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DJANGOH: Event Generation for Deep Inelastic Lepton Scattering Including Radiative Processes

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Abstract

Monte Carlo event simulation of neutral and charged current lepton nucleon scattering with the program DJANGOH is described. DJANGOH was developed originally for deep inelastic electron proton scattering at HERA, but has been extended and includes options for muon scattering, heavy nuclear targets, elastic scattering and polarized protons. The emphasis is put on the inclusion of radiative corrections, comprising single soft and hard photon emission and the complete set of electroweak 1-loop corrections in the Standard Model. Large-mass hadronic final states are generated by an interface to LEPTO which simulates QCD effects. Low-mass hadronic final states are included by an interface to SOPHIA.

1 Introduction

DJANGOH is a Monte Carlo event generator which allows the simulation of deep inelastic scattering (DIS) of leptons and nucleons or nuclei including electroweak radiative corrections of order α, both for neutral-current (NC) and charged-current (CC) scattering. The $O(\alpha)$ corrected cross section $eq \to eq(+\gamma)$ is generated using routines originally packaged in a program called HERACLES [1]. The events generated by HERACLES are events at the parton level. They are completely described by the flavor of the scattering quark and the 4-momenta of the final electron, the final quark, and the potentially radiated photon. HERACLES includes the leptonic corrections as well as the complete one-loop virtual corrections. Higher-order effects due to the strong interaction, described in perturbative Quantum Chromodynamics (QCD) are included in the leading logarithmic approximation using a parton shower algorithm, implemented in the program LEPTO [2]. In this approximation, corrections from QED and QCD can be factorized which allows us to develop a step-by-step procedure where the QED and QCD effects are described in separate programs [3, 4].

The calculation of the electroweak radiative corrections included in DJANGOH [5–7] had been performed in the on-mass-shell scheme of the electroweak Standard Model which uses, as basic parameters, the particle masses M_W and M_Z of the vector bosons besides the Higgs boson mass M_H and the fermion masses m_f together with the fine structure constant α. A detailed discussion of the renormalization procedure and the basic building blocks needed in the calculation of electroweak one-loop corrections can be found in [8]. The complete $\mathcal{O}(\alpha)$ electromagnetic and weak corrections implemented in DJANGOH have been cross-checked against other calculations, first of all those described in Refs. [9–11].

At low $Q^2 \ll M_Z^2$ electromagnetic corrections are the only important ones. For the pure photon exchange they have been calculated first by Mo and Tsai [12, 13] and later by Akhundov et al. [14]. There is also extensive literature concerning the radiative corrections to charged current deep inelastic neutrino scattering at low Q^2 [15–18]. In this context de Rújula et al. [19] have shown how quark mass singularities should be factorized and absorbed into the parton distribution functions (see [20, 21] for the case of NC scattering).

The main purpose of this note is to provide a complete and up-to-date information of the possible ways to use the program DJANGOH. Most of the text below is written in the style of a user manual. The main physics aspects underlying the simulations implemented in DJANGOH are reviewed in a short introductory part.

While the physics underlying the calculation of the electroweak interaction and its first-order corrections is of course still valid, some parts of the event simulation in DJANGOH may require a revision. This concerns the model assumptions implemented in those parts which are responsible for the generation of the hadronic final state. The basic ideas and some specific choices of model parameters date back to the first days of HERA. DJANGOH may require a major update towards revised hadronization models, like PYTHIA or HERWIG, or new concepts for the description of scattering in the transition region between the non-perturbative low-Q^2 regime and the regime of large Q^2 where the parton model is believed to work well. High-precision measurements at future facilities like the EIC may also require to include higher-order corrections, for example multi-photon radiation or soft-photon exponentiation, presently not available in a form suited for an implementation in a Monte Carlo simulation program for DIS.

Part A: Theoretical foundation

2 Leading order cross sections

The cross section for deep inelastic neutral-current scattering is ususally written in terms of generalized structure functions $F_2^{\pm}$, $xF_3^{\pm}$ and $F_{\rm L}^{\pm}$ which depend on the Bjorken variable x and the momentum transfer squared, Q^2,

$$\frac{d^2\sigma^{\rm NC}(e^{\pm}\mathcal{N})}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+F_2^{\pm}(x,Q^2) \mp Y_- xF_3^{\pm}(x,Q^2) - y^2 F_{\rm L}^{\pm}(x,Q^2)\right] . \tag{1}$$

Upper and lower signs refer to positron and electron scattering, respectively, y is the inelasticity and $Y_{\pm} = 1 \pm (1-y)^2$. This representation relies on the assumption that the interaction is mediated by the exchange of a single boson, photon for the electromagnetic and Z-boson for the neutral-current weak interaction. The longitudinal structure function $F_{\rm L}$ enters at large y. It is zero in the naive parton model and appears at order $O(\alpha_s)$ as a correction from perturbative QCD. We omit it in the following[1].

For scattering of polarized electrons or positrons it is convenient to separate the cross section into an unpolarized part and a polarization-dependent contribution. Denoting the degree of longitudinal polarization[2] by P_ℓ, one can write:

$$\frac{d^2\sigma^{\rm NC}(e^{\pm}\mathcal{N})}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left(\tilde{\sigma}_0^{\pm} + P_\ell\tilde{\sigma}_\ell^{\pm}\right) , \tag{2}$$

and at LO QCD the separation of the reduced cross sections $\tilde{\sigma}_i^{\pm}$ of Eq. (2) in terms of structure functions can be written as

$$\begin{aligned}\tilde{\sigma}_0^{\pm} &= \left(1+(1-y)^2\right)F_2^0 \pm \left(1-(1-y)^2\right)xF_3^0 ,\\ \tilde{\sigma}_\ell^{\pm} &= \left(1+(1-y)^2\right)F_2^\ell \mp \left(1-(1-y)^2\right)xF_3^\ell .\end{aligned} \tag{3}$$

[1]A longitudinal structure function contributing to the electromagnetic part of the cross section, i.e. to the one-photon exchange, can be included in DJANGOH, but this can not be combined with QED corrections from photon radiation off quarks, see below.

[2]For electrons, $P_\ell = -1$ corresponds to a purely left-handed beam polarization.

In the leading logarithmic approximation of QCD the structure functions are related to sums and differences of parton distribution functions, $q_f(x)$ for quarks and $\bar{q}_f(x)$ for anti-quarks:

$$\begin{aligned} F_2^{0,\ell} &= \sum_f A_{0,\ell}^f(Q^2)\, x\, (q_f(x) + \bar{q}_f(x)) \;, \\ xF_3^{0,\ell} &= \sum_f B_{0,\ell}^f(Q^2)\, x\, (q_f(x) - \bar{q}_f(x)) \;, \end{aligned} \tag{4}$$

with

$$\begin{aligned} A_0^f =&\ Q_f^2 - 2v_e Q_f v_f\, \chi_Z(Q^2) + (v_e^2 + a_e^2)(v_f^2 + a_f^2) \left(\chi_Z(Q^2)\right)^2 \;, \\ A_\ell^f =&\ \qquad -2a_e Q_f v_f\, \chi_Z(Q^2) + 2v_e a_e (v_f^2 + a_f^2) \left(\chi_Z(Q^2)\right)^2 \;, \\ B_0^f =&\ \qquad -2a_e Q_f a_f\, \chi_Z(Q^2) + 2v_e a_e 2v_f a_f \left(\chi_Z(Q^2)\right)^2 \;, \\ B_\ell^f =&\ \qquad -2v_e Q_f a_f\, \chi_Z(Q^2) + (v_e^2 + a_e^2) 2v_f a_f \left(\chi_Z(Q^2)\right)^2 \;. \end{aligned} \tag{5}$$

In (5) one recognizes the different contributions of the pure γ exchange (where only $A_0^f = Q_f^2$ is non-zero and therefore the cross section does not depend on the charge and polarization of the leptons), the γZ interference and the pure Z exchange which contain the reduced Z propagator

$$\chi_Z(Q^2) = \frac{Q^2}{Q^2 + M_Z^2} \;, \tag{6}$$

where M_Z is the mass of the Z-boson. v_f and a_f are the vector and axial vector coupling constants of the fermions to the Z-boson, given by their charge Q_f and isospin I_3^f:

$$v_f = \frac{I_3^f - 2s_W^2 Q_f}{2s_W c_W} \;, \quad a_f = \frac{I_3^f}{2s_W c_W} \;, \tag{7}$$

where $s_W = \sin\theta_W$, $c_W = \cos\theta_W$ and θ_W is the weak mixing angle. In the on-shell scheme s_W is determined by the W and Z gauge boson masses:

$$s_W^2 = 1 - c_W^2, \quad c_W = \frac{M_W}{M_Z} \;. \tag{8}$$

One could also parametrize the cross section in terms of the μ decay constant G_μ using the relation [22]

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}} \frac{1}{s_W^2 M_W^2} \frac{1}{1 - \Delta r} \;. \tag{9}$$

The prescription for the calculation of radiative corrections based on (9) leads to the so-called modified on-mass-shell scheme. In particular, the parameters of the weak interaction determine the normalization factors for Z and W exchange,

$$\frac{\pi\alpha^2}{2s_W^2 c_W^2} \frac{1}{Q^2 + M_Z^2} = \frac{\alpha G_\mu (1 - \Delta r)}{\sqrt{2}} \frac{M_Z^2}{Q^2 + M_Z^2} \qquad \text{NC, } Z\text{exchange} \tag{10}$$

and

$$\frac{\pi\alpha^2}{2s_W^2} \frac{1}{Q^2 + M_W^2} = \frac{\alpha G_\mu (1 - \Delta r)}{\sqrt{2}} \frac{M_W^2}{Q^2 + M_W^2} \qquad \text{CC, } W\text{exchange.} \tag{11}$$

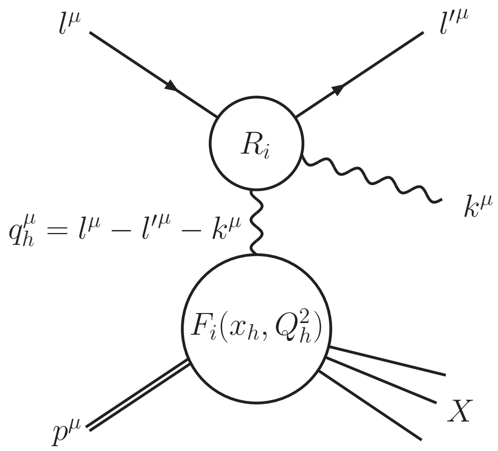

Figure 1: Kinematic variables for lepton nucleon scattering with radiation of a photon from the lepton. Structure functions $F_i(x_h, Q_h^2)$ describe the interaction of the nucleon with a virtual boson, R_i denote radiator functions describing the emission of a photon from the lepton.

3 Bremsstrahlung

3.1 Kinematics

At leading order it is sufficient to use the kinematic variables

$$\begin{aligned} Q^2 &= -(l-l')^2, \\ x &= \frac{Q^2}{2p\cdot(l-l')}, \\ y &= \frac{p\cdot(l-l')}{p\cdot l} = \frac{Q^2}{x(S-m_N^2)} \end{aligned} \tag{12}$$

where $S = (p+l)^2$, l and l' are the 4-momenta of the incoming and scattered lepton and p that of the incoming nucleon of mass m_N (proton or neutron). These variables are determined experimentally by a measurement of the energy and angle of the scattered lepton[3].

Radiative corrections include contributions from processes with an additional photon that can not be identified by the measurement. The bremsstrahlung process

$$e(l) + N(p) \to e'(l') + \gamma(k) + X(p_X) \tag{13}$$

requires to distinguish different possibilities to define kinematic variables for the hadron-inclusive scattering. The standard DIS variables, as defined above, suited for a measure-

[3]The lepton mass is neglected in the formulas shown here and below, but taken into account in the numerical calculations of DJANGOH.

ment of the scattered lepton only, are defined by

$$Q_l^2 = -q_l^2\,, \quad q_l = l - l'\,, \tag{14}$$
$$x_l = \frac{Q_l^2}{2p \cdot q_l}\,, \quad y_l = \frac{p \cdot q_l}{p \cdot l}\,.$$

These leptonic variables can be used to calculate the invariant mass of the hadronic final state including the radiated photon, i.e.

$$W^2 = (p + q_l)^2 = (p_X + k)^2\,. \tag{15}$$

At given squared center-of-mass energy S only two variables in Eq. (14) are independent by virtue of the relations

$$Q_l^2 = x_l y_l (S - m_N^2)\,; \quad W^2 = (1 - x_l) y_l S + m_N^2\,. \tag{16}$$

The kinematics can also be defined through the hadronic final state, provided the measurement allows one to extract the required information from energy and momenta of all hadrons in the final state. If the photon in the bremsstrahlung process (13) can be identified, one can actually measure the hadronic variables

$$Q_h^2 = -q_h^2\,, \quad q_h = p_X - p\,, \tag{17}$$
$$x_h = \frac{Q_h^2}{2p \cdot q_h}\,, \quad y_h = \frac{p \cdot q_h}{p \cdot l}\,, \tag{18}$$

and

$$W_h^2 = (p + q_h)^2 = p_X^2\,. \tag{19}$$

Note that the variable y_h is defined in terms of $p \cdot l$ rather than in terms of $p \cdot (l - k)$. The hadronic variables obey relations analogous to Eq. (16),

$$Q_h^2 = x_h y_h (S - m_N^2)\,; \quad W_h^2 = (1 - x_h) y_h S + m_N^2\,. \tag{20}$$

These hadronic variables are well-suited for a description of the process in the presence of photon radiation from the incoming and scattered lepton. The hadronic variables x_h and Q_h^2 have to be used as the arguments of the structure functions, see Fig. 1. The 'true' momentum transfer is shifted due to bremsstrahlung,

$$Q_h^2 = Q_l^2 + 2l \cdot k - 2l' \cdot k\,. \tag{21}$$

Q_h^2 can be smaller or larger than Q_l^2 depending on the angle of the radiated photon relative to the lepton momenta. x_h is shifted by photon radiation always to larger values compared with x_l, $x_h > x_l$.

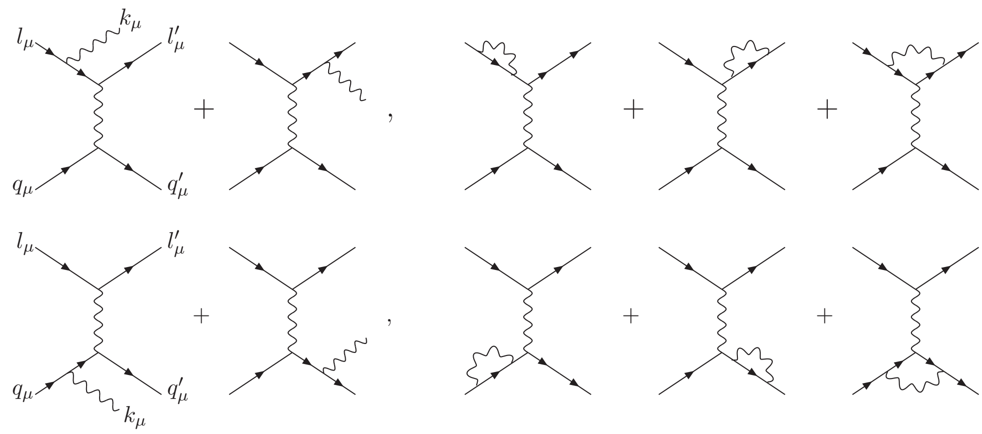

Figure 2: Single-photon bremsstrahlung diagrams for neutral-current lepton quark scattering, shown together with photonic loop diagrams (fermion self energies and vertex graphs) needed to cancel infrared divergences.

