UNIT - 2

Merge Sort

$T(n) = T(n/2) + T(n/2)$

$T(n) = 2T(n/2) + n$ $\qquad$ $T(n) = 2T(n/2) + n$

$= 2[2T(n/4) + n/2] + n$ $\qquad$ $T(n/2) = 2T(n/4) + n/2$

$= 4T(n/4) + n + n$ $\qquad$ $T(n/4) = 2T(n/8) + n/4$

$= 4[2T(n/16) + n/16] + n + n$

$= 8T(n/8) + n + n + n$

$= 2^3 T(n/8) + n\left[1+1+1\right]$ $\quad 3n$

$= 2^3 T(n/8) + 3n$

$\vdots$

$= 2^i T(n/2^i) + n\left[\frac{2^i - 1}{2}\right]$

$= 2^i T(n/2^i) + in$

$= \cancel{n\,T(n/n) + n^2}$ $\qquad$ $2^i = n$

$= n\,T(n/n) + \log_2 n \times n$ $\qquad$ $i = \log_2 n$

$= 0 + \log_2 n \times n$

$= n \log n$

{$\log_2 n$}

Quick sort

- $\Omega, \Theta \rightarrow n \log n$ $\qquad$ $T(n) = 2T(n/2) + n$
- $O \rightarrow n^2$ $\qquad$ Ex 1, 2, 3, 4, 5, 6

$T(n) = T(n-1) + n$

$\vdots$

n^2

Binary Search

$T(n) = T(n/2) + 1$

$\simeq O(\log_2 n)$

General Recurrance for Divide and Conquer

$T(n) = aT(n/b) + f(n)$

MASTER's METHOD / THEOREM

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

$T(n) = T(n/2) + 1 \quad \Rightarrow T(n/2) + n^0$

$a = 1 \qquad b = 2 \qquad \therefore d = 0$

$b^d = 2^0 = 1$

$a = b^d$

$\Theta(n^d (\log n))$

$\Theta(n^0 \log n)$

$\Theta(\log_2 n)$

(i) $T(n) = 2T(n/2) + n$

$a=2 \quad b=2 \quad d=1$

$b^d = 2^1 = 2$

$a = b^d$

$\Theta(n^d (\log n))$

$\Theta(n^1 \log n)$

$\Theta(n \log_2 n)$

(ii) $T(n) = 3T(n/2) + n^2$

$a=3 \quad b=2 \quad d=2$

$b^d = 2^2 = 4$

$a < b^d$

$\cancel{\Theta}(n^{\log_b a}) \qquad \Theta(n^d)$

$\cancel{\Theta(n^{\log_2 3})} \quad \cancel{\in \Theta(n^2) \log} \quad \Theta(n^2)$

(iii) $T(n) = 4T(n/2) + n$

$a=4 \quad b=2 \quad d=1$

$b^d = 2^1 = 2$

$a > b$

$\Theta(n^{\cancel{d}} \log_2 4)$

$\Theta(n^2)$

$\cancel{\Theta(n)}$

(iv) $4T(n/2) + n^2$

→ $a=4 \quad b=2 \quad d=2$

$b^d = 2^2 = 4$

$a = b^d$

$\Theta(n^d \log n)$

$\Theta(n^2 \log n)$

(v) $4T(n/2) + n^3$.

→ $a=4 \quad b=2 \quad d=3$

$b^d = 2^3 = 8$

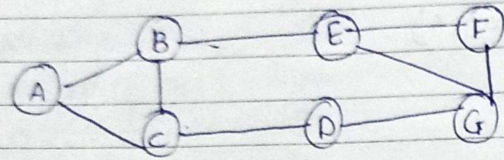
$a < b^d$.

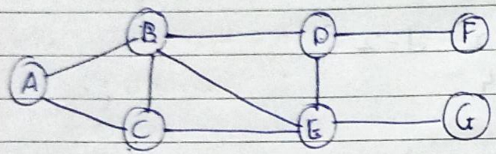
$\Theta(n^3)$.

D

GRAPH TRAVERSAL

1 DEPTH FIRST SEARCH : Blind Search





F

D

E G

C

BA

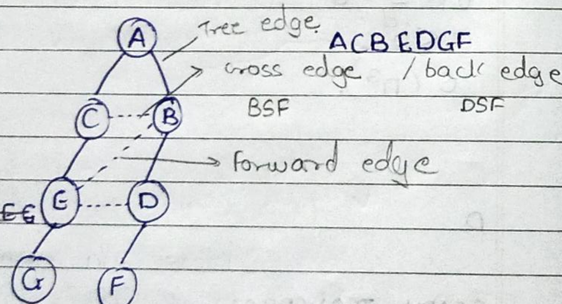
A

DFS

F D G E C B A

~~BREATH~~

BFS



A	C	B	E	D	G	F

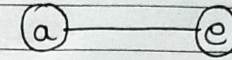
ACBEDGF

Graph → has cycle, articulation

Tree → does't have cycle

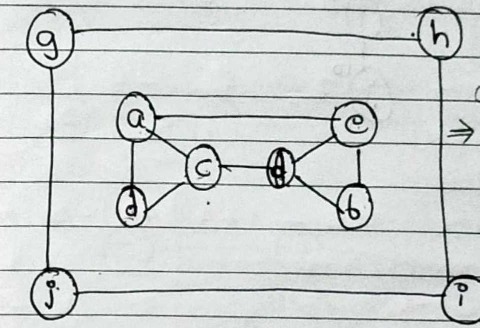
b^d b → breadth d = depth

Examples



a c

(1)

