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UNIT-3

CONTEXT-FREE GRAMMARS AND LANGUAGES

Chomsky Hierarchy

Type 0	Type 1	Type 2	Type 3
- Structured	- Context-sensitive	- Content free	- Regular

compound sentence → simple sentence conjuction simple sentence

conjuction → and / or / but

simple sentence → <subject> <verb> <object>

subject - noun

object - noun / pronoun

Context Free Grammar - CFG is grammar which is used to generate all the possible patterns of strings in a given formal lang.

~~CFG has~~ A CFG $G = (V, T, P, S)$ has following

V is set of variables or non terminals - uppercase

T is set of terminals - lowercase

P is set of productions or rules

S is start symbol

For a simple sentence V includes: simple sentence, subject, object

T includes: Noun, pronoun and verbs

P includes: simple sentence, subject (noun), object (noun / pronoun).

S includes: simple sentence

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Write a CFG for any number of a's.

→ $S \to aS$

$S \to \epsilon$

Write a CFG for atleast one a.

→ $S \to aS$

$S \to a$

Write a CFG for any number of a's and b's.

→ $S \to aS$ or $S \to aS / bS / \epsilon$

$S \to bS$

$S \to \epsilon$

Write a CFG ~~for~~ beginning with a

→ $S \to aA$

$A \to aA$

$A \to bA$

$A \to \epsilon$

Write CFG for substring ab

→ $S \to A\ ab\ A$

$A \to aA / bA / \epsilon$

$G = (V, T, P, S)$

$V = \{S, A\}$

$T = \{a, b\}$

$P = \{ S \to AabA \quad A \to aA / bA / \epsilon \}$

S = Start symbol

Write CFG for palindrome, assume that palindrome has 0's and 1's

→ $S \to$ 1) $0S0$ / $1S1$ 2)

3) $S \to 0$

4) $S \to 1$

5) $S \to \epsilon$

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For 10101 2^{nd} rule, 1^{st} rule, 4^{th} rule

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CFG for equal number of a's followed by equal number of b's.

→ $S \rightarrow aSb$ $V = \{S\}$ $P = \{S \rightarrow aSb \quad S \rightarrow \epsilon\}$

$S \rightarrow \epsilon$ $T = \{a, b\}$ S = Start symbol

CFG for $a^n b^n$ $n \geq 1$ $L = \{a^n b^n / n \geq 1\}$

→ $S \rightarrow aSb$

$S \rightarrow ab$

Write a CFG that has atleast 2a's [no b's]

→ $S \rightarrow aS$

$S \rightarrow aa$

Even num of a's. [no b's]

→ $S \rightarrow aaS$

$S \rightarrow \epsilon$

Odd num of a's

→ $S \rightarrow aaS$

$S \rightarrow a$

Number of a's is divisible by 3. / $L = \{w / n_a(w) \bmod 3 = 0, w \in a^*\}$

→ $S \rightarrow aaaS$

$S \rightarrow \epsilon$

$L = \{w / |w| \bmod 3 = 0, w \in \{a, b\}^*\}$

→ $S \rightarrow aaaS$ $S \rightarrow AAAS$

$S \rightarrow bbbS$ $S \rightarrow \epsilon$ ✓

$S \rightarrow \epsilon$ ✗ $A \rightarrow a|b$

this takes consecutive

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Write a CFG for $|w| \bmod 3 > 0$, $w \in a^+$ 1, 2, 4, 5, 7, 8

