

2.7 Acceleration

- ❖ Kenny CY Ng
- ❖ kcyng@cuhk.edu.hk
- ❖ Sci Cen North Black 345
- ❖ CUHK



How to accelerate cosmic rays

PHYSICAL REVIEW

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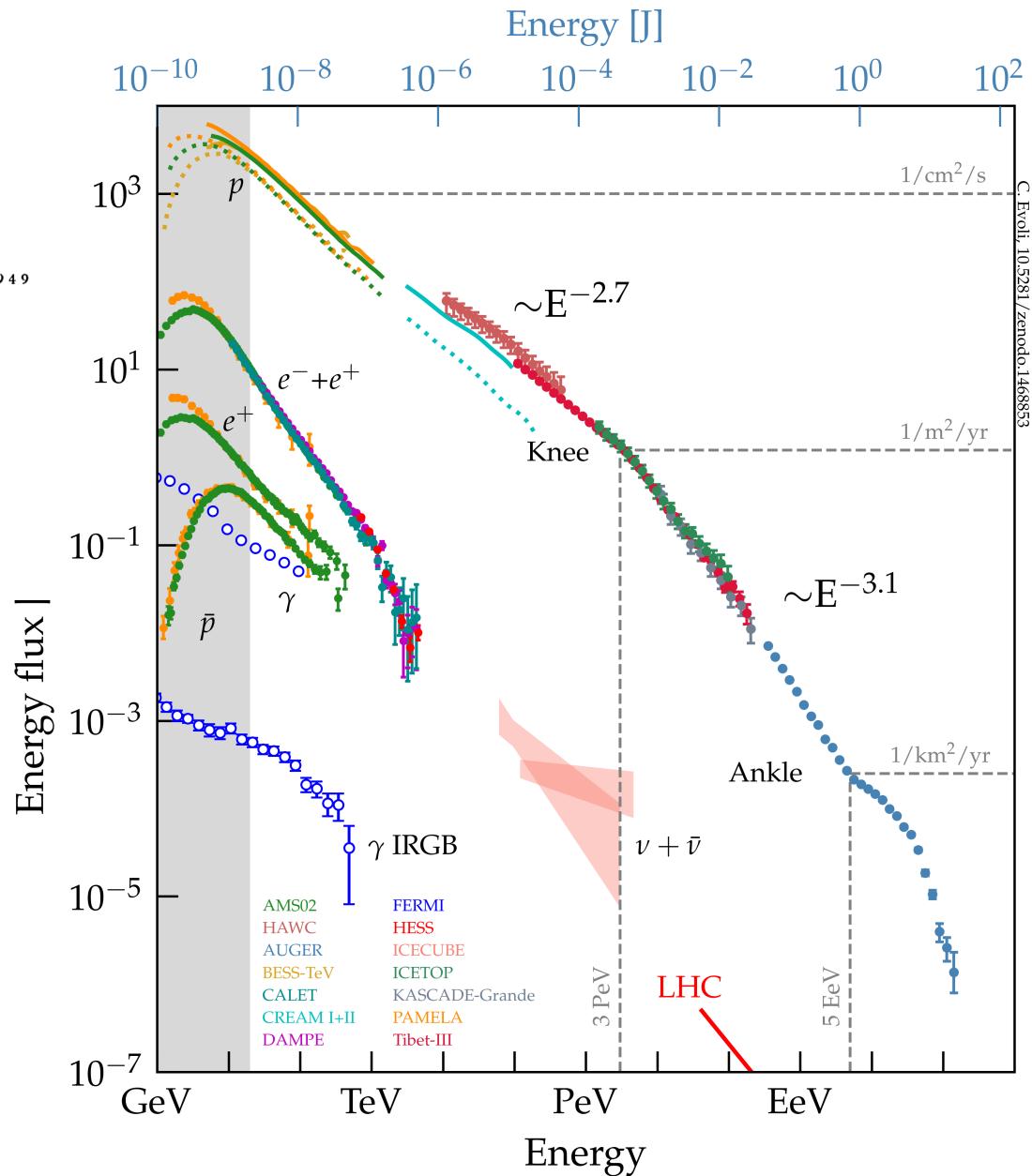
APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI

*Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)*

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.





Wikipedia

https://en.wikipedia.org/wiki/Enrico_Fermi

Enrico Fermi

He was one of very few physicists to excel in both theoretical and experimental physics. Fermi was awarded the 1938 Nobel Prize in Physics for his work on ...



Fermi's theory of weak interaction

History of initial rejection and later publication [edit]

Fermi first submitted his "tentative" theory of beta decay to the prestigious science journal *Nature*, which rejected it "because it contained speculations too remote from reality to be of interest to the reader."^{[5][6]} It has been argued that *Nature* later admitted the rejection to be one of the great editorial blunders in its history, but Fermi's biographer David N. Schwartz has objected that this is both unproven and unlikely.^[7] Fermi then submitted revised versions of the paper to Italian and German publications, which accepted and published them in those languages in 1933 and 1934.^{[8][9][10][11]} The paper did not appear at the time in a primary publication in English.^[5] An English translation of the seminal paper was published in the *American Journal of Physics* in 1968.^[11]

Fermi found the initial rejection of the paper so troubling that he decided to take some time off from theoretical physics, and do only experimental physics. This would lead shortly to his famous work with activation of nuclei with slow neutrons, for which he received a Nobel Prize in Physics in 1938.

Scientist of the Day

FERMIONS

baryons leptons

proton electron

neutron muon

tauon tauon neutrino

electron neutrino

muon neutrino

tauon neutrino

IG : cosmological astrophysics

e u

τ ν

ν ν

ν ν

ν ν

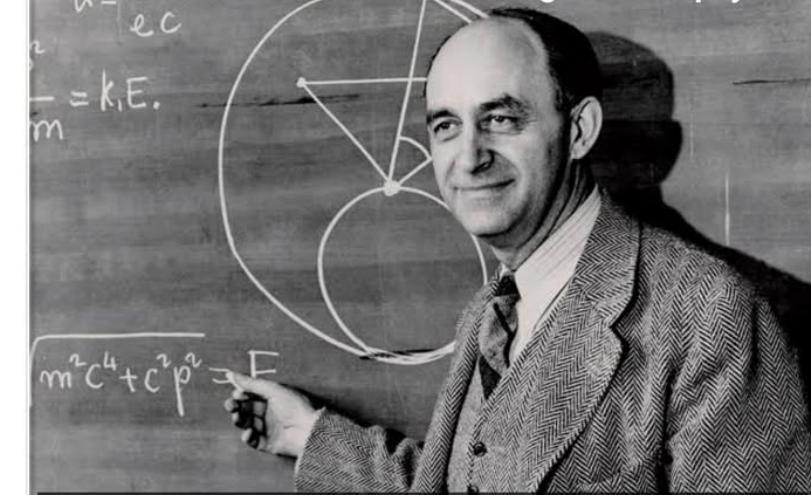
ν ν

$$\alpha = \frac{\hbar}{e c}$$

$$= k, E.$$

$$m^2 c^4 + c^2 p^2 = E$$

Fb / Cosmological Astrophysics



Italian & naturalized American physicist
(Born - 29 Sep 1901, Died - 28 Nov 1954)

Credit - Radha Mohan.

