Why Break Condorcet Cycles When We Can Make Them Disappear?

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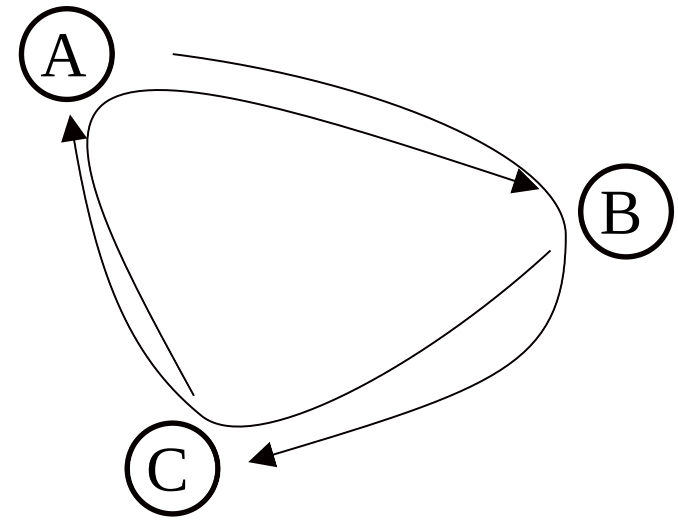
Brief overview: Using preferential ballots, some elections produce cycles. However, using rated ballots, we propose to maintain most of Arrow's basic democratic criteria, making Condorcet's cycles disappear to obtain a coherent sorted list, with grades if needed.

Summary

1. Condorcet’s paradox
2. Advantages and flaws of evaluation methods
3. A new criterion: median of differences
4. Rank of a median
5. A generic method
6. Examples with scores as outputs
7. Conclusion

1. Condorcet's paradox



- Plenty of examples in daily life
- In terms of preferences, a group can be inherently incoherent
- Respect of Condorcet's criterion – or coherency in general – implies a stronger resistance to strategic voting
- Traditional methods of evaluation (average and median) do not respect Condorcet's criterion
- But they are independant of irrelevant alternatives
- Could we have both ?

1. Kenneth Arrow's Impossibility Theorem

- A group can be incoherent when the individual preferences of its members are considered

- We cannot resolve Condorcet's paradox and respect the independence of irrelevant alternatives criteria, using preferences

- Could we if we use evaluations (grades or rated ballots) ?

2. Advantages and Flaws of Traditional Grade Methods

- Average and median methods

- Average method is sensitive to strategic voting by exaggeration.

- The median method, described by Balinski & Laraki, does not present this flaw.

- Both methods are combined in the scoring of Olympic diving and figure skating competitions, where the lowest and highest scores are removed before the remaining scores are averaged out.

- However, neither method guarantees the election of a Condorcet winner.

3. Median of Differences

- The margin criterion represents the number of voters that prefer one option minus the number of voters that prefer another option.

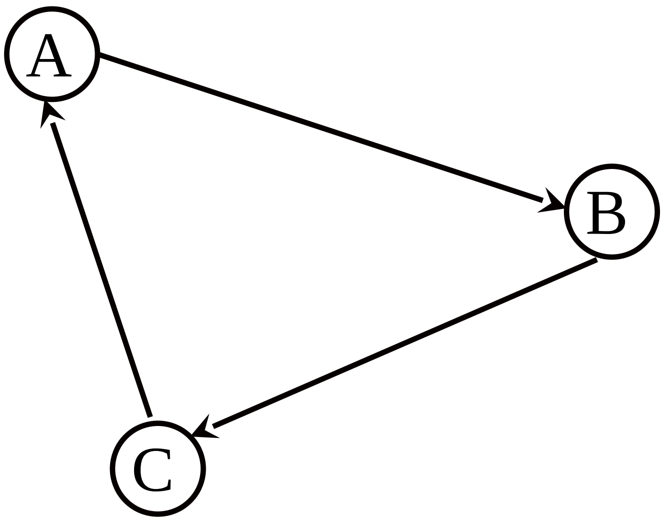
- The median of differences represents the strength by which the median voter prefers one option over another: eval(A) – eval(B)