$S \to$ ~~aaa~~S / aa / aaaS

~~S→~~

Write CFG for number of a's divisible by 2. [consists b's] $w \in \{a, b\}^*$ }

$L = \{ w \mid n_a(w) \bmod 2 = 0$ $(b^* a b^* a b^+)^*$

~~S → aaS / bS / ε~~

$S \to SaSaS$

$S \to bS$

$S \to \epsilon$

Number of a's equal to number of b's

$S \to aSb / bSa / \epsilon$

works ~~only for~~ few conditions

$S \to SS$

$S \to aSb$

$S \to bSa$

$S \to \epsilon$

Write a CFG for a balanced paranthesis

$S \to (S)$

$S \to SS$

$S \to \epsilon$

Write a CFG $L = \{0^m 1^m 2^n$ such that $m \geq 1, n \geq 0\}$

~~S→~~ ~~a~~ ~~01~~ $S \to AB$

$A \to 0A1 / 01$

$B \to 2B \mid \epsilon$

$L = \{ a^n b^m \mid n \geq 0, m > n \}$

~~S→ abbS~~

~~S→ ab~~

~~S → AB~~

~~A → aS~~ / ε

~~B → bbS~~ / b

$S \to aSb \mid B$

$B \to bB \mid b$

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$L = \{a^n b^{n-3} \mid n \geq 3\}$

→ $S \rightarrow aSb$

$S \rightarrow aaa$

Write a CFG for the following:

$L = \{0^i 1^j / i \neq j, i \geq 0, j \geq 0\}$

→ $S \rightarrow 0S1 / A / B$

$A \rightarrow 0A / 0$

$B \rightarrow 1B / 1$

* $L = \{a^n b^m c^k / n+2m = k \text{ for } n \geq 0, m \geq 0\}$

→ $a^n b^m c^k$

$a^n b^m c^{n+2m}$

$a^n b^m c^n c^{2m}$

$a^n b^m c^{2m} c^n$

$S \rightarrow aSc / A$

$A \rightarrow bAcc / \epsilon$

$L = \{a^n b^m c^k / m = n+k \text{ for } n \geq 0 \; k \geq 0\}$

→ $a^n b^m c^k$

$a^n b^{n+k} c^k$

$a^n b^n b^k c^k$

$S \rightarrow AB$

$S \rightarrow aAb / \epsilon$

$B \rightarrow bBc / \epsilon$

$L = \{w / |w| \bmod 3 \neq |w| \bmod 2\}, w \in \{a,b\}^*\}$

→ 2, 3, 4, 5, 8, 9, 10, 11, 14...

Invalid = 1, 6, 7, 12, 13

$S \rightarrow aa / aaa / aaaa / aaaaa / aaaaaaS$

$L = \{w \mid |w| \bmod 3 \geq |w| \bmod 2,\ w \in \{a\}^*\}$

3, 9, 15 - Invalid

→ 0, 1, 2, 4, 5, 6, 7, 8, 10, 11, ...

$S \rightarrow \epsilon \mid a \mid aa \mid aaaa \mid aaaaa \mid aaaaaaS$

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Derivations Using a Grammar
↳ Leftmost Derivation (LMD)
↳ Rightmost Derivation (RMD)

Derivation tree is ordered rooted tree that graphically represents semantic info of strings derived from CFG

1. Derive a string 3a's followed by 3b's

→ aaabbb

$S \rightarrow aSb \mid \epsilon$

$S \Rightarrow aSb$ $\quad \therefore S \rightarrow aSb$

$\Rightarrow aaSbb$ $\quad \therefore S \rightarrow aSb$

$\Rightarrow aaaSbbb$ $\quad \therefore S \rightarrow aSb$

$\Rightarrow aaabbb$ $\quad \therefore S \rightarrow \epsilon$

2. ababba

→ $S \rightarrow SS$

$S \rightarrow aSb$

$S \rightarrow bSa$

$S \rightarrow \epsilon$

LMD	RMD
$S \underset{lm}{\Rightarrow} SS$	$S \underset{rm}{\Rightarrow} SS$
$\underset{lm}{\Rightarrow} SSS$	$\underset{rm}{\Rightarrow} SSS$
$\underset{lm}{\Rightarrow} aSbSS$	$\underset{rm}{\Rightarrow} SSbSa$
$\underset{lm}{\Rightarrow} abSS$	$\underset{rm}{\Rightarrow} SSba$
$\underset{lm}{\Rightarrow} abaSbS$	$\underset{rm}{\Rightarrow} SaSbba$
$\underset{lm}{\Rightarrow} ababS$	$\underset{rm}{\Rightarrow} Sabba$
$\underset{lm}{\Rightarrow} ababbSa$	$\underset{rm}{\Rightarrow} aSbabba$
$\underset{lm}{\Rightarrow} ababba$	$\underset{rm}{\Rightarrow} ababba$

