3.1 Neutrino Cosmology

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- Course webpage: https://blackboard.cuhk.edu.hk
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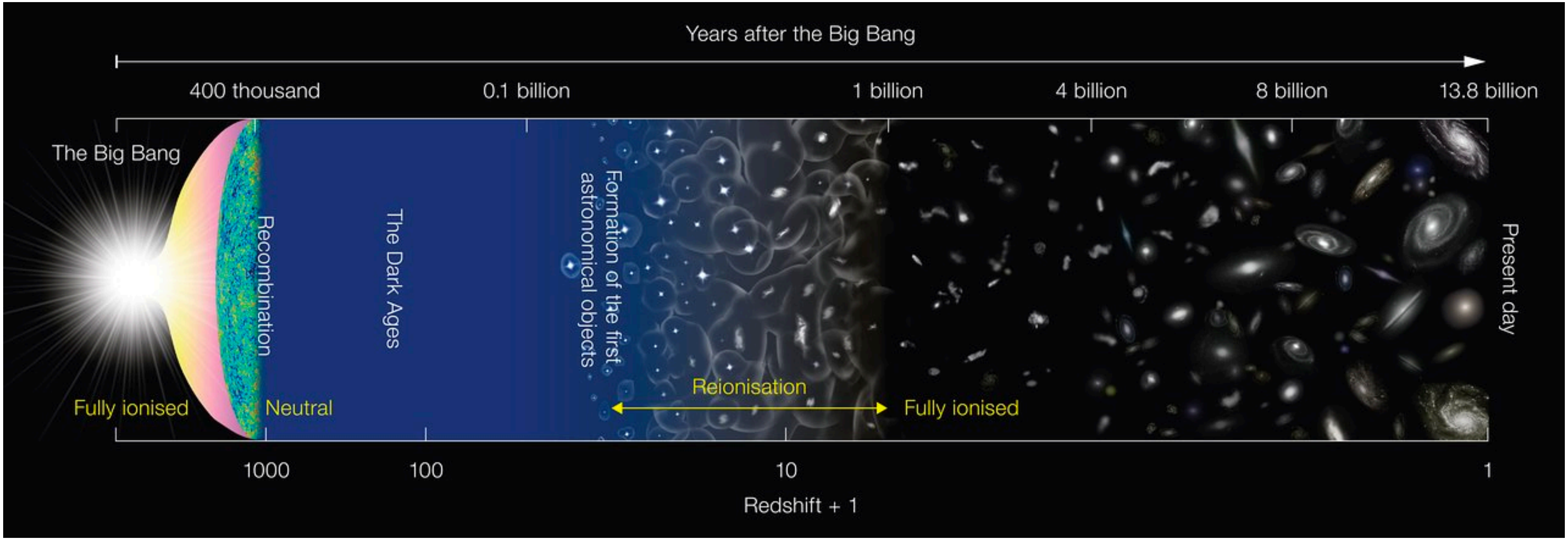
Quck overview of our Universe:
ΛCDM model



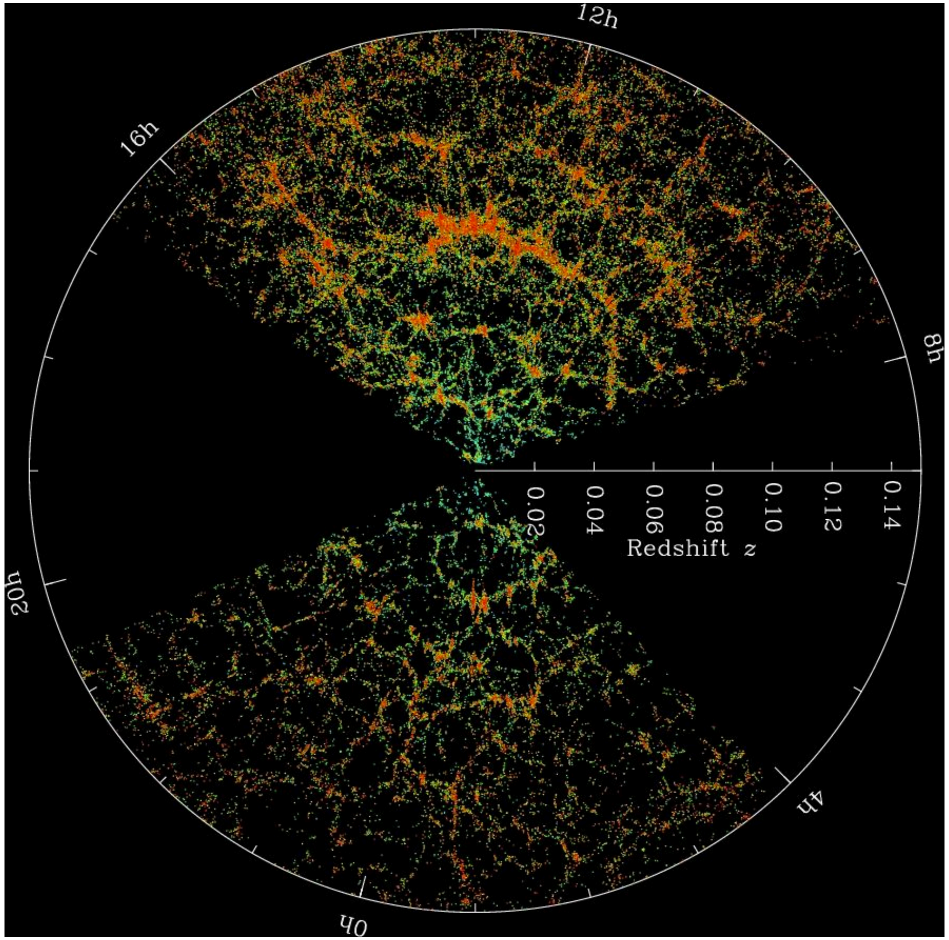
https://www.eso.org/public/images/eso1620a/

- Inflation -> Hot soup (big bang) -> Recombination (CMB) ->
- Reionization -> Structure formation -> Star formation
- Planets -> Life

The cosmological principles

- On large scales, the universe is
  - Isotropic
  - Homogeneous

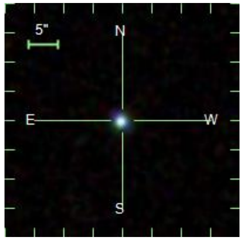
Hubble deep field

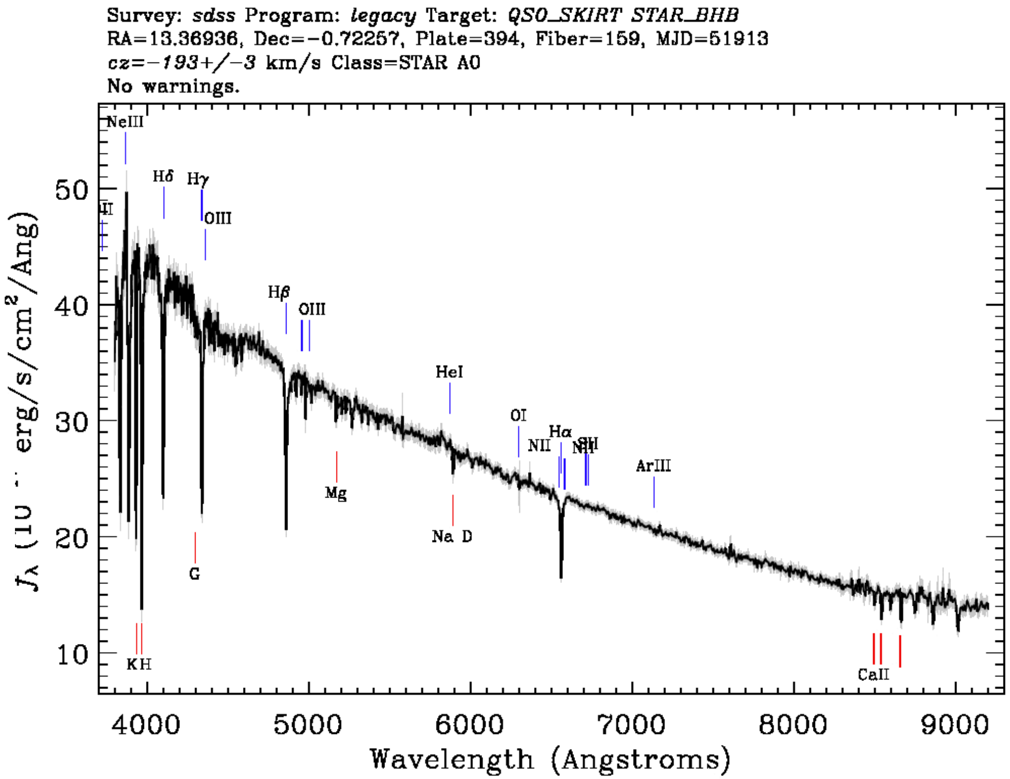


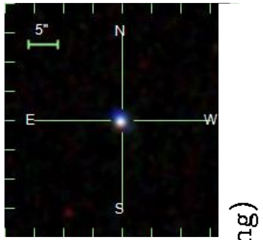
SDSS galaxies

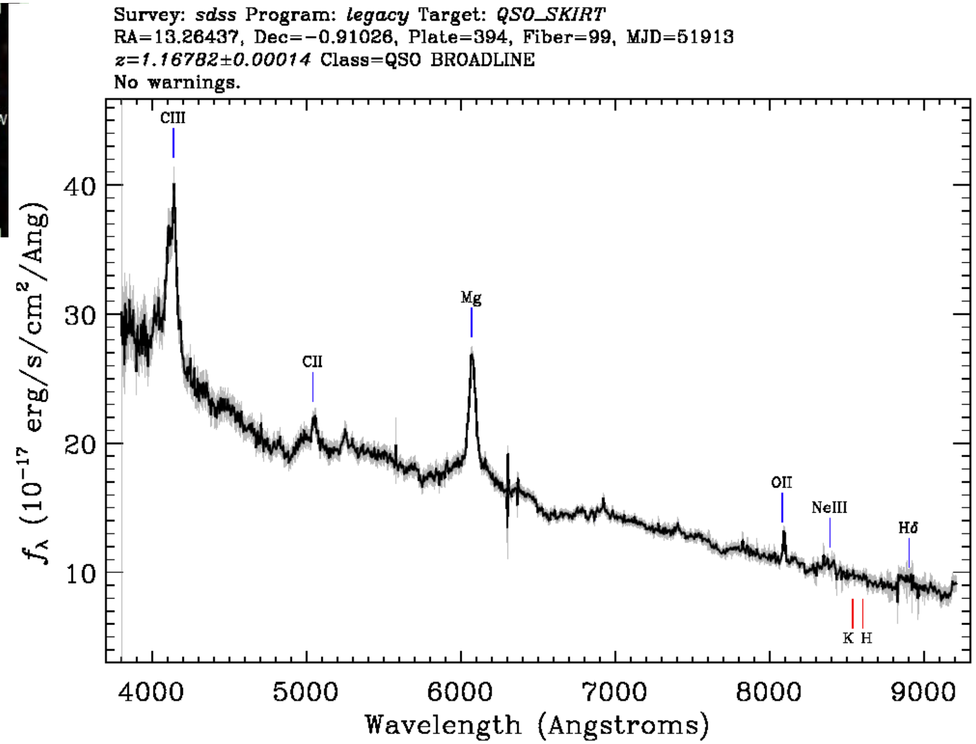
Hubble's discovery

- Measure velocity with line shifts
- Redshift -> moving away
- Blueshift -> moving towards observer







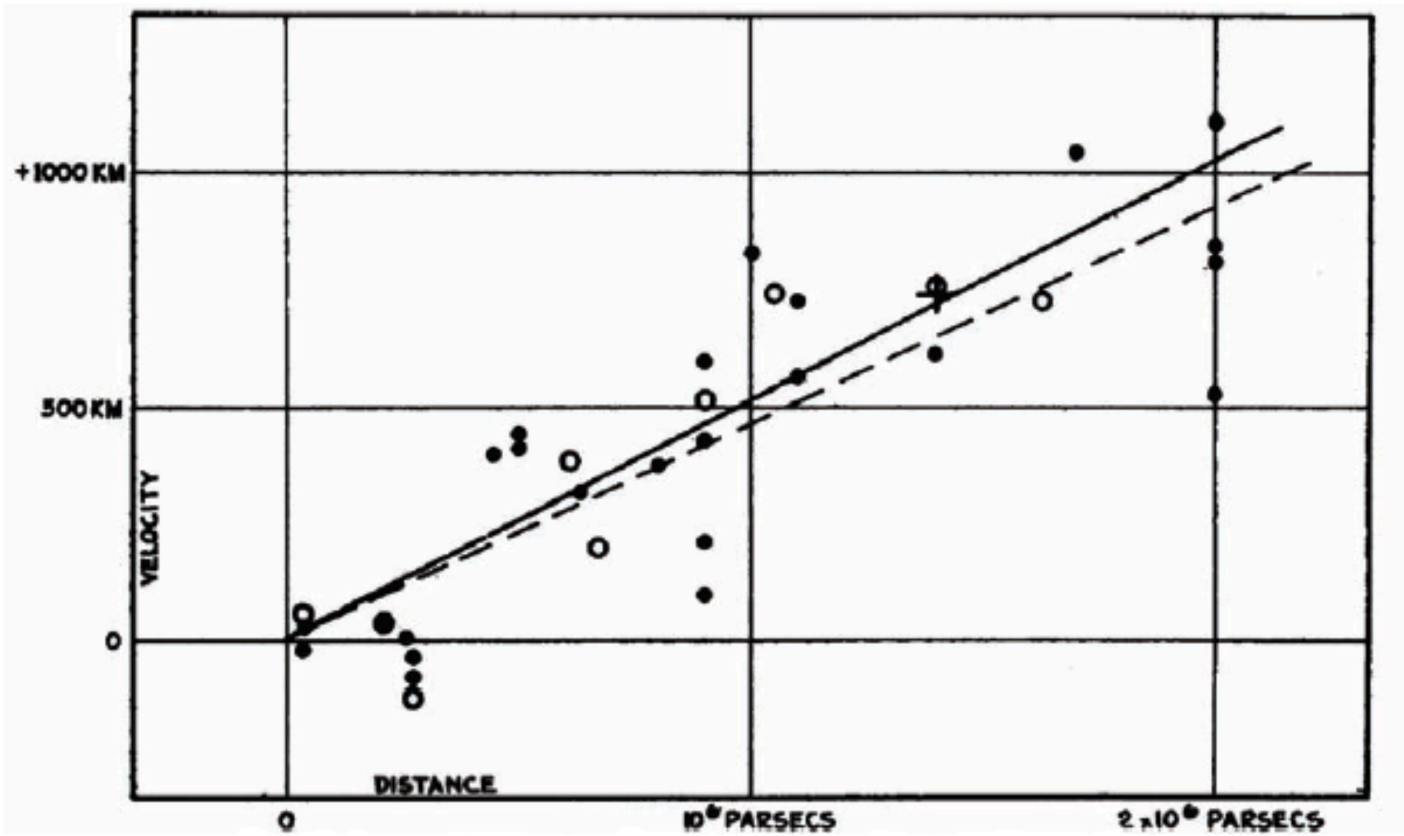


- $$z = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$$

http://voyages.sdss.org/preflight/light/spectra/

Hubble's discovery

- Velocity ∝ distance
- $v = Hd$
- The Universe must be expanding!