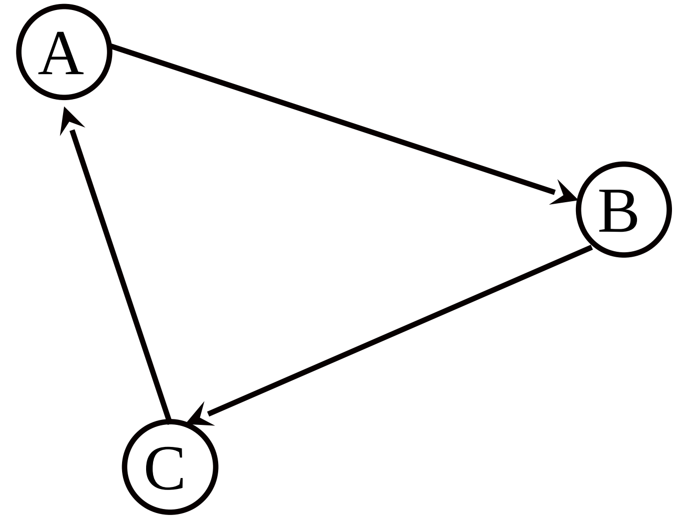
7 ballots:{-90; -60; -10; 12; 12; 33; 40}

Margin: 4 - 3 = 1

Median: +12

3. from Margin to Median of Differences





Margin: 3 - 2 = 1

Median: 0

{-90; -60; 0; 0; 12; 33; 40}

Only for an even number of ballots:{-90; -60; -10; 12; 33; 40}

Margin: 3 - 3 = 0

Median: (12 – 10) / 2 = +1

