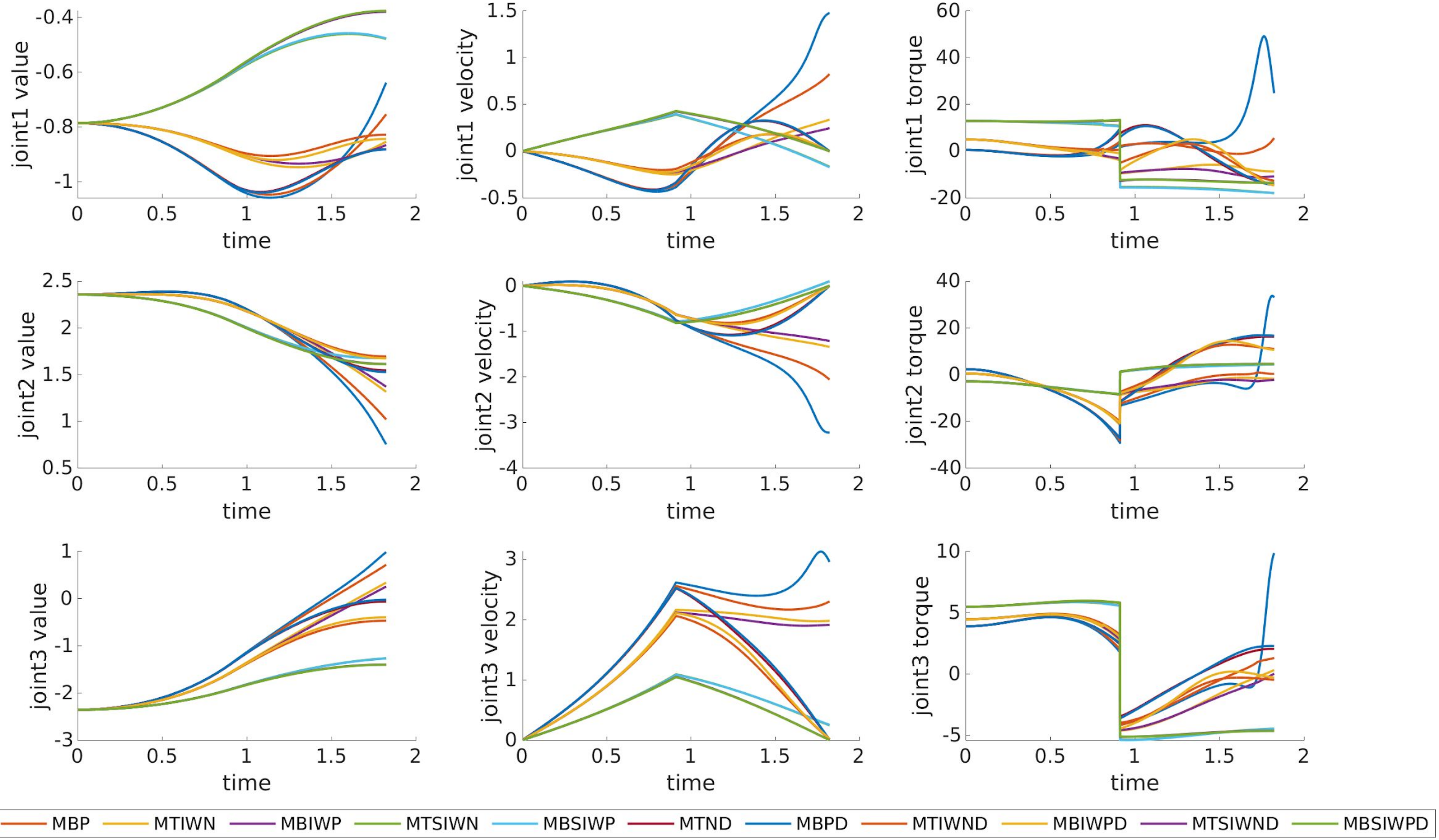
- $T_s = 0.001$ s
- $D_k = 10\ I$
- $K_P = 10\ I$
- $K_D = I$
- $p = 100 \rightarrow T = 100\ T_s = 0.1$ s
- $A = 1.4142\ m/s^2$

