2.1 Cosmic ray propagation

- PHYS 5562 Topics in Theoretical Physics (Astroparticle Physics Module)

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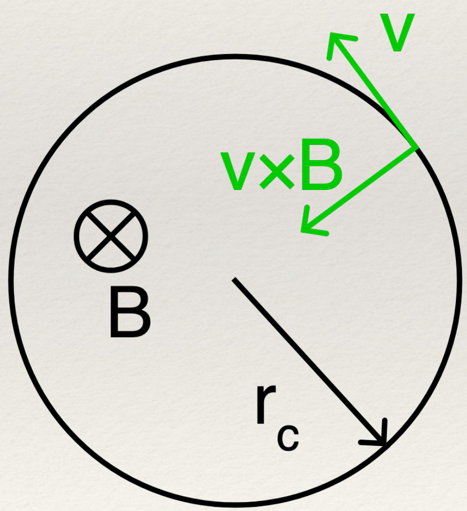
Cosmic Ray arrival directions

- Cosmic rays arrive more or less isotropically
    - => their directions are "isotropized" during propagation

- Lorentz force
- $\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$
    - Neutrality => Electric fields are zero
    - Under constant and perpendicular B-field
- Lamor Radius (Gryro-radius)
- $r_g = \frac{p_\perp}{|q|B}$
- $r_g \simeq 3.3\text{m} \left(\frac{E}{\text{GeV}}\right)\left(\frac{e}{|q|}\right)\left(\frac{1\,\text{T}}{B}\right)$



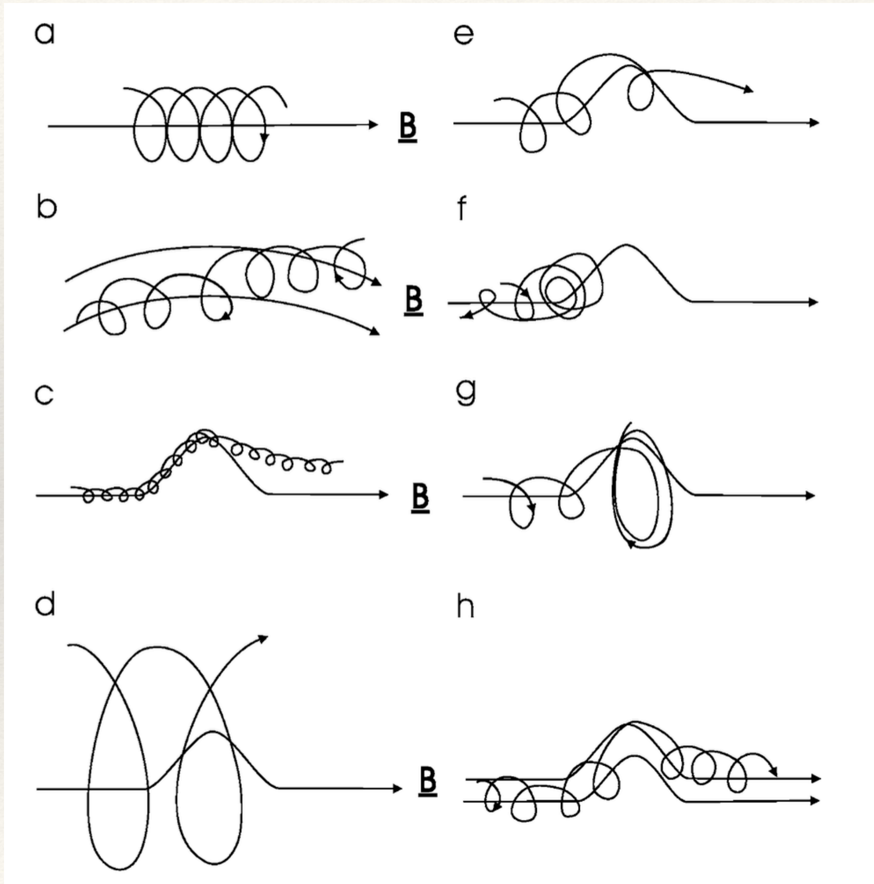
Magnetic Irregularities

Space Sci Rev (2013) 176:299–319
DOI 10.1007/s11214-011-9819-3

Cosmic-Ray Modulation Equations

H. Moraal

- a) Particles moves in a spiral in uniform field.
- b) If there is a field gradient, particles may move across field line.
- Let's consider magnetic irregularities with scale length l_B
- c) If $r_g << l_B$, the particle will mostly follow the field line, and have a chance to drift off.
- d) If $r_g >> l_B$, the irregularities are ignored.
- e, f, g) If $r_g \sim l_B$, then it is possible that the particle's trajectories are significantly deflected.
- This will allows us to approximate cosmic-ray propagation as a diffusion problem.



Diffusion equation (1D)

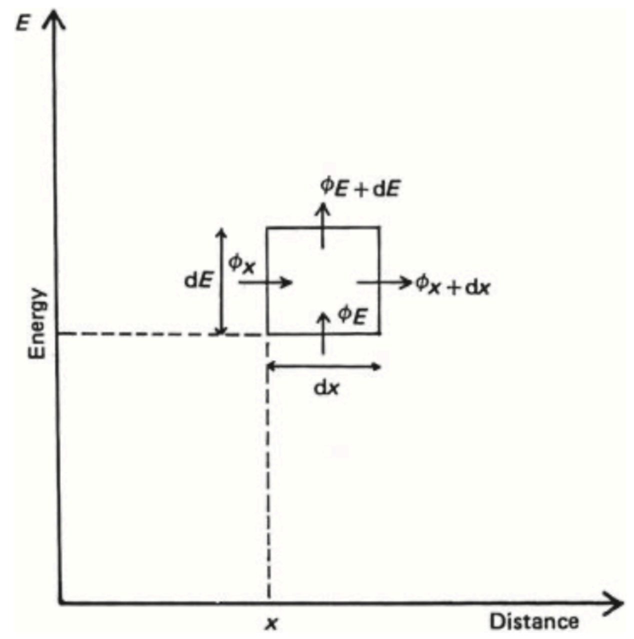
Longair

- Consider $n = \dfrac{dN}{dxdE}$

- $\dfrac{d}{dt}\left(ndxdE\right) = -\left(\phi_{x+dx} - \phi_x\right) dE - \left(\phi_{E+dE} - \phi_E\right) dx + (\text{Source})dxdE$
- ϕ_x : "number of particles/ (dE)/(time)" flowing from (x) to (x+dx)
- ϕ_E**: "**number of particle / (dx)/(time)" flowing from (E) to (E+dE)

- $\phi_x = V\dfrac{dN}{dxdE}$, this is our 1D differential "flux"
    - "V" is the **net** speed of the particles
    - This is macroscopic flux flowing through volume

