How to Make Condorcet Cycles Vanish and Obtain Grades without Combinatorial Exposure?

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Keywords: Condorcet's Paradox, Arrow's Impossibility Theorem, Median of Differences, Combinatorial Exposure, Incomplete Ballots, Computer Networks, Clustering.

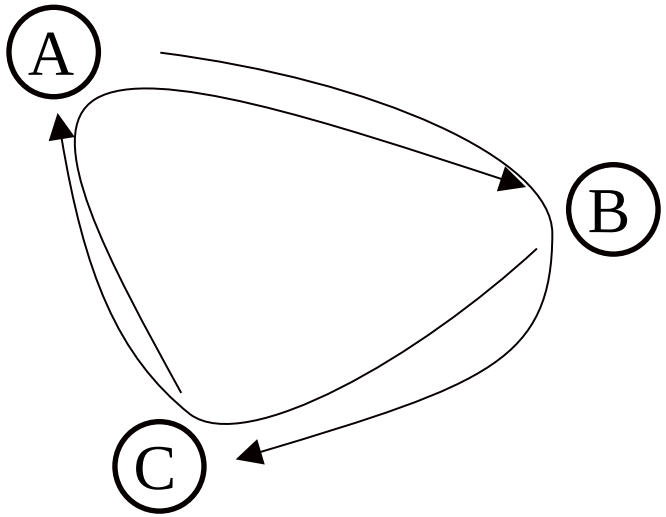
Brief overview: We use rated ballots rather than preferential ones to address Arrow's impossibility theorem, introducing a new method that eliminates Condorcet cycles and obtains a coherent sorted list that can produce grades, even for a huge number of ballots

Summary

1. Condorcet’s paradox
2. Advantages and flaws of evaluation methods
3. A new criterion: median of differences
4. Rank of a median
5. A generic method
6. Examples with scores as outputs
7. Conclusion

1. Condorcet's paradox

- Plenty of examples in daily life



- In terms of preferences, a group can be inherently incoherent

- Respect of Condorcet's criterion – or coherency in general – implies a stronger resistance to strategic voting

- Traditional methods of evaluation (average and median) do not respect Condorcet's criterion

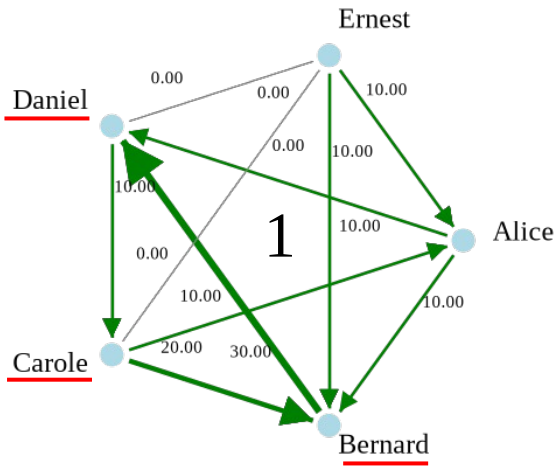
- But they are independant of irrelevant alternatives

- Could we have both ?

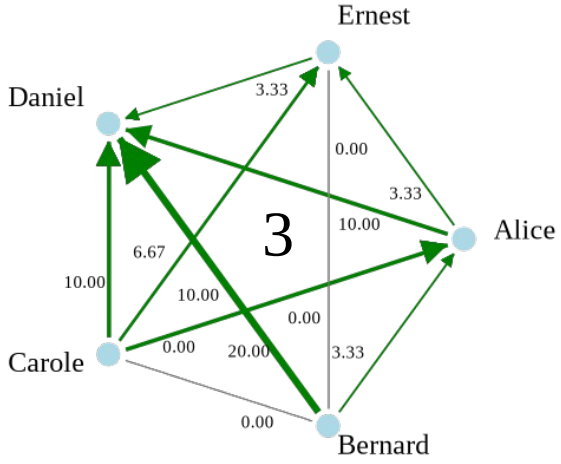
2. A simple example

Professors \ Students	Alice	Bernard	Carole	Daniel	Ernest
Valérie	60	90	10	20	10
Walter	70	60	80	30	30
Xavier	20	10	60	10	30
Yoshua	30	80	40	50	90
Zhou	30	20	40	80	50