Results: Long Path - Velocity and Torque Norm

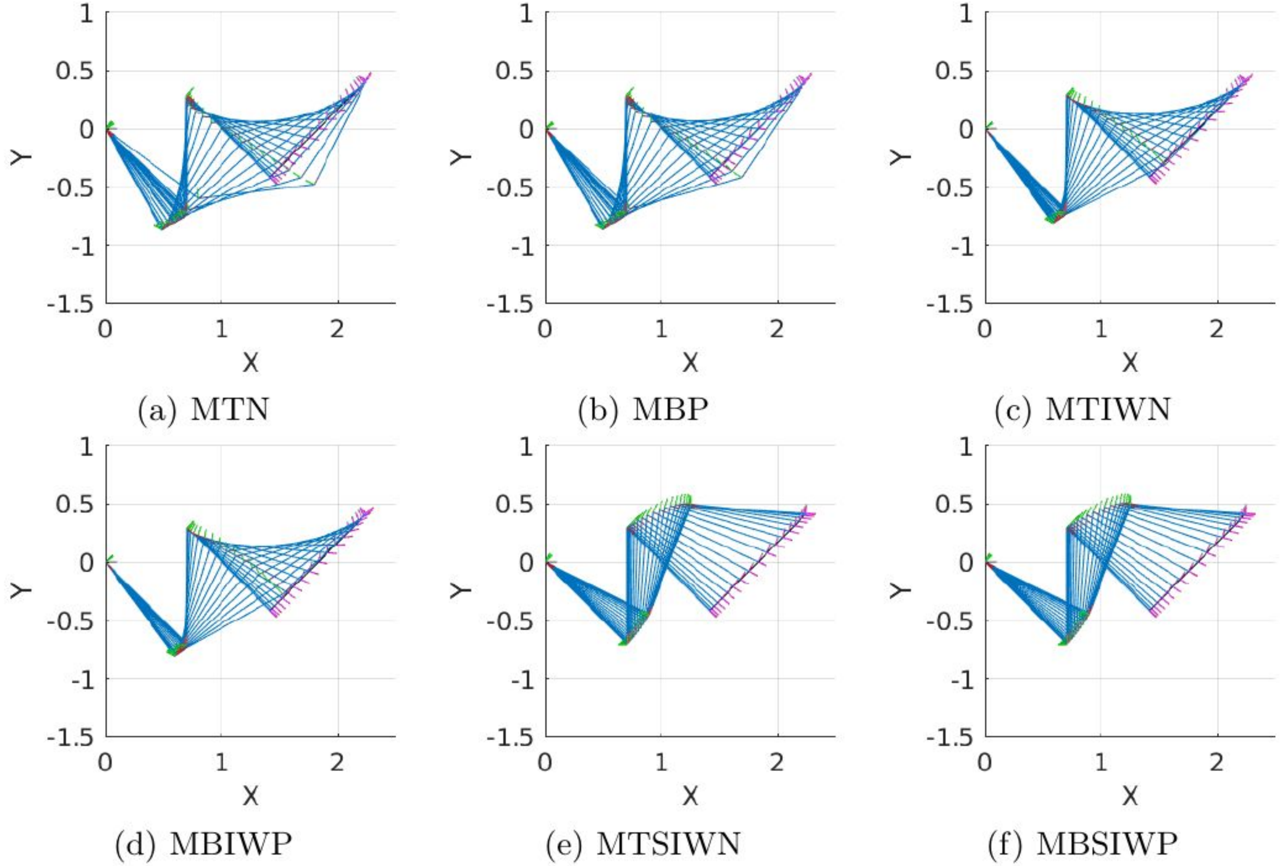


Results: Long Path - Joint Profiles

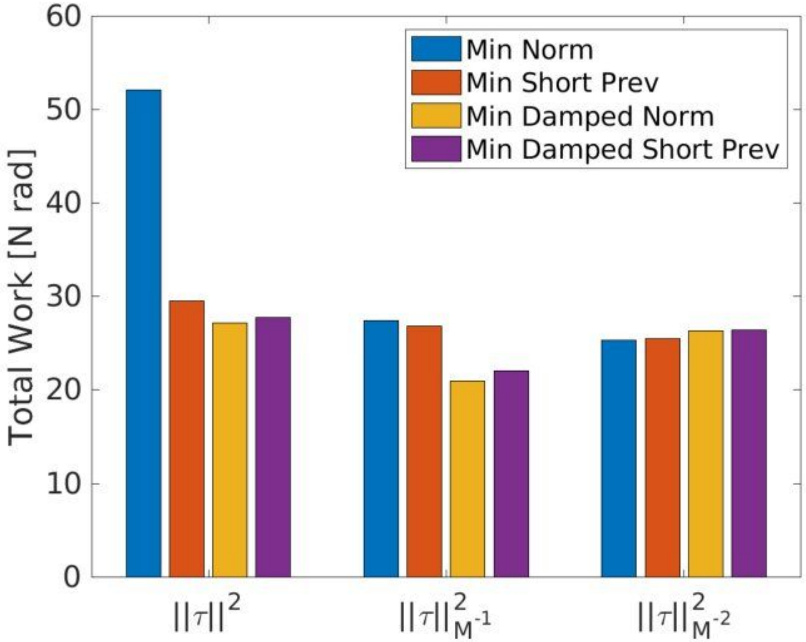
Only for general differences in behavior. Cannot pinpoint specific methods.



