Design and Analysis of Algorithms Lab (18ISL47)

Department of Information Science and Engineering
Gogte Institute of Technology

Experiment-2
Quicksort

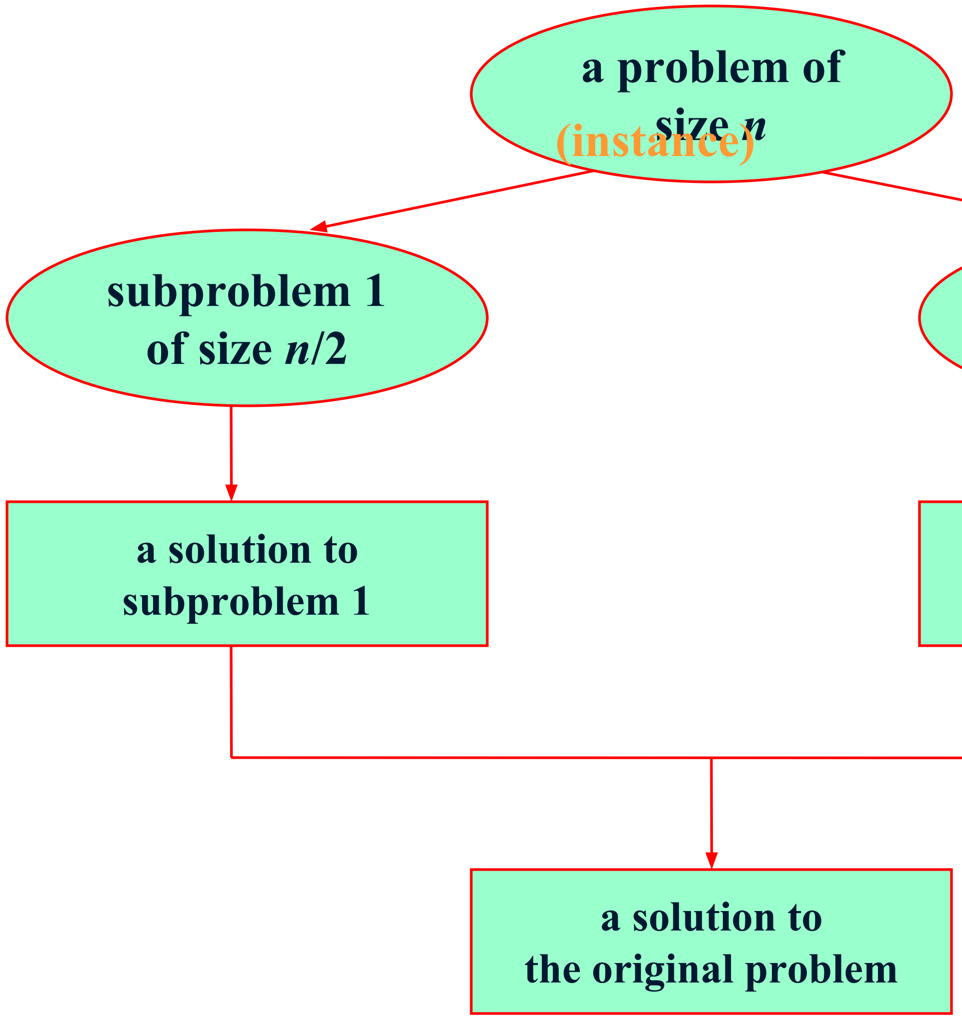
Sort a given set of elements using the Quicksort method and determine the time required to sort the elements. Repeat the experiment for different values of n, the number of elements in the list to be sorted and plot a graph of the time taken versus n. The elements can be read from a file or can be generated using the random number generator.

Divide-and-Conquer

The most-well known algorithm design strategy:

- Divide instance of problem into two or more smaller instances
- Solve smaller instances recursively
- Obtain solution to original (larger) instance by combining these solutions

Divide-and-Conquer



It general leads to a recursive algorithm!

Quicksort Algorithm

Given an array of *n* elements (e.g., integers):

- If an array contains only one element, then return
- Else
  - pick one element to use as *pivot.*
  - Partition elements into two sub-arrays:
    - First array that contains elements less than or equal to pivot
    - Second array that contains elements greater than pivot
  - Quicksort two sub-arrays
  - Return results

Example

We are given an array of n integers to sort:

40	20	10	80	60	50	7	30	100

Pick Pivot Element

There are a number of ways to pick the pivot element. In this example, we will use the first element in the array:

40	20	10	80	60	50	7	30	100

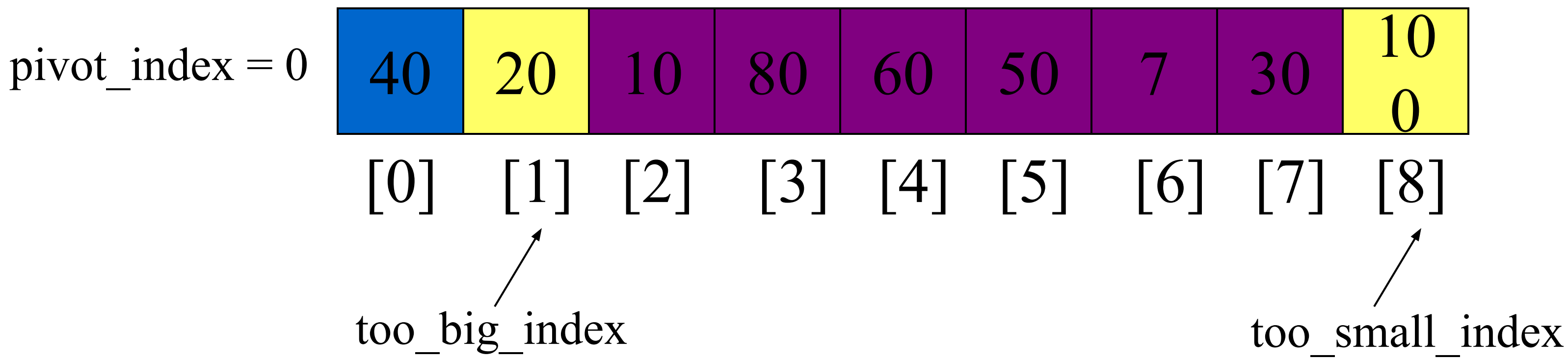
Partitioning Array

Given a pivot, partition the elements of the array such that the resulting array consists of:

1. One sub-array that contains elements <= pivot
2. Another sub-array that contains elements > pivot

The sub-arrays are stored in the original data array.

Partitioning loops through, swapping elements less than or greater than pivot.


pivot_index = 0
40
20
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80
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[0]
[1]
[2]
[3]
[4]
[5]
[6]
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too_small_index

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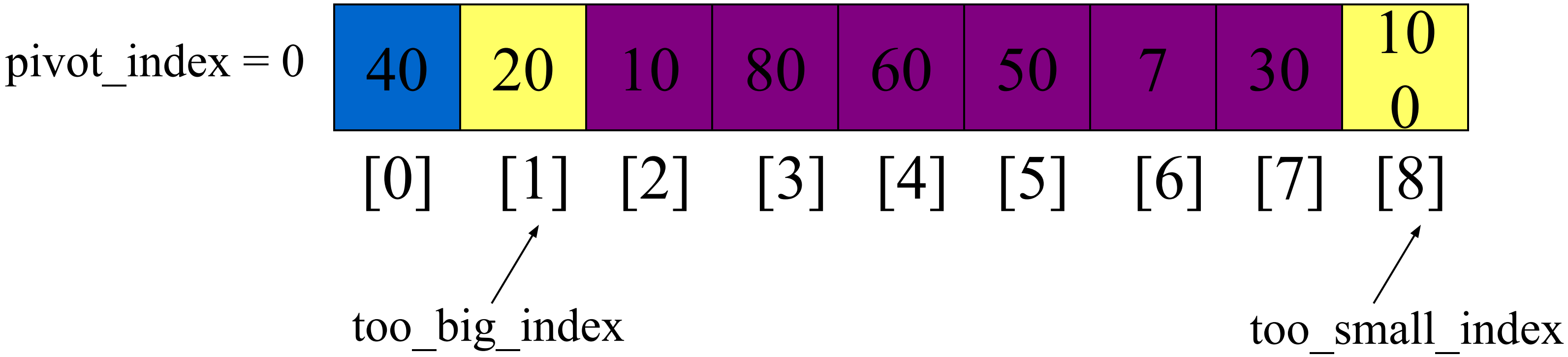

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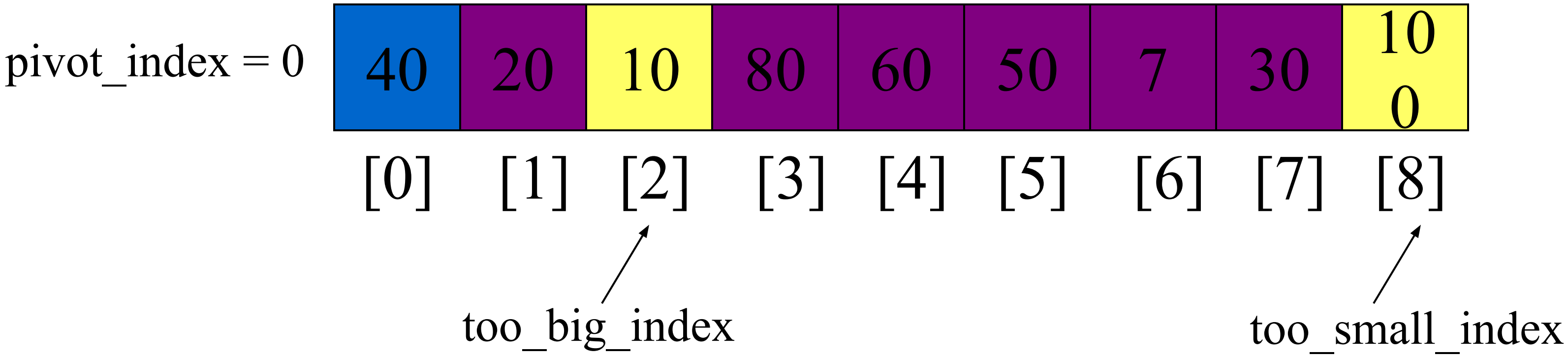