Table 1 – Evaluations of all students by professors



Candidate	Alice	Bernard	Carole	Daniel	Ernest
Alice	-	10.00	-10.00	10.00	-10.00
Bernard	-10.00	-	-20.00	30.00	-10.00
Carole	10.00	20.00	-	-10.00	0.00
Daniel	-10.00	-30.00	10.00	-	0.00
Ernest	10.00	10.00	0.00	0.00	-



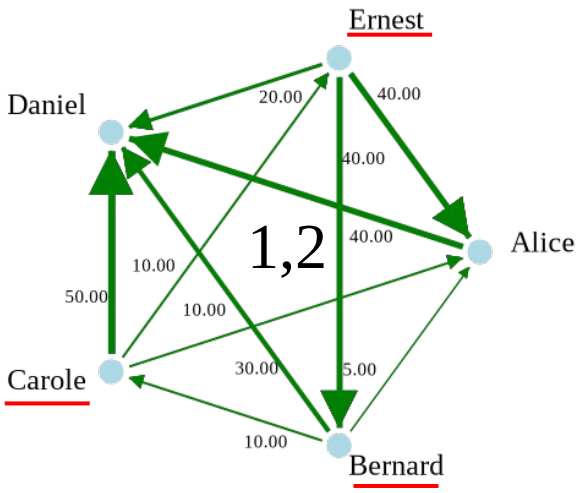
Candidate	Alice	Bernard	Carole	Daniel	Ernest
Alice	-	-3.33	-10.00	10.00	3.33
Bernard	3.33	-	0.00	20.00	0.00
Carole	10.00	0.00	-	10.00	6.67
Daniel	-10.00	-20.00	-10.00	-	-3.33
Ernest	-3.33	0.00	-6.67	3.33	-

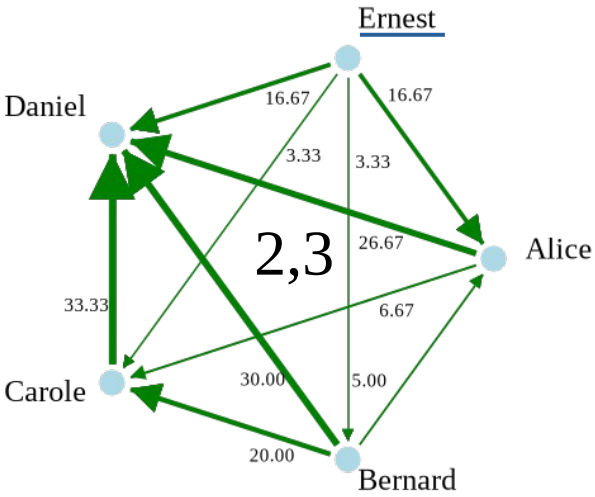
3. Now with mixed parities

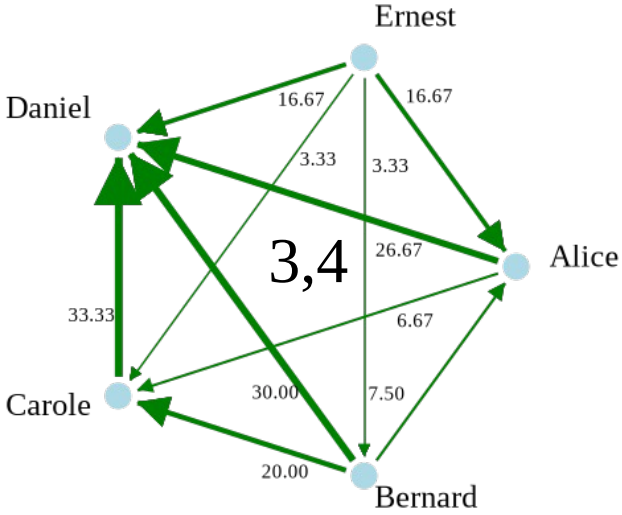
A. Mixed ranks

Professors \ Students	Alice	Bernard	Carole	Daniel	Ernest
Valérie	60	90	20	20	10
Walter	70	60	80	30	
Xavier			60	10	30
Yoshua	30	50	40		90
Zhou	30	20		30	70

