FORMATIVE ASSESSMENT 2

APM1111 – STATISTICAL THEORY

3.49 Prove that $\sum_{j=1}^{N} (X_j - 1)^2 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N$.

Solution:

- To prove this, we can start by expanding the left side

$$\sum_{j=1}^{N}(X_j^2 - 2X_j + 1) = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N$$

Note: both sides must be equal (left side and right side)

- Distribute the summation on each term

$$\sum\nolimits_{j=1}^{N} X_j^2 - \sum\nolimits_{j=1}^{N} 2X_j + \sum\nolimits_{j=1}^{N} 1 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N$$

This can be simplified to

$$\sum\nolimits_{j=1}^{N} X_j^2 - 2\sum\nolimits_{j=1}^{N} X_j + \sum\nolimits_{j=1}^{N} 1 = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N$$

- The summation of 1 from j=1 to N is simply N. We are just basically adding the number 1 over and over again until we reach N.

Since we are adding 1 exactly N time, we can rewrite this as

$$\sum\nolimits_{j=1}^{N} X_j^2 - 2\sum\nolimits_{j=1}^{N} X_j + \sum\nolimits_{j=1}^{N} 1 \times N$$

We can rewrite again this as

$$\sum\nolimits_{j=1}^{N} X_j^2 - 2\sum\nolimits_{j=1}^{N} X_j + N$$

- Since we have shown that the left side is equal to the right side, this identity is proved

$$\sum\nolimits_{j=1}^{N} X_j^2 - 2\sum\nolimits_{j=1}^{N} X_j + N = \sum_{j=1}^{N} X_j^2 - 2\sum_{j=1}^{N} X_j + N$$

3.51 Two variables, U and V, assume the values $U_1 = 3$, $U_2 = -2$, $U_3 = 5$, and $V_1 = -4$, $V_2 = -1$, $V_3 = 6$, respectively. Calculate (*a*) $\sum UV$, (*b*) $\sum (U+3)(V-4)$, (*c*) $\sum V^2$, (*d*) $(\sum U)(\sum V)^2$, (*e*) $\sum UV^2$, (*f*) $\sum (U^2 - 2V^2 + 2)$, and (*g*) $\sum (U/V)$.

Given that:

$$\mathrm{U_1} = 3, \mathrm{U_2} = -2, \mathrm{U_3} = 5$$

$$\mathrm{V_1} = -4, \mathrm{V_2} = -1, \mathrm{V_3} = 6$$

(a) $\sum \mathrm{UV}$

$$\sum \mathrm{UV} = (\mathrm{U_1V_1}) + (\mathrm{U_2V_2}) + (\mathrm{U_3V_3})$$

$$\sum \mathrm{UV} = (3)(\text{-}4) + (\text{-}2)(\text{-}1) + (5)(6)$$

$$\sum \mathrm{UV} = \text{-}12 + 2 + 30$$

$$\sum \mathrm{UV} = 20$$

(b) $\sum(\mathrm{U}+3)(\mathrm{V}-4)$

$$\sum(\mathrm{U}+3)(\mathrm{V}-4) = ((\mathrm{U_1}+3)(\mathrm{V_1}-4)) + ((\mathrm{U_2}+3)(\mathrm{V_2}-4)) + ((\mathrm{U_3}+3)(\mathrm{V_3}-4))$$

$$\sum(\mathrm{U}+3)(\mathrm{V}-4) = ((3+3)(\text{-}4-4)) + ((\text{-}2+3)(\text{-}1-4)) + ((5+3)(6-4))$$