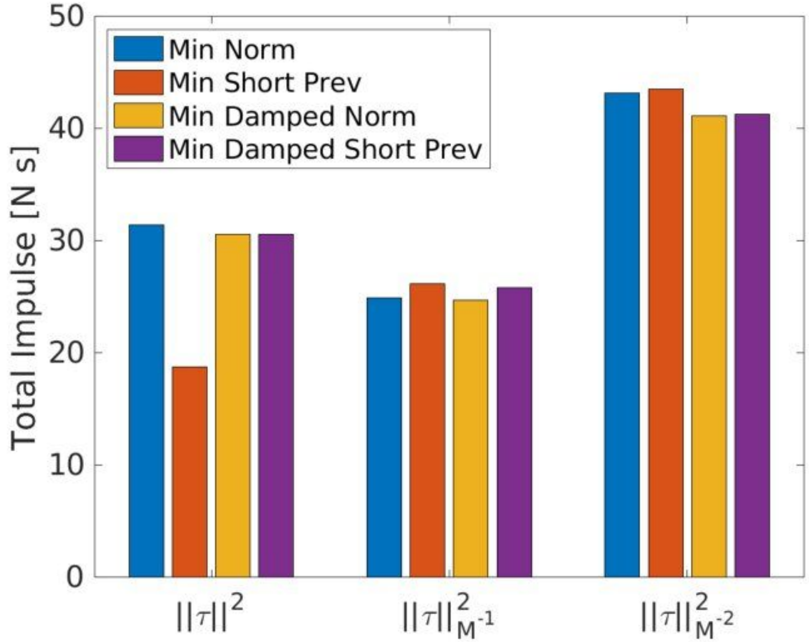
Results: Long Path - Motions



Results: Long Path - Work & Impulse

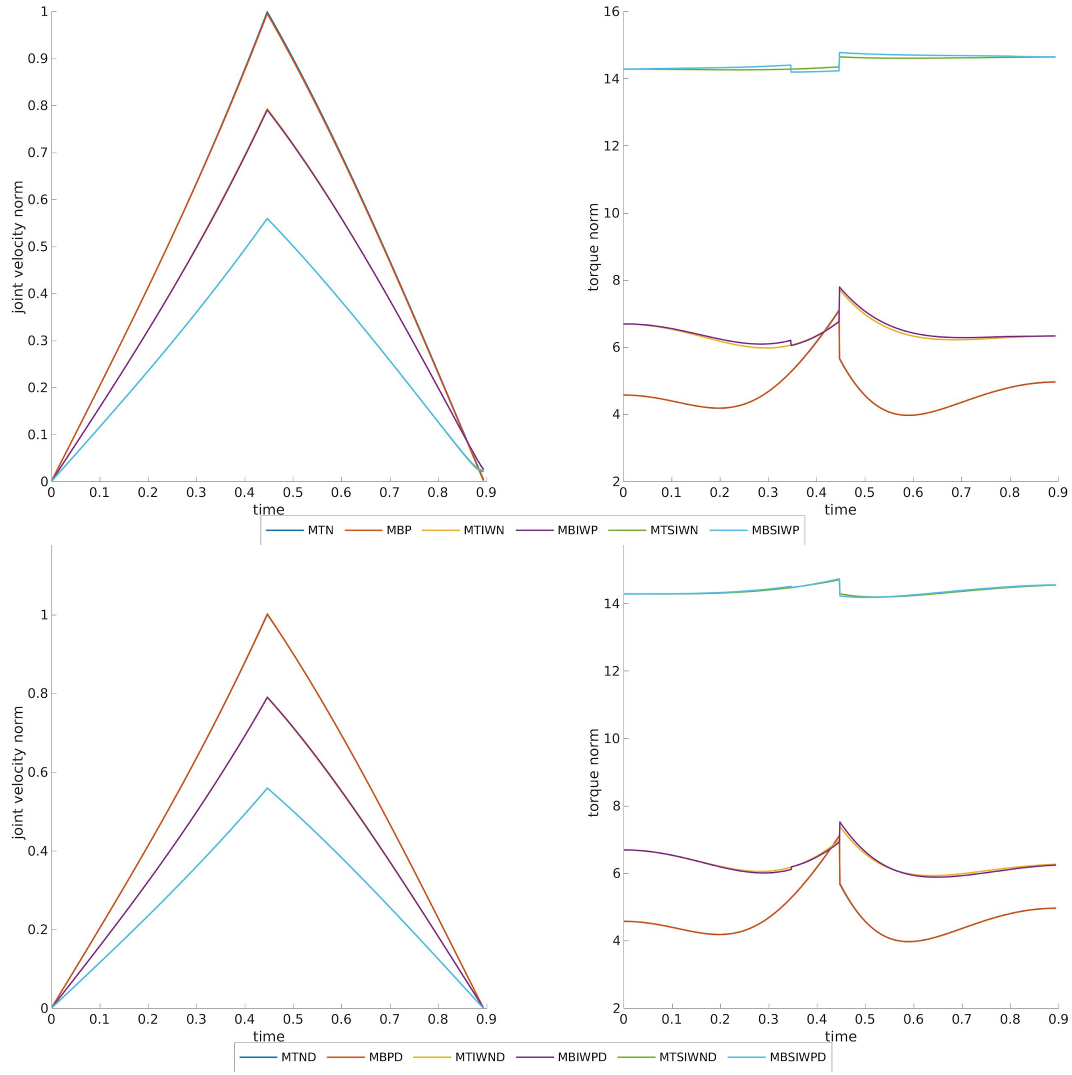


(a)



