# UNIT - IV
# PROBABILITY

* A random variable $X$ has the following probability function for various values of $x$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

i) Find $k$ ii) Evaluate $P(x<6)$, $P(x\geq 6)$ and $P(3\leq x\leq 6)$

**Soln**: we have i) $\boxed{\sum P(x) = 1}$

$\therefore 0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$

$\Rightarrow 10k^2+9k-1 = 0 \quad \therefore k = \frac{1}{10}$

Hence

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0 | $1/10$ | $2/10$ | $2/10$ | $3/10$ | $1/100$ | $2/100$ | $17/100$ |

ii) $P(x<6) = 0+\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}+\frac{1}{100} = 0.81$

i $P(x\geq 6) = P(6)+P(7) = \frac{2}{100}+\frac{17}{100} = 0.19$

$P(3\leq x\leq 6) = P(3)+P(4)+P(5)+P(6)$

$= \frac{2}{10}+\frac{3}{10}+\frac{1}{100}+\frac{2}{100}$

$= 0.53$

* A Random variable x has the following probability function

| x | : | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| P(x) | : | 0.1 | K | 0.2 | 2K | 0.3 | K |

Find K and find mean and variance.

Soln: we have $\sum P(x) = 1$

$0.1 + K + 0.2 + 2K + 0.3 + K = 1$

$\Rightarrow K = 0.1$

Mean $= E(X) = \sum x\, P(x)$

$= -2(0.1) + (-1)(0.1) + 0(0.2) + (1)(0.2) + 2(0.3) + 3(0.1)$

$E(X) = 0.8$

Variance $= E(X^2) - [E(X)]^2$

Now $E(X^2) = \sum x^2 \cdot P(x)$

$= (-2)^2(0.1) + (-1)^2(0.1) + 0(0.2) + (1)^2(0.2) + (2)^2(0.3) + (3)^2(0.1)$

$= 2.8$

$\therefore$ Variance $= 2.8 - (0.8)^2$

$= 2.16$

* From a sealed box containing a dozen apples it was found that 3 apples are perished. Obtain the probability distribution of the no. of perished apples when 2 apples are drawn at random. Also find mean and variance of this distribution.

Soln: Let X : No. of perished apples

2 apples out of 12 can be selected in $^{12}C_2$ ways

Good apples are 9

$\therefore X = 0, 1, 2$

$P(X=0)$ = Probability of getting 0 perished apple $= \frac{^3C_0 \cdot ^9C_2}{^{12}C_2} = \frac{6}{11}$

$P(X=1) = \frac{^3C_1 \cdot ^9C_1}{^{12}C_2} = \frac{9}{22}$

$P(X=2) = \frac{^3C_2 \cdot ^9C_0}{^{12}C_2} = \frac{1}{22}$

The prob. distr$^n$ is

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(x)$ | 6/11 | 9/22 | 1/22 |

Mean $= \Sigma x \cdot P(x) = 1/2$ Var$(X) = E(X^2) - [E(X)]^2 = 15/44$

# Binomial Distribution

*

$m$ = Mean = $np$ Var = $npq$

Probability fun of B.D is $P(x) = {}^{n}C_x\, p^x q^{n-x}$

p - success, q = failure, n - trials.

---

* 256 set of 12 tosses of a coin in how many cases one can expect 8 heads & 4 tails ?

Sol?: $p = 0.5$, $q = 0.5$, $n = 12$,

$P(x) = {}^{n}C_x\, p^x q^{n-x}$

$P(x=8) = {}^{12}C_8\, (0.5)^8\, (0.5)^4 = 0.30$

Expected no of such cases in 256 sets

is = $256 \times 0.30$

$= 76.80 \simeq 77$

* In sampling a large no of parts manufa-ctured by a company, the mean no, of defectives in samples of 20 is 2. Out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

Sol?: Mean $= np = 2$ when $n = 20$

$\Rightarrow 20p = 2 \Rightarrow p = 1/10 = 0.1$

$P(x) = {}^{n}C_x\, p^x q^{n-x}$

$= {}^{20}C_x\, (0.1)^x (0.9)^{20-x}$

Prob' of atleast 3 defectives is $= P(3) + P(4) \ldots + P(20)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \{ {}^{20}C_0 (0.1)^0 + (0.9)^{20-0} + {}^{20}C_1 (0.1)^1 (0.9)^{19} + {}^{20}C_2 (0.1)^2 (0.9)^{18} \}$

$= 0.323$

No of defectives in 1000 samples =

$1000 \times 0.323 = 323$

* Five dice were thrown 96 times and the number of times an odd no actually turned out in the experiment is given. Fit B.D to this data & calculate expected frequencies.

| No of dice showing 1 or 3 or 5 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| observed freq | 1 | 10 | 24 | 35 | 18 | 8. |

Soln: Prob' of getting 1 or 3 or 5 = $3/6 = 1/2$

$P(x) = {}^nC_x\, p^x q^{n-x}$ ; $n = 5$

$= {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x} =$

Now $f(x) = 96 \times P(x)$

$f(0) = 96 \times {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} = 3$

$f(1) = 96 \times {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1} = 15$

$f(2) = 96 \times {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2} = 30$

$f(3) = 96 \times {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} = 30$

$f(4) = 96 \times {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} = 15$

$f(5) = 96 \times {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5} = 3$

Expected frequencis are

3, 15, 30, 30, 15, 3.