Table 2 – Evaluations of all students by professors





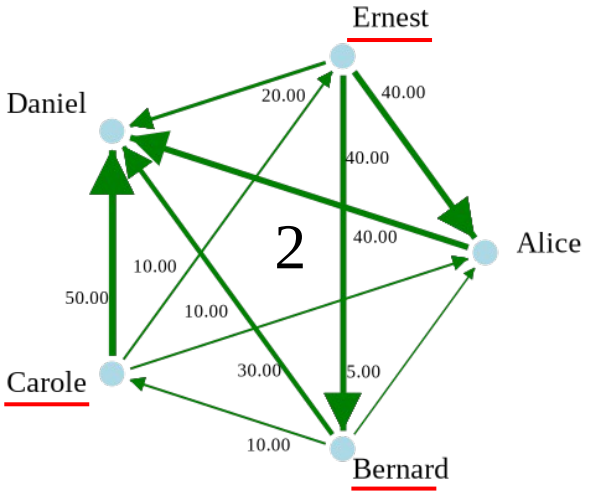


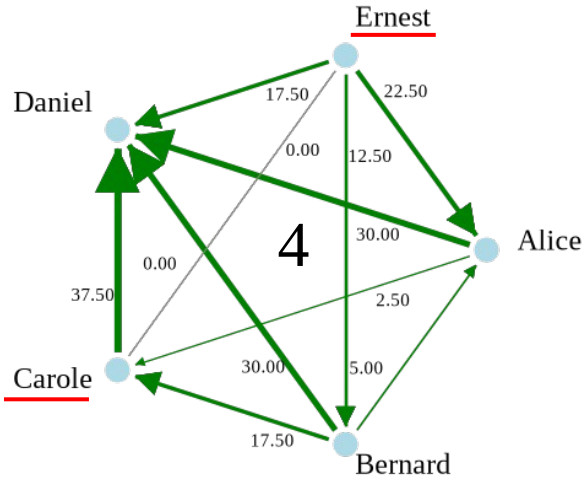
3. Now with mixed parities

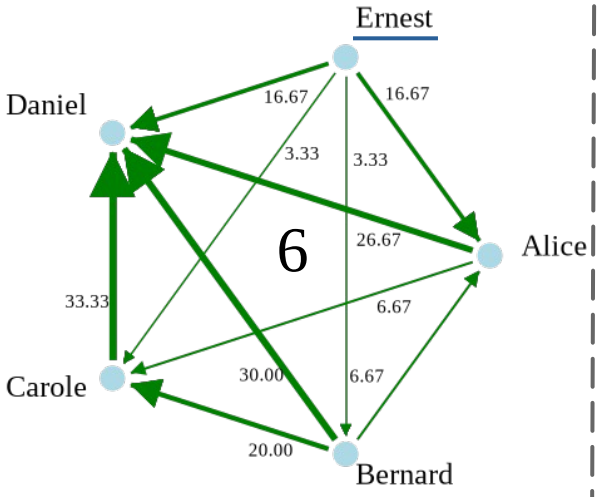
B. Ballots Doubling

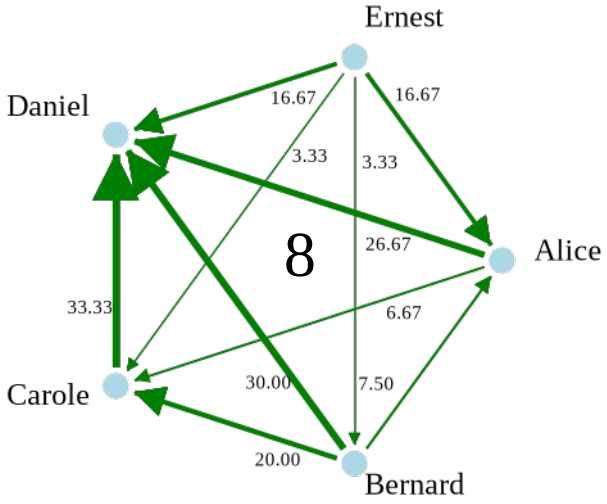
Professors \ Students	Alice	Bernard	Carole	Daniel	Ernest
Valérie	60	90	20	20	10
Walter	70	60	80	30	
Xavier			60	10	30
Yoshua	30	50	40		90
Zhou	30	20		30	70

Table 2 – Evaluations of all students by professors









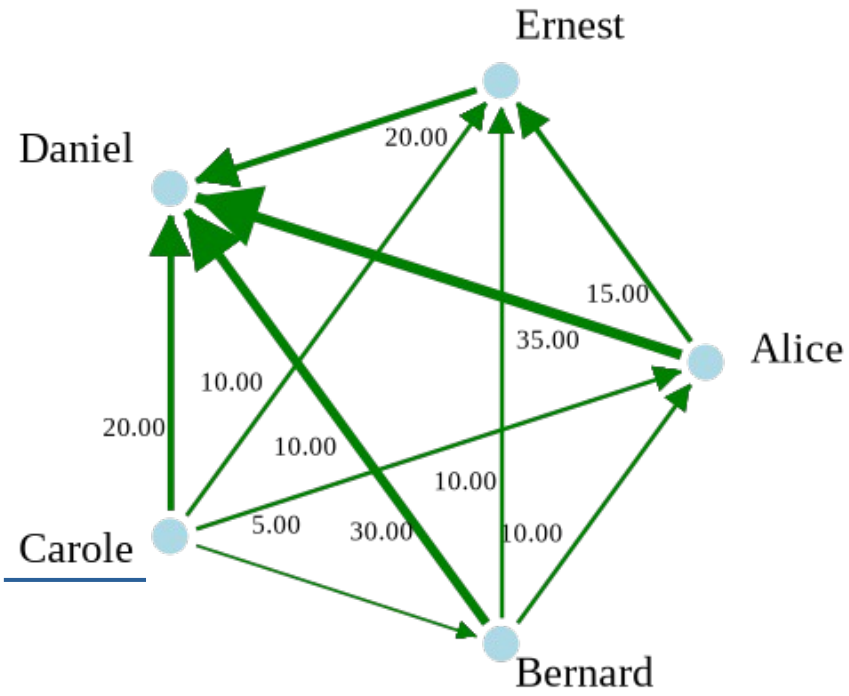
3. Now with mixed parities

C. Values filling

Professors \ Students	Alice	Bernard	Carole	Daniel	Ernest
Valérie	60	90	20	20	10
Walter	70	60	80	30	**50**
Xavier	**45**	**55**	60	10	30
Yoshua	30	50	40	**25**	90
Zhou	30	20	**50**	30	70

Table 2 – Evaluations of all students by professors

Candidate	Alice	Bernard	Carole	Daniel	Ernest
Alice	-	-10.00	-10.00	35.00	15.00
Bernard	10.00	-	-5.00	30.00	10.00
Carole	10.00	5.00	-	20.00	10.00
Daniel	-35.00	-30.00	-20.00	-	-20.00
Ernest	-15.00	-10.00	-10.00	20.00	-



4. a. Playing with higher ranks

Candidate	Coefficient	A	B	C	D	+
Vote #1	10	10	5	0	11	-
Vote #2	11	0	10	6	11	-
Vote #3	12	4	0	10	11	-
Vote #4	11	100	0	50	10	-
Vote #5	12	60	100	0	10	-
Vote #6	10	0	40	100	10	-
Vote #7	12	0	4	2	1	-
Vote #8	10	3	0	4	1	-
Vote #9	11	4	1	0	1	-
+		-	-	-	-	