7.5 The diffusion-loss equation for high energy particles



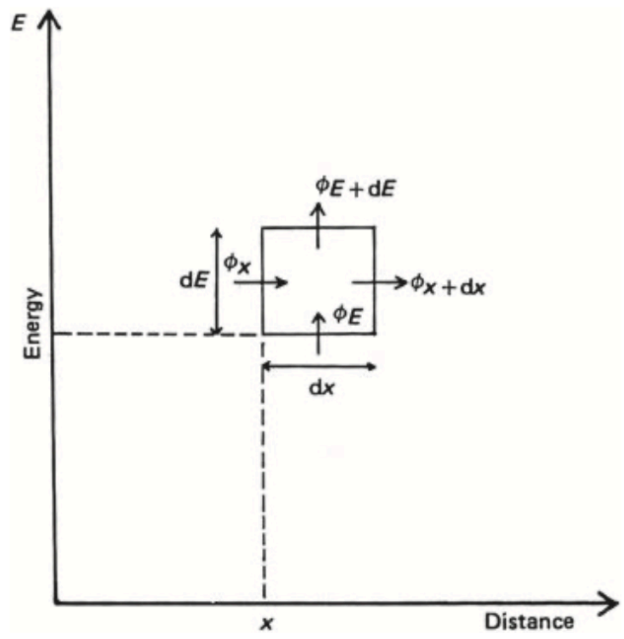
Diffusion equation (1D -> 3D)

- Consider $n = \dfrac{dN}{dxdE}$

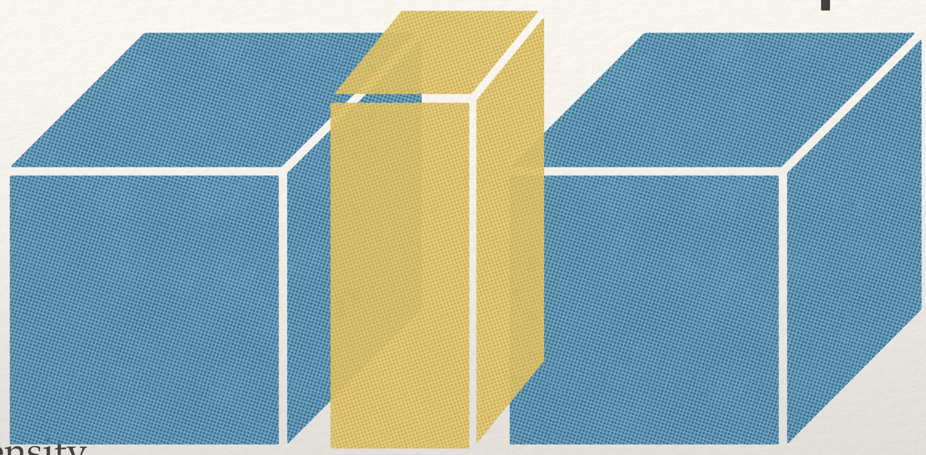
- $\dfrac{d}{dt}\left(ndxdE\right) = -\left(\phi_{x+dx} - \phi_x\right) dE - \left(\phi_{E+dE} - \phi_E\right) dx + (\text{Source})dxdE$
- $\dfrac{d}{dt}\left(ndxdE\right) = -\dfrac{\partial \phi_x}{\partial x}dxdE - \dfrac{\partial \phi_E}{\partial E}dxdE + (\text{Source})dxdE$
- dx, dE are just coordinates
- $\dfrac{d}{dt}n = -\dfrac{\partial \phi_x}{\partial x} - \dfrac{\partial \phi_E}{\partial E} + Q$
- $\dfrac{\partial}{\partial t}n = -\nabla \cdot \overrightarrow{\phi} - \dfrac{\partial \phi_E}{\partial E} + Q$ **generalising to 3D**, $n = \dfrac{dN}{d^3xdE}$

You may recognise the look of a continuity equation

7.5 The diffusion-loss equation for high energy particles



Diffusion equation

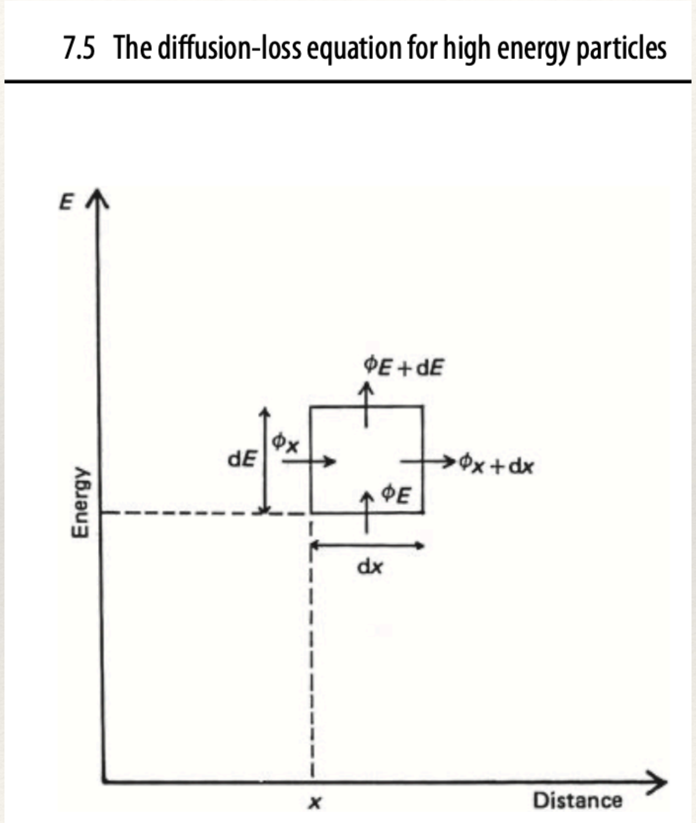


- Let's look at ϕ_x , ϕ_y , ϕ_z
- Now goes to microscopic picture
  - (Feynman lecture: diffusion)
- Particles travel a **"mean free path [l]"**, then scatter
- There can only be non-zero ϕ_x , when there is a difference in number density
- Net flow of particles $\phi_x = \left(n_- - n_+\right) v$
  - "v" is the microscopic particle velocity
- $\phi_x = \left(n_- - n_+\right) v \simeq -\dfrac{dn}{dx}\Delta x\, v \simeq -\dfrac{dn}{dx}\ell v \equiv -D\dfrac{dn}{dx}$, *Phenomenologically, Fick's law*
- $\vec{\phi} = -D\vec{\nabla} n$ (3D)

