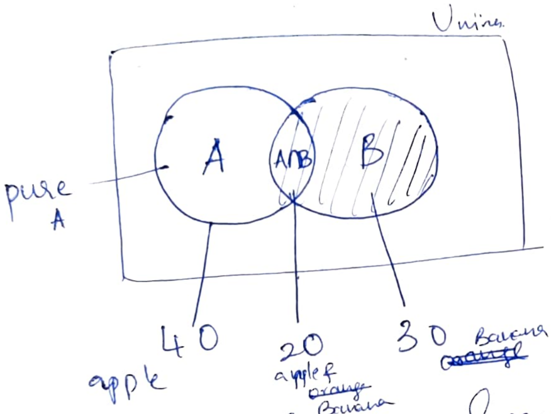
Baye's Theorem

Pre-Requisite - Conditional Probability.

→ Finding out probability of Event A given that Event B has already occurred (true) → $P(A|B) = \frac{P(A \cap B)}{P(B)}$

(Probability of A and B by probability of B)

→ Event B is serving as an condition here.



When no condition applied while Universe is my Sample space. Once I apply condition ~~to~~ ons, Sample Space reduces. that B has occurred.

(Our Sample Space Comprises of Events Occurred i.e B & $A \cap B$.)

Let find out the Values of occurred Events.

Let says there are 100 kids, out of which 40 like apples, 30 like Bananas & 20 like Both.

$A \rightarrow 40$, $B \rightarrow 30$, $A \cap B = 20$.

$\therefore$ $P(B) = \frac{30}{100} = 0.3$, $P(A \cap B) = \frac{20}{100} = 0.2$

$\therefore$ $P(A|B) = \frac{0.2}{0.3} = 0.67$ //.

Lets Consider two Events A & B.

$$\boxed{\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ P(B|A) &= \frac{P(B \cap A)}{P(A)} \end{aligned}} \quad (1)$$

We Can Write these as $\Rightarrow$

$$\boxed{\begin{aligned} P(A|B) \cdot P(B) &= P(A \cap B) \\ P(B|A) \cdot P(A) &= P(B \cap A) \end{aligned}} \quad (2)$$

Observe in Equation (2) R.H.S is Same.

$\therefore$ We Can Equate their L.H.S as follows:

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A).$$

$$\Longrightarrow \boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

Posterior Probability: $P(A|B)$

Likelihood: $P(B|A)$

Prior: $P(A)$

Marginal: $P(B)$

Which is your Baye's Theorem.

→ where A is your hypothesis, B is your data or Evidence. Hence we read, ~~proba~~ find probability of the hypothesis, given that B is some data or we have observed some Evidence. So we compute the probality of hypothesis given the Evidence or data.

* Cannot find out posterior which knowing, likelihood and prior probabilities.

(P)

Likelihood → Probability of Evidence, Given the hypothesis is true.

Prior → Probability of hypothesis, before Considering Evidence

Marginal → Pure Probability of data or Evidence.

Ex: $P(\text{King} \mid \text{Face})$ find probability that Card is King, Given that it was face.

$$P(\text{King} \mid \text{Face}) = \frac{P(\text{Face} \mid \text{King}) \cdot P(\text{King})}{P(\text{Face})}$$

$$= \frac{1 \cdot 4/52}{12/52} = \frac{1 \cdot 1/13}{3/13} = \frac{1}{3}$$

2) Does patient have Cancer or not.

→ Two Outcomes +ve and -ve.

→ Test results Correct +ve result in only 98% of Cases in which disease is actually present.

→ Test results. Correct -ve result in only 97% of Cases in which disease is not present.

→ furthermore, 0.008 of the Entire population have Cancer.

- $P(\text{Cancer}) = 0.008$, $P(\neg\text{cancer}) = 0.992 \rightarrow (1-0.008) = 0.992$
- $P(+ \mid \text{cancer}) = 0.98$ $\quad P(- \mid \text{cancer}) = 0.02 \rightarrow (1-0.98)$
- $P(+ \mid \neg\text{cancer}) = 0.03$ $\quad P(- \mid \neg\text{cancer}) = 0.97 \rightarrow (1-0.03)$

* Suppose we now observe a new patient for whom the lab test returns a positive result.
* Should we diagnose the patient as having Cancer or not ?

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

P(B) → (Probability of Entire data set)

maximized here ignore.

$$P(\text{cancer}|+) = \frac{P(+|\text{cancer})\,P(\text{cancer})}{\cancel{P.E.}}$$

$= 0.98 * 0.008 = 0.0078$

$P(\neg\text{cancer}|+) = P(+|\neg\text{cancer}) * P(\neg\text{cancer}) = 0.03 * 0.992$

$= 0.0298$

* Suppose we now observe a new patient for whom the lab test returns a negative result.
* Should we diagnose the patient as having Cancer or not ?

$P(\text{cancer}|-) = P(-|\text{cancer}) * P(\text{cancer})$

$= 0.02 * 0.008 = 0.00016$

$P(\neg\text{cancer}|-) = P(-|\neg\text{cancer}) * P(\neg\text{cancer})$

$= 0.97 * 0.992 = 0.96224$ ✓ greater.

* Naive Bayes Classifier - Ex:

Ex. No	Color	Type	Origin	Stolen?
1	Red	Sports	Domestic	Yes
2	Red	Sports	"	No
3	Red	Sports	"	Yes
4	Yellow	Sports	"	No
5	Yellow	Sports	Imported	Yes
6	Yellow	SUV	"	No
7	Yellow	SUV	"	Yes
8	Yellow	SUV	Domestic	No
9	Red	SUV	Imported	No
10	Red	Sports	Imported	Yes

New Instance = (Red, SUV, Domestic). — No ✓

$P(Yes) = \frac{5}{10} = 0.5$, $P(No) = \frac{5}{10} = 0.5$.

Color	Yes	NO
Red	3/5	2/5
Yellow	2/5	3/5

Type	Yes	NO
Sports	4/5	2/5
SUV	1/5	3/5

Origin	Yes	No
Domestic	2/5	3/5
Imported	3/5	2/5

$P(Yes|\text{New instance}) = P(Yes) * P(Red|Yes) * P(SUV|Yes) * P(Domestic|Yes)$

$= 0.5 * 3/5 * 1/5 * 2/5 = 0.5 * 0.6 * 0.2 * 0.4$

$= 0.024$

$P(No|\text{New instance}) = P(No) * P(Red|No) * P(SUV|No) * P(Domestic|No)$

$= 0.5 * 2/5 * 3/5 * 3/5 = 0.5 * 0.4 * 0.6 * 0.6$

$= 0.072$.

$\therefore P(No|\text{New instance}) > P(Yes|\text{New instance})$.