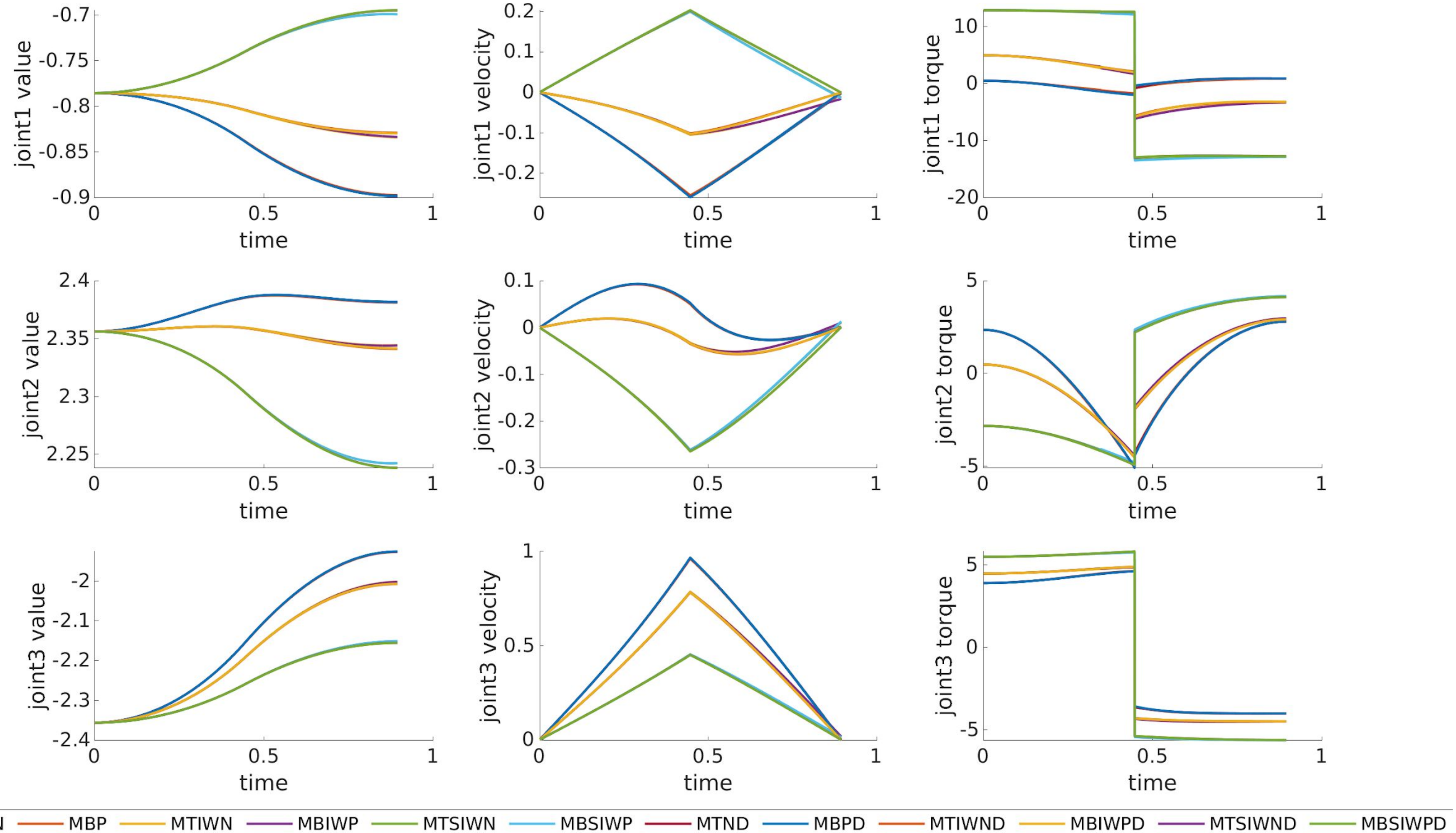
(b)

