book

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Contents

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theory IbookCh0
  imports Main
begin

lemma evens: ∃ (n::nat) . 2∗n > (k::nat)
  by presburger

lemma evens2: ∃ (n::nat) . 2∗n > (k::nat)
  using evens by auto

lemma evens3: ∃ (n::nat) . 2∗n > (k::nat)
proof −
  have example:2∗(k+1) > k
    by simp
  show ?thesis
    using example by blast
qed

lemma evens4: ∃ (n::nat) . 2∗n > (k::nat)
proof (cases k = 0)
  case True
  have ex: 2 ∗ 1 > k using True by simp
  then show ?thesis using ex by blast
next
  case False
  have ex: 2 ∗ k > k using False by auto
  then show ?thesis using ex by blast
qed


lemma evens5: ∃ (n::nat) . 2∗n > (k::nat)
proof (induction k)
  case 0
  then show ?case by auto
next
```

case (*Suc k*)
then obtain *n* **where** *nFact*: *2∗n > k* **by** *blast*
let *?m = Suc n*
from *nFact* **have** *2 ∗ ?m > Suc k* **by** *auto*
then show *?case* **by** *blast*
qed

lemma *T5*: ∃ (*n*::*nat*) . (*k*::*nat*) + *5* < *n*
proof (*induction k*)
case *0*
then show *?case* **by** *auto*
next
case (*Suc k*)
then obtain *n* **where** *n*: ‹*k* + *5* < *n*› — From this, I'd like to say "OK, so there's some n with that property. Let's call it m.
by *blast*
let *?p* = ‹*Suc n*›
from *n* **have** ‹*Suc k* + *5* < *?p*› — I show that Suc k + 5 < p for some p (as I did in the k = 0 case)
by *auto*
then show *?case* **try0**
by (*rule exI*[*of* - ‹*Suc n*›]) — Isabelle can infer the "there-exists" result from that instance, using P(a) => EX x . P(x).
qed

definition *nand* :: *bool* ⇒ *bool* ⇒ *bool* (**infixr** *N 35*)
where *A N B* ≡ ¬(*A* ∧ *B*)

lemma ¬*A* ⟷ *A N A*
unfolding *nand-def*
apply *simp*
done

lemma *odds1*: ∃ (*k*::*nat*) . *2∗k+1* > (*n*::*nat*)
proof −
have *2∗n+1* > *n* **by** *auto*
thus *?thesis* **by** *blast*
qed

lemma *positiveSquare*: (*n*::*nat*) ∗ (*n*::*nat*) ≥ *0*
by *auto*

end