

Cartesian Plane: Point-Slope Formula for Lines

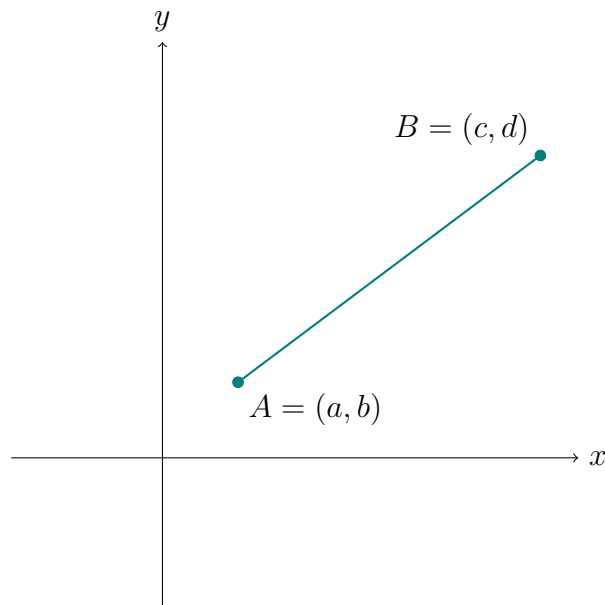
Video companion

1 Introduction

In this video: Demystify formulas for equations of lines

$$\begin{array}{ll} y - y_0 = m(x - x_0) & \text{Point-slope form} \\ y = mx + b & \text{Slope-intercept form} \end{array}$$

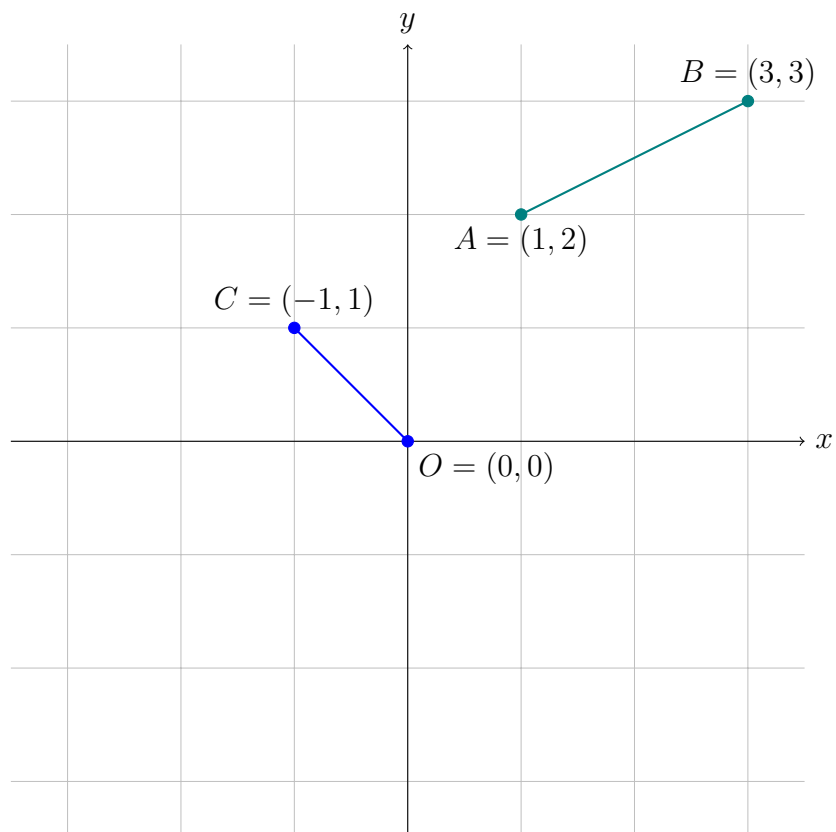
2 Slope of a line segment



Slope of \overrightarrow{AB} :

$$m = \frac{d - b}{c - a} = \frac{\text{“rise”}}{\text{“run”}}$$

3 Examples



Slope of \overrightarrow{AB} :

$$m = \frac{3 - 2}{3 - 1} = \frac{1}{2}$$

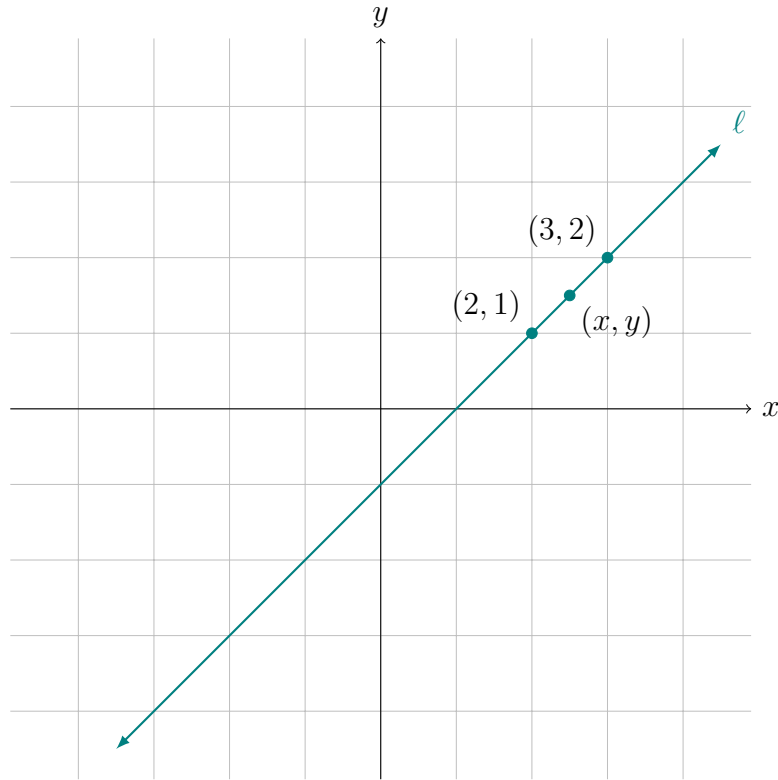
$m = \frac{1}{2}$ is a positive slope.

Slope of \overrightarrow{CO} :

$$m = \frac{0 - 1}{0 - (-1)} = -1$$

$m = -1$ is a negative slope.

4 Equation of a line



For a point (x, y) to be on the line, the line segment from $(2, 1)$ to (x, y) need to have a slope of 1.

$$1 = \frac{y - 1}{x - 2}$$
$$y - 1 = 1(x - 2)$$

The line is defined by this formula:

$$\ell = \{(x, y) \in \mathbb{R}^2 : y - 1 = 1(x - 2)\}$$

Check that $(3, 2)$ is on the line:

$$(3, 2) \in \ell ?$$
$$2 - 1 \stackrel{?}{=} 1(3 - 2)$$
$$1 \stackrel{?}{=} 1 \quad \checkmark$$

Check if $(5, 1)$ is on the line:

$$(5, 1) \in \ell ?$$

$$1 - 1 \stackrel{?}{=} 1(5 - 2)$$

$$0 \stackrel{?}{=} 3 \quad \times$$

5 Point-slope formula

If a line ℓ has slope m , *and* if (x_0, y_0) is *any* point on ℓ , then ℓ has the equation

$y - y_0 = m(x - x_0).$