Results: Short Path - Velocity and Torque Norm

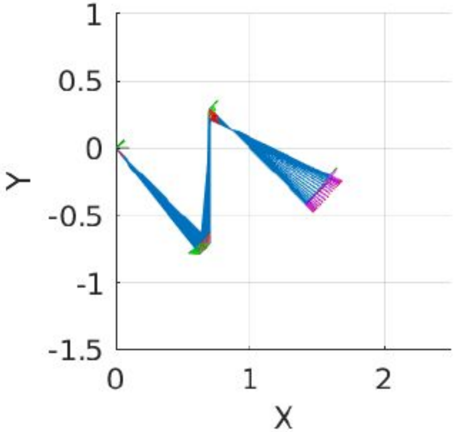


Results: Short Path - Joint Profiles

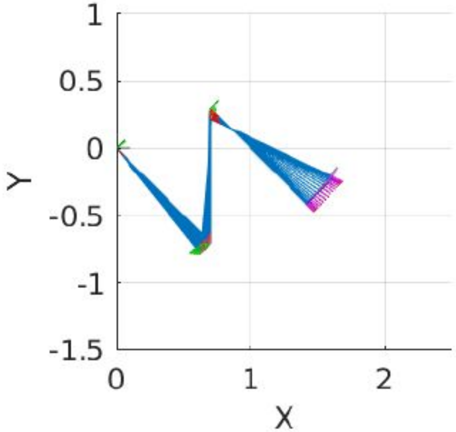
Only for general differences in behavior. Cannot pinpoint specific methods.