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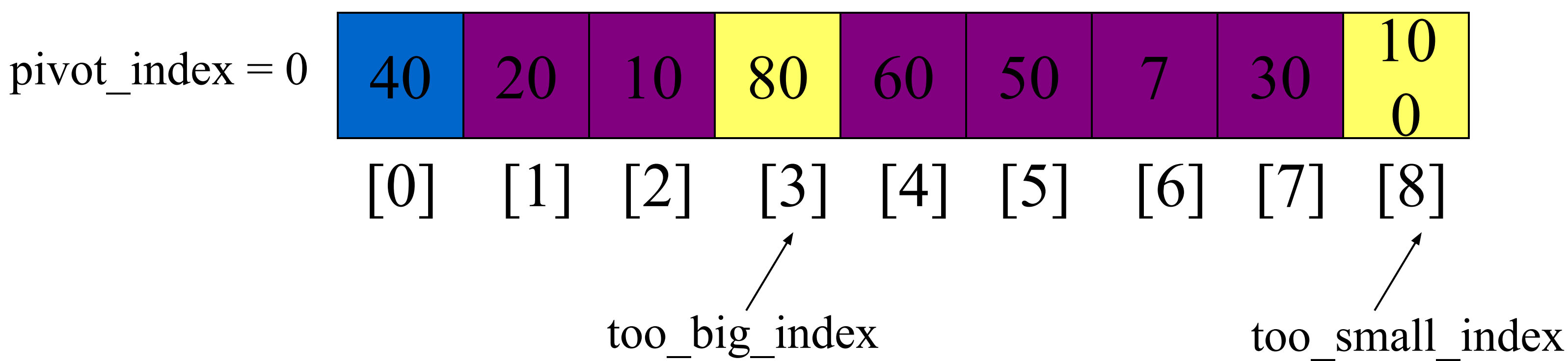

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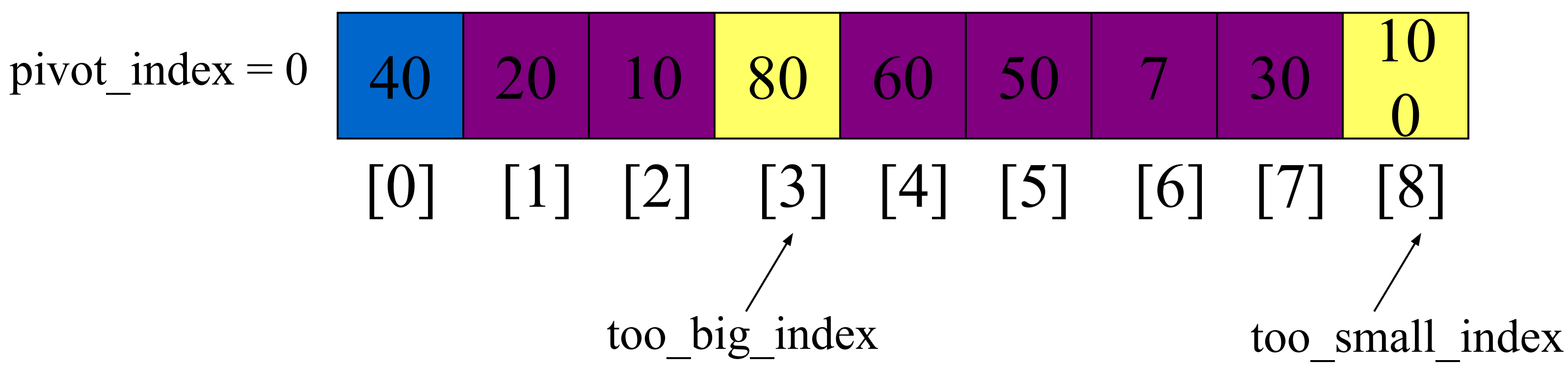

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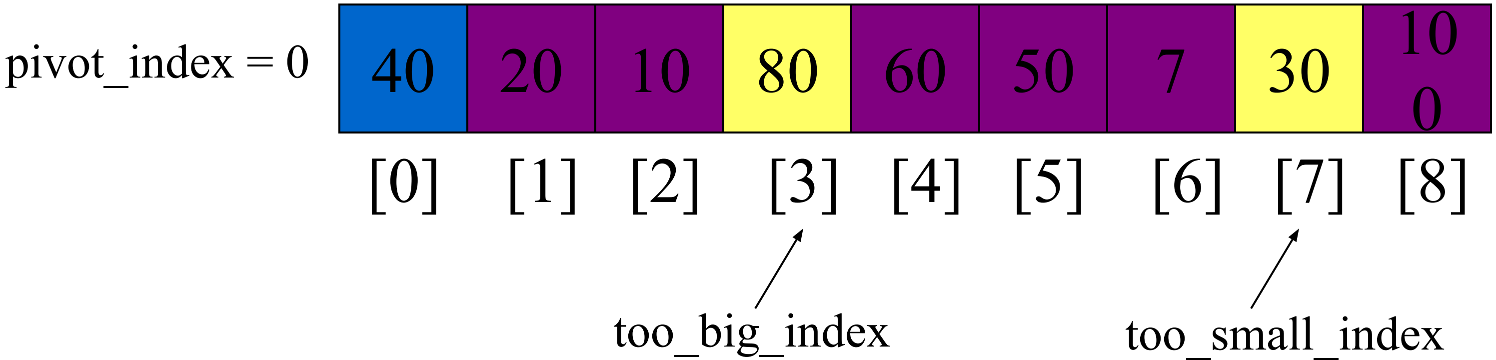

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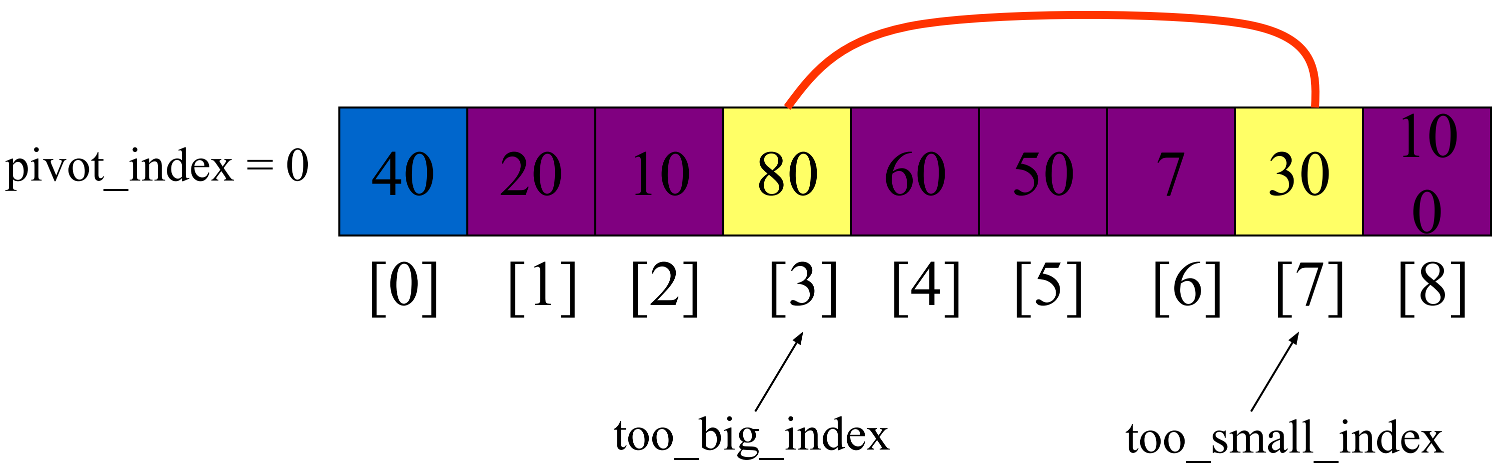