3.2 Properties of bremsstrahlung

The diagrams of Fig. 2 describe the bremsstrahlung for the neutral current lepton-quark scattering process. The corresponding squared matrix element has been given in Ref. [5]. The corrections from radiation at the lepton line and at the quark line (upper and lower rows of Fig. 2, respectively) are gauge invariant by themselves. Therefore leptonic and quarkonic radiation, and also their interference, can be treated separately.

A common feature of the radiative corrections is the appearance of infrared divergences which are found upon integration over the photon phase space. These divergences are cancelled by virtual photonic corrections, i.e. Feynman diagrams with loops containing a photon. The loop diagrams required for this cancellation are shown in Fig. 2 together with the corresponding diagrams for real radiation. For the cancellation of infrared divergences in the interference terms, also box graphs with an additional photon connecting the lepton and the quark lines are needed, see Fig. 3 below. Technically, the infrared divergences are isolated by separating the photon phase space into a soft region, below a cut, ΔE, of the photon energy, and a hard region. The infrared divergent contributions of the cross section for the radiative process in the soft region, as well as the loop corrections are regulated by a finite photon mass, λ. The divergences are cancelled analytically and the photon mass regulator is removed afterwards. The soft region is chosen such that the photon momentum can be neglected compared with the momenta of the other particles. The separation of the soft and hard region is arbitrary. The final result must not depend on the prescription for this separation. The verification of the independence of the results on ΔE is an important check of the correctness of the calculations.

The soft-photon approximation has often been used in the past to calculate radiative corrections, but high precision requires to also take into account hard radiation, i.e. processes where the photon has a large energy but can not be observed, for example because it is collinear with the emitting fermion. The integral over the hard region is finite and has to be done numerically since the integration involves a non-trivial dependence of structure functions or parton distribution functions on the photon momentum, via their arguments x_h and Q_h^2. In addition, bremsstrahlung has to be counted as a correction if the radiated photon is not observed and experimental conditions may lead to direct or indirect restrictions on the allowed photon phase space which are difficult to treat analytically.

The matrix element for the radiative process contains terms of the form

$$\frac{m_i^2}{(k \cdot p_i)^2} \quad \text{and} \quad \frac{p_i \cdot p_j}{(k \cdot p_i)(k \cdot p_j)} \, ,$$

where p_i denote the momenta of the fermions with mass m_i. The integration over the emission angle of the photon, relative to the momentum of the emitting fermion, leads to terms of the form

$$\int_{-1}^{1} \frac{dz}{E_i - |\vec{p}_i\,| z} \tag{22}$$

which give rise to mass-singular logarithms

$$\ln \frac{m_i^2}{4E_i^2} \, . \tag{23}$$

They are due to configurations where the photon is emitted parallel to one of the fermion momenta (collinear logarithms). The numerical value of these logarithms can be large. For example for leptonic radiation, where m_i is the electron mass, one has $\frac{\alpha}{\pi} \ln \frac{Q^2}{m_e^2} \simeq \mathcal{O}(10\,\%)$ for values of Q^2 typical for the HERA experiments. However, additional considerations are important to reach a full understanding of the important features of photonic corrections:

- **Radiation from the lepton line**.

  The emission of an energetic photon can shift the value of the momentum transfer Q_h^2 seen from the nucleon to very small values. In this case the bremsstrahlung contribution gets enhanced through the photon propagator $1/Q_h^4$. This effect is similar to the radiative tail effect above the peak of a resonant cross section. It is responsible for the large increase of the corrections at large y which is a typical feature of radiative corrections for deep inelastic scattering. In principle its magnitude can be reduced by applying suitable cuts, e.g., directly on the energy of the emitted photon or, indirectly, through a minimum requirement on the mass of the hadronic final state.

  In addition, the phase space available for photon radiation depends strongly on the lepton kinematic variables x and y. The maximal photon energy can be large at small x and large y, leading to an additional enhancement by a logarithm of the ratio of the maximal photon energy and the center of mass energy. In contrast, at small values of y and large x the photon phase space volume shrinks, leaving large negative contributions from loop diagrams.

These observations explain why leptonic corrections show a very pronounced increase at small x and large y and why they can become large but negative for small y and large x [23].

The purely leptonic corrections can be calculated with a good precision in the collinear approximation where only the large leading logarithmic contributions proportional to $(\alpha/\pi) \times \ln(Q^2/m_e^2)$ are kept (for early applications of this approximation to the case of DIS at HERA, see Refs. [24–26]). Since these terms are due to collinear radiation, the technique is also often called "peaking approximation". The approach allows one to factorize the corrections into radiator functions. A calculation of leptonic QED corrections in terms of radiator functions can also be generalized to include non-logarithmic corrections. It can also be used as a basis for the resummation of higher-order leading logarithms. This has first been discussed by Consoli and Greco in [27] and later by Kuraev et al. in [28]. The program DJANGOH, however, includes also non-logarithmic terms and is not based on the collinear approximation.

Finally we note that the leptonic corrections can be calculated in a way that is independent of a model for the hadronic part of the interaction. Their implementation in DJANGOH is done in such a way that not only PDFs, but also general structure functions can be used, without relying on a description of the hadronic side of the process in perturbative QCD.

- **Radiation from the quark line**. A naive application of the same techniques for the calculation of photon radiation from quarks would lead to a dependence on ill-defined quark masses. The appearance of quark mass singularities is, however, an artifact of a naive interpretation of the parton distribution functions as "bare" ones [19]. The mass singularities can be factorized from the cross section and are absorbed into the definition of the parton distribution functions. In fact, any experiment can only measure the "dressed" distribution functions which contain these corrections. The only observable effect of the photonic quark line corrections is an additional Q^2 dependence. This can be described in complete analogy to the Q^2 dependence arising from gluonic corrections, e.g., with the help of the generalized DGLAP evolution equations [20, 21]. Some modern PDF parametrizations include QED-evolved distribution functions, see e.g. [29, 30]. Remaining non-logarithmic corrections are small, in particular because the emission of a photon from the quark line cannot shift Q^2 to small values. Therefore it is often justified to neglect photon radiation from the quark altogether.

- **The lepton-hadron interference** of the diagrams in the upper and lower rows of Fig. 2 have to be combined with the box graphs of Fig. 3 which contain an additional photon. The combination of real and virtual corrections does not contain a logarithmic dependence, neither on the lepton nor on the quark mass. Therefore this contribution remains numerically small except at extreme values of x and y. These corrections cannot be absorbed by a redefinition of the quark distribution functions but have to be included as explicit corrections to the cross section. They are also essential for an accurate determination of charge and polarization asymmetries.

DJANGOH is restricted to single-photon radiation and 1-loop corrections. Multi-photon corrections from hard photon radiation are expected to be important at large y_l. At small y_l, in contrast, soft photons will dominate. Soft-photon corrections can be exponentiated [31], but this is not included in DJANGOH. Therefore the kinematic region at very small y_l should be avoided.

4 One-loop corrections

For measurements at larger values of the momentum transfer as well as for high-precision measurements at lower Q^2, one will have to include weak corrections in addition to QED radiative corrections. The corresponding one-loop diagrams for neutral-current electron quark scattering are shown in Fig. 3. Explicit formulas for the $\mathcal{O}(\alpha)$ weak radiative corrections can be found e.g. in Ref. [32].

The boson self energy diagrams (last row of Fig. 3) contain loop diagrams built with all particle degrees of freedom that couple to the gauge bosons. Therefore they contain information on the whole theory. In particular, they are responsible for a dependence of radiative corrections on the top-quark mass and the Higgs-boson mass. They would also be affected by masses and couplings of new particles in models beyond the standard model. Vertex corrections depend on the momentum transfer and can be absorbed in a redefinition of the vector- and axial-vector coupling constants of the gauge bosons. Box graphs give rise to correction factors which depend on two kinematic variables, for example x and Q^2.

The self energies are dominated by the fermion loops. For the photon self energy it can be accounted for by the use of the running fine structure constant

$$\alpha(Q^2) = \frac{\alpha(0)}{1 - \Pi^\gamma(Q^2)}\,, \tag{24}$$

where $\Pi^\gamma = \hat{\Sigma}^\gamma / Q^2$ is the vacuum polarization derived from the renormalized photon self energy $\hat{\Sigma}^\gamma$ and $\alpha(0) = 1/137.036$. Its hadronic part is connected with the total hadronic cross section of e^+e^- annihilation [33] by a dispersion relation. It is therefore usually parametrized or obtained from an interpolation over a grid of data.

The dominating constant part of the Z self energy can be taken into account by normalizing the Z-boson couplings in terms of the muon decay constant. At leading order, one has the relation

$$\frac{\alpha}{s_W^2(1 - s_W^2)} = \frac{\sqrt{2} M_Z^2 G_\mu}{\pi}\,, \tag{25}$$

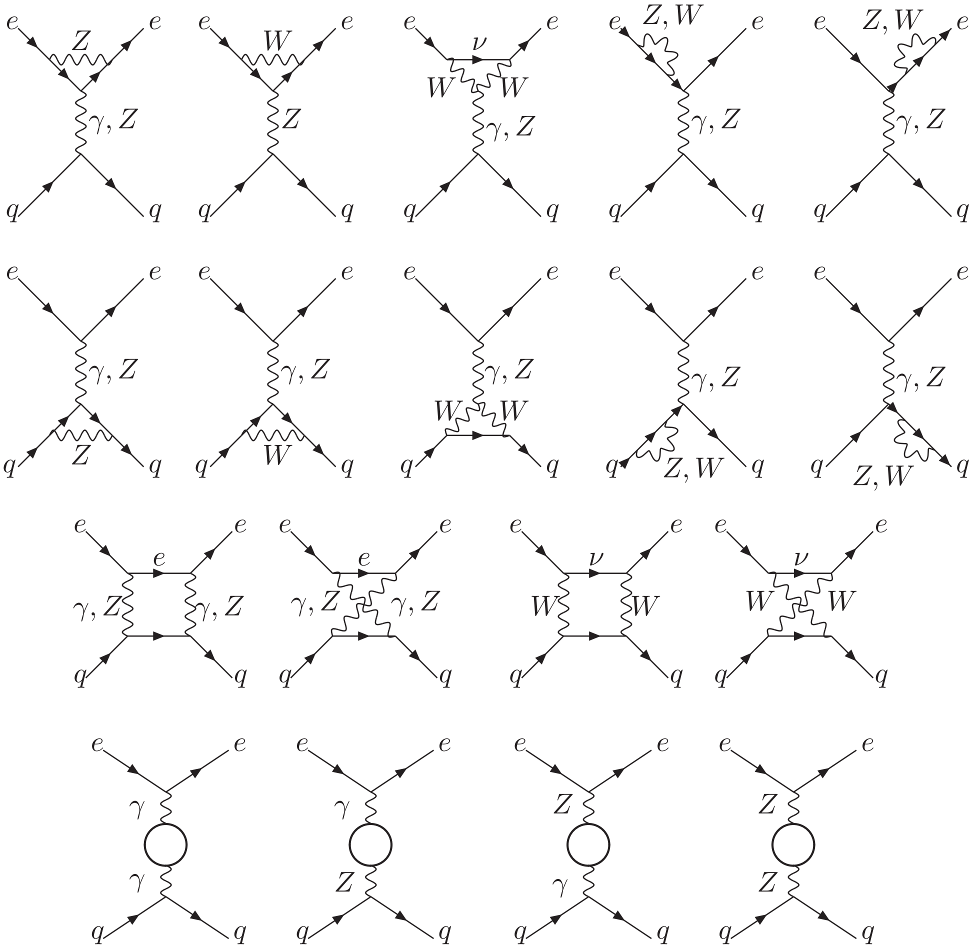

Figure 3: One-loop diagrams for neutral-current lepton quark scattering.

but this has to be modified at next-to-leading order by taking into account also the loop corrections for the muon decay, described by Δr,

$$\frac{\alpha}{s_W^2(1-s_W^2)}\frac{1}{1-\Pi^Z} = \frac{\sqrt{2}M_Z^2 G_\mu}{\pi}\frac{1-\Delta r}{1-\Pi^Z}\,. \tag{26}$$

Here, $\Pi^Z = \hat{\Sigma}^Z(Q^2)/(Q^2+M_Z^2)$ is given by the Q^2-dependent Z-boson self energy. The remaining correction factor $(1-\Delta r)/(1-\Pi^Z)$ is often close to unity. This prescription is the basis of the modified on-mass-shell scheme where weak corrections are usually smaller than in the on-shell scheme where the normalization of the Z-exchange contributions is determined by the left-hand side of Eq. (26).

The γZ mixing self energy (see the two Feynman diagrams in the middle of the last row of Fig. 3) can be absorbed by a redefinition of the weak mixing angle. This gives rise to a scale-dependent effective weak mixing angle,

$$\sin^2\theta_{W,f}^{\mathrm{eff}}(Q^2) = \kappa_f(Q^2)s_W^2\,, \tag{27}$$

where the universal contribution to $\kappa_f(Q^2)$ is determined by the γZ mixing $\hat{\Sigma}^{\gamma Z}(Q^2)$. Additional non-universal contributions which depend on the fermion type are often included in the definition of the effective mixing angle as well.

5 Charged current

5.1 Lowest Order

The cross section formulas for the charged current processes

$$e^-\mathcal{N} \to \nu_e X, \qquad e^+\mathcal{N} \to \bar{\nu}_e X \tag{28}$$

are simpler than for the neutral current since only one diagram appears at lowest order and because the CC interaction is purely left-handed. The Born cross section can be written as

$$\begin{aligned}\frac{d^2\sigma^{\mathrm{CC}}(e^\pm\mathcal{N})}{dxdQ^2} = \frac{1 \pm P_\ell}{2}\frac{\pi\alpha^2}{4\sin^4\theta_W}\frac{1}{x}\left[\frac{1}{Q^2+M_W^2}\right]^2 \times \\ \left(Y_+W_2^\pm(x,Q^2) \mp Y_- xW_3^\pm(x,Q^2) - y^2 W_{\mathrm{L}}^\pm(x,Q^2)\right) .\end{aligned} \tag{29}$$

The incoming electron can scatter only with positively charged quarks. Therefore, in the naive quark-parton model the structure functions $W_2^\pm$ and $xW_3^\pm$ are obtained from parton distribution functions for up-type quarks and down-type anti-quarks. For the proton one has

$$W_2^- = x\left(U + \overline{D}\right) , \quad xW_3^- = x\left(U - \overline{D}\right) , \tag{30}$$

where for $n_f = 4$ active quark flavors $U = u + c$ and $\overline{D} = \bar{d} + \bar{s}$. For positron scattering, the combinations $\overline{U} = \bar{u} + \bar{c}$ and $D = d + s$ are needed and one has

$$W_2^+ = x\left(\overline{U} + D\right) , \quad xW_3^+ = x\left(D - \overline{U}\right) . \tag{31}$$

At LO of QCD, the longitudinal structure function vanishes, $W_{\mathrm{L}}^\pm = 0$.

The cross section can also be normalized with the help of the Fermi constant G_μ using the relation (9). Then again the cross section does not receive large radiative corrections.

5.2 Radiative Corrections

The one-loop diagrams for charged-current lepton quark scattering are collected in Fig. 4. They can be combined in a correction factor $(1 + \delta^{\mathrm{CC}}_{1-\mathrm{loop}})$ which depends on x and Q^2 and differs between electron-quark and electron-antiquark scattering. The corresponding formulas can be found in Ref. [6, 11].