Enrico Fermi
(The last man who knew everything)

He won the 1938 Nobel Prize in Physics.

In his honor, Particles that obey the Pauli exclusion principle are called fermions.

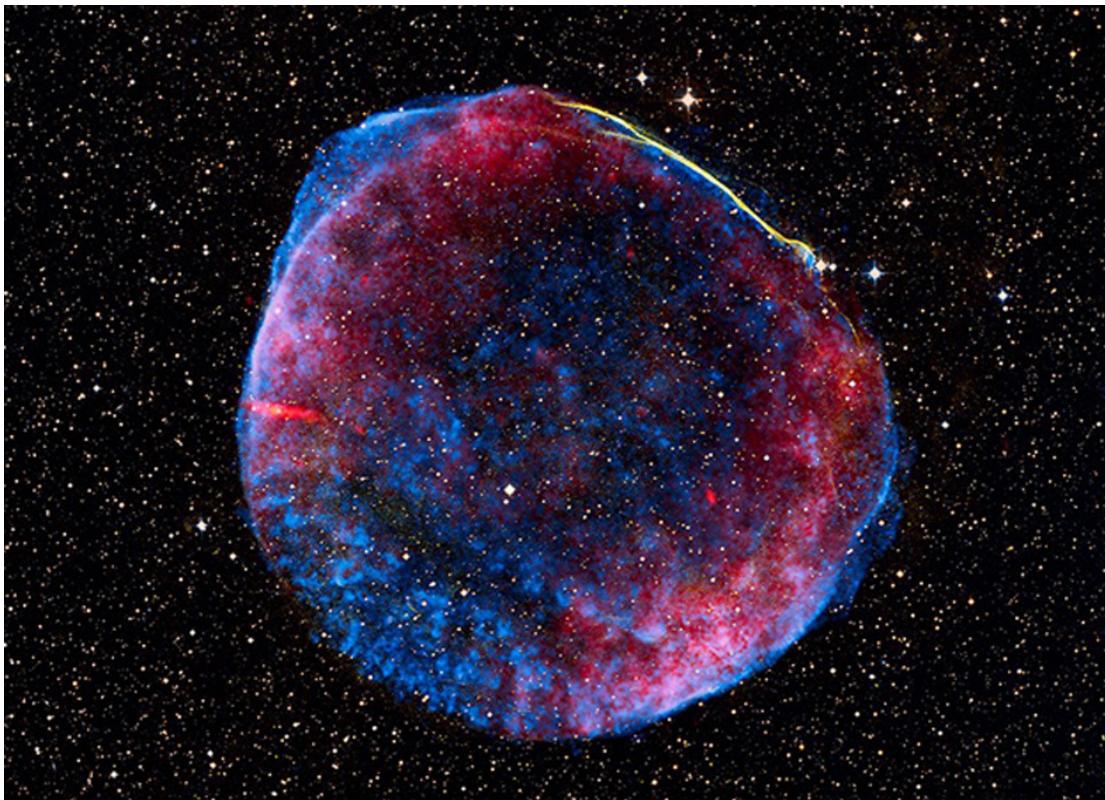
Cosmic ray acceleration

- Suppose there is a process that a particles gains some energy after each encounter
- $E = \beta E_0$
- And the probability that the particle remains in the accelerating region as P
- Then after k encounters, the number of particles remained is
- $N(>E) = N_0 P^k$
- and
- $E = E_0 \beta^k$

Cosmic ray acceleration

- $N(>E) = N_0 P^k$
- $E = E_0 \beta^k$
- eliminate k , you get
- $\frac{\ln N/N_0}{\ln E/E_0} = \frac{\ln P}{\ln \beta}$
- $\frac{N}{N_0} = \left(\frac{E}{E_0} \right)^{\ln P / \ln \beta}$
- Then the particle number spectrum is
- $\frac{dN}{dE} \propto E^{-1 + \frac{\ln P}{\ln \beta}}$

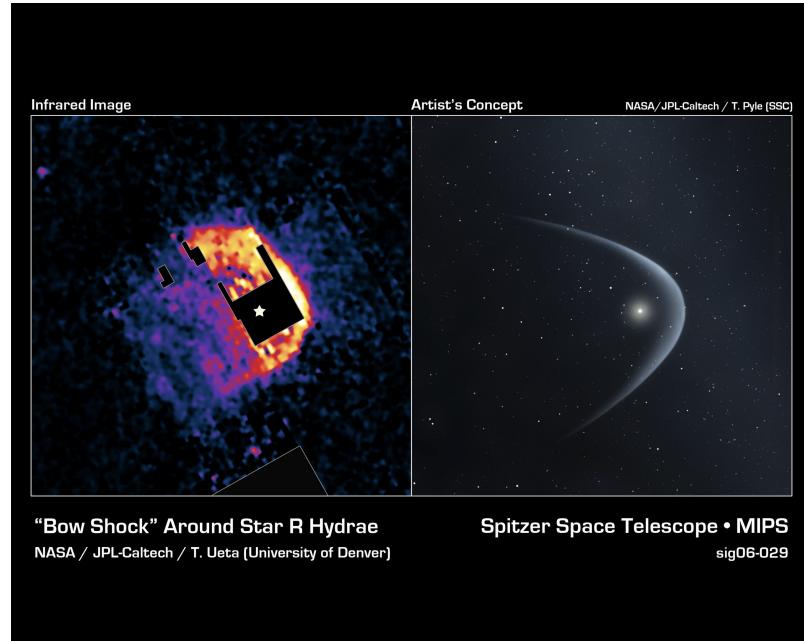
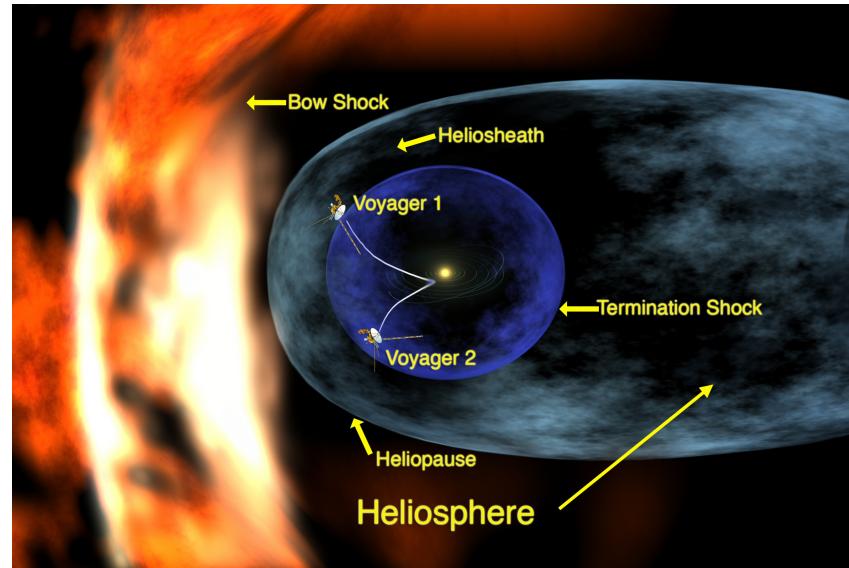
Astrophysical shocks