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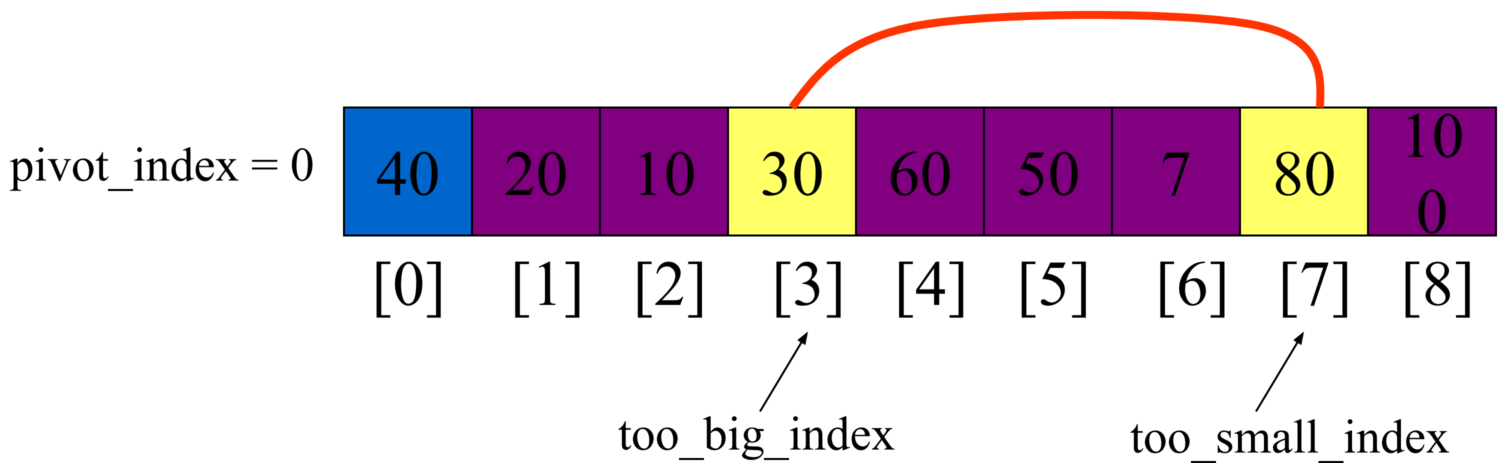

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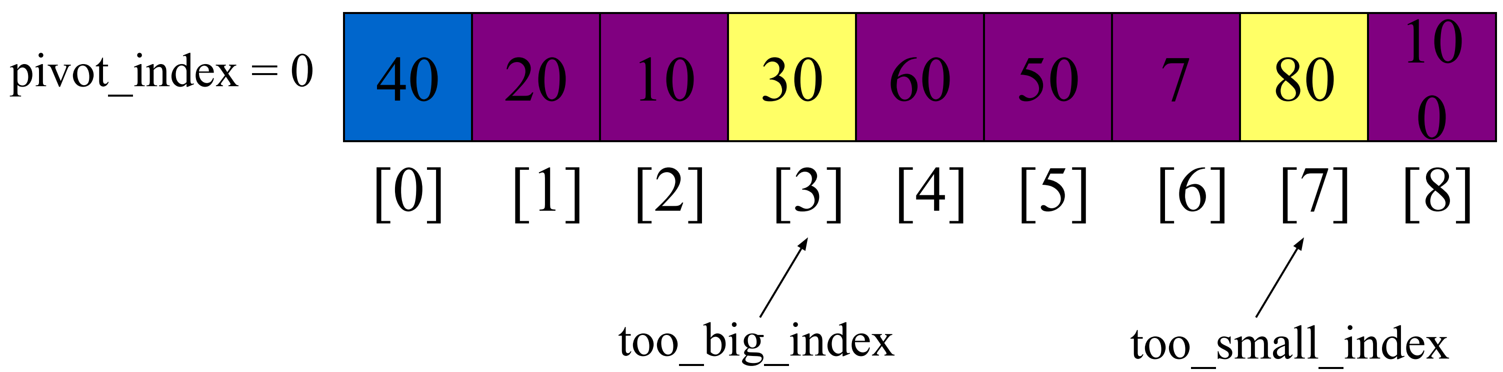

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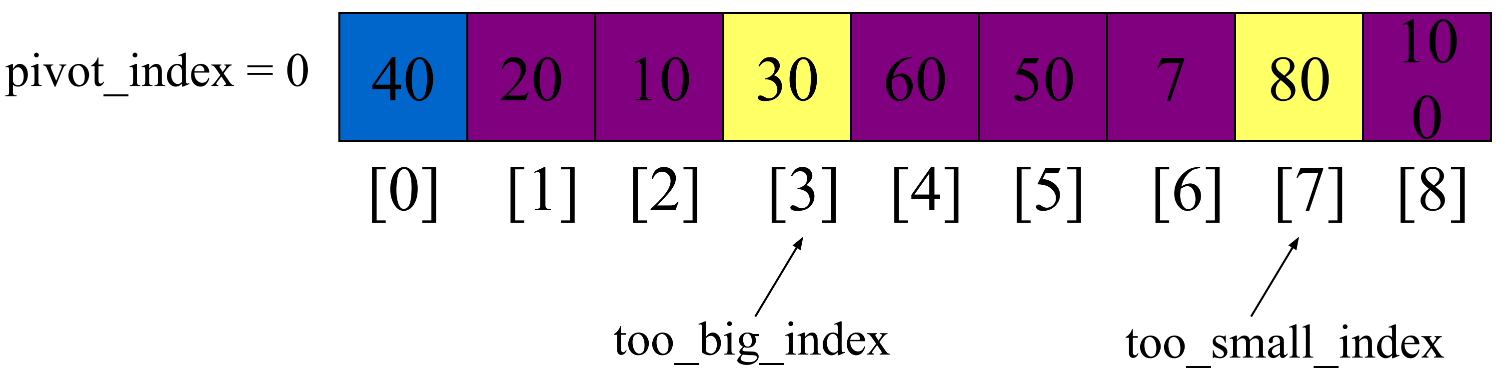

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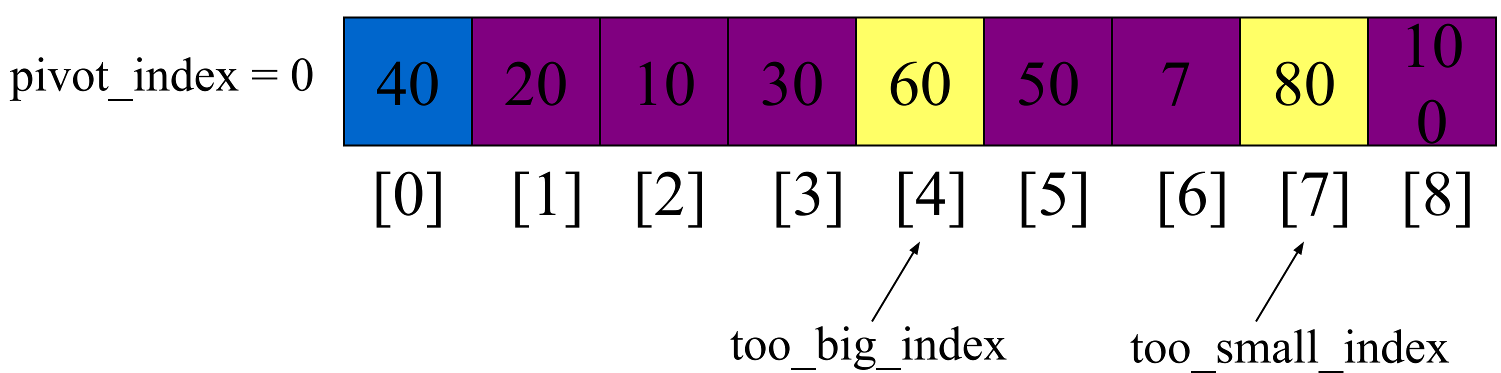

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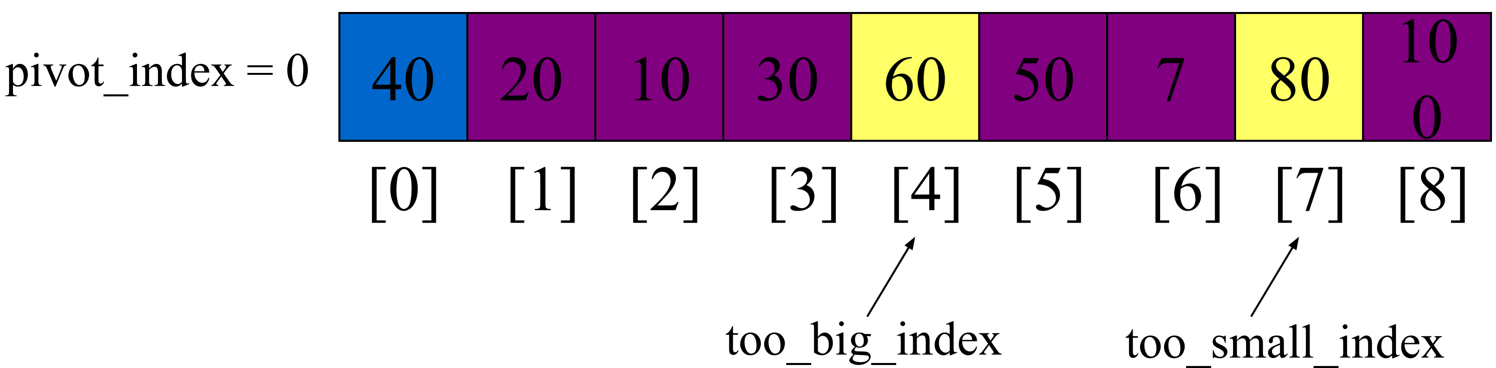