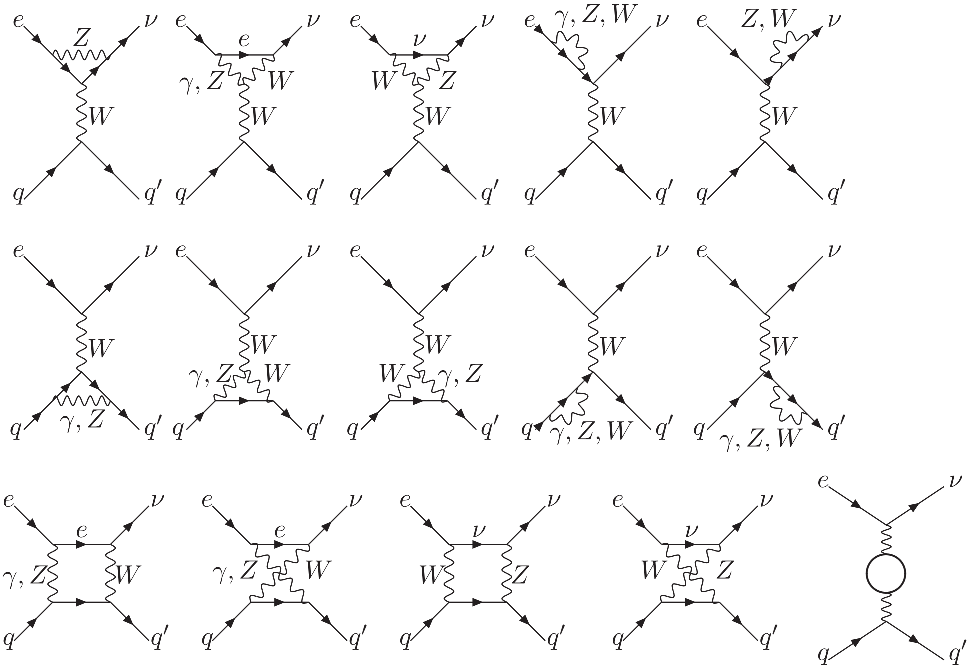

Figure 4: Feynman diagrams contributing to the $\mathcal{O}(\alpha)$ radiative corrections for charged-current scattering.

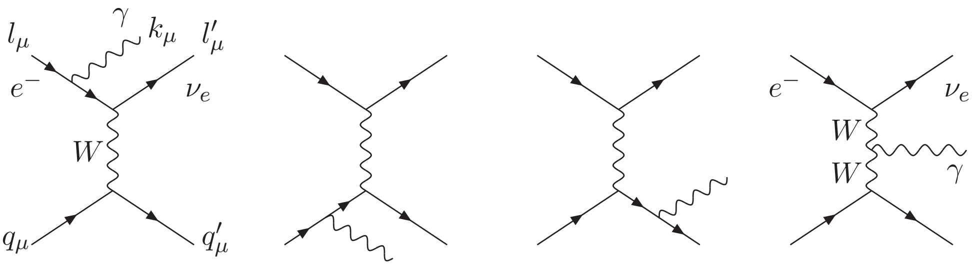

Figure 5: Single-photon bremsstrahlung diagrams for charged-current lepton quark scattering.

Compared to the NC cross section the real bremsstrahlung has, however, a more complicated structure. This is due to the last of the four diagrams shown in Fig. 5 which contains the non-Abelian γWW vertex. Due to the presence of this diagram the separation of the radiative corrections into gauge invariant classes for leptonic and quarkonic radiation and their interference can not be done in terms of Feynman diagrams. In order to obtain such a well-defined separation one has to sort the contributions to the cross sections according to their dependence on the fermion charges, using the relations between the fermion charges and the charge of the W,

$$|Q_e| = |Q_W|\,, \quad Q_d = Q_e + Q_u\,. \tag{32}$$

This allows one to express the cross section in terms of the electron charge and the charge of the incoming quark. The bremsstrahlung contribution then has the form

$$|\mathcal{M}|^2_{eq\to\nu q'\gamma} \sim \left(Q_e^2 I_{\rm lep} + Q_e Q_q I_{\rm int} + Q_q^2 I_{\rm qua}\right). \tag{33}$$

After integration over the photon phase space and adding the virtual photonic corrections separated in the same way one can see that the collinear logarithms of the lepton mass are all contained in the term proportional to Q_e^2, those of the incoming quark in the term proportional to Q_q^2 and the contribution proportional to $Q_e Q_q$ contains no mass logarithms at all. The quark mass singularities appear in the same universal form as in the NC case. It is therefore possible to absorb them into the distribution functions in a process independent way.

6 Polarized nucleon

For a polarized nucleon one has to add a polarization-dependent contribution to the cross section. Allowing for the possibility that both beam and target are polarized, with degrees of polarization P_ℓ and P_H, respectively, one can write

$$\frac{d^2\sigma^{\rm NC}(e^\pm\mathcal{N})}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left(\tilde\sigma_0^\pm + P_\ell\tilde\sigma_\ell^\pm + P_H\tilde\sigma_H^\pm + P_\ell P_H\tilde\sigma_{\ell H}^\pm\right), \tag{34}$$

extending Eq. (2). The reduced cross sections $\tilde\sigma_H^\pm$ and $\tilde\sigma_{\ell H}^\pm$ are expressed in terms of structure functions in analogy to the unpolarized cross section, by convention called G_1, $\tilde G_4 = G_5 - G_4$ and G_3. $\tilde\sigma_H^\pm$ and $\tilde\sigma_{\ell H}^\pm$ are obtained from $\tilde\sigma_0^\pm$ and $\tilde\sigma_\ell^\pm$ by the replacements

$$F_1^{0,\ell} \to -G_3^{0,\ell}, \quad F_2^{0,\ell} \to -\tilde G_4^{0,\ell}. \quad F_3^{0,\ell} \to -2G_1^{0,\ell}, \tag{35}$$

In the parton model, at leading order of QCD, these structure functions are determined from spin-dependent parton distribution functions,

$$\begin{aligned} 2xG_1^{0,\ell} &= \sum_f A^f_{\ell,0}(Q^2)\,x\left(\Delta q_f(x) + \Delta\bar q_f(x)\right), \\ \tilde G_4^{0,\ell} &= \sum_f B^f_{\ell,0}(Q^2)\,x\left(\Delta q_f(x) - \Delta\bar q_f(x)\right), \end{aligned} \tag{36}$$

where the coefficients have been defined in Eq. (5). Note that the role of the indices 0 and ℓ appears interchanged in these expressions. At leading order of QCD, $\tilde G_4^{0,\ell}$ is related with $G_3^{0,\ell}$: $\tilde G_4^{0,\ell} = 2xG_3^{0,\ell}$, but higher-order QCD corrections break this relation. The polarized PDFs, $\Delta q(x)$, are defined as differences of PDFs for partons with spins aligned or anti-aligned with the spin of the longitudinally polarized parent nucleon. They have to be determined from data and their Q^2-dependence is determined by perturbative QCD, i.e. the DGLAP equations, similar to the unpolarized PDFs.

The spin-dependent part of the charged-current cross section for electrons or positrons,

$$\frac{d^2\sigma_H^{\rm CC}(e^\pm\mathcal{N})}{dxdQ^2} = \frac{1 \pm P_\ell}{2}\frac{\pi\alpha^2}{4\sin^4\theta_W}\frac{1}{x}\left[\frac{1}{Q^2+M_W^2}\right]^2 \times$$
$$\left(2Y_- xH_1^\pm - Y_+ H_4^\pm + y^2 H_{\rm L}^\pm\right) \tag{37}$$

can also be obtained from the unpolarized case, Eq. (29), by the replacements

$$W_2^\pm \to -H_4^\pm\,, \quad xW_3^\pm \to -2H_1^\pm\,, \quad W_{\rm L}^\pm \to -H_{\rm L}^\pm\,. \tag{38}$$

Eq. (37) contributes multiplied with the nucleon degree of polarization to the cross section, see e.g. Ref. [34, 35] for details.

The structure functions $H_i^\pm$ in (37) for a proton target and $n_f = 4$ active quark flavors are given by

$$\begin{aligned}
H_1^-(x) &= \Delta u(x) + \Delta\bar{d}(x) + \Delta c(x) + \Delta\bar{s}(x)\,,\\
H_4^-(x) &= 2x\left(-\Delta u(x) + \Delta\bar{d}(x) - \Delta c(x) + \Delta\bar{s}(x)\right)\,,\\
H_1^+(x) &= \Delta\bar{u}(x) + \Delta d(x) + \Delta\bar{c}(x) + \Delta s(x)\,,\\
H_4^+(x) &= 2x\left(-\Delta\bar{u}(x) + \Delta d(x) - \Delta\bar{c}(x) + \Delta s(x)\right)
\end{aligned} \tag{39}$$

at the leading order (LO) or naive parton model approximation. H_4 is related to H_5 by the Dicus relation [36], $H_{\rm L} = H_4 - 2xH_5$, with $H_{\rm L} = 0$ at LO (i.e., the analog to the Callan-Gross relation in unpolarized DIS).

7 Radiative tail from elastic scattering

If only the final scattered electron is observed and no experimental information is available from the hadronic final state, also the radiative tail from elastic lepton nucleon scattering will contribute to the measured cross section. The effect has been calculated for example in Ref. [37]. For typcial HERA energies, the elastic radiative tail would contribute non-negligibly for $x \leq 0.1$ and $y \geq 0.7$, if it had not been possible to impose a lower cut on the mass of the hadronic final state.

Elastic scattering is described by only one kinematic variable. The invariant momentum transfer squared can be determined from the lepton scattering angle, θ_ℓ, alone. The usual way to write the cross section proceeds through the Sachs electric and magnetic form factors, G_E and G_M, or, equivalently the Dirac and Pauli form factors F_1^p, F_2^p, related to each other by $G_E = F_1^p - \tau F_2^p$ and $G_M = F_1^p + F_2^p$ with $\tau = Q^2/(4M^2)$. In terms of invariants, the cross section for the elastic process at leading order can be written as

$$\frac{{\rm d}\sigma(ep\to ep)}{{\rm d}Q^2} = \frac{\pi\alpha^2}{(S-m_\ell^2-M_N^2)^2 - m_\ell^2 M_N^2}\frac{1}{Q^4} \times \tag{40}$$
$$\left\{G(2m_e^2 - Q^2) - H\left[M_N^2 Q^2 + \left(S - m_e^2 - M_N^2\right)\left(Q^2 - S + m_e^2 + M_N^2\right)\right]\right\}\,,$$

with

$$G = -2Q^2 \left(F_1^p + F_2^p\right)^2 \,, \qquad H = 4 \left(F_1^p\right)^2 + \frac{Q^2}{M^2} \left(F_2^p\right)^2 \,. \tag{41}$$

The radiative tail, i.e. the corrections due to the radiative process $ep \to ep+\gamma$, where only photon emission from the lepton is numerically relevant for the applications we have in mind, is easily derived from this representation and can be found for example in Ref. [38].

The form factors have a steep Q^2 dependence, given by a dipole form, $G_E \simeq \left(1 + Q^2/\Lambda^2\right)^{-2}$ and $G_M = \kappa_p G_E$ with $\Lambda^2 = 0.71$ GeV^2 and $\kappa_p = 2.7928$ the proton's magnetic moment in units of the nuclear magneton. High-precision data for the proton form factors have become available over the years which have resulted in slightly more complicated parametrizations, usually valid in a Q^2-range up to about 1 GeV^2, see for example Ref. [39].

8 Hadronic final state

8.1 Factorization

The idea to generate a hadronic final state in DJANGOH is based on the observation that the radiative process (13) is dominated by far by leptonic bremsstrahlung which leads to a shift of the kinematic variables. The HERACLES part of the program is used to generate the lepton side of the process and to define values for x and Q^2 which determine the kinematics of the hadronic final state. This factorization of the process into a leptonic and a hadronic part can be done in such a way that the cross section differential with respect to the lepton in the final state and a bremsstrahlung photon is correct at order $O(\alpha)$, including non-logarithmic contributions.

Events are classified according to the four channels of HERACLES as being either non-radiative, an event with initial-state radiation, an event with final-state radiation, or a Compton event characterized by low hadronic momentum transfer, $Q_h^2 \to 0$ (c.f. [1]). For radiative events we then define a rescaled Born-like process

$$E(L) + p(p) \to E'(L') + X(p_X)\,. \tag{42}$$

In general both E and E' will be virtual particles. Since we restrict our considerations to $O(\alpha^3)$, only either of these will be virtual. For example, in case the event (13) is classified as resulting from initial-state photon radiation we imagine the two-step process

$$\begin{aligned} e(l) &\to \tilde{e}(\tilde{l}) + \gamma(k), \\ \tilde{e}(\tilde{l}) + p(p) &\to e'(l') + X(p_X), \end{aligned} \tag{43}$$

thus $E(L) = \tilde{e}(\tilde{l})$ and $E'(L') = e'(l')$. Here $\tilde{l} = l - k$ and we define DIS variables for the

$\tilde{e}p$ subprocess

$$
\begin{aligned}
\tilde{s} &= (\tilde{l} + p)^2 = S - 2k \cdot (l + p), \\
\tilde{Q}^2 &= -\tilde{q}^2 \qquad \equiv -(\tilde{l} - l')^2, \\
\tilde{x} &= \frac{\tilde{Q}^2}{2p \cdot \tilde{q}} \quad = \frac{\tilde{Q}^2}{y(S - m_N^2) - 2p \cdot k}, \\
\tilde{y} &= \frac{\tilde{Q}^2}{\tilde{x}\tilde{s}}, \\
\tilde{W}^2 &= (p + \tilde{q})^2 = (1 - \tilde{x})\tilde{y}\tilde{s} + m_N^2.
\end{aligned}
\tag{44}
$$

Note that the tilde variables coincide with the hadronic variables (17) except for S and y.

A similar rescaling of variables is performed for events with final-state radiation, now imagining the process as a hard scattering followed by a decay:

$$
\begin{aligned}
e(l) + p(p) &\to \tilde{e}'(\tilde{l}') + X(p_X), \\
\tilde{e}'(\tilde{l}') &\to e'(l') + \gamma(k),
\end{aligned}
\tag{45}
$$

where $\tilde{l}' = l' + k$ and

$$
\begin{aligned}
\tilde{s} &= S, \\
\tilde{Q}^2 &= -\tilde{q}^2 \quad \equiv -(l - \tilde{l}')^2, \\
\tilde{x} &= \frac{\tilde{Q}^2}{2p \cdot \tilde{q}} = \frac{\tilde{Q}^2}{y(S - m_N^2) - 2p \cdot k}, \\
\tilde{y} &= \frac{\tilde{Q}^2}{\tilde{x}\tilde{s}}.
\end{aligned}
\tag{46}
$$

Events from the Compton channel are characterized by typically small Q_h^2 and small W_h. For this reason they require a special treatment. Instead of large-Q^2 DIS events, they are more similar to events of the photoproduction process $\gamma N \to X$. The number of Compton events is small, typically a fraction of a percent if a cut on the mass of the hadronic final state of $W_h \geq W_h^{min}$ with $W_h^{min} \simeq 2\,GeV$ is imposed. These events are treated by SOPHIA, a Monte Carlo simulation program originally written for photohadronic processes in astrophysics [40].