4. Rank of a Median

- Median_k of rank k is equal to the average over the k central values

$$7 \text{ ballots}: \{-90;\ -60;\ -10;\ 12;\ 12;\ 33;\ 40\}$$

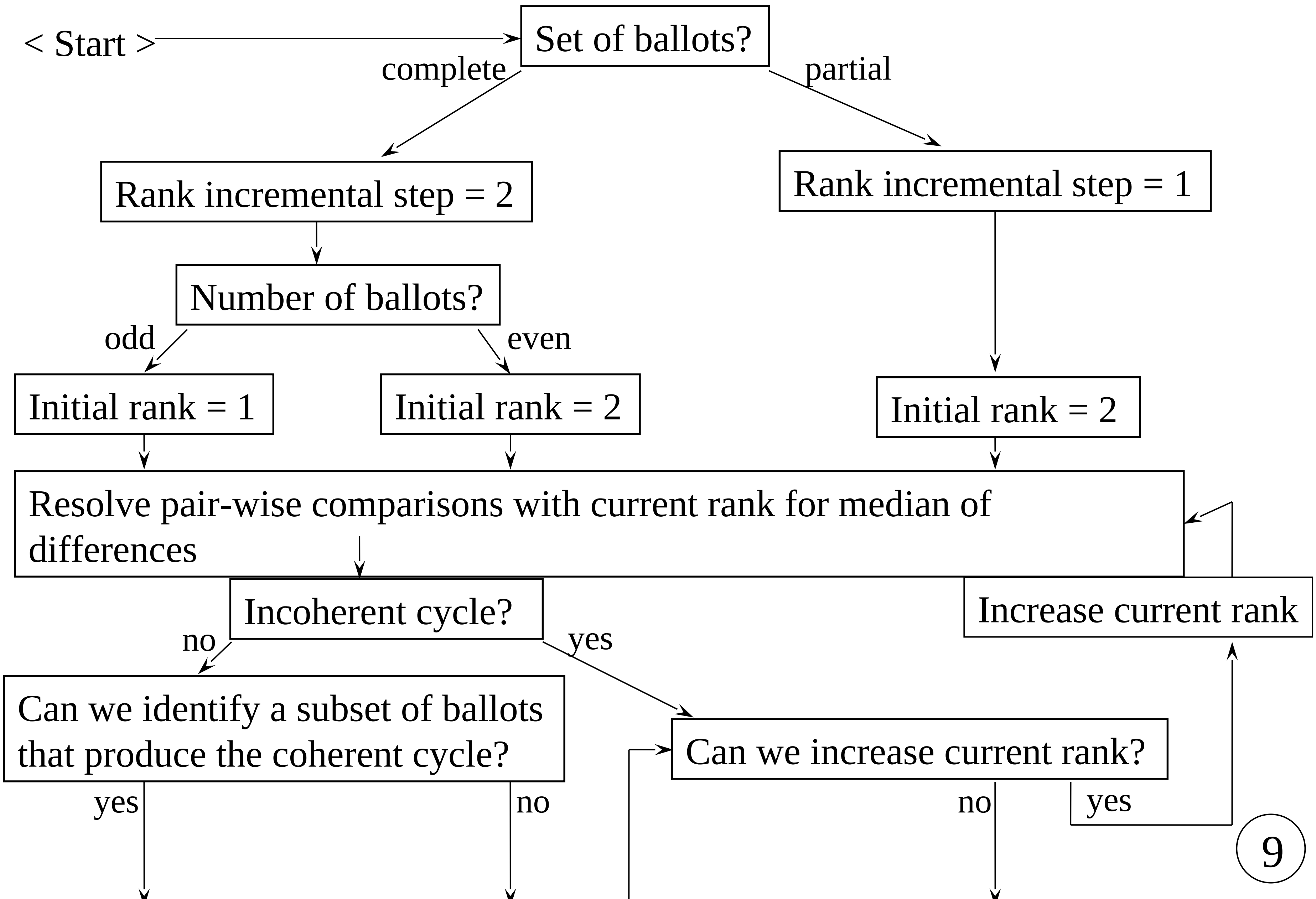
$$\text{Median}_1 = +12$$
$$\text{Median}_3 = +14/3$$
$$\text{Median}_5 = -13/5$$
$$\text{Median}_7 = -63/7$$