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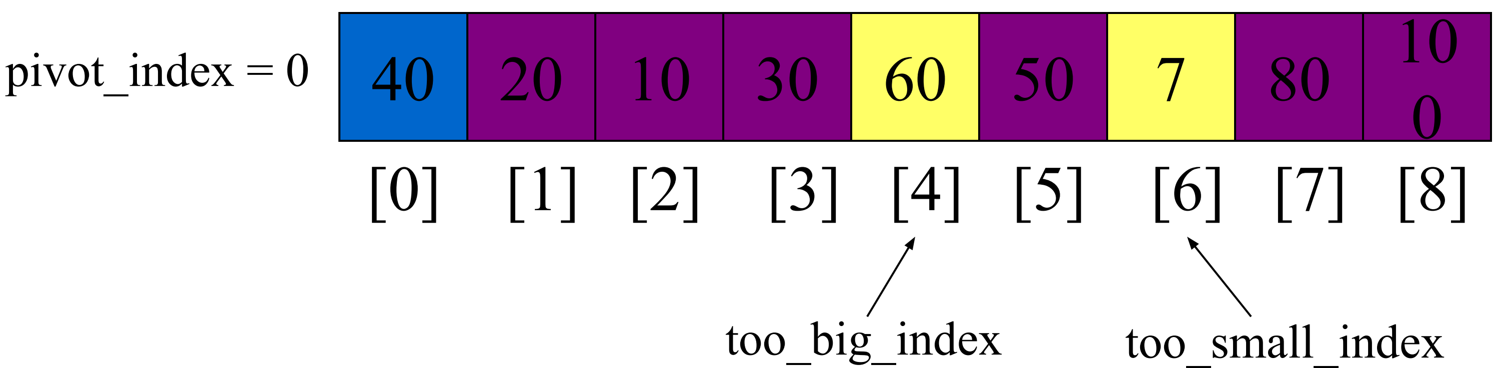

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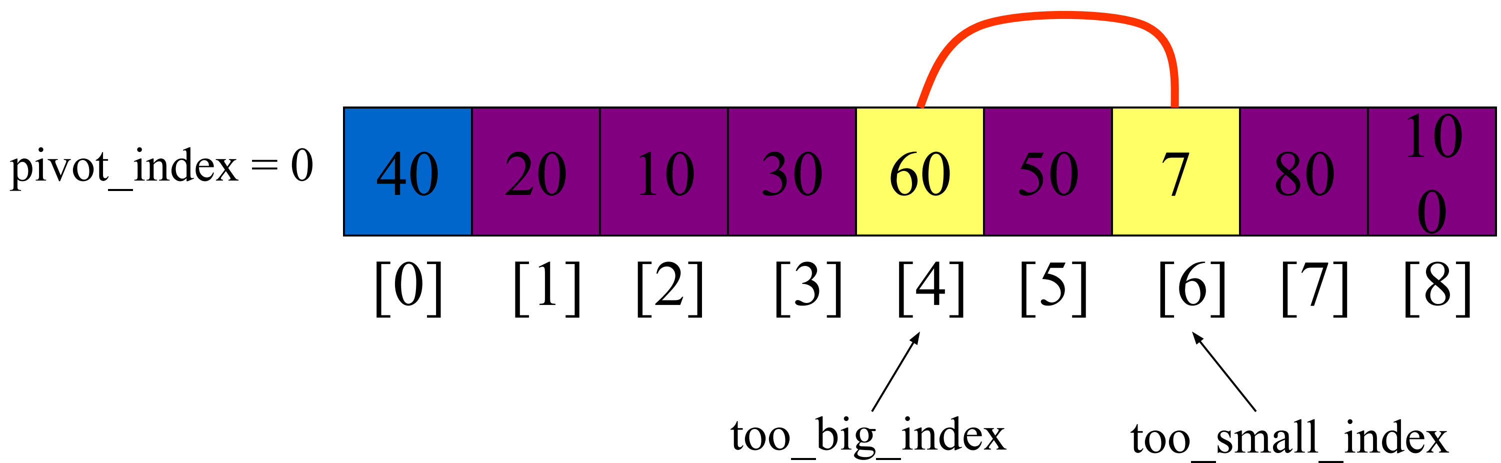

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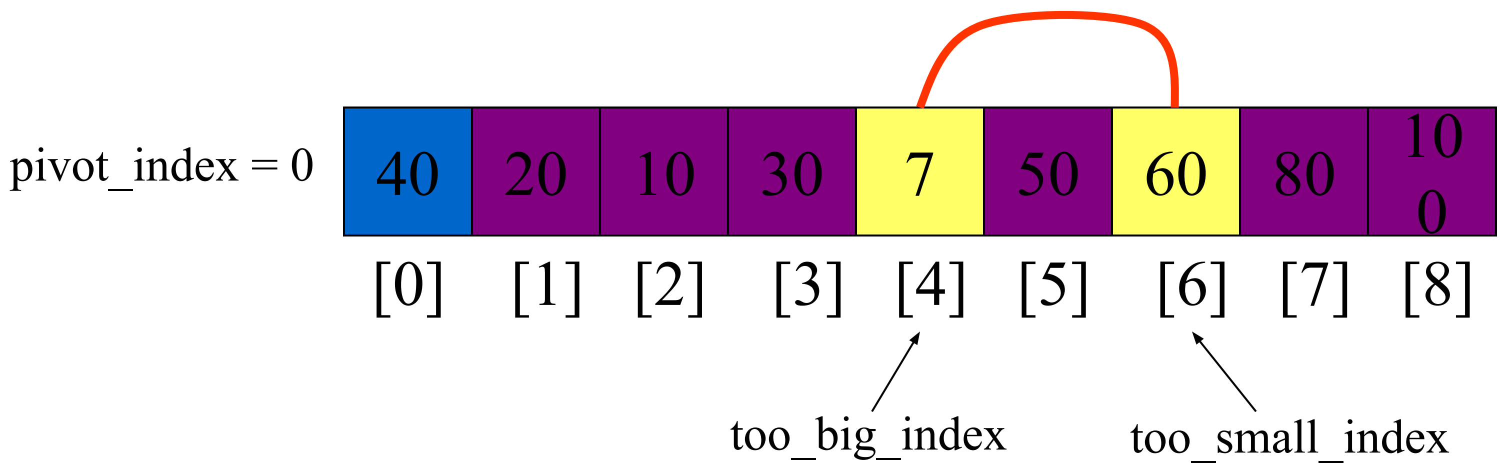

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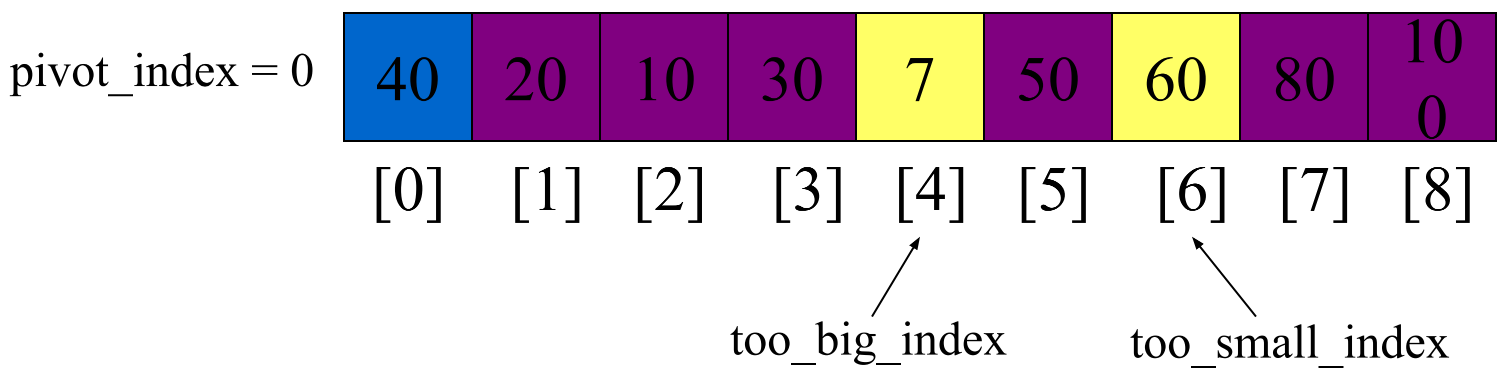

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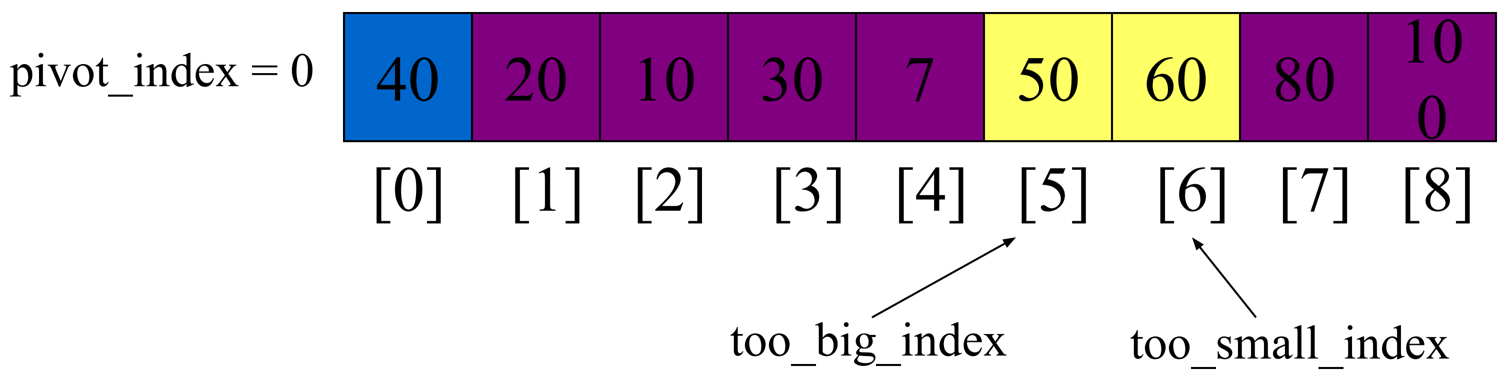