In physics, a **shock wave** (also spelled **shockwave**), or **shock**, is a type of propagating disturbance that moves faster than the local [speed of sound](#) in the medium. Like an ordinary wave, a shock wave carries energy and can propagate through a medium, but is characterized by an abrupt, nearly discontinuous, change in [pressure](#), [temperature](#), and [density](#) of the medium.[\[1\]](#)[\[2\]](#)[\[3\]](#)[\[4\]](#)[\[5\]](#)[\[6\]](#)

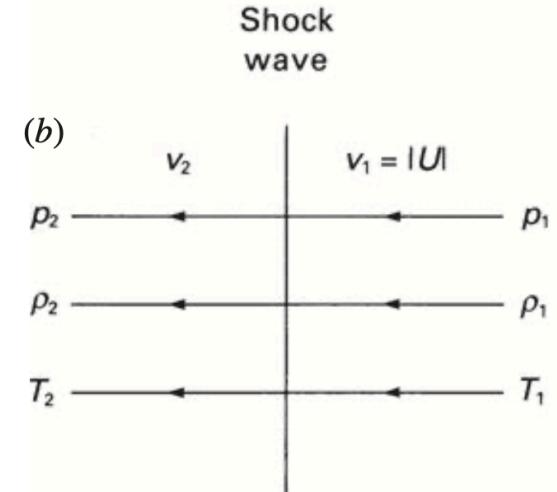
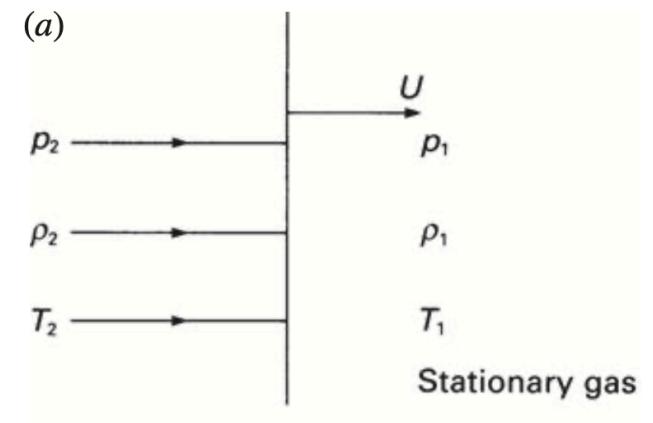
The solar system moving in Interstellar medium

- Termination shock
 - Where solar wind slows down to below the speed of sound (pressure waves)
- Bow Shock
 - When interstellar wind slows down when entering the heliosphere
 - No evidence of it
 - Same can happen for other stars



Astrophysical shocks

- Two gases colliding with speed faster than speed of sound
- U the velocity of the shock
- gas properties, temperature, density, pressure
- Boost to the shock frame, then particles are flowing in with velocity $v_1 = U$
- After the shock (discontinuous layer), particle velocity is v_2



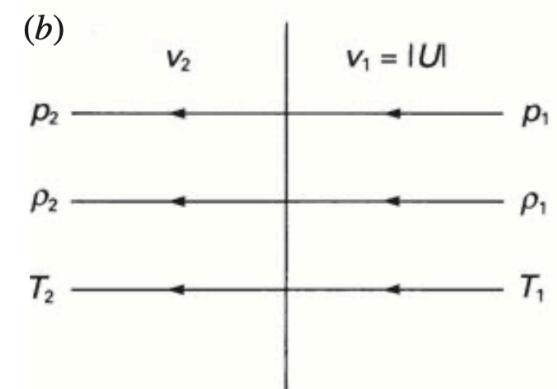
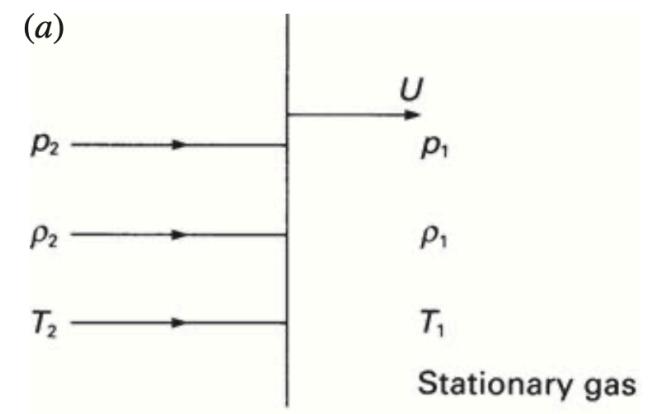
Astrophysical shocks

- (1), up stream
 - $\rho_1, v_1, P_1 = 0$
- (1), down stream
 - ρ_2, v_2, P_2
- Consider a box with Volume = Adx_2 moving from upstream to down stream, becoming a new volume element Adx_1
- Mass in the element is then $\rho_1 Adx_1 = \rho_2 Adx_2$
- Mass conservation
- $\rho_1 v_1 = \rho_2 v_2$