$$6 \text{ ballots}: \{-65;\ -31;\ -2;\ 6;\ 13;\ 91\}$$

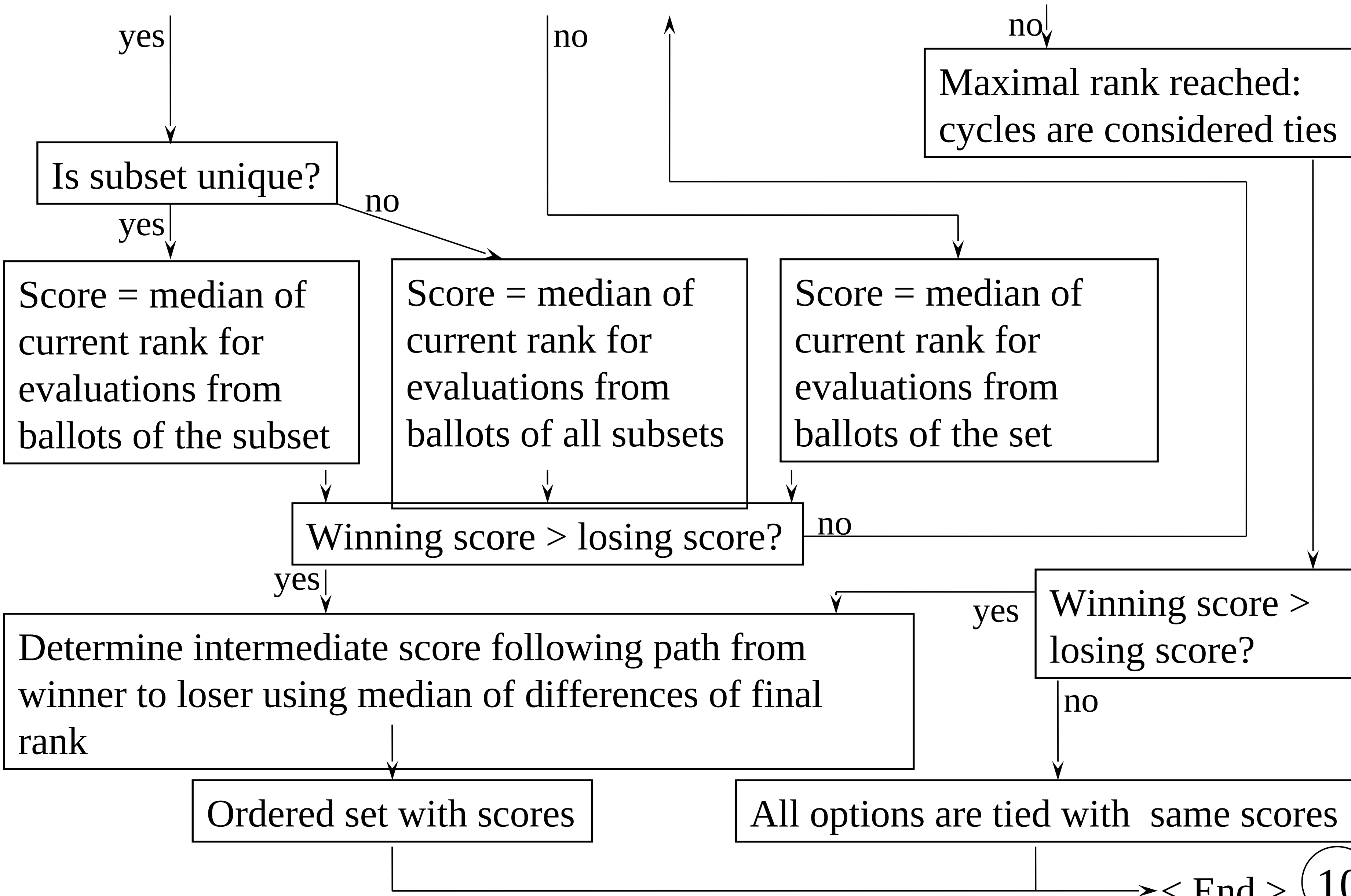
$$\text{Median}_2 = +2$$
$$\text{Median}_4 = -3.5$$
$$\text{Median}_6 = +2$$