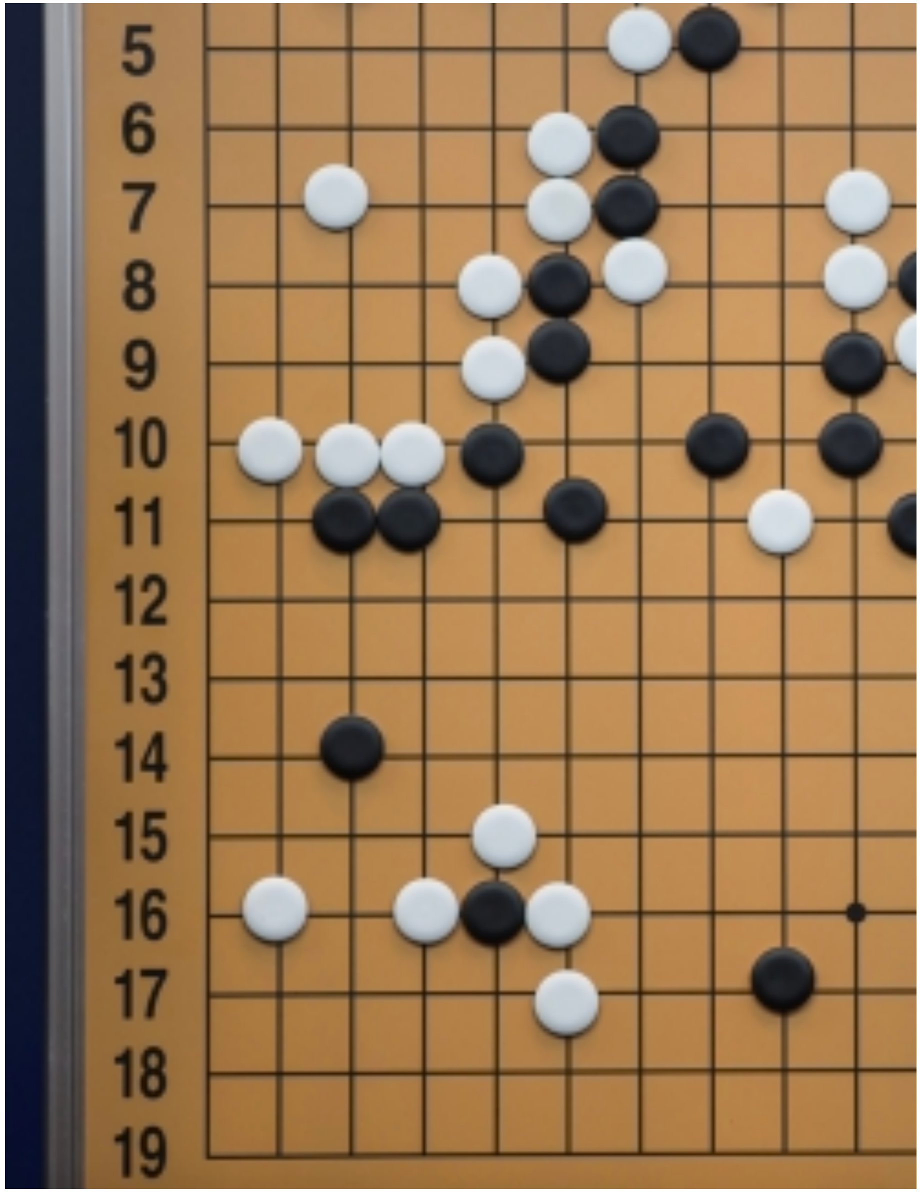
Hubble's graph of redshift versus distance. (Hubble, Proceedings of the National Academy of Sciences, 1929, 15, 168)

Expanding Universe

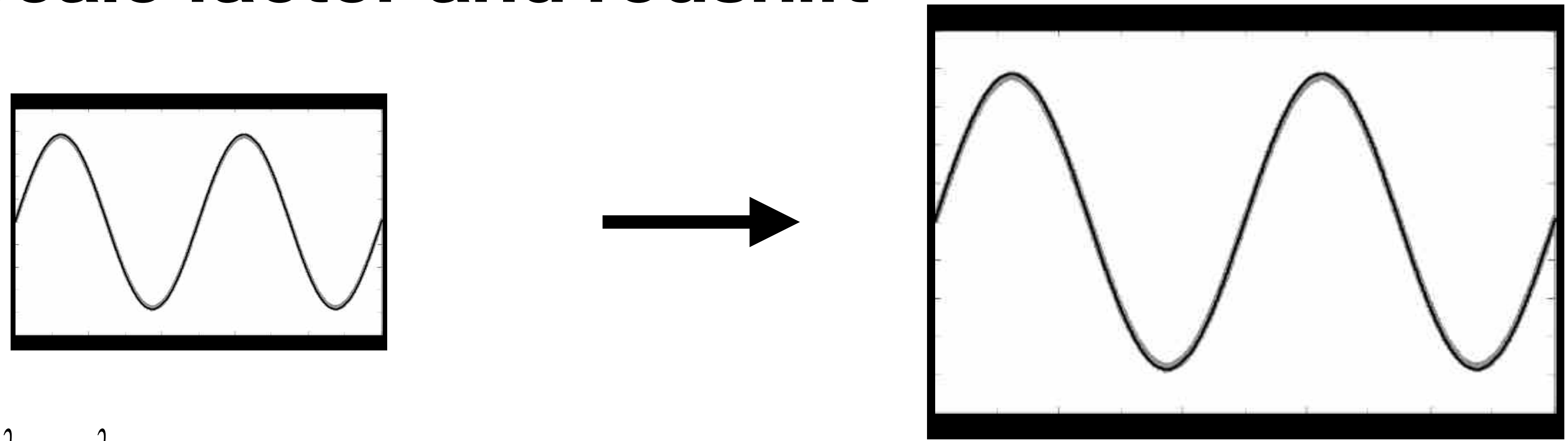
- d: distance between two points
- a: The spacing of of the grid points
  - **The scale factor**
- Δx: the coordinate ‘distance'

- $d = a\Delta x$
- $\dot{d} = \dot{a}\Delta x + \cancel{a\dot{\Delta x}}$
- $\dot{d} = v = \dfrac{\dot{a}}{a}d$ <==> Hubble’s law

- *Expanding (+time) => contracting (-time)*



The Scale factor and redshift



- $z = \dfrac{\lambda_o - \lambda_e}{\lambda_e}$

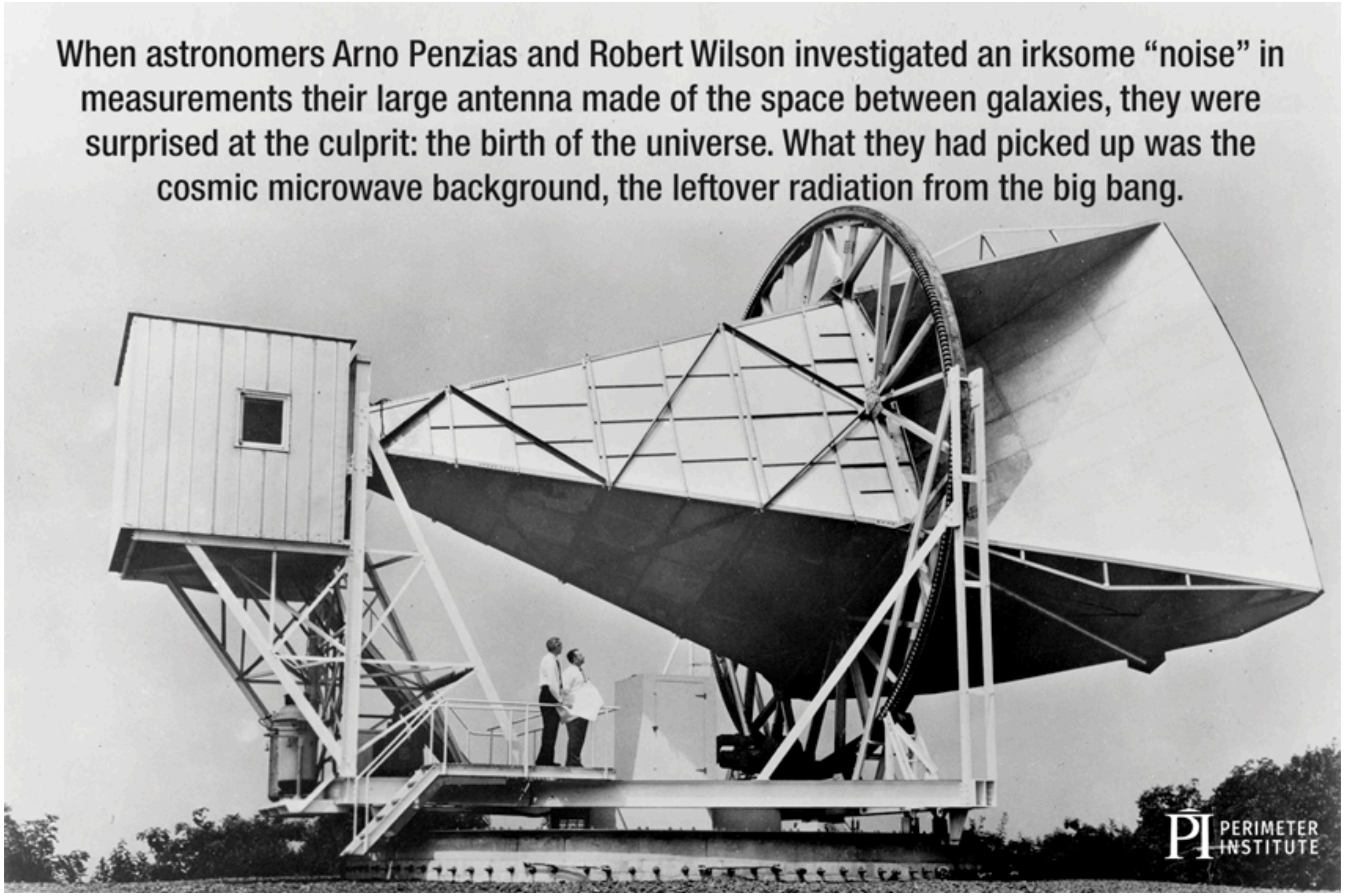
- $\dfrac{\lambda_e}{a(t_e)} = \dfrac{\lambda_o}{a(t_o)}$, $a(t_o) \equiv 1$

- $1 + z = \dfrac{1}{a}$, redshift z can be used to indicate distance, and past time

The Cosmic Microwave Background

- 1960

1978



The Cosmic Microwave Background

- 1960

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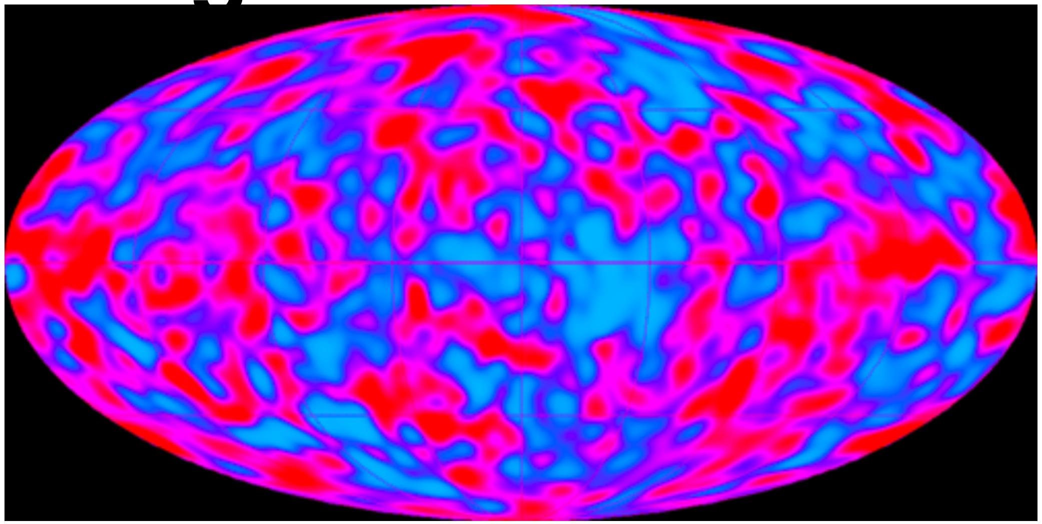
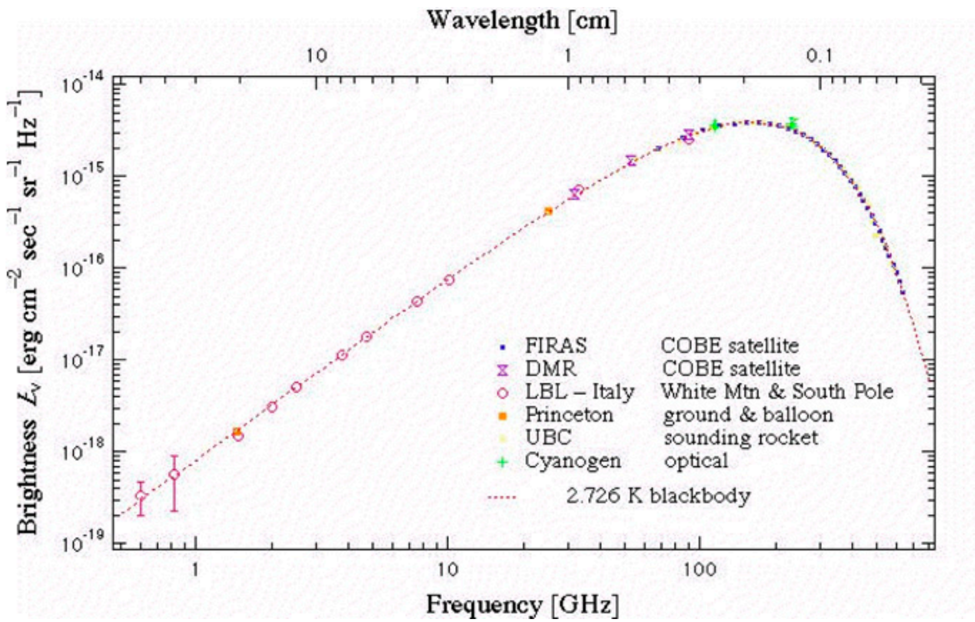
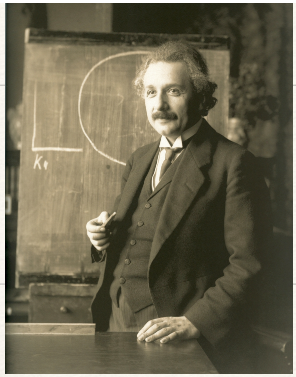


- 1990
- COBE experiment
  - 2.7K
  - smooth at 10^{-5} level

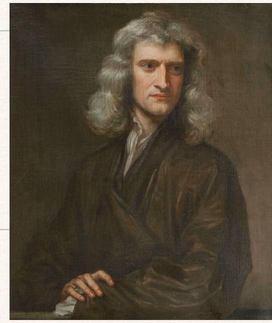
2006



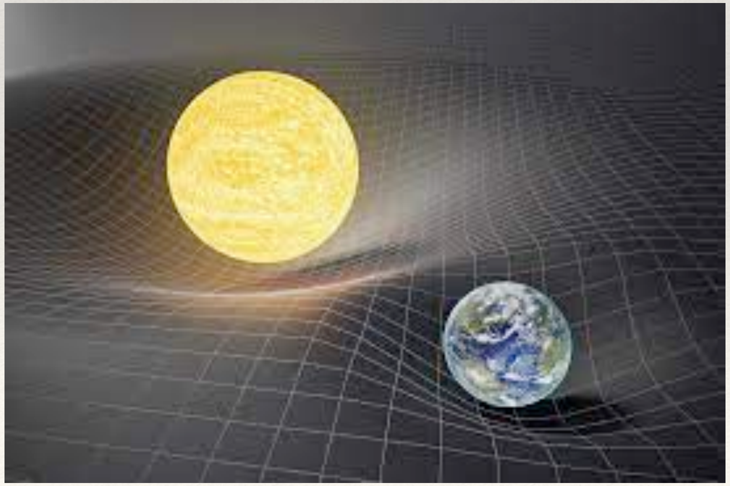
- The early Universe was hot, dense



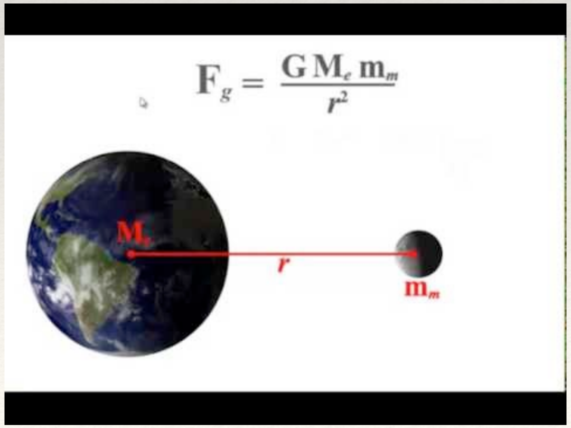
General Relativity



- GR, Einstein told us
- Mass/Energy bends space-time
- space-time determines motion of mass/energy



- Newton's gravity
- Mass generates forces
- Force determines motion
- (massless photons are not affected by Newton's gravity!)



Friedmann Equation

Einstein's equation

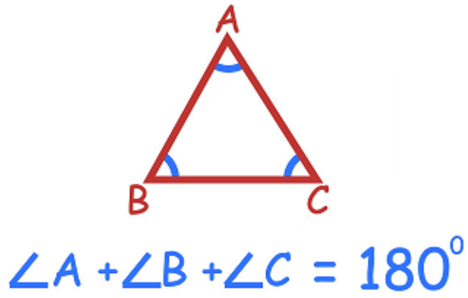
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Space-time geometry
  - solves for the coefficients
- Then stress-energy tensor
  - Mass-energy, pressure

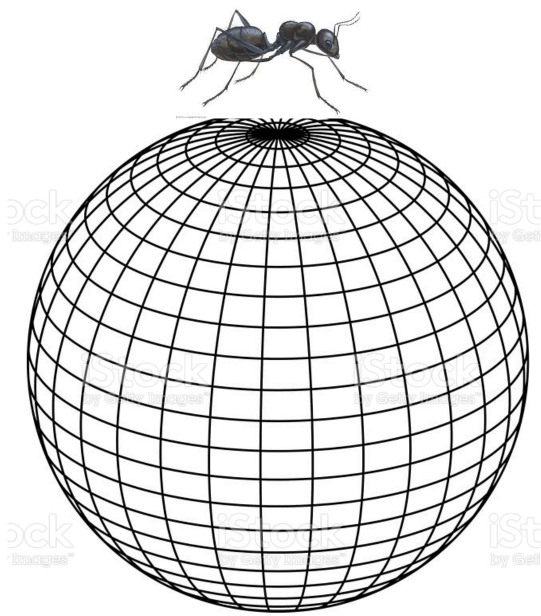
$$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1 - k\frac{r^2}{R^2}} + r^2 d\Omega^2 \right)$$

An expanding spacetime is possible in this theory!