LMD RMD

LMD — At each step we replace the left most variable by one of its production bodies such a derivation is called left most derivation.

RMD — At each step we replace the right most variable by one of its production bodies such a derivation is called right most derivation.

Give LM and RM derivation of the following strings:

a) 00101 b) 1001 c) 00011

$S \rightarrow A1B$

$A \rightarrow 0A \mid \epsilon$

$B \rightarrow 0B \mid 1B \mid \epsilon$

a) 00101

$S \Rightarrow A1B \quad \because S \rightarrow A1B$

$\underset{lm}{\Rightarrow} 0A1B \quad \because A \rightarrow 0A$

$\underset{lm}{\Rightarrow} 00A1B \quad \because A \rightarrow 0A$

$\underset{lm}{\Rightarrow} 001B \quad \because A \rightarrow \epsilon$

$\underset{lm}{\Rightarrow} 0010B \quad \because B \rightarrow 0B$

$\underset{lm}{\Rightarrow} 00101B \quad \because B \rightarrow 1B$

$\underset{lm}{\Rightarrow} 00101 \quad \because B \rightarrow \epsilon$

$S \Rightarrow A1B \quad \because S \rightarrow A1B$

$\underset{rm}{\Rightarrow} A10B \quad \because B \rightarrow 0B$

$\underset{rm}{\Rightarrow} A101B \quad \because B \rightarrow 1B$

$\underset{rm}{\Rightarrow} A101 \quad \because B \rightarrow \epsilon$

$\underset{rm}{\Rightarrow} 0A101 \quad \because A \rightarrow 0A$

$\underset{rm}{\Rightarrow} 00A101 \quad \because A \rightarrow 0A$

$\underset{rm}{\Rightarrow} 00101 \quad \because A \rightarrow \epsilon$

b) 1001

~~$S \Rightarrow 1B$ $A \rightarrow \epsilon$ $\because S \rightarrow A1B$~~

~~$\underset{lm}{\Rightarrow} 10B \quad \because B \rightarrow 0B$~~

~~$\underset{lm}{\Rightarrow} 100B \quad \because B \rightarrow 0B$~~

~~$\underset{lm}{\Rightarrow} 1001B \quad \because B \rightarrow 1B$~~

~~$\underset{lm}{\Rightarrow} 1001 \quad \because B \rightarrow \epsilon$~~

$S \underset{lm}{\Rightarrow} A1B \quad \because S \rightarrow A1B$

$\underset{lm}{\Rightarrow} 1B \quad \because A \rightarrow \epsilon$