- Fluctuations over the sense of preferences depends on the values

5. A Generic Method: the Algorithm



5. A Generic Method: the Algorithm

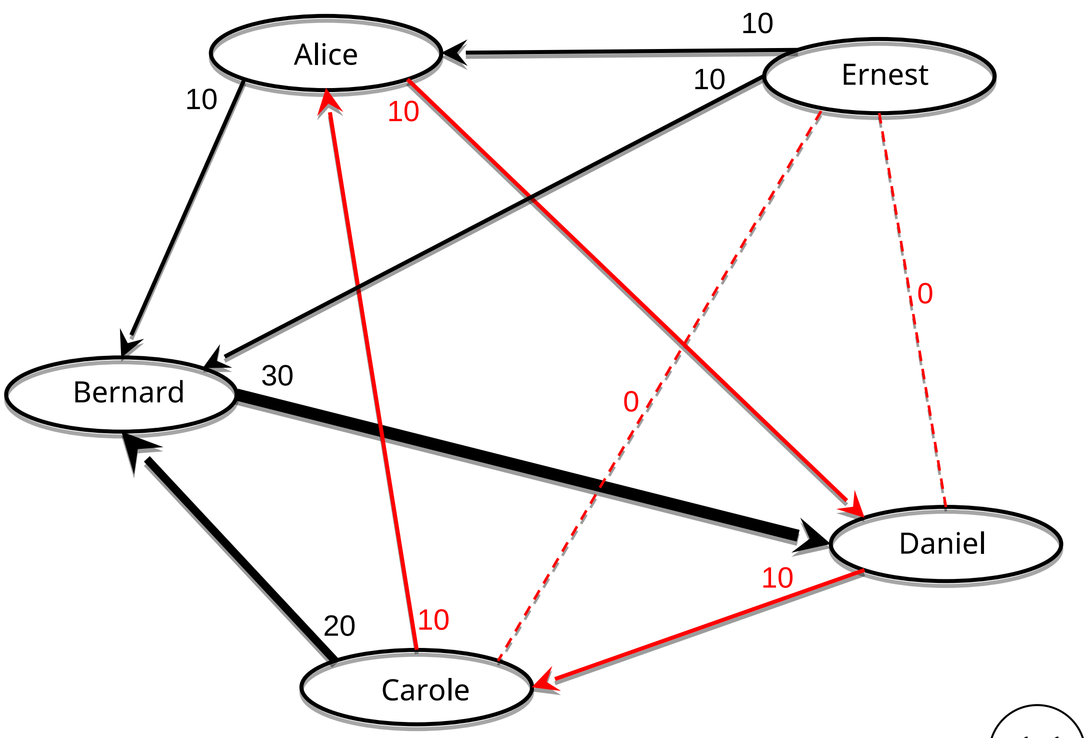


6. An Example for a Set of Complete Evaluations

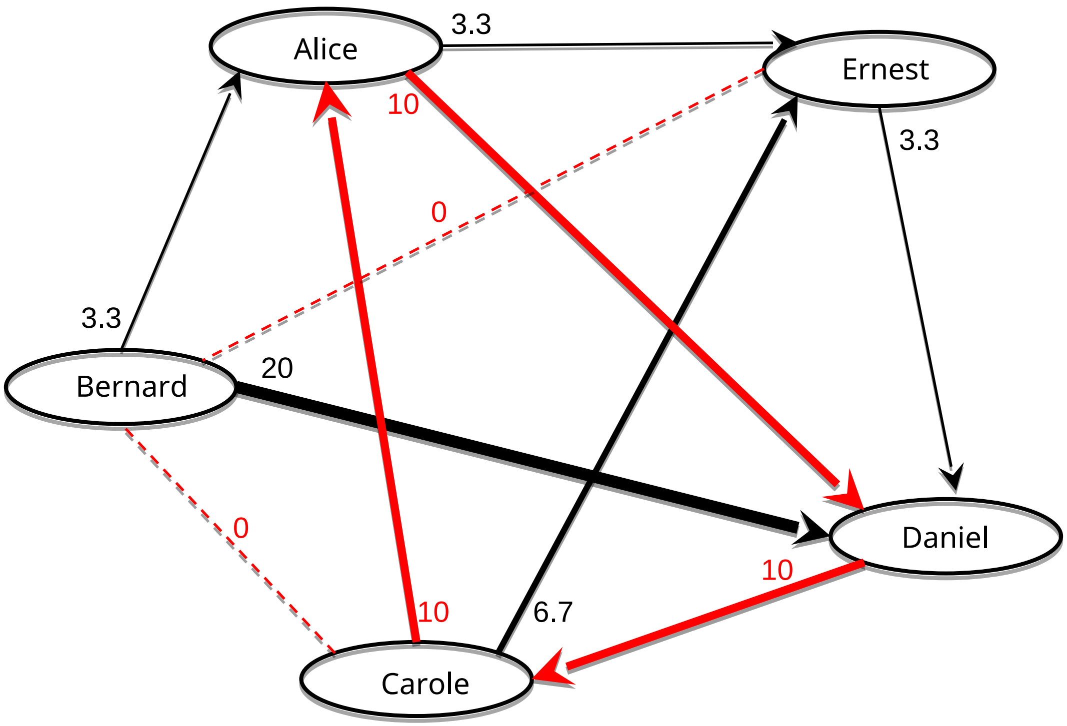
Professors \ Students	Alice	Bernard	Carole	Daniel	Ernest
Valérie	60	90	10	20	10
Walter	70	60	80	30	30
Xavier	20	10	60	10	30
Yoshua	30	80	40	50	90
Zhou	30	20	40	80	50