Metric



- Flat space
- $ds^2 = dx^2 + dy^2 + dz^2$ (3D flat space)
- $ds^2 = dr^2 + r^2 d\Omega^2$ (2D flat space in spherical coordinate)
    - $d\Omega^2 = \sin^2\theta d\phi^2 + d\theta^2$



- Sphere, $x^2 + y^2 + z^2 = R^2$ (2D sphere)
- $ds^2 = R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$
- $ds^2 = dr^2 + R^2 \sin^2 \dfrac{r}{R} d\phi^2$

$x = R\cos\phi\sin\theta$

$y = R\sin\phi\sin\theta$

$z = R\cos\theta$

$r = R\theta$

Metric

- $$ds^2 = \begin{cases} dr^2 + r^2 d\Omega^2 \\ dr^2 + R^2 \sin^2 \frac{r}{R} d\Omega^2 \\ dr^2 + R^2 \sinh^2 \frac{r}{R} d\Omega^2 \end{cases}$$

  - FLAT
  - Positively curved, $d\phi^2 \rightarrow d\Omega^2$ (2D -> 3D)
  - Negatively curved: $x^2 + y^2 - z^2 = -R^2$

- $ds^2 = dr^2 + S_\kappa(r)^2 d\Omega^2$

- $r' = S_\kappa(r)$

We will focus only on Flat space in this course

- $$ds^2 = \frac{dr^2}{1 - \frac{\kappa}{R^2} r^2} + r^2 d\Omega^2 \ , \quad r' \rightarrow r \quad , \quad \kappa = 0, +1, -1$$

The Friedmann-Lemaitre-Robertson-Walker (FLRW) Metric

- $$ds^2 = -c^2dt^2 + a(t)^2\left[\frac{dr^2}{1-\frac{\kappa}{R^2}r^2} + r^2d\Omega^2\right]$$
- $a(t)$ is the scale factor, $a(t = \mathrm{now}) = 1$ by definition.
- $\kappa =$ 0, +1, or -1, describing a flat, closed, or open universe, with R as the radius of the curvature

- Plug this metric into Einstein's equation -> Friedmann equation

Expanding Universe

- An expanding sphere (heuristic Newtonian derivation)
- $m\frac{d^2R}{dt^2} = -G\frac{Mm}{R^2}$
- $\frac{1}{2}\frac{d}{dt}(\frac{dR}{dt})^2 = \frac{dR}{dt}\frac{d^2R}{dt^2} = -G\frac{M}{R^2}\frac{dR}{dt} = \frac{GM}{1}\frac{d}{dt}\frac{1}{R}$
- $\frac{1}{2}\left(\frac{dR}{dt}\right)^2 = \frac{GM}{R} + U$
- $R = a(t)r, M = \frac{4}{3}\rho R^3 \quad ------> \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2U}{a^2r^2}$

Friedmann Equation

- $H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2 a^2}$
- ρ : mass density -> energy density
- $\frac{2U}{a^2 r^2} \rightarrow \frac{\kappa c^2}{R^2 a^2}$, Effect of curvature

Cosmic Inventories

Conservation of energy

- $dU = -W$
- $d(\rho V) = -PdV$
- $\dot{\rho}V + \rho\dot{V} = -P\dot{V}$
- $V \propto a^3$ => $\dfrac{\dot{V}}{V} = 3\dfrac{\dot{a}}{a} = 3H$
- $\dot{\rho} = -3H(\rho + P)$
- **the 'Conservation Equation'**

FLRW model

- Friedmann Equation
  - $H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{R^2 a^2}$ ----- (1)
- Conservation equation
  - $\dot{\rho} = -3H(\rho + P)$ ----- (2)
- Differentiate (1) + (2) =>
- The ‘acceleration equation’
  - $\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3P)$

expansion determined by matter/energy

The cosmic inventory

- Equation of state
- $P = w\rho$
- $\dot{\rho} = -3H(1+w)\rho$ from the conservation eq.
- $\dfrac{\dot{\rho}}{\rho} = -3(1+w)\dfrac{\dot{a}}{a}$
- $d\ln\rho = -3(1+w)(d\ln a)$
- $\rho \propto a^{-3(1+w)}$

- $\dfrac{\ddot{a}}{a} = -\dfrac{4\pi}{3}\rho(1+3w)$ $\ddot{a} < 0$ for w > -1/3

The cosmic inventory $\rho \propto a^{-3(1+w)}$

- Matter ($P = 0, w = 0$)
    - $\rho_m \propto a^{-3} \propto (1+z)^3$
- Radiation ($P = \rho/3, w = 1/3$)
    - $\rho_{rad} \propto a^{-4} \propto (1+z)^4$
- Cosmological constant ($P = -\rho, w = -1$)
    - $\rho_\Lambda = \Lambda$
- The curvature 'energy density'
    - $\rho_\kappa \propto a^{-2} \propto (1+z)^2$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{R^2 a^2}$$

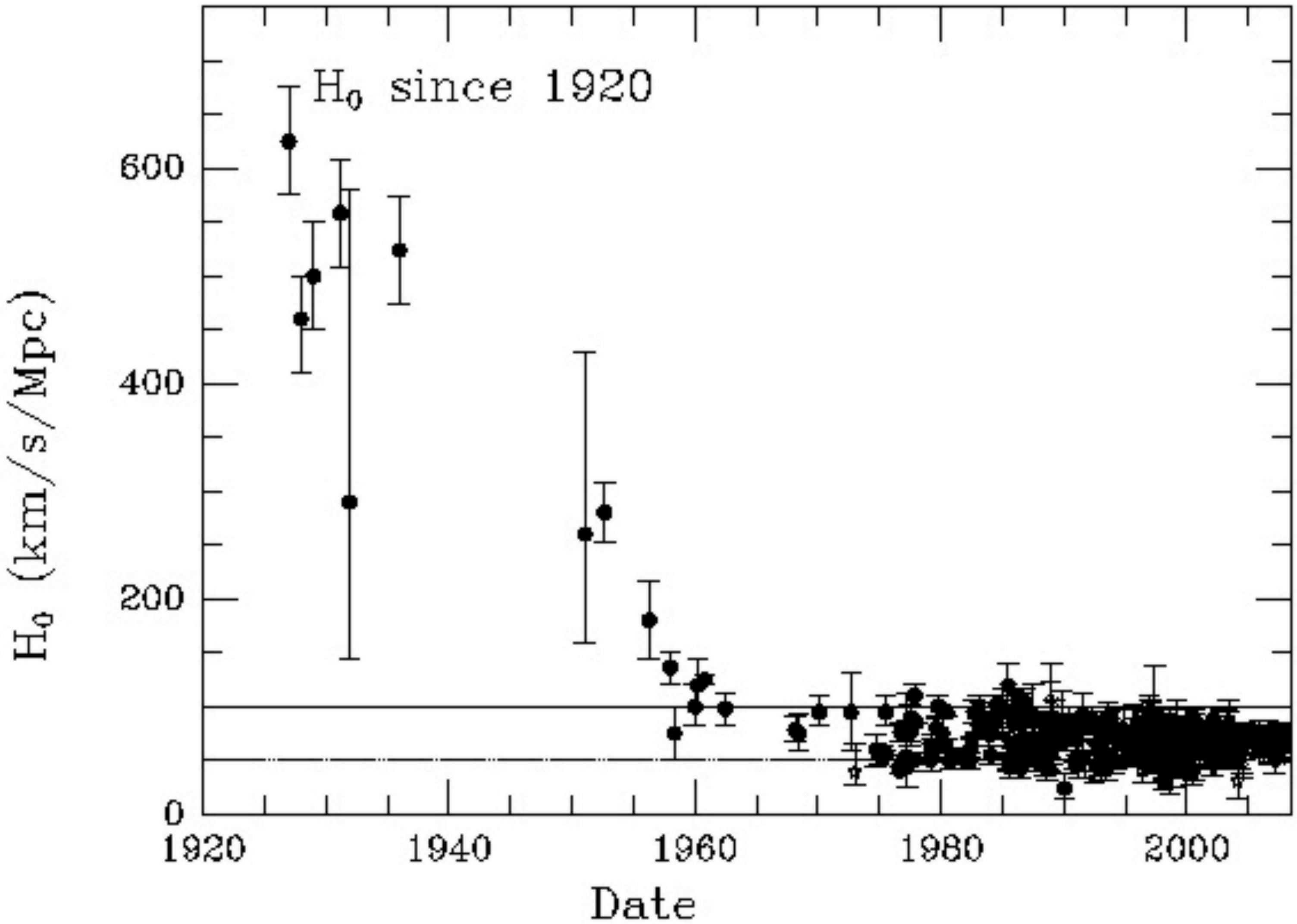
The Critical density

- Define
- $\rho_{cr} = \frac{3H_0^2}{8\pi G}$ (Critical density <=> the universe expand in a flat universe)

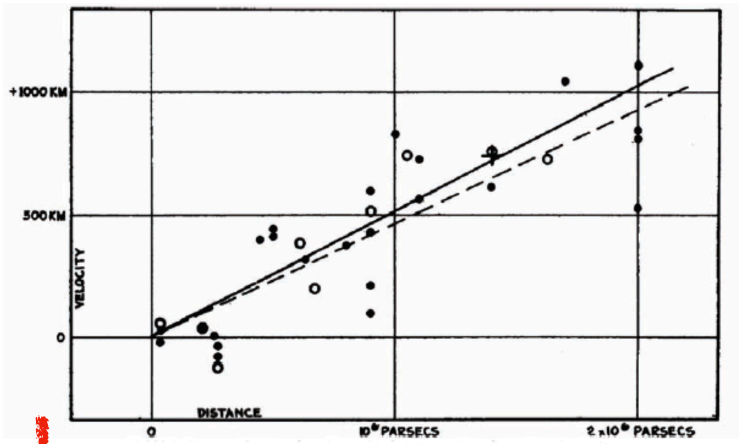
$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{R^2 a^2}$$

- $\Omega_i = \frac{\rho_i}{\rho_{cr}}$
- $H^2/H_0^2 = \sum_i \Omega_i - \frac{\kappa}{R^2 a^2 H_0^2} \implies \frac{\kappa}{R^2 H_0^2} = 1 - \Omega^0_{tot}$
- Rewrite the Friedmann equation
- $H^2 = H_0^2 \left[\Omega_\Lambda + \Omega_m^0 (1+z)^3 + \Omega_{rad}^0 (1+z)^4 + (1-\Omega_{tot}^0)(1+z)^2\right]$
- Just need to know 4 numbers!

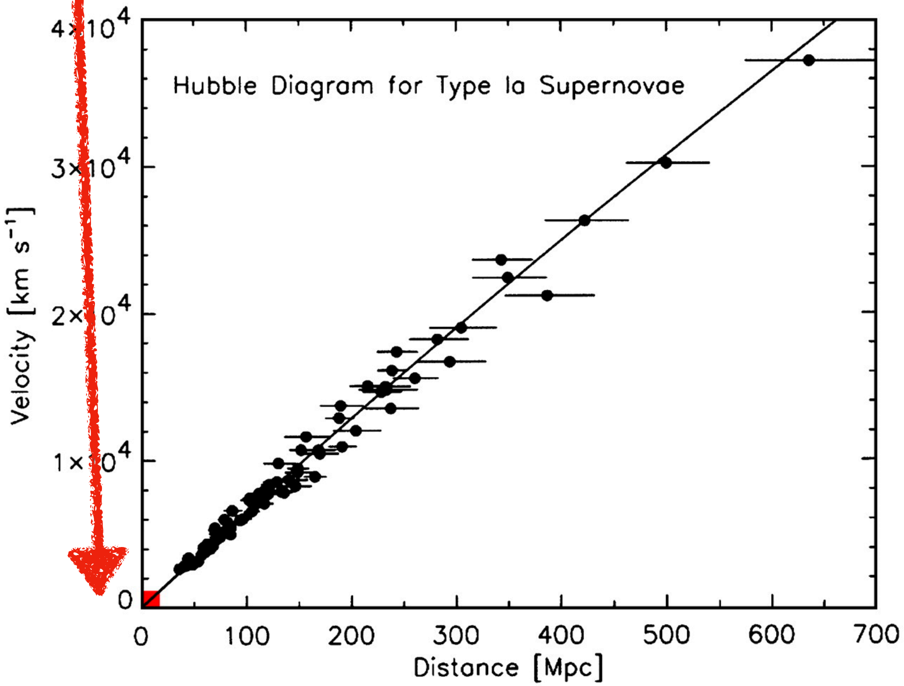
The Hubble Constant

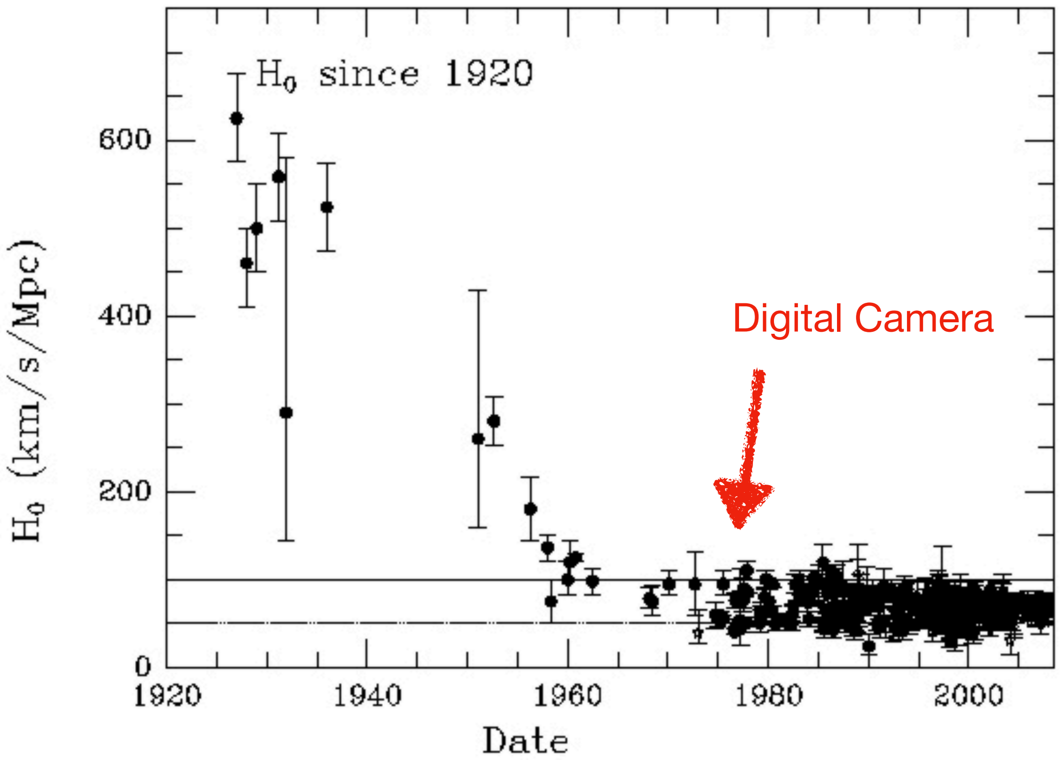


The Hubble Constant



Hubble's graph of redshift versus distance. (Hubble, Proceedings of the National Academy of Sciences, 1929, 15, 168)





$$H_0 \sim 70\mathrm{kms^{-1}Mpc^{-1}}$$

parsec: $\mathrm{pc} \simeq 3\times10^{16}\mathrm{m}$

The age of the Universe t_0

- $\dfrac{\dot{a}}{a} = H(a)$
- $\displaystyle\int dt = \int da \frac{1}{aH}$, $a(t_0) = 1,\ a(0) = 0$

Summaries so far

- The cosmological principle implies metric (for flat space)
- $ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + S_k(r)^2 d\Omega^2\right]$
  - $S_\kappa(r) = r$ for a Flat universe, $\kappa = 0$
- $a(t)$ is called the scale factor
  - $a = 1/(1+z)$
  - Hubbles law $V = Hd = \frac{\dot{a}}{a}d$
  - Note that V here is not a real velocity from Doppler shift
  - The actual physics is that due to expansion of the Universe, the wavelength shifted to be red side (redshift = z). You get the velocity only when you interpret this as the Doppler effect.