For the reduced process (42) we then apply higher-order QCD corrections in a parton shower approach and let the final partons and the nucleon remnants fragment using the LUND string fragmentation implemented for DIS in LEPTO [2]. The main steps of the algorithm are:

1. Generate the lepton side of an event by HERACLES, including a possible radiative photon.

2. Check whether the hadronic mass W_h is large enough to allow hadronization in LEPTO. For low W_h hand over to SOPHIA.
3. Define the kinematic variables for the sub-process (42).
4. Fill the event record with the shifted variables for the sub-process, L, p, $\tilde{q}$ and L'.
5. Transform to the hadronic center-of-mass frame.
6. Choose the flavor of the struck quark and the target remnant.
7. Include parton cascades, target remnant, and primordial k_T.
8. Perform fragmentation.
9. Boost to desired frame.
10. Update and clean the event record.

Part B: Technical description

The computational procedures applied in DJANGOH are based on the methods used in the AXO library [41] for Monte Carlo integration and event generation. AXO itself relies on the Monte Carlo integration algorithm VEGAS by P. Lepage [42]. The following steps have to be performed in actual computations:

1. Integration of the different contributions to be included. Thereby partial cross sections are determined according to the actually defined phase space region. They define the relative weight of the corresponding contribution in the final step of event sampling. Furthermore, the integration procedure supplies information for the construction of the distribution function applied for event generation. This step is controlled by input via code-words `'INT-OPT-NC'` and `'INT-OPT-CC'`, see Sec. 9.1.
2. Estimation of the local maxima of the distribution function in a predefined number of hypercubes, needed since the integration volume is subdivided to optimize the rejection procedure. This step is usually executed together with the integration step 1. For applications where only cross sections are needed and no event generation is required, this step can be suppressed, see code-word `'INT-ONLY'`, Sec. 9.1.
3. According to the partial cross sections determined in step 1, events are then generated randomly from the individual contributions. In HERACLES, only the scattered lepton and sometimes a radiative photon is added to the event record. After construction of this part of the event, routines from DJANGO are called to simulate QCD effects and to generate the hadronic part of the event. Control input for this step is transferred via code-words `'SAM-OPT-NC'` and `'SAM-OPT-CC'`, see Sec. 9.1.

Steps 1 and 2 are necessary for each individual contribution to be included in the event generation. For flexibility (in particular with respect to CPU-time requirements) and test purposes the program allows a treatment of the different steps for each contribution separately. The information obtained in steps 1 and 2 is always written to an external file which has to be assigned in the corresponding run. If step 3 is done in a separate run of the program, a corresponding file has to be provided with information from steps 1 and 2. The random number seeds for refined integration by VEGAS or continued event sampling can also be stored on an external file.

The user can communicate with the program by setting initial conditions (through input flags) and through a user analysis routine. The latter routine allows for (1) additional initialization before generating events (for example booking of histograms), (2) analysis after having accepted an event and (3) a final call after completion of event generation. In the following section 9 input flags will be documented while in section 10 the standard output and possible user analysis is described. Some common blocks used to store output of the program will be described in appendices.

9 Input to the program

The input to DJANGOH consists of two parts: the first part deals with HERACLES options and the second one contains DJANGO input flags. The two parts are separated by the code word `'START'` in the sequence of lines in the input file. The format for the input in the two parts is the same, i.e. an option card defining the expected information (`FORMAT (A10)`) is followed by one card containing the appropriate data (format-free – `READ(5,*)`). All data items except `'START'` (in the HERACLES part) and `'CONTINUE'` (in the DJANGO part) are optional and are set to default values if they are not given as input. The sequence of different options within the two parts does not matter, only the `'CONTINUE'` option triggering the actual operation of the program has to be the last one. Input lines after `'CONTINUE'` are ignored.

In the following we describe the possible input options and the corresponding data expected by the program both for the HERACLES and the DJANGO part. Default values are given in parentheses. For additional information concerning options of LEPTO 6.5.1 and JETSET 7.4 we refer to the corresponding manuals [2] and [43, 44].

9.1 HERACLES part

- `'TITLE'`
  data: user defined heading of the first output page.
- `'OUTFILENAM'`
  data: `character*232`
  user defined base name for output files;

- OUTFILENAM_out.dat is used for standard output. Note that redirection of the output to the file can start only after code word START has appeared in the input file;
- OUTFILENAM_smp.dat is used for sampling information needed for iterated calls;
- OUTFILENAM_rnd.dat takes information about the state of the random number generator;
- OUTFILENAM_evt.dat is used in the sample version of the user routine to store event listings.

If no input to code word OUTFILENAM is given, standard output continues to appear at the console and the default name djangoh-default-output is used as base name for the other output files.

- 'EL-BEAM'
  data: EELE, POLARI, LLEPT
  quantities defining the properties of the lepton beam;

EELE	= energy of the lepton beam in GeV (D=27.5);
POLARI	= degree of beam polarization[4] (D=0.), $-1 \leq$ POLARI ≤ 1;
LLEPT	= lepton charge (D=–1),
	$= -1$, electron beam,
	$= +1$, positron beam,
	$= -3$, muon beam,
	$= +3$, anti-muon beam.

- 'PR-BEAM'
  data: EPRO, HPOLAR
  properties of the proton beam;

  EPRO = energy of the proton beam in GeV (D=920.). This is the relativistic energy, i.e. including the proton mass. If input is smaller than m_p, EPRO is set to m_p.

  HPOLAR = the degree of polarization of the proton beam (D=0.). If not between -1 and $+1$, HPOLAR is set to zero.

- 'KINEM-CUTS'
  data: ICUT, XMIN, XMAX, YMIN, YMAX, Q2MIN, Q2MAX, WMIN
  definition of kinematical cuts. These cuts are applied to the *leptonic* variables x_l, y_l, Q_l^2, defined by the momentum of the final-state lepton. WMIN is the mass of the hadronic final state.

[4]The treatment of events in LEPTO may have problems for values above 0.99 or below -0.99.

`ICUT` $= 1$: cuts in x_l and lower cut in Q_l^2 (`Q2MIN` in GeV^2) are applied, cuts in y_l are ignored,

$= 2$: cuts in x_l, lower cut in Q_l^2 and lower cut in the hadronic mass W_h (`WMIN` in GeV) are applied, cuts in y_l are ignored,

$= 3$: all supplied cuts for x_l, y_l, Q_l^2 and W_h are applied.

Default values: `ICUT` $= 3$, `XMIN` $= 10^{-5}$, `XMAX` $= 1.0$, `YMIN` $= 10^{-2}$, `YMAX` $= 1.0$, `Q2MIN` $= 4.0$, `Q2MAX` $= 10^8$, and `WMIN` $= 5.0$. The final definition of cuts according to the most restrictive conditions is performed in the subroutine `HSPRLG`. Note that the most restrictive limits may differ from the given input values.

In order to avoid large virtual corrections which could lead to a negative non-radiative cross section, x_l and y_l should be restricted to the region $y_l(1-x_l)^2 \geq 0.004$.

- `'THETA-CUT'`
  data: `THEMIN, THEMAX`
  lower and upper limit for the lepton scattering angle in degrees with respect to the incoming lepton (D=0, 180.). `THEMAX` is used only for elastic scattering.

- `'PT-CUT'`
  data: `PTMIN`
  lower cut on the transverse momentum of the scattered lepton in GeV (D=0.).

- `'EGAM-MIN'`
  data: `EGMIN`
  definition of a lower cutoff energy for bremsstrahlung photons (in GeV);

  `EGMIN` $= 0.0$: Both radiative and non-radiative events are generated. The separation of the cross section into non-radiative and radiative parts with the help of a soft-photon cutoff is determined internally as a function of x and y. `EGMIN` $= 0$ has to be used if integration/sampling of both radiative and non-radiative channels is requested.

  `EGMIN` > 0.0: Only hard-photon bremsstrahlung (with energy above `EGMIN`) is considered for event sampling (in this case the data items `ISNC2`, `ISCC2` and `IEL2` from input options `'SAM-OPT-NC'` and `'SAM-OPT-CC'` are ignored!).

  Default: `EGMIN` $= 0$.

- `'THMIN-QRAD'`
  data: `TCUTQ`, `TCUTQS`
  definition of cuts for the separation of photons from the incoming and outgoing quarks (in *rad*, defaults: `TCUTQ=TCUTQS` $= 0.25$). If radiation from quarks is included, only events with photons outside cones defined by `TCUTQ` and `TCUTQS` around

the incoming and outgoing quark are generated. Photon radiation inside these cones is assumed to be part of the parton shower and included in the parametrization of parton distribution functions.

- `'GSW-PARAM'`
  data: `LPARIN(1:11)`
  monitoring the definition of electroweak parameters and the inclusion of different parts of virtual corrections;

`LPARIN(1)` = 1 :	electroweak parameters are calculated from fixed values for the boson masses M_W, M_Z;
= 2 :	electroweak parameters are calculated from fixed M_Z, and the muon decay constant G_μ.

Default: `LPARIN(1)` = 2. Some independent mass parameters of the Standard Model in the on-shell renormalization scheme are taken from input to code word `'GSW-MASS'`, see below. All other electroweak parameters, including boson and fermion masses and coupling constants, are calculated in the subroutine `HSSETP`.

`LPARIN(2)` = 0 :	only Born cross section without electromagnetic or weak corrections is integrated/sampled,
= 1 :	Born cross section including corrections as determined by the values of `LPARIN(3)` – `LPARIN(11)`.

Default: `LPARIN(2)` = 1.

`LPARIN(3)` (D=3) :	flag for inclusion of some known higher-order contributions in parameter relations and virtual corrections:
= 0 :	no higher order corrections,
≥ 1 :	terms of $\mathcal{O}(\alpha^2 m_t^4)$ are included in Δr,
≥ 2 :	terms of $\mathcal{O}(\alpha\alpha_s m_t^2)$ in Δr included,
≥ 3 :	running $\alpha(Q^2)$ is used for the radiative cross section.

The following options define partial corrections to be included in the actual calculation (0 / ≥ 1 = no / yes):

`LPARIN(4)` (D=1) :	leptonic QED corrections;
`LPARIN(5)` (D=0) :	quarkonic QED corrections;
`LPARIN(6)` (D=0) :	lepton-quark interference[2];
`LPARIN(7)` (D=2) :	fermionic contributions to the photon self energy Σ^γ:

[2]The contribution from lepton-quark interference can be requested only if both the leptonic and the quarkonic corrections are activated as well at the same time.

$= 0$:	not included,
$= 1$:	parametrization with the help of quark masses,
$= 2$:	parametrization from [33];
`LPARIN(8)` (D=0) :	fermionic contribution to the $\gamma - Z$ mixing;
`LPARIN(9)` (D=0) :	fermionic contribution to the self energy of the Z boson;
`LPARIN(10)` (D=0) :	fermionic contribution to the self energy of the W boson;
`LPARIN(11)` (D=0) :	purely weak contributions to the self energies, vertex corrections and boxes.

- `'GSW-MASS'`
  data: `MW`, `MZ`, `MH`, `MT`
  the electroweak mass parameters: W and Z boson masses, Higgs mass and the top-quark mass (in GeV). The value given for `MW` is used only if at the same time `LPARIN(1)` $= 1$. Otherwise, M_W is calculated in the program from the μ decay constant. The default values are $M_W = 80.379$ GeV, $M_Z = 91.1867$ GeV, $M_H = 125.25$ GeV and $m_t = 172.5$ GeV [45].

- `'STRUCTFUNC'`
  data: `ILQMOD`, `ILIB`, `ICODE`
  defines the parametrization of parton densities or structure functions applied in the calculation. `ILQMOD` encodes options for the treatment of structure functions in the low Q^2 region, `ILIB` defines a library of subroutines to be used for the calculation of parton distribution or structure functions and the corresponding parametrization is specified by `ICODE`. The parton distribution functions are taken via the modified LEPTO routine `LYSTFU` from PYTHIA (routine `PYSTFU`) [46]. The switches `ICODE` and `ILIB` agree with the PYTHIA parameters `MSTP(51)` and `MSTP(52)`. The meaning of `ICODE` depends on the value of `ILIB`. The allowed values are as follows:

`ILIB` $= 1$:	obsolete, to choose parton distribution functions from the PYTHIA routine `PYSTFU`;
`ILIB` $= 2$:	the only recommended value, to choose parton distribution functions from LHAPDF, version 6 [47,48];

`ILQMOD` $= 0$:	use unmodified parton distribution functions for all Q^2.
`ILQMOD` $= 1$:	apply exponential low Q^2 suppression factor to parton distribution functions [49].

The following options are available only in the HERACLES mode, i.e. without generation of hadronic final states with LEPTO:

`ILQMOD` $= 2$:	for low Q^2 ($Q^2 < 6$ GeV^2) use F_1 and F_2 parametrizations from Brasse [50] (for $W < 2$ GeV) and Stein [51] (for $W > 2$ GeV).

For large Q^2, parton distribution functions as determined by `ICODE` are used.

`ILQMOD` = 3: for $Q^2 < 6$ GeV^2 use parametrizations from Abramowicz, Levy, Levin, and Maor [52] (for $W > 2$ GeV) together with Brasse (for $W < 2$ GeV). If `ICODE = 0`, the ALLM parametrization is used for high Q^2 as well; if `ICODE` $\neq 0$, for large Q^2 the parton distributions as requested by `ICODE` are combined with the ALLM parametrization.

`ILQMOD` = 4: Obsolete. For $x < 0.1$ and $\nu > 10$ GeV, the parametrization by Badełek and Kwieciński [53] is used. For $\nu < 10$ GeV, $x < 0.1$, the Brasse and Stein parametrization is used. At larger x, the parton distribution functions from MRS, set D'_- [54, 55] have to be used, other combinations of BK with PDF's are inconsistent.

`ILQMOD` = 5: The parametrization by Donnachie and Landshoff [56, 57] for $Q^2 < 10$ GeV^2 together with parton distribution functions are used.

`ILQMOD` = 10: User defined structure functions for one-photon exchange, to be defined in the subtroutine `FIUSER`.

The default values are `ILQMOD` = 1 (exponential low Q^2 suppression), `ILIB` = 2 (PDF's from LHAPDF) and `ICODE` = 14400 (CT18NLO). The user himself is responsible for giving consistent input. Some PDF may include already a prescription for a proper small-Q^2 behaviour which should not be combined with an additional suppression by a non-zero input for `ILQMOD`.

- `'POLPDF'`
  data: `IDPVR`
  to select a parametrisation of polarized PDFs. `IDPVR = 100*ISET+MODE`.

  `ISET` = 1: DSSV [58];

  `ISET` = 2: DNS [59];

  `ISET` = 3: DS [60];

  `ISET` = 4: GSLO [61];

  `ISET` = 5: GSNLO [61];

  `ISET` = 6: BB [62];

  `ISET` = 7: AAC [63];

  `ISET` = 8: LSS [64];

  `ISET` = 9: GRSV [65];

  Allowed choices for `MODE` depend on the PDF set, see the corresponding references.

- 'NUCLEUS'
  data: EPRO, HNA, HNZ
  for lepton nucleon scattering. EPRO is the energy per nucleon in the beam (in GeV), HNA and HNZ are atomic and charge numbers of the nucleon. This input item should replace the one with code word PR-BEAM. If both are present, the last value for EPRO will be used for the energy per nucleon and a non-zero value for the proton polarization HPOLAR will be ignored.
  Defaults: HNA = HNZ = 1.