Shock frame
variables

Simplify by
assuming P_1 is zero

Down stream up stream



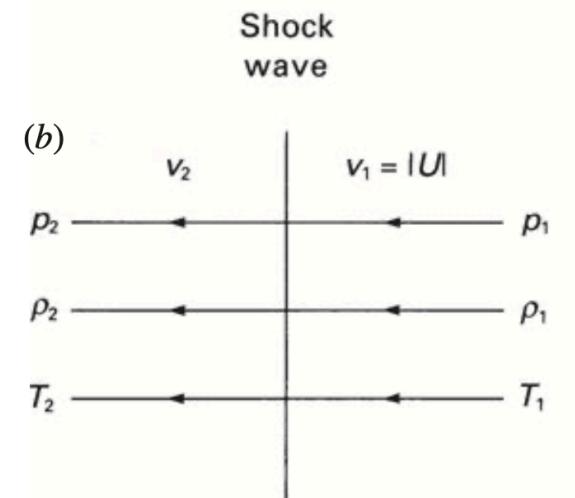
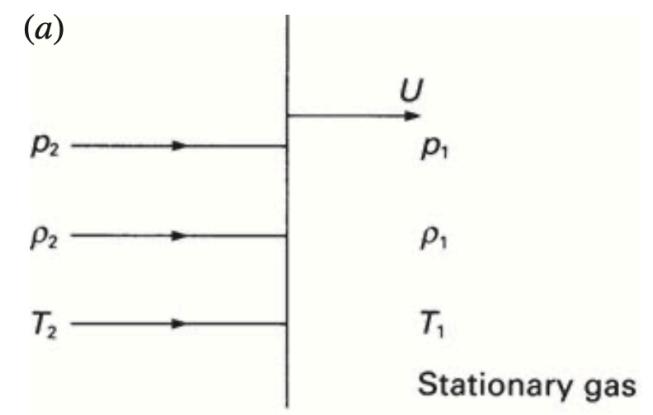
Astrophysical shocks

- (1), up stream
 - $\rho_1, v_1, P_1 = 0$
- (1), down stream
 - ρ_2, v_2, P_2
- Mass conservation
- $\rho_1 v_1 = \rho_2 v_2$
- momentum conservation
 - Consider the Bulk momentum density ρv
 - And the microscopic momentum transfer rate
- Momentum in the volume element
 - $(\rho_2 v_2) A \cdot dx_2 + dp_2$ p_2 momentum contained in the gas particles
- $\Rightarrow (\rho_1 v_1) \cdot v_1 = (\rho_2 v_2) \cdot v_2 + P_2$

Shock frame
variables

Simplify by
assuming P_1 is zero

Down stream up stream



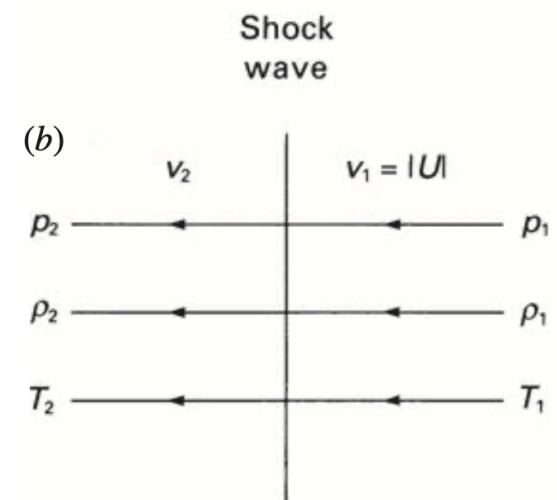
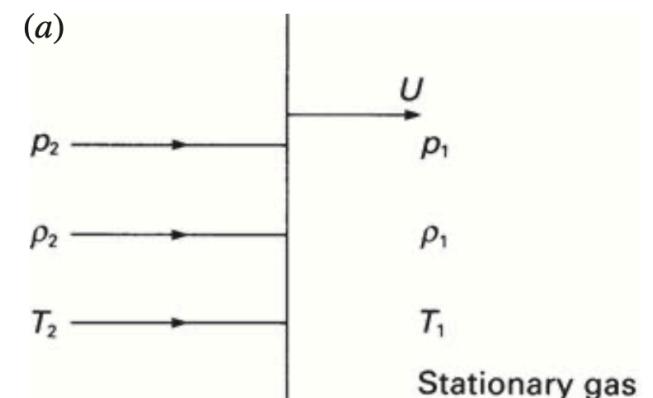
Astrophysical shocks

- (1), up stream
 - $\rho_1, v_1, P_1 = 0$
 - (1), down stream
 - ρ_2, v_2, P_2
 - Mass conservation
 - $\rho_1 v_1 = \rho_2 v_2$
 - momentum conservation
 - $(\rho_1 v_1) \cdot v_1 = (\rho_2 v_2) \cdot v_2 + P_2$
 - energy conservation
 - $\left(\frac{1}{2}\rho_1 v_1^2\right) \cdot v_1 = \left(\frac{1}{2}\rho_2 v_2^2 + \frac{3}{2}P_2\right) \cdot v_2 + P_2 v_2$
- Bulk