- Friedman Equation - (1)
  - $H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2 a^2}$
- Conservation Equation - (2)
  - $\dot{\rho} = -3H(\rho + P)$
- The 'acceleration equation'
  - $\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3P)$
  - (1) + (2) = Acceleration equation
- Equation of state - (3)
  - $P = w\rho$
  - w=0 for matter, w=1/3 for radiation, w=-1 for Cosmological constant
- (2) + (3)
  - $\rho_i = \rho_{i,0} a^{-3(1+w)}$
- (1) + (2) + (3)
- $H^2 = H_0^2\left[\Omega_\Lambda + \Omega_m^0(1+z)^3 + \Omega_{rad}^0(1+z)^4 + (1-\Omega_{tot}^0)(1+z)^2\right]$

Summaries so far

- (1) + (2) + (3)

The script "0" denotes a = 1, or z = 0, or time = Now

- $H^2 = H_0^2 \left[\Omega_\Lambda + \Omega_m^0(1+z)^3 + \Omega_{rad}^0(1+z)^4 + (1-\Omega_{tot}^0)(1+z)^2\right]$

We have defined

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

The critical density is the density required for the Universe to expand at the observed Hubble expansion rate H_0 in a **flat universe**

$$H_0^2 = \frac{8\pi G}{3}\rho_{cr} \quad \text{or} \quad \rho_{cr} = \frac{3H_0^2}{8\pi G}$$

You can then also see that for a flat Universe

$\Omega_{tot}^0 = 1$ <==> $\sum_i \rho_i = \rho_c$

Distance as a cosmological probe

Cosmic distances

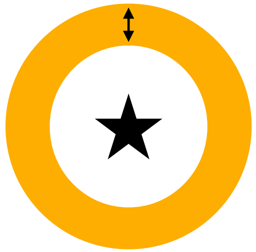
- $ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + r^2d\Omega^2\right]$
- **Proper distance,** radial direction.
  - $d_p(t) = a(t)r$
- **Comoving distance** = proper distance at time = now, t_0
  - $d_p(t_0) = r$
  - $r = \int dr = \int \frac{dt}{a},$ using the metric for light paths having ds = 0
  - $r = \int \frac{1}{a}\frac{dt}{da}\frac{da}{dz}dz$
  - $r = \int \frac{1}{H(z)}dz$

$$\frac{da}{dz} = \frac{d}{dz}\frac{1}{1+z} = -a^2$$

Cosmic distances

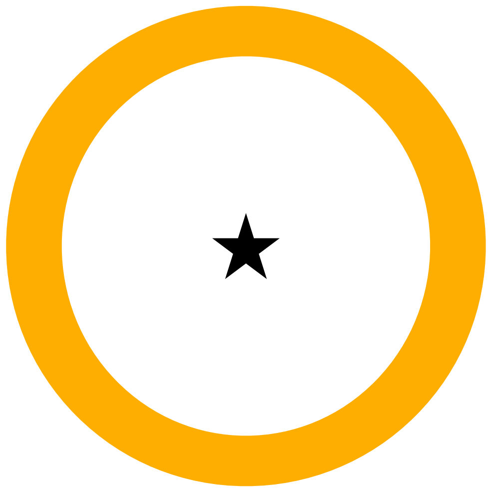
- **Luminosity distance**
- $d_L^2 = \dfrac{L}{4\pi F}$
- L : Luminosity: energy output per unit time
    - **Standard Candle**, known L
- F : Flux: energy per area per time

- Observed energy flux (flat Universe)
- $F = \dfrac{E_o N_{ph}}{4\pi r^2 \Delta t_0} = \dfrac{E_o}{E_e} \dfrac{\Delta t_e}{\Delta t_o} \dfrac{L}{4\pi r^2}$



Emission frame:
Shell width: Δt_e

$$N_{ph} = \frac{\Delta t_e}{E_e} L$$

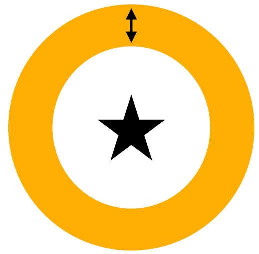


Observer (now):
Shell width: Δt_o

Shell area:
$4\pi \left[a(t_0) S_\kappa(r)\right]^2$
$= 4\pi r^2$

Cosmic distances

- $F = \dfrac{E_o N_{ph}}{4\pi r^2 \Delta t_0} = \dfrac{E_o}{E_e}\dfrac{\Delta t_e}{\Delta t_o}\dfrac{L}{4\pi r^2}$
- $E_e = E_o(1+z)$
- $\dfrac{\Delta t_e}{\Delta t_o} = \dfrac{a(t_e)}{a(t_o)} = \dfrac{1}{1+z}$
- $\dfrac{L}{4\pi F} = d_L^2 = \left[r(1+z)\right]^2$
- $d_L = r(1+z)$



Emission frame:

Shell width: Δt_e

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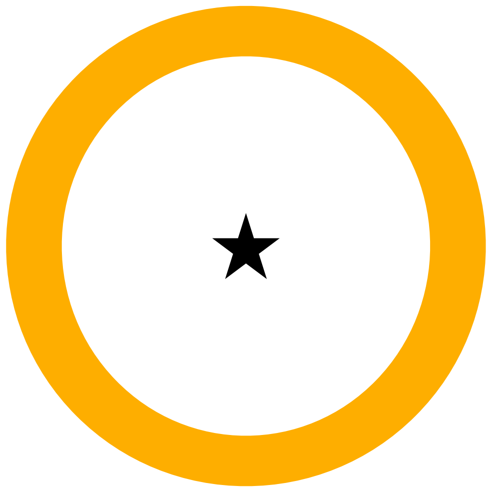


Observer (now):

Shell width: Δt_o

Shell area: $4\pi\left[a(t_0)r\right]^2$

Cosmic distances

- Angular diameter distance
- From a ***standard ruler*** of know physical size, L
- $ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + S_\kappa(r)^2 d\Omega^2\right]$

Distance to the ship = L_{ship}/θ

- $L = a(t_e)S_\kappa(r)\Delta\theta$
- $d_A = \dfrac{L}{\Delta\theta}$
- $d_A = \dfrac{r}{1+z}$ in a flat universe
- $d_A = \dfrac{d_L}{(1+z)^2}$ in general

Computing Luminosity distance (Flat)

$$d_L = r(1+z)$$
$$= (1+z)\int \frac{dz}{H(z)}$$
$$= (1+z)\int dz \left(H_0^{-1} + \frac{d}{dz} H(z)^{-1} \Big|_{z=0} z + \dots \right)$$
$$= \frac{1}{H_0}(1+z)(z + \frac{H_0}{2}\frac{d}{dz} H(z)^{-1} \Big|_{z=0} z^2) + \dots$$

- $$= \frac{1}{H_0}(1+z)(z + \frac{1}{2}\left[-(1+q_0)\right] z^2) + \dots$$

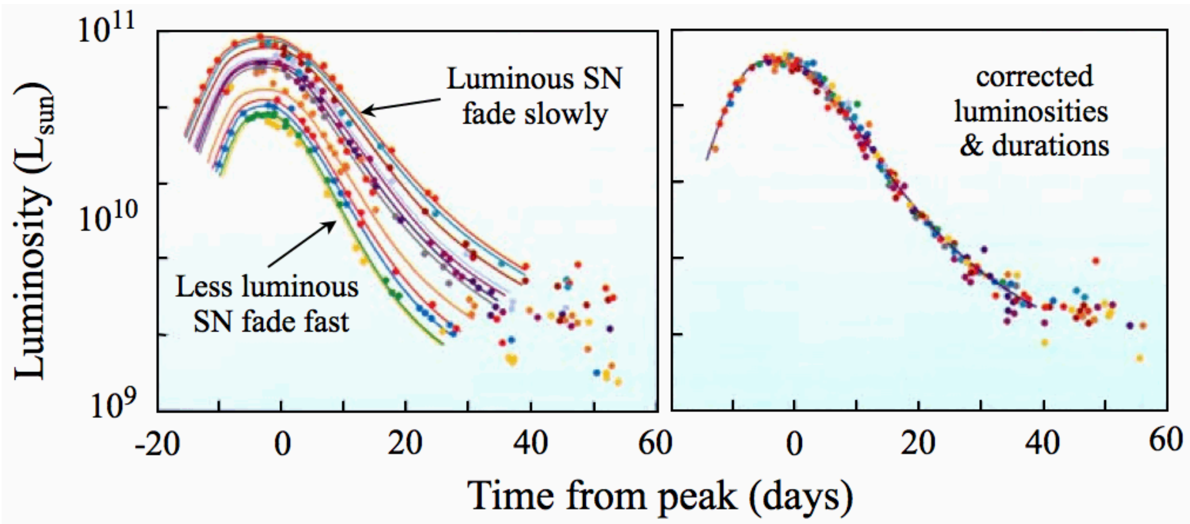
$$= \frac{z}{H_0}(1 + \frac{1-q_0}{2} z) + \dots$$

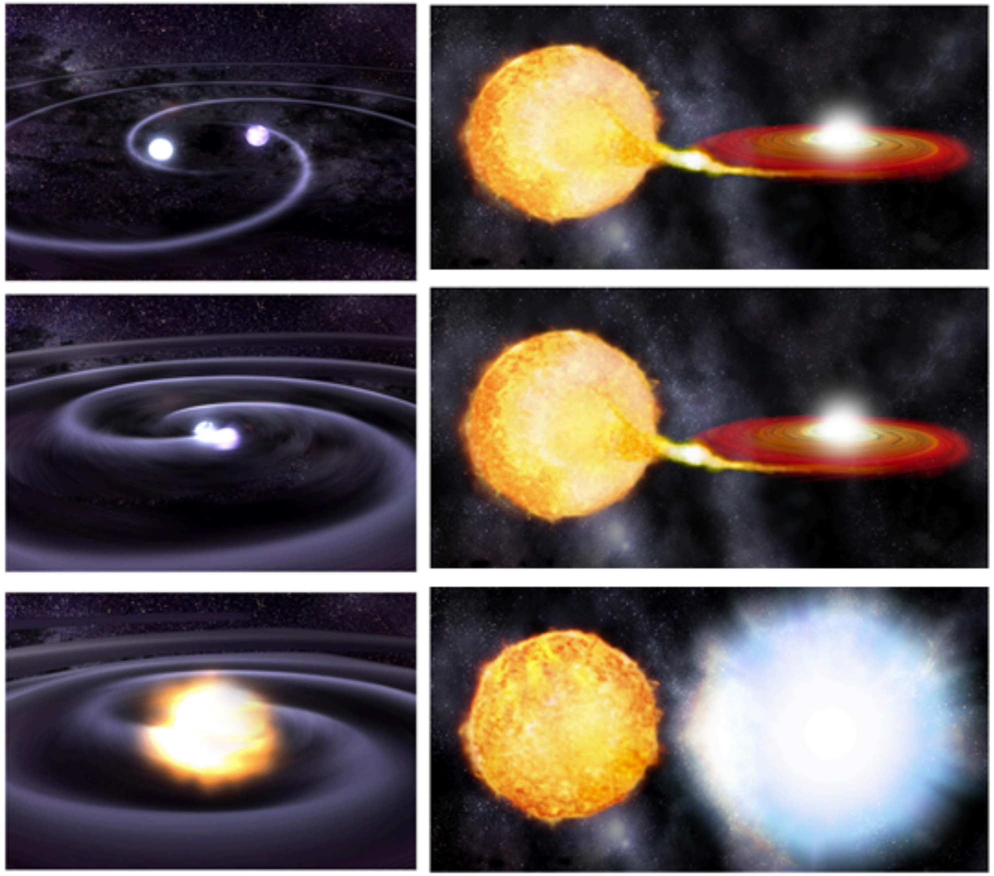
- Using the acceleration equation, you can show that 'Deceleration parameter'
- $$q_0 = -\frac{\ddot{a}}{aH^2}\Big|_{z=0} = \frac{1}{2}\Omega_M^0 + \Omega_r^0 - \Omega_\Lambda$$

Standard Candle

- Type-1a Supernova
- Ignition of white dwarfs
- WD: degenerate star supported by electron pressure (C, O, ….)

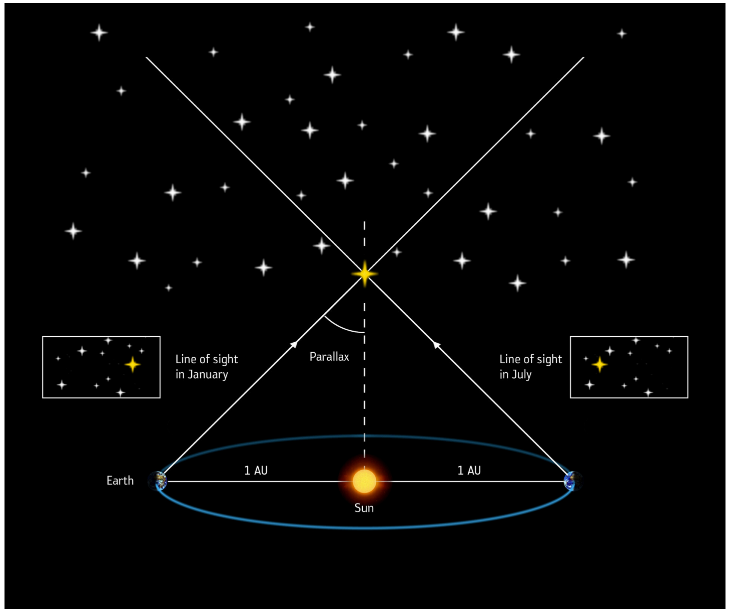
$$^{56}\mathrm{Ni}(\tau = 8.76\,\mathrm{d}) \rightarrow^{56} \mathrm{Co}(\tau = 111.27\,\mathrm{d}) \rightarrow^{56} \mathrm{Fe}$$

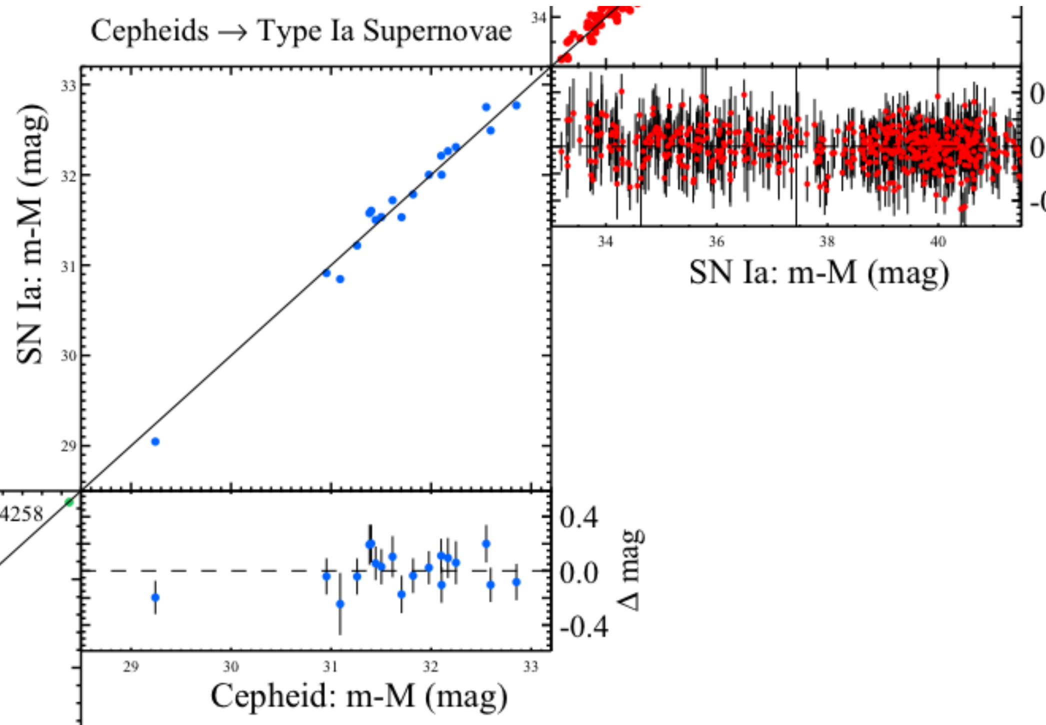


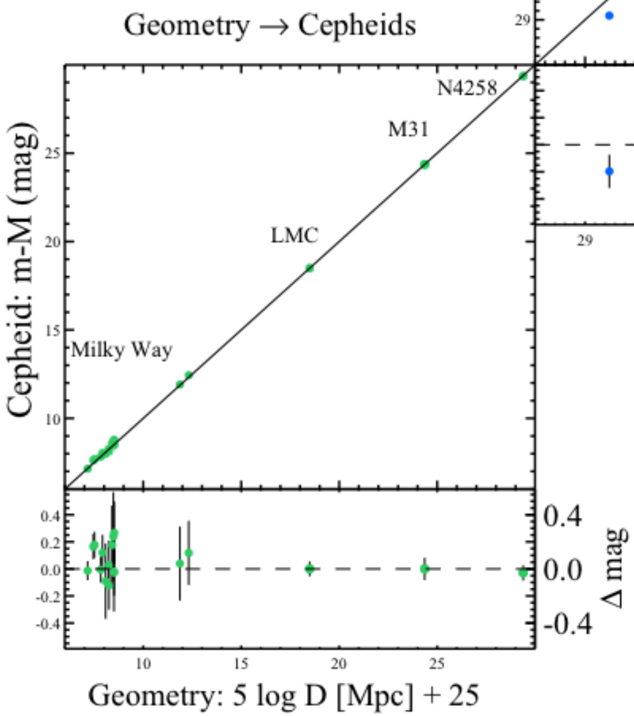


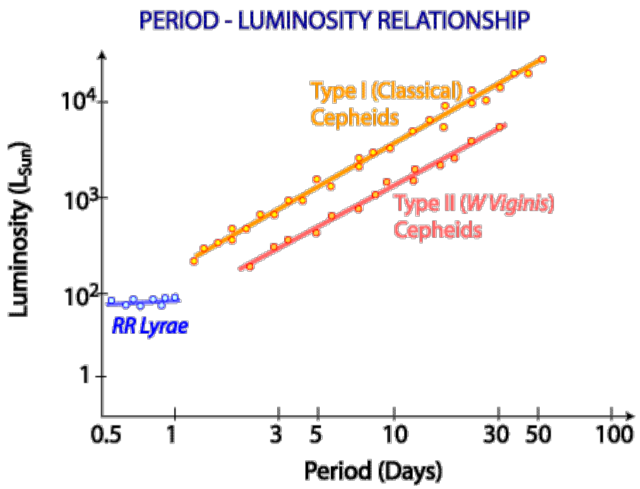
Distance Ladder

- Distance module <=> luminosity distance
- $$m - M = 5\log_{10}\frac{d_L}{Mpc} + 25$$

