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Numbers: Algebra with Inequalities

Video companion

1 Introduction

- Review algebra with equalities ($=$)
  - how?
  - why?
- Learn algebra with inequalities ($<, >, \leq, \geq$)
  - what works
  - A BIG WARNING

2 Algebra with equalities

$$\begin{aligned} 4 &= 4 \\ 4+3 &= 4+3 \\ 7 &= 7 \quad \checkmark \end{aligned}$$

Rule:
If $a = b$, then $a + c = b + c$.

Example:

$$\begin{aligned} x+3 &= 10 \\ (x+3)-3 &= 10-3 \\ x &= 7 \end{aligned}$$

Similarly with multiplication,

$$\begin{aligned} 4 &= 4 \\ 2 \cdot 4 &= 2 \cdot 4 \\ 8 &= 8 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 4 &= 4 \\ (-3)\cdot 4 &= (-3)\cdot 4 \\ -12 &= -12 \quad \checkmark \end{aligned}$$

Rule:
If a, b, and c are numbers, and $c \neq 0$, and $a = b$, then $c \cdot a = c \cdot b$.

Example:

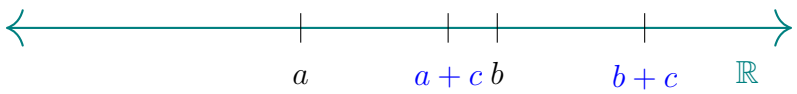
$$\begin{aligned} -5x &= 15 \\ \left(-\frac{1}{5}\right)\cdot(-5x) &= \left(-\frac{1}{5}\right)\cdot 15 \\ x &= -3 \end{aligned}$$

3 Algebra with inequalities

$$\begin{aligned} 4 &< 7 \\ 4+2 &\overset{?}{<} 7+2 \\ 6 &\overset{?}{<} 9 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 4 &< 7 \\ 4-1 &\overset{?}{<} 7-1 \\ 3 &\overset{?}{<} 6 \quad \checkmark \end{aligned}$$

Rule:
If $a < b$, then $a + c < b + c$.



Example:

$$\begin{aligned} x + 3 &< 10 \\ (x+3) - 3 &< 10 - 3 \\ x &< 7 \end{aligned}$$

$x \in (-\infty, 7)$

Test cases with multiplication:

$$\begin{aligned} 5 &< 8 \\ 3 \cdot 5 &\overset{?}{<} 3 \cdot 8 \\ 15 &\overset{?}{<} 40 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 5 &< 8 \\ (-1) \cdot 5 &\overset{?}{<} (-1) \cdot 8 \\ -5 &\overset{?}{<} -8 \quad \times \\ -5 &> -8 \quad ! \end{aligned}$$

Rule:

Suppose $a < b$.

If $c > 0$, then $a \cdot c < b \cdot c$.

If $c < 0$, then $a \cdot c > b \cdot c$.

Example:

$$\begin{aligned} -2x &< 10 \\ \left(-\frac{1}{2}\right) \cdot (-2x) &> \left(-\frac{1}{2}\right) \cdot 10 \\ x &> -5 \end{aligned}$$