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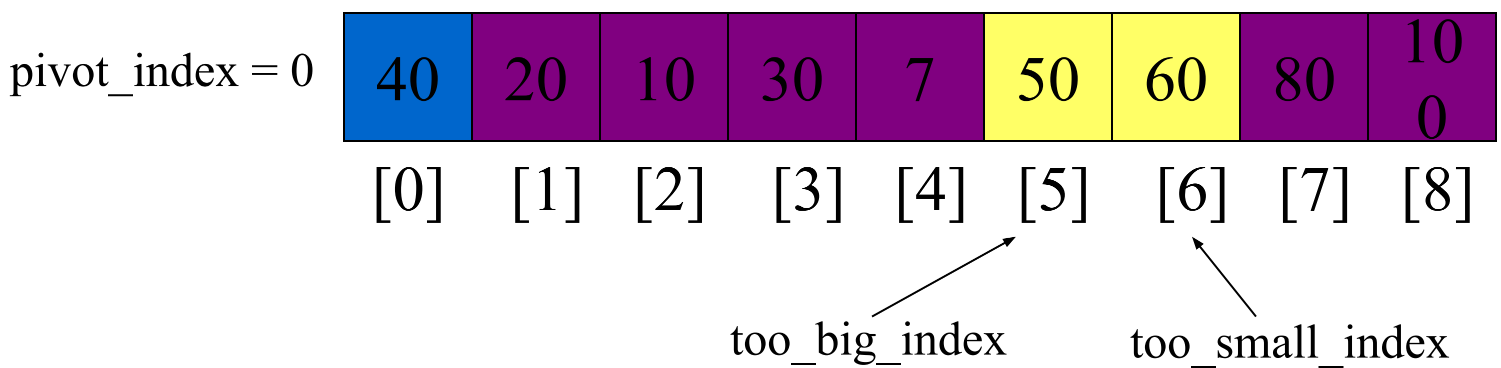

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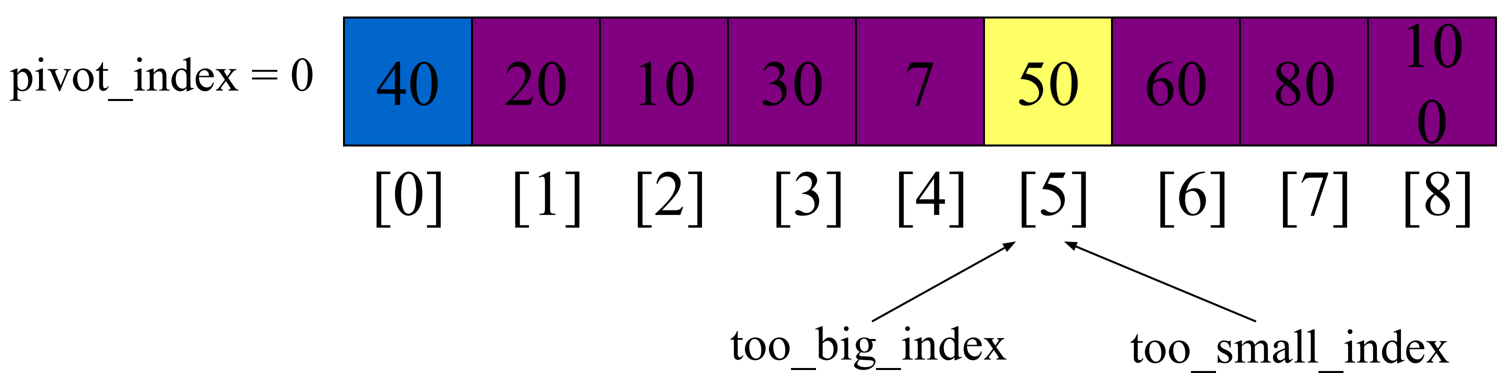

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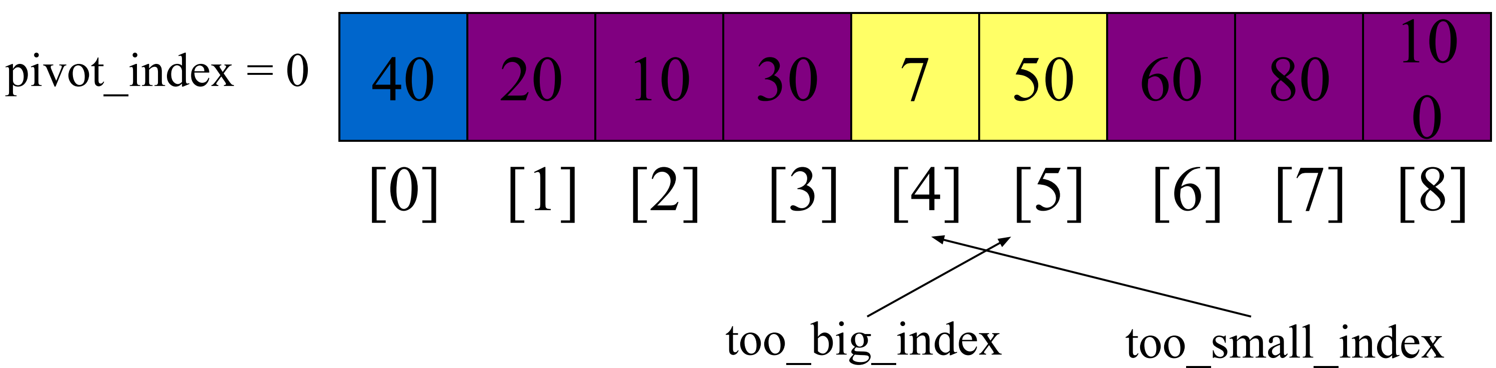

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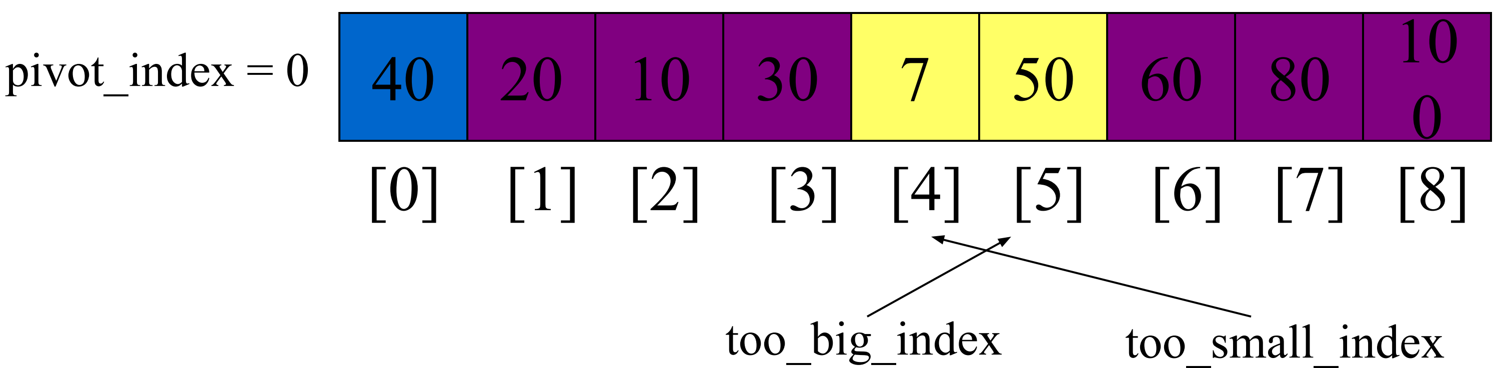

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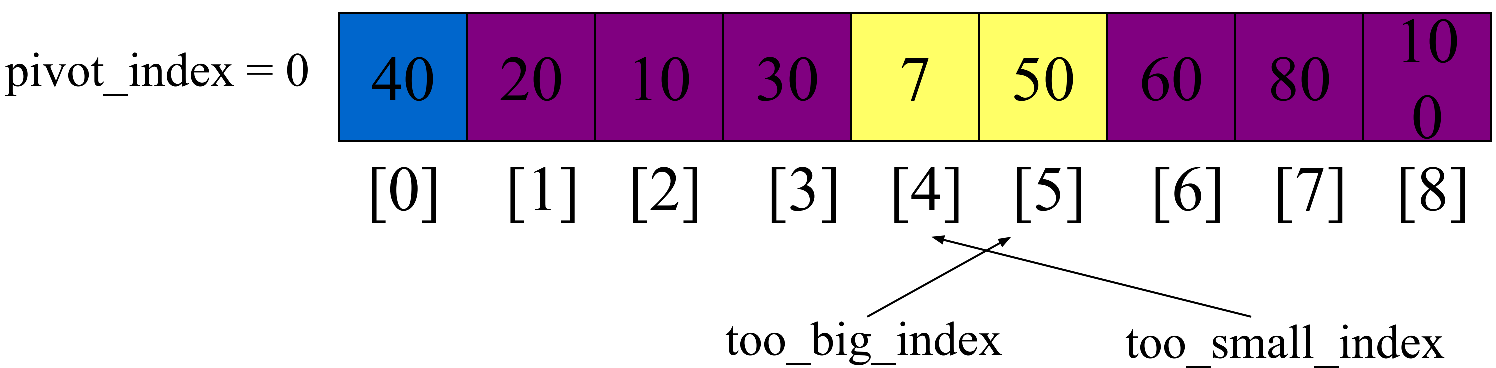