Diffusion equation

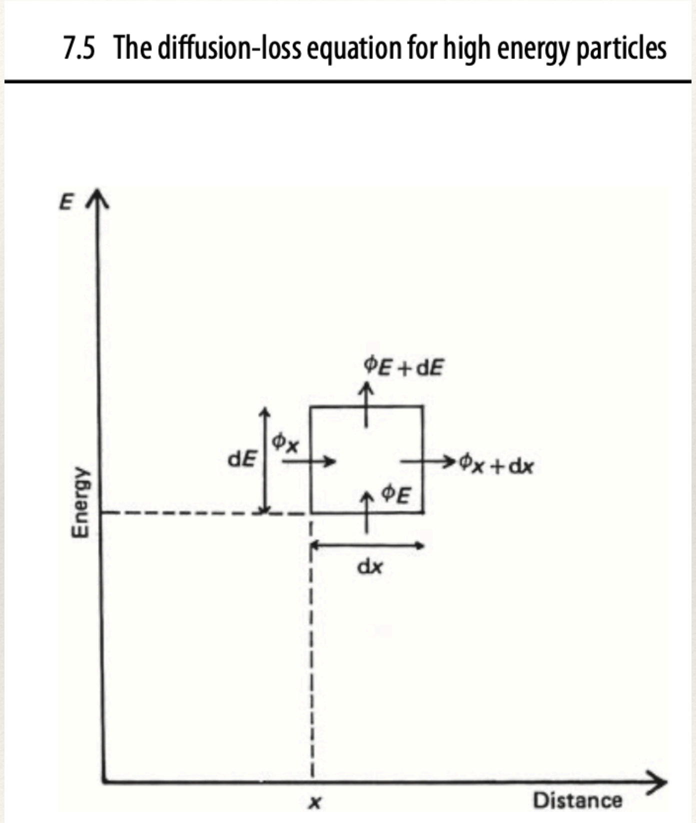
- $\frac{\partial}{\partial t}n = -\nabla \cdot \overrightarrow{\phi} - \frac{\partial \phi_E}{\partial E} + Q$
- ϕ_E : "number of particle / (dx)/(time)" flowing from (E) to (E+dE)
- For each particle with energy E, if their energy change with time "continuously"
  - (The continuous energy loss approximation)
- $\phi_E = \frac{dN}{dxdE}\frac{dE}{dt} = (3D) = n\frac{dE}{dt}$

This is like energy wind



Diffusion equation

- $$\frac{\partial}{\partial t}n = -\nabla \cdot \vec{\phi} - \frac{\partial \phi_E}{\partial E} + Q$$
- $$\vec{\phi} = -D\vec{\nabla}n$$
- $$\phi_E = n\frac{dE}{dt} = n\dot{E}$$
- $$\frac{\partial}{\partial t}n = \nabla \cdot \left(D\vec{\nabla}n\right) - \frac{\partial}{\partial E}\left(n\dot{E}\right) + Q,$$ if D is position independent
- $$\frac{\partial}{\partial t}n = D\nabla^2 n - \frac{\partial}{\partial E}\left(n\dot{E}\right) + Q$$
- **Diffusion-loss equation** (or I like to call "them" Boltzmann eqn.)
- We have considered (spatial diffusion), (continuously energy loss), (source/particle injection)
- *Add more terms when there are more effects*



Diffusion-Convection equation

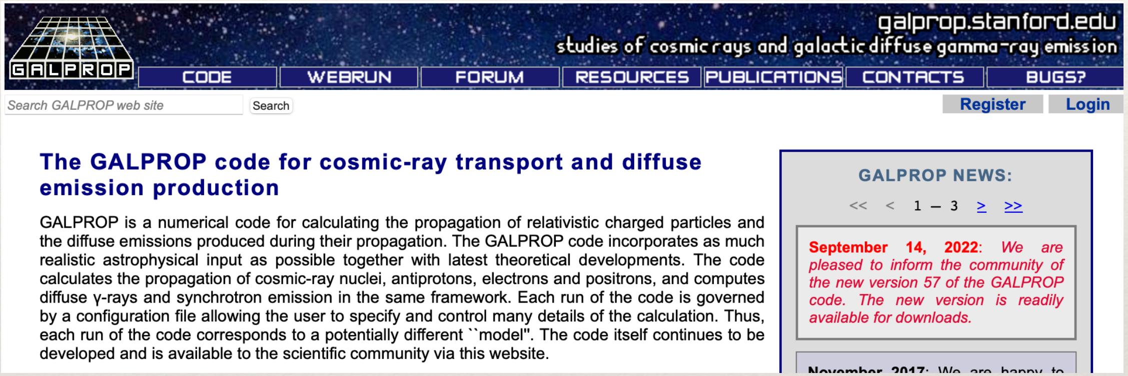
- $\frac{\partial}{\partial t} n = -\nabla \cdot \overrightarrow{\phi} - \frac{\partial \phi_E}{\partial E} + Q$
- Cosmic rays can experience convection due to moving magnetic fields
  - (Solar modulation as an example)
- $\overrightarrow{\phi} = -D\overrightarrow{\nabla} n + \overrightarrow{V} n$, where $\overrightarrow{V}$ here is the convection speed. (See page 4)
- $\frac{\partial}{\partial t} n = \nabla \cdot \left(D\overrightarrow{\nabla} n - \overrightarrow{V} n \right) - \frac{\partial}{\partial E} \left(n\dot{E} \right) + Q,$

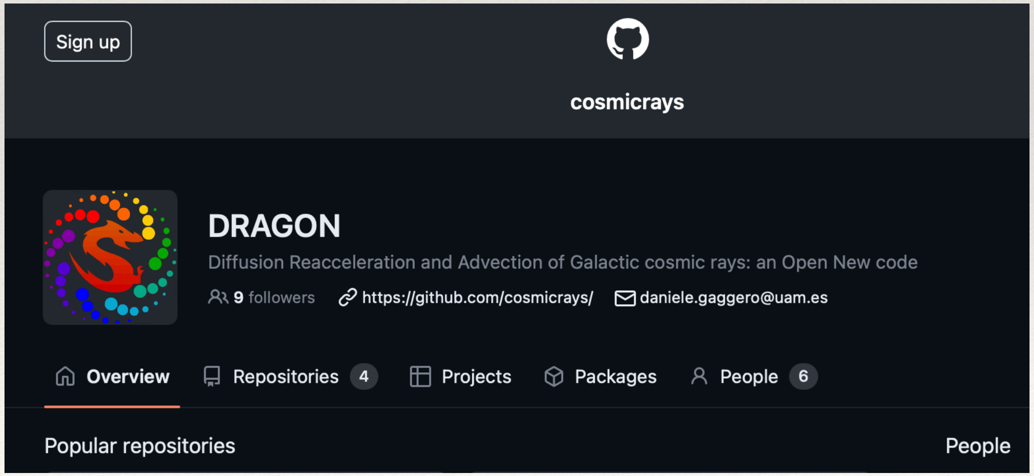
If you are curious, see here
https://articles.adsabs.harvard.edu/pdf/1978Ap%26SS..58...21G

- There is also an momentum loss term due to convection, which we state without proof.
  - Imagine the particles are moving in a cell away from you
  - $\dot{p} = -\frac{1}{3}(\overrightarrow{\nabla} \cdot \overrightarrow{V})p$

Cosmic Ray propagation

- In practice, solved numerically with many effects, empirical relations taken into account, and by matching observations.