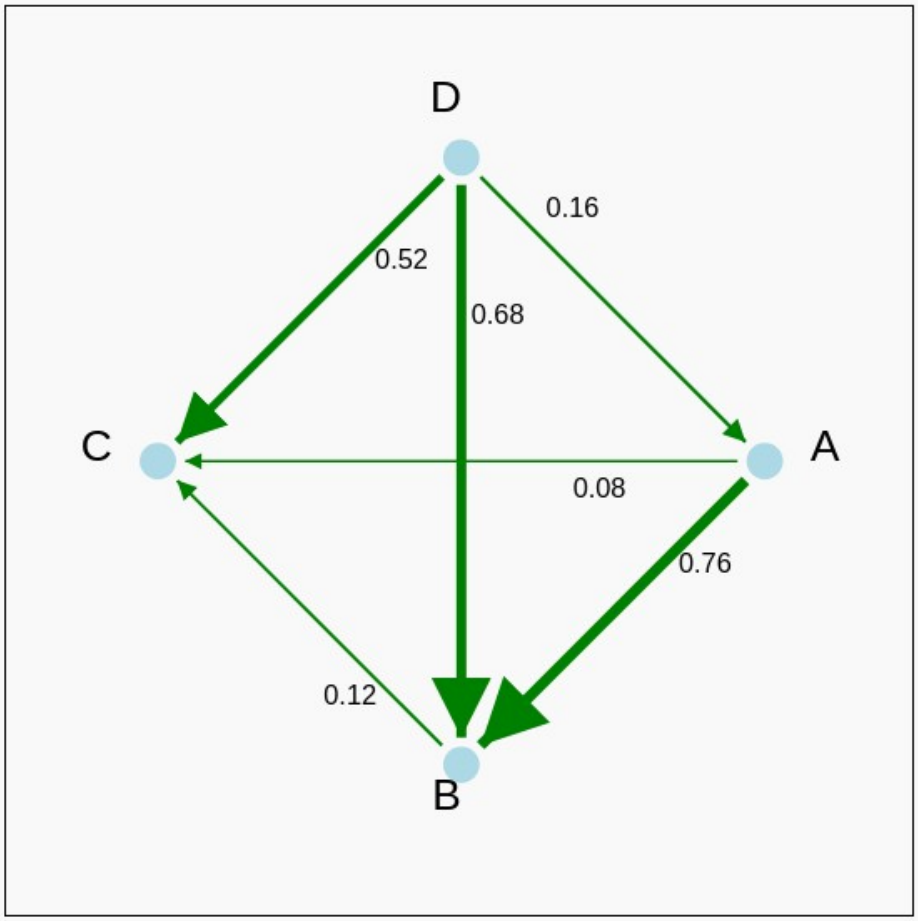
4. a. Playing with higher ranks

< Rank 24 : (m24,m25) > Explore

Candidate	A	B	C	D
A	-	0.76	0.08	-0.16
B	-0.76	-	0.12	-0.68
C	-0.08	-0.12	-	-0.52
D	0.16	0.68	0.52	-