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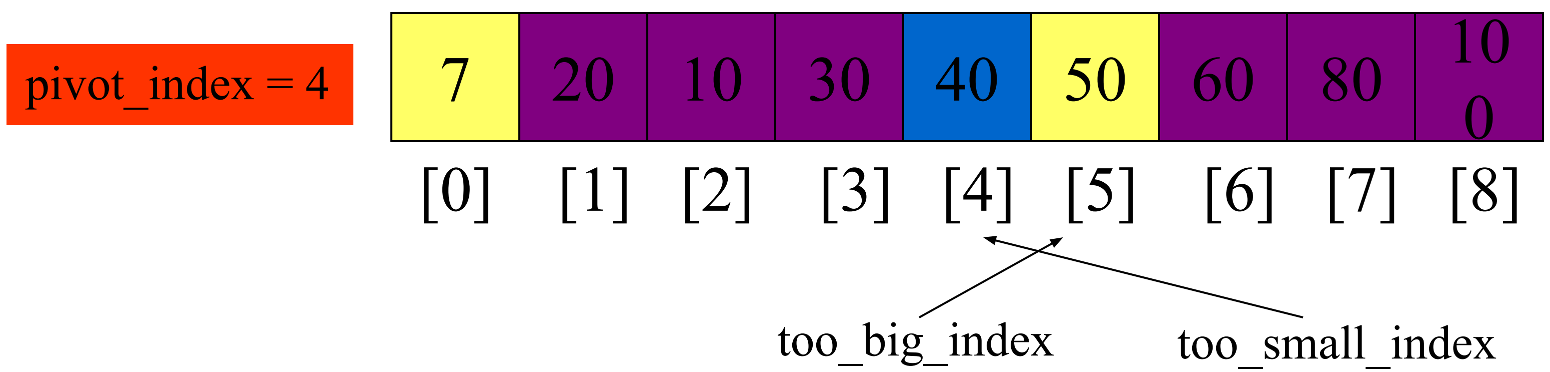

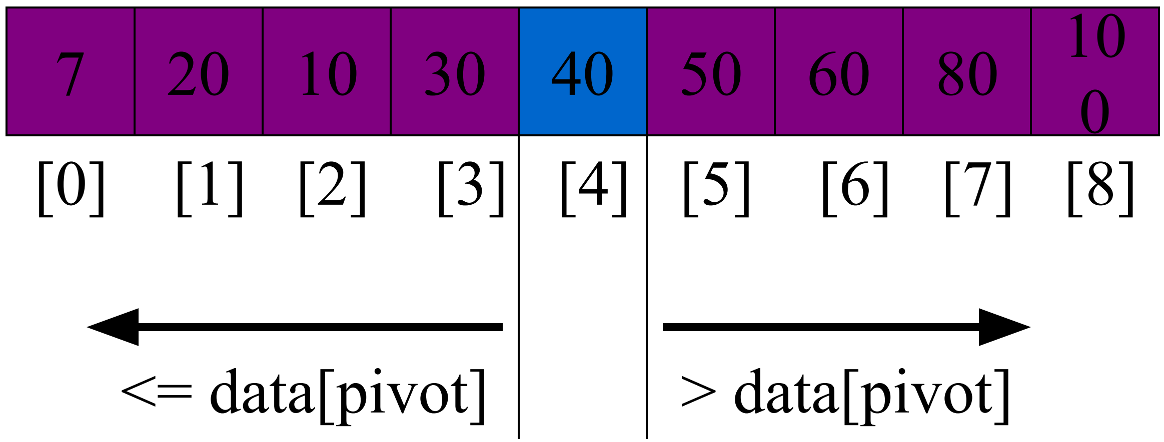
7
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[0]
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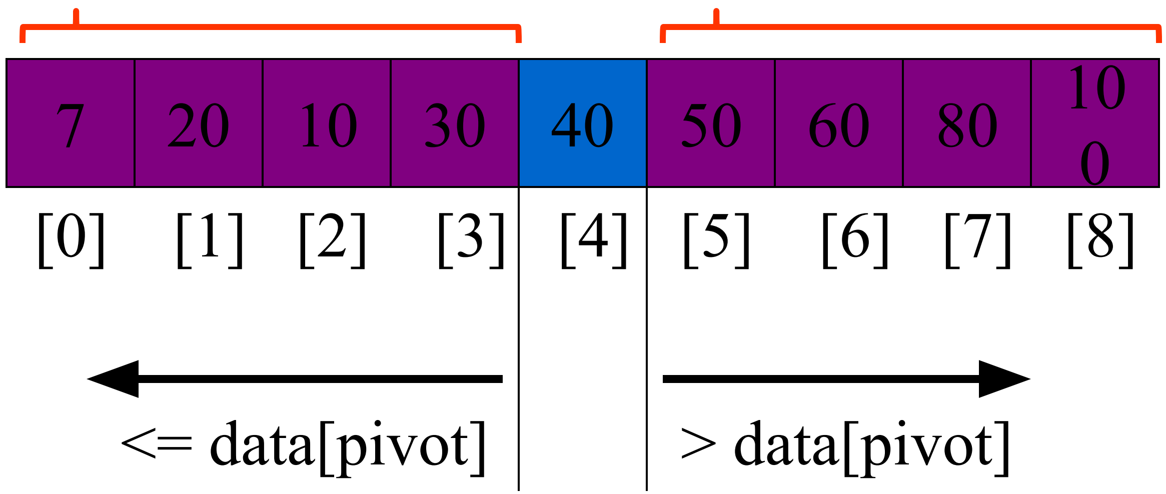
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Quicksort Algorithm

Quiksort(A[L…r])

//Sorts a subarray by quicksort

//input:A subarrayA[L..r] of A[0 ..n-1]defined by its left //and right indices L and r

//Output:The subarray A[L..r] sorted in non //decreasing order

if L<r

s ‹-- partition(A[L..r])   //s is spit position

Quicksort(A[L..s-1])

Quicksort(A[s+1..r])

return

Algorithm *Partition*$(A[l..r])$
//Partitions a subarray by using its first element as a pivot
//Input: A subarray $A[l..r]$ of $A[0..n-1]$, defined by its left and right
//   indices l and r $(l < r)$
//Output: A partition of $A[l..r]$, with the split position returned as
//   this function's value

```
p ← A[l]
i ← l;   j ← r + 1
repeat
    repeat i ← i + 1 until A[i] ≥ p
    repeat j ← j − 1 until A[j] < p
    swap(A[i], A[j])
until i ≥ j
swap(A[i], A[j])   //undo last swap when i ≥ j
swap(A[l], A[j])
return j
```

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- Assume that keys are random, uniformly distributed.
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General Divide-and-Conquer Recurrence

$T(n) = aT(n/b) + f(n)$ where $f(n) \in \Theta(n^d)$, $d \geq 0$

<u>Master Theorem</u>: If $a < b^d$, $T(n) \in \Theta(n^d)$

If $a = b^d$, $T(n) \in \Theta(n^d \log n)$

If $a > b^d$, $T(n) \in \Theta(n^{\log_b a})$

Note: The same results hold with O instead of Θ.

$T(n) = 2T(n/2) + \Theta(n)$, $T(1) = 0$

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Quicksort Analysis

- Assume that keys are random, uniformly distributed.
- Best case running time: $O(n \log_2 n)$
- Worst case running time?

Quicksort: Worst Case

- Assume first element is chosen as pivot.
- Assume we get array that is already in order:

pivot_index = 0

2	4	10	12	13	50	57	63	10 0
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]

too_big_index

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    ++too_big_index

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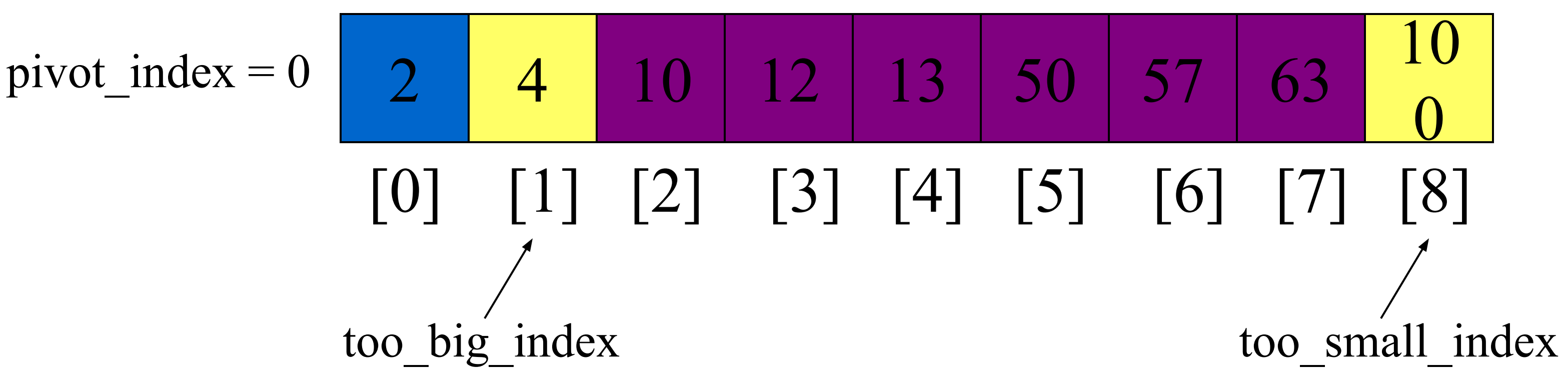