- 'NUCL-MOD'
  data: INUMOD
  to select a model for nuclear shadowing or nuclear parton distribution functions.
  Default: INUMOD = 0.

  INUMOD = 0: No correction. Parton distribution functions are used as selected by ILQMOD, ILIB, ICODE from code word STRUCTFUNC.
  This is the appropriate setting if nuclear parton distribution functions like those from [66] are used which incorporate nuclear effects.

  INUMOD = 1000: Structure functions are calculated from the isoscalar average of unmodified parton distribution functions and a Q^2-independent correction factor. The same correction factor is applied to both F_2 and F_L.

  INUMOD = 2000: Apply correction factors to free parton distribution functions from EKS98 [67].

  INUMOD = 3000: Apply correction factors to free parton distribution functions from EPS08 [68].

  INUMOD > 4000: Apply correction factors to free parton distribution functions from EPS09 [69].
  LO or NLO fits (iorder = 1 or 2) and the error set ipset = 1, ..., 23, are selected according to INUMOD = 100×iorder + ipset.
  The EPS09 analysis is based on CTEQ6.1M partons, but any PDF set can be combined with the nuclear corrections in this scheme.

- 'EP-DIPOLE'
  data: IDIPOL
  for elastic scattering (and its radiative tail) to include (if non-zero) the parametrization of [51] for the deviation of the elastic *ep* form factors from the simple dipole form.
  Default: IDIPOL = 0.

- 'FLONG'
  data: IFLOPT, PARL11, PARL19
  for the inclusion of the longitudinal structure function F_L. The definition of IFLOPT coincides with the definition of LST(11) of LEPTO 6.5.1 [2]:

  IFLOPT = LQCD + LTM*10 + LHT*100

  and LQCD, LTM and LHT specifies whether QCD contributions, target mass effects and higher twist should be included. See the manual of LEPTO 6.5.1 [2] for details. It is assumed that F_L contributes only to the one-photon exchange part of the cross section, i.e. no corrections from Z-boson-gluon fusion, or from target-mass and higher-twist contributions to Z-exchange are taken into account. HERACLES calls modified routines from LEPTO 6.5.1 to calculate F_L either event-by-event when LQCD $= 2$ or LTM $= 2$ (time-consuming) or by using a grid and linear interpolation for LQCD $= 1$ or LTM $= 1$.
  For the calculation of radiative corrections, F_L is needed for small Q^2, also in the limit $Q^2 \to 0$. The prescription for $F_L(Q^2 \to 0)$ is determined by ILQMOD. If ILQMOD ≥ 2, the structure function parametrization includes already a prescription for F_L. For these values, IFLOPT is reset to 0. For other input, F_L is calculated from perturbative QCD and for Q^2 values smaller than $Q_0^2 = 6$ GeV^2, F_L is assumed to be independent of Q^2 and its value is given by $F_L(Q^2 = Q_0^2)$.
  The meaning of the parameters PARL11 and PARL19 coincides with that of PARL(11) and PARL(19) in LEPTO 6.5.1:

PARL11 :	Accuracy for the integration needed to calculate F_L;
PARL19 :	κ^2, the scale parameter for higher-twist contributions in F_L.

  The default values are IFLOPT $= 0$, PARL11 $= 0.01$, PARL19 $= 0.03$.
  Note: There is no F_L contribution for the charged current process. For consistency, IFLOPT is reset to 0 for runs with charged-current contributions turned on.
  Note also that the combined inclusion of F_L and higher twist together with QED corrections at the quark line is, formally, a second-order mixed QED+QCD correction. A complete calculation of these corrections is not yet available. Therefore, quarkonic radiation and lepton-quark interference of photon radiation together with $\gamma\gamma$ and γZ box graphs can not be included if F_L is to be included. If input for IFLOPT is non-zero, LPARIN(i), i=5,6,11 are reset to 0.

- 'ALFAS'
  data: MST111, MST115, PAR111, PAR112
  information defining the value of α_s needed for the inclusion of F_L. α_s enters the calculation of F_L and is evaluated in the routine ULALPS from JETSET 7.4. The meaning of the input data corresponds to the parameters MSTU(111), MSTU(115), PARU(111), PARU(112) of JETSET 7.4 and is as follows:

MST111 :	Order of α_s evaluation in ULALPS: $= 0$: fixed α_s given by PAR111,

= 1: first-order running α_s,
= 2: second-order running α_s,
(D=1);

`MST115` : Treatment of the α_s singularity at $Q^2 \to 0$:
= 0: allow it to diverge like $1/\ln(Q^2/\Lambda^2)$,
= 1: soften the divergence to $1/\ln(1 + Q^2/\Lambda^2)$,
= 2: freeze Q^2 evolution below $Q_0^2 = 4\ \mathrm{GeV}^2$,
(D=0);

`PAR111` : The fixed value of α_s assumed in `ULALPS` when `MST111` = 0 (D=0.20);

`PAR112` : Λ used in running α_s in `ULALPS` (D=0.25).

- `'NFLAVORS'`
  data: `NPYMIN`, `NPYMAX`
  minimal and maximal number of flavors. Used only for the calculation of F_L and in LEPTO for final-state hadronization. The flavors are counted from 1 to 6 in the following order: d, u, s, c, b and t. Defaults are `NPYMIN` = 1 and `NPYMAX` = 6.

- `'INT-ONLY'`
  data: `INTOPT`
  A negative value for `INTOPT` supresses the call to subroutines needed for the preparation of the sampling steps. Only integration of the differential cross section is performed in this case. Default: 0.

- `'ASYM-ONLY'`
  data: `ICPASY`
  To calculate only the polarization-dependent part, or the charge-odd part, of the cross section.

  `ICPASY = 1` : Calculate the lepton-polarization dependent part of the cross section, $\sigma_\ell = (1/2)\,(\sigma(P_\ell = +1) - \sigma(P_\ell = -1))$.

  `ICPASY = 2` : Calculate the charge-odd part of the cross section, defined as $\sigma_C = (1/2)\,(\sigma(e^-) - \sigma(e^+))$. Use this option together with `IFLOPT=0` (no contribution from F_L) and `LPARIN(4)=LPARIN(5)=LPARIN(6)=1`.

  This option will be helpful to calculate polarization or charge asymmetries at low Q^2 where the polarization-dependent and lepton charge-dependent parts are very small. Event generation is not possible together with this option. Default: 0.

- `'INT-OPT-NC'`
  data: `INC2`, `INC31`, `INC32`, `INC33`, `INC34`, `IEL2`, `IEL31`, `IEL32`, `IEL33`
  defines the contribution(s) to neutral current interactions for which the integrated

cross section should be calculated in order to prepare the sampling procedure (including estimation of local maxima of the actual distribution function if `INTOPT` $\geq$ 0):

`INC2` :	Integration for the **non-radiative contribution** (Born term including virtual and soft corrections); integration by Gaussian quadrature, NAGLIB routine `D01FCF`,
`INC31` < 100:	number of iterations for the integration of the contribution from **initial-state leptonic bremsstrahlung** by `VEGAS`,
`INC31` > 100:	(INC31-100) iterations by `VEGAS1`; here the iteration procedure is restarted to estimate the integral; the grid structure is used from the previous run, but estimates of the integral and its error accumulated in the previous run(s) are discarded. The necessary information is read from an external file which had to be defined before by a run with `INCx` < 100,
`INC31` > 200:	(`INC31-200`) iterations by `VEGAS2`: the iteration procedure is restarted using both the grid information and the accumulated estimates of the integral and its error from the previous run(s);
`INC32` :	monitoring the integration of **final-state leptonic bremsstrahlung** with the same conventions as for `INC31`;
`INC33` :	monitoring the integration of **the Compton contribution** with the same conventions as for `INC31`;
`INC34` :	monitoring the integration of **the contribution from radiation from the quark line** with conventions as for `INC31` (no generation of hadronic final states for this contribution);
`IEL2` :	monitoring the integration of **elastic scattering** $ep \to ep$ with conventions as for `INC2`;
`IEL31` :	for the integration of the quasi-elastic tail $ep \to ep\gamma$ from **initial-state leptonic bremsstrahlung**,
`IEL32` :	for the integration of the quasi-elastic tail $ep \to ep\gamma$ from **final-state leptonic bremsstrahlung**,
`IEL33` :	for the integration of the quasi-elastic tail $ep \to ep\gamma$ from the **Compton contribution**,

Defaults are `INC2=INC31=INC32=INC33=INC34` = 0 and `IEL2=IEL31=IEL32=IEL33` = 0, no integration.

- `'INT-OPT-CC'`
  data: `ICC2`, `ICC31`, `ICC32`, `ICC33`
  same as `'INT-OPT-NC'` but for the charged current interaction:

`ICC2` :	Integration for the non-radiative contribution (Born term including virtual and soft corrections);
`ICC31` :	Integration of the contribution from leptonic initial-state bremsstrahlung by `VEGAS`;
`ICC32` :	Integration of the contribution from quarkonic initial-state radiation;
`ICC33` :	Integration of the contribution from the quarkonic final-state radiation (not activated).

Defaults are `ICC2=ICC31=ICC32=ICC33=0`, no integration.

- `'INT-POINTS'`
  data: `NPOVEG`
  upper limit for the number of integration points used by `VEGAS` (D=10000).

- `'HYP-CUBES'`
  data: `NPHYP`
  the number of points in each bin used to estimate the function maxima (D=20).

- `'SAM-OPT-NC'`
  data: `ISNC2`, `ISNC31`, `ISNC32`, `ISNC33`, `ISNC34`, `ISEL2`, `ISEL31`, `ISEL32`, `ISEL33`
  monitoring the inclusion of individual contributions to the neutral current cross section for event sampling. A contribution is included if the corresponding option is set to 1 or 2, resp.; `ISNCx` = 2 triggers continued sampling, i.e. information from a previous sampling run with `ISNCx` $\neq 0$ is expected:

`ISNC2` :	non-radiative contribution;
`ISNC31` :	initial-state leptonic bremsstrahlung;
`ISNC32` :	final-state leptonic bremsstrahlung;
`ISNC33` :	Compton contribution;
`ISNC34` :	quarkonic radiation.
`ISEL2` :	elastic scattering $ep \to ep$;
`ISEL31` :	initial-state leptonic radiation for $ep \to ep\gamma$;
`ISEL32` :	final-state leptonic radiation for $ep \to ep\gamma$;
`ISEL33` :	Compton contribution for $ep \to ep\gamma$;

Defaults: `ISNCx` = 0.

- `'SAM-OPT-CC'`
  data: `ISCC2`, `ISCC31`, `ISCC32`, `ISCC33`
  same as `'SAM-OPT-NC'` but for the charged current. For the physical content of the charged current radiative channels, see option `'INT-OPT-CC'`. Defaults: `ISCCx` = 0.

- 'WEIGHTS'
  data: IWEIGS
  generation of events with externally defined weight function;

  IWEIGS $= 0$: Events are generated according to the true differential cross section calculated by the program including or excluding radiative corrections as requested by the user input;

  IWEIGS $= 1$: Events are generated according to the differential cross section modified by the additional weight factor

  $$\omega = x_l;$$

  IWEIGS $= 2$: Events are generated according to the differential cross section modified by the additional weight factor

  $$\omega = \begin{cases} Q_l^2/Q_0^2 & \text{for } Q_l^2 \leq Q_0^2 = 100\,\mathrm{GeV}^2, \\ 1 & \text{for } Q_l^2 \geq Q_0^2. \end{cases}$$

  Other weight functions, which may depend on the lepton kinematic variables x_l and y_l, can be defined by the user in the routine HSWGTX. This option can be used to increase the efficiency of the event generation. The calculation of the cross sections is not affected.
  Default: IWEIGS $= 0$.

- 'RNDM-SEEDS'
  data: ISDINP, ISDOUT
  monitoring input/output of actual random seeds from/to a file:

  ISDINP < 0 : use ranlux with date and time as seed;

  ISDINP $= 0$: use ranlux with fixed seed;

  ISDINP $= 1$: read seed from file;

  ISDOUT $= 0/1$: (no) output of seeds to file;

  Default: ISDINP = ISDOUT = 0.

- 'TEST-OPT'
  data: IPRINT
  to control the amount of output: a larger value for IPRINT creates more output. For testing.

- 'START'
  data: NEVENT
  starts the execution of the main program;

  NEVENT : number of requested events if any sampling option is activated (D = 0).

9.2 DJANGO part

In the following we describe input options which determine the treatment of QCD effects in the DJANGO, resp. LEPTO part of the program. Only a few selected flags are allowed as input items. Other flags can be changed in the block data subroutine LEPTOD. Exceptions are the flags `LST(1)`, `LST(2)`, `LST(6)`, `LST(17)`, and `LST(18)` which must not be changed in order that DJANGOH works correctly.

- `'OUT-LEP'`
  data: `LST(4)` (integer, D=1)
  regulates which information is written onto the event record:

  `LST(4)` $=$ $\mathrm{I}_{lepton} + 10\times\mathrm{I}_{shower}$;

  $\mathrm{I}_{lepton} = 0/1$ inactive/active scattered lepton;

  $\mathrm{I}_{shower} = 0/1$ exclude/include intermediate partons.

- `'FRAME'`
  data: `LST(5)` (integer, D=3)
  choice of frame for the event:

  `LST(5)` = 1: hadronic cms frame, z-axis along boson,

  = 2: *ep* cms frame, z-axis along lepton,

  = 3: *ep* lab system, z-axis along lepton,

  = 4: *ep* lab system, z-axis along exchanged boson.

- `'FRAG'`
  data: `LST(7)` (integer, D=1)

  `LST(7)` = -1: no generation of parton cascades and no fragmentation,

  = 0: event generation at the parton level, no hadronization,

  = 1: event generation including hadronization and decays of unstable particles.

- `'CASCADES'`
  data: `LST(8)` (integer, D=12)
  describes that part of the event simulation which is determined by perturbative QCD:

  `LST(8)` = 0: no QCD effects, i.e. no gluon radiation or boson-gluon fusion,

  = 1: including QCD processes (gluon radiation and boson-gluon fusion) according to the first-order matrix elements,

  = 2: QCD parton cascade evolution of initial and final quark,

= 3: QCD parton cascade evolution of initial quark only,

= 4: QCD parton cascade evolution of final quark only,

= 5: QCD switched off, but target treatment exactly as in parton cascade case,

= 9: simulating QCD cascades in the colour dipole model as implemented in ARIADNE,

= 12 – 15: as 2 – 5, but parton shower added to the event as obtained from first-order matrix elements (ME+PS).

- `'MAX-VIRT'`
  data: `LST(9)` (integer, D=5)
  maximal virtuality in parton cascades (active only for `LST(8)` = 2 – 5):

  `LST(9)` = 1: Q^2,

  = 2: W^2,

  = 3: $W \times Q$,

  = 4: $Q^2 \times (1 - x)$,

  = 5: $Q^2 \times (1 - x) \times max(1, \ln(1/x))$,

  = 6: $x_0 W^2$,

  = 9: $W^{4/3}$, i.e. similar as in the colour dipole model.

- `'BARYON'`
  data: `LST(14)` (integer, D=4)
  treatment of the target remnant (for more detailed information, see [2]):

  `LST(14)` = 0: baryon production from remnant excluded,

  = 1: baryon production from remnant included,

  = 2, 3: as 1 but with different energy-momentum fractions distributed over remnant parts;

  = 4: using LUZDIS [43, 44].

- `'KT-PARTON'`
  data: `PARL(3)` (real, D=0.44)
  width of Gaussian primordial transverse momentum distribution (in GeV).