Energy change in
microscopic particles

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Down stream up stream



Recall that
 $E = \frac{3}{2}nkTV$
 $P = nkT$

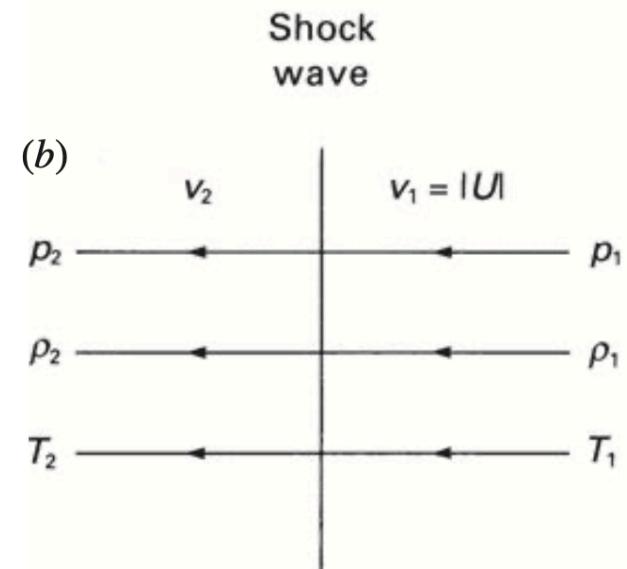
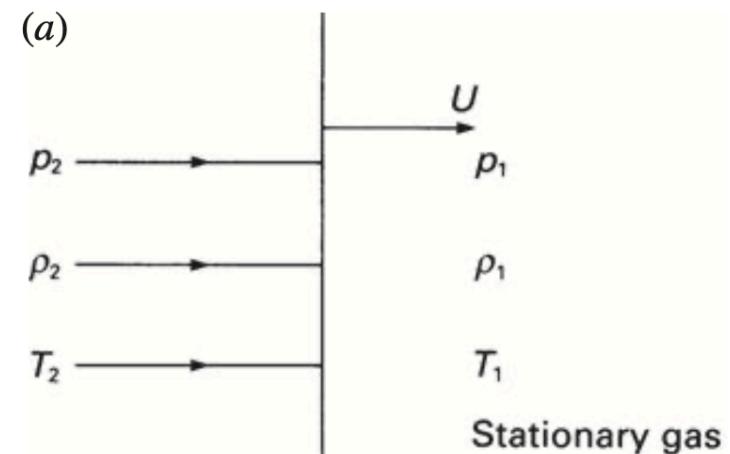
Work done = $F \cdot dx$

Astrophysical shocks

- Solving the 3 equations with 5 unknowns, you get

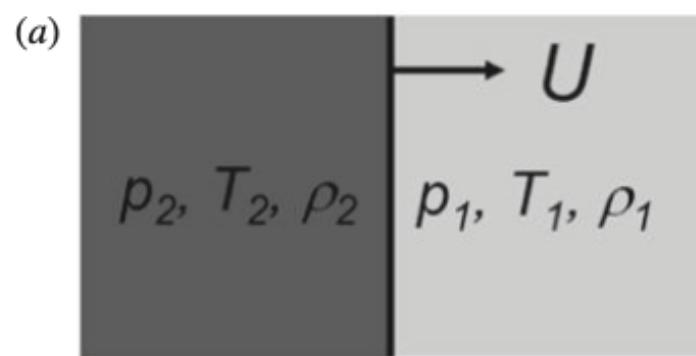
- $\left(\frac{\rho_2}{\rho_1} - 4 \right) \left(\frac{\rho_2}{\rho_1} - 1 \right) = 0$

- Shock conditions
- $\rho_2 = 4\rho_1$ and
- $v_1 = 4v_2$

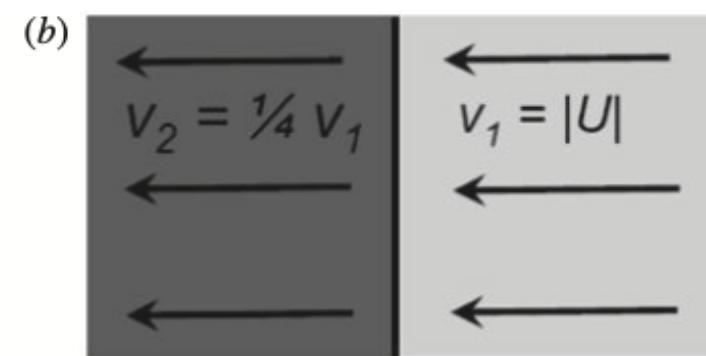


Particle dynamics around shocks

- Shock conditions
- $\rho_2 = 4\rho_1$ and
- $v_1 = 4v_2$



• Lab frame



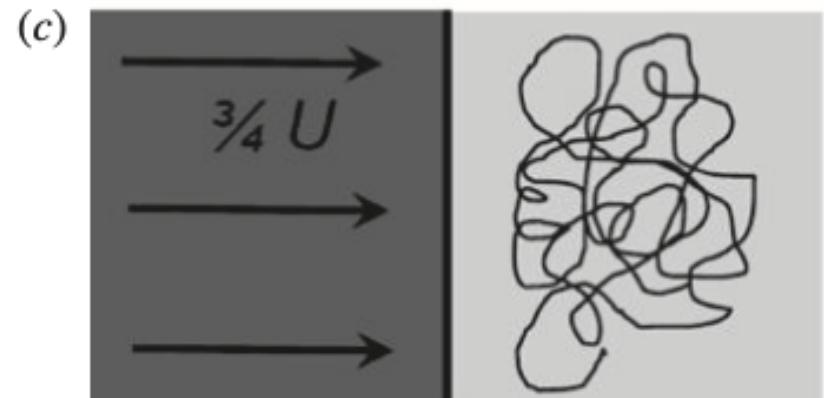
• Shock frame

- a) Lab frame, shock velocity is U , which is supersonic
 - Let's take $U \ll c$; (change your mind to Galileo)
- b) In the shock frame, we have the shock conditions.

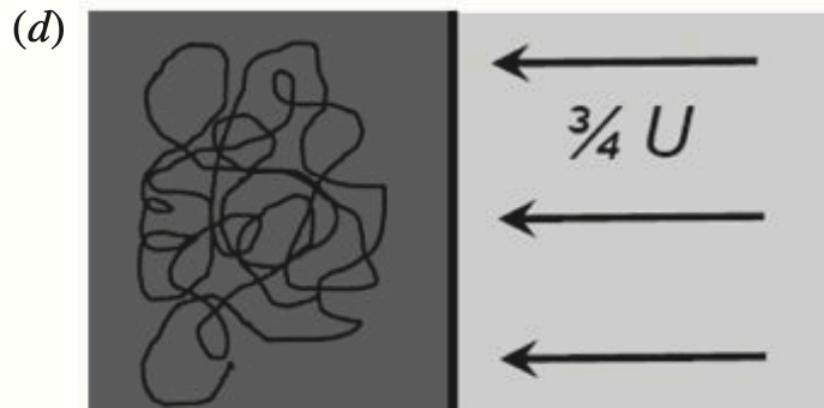
Particle acceleration in shocks

- Now consider relativistic particles that are isotropized in the **up stream** due to say, magnetic diffusion. (diffusion without energy loss)
- When they cross the shock boundary and enter the down stream. They see particles moving with bulk speed $V = \frac{3}{4}U$
- In the **down stream frame**, the particle has energy
- $E' = \gamma_V(E + Vp_x) \simeq E(1 + V\cos\theta) \dots \text{--- } (c = 1)$
- $\frac{E' - E}{E} = \frac{\Delta E}{E} = V\cos\theta$