Export Graph as SVG Export Results as CSV

D > A > B > C

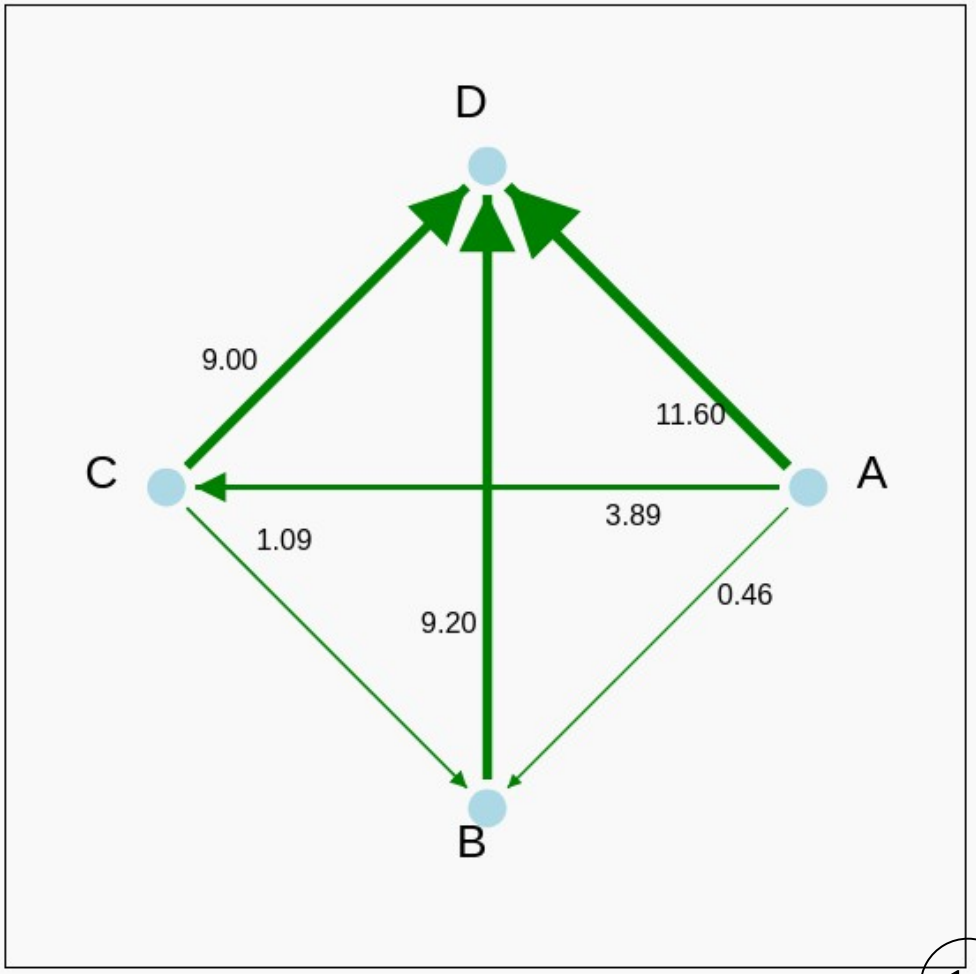


4. a. Playing with higher ranks

< Rank 92 : (m92,m93) > Explore

Candidate	A	B	C	D
A	-	0.46	3.89	11.60
B	-0.46	-	-1.09	9.20
C	-3.89	1.09	-	9.00
D	-11.60	-9.20	-9.00	-

Export Graph as SVG Export Results as CSV



A > C > B > D ?