Table 1 – Evaluations of all students by professors

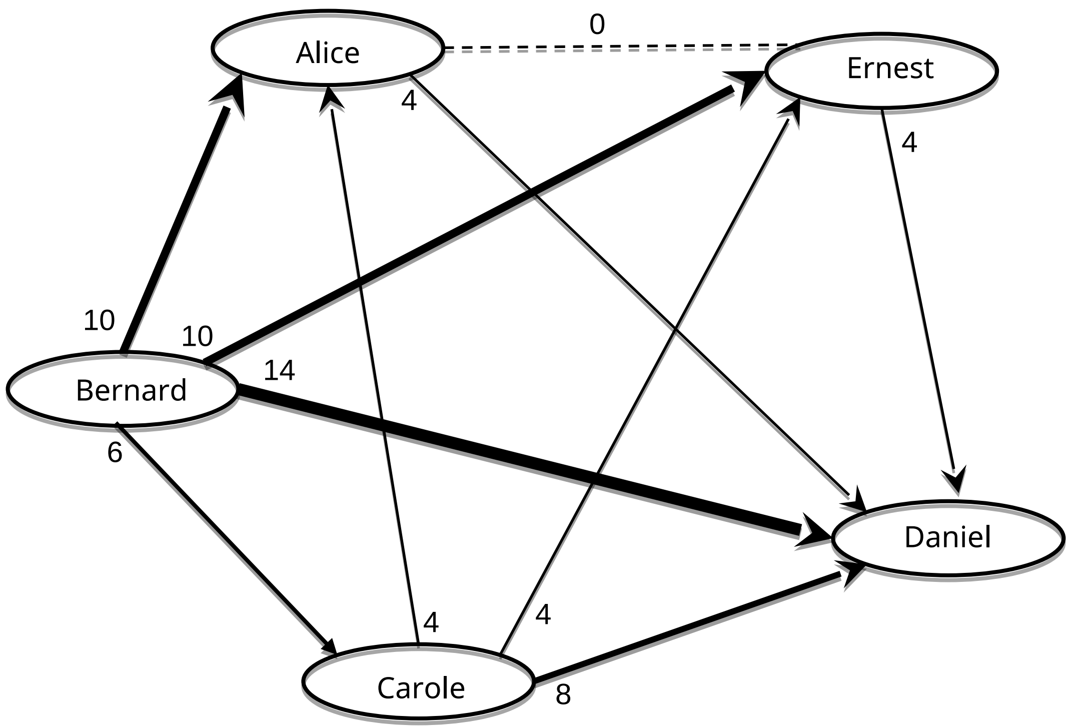
Rank 1



6. Median of Differences of Rank 3



6. Median of Differences of Maximal Rank



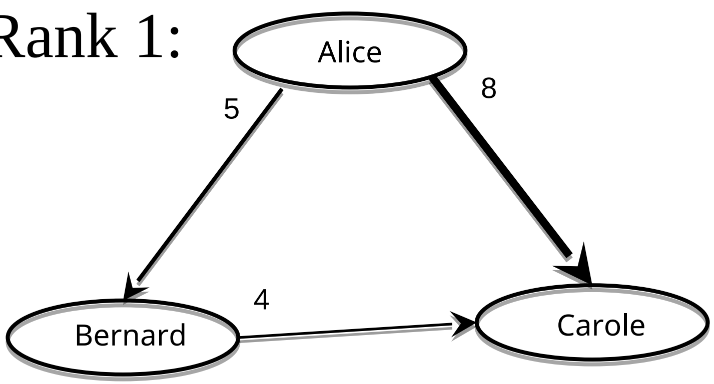
B (52) > C (50) > A (42) = E (42) > D (38)

6. An Example with a Subset of Complete Evaluations

Professors \ Students	Alice	Bernard	Carole
Valérie	100	50	0
Walter	15	10	60
Xavier	44	41	36
Yoshua	55	49	45
Zhou	0	50	100