$V = (3/4)U$
 corresponds to the Lorentz
 boost between up and down
 stream frame



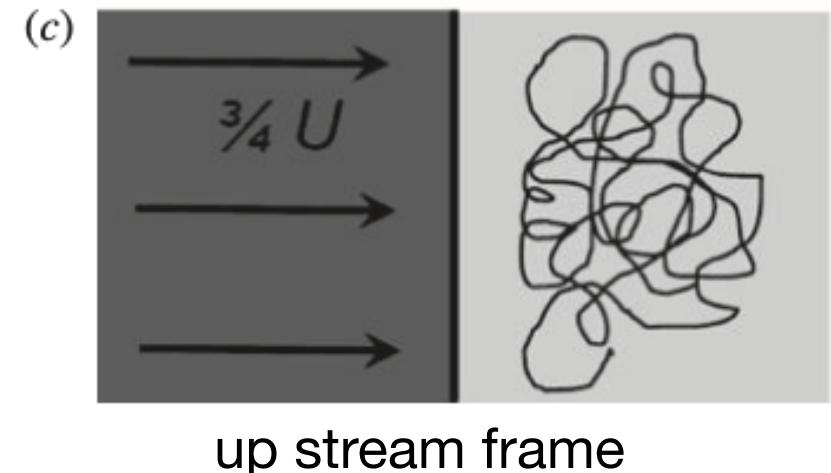
up stream frame (lab frame)



down stream frame

Particle acceleration in shocks

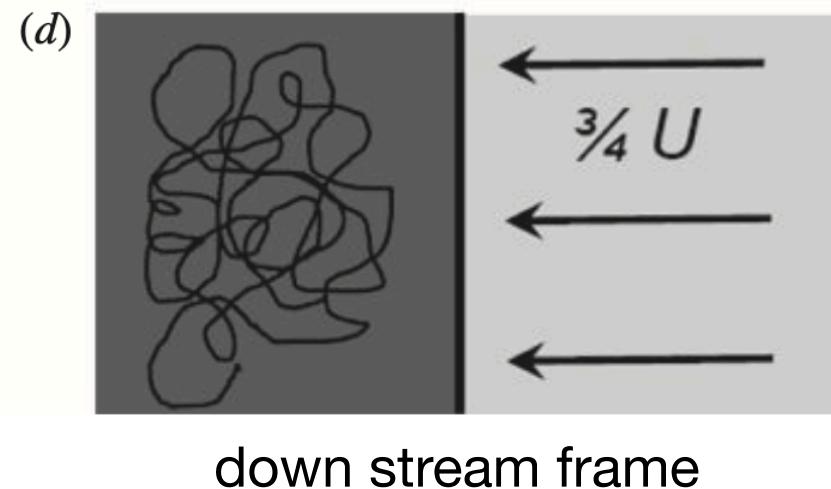
- Rate of crossing \propto Relative velocity x Solid angle
- $dP \propto \cos \theta (\sin \theta d\theta)$, normalize the probability for θ from 0 to $\pi/2$, where 0 is towards the shock
- $dP = 2 \cos \theta d\cos \theta$



- Averaged energy gain from shock crossing

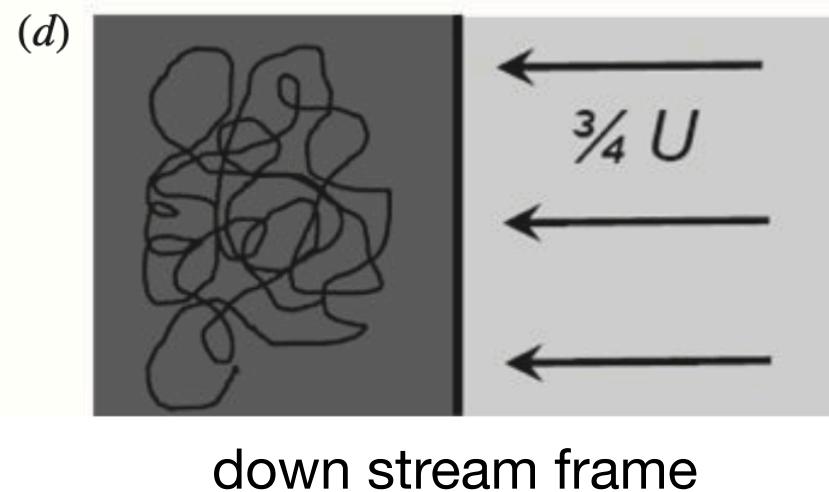
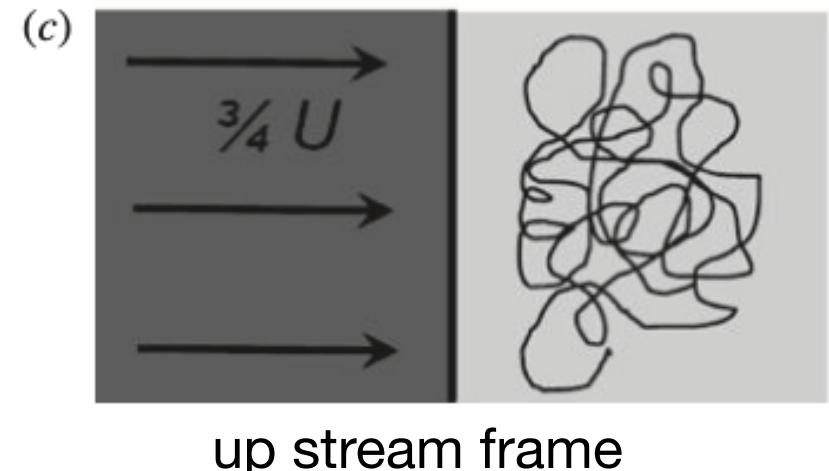
$$\frac{\Delta E}{E} = V \cos \theta$$

$$\langle \frac{\Delta E}{E} \rangle = \int_0^{\pi/2} V \cos \theta \times 2 \cos \theta d\cos \theta = \frac{2}{3} V$$



Particle acceleration in shocks

- The particles are isotropized in each frame, and shock crossing can happen again.
- Each round trip
- $\langle \frac{\Delta E}{E} \rangle = \frac{4}{3}V$
- **First order Fermi acceleration**
- $E = \beta E_0$
- $\beta = 1 + \frac{4}{3}V = 1 + U$