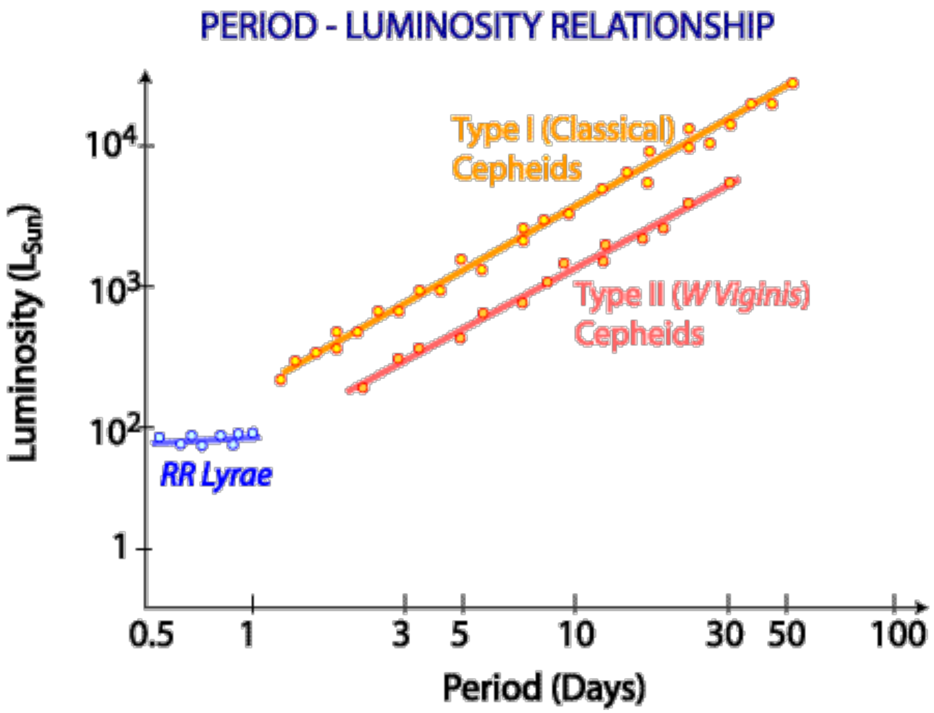






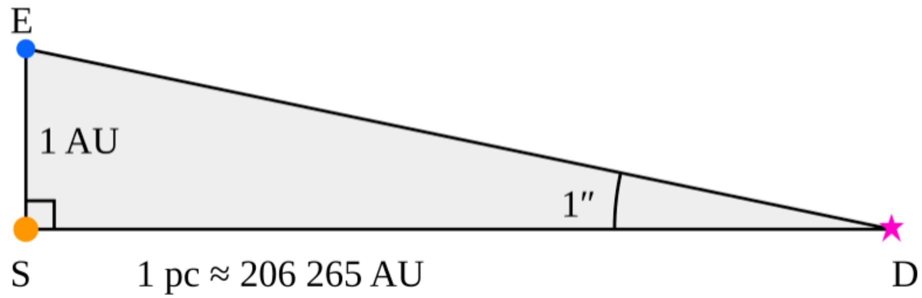


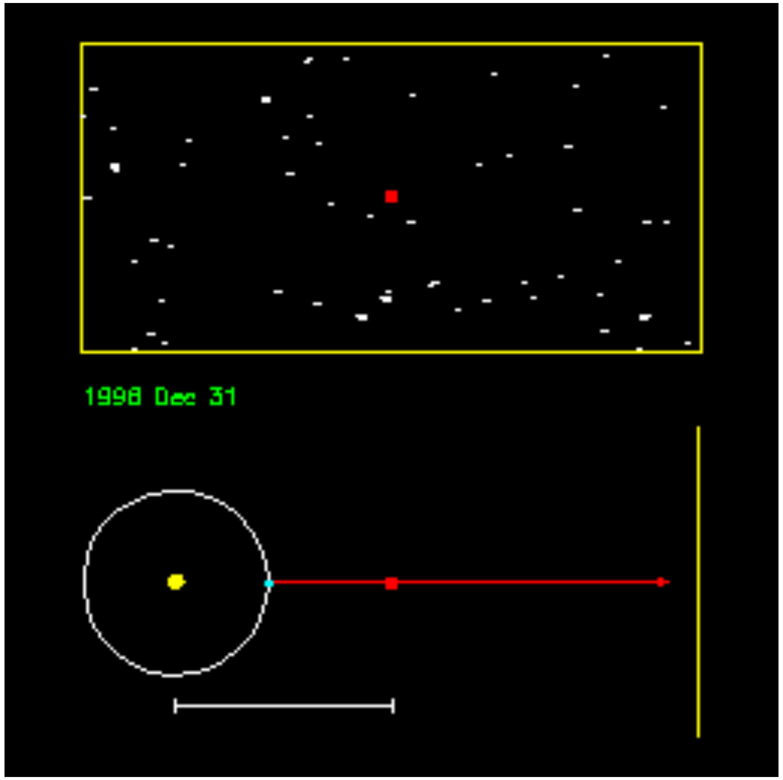
Cepheids

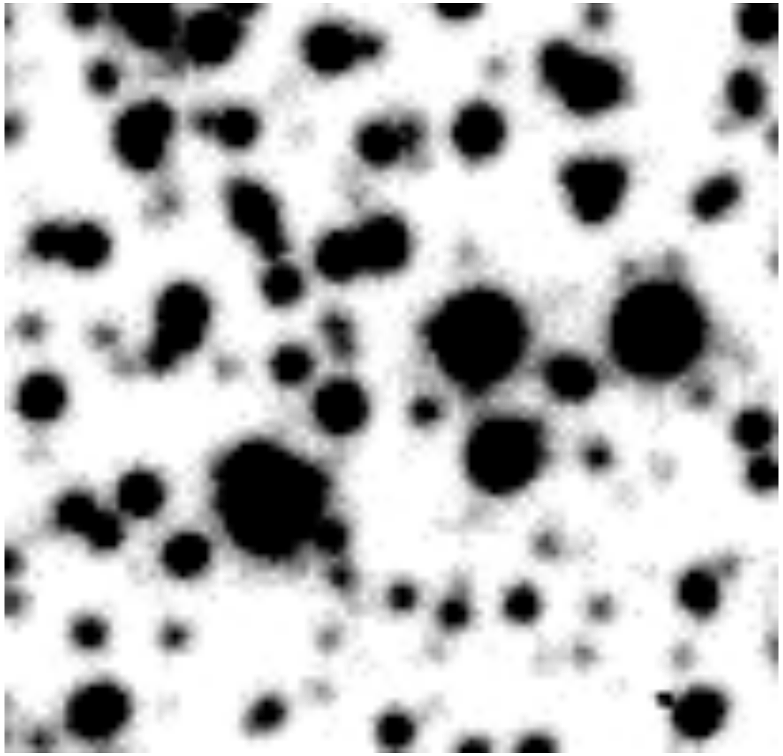

PERIOD - LUMINOSITY RELATIONSHIP
Type I (Classical) Cepheids
Type II (W Viginis) Cepheids
RR Lyrae
Luminosity (L_{Sun})
10^4
10^3
10^2
1
0.5
1
3
5
10
30
50
100
Period (Days)

Parallax

- Definition of Parsec
  - Parallax = 1”







Distance Ladder

arXiv.org > astro-ph > arXiv:1604.01424

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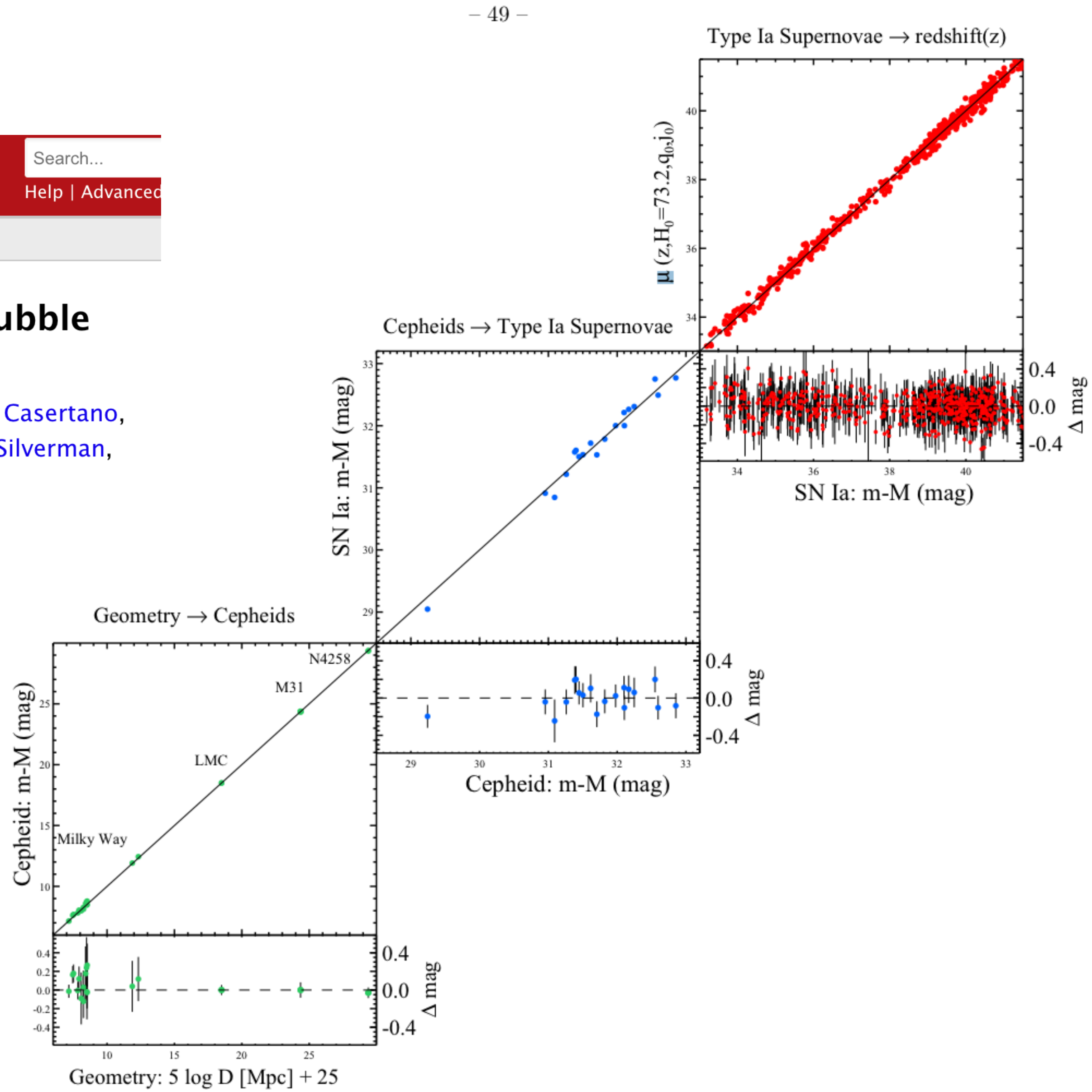
Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 5 Apr 2016 (v1), last revised 9 Jun 2016 (this version, v3)]

A 2.4% Determination of the Local Value of the Hubble Constant

Adam G. Riess, Lucas M. Macri, Samantha L. Hoffmann, Dan Scolnic, Stefano Casertano, Alexei V. Filippenko, Brad E. Tucker, Mark J. Reid, David O. Jones, Jeffrey M. Silverman, Ryan Chornock, Peter Challis, Wenlong Yuan, Peter J. Brown, Ryan J. Foley

- Parallax -> Cepheids -> Supernova
- Fixed absolute distance

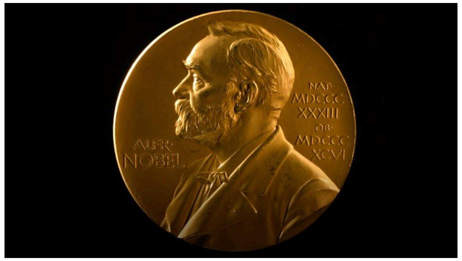


Negative deceleration parameter???

OBSERVATIONAL EVIDENCE FROM SUPERNOVAE FOR AN ACCELERATING UNIVERSE AND A COSMOLOGICAL CONSTANT

Adam G. Riess,[1] Alexei V. Filippenko,[1] Peter Challis,[2] Alejandro Clocchiatti,[3] Alan Diercks,[4] Peter M. Garnavich,[2] Ron L. Gilliland,[5] Craig J. Hogan,[4] Saurabh Jha,[2] Robert P. Kirshner,[2] B. Leibundgut,[6] M. M. Phillips,[7] David Reiss,[4] Brian P. Schmidt,[8,9] Robert A. Schommer,[7] R. Chris Smith,[7,10] J. Spyromilio,[6] Christopher Stubbs,[4] Nicholas B. Suntzeff,[7] and John Tonry[11]

