

Fast Growth, Slow Growth: How Logarithms and Exponents Are Related

Video companion

1 Introduction

Logarithm means “raised to what power?”

If the question is “what power of two is $2 \cdot 2 \cdot 2 = 8$?” then the answer is the logarithm to the base two of eight, which is $\log_2(8) = 3$.

Two general forms

$$b^x = N$$

“exponential form”

$$x = \log_b(N)$$

“logarithmic form”

Examples

If $b = 2$, $x = 3$, and $N = 8$:

$$2^3 = 8$$

$$3 = \log_2(8)$$

If $b = 2$, $x = 4$, and $N = 16$:

$$2^4 = 16$$

$$4 = \log_2(16)$$

2 Logs of one

Recall that raising any number to the power of zero is one, $b^0 = 1$. Therefore, the log, to any base, of one is zero.

$$\log_2(1) = 0$$

$$2^0 = 1$$

$$\log_{10}(1) = 0$$

$$10^0 = 1$$

$$\log_{20}(1) = 0$$

$$20^0 = 1$$

3 General rules

1. Product rule

$$\log(xy) = \log(x) + \log(y)$$

2. Quotient rule

$$\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$

3. Power and root rule

$$\log(x^n) = n \log(x)$$

Examples

$$\begin{aligned}\log_b(35) &= \log_b(5) + \log_b(7) \\ &= \log_b(70) - \log_b(2)\end{aligned}$$

$$\log_2\left(\frac{16}{4}\right) = \log_2(16) - \log_2(4) = 4 - 2 = 2$$

$$\log_2(1000)^{\frac{1}{3}} = \frac{1}{3} \log_2(1000)$$

$$\log_{10}(7)^5 = 5 \log_{10} 7$$

$$\log_b(x)^{-1} = -\log_b(x)$$

$$\begin{aligned}\log_b x^2 y^{-3} &= \log_b x^2 + \log_b y^{-3} \\ &= 2 \log_b x - 3 \log_b y\end{aligned}$$

$$\begin{aligned}\log_b \frac{x^2}{y^{-\frac{1}{2}}} &= \log_b x^2 - \log_b y^{-\frac{1}{2}} \\ &= 2 \log_b x + \frac{1}{2} \log_b y\end{aligned}$$

4 Problem-solving technique

Problem-solving technique: Treat both sides of an equation as though they were exponents.

$$x = y$$

$$z^x = z^y$$

Example

$$\log_2\left(\frac{39x}{(x-5)}\right) = 4$$

$$2^{\log_2\left(\frac{39x}{(x-5)}\right)} = 2^4$$

$$\frac{39x}{(x-5)} = 16$$

$$39x = 16x - 80$$

$$23x = -80$$

$$x = -\frac{80}{23}$$