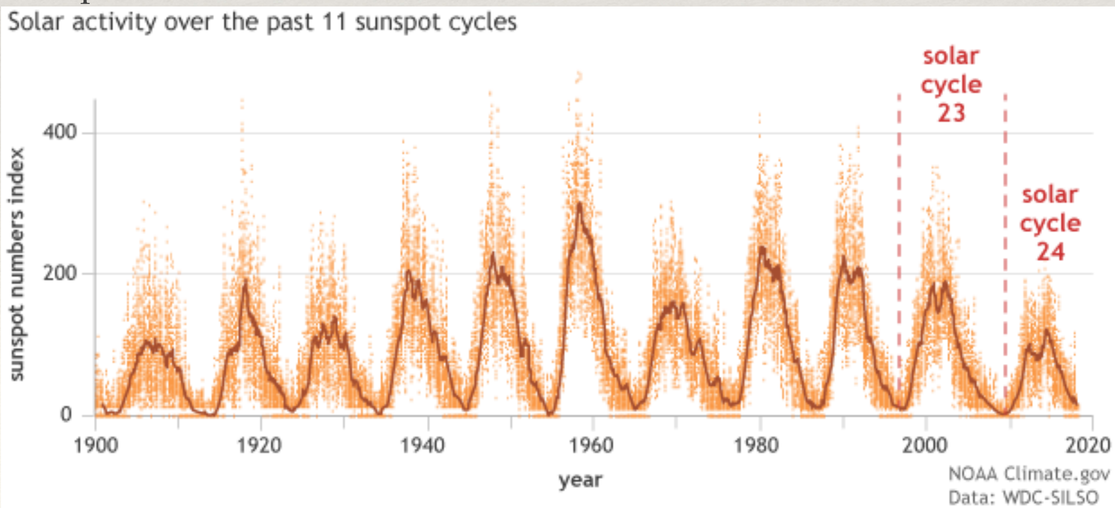


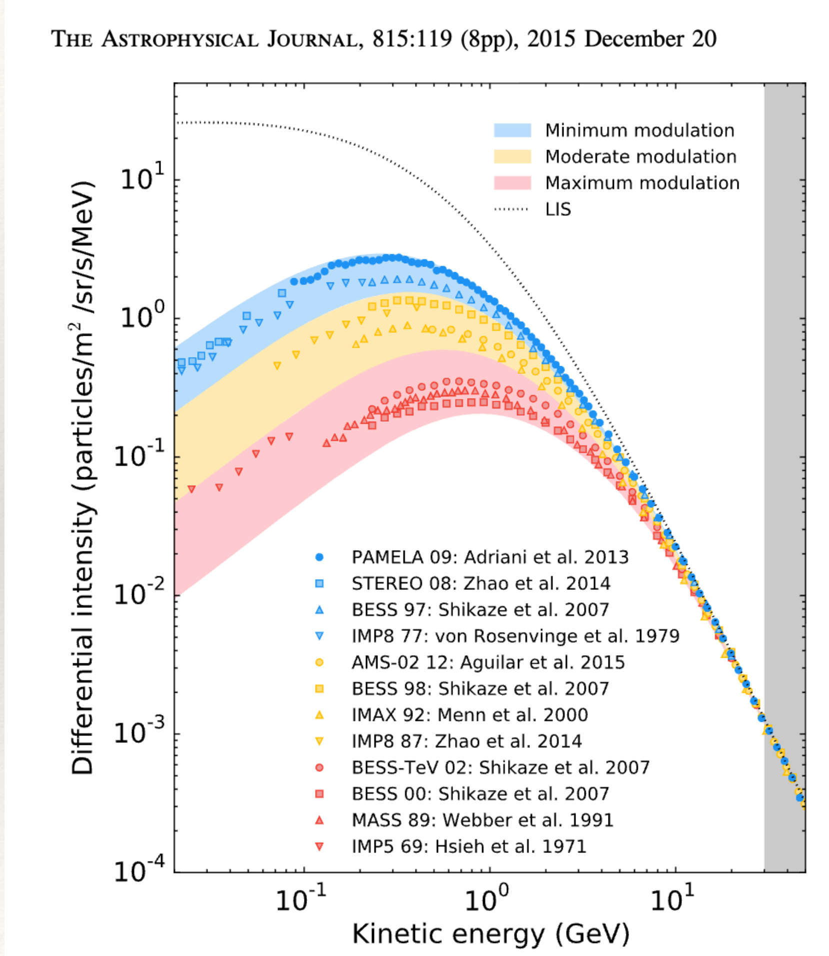


Cosmic Rays in the solar system
Solar Modulation

Solar modulation of cosmic rays

- For galactic cosmic rays, it is well known that they are affected by the Sun below 10 GeV.
- The Sun has a ~ 11 year cycle.
- More cosmic rays arrive when the Sun is less active
- Less cosmic rays arrive when the Sun is more active
- LIS = Local Interstellar Spectrum,
    - CR spectrum without solar modulation