$$\sum(U+3)(V-4) = (6)(-8) + (1)(-5) + (8)(2)$$

$$\sum(U+3)(V-4) = \text{-}48 + \text{-}5 + 16$$

$$\sum(U+3)(V-4) = \text{-}37$$

(c) $\sum V^2$

$$\sum V^2 = \mathrm{V_1^2} + \mathrm{V_2^2} + \mathrm{V_3^2}$$

$$\sum V^2 = (-4)^2 + (-1^2) + (6)^2$$

$$\sum V^2 = 16 + 1 + 36$$

$$\sum V^2 = 53$$

(d) $\sum \mathrm{U} \sum V^2$

$$\sum \mathrm{U} \sum V^2 = (\mathrm{U}_1 + \mathrm{U}_2 + \mathrm{U}_3)(\mathrm{V}_1^2 + \mathrm{V}_2^2 + \mathrm{V}_3^2)$$

$$\sum \mathrm{U} \sum V^2 = (3 + \text{-}2) + 5)((-4)^2 + (-1^2) + (6)^2)$$

$$\sum \mathrm{U} \sum V^2 = 6 \times 53$$

$$\sum \mathrm{U} \sum V^2 = 318$$

(e) $\sum UV^2$

$$\sum UV^2 = (U_1\mathrm{V}_1^2) + (\mathrm{U}_2\mathrm{V}_2^2) + (\mathrm{U}_3\mathrm{V}_3^2)$$

$$\sum UV^2 = (3)(-4)^2 + (-2)(-1)^2 + (5)(6)^2$$

$$\sum UV^2 = 226$$

(f) $\sum(U^2 - 2V^2 + 2)$

$$\sum(U^2 - 2V^2 + 2) = (\mathrm{U}_1^2 - 2\mathrm{V}_1^2 + 2) + (\mathrm{U}_2^2 - 2\mathrm{V}_2^2 + 2) + (\mathrm{U}_3^2 - 2\mathrm{V}_3^2 + 2)$$

$$\sum(U^2 - 2V^2 + 2) \\ = ((3)^2 - 2(-4)^2 + (2)) + ((-2)^2 - 2(-1)^2 + (2)) + ((5)^2 - 2(6)^2 + (2))$$

$$\sum(U^2 - 2V^2 + 2) = (9 - 32 + 2) + (4 - 2 + 2) + (25 - 72 + 2)$$

$$\sum(U^2 - 2V^2 + 2) = (-21) + (4) + (-45)$$

$$\sum(U^2 - 2V^2 + 2) = -62$$

(g) $\sum \frac{\mathrm{U}}{\mathrm{V}}$

$$\sum \frac{\mathrm{U}}{\mathrm{V}} = (\frac{\mathrm{U}_1}{\mathrm{v}_1}) + (\frac{\mathrm{U}_2}{\mathrm{v}_2}) + (\frac{\mathrm{U}_3}{\mathrm{v}_3})$$

$$\sum \frac{\mathrm{U}}{\mathrm{V}} = (\frac{3}{\text{-}4}) + (\frac{\text{-}2}{\text{-}1}) + (\frac{5}{6})$$

$$\sum \frac{\mathrm{U}}{\mathrm{V}} = \frac{25}{12}$$

3.90 Find the geometric mean of the sets (*a*) 3, 5, 8, 3, 7, 2 and (*b*) 28.5, 73.6, 47.2, 31.5, 64.8.

Set A: 3,5,8,3,7,2

$$Geometric\ Mean\ =\ \sqrt[N]{X_1}X_1X_1 \ldots X_N$$

$$Geometric\ Mean\ =\ \sqrt[6]{(3)(5)(8)(3)(7)(2)}$$

$$Geometric\ Mean\ =\ \sqrt[6]{5040}$$

$$Geometric\ Mean\ = 4.14$$

Set B: 28.5, 73.6, 47.2, 31.5, 64.8

$$Geometric\ Mean\ =\ \sqrt[N]{X_1}X_1X_1 \ldots X_N$$

$$Geometric\ Mean\ =\ \sqrt[5]{(28.5)(73.6)(47.2)(31.5)(64.8)}$$

$$Geometric\ Mean\ =\ \sqrt[5]{202092516.9}$$

$$Geometric\ Mean\ =\ 45.8$$