Particle acceleration in shocks

- Rate of relativistic particles crossing the shock:

- $\frac{1}{4}nc$ (c explicit)

- The $\frac{1}{4}$ comes from $\frac{1}{4\pi} \int \cos \theta d\phi d\cos \theta$, which calculates the fraction of particles passing through the shock over an isotropic distribution

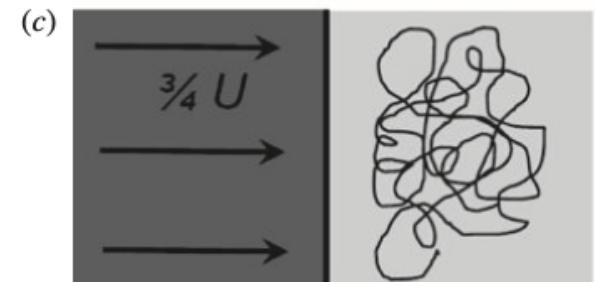
- Rate of particles moving away from the shock

- $n(\frac{1}{4}U)$, which is the speed particles are leaving the shock in downstream frame

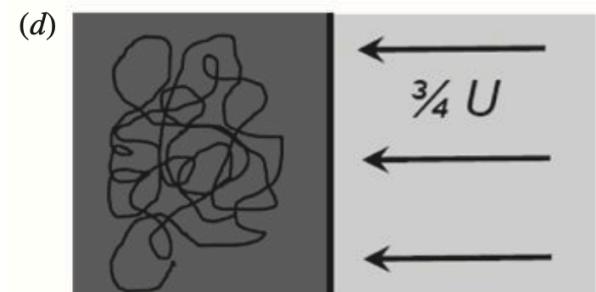
- Probability of losing the particles: $\frac{U}{c} = U$ (c implicit)

- Probability of remaining

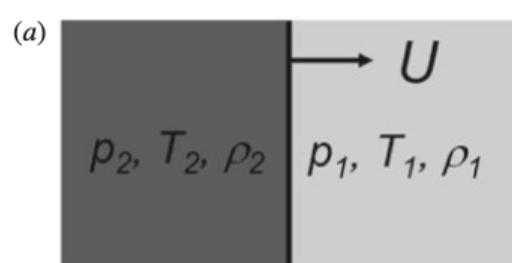
- $P = 1 - U$



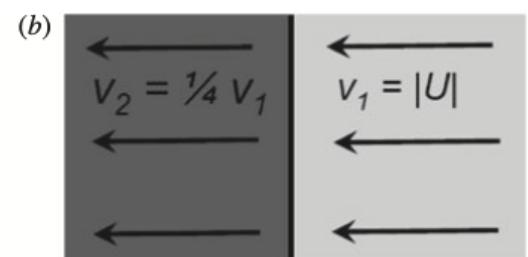
up stream frame



down stream frame



• Lab frame

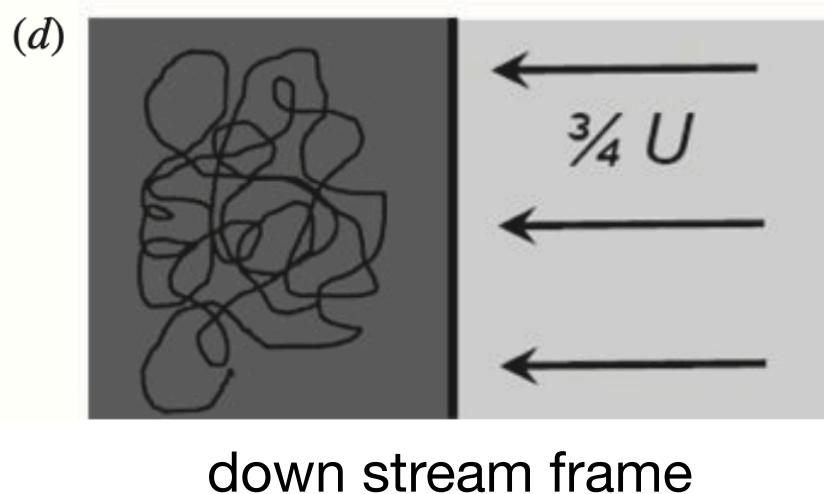
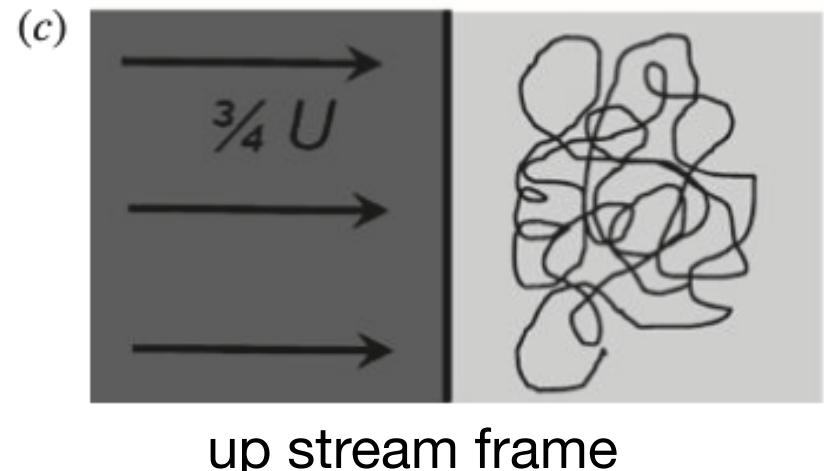


• Shock frame

Particle acceleration in shocks

- $\beta = 1 + U$
- $P = 1 - U$

For small U
- $\frac{\ln P}{\ln \beta} = \frac{\ln(1 - U)}{\ln(1 + U)} \simeq \frac{-U}{U} = -1$
- $\frac{dN}{dE} \propto E^{-1 + \frac{\ln P}{\ln \beta}} = E^{-2}$
- A power-law solution for accelerated spectrum
- also called [Diffuse shock acceleration]



Cosmic ray acceleration

- Particle accelerated with in a source with some spectrum, $E^{-\gamma}$ with index $\gamma \sim 2 - 2.2$
- Propagation effect (diffusion and energy loss) leads observed spectrum $E^{-2.7}$ or $E^{-3.3}$
- Particle accelerations a huge and important topic in both astrophysics and plasma physics

<https://arxiv.org/abs/1506.02034>

- relativistic shocks
- magnetic fields
-

List of things named after Enrico Fermi

1 language

Article

Talk

Read Edit View history Tools

From Wikipedia, the free encyclopedia

Enrico Fermi (1901–1954), an Italian-born, naturalized American physicist, is the eponym of the topics listed below.