# Poisson Distribution

$$P(x) = \frac{e^{-m} m^x}{x!}$$

[Mean = m] [Var = m]

* The no of persons joining a cinema que. in a minute has poisson distribution with parameters 5.8. Find the probability i) no one joins the queue in a particular minute ii) 2 or more persons join the queue

Sol?: X : No of persons joining the queue

$$P(x) = \frac{e^{-m} m^x}{x!}$$ where $m = 5.8$ & $x = 0, 1, 2 \ldots$

$$P(x) = \frac{e^{-5.8} (5.8)^x}{x!}$$

i) $P(x=0) = \frac{e^{-5.8} (5.8)^0}{0!} = 0.003$

ii) $P(x \geq 2) = 1 - P(x < 2)$

$= 1 - \{ P(x=0) + P(x=1) \}$

$= 1 - [0.003 + 6.8]$

$= 0.9796$

* For a Poisson variable ⑧

$3x\ P[X=2] = P[X=4]$ Find standard deviation.

Sol?: $3x\ P[X=2] = P[X=4]$

$$3 \times \frac{e^{-m} m^2}{2!} = \frac{e^{-m} m^4}{4!}$$

$$m^2 = 36 \Rightarrow m = 6$$

$$S.D = \sqrt{Var} = \sqrt{6} = 2.449 \quad [Mean = Var = 6]$$

* Fit a Poisson distribution

| $x$ : | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f$ : | 122 | 60 | 15 | 2 | 1 |

Sol$^n$ : Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{0+60+30+6+4}{200} = 0.5$

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \& \quad f(x) = 200 \times P(x)$$

$$f(0) = 200 \times \frac{e^{-0.5}(0.5)^0}{0!} \simeq 121$$

$$f(1) = 200 \times \frac{e^{-0.5}(0.5)^1}{1!} \simeq 61$$

$$f(2) = 200 \times \frac{e^{-0.5}(0.5)^2}{2!} \simeq 15$$

$$f(3) = 200 \times \frac{e^{-0.5}(0.5)^3}{3!} \simeq 3$$

$$f(4) = 200 \times \frac{e^{-0.5}(0.5)^4}{4} \simeq 0.$$

$\therefore$ Expected freq' are 121, 61, 15, 03, 0.

# Cumulative distribution fun

* The diameter of electric cable is assumed to be continuous with P.d.f $f(x) = \begin{cases} 6x(1-x) & \text{otherwise} \\ 0 & \end{cases}$

Verify that $f(x)$ is a P.d.f & find mean & var

Sol:

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx.$$

$$= 0 + \int_0^1 6x(1-x)\,dx + 0$$

$$= \int_0^1 (6x - 6x^2)\,dx = \left[3x^2 - 2x^3\right]_0^1 = 1$$

$\therefore f(x)\,dx$ is Pdf.

$$\text{Mean} = \int_{-\infty}^{\infty} x\, f(x)\,dx = \int_0^1 x \cdot 6x(1-x)\,dx$$

$$= \int_0^1 (6x^2 - 6x^3)\,dx = \left[2x^3 - \frac{3}{2}x^4\right]_0^1$$

$$= 1/2$$

$$\text{Var} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \int_0^1 (x - 1/2)^2\, 6x(1-x)\,dx.$$

$$= \int_0^1 \left(-6x^4 + \frac{9}{2}x^2 + \frac{3}{2}x\right)dx$$

$$= \frac{1}{20}$$

# Normal Distribution

* If $x$ is normally distributed with mean 12 and S.D 4 find the following

i) $P(x \geq 20)$ ii) $P(x \leq 20)$

Soln: Let $z = \frac{x-\mu}{\sigma} = \frac{x-12}{4}$

$$P(x \geq 20) = P\left[\frac{x-12}{4} \geq \frac{20-12}{4}\right]$$

$$= P[z \geq 2]$$

= area from 2 to $\infty$


= [area from 0 to $\infty$] − [area from 0 to 2]

$= 0.5 - 0.52$

$= 0.0228$

$$P(x \leq 20) = P\left[\frac{x-12}{4} \leq \frac{20-12}{4}\right]$$

$= 0.9772$