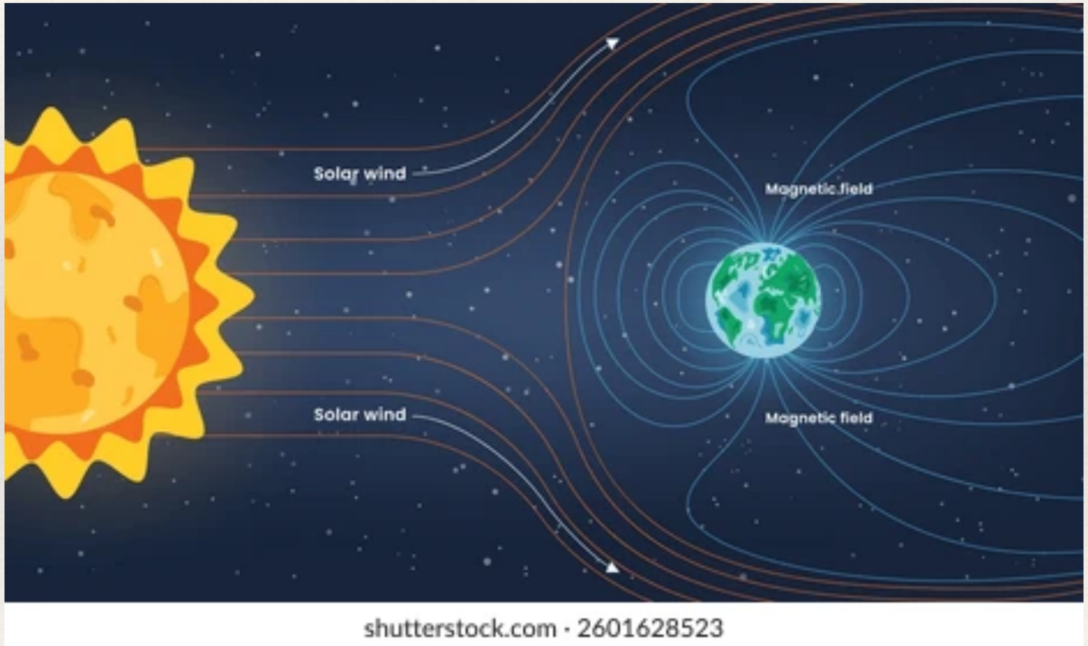




Solar Wind

- Streams of charged particles flowing from the Sun
    - (Low energy solar cosmic rays)

- They carry magnetic fields together with them
    - (Imagine magnetic fields in super conductors)



Rigidity

- Recall
  - Lamor Radius (Gryro-radius)
  - $r_g = \frac{p_\perp}{|q|B}$
  - $r_g \simeq 3.3\text{m} \left(\frac{E}{\text{GeV}}\right) \left(\frac{e}{|q|}\right) \left(\frac{1\,\text{T}}{B}\right)$
- Because magnetic propagation only cares about momentum/charge
- It is convenient to define rigidity as
- $R = \frac{p}{q}$
- In this variable, different particle species have the same trajectory

cosmic rays propagating in the solar system

- $\frac{\partial}{\partial t}n = \nabla \cdot \left(D\vec{\nabla}n - \vec{V}n\right) - \frac{\partial}{\partial E}\left(n\dot{E}\right) + Q$
- $\dot{p} = -\frac{1}{3}(\vec{\nabla} \cdot \vec{V})p$
- First change to momentum space, by redefining "n"
- $n = \frac{dN}{d^3xdp}$
- $\frac{\partial}{\partial t}n = \nabla \cdot \left(D\vec{\nabla}n - \vec{V}n\right) - \frac{\partial}{\partial p}\left(n\dot{p}\right) + Q$
- $\dot{p} = -\frac{1}{3}(\vec{\nabla} \cdot \vec{V})p$

- Simply our assumptions
- Steady state solution
  - $\frac{\partial}{\partial t}n = 0$
- No sources/sinks
  - Only consider Galactic CR
  - Q = 0
- D is position independent.

Flux, Phase Space density, number density

- Number of particles per phase space
- $f = \dfrac{dN}{d^3p d^3x}$

- Differential number density of particles, e.g.,
- $n = \dfrac{dN}{dp d^3x} = 4\pi p^2 f$
  - Note that sometimes "n" is simply the number density, need to pay attention to context. Writing $n(p), n(E), n(KE)$ definitely means differential

- Differential Flux/Intensity: number of particles per area per time per solid angle per energy
- Consider particles moving isotropically in a region, and you are sitting in the middle.

- $I = \dfrac{vn}{4\pi}$, where "v" is the speed of the particles. For relativistic particles, it is the speed of light.

Updated Sept 14

- $0 = \nabla \cdot \left(D \vec{\nabla} n - \vec{V} n \right) - \frac{\partial}{\partial p} \left(n \dot{p} \right)$
- $\dot{p} = -\frac{1}{3}(\vec{\nabla} \cdot \vec{V}) p$
- $=> \nabla \cdot \left(D \vec{\nabla} n - \vec{V} n \right) + \frac{\partial}{\partial p} \left(\frac{1}{3}(\vec{\nabla} \cdot \vec{V}) p n \right) = 0$
- Now consider the "Phase space density"
- $f = \frac{dN}{d^3 p d^3 x} = \frac{1}{4\pi p^2} \frac{dN}{dp d^3 x} = \frac{1}{4\pi p^2} n => n = 4\pi^2 p^2 f$
-

- $0 = \nabla \cdot \left(D \vec{\nabla} n - \vec{V} n \right) - \frac{\partial}{\partial p} \left(n \dot{p} \right)$
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- $=> \nabla \cdot \left(D \vec{\nabla} p^2 f - \vec{V} p^2 f \right) + \frac{\partial}{\partial p} \left(\frac{1}{3}(\vec{\nabla} \cdot \vec{V}) p^3 f \right) = 0$
- $=> \nabla \cdot \left(D \vec{\nabla} p^2 f \right) - (\nabla \cdot \vec{V}) p^2 f - \vec{V} \cdot p^2 \vec{\nabla} f + (\nabla \cdot \vec{V}) p^2 f + \frac{1}{3}(\vec{\nabla} \cdot \vec{V}) p^3 \frac{\partial f}{\partial p} = 0$
- $=> \nabla \cdot \left(D \vec{\nabla} p^2 f \right) - \vec{V} \cdot p^2 \vec{\nabla} f + \frac{1}{3}(\vec{\nabla} \cdot \vec{V}) p^3 \frac{\partial f}{\partial p} = 0$
- $=> \nabla \cdot \left(D \vec{\nabla} f \right) - \vec{V} \cdot \vec{\nabla} f + \frac{1}{3}(\vec{\nabla} \cdot \vec{V}) p \frac{\partial f}{\partial p} = 0$
- (Historical Force Field approximation), trying to get the total divergence
- $=> \nabla \cdot \left(D \vec{\nabla} f \right) - \vec{V} \cdot \vec{\nabla} f + \frac{1}{3}(\vec{\nabla} \cdot \vec{V}) p \frac{\partial f}{\partial p} + \frac{1}{3} \vec{V} p \frac{\partial}{\partial p} \nabla f = 0$
- $=> \nabla \cdot \left(D \vec{\nabla} f \right) + \vec{\nabla} \cdot \left(\frac{1}{3} \vec{V} p \frac{\partial f}{\partial p} \right) = 0$

L.J. Gleeson, W.I. Axford, Astrophys. J. **154**, 1011 (1968)
L.J. Gleeson, I.A. Urch, Astrophys. Space Sci. **11**, 288 (1971)
L.J. Gleeson, I.A. Urch, Astrophys. Space Sci. **25**, 387 (1973)
L.J. Gleeson, G.M. Webb, Astrophys. Space Sci. **58**, 21 (1978)