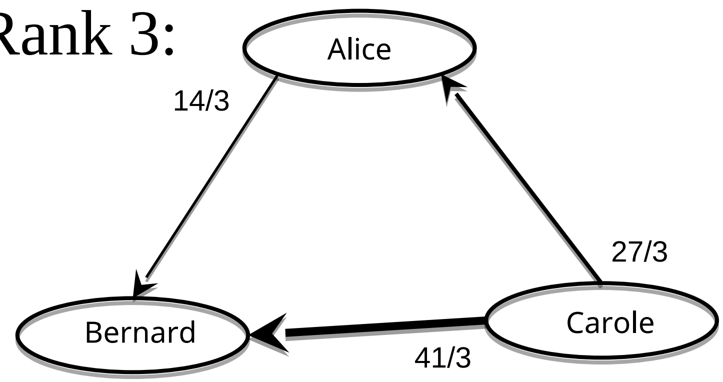
Evaluations of all students by professors

Rank 1:



winning score A(44)
< losing score C(45)

Rank 3:



winning score C(141/3)
> losing score B(100/3)

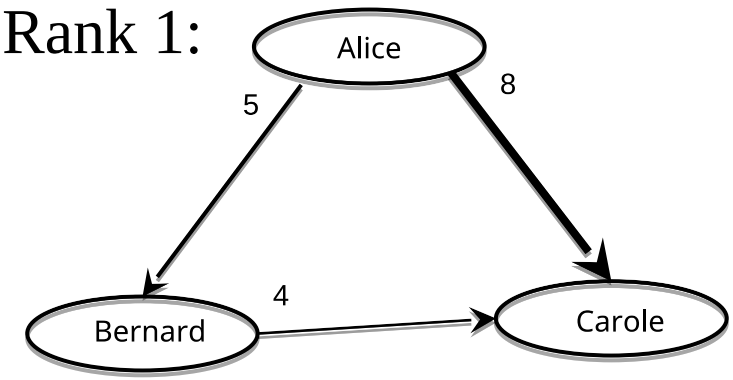
C (60.3) > A (38) > B (33.3)

6. Example with no Subset of Complete Evaluations

Professors \ Students	Alice	Bernard	Carole
Valérie	100	50	0
Walter	15	10	100
Xavier	44	41	36
Yoshua	55	49	45
Zhou	0	50	100

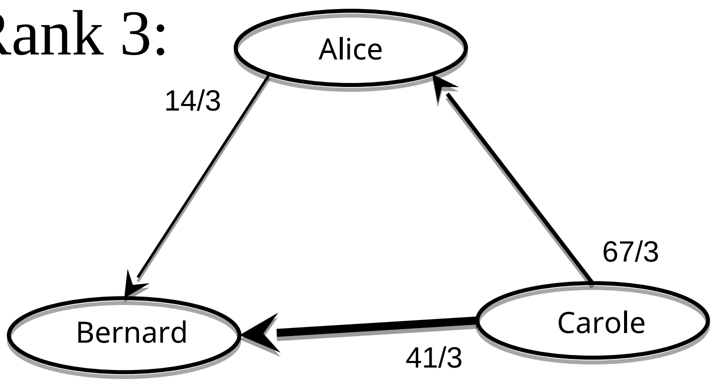
Evaluations of all students by professors

Rank 1:



winning score A(44)
< losing score C(45)

Rank 3:



winning score C(181/3)
> losing score B(140/3)

C (60.3) > A (49) > B (46.7)

7. Conclusion

- Condorcet’s paradox increases the likelihood of strategic voting opportunities.

- We developped a criteria and a Condorcet method based on evaluations to circumvent Arrow's impossibility theorem. The result is much less sensitive to strategic voting. With this generic method, a full set of evaluations produces a coherent ordered set with only pure ties to determine the single or multiple winner(s).

- Finally, the result can produce scores for all options.

- This scoring mechanism can also work with other arbitrary defined values, which can prove useful in deciding on a leader or a leader list in distributed computer systems with hundreds of devices.