Received 1998 March 13; revised 1998 May 6



The Nobel Prize in Physics 2011

Saul Perlmutter

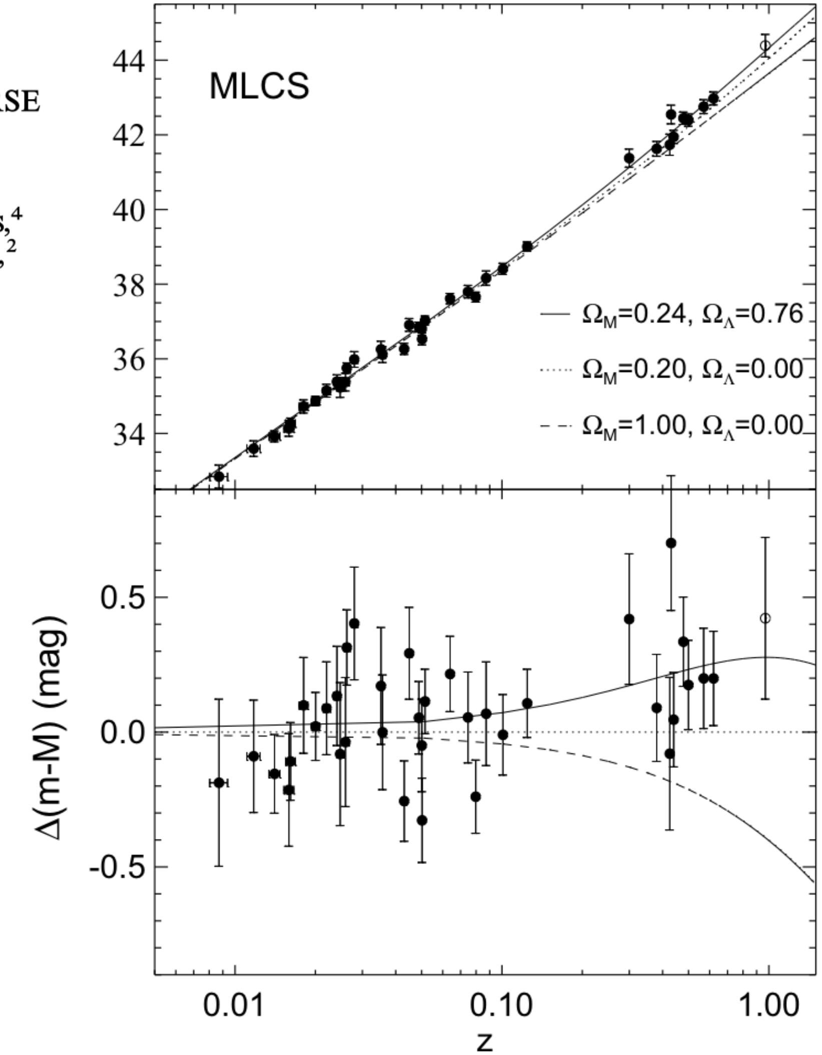
Prize share: 1/2

Brian P. Schmidt

Prize share: 1/4

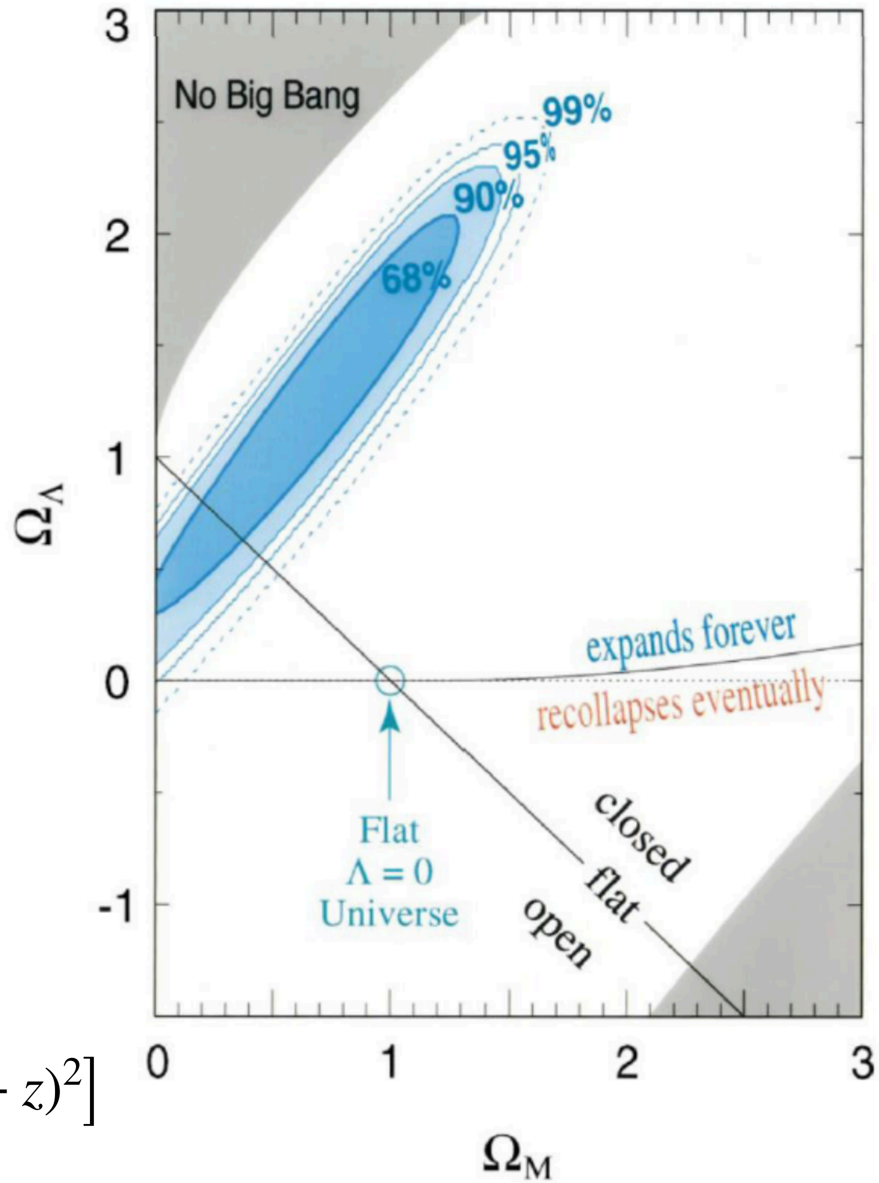
Adam G. Riess

Prize share: 1/4



Supernova cosmology

- Measure Hubble constant
- deceleration parameter $q_0(\Omega_\Lambda, \Omega_m^0)$

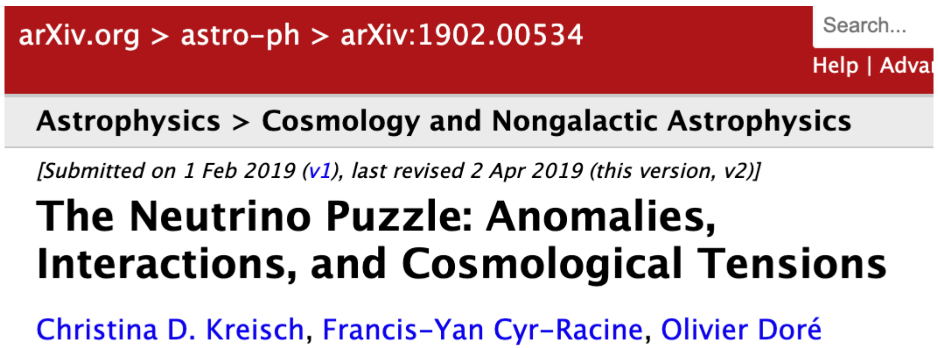


$$H^2 = H_0^2\left[\Omega_\Lambda + \Omega_m^0(1+z)^3 + \Omega_{rad}^0(1+z)^4 + (1-\Omega_{tot}^0)(1+z)^2\right]$$

Hubble tension?

Local measurement vs Early Universe + Model

- very hot topic now



arXiv.org > astro-ph > arXiv:1902.00534

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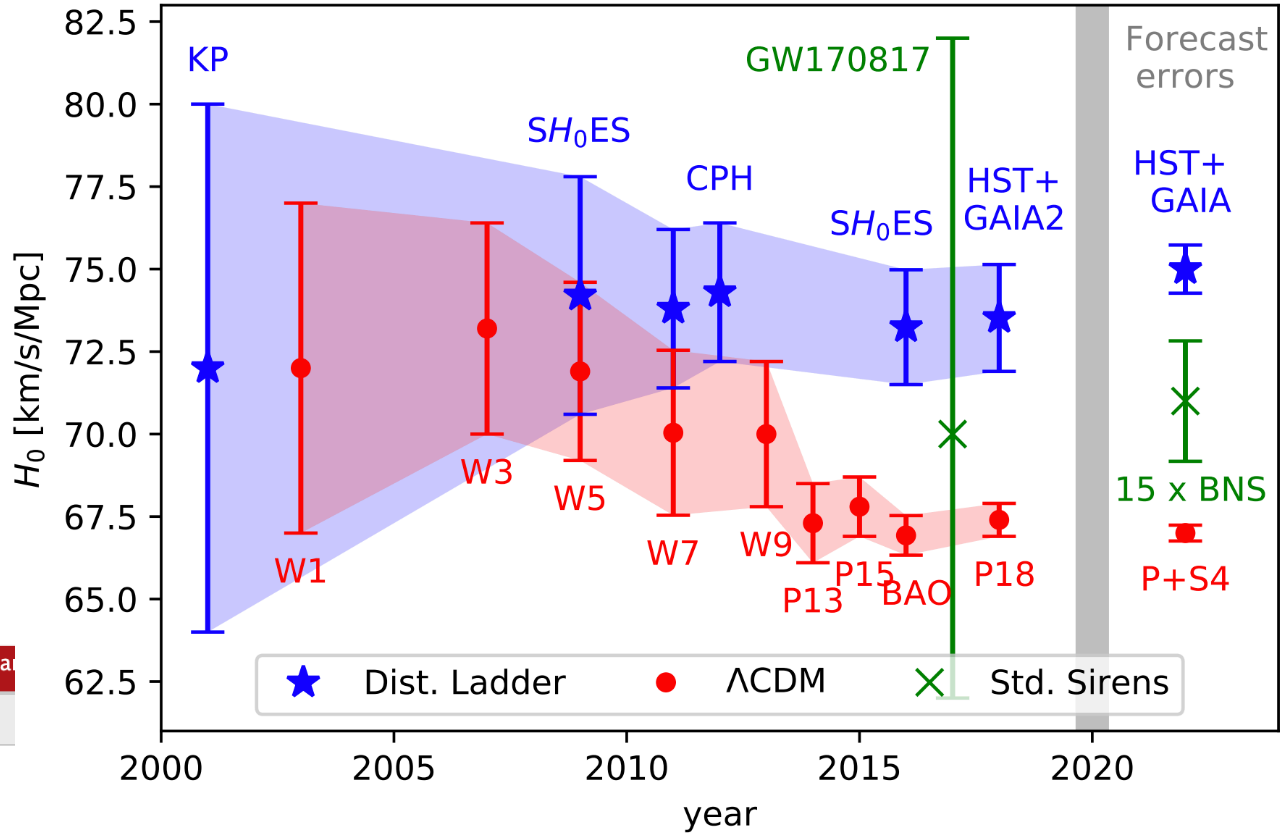
Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 1 Feb 2019 (v1), last revised 2 Apr 2019 (this version, v2)]

The Neutrino Puzzle: Anomalies, Interactions, and Cosmological Tensions

Christina D. Kreisch, Francis-Yan Cyr-Racine, Olivier Doré

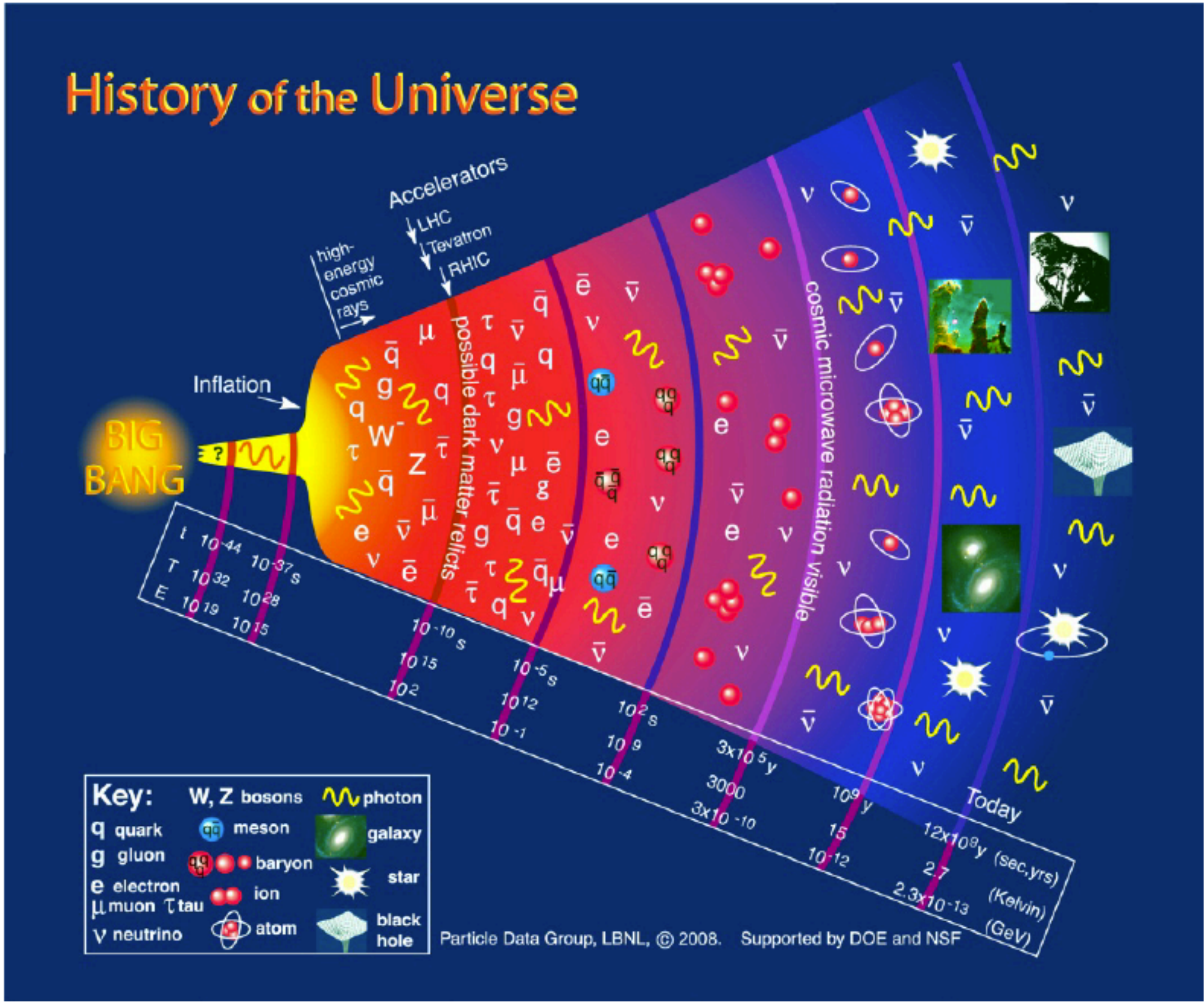


https://arxiv.org/pdf/1807.09241.pdf

Equilibrium Physics

Cosmic hot soup

- When $T > m$, such particles are in thermal equilibrium
- E.g., T > 1 MeV
- $\gamma\gamma \rightleftharpoons e^+e^-$
- So the universe is full of electrons and positron



Equilibrium Physics

- The distribution function of a species, ($\mu = 0$)
- $f = g_i \dfrac{1}{e^{\frac{E}{kT}} \pm 1}$
  - +1: Fermi Dirac, -1: Bose Einstein. 0: Maxwell Boltzmann
  - g_i : degrees of freedom

- $n = \displaystyle\int \frac{d^3p}{(2\pi\hbar)^3} f$ — number density
- $\rho = \displaystyle\int \frac{d^3p}{(2\pi\hbar)^3} f \times E$ — energy density
- $P = \displaystyle\int \frac{d^3p}{(2\pi\hbar)^3} f \times \frac{p^2}{3E}$ — Pressure

Pressure

- Force of the gas on a wall

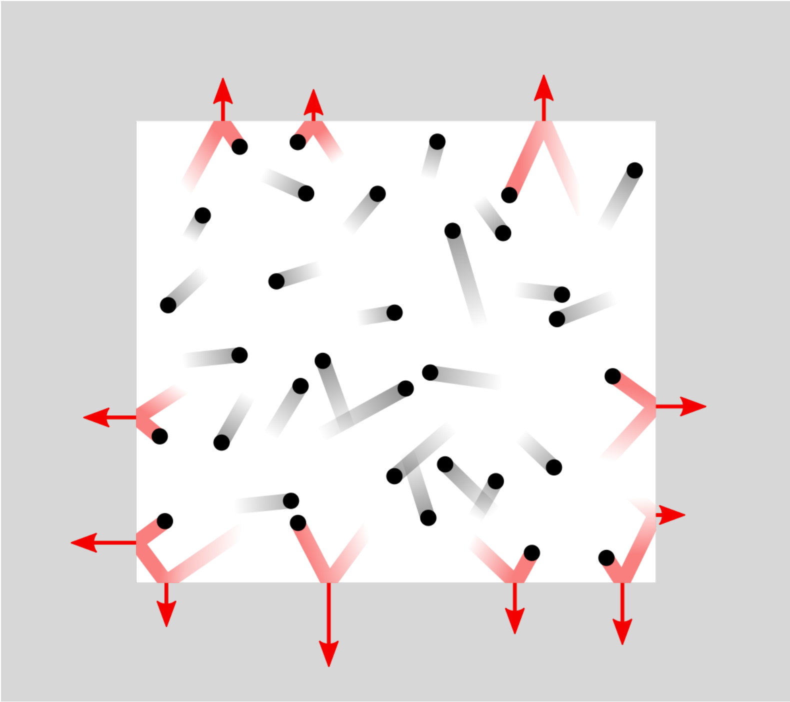
$$P = N\frac{F}{A}$$

$$= N\frac{\Delta p_x}{A\Delta t}$$

$$= N\frac{(2p_x)}{A}\frac{v_x}{2L}$$

- $$= \frac{N}{V}\frac{p}{\sqrt{3}}\frac{v}{\sqrt{3}}$$

$$= \frac{N}{V}\frac{p^2}{3E}$$



Pressure

- $P = \int \frac{d^3p}{(2\pi\hbar)^3} f \times \frac{p^2}{3E}$
- For relativistic species $\frac{p^2}{3E} \rightarrow \frac{p}{3}$
- $P = \frac{\rho}{3}$ $--$ the equation of state