- $=> \nabla \cdot \left(D\vec{\nabla} f\right) + \vec{\nabla} \cdot \left(\frac{1}{3}\vec{V}p\frac{\partial f}{\partial p}\right) = 0$
- Integrate both sides, and set integration constant to be zero
- Consider Spherical coordinate, and $V = V_r$
- $D\frac{\partial f}{\partial r} + \frac{Vp}{3}\frac{\partial f}{\partial p} = 0$
- This implies
- $df(r,p) = \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial p}dp = 0$, with $\frac{dp}{dr} = \frac{Vp}{3D}$

- $df(r,p) = \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial p}dp = 0$, with $\frac{dp}{dr} = \frac{Vp}{3D}$

- This means that $f(r_1, p_1) = f(r_2, p_2)$, the phase space density is the same in two places.
- Recall $n = 4\pi p^2 f$, where n is the number density, and flux $I = vn/(4\pi) = vp^2 f$.
  - Here v is the particle velocity

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- Consider cosmic rays travel from the interstellar space, let's call it the
  - Local Interstellar Spectrum (LIS), we have

- $f(r_{Sun}, p) = f(r_{lis}, p_{lis}) = f(p_{lis})$, thus
- $\frac{I(p)}{vp^2} = \frac{I(p_{lis})}{v_{lis}p_{lis}^2}$

- $f(r_{Sun}, p) = f(r_{lis}, p_{lis}) = f(p_{lis})$, thus
- $\dfrac{I(p)}{vp^2} = \dfrac{I(p_{lis})}{v_{lis} p_{lis}^2}$ Updated Sept 14

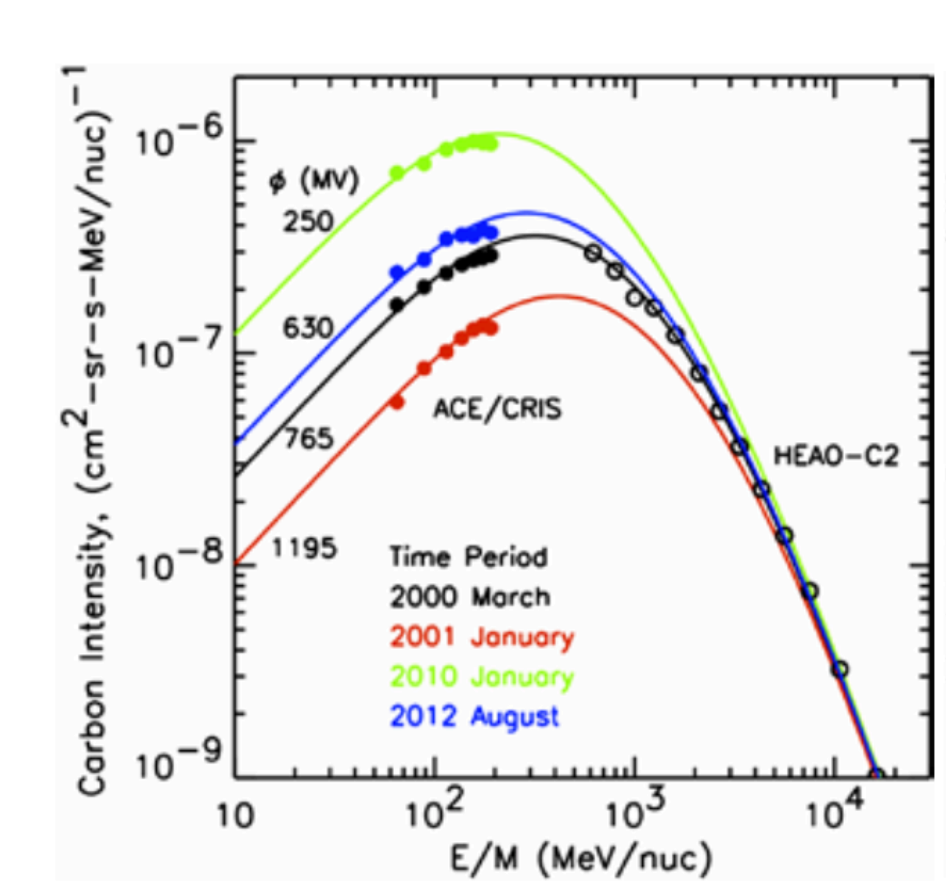
- Consider $\dfrac{dp}{dr} = \dfrac{Vp}{3D}$
- Historically, it is assumed that $D = \beta D_r(r) D_p(p)$ is separable, and $\beta = v/c$
- Then we have
- $\displaystyle\int_{P_1}^{P_2} \frac{\beta D_p}{p} dp = \int_{r_1}^{r_2} \frac{V(r)}{3D_r} dr \equiv \phi$
- If we further assume $D_p \propto p$ and $\beta \simeq 1$
- Then $\phi = P_2 - P_1$
- ϕ is called the modulation potential, and is considered as the momentum (or rigidity) loss as charged particles propagate in solar system from interstellar space to us, against the solar wind.

- $f_{sun}(p) = f(p_{lis} = p + \phi)$
- $\dfrac{I(p)}{vp^2} = \dfrac{I_{LIS}(p+\phi)}{v_{lis}(p+\phi)^2}$
- If you change variable to energy, a more common form (Homework)

$$J_{Earth}(E) = J_{LIS}(E+\Phi)\frac{(E^2-m^2)}{(E+\Phi)^2-m^2},$$

  - (Note Φ here is energy loss)
- Extremely simple model, but works quite well

- Of course, for extremely, precise application, people would solve the full propagation equation.



