Sigma Notation: Mean and Variance

Video companion

1   Introduction

Important equations for this video:

$$X = \{x_1, ..., x_n\}$$

$$\mu_x = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\sigma_x{}^2 = \frac{1}{n} \left[\sum_{i=1}^{n} (x_i - \mu_x)^2 \right]$$

The symbol μ_x is the "mean of x," and $\sigma_x{}^2$ is the "variance of x." The standard deviation is denoted σ_x.

2   Mean

Example:

$$Z = \{1, 5, 12\}$$

$$|Z| = 3$$

$$\mu_z = \frac{1 + 5 + 12}{3} = \frac{18}{3} = 6$$

The mean μ_z is also denoted $\mu(z)$ or simply μ.

Symbolic example:

$$Y = \{y_1, y_2, y_3, y_4\}$$

$$\mu_y = \frac{1}{4} (y_1 + y_2 + y_3 + y_4)$$

$$= \frac{1}{4} \left(\sum_{i=1}^{4} y_i \right)$$

In general, suppose you have a set

$$X = \{x_1, x_2, ..., x_n\},$$
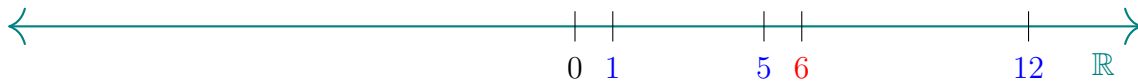
then the mean of X is

$$\mu_x = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right).$$

The variable i is a counter. The variable n is a number, which tells you when to stop counting.
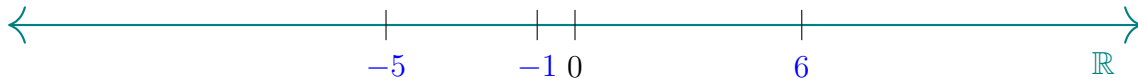
3   Mean centering

$Z = \{1, 5, 12\}$
$\mu_z = 6$

$Z' = \{1 - 6, 5 - 6, 12 - 6\}$
$\quad = \{-5, -1, 6\}$
$\mu_{z'} = 0$

Mean centering data produces a new data set, which has the same relationships, but the mean is zero.
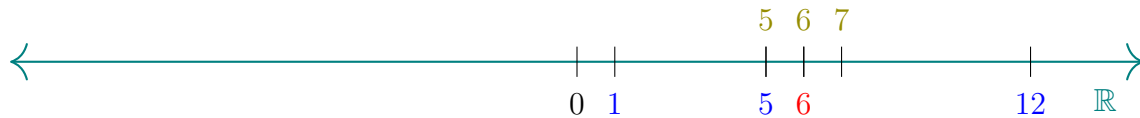
4 Variance

$Z = \{1, 5, 12\}$

$\mu_z = 6$

$W = \{5, 6, 7\}$

$\mu_w = 6$

Set Z (blue) is more "spread out" than set W (olive).

If $X = \{x_1, ..., x_n\}$, the variance of X is

$$\sigma_x{}^2 = \frac{1}{n}\left[\sum_{i=1}^{n}(x_i - \mu_x)^2\right].$$

The standard deviation is given by

$$\sigma_x = \sqrt{\sigma_x{}^2}.$$

Z and W have the same mean, but Z is more spread out, so σ_z should be greater than σ_w.

$$\sigma_w^2 = \frac{1}{3}\left[\sum_{i=1}^{3}(w_i - \mu_w)^2\right]$$

$$= \frac{1}{3}\left[(5-6)^2 + (6-6)^2 + (7-6)^2\right]$$

$$= \frac{1}{3}\left[(-1)^2 + 0^2 + 1^2\right]$$

$$= \frac{2}{3}$$

$$\sigma_w = \sqrt{\frac{2}{3}}$$

$$\sigma_z^2 = \frac{1}{3}\left[\sum_{i=1}^{3}(z_i - \mu_z)^2\right]$$

$$= \frac{1}{3}\left[(1-6)^2 + (5-6)^2 + (12-6)^2\right]$$

$$= \frac{1}{3}\left[(-5)^2 + (-1)^2 + 6^2\right]$$

$$= \frac{62}{3}$$

$$\sigma_z = \sqrt{\frac{62}{3}}$$

$\sigma_z^2 \gg \sigma_w^2$, so Z is much more spread out than W.