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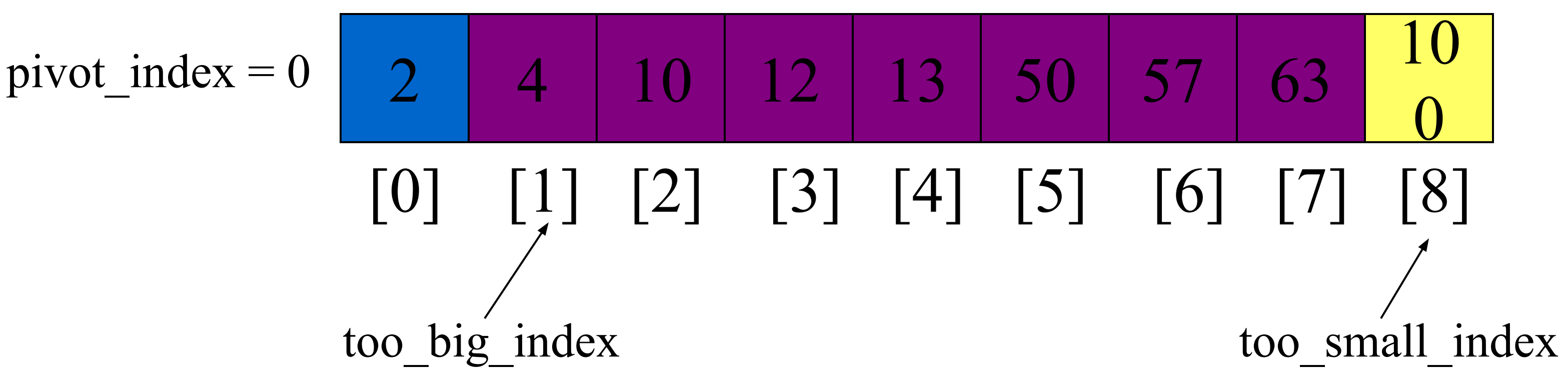

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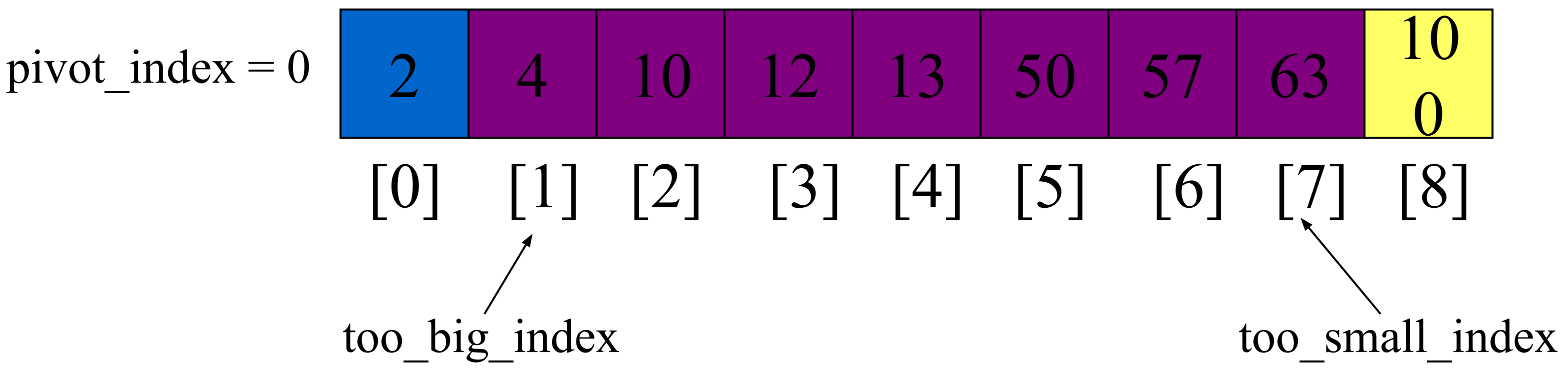

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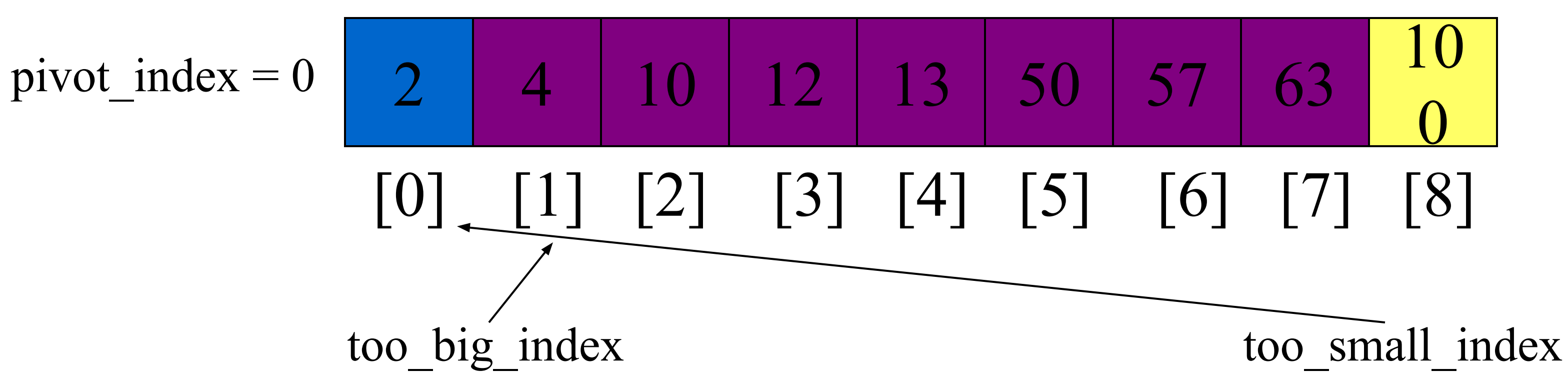

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2. While data[too_small_index] > data[pivot]
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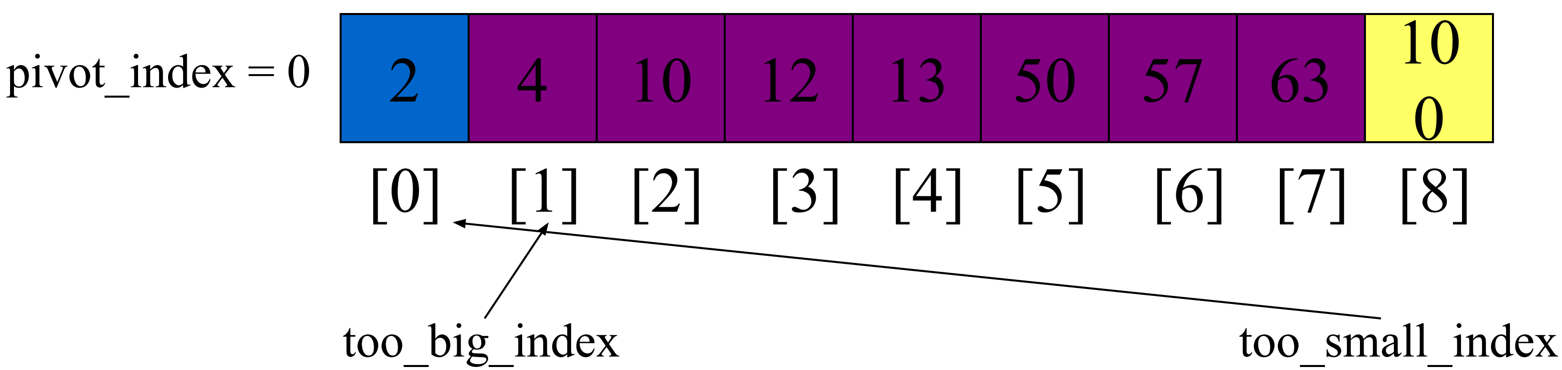
3. If too_big_index < too_small_index
   swap data[too_big_index] and data[too_small_index]
4. While too_small_index > too_big_index, go to 1.
5. Swap data[too_small_index] and data[pivot_index]



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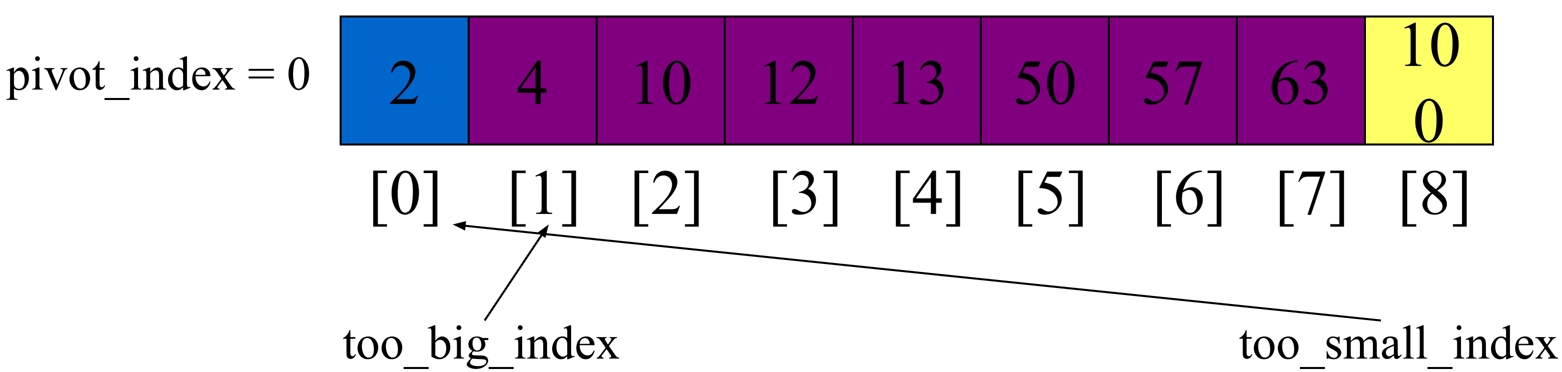