Local Interstellar spectrum

- De-modulating to get the actual CR spectrum in space

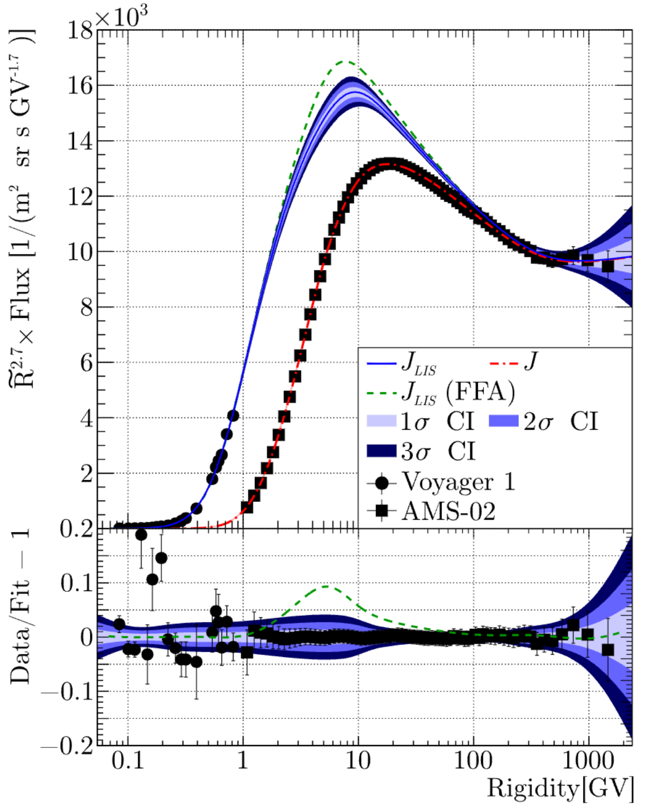


Figure 5. Same as Figure 3, but using the rigidity-dependent solar modulation parameter of Equation (7). For comparison, the LIS derived in Section 2 with the force-field approximation, J_{LIS} (FFA), is shown as a dashed line.

Local Interstellar spectrum

- De-modulating to get the actual CR spectrum in space

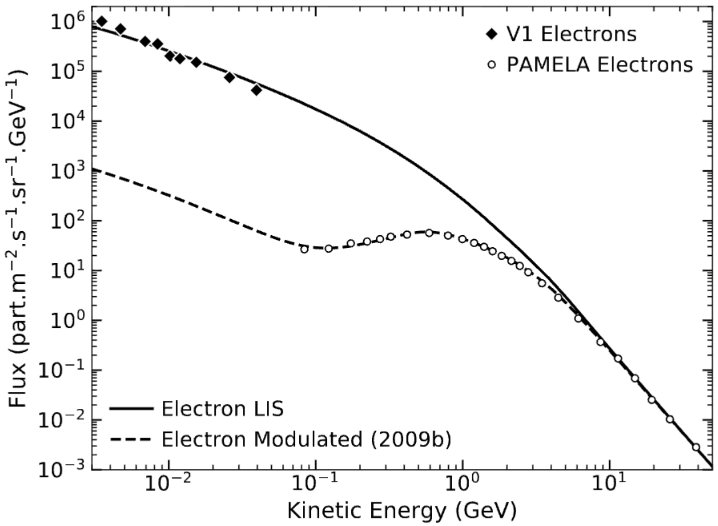


Figure 3. Computed electron LIS (solid black curve) and the corresponding modulated electron spectrum at the Earth (dashed black curve) compared to the V1 electron observations at 122 AU (Cummings et al. 2016) and PAMELA observations at the Earth (1 AU) for the second half of 2009 (Adriani et al. 2015).

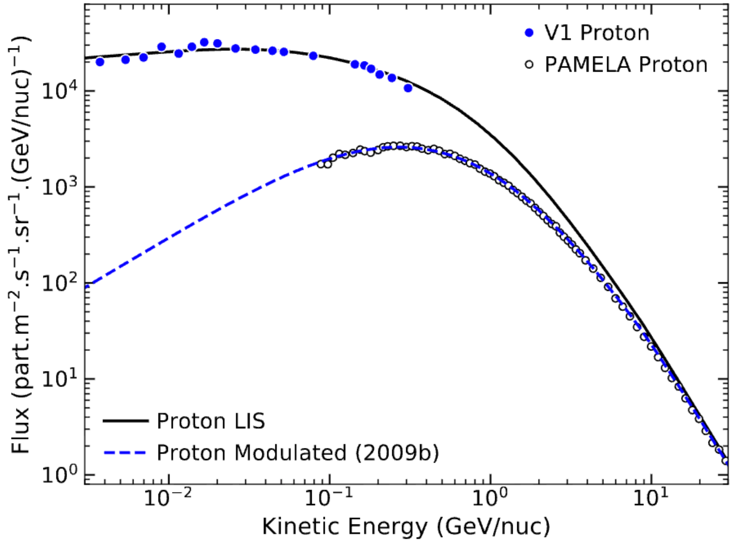
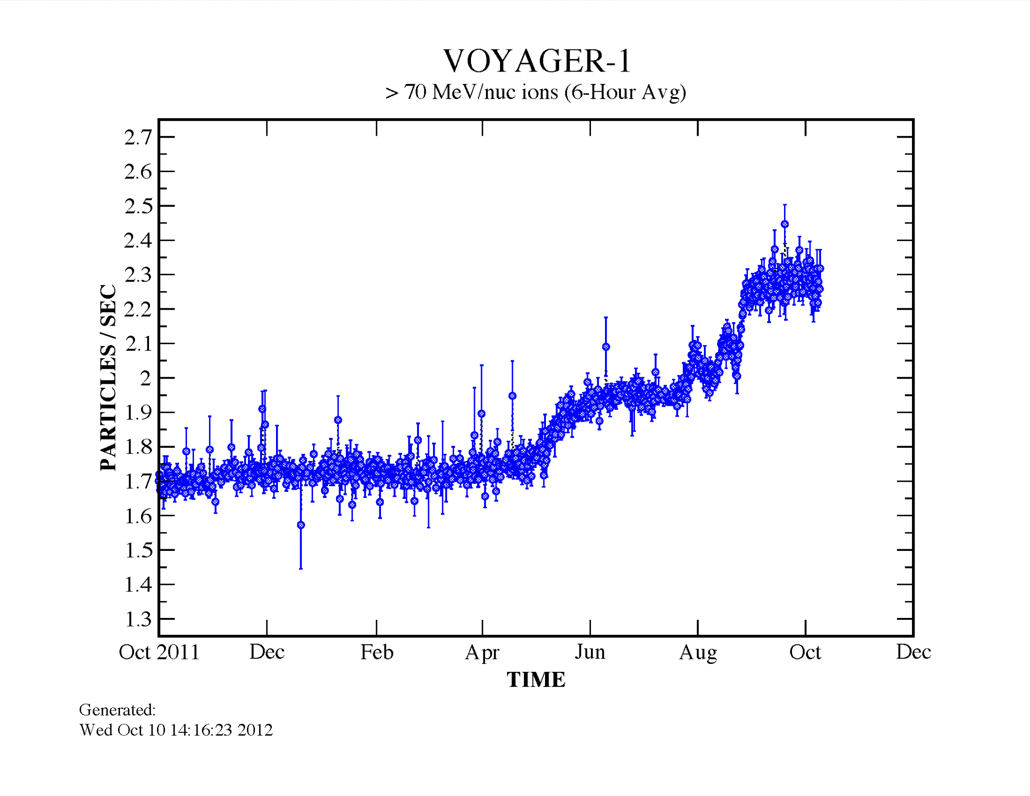


