Fast Growth, Slow Growth: Simplification Rules for Algebra Using Exponents

Video companion

1 Exponent simplification rules

Five rules for simplifying algebraic expressions with exponents

1. Multiplication rule
   $x^n x^m = x^{(n+m)}$
2. Power to a power
   $(x^n)^m = x^{nm}$
3. Product to a power
   $(xy)^n = x^n y^n$
4. Fraction to a power
   $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$
5. Division and negative powers
   $\frac{x^n}{x^m} = x^{(n-m)}$

2 Examples

Simple examples

$$\left(7^3\right)\left(7^7\right) = 7^{(3+7)} = 7^{10}$$

$$\left(4^3\right)^5 = 4^{(3\cdot 5)} = 4^{15}$$

$$(8 \cdot 9)^7 = \left(8^7\right)\left(9^7\right) = 1.00306 \times 10^{13}$$

$$\left(\frac{2}{7}\right)^3 = \frac{2^3}{7^3} = 0.023323615$$

$$\frac{10^5}{10^3} = 10^{(5-3)} = 10^2 = 100$$

Complex examples

$$\frac{x^3y^4z^5}{x^3y^5z^2} = \frac{x^3}{x^3}\frac{y^4}{y^5}\frac{z^5}{z^2} = x^{(3-3)}y^{(4-5)}z^{(5-2)} = y^{-1}z^3 = \frac{z^3}{y}$$

$$\left[\frac{(xy)^2}{x^{-3}y^2}\right]^{-1} = \left[\frac{x^2y^2}{x^{-3}y^2}\right]^{-1} = \left[x^{(2--3)}y^{(2-2)}\right]^{-1} = \left[x^5\right]^{-1} = x^{-5} = \frac{1}{x^5}$$

3 Fractional exponents

In general

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

Examples

$$\begin{aligned} 8^{\frac{2}{3}} &= \left[\sqrt[3]{8}\right]^2 \\ &= \left[\sqrt[3]{2\cdot 2\cdot 2}\right]^2 = 2^2 = 4 \end{aligned}$$

or

$$\begin{aligned} &= \sqrt[3]{8^2} \\ &= \sqrt[3]{64} = \sqrt[3]{4\cdot 4\cdot 4} = 4 \end{aligned}$$

$$\begin{aligned} 125^{\frac{4}{3}} &= \left[\sqrt[3]{125}\right]^4 \\ &= \left[\sqrt[3]{5\cdot 5\cdot 5}\right]^4 = 5^4 = 625 \end{aligned}$$