

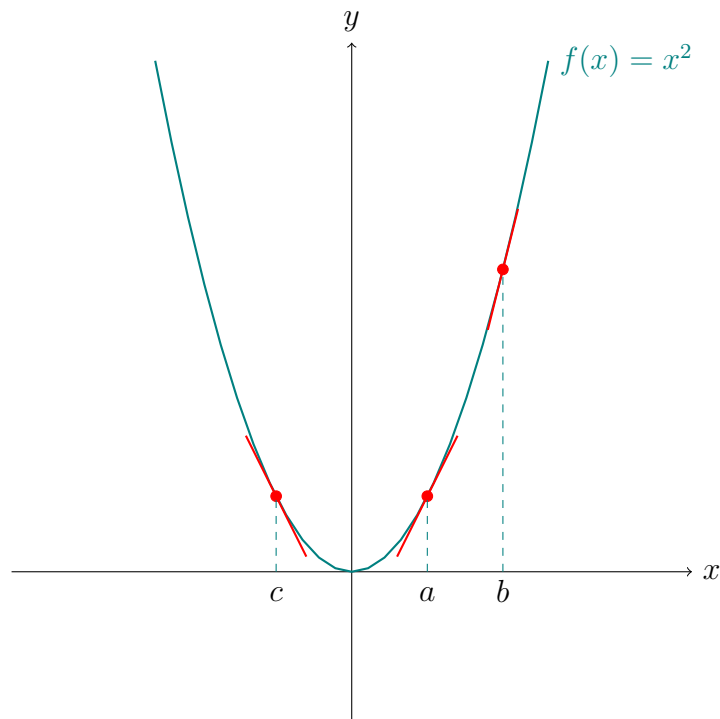
# Tangent Lines: The Derivative Function

Video companion

## 1 Introduction

Derivative formula:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$



- Slope is positive at  $a$ :  $f'(a) > 0$
- Slope is positive at  $b$  and greater than at  $a$ :  $f'(b) > f'(a)$
- Slope is negative at  $c$ :  $f'(c) < 0$

## 2 Calculate derivative

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^2 + 2ah + h^2 - a^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2a + h)}{h} \\ &= \lim_{h \rightarrow 0} (2a + h) \\ &= 2a \end{aligned}$$

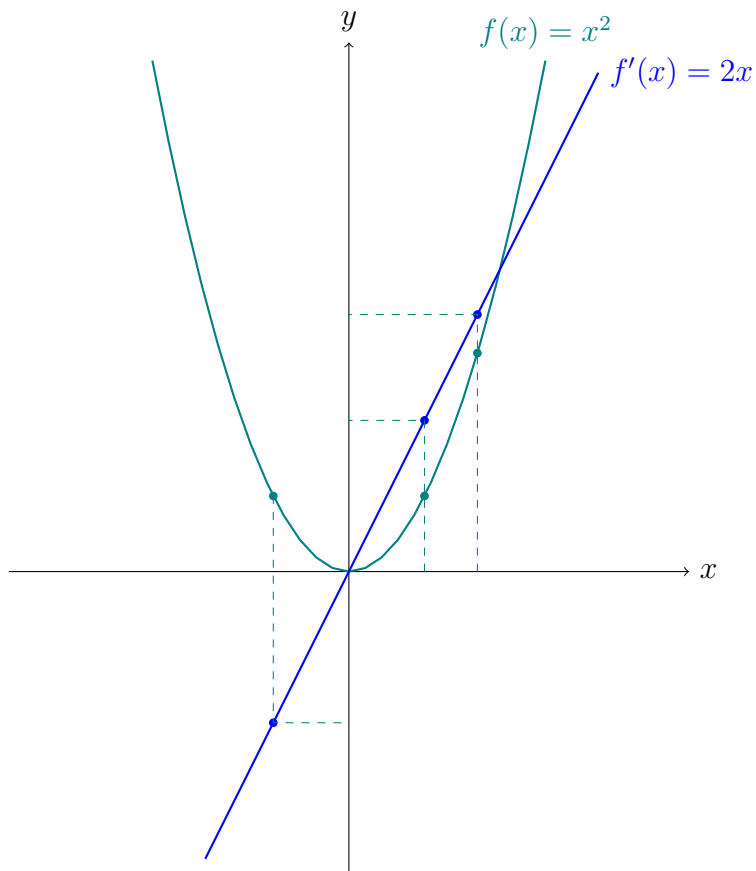
$$f'(a) = 2a$$

$$f'(b) = 2b$$

$$f'(c) = 2c$$

Can verify  $2a > 0$ ,  $2b > 2a$ , and  $2c < 0$

### 3 Graph of derivative function



Next video: Finding where derivative is zero (where the tangent line to the function is horizontal) is important for optimization problems.