- For non-relativistic particles, $P \rightarrow 0$

How to do these integrals? E.g., Bose Einstein Integrals

- $$I_{BE}(k) = \int_0^\infty dx \frac{x^{k-1}}{e^x - 1}$$
  $$= \int dx e^{-x}(x^{k-1})(1 - e^{-x})^{-1}$$
  $$= \int dx e^{-x}(x^{k-1}) \sum_{j=0} (e^{-x})^j$$
  $$= \sum_{j=0} \int dx e^{-x(j+1)}(x^{k-1})$$
  $$= \sum_{j=0} \frac{1}{(j+1)^k} \int dy e^{-y}(y^{k-1}) \qquad y = x(j+1)$$
  $$= \zeta(k)\Gamma(k)$$
- Riemann Zeta function and Gamma function

- Gamma function
  - $\Gamma(k) = (k-1)!$
- Riemann Zeta function
  - $\xi(2) = \dfrac{\pi^2}{6}$
  - $\xi(3) = 1.202$
  - $\xi(4) = \dfrac{\pi^4}{90}$

Fermi-Dirac integrals

$$
\begin{aligned}
I_{FD}(k) &= \int dx \frac{x^{k-1}}{e^x + 1} \\
&= \int dx e^{-x}(x^{k-1})(1 + e^{-x})^{-1} \\
&= \int dx e^{-x}(x^{k-1}) \sum_{j=0} (-1)^j (e^{-x})^j \\
&= \sum_{j=0} (-1)^j \int dx e^{-x(j+1)}(x^{k-1})
\end{aligned}
$$

- $$= \sum_{j=0} \frac{(-1)^j}{(j+1)^k} \int dy e^{-y}(y^{k-1}) \qquad y = x(j+1)$$
  $$= \eta(k)\Gamma(k)$$

-

How to do these integrals? For Fermi-Dirac

- $\eta(k) = \sum_{j=0} \frac{(-1)^j}{(j+1)^k}$

- $\eta(k) = \frac{1}{1^k} - \frac{1}{2^k} + \frac{1}{3^k} - \frac{1}{4^k} + \ldots$
- $\xi(k) = \frac{1}{1^k} + \frac{1}{2^k} + \frac{1}{3^k} + \frac{1}{4^k} + \ldots$
- $\xi(k) - \eta(k) = \frac{2}{2^k} + \frac{2}{4^k} + \frac{2}{6^k} + \ldots = \frac{2}{2^k}\xi(k)$
- $\eta(k) = (1 - \frac{1}{2^{k-1}})\xi(k)$

Bose-Einsten and Fermi-Dirac Integrals

- $I_{BE}(k) = \int dx \frac{x^{k-1}}{e^x - 1} = \zeta(k)\Gamma(k)$
- $I_{FD}(k) = \int dx \frac{x^{k-1}}{e^x + 1} = \left(1 - \frac{1}{2^{k-1}}\right) \xi(k)\Gamma(k)$

Number density

- $n = \int \frac{d^3p}{(2\pi)^3} f$
- Bosons:
  - $n_{BE} = g_i \frac{\zeta(3)}{\pi^2} T^3$ $\quad \zeta(3) \simeq 1.202$
- Fermions:
  - $n_{FD} = \frac{3}{4} n_{BE}(g_i)$
- Non-relativistic matter:
  - $n_{MB} = g_i \left(\frac{mT}{2\pi} \right)^{3/2} e^{-\frac{m}{T}}$

$$1 = c \simeq 3 \times 10^8 \, \mathrm{ms}^{-1}$$

$$1 = \hbar c \simeq 200 \, \mathrm{MeVfm}$$

$$1 = k \simeq 10^{-4} \, \mathrm{eVK}^{-1}$$

Energy density

- $\rho = \int \frac{d^3p}{(2\pi)^3} f \times E$
- Relativistic Bosons:
  - $\rho_{BE} = g_i \frac{\pi^2}{30} T^4$
- Relativistic Fermions:
  - $\rho_{FD} = \frac{7}{8} \rho_{BE}(g_i)$
- Non-relativistic matter:
  - $\rho_{MB} = m n_{MB}$

$$\rho_{rad} \propto (1+z)^4 = a^{-4}$$

$$T \propto (1+z)$$

CMB photon density

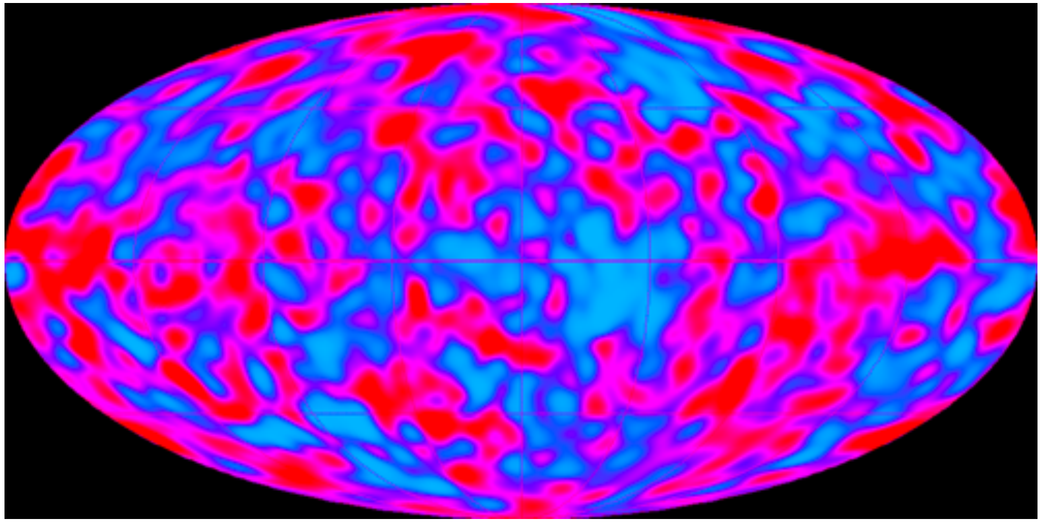
- T = 2.7 K
- $g_\gamma = 2$ for two polarisation states
- $\rho_{BE} = g_i \frac{\pi^2}{30} T^4$

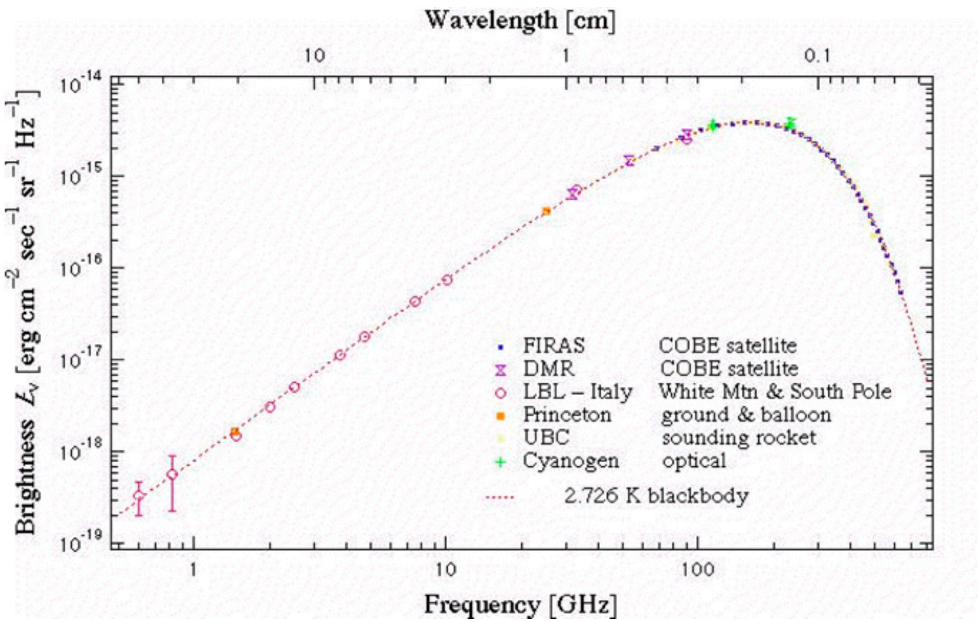
$1 = c \simeq 3 \times 10^8\,\mathrm{ms^{-1}}$

$1 = \hbar c \simeq 200\,\mathrm{MeVfm}$

$1 = k \simeq 10^{-4}\,\mathrm{eVK^{-1}}$

- Plug in the numbers
- $\rho_{CMB} \simeq 0.26\,\mathrm{eVcm^{-3}}$
- $n_{CMB} \simeq 411\mathrm{cm}^{-3}$





CMB photon density

- $\rho_{CMB} \simeq 0.26\,\mathrm{eV cm^{-3}}$
- $\rho_{cr} = \dfrac{3H_0^2}{8\pi G}$
- $H_0 = 100h\ \mathrm{km s^{-1} Mpc^{-1}}\,, \qquad h = 0.67$
- $\rho_{cr} \simeq 1.05 h^2 \times 10^{-5}\,\mathrm{GeV cm^{-3}}$
- $\Omega_\gamma \simeq 0.5 \times 10^{-5}$

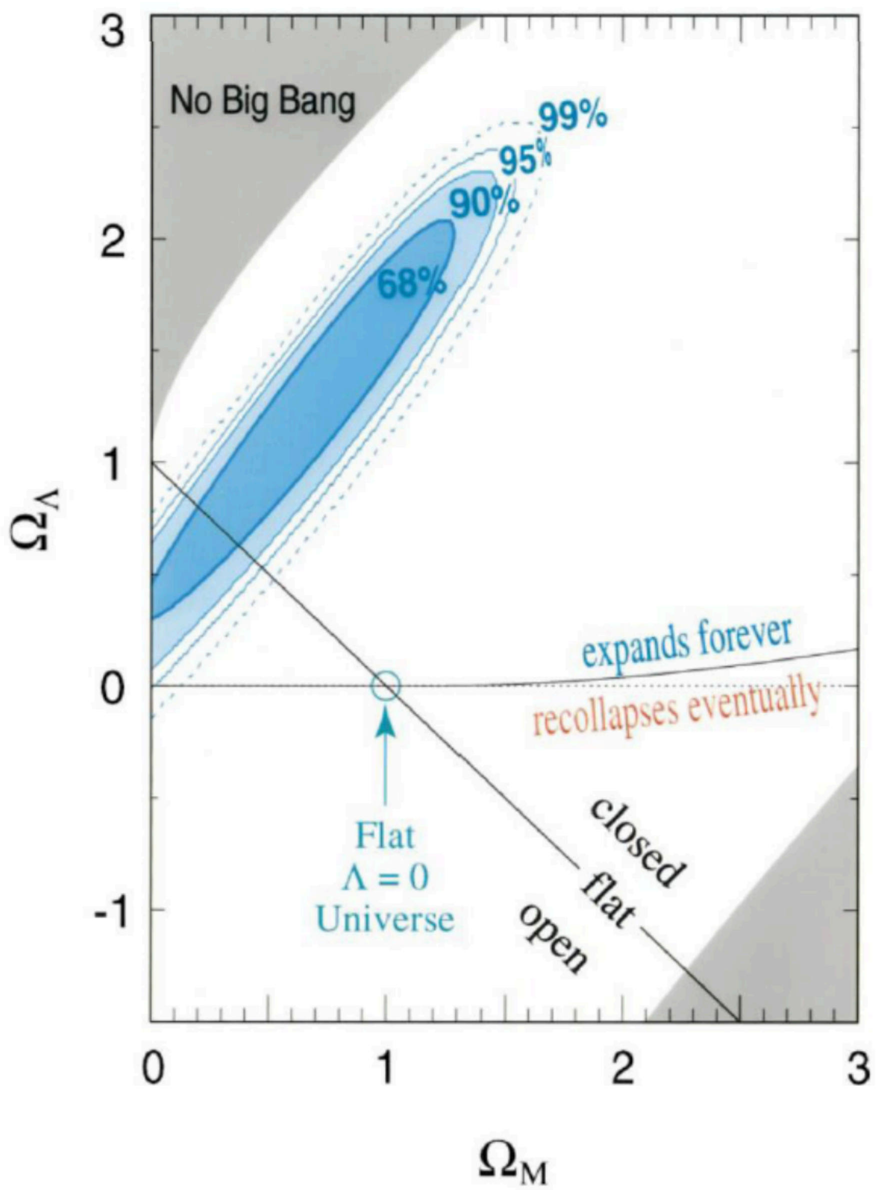
https://pdg.lbl.gov/2021/reviews/rpp2020-rev-astrophysical-constants.pdf

Cosmological parameters

- h ~ 0.67
- $\Omega_\gamma \simeq 0.5 \times 10^{-5}$

$$H^2 = H_0^2 \left[\Omega_\Lambda + \Omega_m^0(1+z)^3 + \Omega_{rad}^0(1+z)^4 + (1-\Omega_{tot}^0)(1+z)^2\right]$$

- *In the early Universe, it is radiation dominated*



Radiation dominated Universe

- The total energy density
- $\rho = \sum \rho_{BE} + \sum \rho_{FD}$
- $\rho = \sum_{\text{bosons}} g_i \frac{\pi^2}{30} T^4 + \frac{7}{8} \sum_{\text{fermions}} g_i \frac{\pi^2}{30} T^4$
- $\rho = g_* \frac{\pi^2}{30} T^4$
- Effective relativistic degrees of freedom
- $g_* = \sum_{\text{bosons}} g_i + \frac{7}{8} \sum_{\text{fermions}} g_i$

Radiation dominated Universe

- Effective relativistic degrees of freedom
- $$g_* = \sum_{\text{bosons}} g_i + \frac{7}{8} \sum_{\text{fermions}} g_i$$
- T ~ 1MeV, when photons (2 polarisations), electrons (2 spins), positrons (2 spins), neutrino (3 flavours, nu + nubar, no spins (chirality)!) were in equilibrium
- $g_* = (2) + \frac{7}{8}\left[2 + 2 + 3 \times (2)\right]$ = 10.75
- T ~ 150 MeV, $\gamma, e^{\pm}, \mu^{\pm}, \nu, \pi^{\pm,0}$, $g_* = 2 + 3 + \frac{7}{8}\left[4 + 4 + 6\right]$ = 17.25
- T > 200 GeV, all known Standard Model particles are relativistic
- $g_* = 107.75$

Entropy density

- $d(\rho V) = Td(sV) - PdV$
- $\rho dV + V\dfrac{d\rho}{dT}dT = TsdV + TV\dfrac{ds}{dT}dT - PdV$
- =>
- $\rho = Ts - P$ => $s = \dfrac{\rho + P}{T}$
- $\rho = g_*\dfrac{\pi^2}{30}T^4$, $w = 1/3$
- $T \propto 1/a$
- $s = g_*\pi^2\left(\dfrac{1}{30} + \dfrac{1}{90}\right)T^3 = g_*\dfrac{2\pi^2}{45}T^3$

- $d(sV) = 0$
  - when considering energy conservation
  - And Zero Chemical potential

=> sa^3 is a constant wrt time.

Problem 3: Chemical potential and antimatter

a) Show that for a relativistic Fermion species, i, with non-zero chemical potential, the following expression for number densities is exact,

$$n_i - \overline{n}_i = \frac{g_i}{6\hbar^3 c^3}(kT)^3 \left[\frac{\mu}{kT} + \frac{1}{\pi^2}\left(\frac{\mu}{kT}\right)^3\right], \tag{1}$$

where n, $\overline{n}$ are number densities of matter and antimatter of the species, respectively. *(20pts)*

Note: you will need to use $\int_0^\infty \frac{x}{e^x+1}dx = \pi^2/12$ or $\int_0^\infty \frac{x}{e^x-1}dx = \pi^2/6$.

b) What is the corresponding $n - \overline{n}$ formula for bosons? *(10pts)*

c) At temperature $kT \sim 10\,\mathrm{MeV}$, given the baryon fraction $n_{baryon}/n_\gamma = 5.5 \times 10^{-10}$, what is the value of μ/kT for electrons?*(10pts)*

Cosmic neutrino background

- At high temperature, neutrinos were in thermal equilibrium

- $\nu + \bar{\nu} \rightleftharpoons e^+ + e^-$

Dont worry if you dont know this
We will see how we can expect this later

- Rate of interaction $\Gamma \simeq \sigma_w n_\nu \simeq G_F^2 E_{cm}^2 \times T^3 \simeq G_F^2 T^5$

- The expansion rate of the Universe $H \simeq \left(G g_* T^4\right)^{1/2}$

- $\dfrac{\Gamma}{H} \simeq \dfrac{G_F^2 T^3}{\sqrt{G g_*}}$

Cosmic neutrino background

- $G_F \simeq \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2}$ <--- effective coupling when $q^2 \ll M_W^2$
- $G_F \simeq 10^{-5}\mathrm{GeV}^{-2}$

- $G \simeq 6.67 \times 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$
- $G \simeq \frac{1}{M_{PL}^2},$

Cosmic neutrino background

- $\frac{\Gamma}{H} \simeq \frac{G_F^2 T^3}{\sqrt{G g_*}}, \quad g_* \simeq 10$

- $\frac{\Gamma}{H} \simeq \left(\frac{T}{\mathrm{MeV}}\right)^3$

- When temperature drops below 1MeV, rate of expansion >> interaction.