$\underset{lm}{\Rightarrow} 10B \quad \because B \rightarrow 0B$

$\underset{lm}{\Rightarrow} 100B \quad \because B \rightarrow 0B$

$\underset{lm}{\Rightarrow} 1001B \quad \because B \rightarrow 1B$

$\underset{lm}{\Rightarrow} 1001 \quad \because B \rightarrow \epsilon$

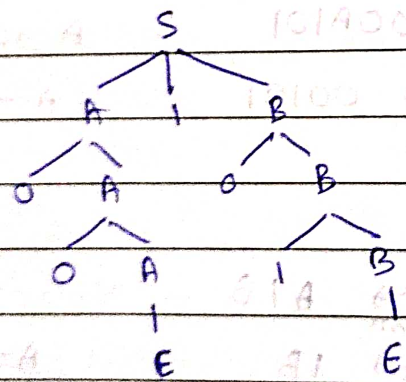
$S \underset{rm}{\Rightarrow} A1B$	$\because S \to A1B$
$\underset{rm}{\Rightarrow} A10B$	$B \to 0B$
$\underset{rm}{\Rightarrow} A100B$	$B \to 0B$
$\underset{rm}{\Rightarrow} A1001B$	$B \to 1B$
$\underset{rm}{\Rightarrow} A1001$	$B \to \epsilon$
$\underset{rm}{\Rightarrow} 1001$	$A \to \epsilon$

c) 00011

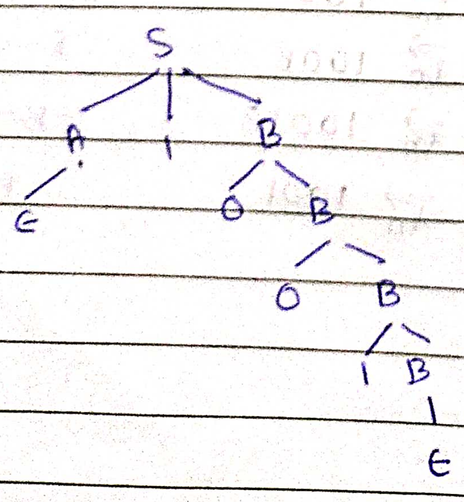
$S \underset{lm}{\Rightarrow} A1B$	$\because S \to A1B$	$S \underset{rm}{\Rightarrow} A1B$	$S \to A1B$
$\underset{lm}{\Rightarrow} 0A1B$	$\because A \to 0A$	$\underset{rm}{\Rightarrow} A11B$	$B \to 1B$
$\underset{lm}{\Rightarrow} 00A1B$	$A \to 0A$	$\underset{rm}{\Rightarrow} A11$	$B \to \epsilon$
$\underset{lm}{\Rightarrow} 000A1B$	$A \to 0A$	$\underset{rm}{\Rightarrow} 0A11$	$A \to 0A$
$\underset{lm}{\Rightarrow} 0001B$	$A \to \epsilon$	$\underset{rm}{\Rightarrow} 00A11$	$A \to 0A$
$\underset{lm}{\Rightarrow} 00011B$	$B \to 1B$	$\underset{rm}{\Rightarrow} 000A11$	$A \to 0A$
$\underset{lm}{\Rightarrow} 00011$	$B \to \epsilon$	$\underset{rm}{\Rightarrow} 00011$	$A \to \epsilon$

* Parse Tree (Derivation Tree)

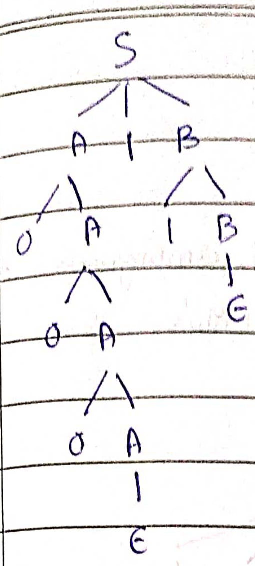
a)



b)



c)



Yield of a parse tree - concatenation of leaves from left to right, we get a string ie yield of a parse tree

21/1/21 Ambiguity in Grammars and Languages

$E \rightarrow E + E$

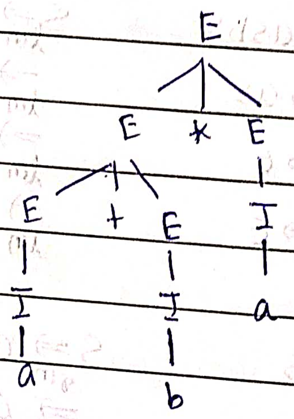
$E \rightarrow E * E$

$E \rightarrow I$

$I \rightarrow a$

$I \rightarrow b$

$a + b * a$



Ambigiuous grammar :- Let $G = (V, T, P, S)$ is a CFG, we say G is ambiguous if there is atleast one string w in T^* for which we can find two different parse trees each with root labeled S and yield w.