- `'DIQUARK'`
  data: `PARL(4)` (real, D=0.75)
  probability that a ud-diquark in target remnant has spin and isospin equal to zero, i.e. I=S=0.

- `'KT-REMNANT'`
  data: `PARL(14)` (real, D=0.35)
  width of Gaussian p_T when non-trivial target remnant is split into a particle and a jet (in GeV).

- `'AR-REMNANT'`
  data: `MSTA(30)` (integer, D=1)
  extendedness of initial partons in ARIADNE (see [70]);

  `MSTA(30)` = 0: struck quark is pointlike and proton remnant extended with $\mu =$ `PARA(11)` (= 1 GeV by default), where μ is the inverse extension of the proton remnant,

  = 1: as for 0 but $\mu =$ `PARA(11)`$/(1-x)$,

  = 2: as for 1 but also struck quark is extended with $\mu = \sqrt{Q^2}$.

- `'WREM-MIN'`
  data: `PYPAR(12)` (real, D=2.0)
  parameter to determine the minimum mass required for the fragmentation of the target remnant. Used together with `PARL(3)` to decide whether the event can be hadronized with LEPTO.

  An event has the chance to be hadronized in LEPTO if the invariant mass available for the remnant fulfils the condition

  $$W_R \geq m_p + m_q + M_0 + \sqrt{(k_T^{\min})^2 + m_q^2}$$

  where m_p is the proton mass, m_q the mass of the lightest quark, $k_T^{\min}$ the minimum transverse momentum of the quark (input from `'KT-PARTON'`) and M_0 equal to the input for `'WREM-MIN'`, i.e. `PYPAR(12)`. The default value of `PYPAR(12)` is 2.0 GeV, but a lower value may still lead to realistic events.

  Note that events with small W should be generated by SOPHIA, choosing an appropriate value for the input item `WSOPHIA`. If `WSOPHIA` is chosen too small, i.e. below the limit for W_R shown above, there will be a range of W values for which events are not hadronized. See next input item.

- `'SOPHIA'`
  data: `WSOPHIA` (real, D=1.5)
  defines the upper limit for the hadronic mass, below which SOPHIA [40] is used to generate the hadronic final state. Note that SOPHIA works only for neutral-current scattering.

- `'CONTINUE'`
  continue execution of the main program, start event generation.

10 Output

10.1 Standard output

All quantities transferred to the program via the input options are echoed immediately after reading the input. A consistency check is performed and the actual values of all important parameters is printed before the start of the integration and sampling procedure. The results of the integration step are:

(i) Non-radiative contribution:
The resulting estimate of the integrated non-radiative cross section and its error is printed. Integration is done with an adaptive Gauss algorithm. The requested relative accuracy is fixed at $\Delta I/I = 10^{-4}$.

(ii) Bremsstrahlung contributions:
The program gives the standard output generated by the VEGAS routines, including

- number of function evaluations per iteration;
- integral and error estimates for the actual iteration;
- accumulated integral and error estimates taking into account results from previous iterations (depending on the entry chosen for the VEGAS routine, compare input option 'INT-OPT-NC').

After event sampling, the actually applied cross sections and the numbers of generated events are given for each individual contribution. For each run, a header record and a terminating record is written to the common block /HEPEVT/ according to the standards proposed in [71, 72]. The header record contains in addition to the standard quantities parameter definitions and option flags, the final record includes some partial results. Details are given in appendix 13.4.

10.2 Event record and user's analysis

Any user action is expected in the user supplied subroutine `HSUSER(ICALL,X,Y,Q2)`. The routine allows for user action in three different phases of running DJANGOH:

- before generating events (`ICALL=1`),
- after each generated event (`ICALL=2`),
- after completion of event generation (`ICALL=3`).

The arguments `X`, `Y`, `Q2` correspond to the lepton kinematic variables of the actual event.

For each sampled event the information about the $eq \to eq(\gamma)$ subprocesses is transferred to `HSUSER` via the HERACLES common blocks `/HEPEVT/, /HSIKP/` and `/HSCHNN/`. The information about the hadronic final state is contained in the common block `/LUJETS/` of JETSET 7.4 [43,44]. In case of an event with hard QED bremsstrahlung, the information about the radiated photon is found in `/LUJETS/` right after the scattered lepton, i.e. in line 5. Additional information is contained in the common blocks `/DJPASS/` and `/LEPTOU/` (of LEPTO). Below we describe those common blocks which can be useful for the analysis of the event. For details about the content of the common block `/LUJETS/` which contains the event record we refer the reader to the manual of JETSET 7.4 [43,44].

The event record is also stored on the common block `LUJETS` when the hadronization step failed (or was not attempted). In this case the event record contains, besides the scattered electron and a potential bremsstrahlung photon, the scattered quark and the proton remnant.

11 Usage

No special installation procedure is required. The files with the fortran source code have to be copied into a directory where the compilation can be done. The CERN libraries `mathlib` and `kernlib` are needed for the random number generator and the timer routine. Also the library `LHAPDF`, version 6, for parton distribution functions has to be made visible for the linker. The calculation of polarization-dependent structure functions and parton distribution functions requires grid files; they have to be stored in a sub-directory `polpdf-gridfiles`.

The code was tested with GNU Fortran 6.3.0 on a Debian system. It is recommeded to use the compiler options `-fbacktrace` and `-ffpe-trap=invalid,zero` to be able to identify (unlikely, but still possible) numerical problems.

The initialization steps for integration and estimation of maxima takes few minutes, roughly 10 seconds per channel and per 10^6 integration points. Typical times for event generation are 3×10^{-4} seconds per event (on a i7-3770 CPU at 3.4 GHz).

Part C: Appendices

12 Files in the package

12.1 File django_h.f

The main routines, originating from HERACLES, Refs. [1,73]. Contains routines for input and output, steering, initialization and integration of the cross sections. Many routines and functions for the calculation of bremsstrahlung and loop corrections.

12.2 File djangoh_l.f

The code needed for the generation of hadronic final states with sufficiently large hadronic mass. Routines were taken from LEPTO, version 6.5.1 [2]. Many are modified to work together with the HERACLES part of the program. Details have been first described in Refs. [3, 4].

12.3 File djangoh_u.f

This file contains a sample of the user routine `HSUSER` called for each generated event. The user should adjust the code for the specific application.

12.4 File djangoh_t.f

A sample routine, `tstrrr` which can be called after initialization, but before event generation, to perform tests, for example to print internal parameters or a table of parton distribution functions. A corresponding line in `djangoh_h.f` has to be un-commented to activate the call to `tstrrr`.

12.5 File djangoh_p.f

A collection of routines for the calculation of polarized parton distribution functions, adjusted for usage in DJANGOH. This part is based on the thesis of Till Martini [35] and was first used in Ref. [74]. In previous versions, this file was called `polpdf.f`. Some of the programs were obtained from `http://hepdata.cedar.ac.uk/pdfs`.

Most of the parametrizations of polarized PDFs require grid files on which an interpolation is done. The grid files are contained in the package `polpdf-gridfiles.tgz` which should be unpacked into a subdirectory named `polpdf-gridfiles`.

12.6 File `sophia-dj.f`

Source code of SOPHIA, version 1.5, Ref. [40], including one additional routine to transfer DJANGOH variables to SOPHIA. Responsible for the generation of hadronic final states in the low-W region.

12.7 File `pythia-6.4.24-dj.f`

The source code of PYTHIA version 6.4, Ref. [46, 75], including a modification to fix Λ_{QCD} for some modern PDF parametrizations, but otherwise unmodified.

12.8 File `jetset7409.f`

The source code of JETSET version 7.4, unmodified, Refs. [43, 44].

12.9 File `gmc_random.f`

Random number generator `ranlux` from Cernlib. The function `PYR` has been replaced by an interface to `ranlux`.

12.10 Files `Makefile-sample` and `config.mk`

A sample makefile for a Linux system. If the LHAPDF library is not in the standard path, provide its directory name through the environment variable `LHAPDF`.

12.11 File `testrun-sample.in`

A sample input file.

13 Useful variables, common blocks

13.1 Common block LEPTOU

```
COMMON /LEPTOU/ CUT(14),LST(40),PARL(30),X,Y,W2,Q2,U
```

Content: contains input switches (`LST(1)` – `LST(20)`) and input parameters (`PARL(1)` – `PARL(20)`) for LEPTO to specify physics, kinematic cuts and numerical procedures, as well as output flags (`LST(21)` – `LST(40)`) and output variables (`PARL(21)` – `PARL(30)`). Many of the input flags and parameters are initialized in HERACLES. Additional ones are determined either through explicit input of DJANGOH or by default values in `BLOCK DATA LEPTOD`. Some switches of LEPTO are not optional anymore in DJANGOH, but have to be kept fixed at values given in the list below.

Parameters:

`CUT(1)` – `CUT(14)`: lower and upper limits on x_l, y_l, Q_l^2, W_h^2, ν, E', θ'; included for documentation purposes only (passed from HERACLES).

`LST(1)` : (D=2) not optional: independent variables are x and y;
`LST(2)` : (D=3) not optional: scaled variables $(\tilde{x}, \tilde{y})$ passed via `LEPTOU`;
`LST(3)` : (D=2) only warnings printed, execution stopped on error;
`LST(4)` : (D=1) user input via code-word `'OUT-LEP'`, cf. section 9.2;
`LST(5)` : (D=3) user input via code-word `'FRAME'`, cf. section 9.2;
`LST(6)` : (D=0) not optional: no rotation of the azimuthal angle, ϕ, of the lepton scattering plane; random azimuthal orientation passed from HERACLES;
`LST(7)` : (D=1) user input via code-word `'FRAG'`, cf. section 9.2;
=0: parton level events generated, optionally including QCD effects according to `LST(8)`;
=1: full event generated, i.e. including hadronization and decays;
`LST(8)` : (D=12) user input via code-word `'CASCADES'`, cf. section 9.2;
`LST(9)` : (D=5) user input via code-word `'MAX-VIRT'`, cf. section 9.2;
`LST(10)` : (D=1) not used;
`LST(11)` : (D=0) user input via code-word `'FLONG'`, cf. section 9.1;
`LST(12)` : (D=6) `NPYMAX`, maximum flavor used in the sea structure function parametrizations, passed from HERACLES;
`LST(13)` : (D=5) heaviest flavour produced in boson-gluon fusion;
`LST(14)` : (D=4) user input via code-word `'BARYON'`, cf. section 9.2;
`LST(15)` : (D=9) choice of the parametrization for parton densities in the nucleon, `NPYMOD`, passed from HERACLES, cf. code-word `STRUCTFUNC`, section 9.1;
`LST(16)` : (D=1) choice of the structure function library, passed from HERACLES, cf. code-word `STRUCTFUNC`, section 9.2;
`LST(17)` : (D=1) not optional: varying energies of initial particles from event to event;
`LST(18)` : (D=0) not optional: running electromagnetic coupling α and boson masses passed from HERACLES;
`LST(19)` : (D=−10) choice of grid for first-order QCD weights;
`LST(20)` : (D=5) scheme for cut-offs against divergences in the QCD matrix elements;
`LST(21)` : error flag, =0 for properly generated event;
`LST(22)` : (D=1) information about chosen target nucleon in current event (proton);
`LST(23)` : specifies process simulated, passed from HERACLES;

=2: weak charged current (CC), i.e. $W^{\pm}$ exchange;

=4: neutral current (NC), i.e. γ/Z^0 exchange;

`LST(24)` : information about first-order QCD process in current event;

=1: q-event, i.e. no first-order QCD;

=2: qg-event, i.e. gluon radiation in first-order QCD;

=3: $q\bar{q}$-event, i.e. boson-gluon fusion in first-order QCD;

`LST(25)` : information about flavor of the struck quark in current event: 1=d, 2=u, 3=s, 4=c, 5=b, –1=$\bar{d}$, –2=$\bar{u}$, –3=$\bar{s}$, –4=$\bar{c}$, –5=$\bar{b}$;

`LST(26)` : entry line in event record of outgoing struck quark. In parton shower case, quark at boson vertex before final state shower;

`LST(27)` : signals split of non-trivial nucleon remnant, cf. `LST(14)`;

=0: no split, simple diquark or `LST(14)` = 0;

=1: split into parton and particle, $qq+M$ or $q+B$, occurs when sea (anti)quark removed through the interaction;

=2: split into quark and diquark, $q+qq$, occurs when a gluon is removed;

`LST(28)` : specifies the frame in which the current event is given with code as for `LST(5)`;

`LST(29)` : specifies azimuthal angle rotation with code as for `LST(6)`;

`LST(30)` : chosen helicity for beam lepton in current event;

`LST(31)` : not optional, fixed by `LST(1)`;

`LST(32)` : for internal use;

`LST(33)` : for internal use;

`LST(34)` : (D=1) used for soft color interaction in LEPTO 6.5.1;

`LST(35)` : (D=1) used for new sea quark treatment in LEPTO 6.5.1;

`LST(36)` – `LST(40)` : (D=0) not used.

`PARL(1)` : (D=1.) for internal use;

`PARL(2)` : (D=1.) for internal use;

`PARL(3)` : (D=0.44) user input via code-word `'KT-PARTON'`, cf. section 9.2;

`PARL(4)` : (D=0.75) user input via code-word `'DIQUARK'`, cf. section 9.2;

`PARL(5)` : $\sin^2\theta_W$ (weak mixing angle), passed from HERACLES (`SW2`);

`PARL(6)` : polarization of lepton beam, passed from HERACLES (`POLARI`);

`PARL(7)` : (D=0.5) used for soft color interaction in LEPTO 6.5.1;

`PARL(8)`, `PARL(9)` : (D=0.04, 4.) cut-offs against divergences in QCD matrix elements, see the remarks in the LEPTO manual [2];

`PARL(10)` : not used;

`PARL(11)` : (D=0.01) required relative accuracy for one-dimensional integration, used for first-order QCD matrix element weights and longitudinal structure function integrals;

`PARL(12)` : (D=0.01) probability for intrinsic charm;

`PARL(13)` : (D=0.1) internal parameters used for adjustment of y_{cut} for integration of QCD matrix elements;

`PARL(14)` : (D=0.35) user input via code-word `'KT-REMNANT'`, see section 9.2;

`PARL(15)` : (D=0.01) not used;

PARL(16) : finestructure constant α, passed from HERACLES (ALPHA);
PARL(17) : Fermi constant G_F, passed from HERACLES (GF);
PARL(18) : Δr for radiative corrections, passed from HERACLES (DELTAR);
PARL(19) : (D=0.03) scale κ^2 in GeV^2 for higher twist correction;
PARL(20) : (D=0.1) used for the treatment of a more complicated nucleon remnant than a simple diquark;
PARL(21) : scaled $\tilde{s}$;
PARL(22) : scaled $\tilde{y}\tilde{s}$ ($= y_h s_h$);
PARL(23) : total cross section in nb, passed from HERACLES (= SIGTOT);
PARL(24) : Monte Carlo estimate of the cross section in nb associated with the generated event sample. May be reduced as compared to the value of PARL(23) when hadronization of some of the events failed;
PARL(25) : value of α_s in current event;
PARL(26) : value of Λ used in last structure function call, passed from HERACLES (filled only when LST(16) = 1);
PARL(27) : depending on the value of LST(20), present value of the cut variable for first-order QCD;
PARL(28) – PARL(30) : values of x_p, z_q, ϕ in first order massless QCD matrix elements, see LEPTO manual;

X, Y, W2, Q2, U : **rescaled** variables $\tilde{x}$, $\tilde{y}$, $\tilde{W}^2$, $\tilde{Q}^2$, $\tilde{\nu}$, see section 8. Except for $\tilde{y}$, the rescaled variables are identical to the hadron variables.