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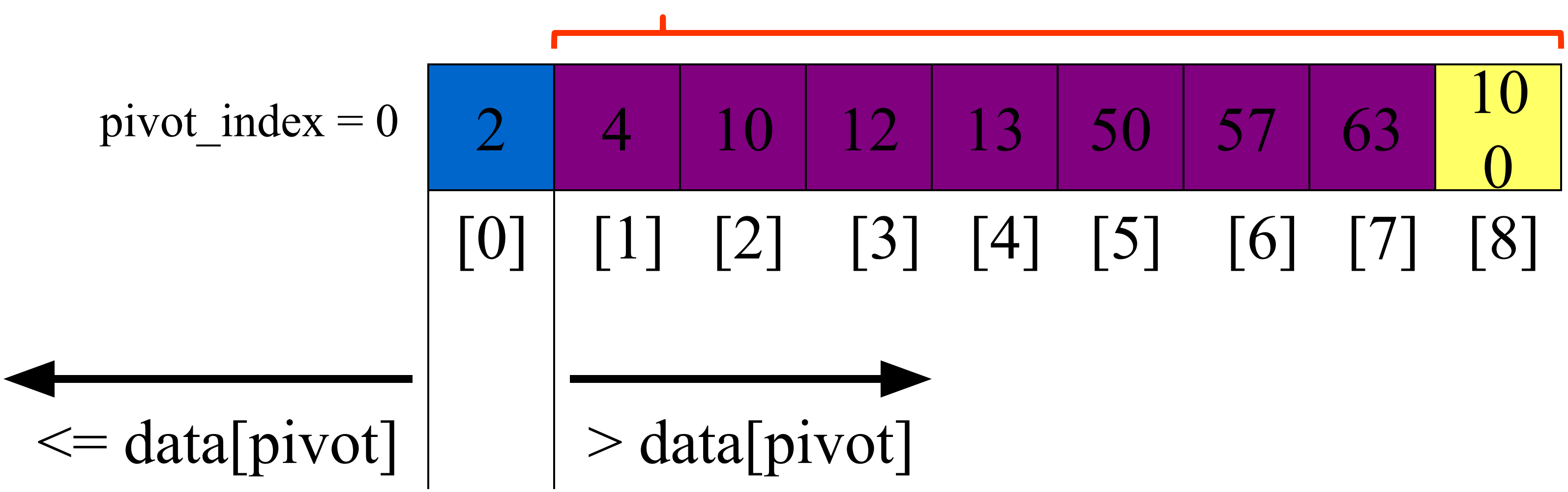

Quicksort Analysis

- Assume that keys are random, uniformly distributed.
- Best case running time: $O(n \log_2 n)$
- Worst case running time?
  - Recursion:
    1. Partition splits array in two sub-arrays:
       - one sub-array of size 0
       - the other sub-array of size n-1
    2. Quicksort each sub-array
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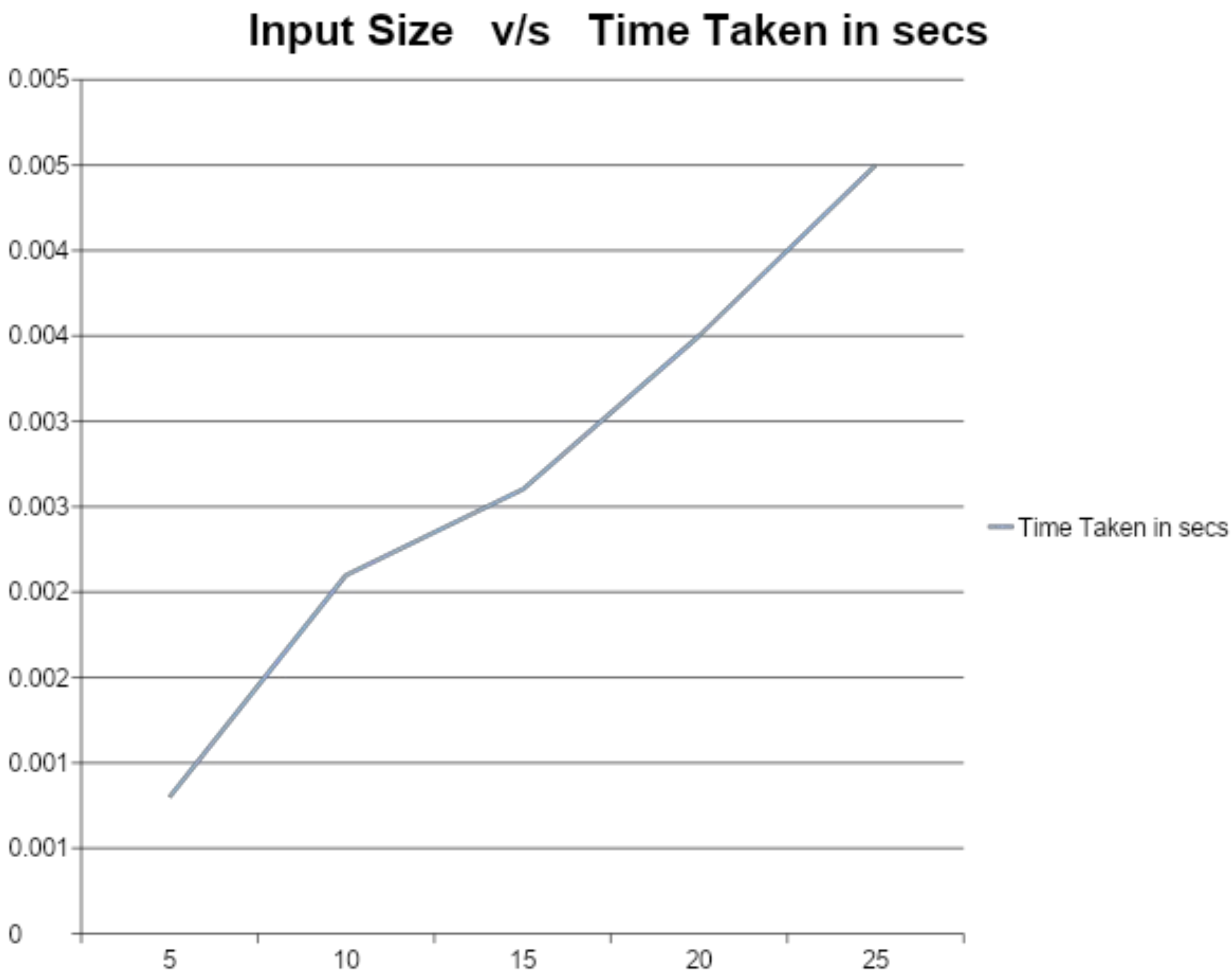
```
 const int MAX_ELEMENTS = 500;
int main(int argc, char *argv[])
{
    time_t  t1,t2;
    int list[MAX_ELEMENTS];
    int j=0,i = 0;
       // generate random numbers and fill them to the list
            for(i = 0; i < MAX_ELEMENTS; i++ ){
                                        list[i] = rand();
                              }
     printf("The list before sorting is:\n");
      printlist(list,MAX_ELEMENTS);
      t1=time(&t1);
              // sort the list using quicksort
         for(i=0;i<MAX_ELEMENTS;i++)
                for(j=0;j<MAX_ELEMENTS;j++)
                     quicksort(list,0,MAX_ELEMENTS - 1);
      t2=time(&t2);
        // print the result
     printf("The list after sorting using quicksort algorithm:\n");
     printlist(list,MAX_ELEMENTS);
     printf("time taken:%f\n",(float)(t2-t1)/CLK_TCK);
     return 0;
}
```

Size of input and Time Taken for Quick sort

Input Size	Time taken in Secs
500	0.0008
600	0.0021
700	0.0026
800	0.0035
900	0.0045

Graph for Quick Sort



```
void quicksort(int list[],int m,int n)
{
  int key,i,j,k;
  if( m < n)
  {          k = choose_pivot(m,n);
             swap(&list[m],&list[k]);
             key = list[m];
             i = m+1;        j = n;
             while(i <= j)
             {
                 while((i <= n) && (list[i] <= key))
                             i++;
                 while((j >= m) && (list[j] > key))
                        j--;
                 if( i < j)
                     swap(&list[i],&list[j]);    // swap two elements
             }
             swap(&list[m],&list[j]);     // swap pivot element in final position
             quicksort(list,m,j-1);
             quicksort(list,j+1,n);
  }
}
```

```c
void printlist(int list[],int n)
{
   int i;
   for(i=0;i<n;i++)
      printf("%d\t",list[i]);
}

void swap(int *x,int *y)
{
   int temp;
   temp = *x;
   *x = *y;
   *y = temp;
}

int choose_pivot(int i,int j )
{
   return((i+j) / 2);
}
```

```
int main(int argc, char *argv[])
{
        time_t t1,t2;
        const int MAX_ELEMENTS = 700;
        int list[MAX_ELEMENTS];
        int j=0,i = 0;
    // generate random numbers and fill them to the list
        for(i = 0; i < MAX_ELEMENTS; i++ ){
            list[i] = rand();
            }
        printf("The list before sorting is:\n");
        printlist(list,MAX_ELEMENTS);
        t1=time(&t1);
        // sort the list using quicksort
        for(i=0;i<MAX_ELEMENTS;i++)
        for(j=0;j<MAX_ELEMENTS;j++)
        quicksort(list,0,MAX_ELEMENTS-1);
        t2=time(&t2);
        // print the result
        printf("The list after sorting using quicksort algorithm:\n");
        printlist(list,MAX_ELEMENTS);
        printf("time taken:%f\n",(float)(t2-t1)/CLK_TCK);
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