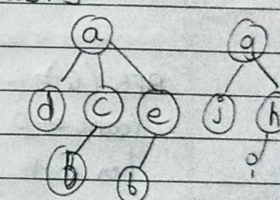


DFS

a c f b e | g h i j

⇒ d e b f c a | j i h g

BFS



a d c e f b g j h i

DFS (G)

count ← 0

for each vertex in V do

if v is marked with 0

dfs (v)

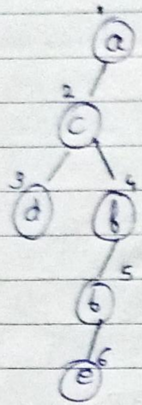
dfs (v)

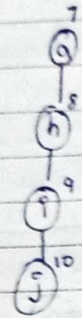
count ← count + 1 ; mark v with count

for each vertex w in V adjacent ~~to~~ v do

if w is marked with 0

dfs(w)



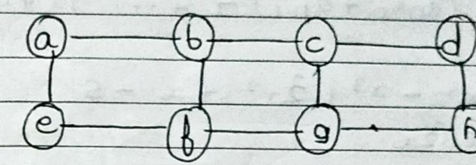


BFS (G)
count ← 0
for each vertex v in V do
  if v is marked with 0
    bfs (v)

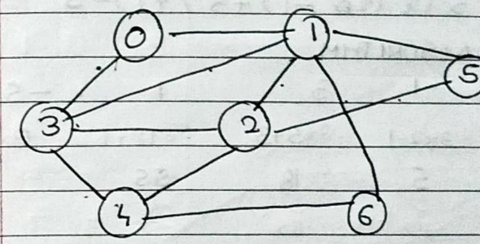
bfs (v)
count ← count + 1 ; mark v with count and initial queue with v

while queue not empty do
  for each vertex w in V adjacent to the front vertex do
    if w marked with zero then
      count ← count + 1 ;
      mark w with the count
      add w to the queue
  remove the front vertex from the queue

(2)



(3)



TRANSFORM & CONQUER

$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x_1 + a_0$ — (1)

$P(x) = (\quad (a_n x_n + a_{n-1}) x + \dots)x + a_0$ — (2)

$P(x) = 2x^4 - x^3 + 3x^2 + x - 5$

$= x(2x$

$= x(x$

$= x(x(x(2\cancel{x} - 1) + 3) + 1) - 5$ — (3)

HORNER'S ALGORITHM.

coeff →	2	-1	3	1	-5
at $x=3$	↓	$3\times2-1$	$3\times5+3$	$3\times18+1$	$3\times55-5$
$x-3$ ↓ divisor	2	5	18	55	60 Remainder
	$2x^2$	$5x^2$	$18x$	55	

quotient

$P \leftarrow P[n]$

for $i = n-1 \dots$ down to 0 do

$P \leftarrow x * P + P[i]$

(1) $P(x) = 3x^5 + 2x^4 - 5x^3 + x^2 + 7$ at $x = -3$

	3	2	-5	1	0	7
at $x=-3$		$-3\times3+2$	$(-3\times-7)-5$	$-3\times16+1$	141	$\cancel{47}141\times-3+7$
	3	-7	16	-47	141	$\cancel{148}$ -416

(2) $P(x) = 2x^5 + 2x^4 + 3x^3 - x^2 - x + 3$ at $x = +1$

→

coeff	2	2	3	-1	-1	3
$x=1$		$2\times1+2$	$4+3$	$7-1$	$6-1$	$5+3$
	2	4	7	6	5	8

TRANSFORM & CONQUER

CLUSTERING

K-Mean

$K = \{2, 3, 4, 10, 11, 12, 20, 25, 30\}$

$K=2$

$m_1 = 4$ $m_2 = 12$

$k_1 = \{2, 3, 4\}$ $=$ $m_1 = \frac{9}{3} = 3$

$K_2 = \{10, 11, 12, 20, 25, 30\}$ $m_2 = \frac{108}{6} = 18$

$m_1 = \cancel{4}\,3$ $m_2 = 18$

$K_1 = \{2, 3, 4, 10\}$ $K_2 = \{11, 12, 20, 25, 30\}$

$m_1 = \frac{19}{4} = 4.75 \simeq 5$ $m_2 = 19.6 \simeq 20$

$K_1 = \{2, 3, 4, 10, 11, 12\}$ $K_2 = \{20, 25, 30\}$

$m_1 = \frac{42}{6} = 7$ $m_2 = 25$

$K_1 = \{2, 3, 4, 10, 11, 12\}$ $K_2 = \{20, 25, 30\}$