13.2 Common blocks DJPASS and SPPASS

```
COMMON /DJPASS/ NTOT,NPASS,NQELAS,NFAILL,NFAILQ
```

Content: contains statistics of events treated in HERACLES and LEPTO:
Parameters:
NTOT : Total number of events generated by HERACLES;
NPASS : Number of events which have been hadronized successfully in LEPTO;
NQELAS : Number of events for (quasi-)elastic scattering, not treated in LEPTO;
NFAILL : Number of events which failed hadronization in LEPTO. These events usually have a hadronic mass above the cut for hadronization in SOPHIA and were identified as candidates for hadronization in LEPTO, but failed there. Their event record is given back at the parton level.
Note that hadronization can fail in LEPTO more often for radiative than for non-radiative events. Photon radiation can lead to a substantial reduction of W_h.
In order to increase the efficiency of the program, there is a check on the value of W_h already in the HERACLES part of the program, before a call to LEPTO. This is done using mass cut-offs of LEPTO in the subroutine

HSWCUT. The effective cut-off depends on the flavor of the struck quark and is controlled by the values of the parameters PYPAR(12) and PARL(3). The number of events recognized by HERACLES to have too small a hadronic mass is used to correct the total cross section accordingly. The corrected cross section is written to PARL(24) in the common block /LEPTOU/.

NFAILQ : Number of events not accepted for fragmentation in LEPTO. NFAILQ can be non-zero if there are events in the channel for photon radiation from quarks and for elastic scattering.

```
COMMON /SPPASS/ NSOPH,NSPOUT,NFAILP,NSPACC
```

Content: contains statistics of events treated in SOPHIA:

Parameters:

NSOPH : Total number of events with successful hadronization in SOPHIA;

NSPOUT : *not used*;

NFAILP : Number of events where hadronization in SOPHIA failed;

NSPACC : Flag to identify events treated by SOPHIA: equal to 1 if SOPHIA was successful, equal to 99 if hadronization in SOPHIA failed.

13.3 Common block HSCHNN

```
COMMON /HSCHNN/ ICHNN
```

Content: contains information on the origin of the event from the channels of HERACLES

Parameters:

ICHNN : flag of channel, passed from HERACLES:

- =1: non-radiative neutral current (NC) event,
- =2: non-radiative charged current (CC) event,
- =3: non-radiative elastic scattering event (EL),
- =6: leptonic initial state photon radiation (NC),
- =7: leptonic final state photon radiation (NC),
- =8: Compton event (NC),
- =9: quarkonic radiation (NC),
- =12: radiative charged current event (CC),
- =13: initial-state quarkonic radiation (CC, not active),
- =14: final-state quarkonic radiation (CC, no hadronization),
- =15: quasi-elastic event with initial state radiation,
- =16: quasi-elastic event with final state radiation,
- =17: quasi-elastc event (Compton type).

13.4 Common block HEPEVT: header and final record

Before event generation, a header record is written to the common block `/HEPEVT/` with `NEVHEP` $= -1$. The content of its first entry (`IHEP = 1`) is defined according to the standard [71, 72]. The other entries have the following meaning:

NHEP:	= 73, number of entries in the header record;
PHEP(1,2):	= EELE, energy of the initial electron in GeV;
PHEP(1,3):	= POLARI, polarization of the initial electron;
PHEP(1,4):	= EPRO, energy of the initial proton in GeV;
PHEP(1,5):	= XMIN, minimum of the leptonic x;
PHEP(1,6):	= XMAX, maximum of the leptonic x;
PHEP(1,7):	= YMIN, minimum of the leptonic y;
PHEP(1,8):	= YMAX, maximum of the leptonic y;
PHEP(1,9):	= Q2MIN, minimum of the leptonic momentum transfer Q^2 in GeV^2;
PHEP(1,10):	= WMIN, minimum of the invariant mass of the hadronic final state in GeV;
PHEP(1,11):	= EGMIN, minimum of the photon energy in GeV;
PHEP(1,12):	= MW, W boson mass in GeV;
PHEP(1,13):	= MZ, Z boson mass in GeV;
PHEP(1,14):	= MH, Higgs boson mass in GeV;
PHEP(1,15):	= MT, top-quark mass in GeV;

ISTHEP(2):	= LLEPT, charge of the initial lepton;
ISTHEP(3):	= ICUT, flag for kinematical cuts;
ISTHEP(4:15):	= LPARIN(1:12), flags for the definition of electroweak parameters and partial electroweak corrections;
ISTHEP(16):	= ISTRFC, flag for the parametrization of quark distribution functions or structure functions;
ISTHEP(17):	= NPYMIN, minimal number of flavors;
ISTHEP(18):	= NPYMAX, maximal number of flavors;
ISTHEP(19):	= LUNIN, logical unit number for parameter input;
ISTHEP(20):	= LUNOUT, logical unit number for standard output;
ISTHEP(21):	= LUNTES, logical unit number for test output;
ISTHEP(22):	= LUNRND, logical unit number for in- and output of the random number generator;
ISTHEP(23):	= LUNDAT, logical unit number for in- and output of results of the integration step;
ISTHEP(24):	= NINP, logical unit number for VEGAS input;
ISTHEP(25):	= NOUTP, logical unit number for VEGAS output;
ISTHEP(26:30):	= flags for integration of non-radiative partial cross sections (internally denoted by INT2(1:5);
ISTHEP(26):	= INC2 from input code word 'INT-OPT-NC';

ISTHEP(27): = ICC2 from input code word 'INT-OPT-CC';
ISTHEP(28:30): = not used;
ISTHEP(31:45): = flags for integration of radiative partial cross sections (internally denoted by INT3(1:15);
ISTHEP(31): = INC31 from input code word 'INT-OPT-NC';
ISTHEP(32): = INC32 from input code word 'INT-OPT-NC';
ISTHEP(33): = INC33 from input code word 'INT-OPT-NC';
ISTHEP(34:36): = not used
ISTHEP(37): = ICC31 from input code word 'INT-OPT-CC';
ISTHEP(38): = ICC32 from input code word 'INT-OPT-CC';
ISTHEP(39): = ICC33 from input code word 'INT-OPT-CC';
ISTHEP(40:45): = not used;
ISTHEP(46): = NPOVEG, maximal number of points used in the VEGAS integration;
ISTHEP(47): = NUMINT, not active in version 4.6.4;
ISTHEP(48): = NPHYP, not active in version 4.6.4;
ISTHEP(49:53): = flags for the inclusion of non-radiative partial cross sections in event generation (internally denoted by ISAM2(1:5));
ISTHEP(49): = ISNC2 from input code word 'SAM-OPT-NC';
ISTHEP(50): = ISCC2 from input code word 'SAM-OPT-CC';
ISTHEP(51:53): = not used;
ISTHEP(54:68): = flags for the inclusion of radiative partial cross sections in event generation (internally denoted by ISAM3(1:15));
ISTHEP(54): = ISNC31 from input code word 'SAM-OPT-NC';
ISTHEP(55): = ISNC32 from input code word 'SAM-OPT-NC';
ISTHEP(56): = ISNC33 from input code word 'SAM-OPT-NC';
ISTHEP(57:59): = not used;
ISTHEP(60): = ISCC31 from input code word 'SAM-OPT-CC';
ISTHEP(61): = ISCC32 from input code word 'SAM-OPT-CC';
ISTHEP(62): = ISCC33 from input code word 'SAM-OPT-CC';
ISTHEP(63:68): = not used;
ISTHEP(69): = INTOPT, flag for integration in- or excluding preparation of the sampling step;
ISTHEP(70): = IPRINT, flag for test ouput;
ISTHEP(71): = ISDINP, flag for input of random number seeds;
ISTHEP(72): = ISDOUT, flag for output of random number seeds;
ISTHEP(73): = NEVENT, requested number of events;

The following list shows the content of the final record with NEVHEP $= -2$. This record contains output of the program.

NHEP: = 44, number of entries in the final record;
PHEP(1,1): = SIGTOT, total cross section;
PHEP(1,2): = SIGTRR, estimated error of the total cross section;

PHEP(1,3:22): = SIGG(1:20), partial cross sections;
PHEP(1,23:42): = SIGGRR(1:20), estimated errors of partial cross sections;
PHEP(1,43): = SW2, weak mixing angle s_W^2;
PHEP(1,44): = MW, W boson mass; may be changed from its input value;

ISTHEP(1): = NEVENT, total number of generated events;
ISTHEP(2:22): = NEVE(1:21), partial event numbers for each channel.

14 Definition of kinematical cuts

The basic constraint on the momentum transfer $Q^2 \geq Q^2_{min}$ is superimposing all further kinematical constraints. Q^2_{min} can be defined by the user via the option 'KINEM-CUTS'. In addition, limits on x and y and a minimal hadronic mass W can be given. In addition, several definitions of the kinematical region may be requested by the user via the input parameters of the option 'KINEM-CUTS'. In the following we list in detail the kinematical cuts as defined in the program for non-radiative scattering ($S = 4E_eE_p$).

- ICUT=1 : x-limits and Q^2_{min} from input accepted, y-limits and W_{min} ignored;

$$Q^2_{min} \leq Q^2 \leq Q^2_{max} = x^{input}_{max} \cdot S,$$
$$x_{min} = \mathrm{Max}\{x^{input}_{min}, Q^2_{min}/S\} \leq x \leq x^{input}_{max}.$$

- ICUT=2 : As ICUT=1 with an additional lower cut on the mass of the final state hadrons,

$$W^2_{min} \leq W^2 = (1-x)yS + M^2_P.$$

  W_{min} will be set automatically to M_P if the input is smaller. The additional restriction of W^2 translates into a modification of the lowest allowed Q^2 value in the actual calculation:

$$Q^2_{min} = \mathrm{Max}\left\{(Q^2_{min})^{input}, \frac{x}{1-x}(W^2_{min} - M^2_P)\right\},$$

  and

$$x_{max} = \mathrm{Min}\{x^{input}_{max}, 1 - (W^2_{min} - M^2_P)/S\}.$$

- ICUT=3 : All given limits ($x_{min,max}$, $y_{min,max}$, Q^2_{min}, and W_{min}) are accepted. The actually applied limits are calculated from the most restrictive input conditions resulting in:

$$Q^2_{min} = \mathrm{Max}\left\{x_{min}y_{min}S, (Q^2_{min})^{input}, \frac{x_{min}}{1-x_{min}}(W^2_{min} - M^2_P)\right\} \leq Q^2,$$

$$x_{min} = \mathrm{Max}\{x_{min}^{input}, Q_{min}^2/y_{max}S\},$$

$$x_{max} = \mathrm{Min}\{x_{max}^{input}, 1 - (W_{min}^2 - M_P^2)/y_{max}S\},$$

$$y_{min} = \mathrm{Max}\left\{y_{min}^{input}, Q_{min}^2/x_{max}S, \frac{W_{min}^2 - M_P^2}{(1 - x_{min})S}\right\}.$$

For radiative events, analogous constraints are respected depending on the value of ICUT. In this case the limits on the hadronic final state mass does not change the input values for the cuts on x and y or Q^2, but restricts the phase space of the bremsstrahlung photon.

15 Update history

Some of the important improvements and bug fixes of previous versions are summarized in this appendix. The list is not complete. The information provided here might be helpful for those users who still use an older version and who wish to know whether an upgrade is required for their specific application. Some of the modifications in older versions may have become obsolete by more recent changes.

15.1 Update for version 4.6.22

- Corrected code to correctly include target polarization also when corrections from the longitudinal structure function are to be included with `IFLOPT` $\neq 0$.
  Note that the corresponding QCD correction to the polarized structure functions, i.e. G_L, the analogue of F_L, is not included.

- Modified declarations of local variables with array index in `gmc_random.f`.

- Corrected a type mismatch in routine `HSWPDF` of file `djangoh_h.f`.

- Duplicated subroutine `DFINT` in `djangoh_p.f` with different array sizes to be used in different parametrizations of polarized PDFs.

- Modified routine `rdarry` in `djangoh_p.f` to avoid a runtime error when the upper limit of Q^2 is reached during interpolation of the GSLO polarized PDFs grid.

15.2 Update for version 4.6.20 and 4.6.21

- Added a correction to properly initialize the calculation of the longitudinal structure function during event simulation.
- Secured reconstruction of event kinematics for the rare cases when transverse momenta are zero (in `HSFLAB` and in `HSACPT`).

15.3 Updates for version 4.6.19

- Activated the option to calculate the charge-odd part of the cross section (integration only, no event generation) with `ICPASY=2`.
- Allowed to calculate the complete set of electroweak corrections for scattering off a nuclear target, other than the proton, in the integration-only mode. In the previous versions, some small corrections were included only partly.

15.4 Updates for version 4.6.18

- Introduced a test version to calculate the charge-odd part of the cross section (integration only, no event generation) with `ICPASY=2`. This option is still preliminary.
- Removed a bug which caused the program to produce wrong $O(\alpha)$ corrected cross sections if lepton polarization was non-zero and `LPARIN(11)=0`.
- Added some extra comments on the usage of the option to include F_L (see input for `'FLONG'`) and on the meaning of input for `'WREM-MIN'` to this manual.
- Added the file `cernlibs.f` to the package, containing those routines of the CERN library which are needed by DJANGOH. A reference to an external cernlib is not needed anymore.
- Modified `gmc_random.f` to avoid a compiler warning.
- Modified the makefile sample, removed reference to explicit directory names.
- Thanks to Andrea Bressan for suggestions.

15.5 Updates for version 4.6.17

- Introduced the option `ICPASY` to calculate the polarization-dependent part of the cross section (integration only, no event generation).

- Corrected error flag for Sophia events: LST(21) was not reset if event generation failed in a previous call to Lepto.

- Corrected counting of failed Sophia events for error flag NSPACC=99.

- Changed units for the input for the minimum lepton scattering angle: a value in degrees is expected now.

- Renamed files djangoh_p.f (was polpdf.f) and sophia_dj.f (was sophia.f).

15.6 Updates for version 4.6.16

- Linking to LHAPDF6 [48] is straightforward. A corrected copy of lhaglue.f, which contained a bug in version 5 of lhapdf, is not needed anymore.

  Input for the code word LHAPATH can be used to set the path name for the PDF data. However, this is not needed if the path contains a link to the directory where the command lhapdf-config is installed.

  If a PDF set has more than one member, the last two digits of the input variable ICODE (see code word STRUCTFUNC) can be used to specify the member.

- Some modern PDF sets do not provide a value for $\Lambda_{\rm QCD}$ anymore since the definition of this parameter is ambiguous. Instead, the strong coupling constant is fixed at an initial value, $\alpha_s(Q_0^2)$. However, the initialization of pythia6 requires a given value for $\Lambda_{\rm QCD}$. As a work-around, the routine PYINIT is modified to fix $\Lambda_{\rm QCD}^{N_f=4} = 0.326$ GeV if no value can be found in the info-file of the chosen PDF set.

  DJANGOH was written to describe lepton nucleon scattering including QED and electroweak higher-order corrections. It does not include higher-order QCD corrections. The exception is the longitudinal structure function. There is the option to calculate F_L from perturbative QCD. The strong coupling and a value for $\Lambda_{\rm QCD}$ is needed only here and for the simulation of the hadronic final state, in particular for parton showers.