Taking averages is biased with « strong » opinions

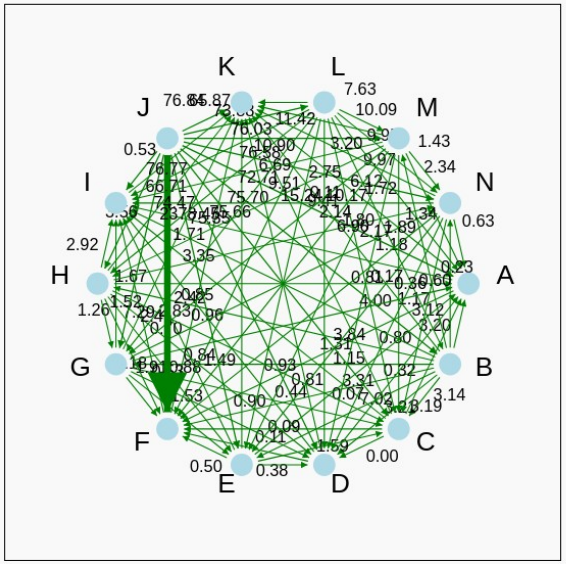
4. b. Exploring more voters

Candidate	Coefficient	A	B	C	D	E	F	G	H	I	J	K	L	M	N	+
Vote #1	64	4	5	7	7	7	1	2	9	3	8	2	3	1	9	-
Vote #2	56	1	3	4	5	7	5	3	3	1	5	7	3	3	3	-
Vote #3	68	1	2	4	4	4	5	5	6	8	6	3	9	4	8	-
Vote #4	65	9	6	3	2	4	1	2	4	5	3	3	2	3	8	-
Vote #5	42	1	4	8	3	1	2	6	8	5	6	4	5	2	1	-
Vote #6	97	2	2	6	7	4	6	5	3	4	3	5	8	2	9	-
Vote #7	74	9	4	9	8	8	7	7	7	2	5	2	9	1	2	-
Vote #8	89	6	1	2	3	8	2	9	7	3	5	5	6	7	3	-
Vote #9	36	5	4	9	7	5	5	2	9	4	1	3	5	6	8	-
Vote #10	29	2	4	1	4	2	6	5	1	8	8	6	1	9	1	-
+		-	-	-	-	-	-	-	-	-	-	-	-	-	-	

4. b. Exploring more voters

< Rank 491 : (m491,m492) > Explore

Candidate	A	B	C	D	E	F	G	H	I	J	K	L	M	N
A	-	-3.20	-0.32	-0.07	-0.44	0.80	-0.96	-3.35	0.81	-76.58	1.18	-9.95	-2.34	-0.63
B	3.20	-	3.14	3.19	3.21	7.02	3.31	1.15	3.84	-72.71	4.00	-9.97	1.17	3.12
C	0.32	-3.14	-	0.00	-0.09	1.59	-0.84	-2.42	0.81	-75.70	1.31	-6.12	-1.34	-0.23
D	0.07	-3.19	-0.00	-	-0.11	0.38	-0.88	-2.83	0.90	-75.66	0.93	-10.17	-1.89	-0.60
E	0.44	-3.21	0.09	0.11	-	0.50	-0.53	-2.41	1.53	-75.35	1.49	-9.41	-2.17	-0.36
F	-0.80	-7.02	-1.59	-0.38	-0.50	-	-1.91	-7.29	0.18	-2378.45	0.70	-15.24	-6.90	-0.17
G	0.96	-3.31	0.84	0.88	0.53	1.91	-	-1.52	1.26	-74.47	1.67	-9.51	-2.14	0.85
H	3.35	-1.15	2.42	2.83	2.41	7.29	1.52	-	2.92	-66.71	3.56	-6.69	-0.11	1.71
I	-0.81	-3.84	-0.81	-0.90	-1.53	-0.18	-1.26	-2.92	-	-76.77	0.53	-10.90	-2.75	-1.80
J	76.58	72.71	75.70	75.66	75.35	2378.45	74.47	66.71	76.77	-	76.84	65.87	73.88	76.03
K	-1.18	-4.00	-1.31	-0.93	-1.49	-0.70	-1.67	-3.56	-0.53	-76.84	-	-11.42	-3.20	-1.72
L	9.95	9.97	6.12	10.17	9.41	15.24	9.51	6.69	10.90	-65.87	11.42	-	7.63	10.09
M	2.34	-1.17	1.34	1.89	2.17	6.90	2.14	0.11	2.75	-73.88	3.20	-7.63	-	1.43
N	0.63	-3.12	0.23	0.60	0.36	0.17	-0.85	-1.71	1.80	-76.03	1.72	-10.09	-1.43	-



J > I > F > A > D > C > E > N > G > H > M > B > L > K

4. c. Extend to other domains

Drones Swarms :

- Can apply to leader election in drone swarms
- Limits adversary voting

AI Agents :

- « Wrong » AI Agents are often massively wrong
- Can be used to find balance between AI « opinions »

5. Conclusion

- A novel method that cuts through voting complexity with grade ballots addressing Arrow’s impossibility theorem
- Drastically reduces manipulation opportunities in collective decision-making, minimizing strategic voting
- Handles incomplete ballots with intelligent scoring algorithms producing either scores or ranks
- Comprehensive testing empirically demonstrates robust performance with both partial ballot scenarios and huge number of ballots

If you want to run your own numbers : https://voting.a-bernard.fr