Physics [edit]

- Fermi (unit), unit of length in particle physics equivalent to the femtometre
- Fermi arc, a phenomenon in superconductivity
- Fermi constant, constant that gives the strength of Fermi's interaction
- Fermi contact interaction, the magnetic interaction between an electron and an atomic nucleus when the electron is inside that nucleus
- Fermi energy
- Fermi's four factor formula
- Fermi gas
- Fermi's interaction, an explanation of the beta decay
- Fermi level
- Fermi liquid theory
 - Quasi Fermi level, also called *imref* which is "fermi" spelt backwards
- Fermi heap and Fermi hole
- Fermi-Kurie plot of beta decay
- Fermi paradox, a fundamental issue in SETI
- Fermi point
- Fermi pseudopotential
- Fermi's golden rule
- Fermi motion, the quantum motion of nucleons bound inside a nucleus
- Fermi resonance
- Fermi surface
- Fermi acceleration
 - Fermi-Ulam model
 - Fermi-Pasta-Ulam model, a model of the Fermi acceleration mechanism
- Fermi glow
- Fermi ball
- Fermi sea
- Fermi transition
- Fermi-Walker transport
- Fermi-Dirac statistics
 - Fermi-Dirac condensate
- Fermion, the class of particles that obey Fermi-Dirac statistics, a name coined by Dirac in 1945
- Fermium, a synthetic element with symbol Fm and atomic number 100.
- Fermionic field
- Thomas-Fermi model approximation
- Thomas-Fermi model
 - Thomas-Fermi equation
- Thomas-Fermi screening, an approximate method for describing screening of electric field by mobile charge
- International School of Physics "Enrico Fermi", an annual summer school hosted by the Italian Physical Society

Mathematics [edit]

- Complete Fermi-Dirac integral
- Incomplete Fermi-Dirac integral
- Fermi-Walker differentiation
- Fermi coordinates, local coordinates that are adapted to a geodesic in Riemannian geometry
- Fermi-Pasta-Ulam-Tsingou problem
- Fermi problem, estimation problem designed to teach dimensional analysis, and approximation
- Fermi paradox, typical example of a Fermi problem pertaining to extraterrestrial civilizations

Research [edit]

- Fermilab, Fermi National Accelerator Laboratory
- Enrico Fermi Institute, Chicago, Illinois
- FeRMI, a research federation in Toulouse, France ^[1]
- Fermi Avenue, Harwell Campus

Technology [edit]

- GeForce 400 series and GeForce 500 series, also "Fermi", the codename for a graphics card architecture developed by NVIDIA
- Fermi Gamma-ray Space Telescope
- Enrico Fermi Nuclear Generating Station, Monroe, Michigan
- Enrico Fermi Nuclear Power Plant (Italy), Trino Vercellese, Italy
- RA-1 Enrico Fermi, Argentinian research reactor
- FERMAC
- Fermi filter
- Fermi Linux, distributions produced Fermilab
- Fermi-Szilard Neutronic Reactor, US patent 2,708,656
- Fermi-ization, method of estimation that involves breaking a problem into component parts, estimating each part separately and then combining the result. This method tends to produce substantially more accurate answers, even with components that are largely unknown, than less systematic approaches ^[2]

Fermi problem

17 languages

A **Fermi problem** (or **Fermi question**, **Fermi quiz**), also known as an **order-of-magnitude problem**, is an **estimation** problem in **physics** or **engineering** education, designed to teach **dimensional analysis** or **approximation** of extreme scientific calculations. Fermi problems are usually **back-of-the-envelope calculations**. Fermi problems typically involve making justified guesses about quantities and their **variance** or lower and upper bounds. In some cases, order-of-magnitude estimates can also be derived using **dimensional analysis**. A **Fermi estimate** (or **order-of-magnitude estimate**, **order estimation**) is an estimate of an extreme scientific calculation.

Maximum energies

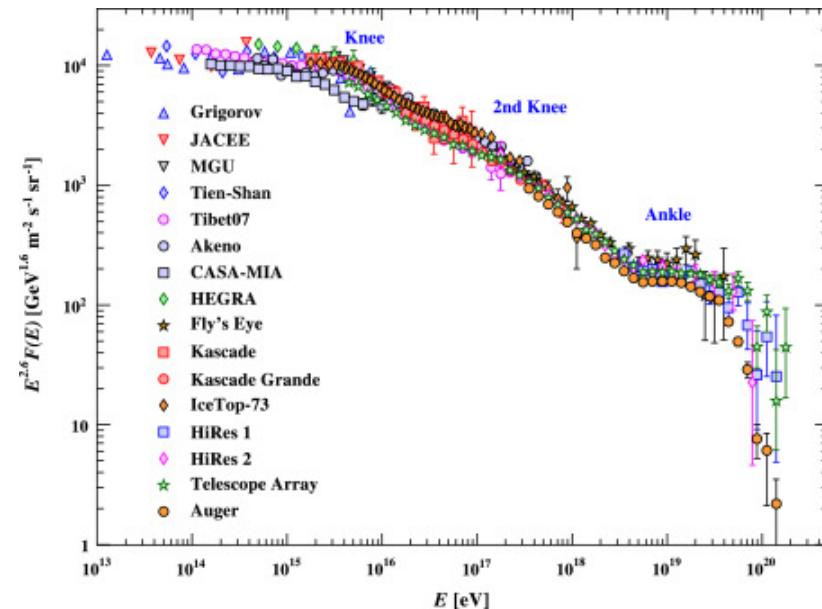
- Estimate using Maxwell equation
- $\nabla \times \mathcal{E} = - \partial_t B$
- U: shock speed
- L: length scale of the system
- B: magnetic field of the system

$$\frac{\mathcal{E}}{L} \sim \frac{B}{L/U}$$

- $E_{max} = Z\mathcal{E}L = ZBU$
- Z, atomic number of the species

- young Supernova remnant
- $B \sim 1\mu G$
- $U \sim 10^4 \text{ km s}^{-1}$
- $t \sim 1000 \text{ year}$

$$E_{max} \sim 10^{16} \text{ eV}$$



The Hillas plot

- $10^{20} \frac{\text{eV}}{\text{ZU}}$
- Gives us some idea for potential Ultra high energy CR sources

