The Noisy-V Function

Take the same example of Medical Diagnosis

$P(A)$ → Probability of having a high temperature
$P(B)$ → Probability of having a cold
$P(A|B)$ → Probability that one will have a high temperature if one has a cold

$P(A|C)$ → Probability that one will also have a high temperature if one has the plague

i.e $P(A|B) = 0.8$ & $P(A|C) = 0.99$

* Noisy-V f^2 is based on the assumption that only possible causes of a high temperature are a cold & the plague
  i.e. $P(A|B \vee C) = 1$
* Since it is not true, let us fix this by adding a [leak node] → which is all possible causes
  i.e $P(A|D) = 0.9$ where D is the leak node
  Here, D represents all other causes of a high temperature
* Need to define the [noise parameters] → conditional probabilities for $\neg A$ → which is a negation of A

i.e.
$$P(\neg A|B) = 1 - P(A|B) = 1 - 0.8 = 0.2$$
$$P(\neg A|C) = 1 - P(A|C) = 1 - 0.99 = 0.01$$
$$P(\neg A|D) = 1 - P(A|D) = 1 - 0.9 = 0.1$$

* Note that the Noise parameters are the ones which stop each illness from causing a high temperature

If B, C & D are all false then Probability of having a high temperature i.e.

$P(A) = 0$ since all the causes B, C & D are false.

∴ Probability of not having a high temperature i.e. $P(\neg A) = 1$

Else $P(\neg A)$ = Product of the noise parameters [i.e. $P(\neg A|B)$, $P(\neg A|C)$ & $P(\neg A|D)$] for all the variables that are true.

Case 1: B = false, C = false, D = false $\Rightarrow P(A) = 0$

∴ $P(\neg A) = 1 - 0 = 1$

↳ Since all noise parameters are false. ∴ The product is 0

Case 2: B = false, C = false, D = true

$P(\neg A) = P(\neg A|D) = 0.1$ // ∴ $P(A) = 1 - 0.1 = 0.9$

↳ Because only the noise parameter for D is true.

Case 3: B = False, C = True, D = False

$P(\neg A) = P(\neg A|C) = 0.01$ // ∴ $P(A) = 1 - 0.01 = 0.99$

Case 4: B = False, C = True, D = True

$P(\neg A) = P(\neg A|C) * P(\neg A|D) = 0.01 * 0.1 = 0.001$ //

Product of the Noise parameters for C & D which are true

$P(A) = 1 - 0.001 = 0.999$

Case 5: B = True, C = False, D = False

$P(\neg A) = P(\neg A|B) = 0.2$ // $P(A) = 1 - 0.2 = 0.8$

Case 6: B = True, C = False, D = True

$P(\neg A) = P(\neg A|B) * P(\neg A|D) = 0.2 * 0.1 = 0.02$ //

$P(A) = 1 - 0.02 = 0.98$

Case 7: B = True, C = True, D = False

$P(\neg A) = P(\neg A|B) * P(\neg A|C) = 0.2 * 0.01 = 0.002$ //

$P(A) = 1 - 0.002 = 0.998$

Case 8: B = True, C = True, D = True

$P(\neg A) = P(\neg A|B) * P(\neg A|C) * P(\neg A|D)$

$= 0.2 * 0.01 * 0.1$

$P(\neg A) = 0.0002$ //

$\therefore P(A) = 1 - 0.0002 = 0.9998$