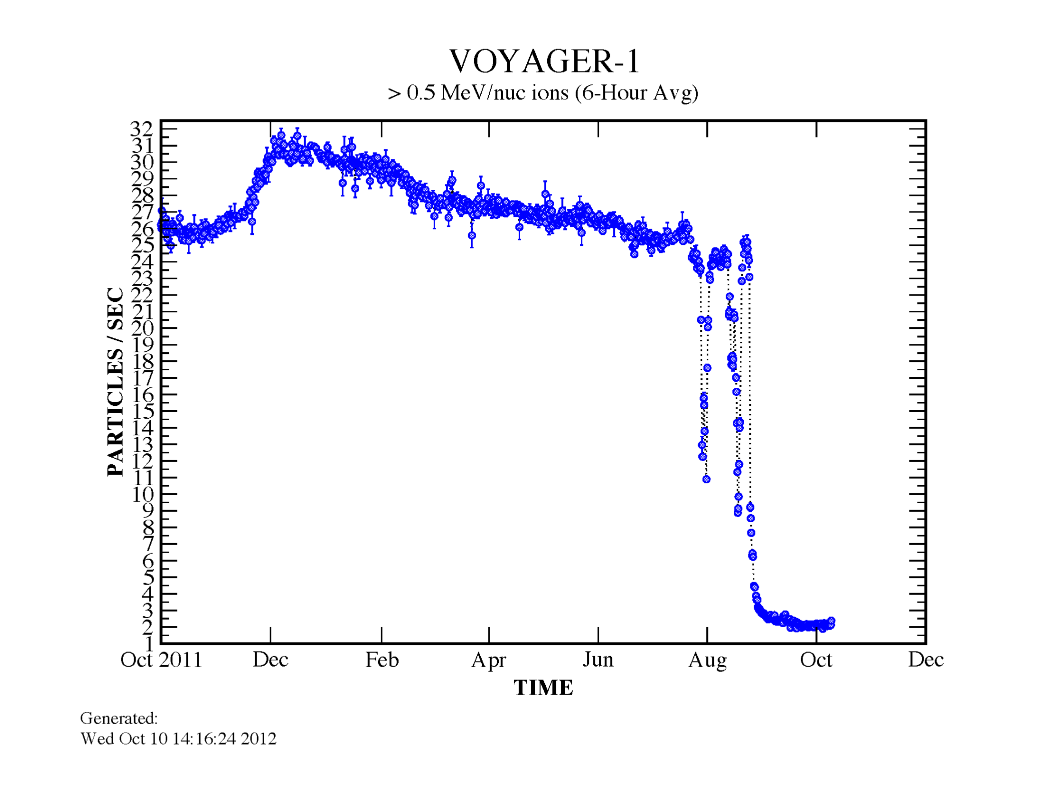
Figure 5. Computed proton LIS (solid black curve) and the computed modulated proton spectrum at the Earth (dashed black curve) compared to the V1 (Stone et al. 2013) and PAMELA observations for 2009b (Adriani et al. 2013b).

Voyager 1

- Cosmic rays

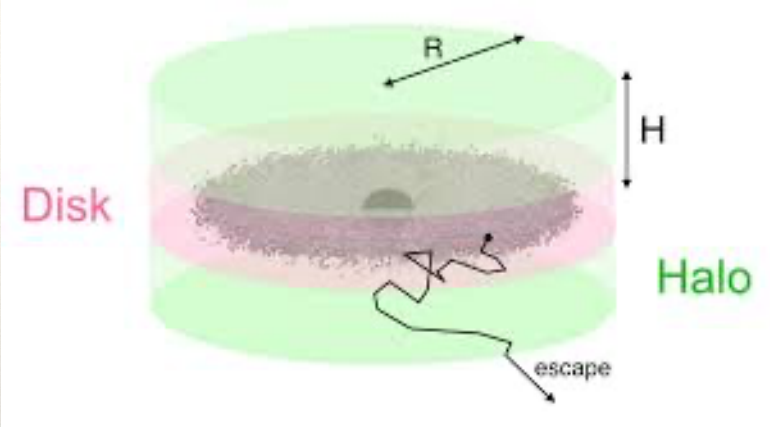


- Solar winds



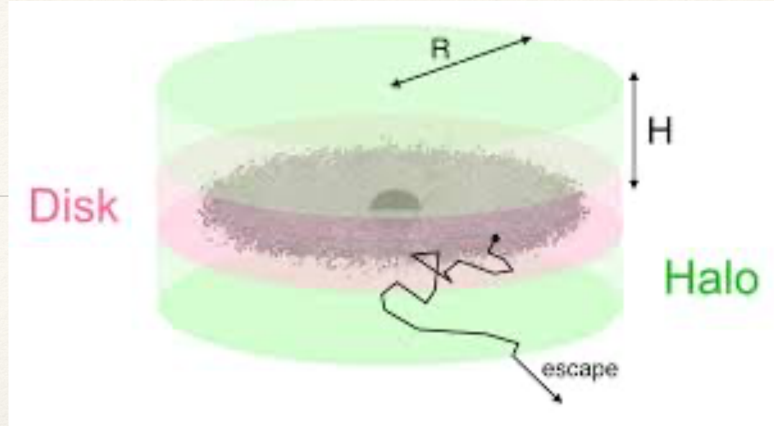
Cosmic Rays in the Milky Way
The leaky box

Let's try to solve it



- By aggressively simplify the problem.
- $\frac{\partial}{\partial t} n = D \nabla^2 n - \frac{\partial}{\partial E} \left(n \dot{E}\right) + Q$
- Equilibrium ($\frac{\partial}{\partial t} n = 0$)
- No energy loss, $\dot{E} = 0$
  - This is a good approximation for cosmic ray protons
- $D \nabla^2 n \rightarrow -\frac{n}{T_{esc}(E)}$ [Leaky Box approximation]
  - Assume a cosmic-ray halo, such that cosmic rays are sufficiently diffused, and have a constant density inside the halo
  - + cosmic rays slowly leak out of the halo

Leaky Box Model



- By aggressively simplify the problem.
- $$\frac{\partial}{\partial t}n = D\nabla^2 n - \frac{\partial}{\partial E}\left(n\dot{E}\right) + Q$$
- =>
- $$\frac{n}{T_{esc}} = Q$$
- =>
- $n = Q(E) \times T_{esc}(E)$
- So a source term of $Q(E) = Q_0 E^{-\alpha}$ and escape term $T_{esc} = T_0 E^{-\delta}$
- Would be sufficient to explain the cosmic-ray data.
- **But both the normalization and spectral index are degenerate**