- Reaction stops $\nu + \bar{\nu} \rightleftharpoons e^+ + e^-$

Cosmic neutrino background

- After neutrino decoupled $T_\nu \propto 1/a$
- The rest of the Universe, however, were heated up when $e^\pm$ annihilates

- Before $e^\pm$ annihilation: $g_* = 5.5$ (tracking only the EM sector)
- after $e^\pm$ annihilation: $g_* = 2$

- Conservation of entropy before/after ($e^\pm$ ann.) =>
  - $g_* T_\nu^3 \Big|_{\text{before}} = g_* T_\gamma^3 \Big|_{\text{after}}$ $\quad \left(T_\nu \Big|_{\text{before}} = T_\gamma \Big|_{\text{before}}, \quad T_\nu \Big|_{\text{after}} \neq T_\gamma \Big|_{\text{after}} \right)$

Cosmic neutrino background

- After neutrino decoupled $T_\nu \propto 1/a$, temperature evolved "boringly"
- The rest of the Universe, however, were heated up when $e^{\pm}$ annihilates
- But neutrinos are not heated, so their temperature still evolved "boringly"

- Before $e^{\pm}$ annihilation: total $s = (10.75)\dfrac{2\pi^2}{45}T_1^3$
- after $e^{\pm}$ annihilation: total $s = (2)\dfrac{2\pi^2}{45}T_\gamma^3 + (5.25)\dfrac{2\pi^2}{45}T_\nu^3$

$$\frac{7}{8} \times 6 = 5.25$$

- Consider conservation of entropy
- $a_a^3 T_1(a_1) = a_2^3 T_2(a_2) \quad ---> \quad T_{1,\,\mathrm{after}} = T_{\nu,\,\mathrm{after}}$

Cosmic neutrino background

- $T_\nu = (\frac{4}{11})^{\frac{1}{3}} T_\gamma \simeq 1.9K$

- Current neutrino number density, per flavor

- $n_\nu = \frac{3}{4}\frac{4}{11} n_\gamma \simeq 112\,\mathrm{cm}^{-3}$

- Energy density

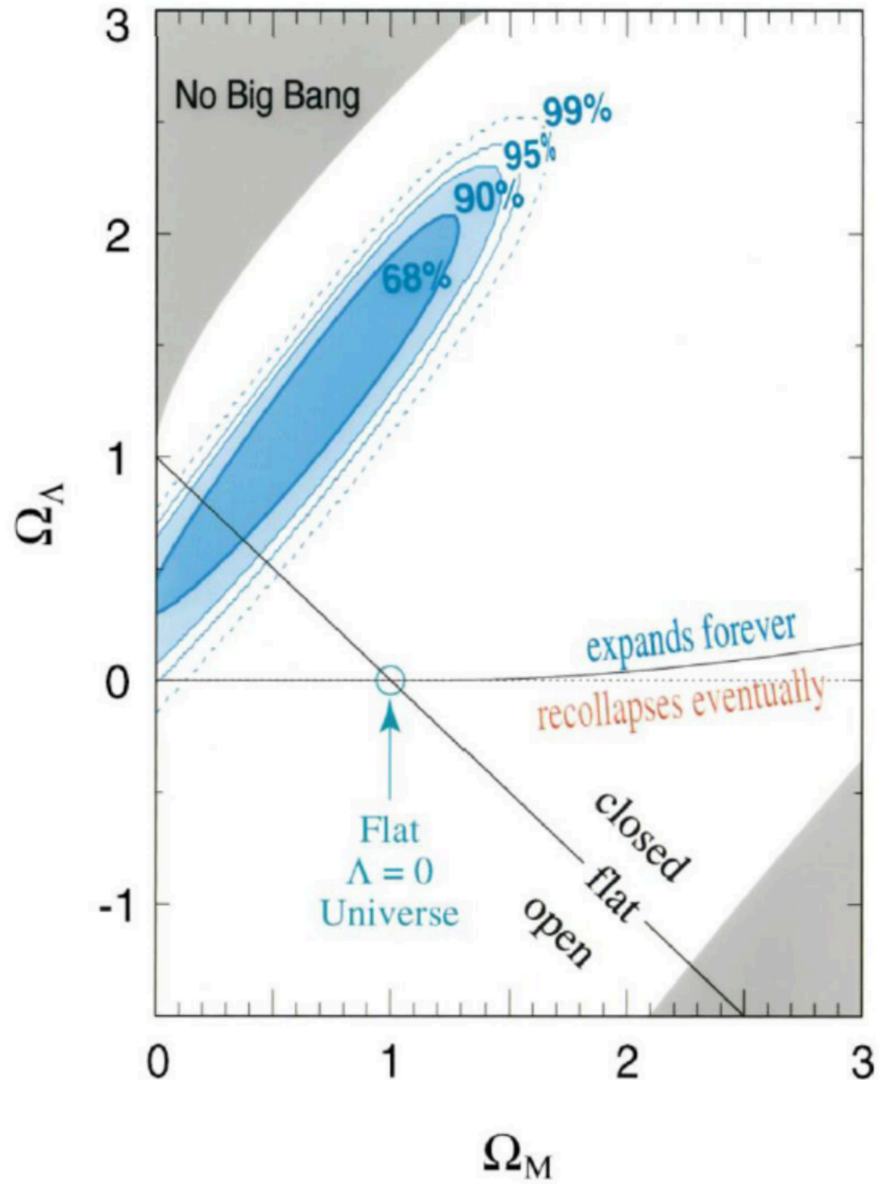
- $\Omega_\nu = \frac{m_\nu n_\nu}{\rho_{cr}}$ => $\Omega_\nu h^2 = \frac{m_\nu}{94\mathrm{eV}}$

Cosmological parameters

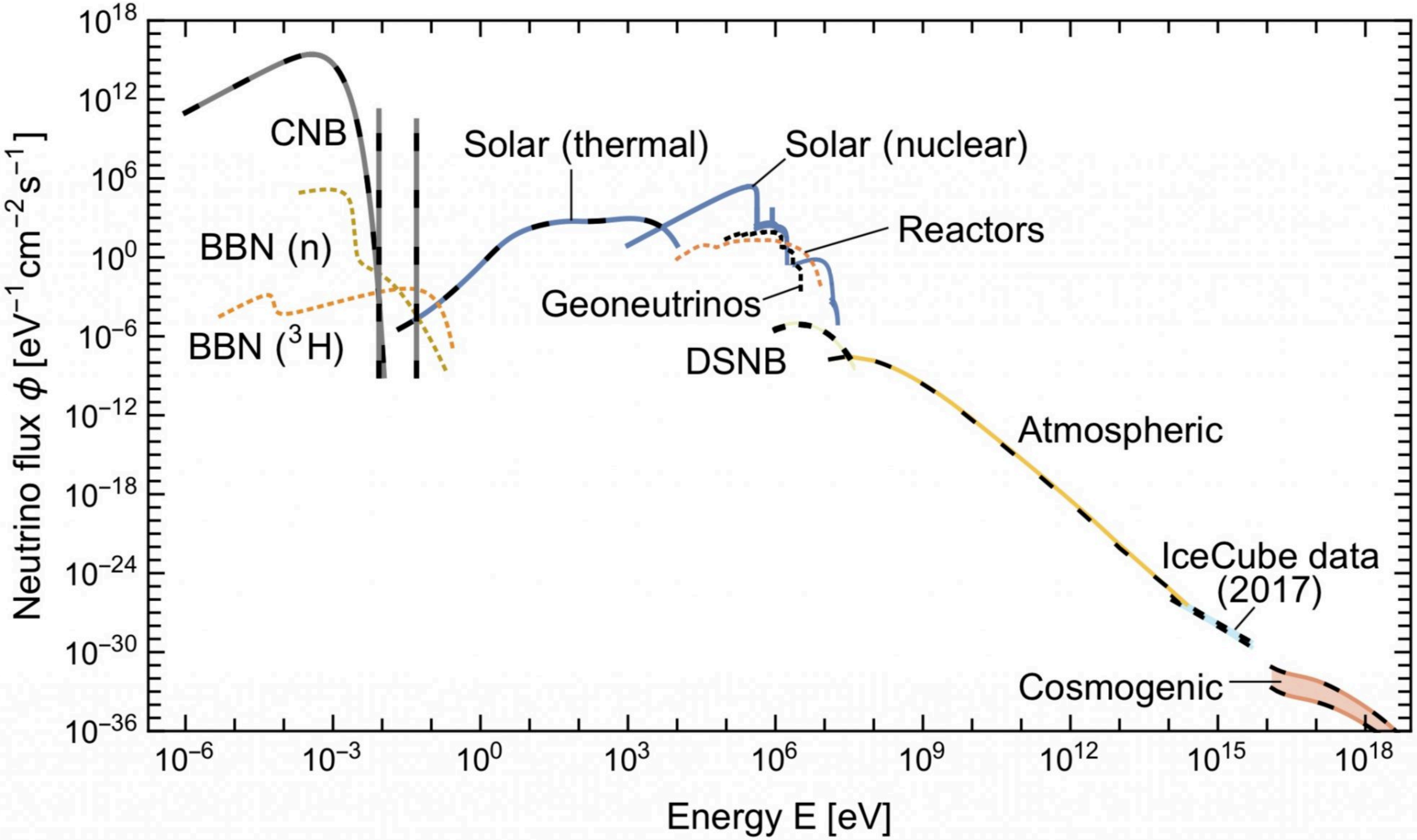
- h ~ 0.67
- $\Omega_\gamma \simeq 0.5 \times 10^{-5}$
- $\Omega_\nu h^2 = \dfrac{m_\nu}{94\mathrm{eV}}$

$$H^2 = H_0^2 \left[\Omega_\Lambda + \Omega_m^0(1+z)^3 + \Omega_{rad}^0(1+z)^4 + (1-\Omega_{tot}^0)(1+z)^2\right]$$

- *In the early Universe, it is radiation dominated*

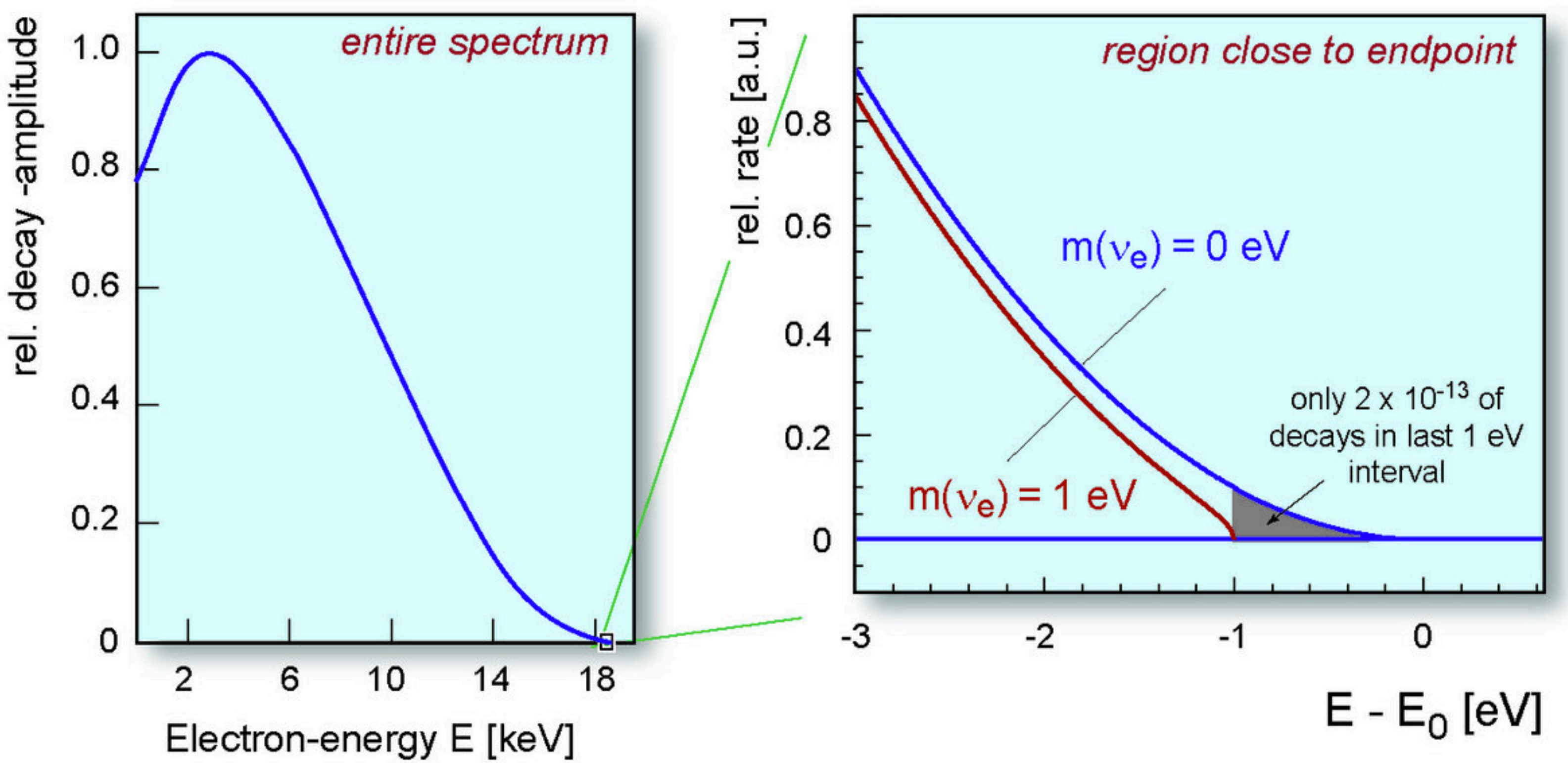


- $n \sim 100\mathrm{cm}^{-3}$
- $\dfrac{dF}{dE} \simeq \dfrac{c}{4\pi}\dfrac{dn}{dE}$
-



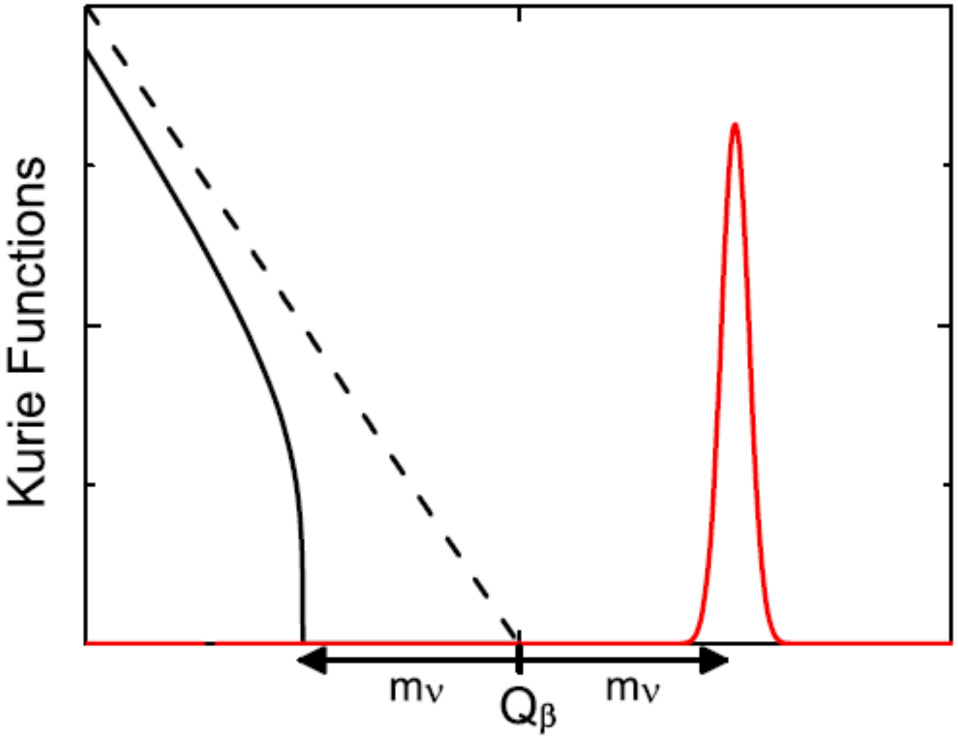
Cosmic neutrino background, direct detection?

- Beta decay end point $n \rightarrow p^+ + e^- + \bar{\nu}_e$



Cosmic neutrino background, direct detection?

- neutrino capture, beta decay: $\nu_e + n \rightarrow p^+ + e^-$



Neutrino physics with the PTOLEMY project: active neutrino properties and the light sterile case

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