- Extensive code cleaning to avoid too many warnings. Re-ordered some common blocks to avoid padding or the need to use the compiler option -fno-align-commons. Resolved conflicting size of arguments for calls to subroutines. Version 4.6.16 complies with GNU Fortran (GCC) 8.1.0.

- Code cleaning in polpdf.f. Resolved conflicting names for common blocks of different size.

- Use NextUn() everywhere to select I/O units. Some parts of third-party code for the parametrization of polarized PDFs was modified to avoid I/O units in conflict with the rest of the program.

- Removed all references to the CERN `Hbook` library from the sample user routine in `djangoh_u.f`.

- Fixed a problem with reading input from file: empty lines are allowed in the new version. If no input for the Lepto part is given, the program will generate events without fragmentation and hadronization.

- Added a safety check of kinematic limits for initial-state radiation in `HSLZK1`.

- Changed the units for the printout of the final total cross section stored in `PARL(23)` and `PARL(24)` to nano-barns, consistent with other printed cross section values. The corrected cross section takes into account that the phase space is restricted by a minimum requirement for the invariant mass of the hadronic remnant which is not described by the input limits for kinematic variables; it does not take into account that events may fail hadronization due to problems in LEPTO or SOPHIA.

- Introduced new input item for the LEPTO part, `'WREM-MIN'`, to allow the user access to the PYTHIA parameter `PYPAR(12)` and more flexibility to control hadronization of low-W events.

  Properties of the hadronic final state in the small W range have not been investigated so far.

- Fixed event record for scattering with nuclei other than a proton: contains now either a proton or a neutron which are chosen according to the relative cross sections.

- Fixed event record for (quasi-)elastic scattering.

- There is no public release of version 4.6.15.

- Thanks to Xiaochao Zheng for many suggestions and for testing.

15.7 Updates for version 4.6.14

- Corrected `sophia.f` to treat muon scattering (flavor flag and calculation of momenta).

- Calculate efficiencies only if event number is non-zero.

- Introduced safety margin for proton energy (now 10^{-5} above proton mass at minimum).

15.8 Updates for version 4.6.13

- New option for muon scattering. Input flag LLEPT can be ± 3 for $\mu^{\pm}$. Variables MEI, MEF are used for the mass of incoming and scattered lepton.
- Removed unused routine DJGNOE and common block LEPTOU from sophia.f.

15.9 Updates for version 4.6.12

- Updated version number and date in djangoh_l.f.
- Set momentum of initial state always for a proton, also for scattering of a nucleus (target nucleon is proton or neutron and chosen event by event).

15.10 Updates for version 4.6.11

- Updated values for some electroweak parameters (M_Z, M_W, M_H, $m_{\rm top}$).
- Allow electron beam energy below 1 GeV.
- Allow proton beam energy also for fixed-target kinematics, i.e. minimal value is $E_p = m_p$. Default is set to 920 GeV.
- Default value for the minimum of x is now 10^{-5}.
- For elastic scattering (P2@MESA), added new option for maximal lepton scattering angle:
  Code word THETA-CUT expects input THEMIN, THEMAX.
  Changed code in HSELK1, HSELK2, HSELCO.
  THEMAX is not used for inelastic scattering.
- Added extra consistency check for allowed phase space for initial-state radiation to avoid unphysical events and numerical problems.
- In file polpdf.f: renamed LSPLINE and ISERCH to LSPLINE1 and ISERCH1 to avoid name conflict with external libraries.
- In file djangoh_l.f: set mass of target according to input for HNA, HNZ and set momentum of the initial state using the nucleus mass. Event generation is always for either proton or nucleon, as determined by the relative cross sections.

15.11 Updates for version 4.6.8 - 4.6.10

- Parton distribution functions from LHAPDF, version 5, required a modification in `lhaglue.f`. The modified version, `lhaglue-copy.f` became part of the DJANGOH source code.
  - Removed reference to common block `/ludat1/` because of type mismatch with JETSET 7.4; removed the corresponding assignment to unused `mstu5(11)`.
  - To allow use of nuclear corrections from [67–69] via calls to `structm` from PYTHIA, `structm` in `lhaglue-copy.f` has been renamed to `structf` and `structa` into `structm`. The argument list of `structa` is shortened: the nuclear number `a` is transfered via the common block `/HSNUCL/` from DJANGOH.
- To allow linkage to the LHAPDF library [47], version 5.8.6, initialization of the path to the PDFsets folder has been introduced in subroutine `HSWPDF` and the path name has to be provided in the input file with code word `LHAPATH`:

  `LHAPATHI`: Path name to the PDFsets folder of LHAPDF, character*80, no default defined.
- The new code word `NUCL-MOD` with input `INUMOD` (integer, default = 0) has been introduced to select a model for nuclear shadowing or nuclear parton distribution functions.
- Since version 4.6.10, the proton can be polarized [35]. The prescription for the calculation of the cross section is based on the parton model and distribution functions for polarized partons are used. New input options for `HPOLAR` and `POLPDF` are introduced. The new subroutine `HSDPVR` provides an interface to the necessary code which is collected in a separate file. The routine (`HSCPVR`) is called after integration and after sampling to print out the number of counts for which x and Q^2 were outside the range of validity of the parametrisation for polarized PDFs.
- Introduced the code word `OUTFILENAM` to give a user-defined base name for output files.
- If a negative value for input variable `ISDINP` (with code word `RNDM-SEEDS`) is given, the Cernlib random number generator `ranlux` is used with date and time as seed.
- A number of modifications was introduced to comply with gnu-fortran standards. The argument list of the subroutines `DX1FCF` and `D01AJF` was shortened and includes now used variables only.
- The target nucleon, proton or neutron, is chosen event by event according to its cross sections and using input from `HNA` and `HNZ`.
- Random number generator `ranlux` from Cernlib. The function `PYR` has been replaced by an interface to `ranlux`.

- Filling of histograms is done with HFILL after a preceding call to HBARX. The present version results in more accurate statistics with large event numbers. Output is written to file OUTFILENAM_evt.dat.

- Input to code word NUCLEUS is EPRO, the energy per nucleon, and HNA (nucleon number) and HNZ (proton number).

- Parton distribution functions are used from LHAPDF [47, 48] if ILQMOD=0 and ILIB=2.

15.12 HERACLES, version 4.6.7

- For Sophia, in routine DJGSPH: status of beam particles on K(IP1,1) set to 201 according to convention used in HSACPT and elsewhere.

- For Sophia, in routine DJGSPH: momentum of virtual boson on event record corrected.

15.13 HERACLES, version 4.6.6

- Value of PARL(24) (corrected cross section to take into account possibly reduced phase space due to failing hadronization) is determined in the main program of HERACLES and not any more updated for every event generation step.

- In djangoh_l.f, do not allow values for the degree of polarization transfered to PARL(6) larger than 0.99 (smaller than -0.99). A corresponding IF-statement in LEPTOX is made safe against roundoff errors.

- In HSACPT, added variable IDJSPH to tag events with W below the cutoff for hadronization in SOPHIA. The LEPTO-related routine HSWCUT is called only if needed to test for possible fragmentation in LEPTO.

- On common block DJPASS, added variable NFAILL to count events where hadronization failed in LEPTO. Also, counting on variable NTOT is corrected: counts events with successful hadronization in LEPTO. Affects also djangoh_u.f.

- On common block SPPASS, added variables NSPACC to tag successful hadronization in SOPHIA. Affects also djangoh_u.f.

- Default of LST(3) reduced to 2: only warnings printed, execution stopped on error.

15.14 HERACLES, version 4.6.5

- Following suggestions by V. Lendermann, the code to specify structure functions and parton distribution functions was modified: The variables `IPART` (on common block `HSPARL`) and `NPYMOD` (on common block `HYSTFU`) are not used anymore and are removed from the code. Also `LPAR(6)` is not used anymore. A new common block `HSSTRP` with the three integer variables `ICODE, ILIB, ILQMOD` is introduced and keeps identification code for the structure functions. Input is expected now for these three variables separately after code word `'STRUCTFUNC'`.

- The default value for the infrared cutoff `XIRDEL` used to separate radiative from non-radiative elastic $ep \to ep(\gamma)$ scattering is increased from 2×10^{-4} to 2×10^{-2}. The consistency check on the value of `XMAX` is modified to take into account this cutoff.

- In `djangoh_l.f`, subroutine `HSWCUT`: corrected name of common block `LYPARA` (was `PYPARA`).

- The undocumented input variable `IOPLOT` is removed from the list of input data for code word `'TEST-OPT'` (but still used for internally).

15.15 HERACLES, version 4.6.4

- The program SOPHIA is interfaced to DJANGOH through the new subroutine `DJGSPH`. The new input code word `'SOPHIA'` allows to change the value of the cutoff for the hadronic mass `WSOPHIA` (on common block `SOPHCT`). The default is 1.5 GeV.

15.16 HERACLES, version 4.6.3

- Counters of event numbers in separate channels are now of type `real*8` to avoid overflows.

- Corrected call of `HSRNDM` (missing argument).

- Corrected normalization factor for radiative charged current scattering with polarized positrons in `HSCCKL` and `HSCCQF`.

- Corrected routine `D01AJF` which was used for initialization of non-radiative elastic scattering cross section: now calls function `F` as provided as input argument, not directly `HSELG1`.

- Clean-up of code and output.

15.17 HERACLES, version 4.6.2

- Most of the changes werde made to improve the calculation of radiative contributions for charged current scattering. The subroutine `HSCCKL` has been rewritten and new routines were added (`HSCKMX`, `HSCXSM`, `HSCLAB`, `HSCCMS` and `HSCPHL`).

- In `HSSTDL` the missing declaration for `HSF2DL` was added (thanks to Masahida Inuzuka).

- Variable names containing "$" were replaced by "_" (underscore) in `HSSFZZ`.

15.18 HERACLES, version 4.6.1 and DJANGO6, version 2.5

- New channels for the radiative elastic tail, $ep \to ep\gamma$ are included. Accordingly, additional input for the code words `'INT-OPT-NC'` and `'SAM-OPT-NC'` is expected and the new code words `'NUCLEUS'` and `'EP-DIPOLE'` were introduced.

- HERACLES and DJANGO6 are now combined in one single program which is run in either HERACLES mode or in DJANGO6 mode. The HERACLES mode is chosen if
  (1) no input for DJANGO6 is given, or
  (2) `LST(7)` < 0 is given as input to the code word `FRAG`.
  In the HERACLES mode, no calls to DJANGO6 routines are performed and the HERACLES options are allowed. These are in particular: combined NC and CC event generation, low-Q^2 structure functions with `ILQMOD` > 1, quarkonic radiation and interference of radiation from the lepton and the quark line. In DJANGO6 mode only channels for either neutral or charged current processes are allowed and the execution stops if `ILQMOD` > 1. Note also, that a check on whether there is enough hadronic energy in the final state to allow for fragmentation and/or hadronization is performed only in the DJANGO6 mode. Internally, the two different modes are flagged by the variable `IHSONL`.

- An upper limit on Q^2 can be given. Input for code word `'KINEM-CUT'` is expected to contain a corresponding value after `Q2MIN` and before `WMIN`.

- The input option for code word `'HYP-CUBES'` with input item `NPHYP` is activated. It can be used to control the number of function evaluations during the estimation of function maxima. The default value for `NPHYP` has been increased.

- For `ILQMOD` $= 10$, the program calls the routine `FIUSER` to calculate user-defined structure functions F_1, F_2.

- The ALLM parametrization for structure functions (`ILQMOD` $= 3$) has been replaced by the new version described in Ref. [52].

- The low-Q^2 suppression of parton distribution functions has been put into a separate function `HSLOQS` to allow for more convenient possible future modifications.

- Calls to parton distributions from PAKPDF (`ILIB` = 3) are disabled.

- The new default for parton distributions is `ISTRFC` = 123041, *i.e.* MRS(G) taken from PDFLIB with exponential low-Q^2 suppression.

- According to the range of validity given by the authors, the parametrization of Badełek and Kwiecinski [53] (`ILQMOD = 4`) is restricted to $x > 10^{-5}$ and execution is stopped if it is requested together with $x_{\rm min} < 10^{-5}$.

- Events from channel `INC33` (Compton events) are not excluded anymore automatically from hadronization. Fragmentation and hadronization is performed if enough hadronic energy is available.

- A new version of the routine `HSDQDV` (for the calculation of some infrared photonic contributions) should avoid floating point exceptions which appeared for $y_l \to 1$ in the former version.

- The requested accuracies for integration in `DX1FCF` and VEGAS (`EPSO` and `ACCVEG`) are increased.

- The timer routine `LTIMEX` is activated.

- The initialization for `PYSTFU` is corrected, reset `MSTU(112) = NPYMAX` and e^+ versus e^-.

- The initialization for F_L is made consistent: new routines `DIALFS` and `DIFLOP` are used to transfer options and parameters from input to LEPTO and PYTHIA. `LST(12)` is reset after having been used in `HSLUFL`.

- The check on negative values F_1 after inclusion of F_L is changed in `HSSTR1`.

- In `HSACPT`, the flavor of the struck quark is chosen by calling `HSFLAV` also for `IPDFOP` = 2, *i.e.* if F_L is included.

- The charged current radiative channel `ICC33` (part of the $1/(k \cdot q')$-pole) is disabled since it gave unphysical results for small $x < 10^{-2}$.

- Contributions from incoming b quarks to the CC process are excluded (the correct treatment would require threshold factors for the t quark mass).

- The treatment of electron and proton masses in relations of kinematic variables is made consistent ($Q^2 = xy\hat{s}$ with $\hat{s} = 2l \cdot p$ also in the calculation of F_L: `PARL(21)` $= \hat{s}$).

- The definition of CUT(10) is corrected (was never used, but when checking consistency of kinematic limits).
- To test whether IX is out of range, modifications were done in HSINIT.
- Execution is stopped in HSGENM if a wrong correction of maxima occurs.
- The OPEN statements for data file and file for random number generator seeds are corrected.
- Routines related to the random number generator are renamed, as well as the corresponding common block /RANDOM/ which became /HSRNDC/.
- D01FCF is renamed to DX1FCF.
- NFLMAX, the number of trials for fragmentation in LEPTO is increased to 10.
- The initialization of ARIADNE is corrected.
- The true $W_{\rm min}$ is used when printing a warning on a reduced cross section written to PARL(24).
- A write statement in routine LWEITS of LEPTO is corrected.
- The output header and many other format statements are changed.
- Two commented lines with a call to a test routine TSTRRR locate the place where the initialization of HERACLES/DJANGO6 is completed. TSTRRR can be used, for example, to produce a listing of values of structure functions and other intermediate results.

16 Concluding remarks

The present version of DJANGOH is the result of a long-lasting history of code development, starting before the experimental collaborations at the HERA collider took their first events, and continuing much beyond the life of HERA. It has been cross-checked against other calculations and, most importantly, it has been tested with success against a large amount of electron proton deep inelastic scattering data. Some recent features of the program, however, have not yet been used very extensively and it is likely that further improvements will be needed.

I hope that studies of possible future experiments both at larger energies (like the LHeC under discussion) or at smaller energies at the EIC or at the Jefferson Lab do profit from this tool. New kinematic regimes and higher precision at these facilities will make it likely that another series of improvements of the Monte Carlo simulation will be needed. Among them, I should mention the model for the hadronization of the final state used in the present version, still based essentially on knowledge of the early days of HERA experiments; the inclusion of higher-order QCD corrections; the extension to include corrections of higher than one-loop order and beyond one-photon radiation.

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