Horner’s Rule

Introduction

In mathematics and computer science, **Horner's method** (or **Horner's scheme**) is an algorithm for polynomial evaluation.

Given the polynomial

$$p(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n,$$

The algorithm is based on **Horner's rule:**

$$\begin{aligned} & a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n \\ & \quad = a_0 + x\Big(a_1 + x\Big(a_2 + x\big(a_3 + \cdots + x(a_{n-1} + x\, a_n)\cdots\big)\Big)\Big). \end{aligned}$$

Algorithm

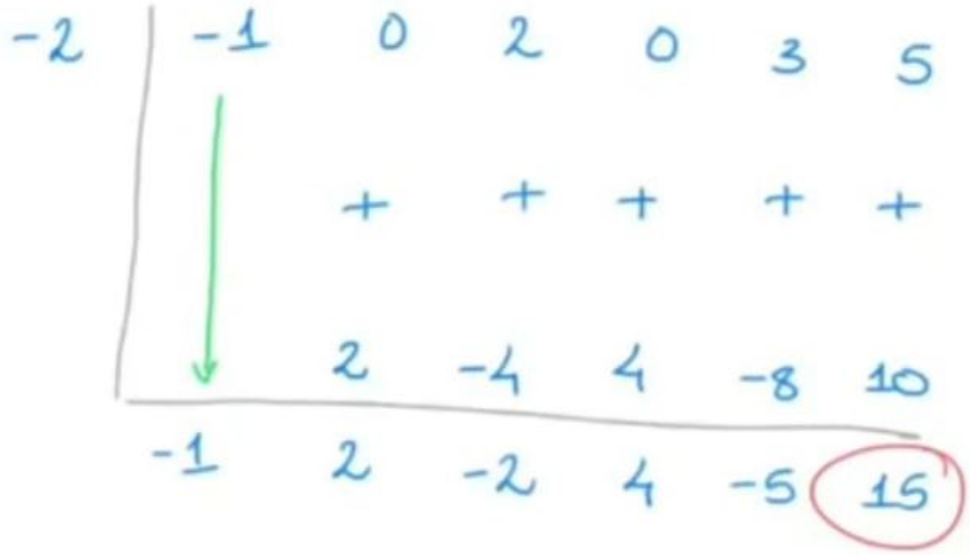
```
Horners(p[0..n], x)
    k=p[n]
        for i=n-1 down to 0 do
            k=k*x+p[i]
```

Solved example 1

Evaluate:

$f(x) = -x^5 + 2x^3 + 3x + 5$

When $x = -2$

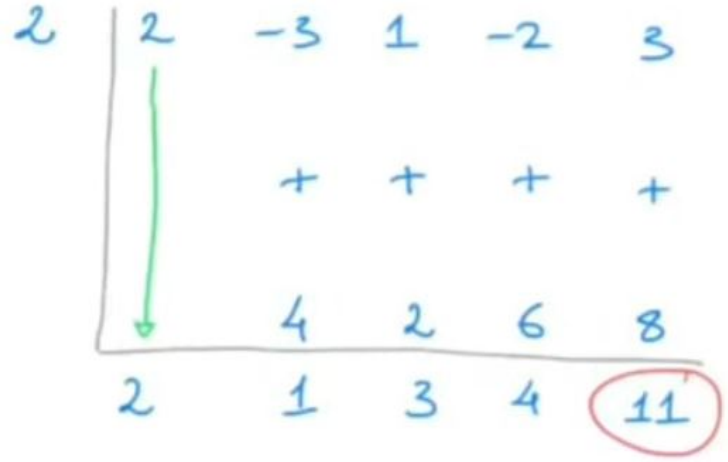


Solved example 2

Evaluate:

$f(x) = 2x^4 - 3x^3 + x^2 - 2x + 3$

When $x = 2$



$f(2) = 11$

$f(2) = 2\times2^4 - 3\times2^3 + 2^2 - 2\times2 + 3$
$= 2\times16 - 3\times8 + 4 - 4 + 3$
$= 11$