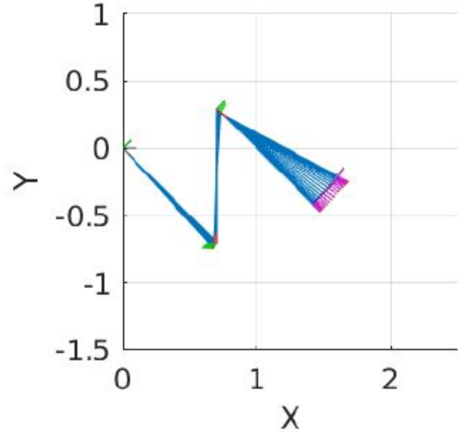
Results: Short Path - Motions



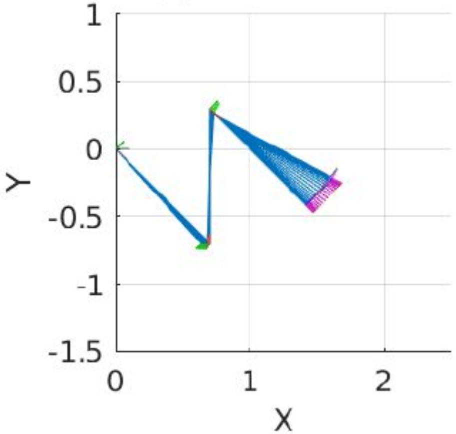
(a) MTN



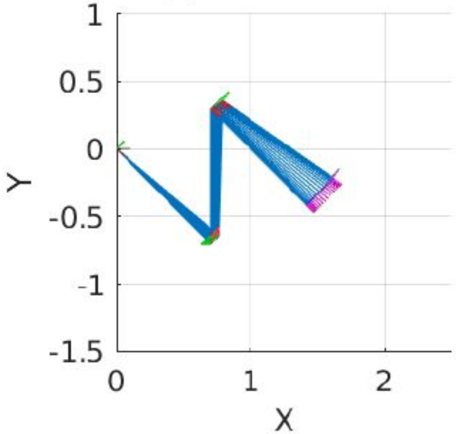
(b) MBP



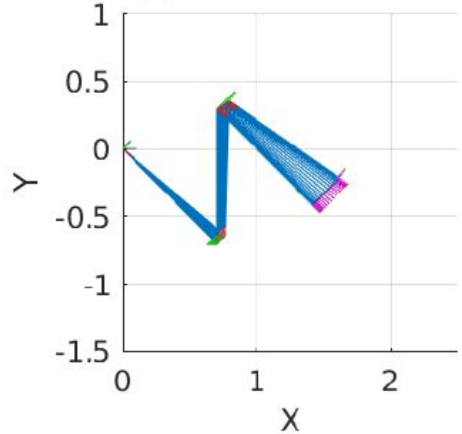
(c) MTIWN



(d) MBIWP

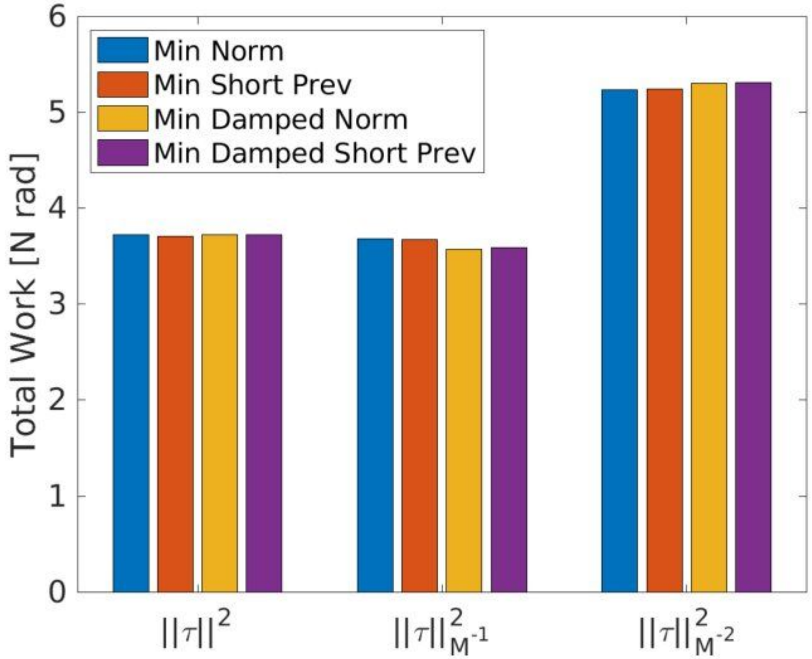


(e) MTSIWN

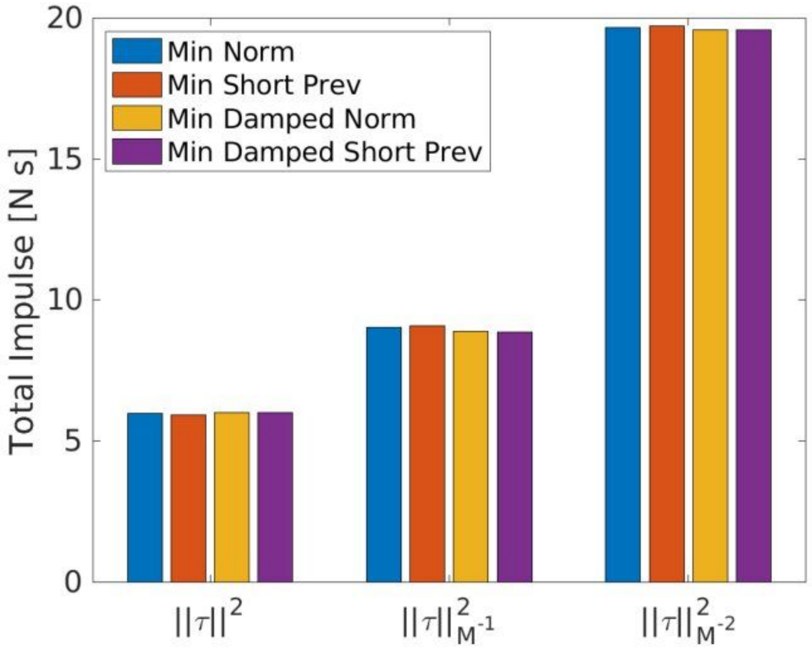


(f) MBSIWP

Results: Short Path - Work & Impulse



(a)



(b)

Conclusions

The tasks we experimented on, may be too simple to evaluate more complex behavior that can arise from the application of the different methods.

Minimizing the torque, taking into account the estimation of the future state of the robot, can lead to a smoother motion.

We see that using the damping and a weighted torque norm already results in a less sharp motion.

Future Work

Using an adaptive time window might further help the effectiveness of the estimation, specifically by choosing a smaller window when possible in order to not approximate more than necessary.

On the other hand, a tiny window may not be helpful as it would simply degenerate to the vanilla approach.

We assume there is a function of velocity and acceleration that can optimally predict the best value for the preview-window size (p), hence it may be possible to train a RL agent, to approximate that function.