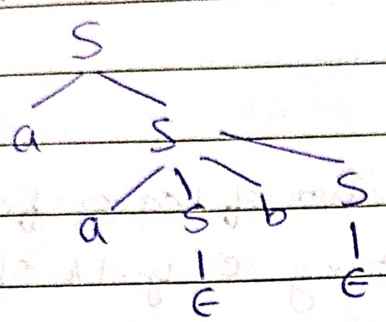
Pbm-X. Consider the following grammar

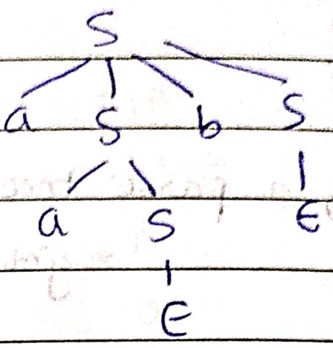
$S \rightarrow aS$

$S \rightarrow aSbS$

$S \rightarrow \epsilon$ check whether this grammar is ambiguous. If so construct two parse trees, two LMD and two RMD's.

→ aab





Yes the grammar is ambiguous

$S \underset{lm}{\Rightarrow} aSbS$	∵ $S \rightarrow aSbS$	$S \underset{lm}{\Rightarrow} aS$	$S \rightarrow aS$
$\underset{lm}{\Rightarrow} aaSbS$	$S \rightarrow aS$	$\underset{lm}{\Rightarrow} aaSbS$	$S \rightarrow aSbS$
$\underset{lm}{\Rightarrow} aabS$	$S \rightarrow \epsilon$	$\underset{lm}{\Rightarrow} aabS$	$S \rightarrow \epsilon$
$\underset{lm}{\Rightarrow} aab$	$S \rightarrow \epsilon$	$\underset{lm}{\Rightarrow} aab$	$S \rightarrow \epsilon$

$S \underset{rm}{\Rightarrow} aSbS$	$S \rightarrow aSbS$	$S \underset{rm}{\Rightarrow} aS$	$S \rightarrow aS$
$\underset{rm}{\Rightarrow}$ ~~a~~aSb~~S~~	$S \rightarrow \epsilon$	$\underset{rm}{\Rightarrow} aaSbS$	$S \rightarrow aSbS$
$\underset{rm}{\Rightarrow}$ aaSb~~S~~	$S \rightarrow aS$	$\underset{rm}{\Rightarrow} aaSb$	$S \rightarrow \epsilon$
$\underset{rm}{\Rightarrow} aab$	$S \rightarrow \epsilon$	$\underset{rm}{\Rightarrow} aab$	$S \rightarrow \epsilon$

Removing ambiguity from Grammars

(context free grammar, [illegible])

Inherent Ambiguity — A context free language L is said to be inherently ambiguous if all its grammars are ambiguous.

Sentential Form — If $G = (V, T, P, S)$ is a CFG then any string α in $(V \cup T)^*$ such that $S \xrightarrow{*} \alpha$ is a sentential form.

Applications of CFG:

3/1/22. Properties of context free language:

- Normal forms for CFG (10m) [Conversion from CFG to CNF]
  (Context free grammar) (Chomsky Normal Form)

The goal is to convert all productions of the form
$A \to BC$
$A \to a$ where A, B, C are variables and a is terminal. This form is called Chomsky Normal Form. (CNF)

To convert the Grammar to CNF we must make a number of preliminary simplifications:
1) We must eliminate useless symbols, those variables or terminals that do not appear in any derivation of a terminal string from the start symbol.
2) We must eliminate ϵ-productions those of the form $A \to \epsilon$ for some variable A.
3) We must eliminate unit productions, those of the form $A \to B$ for some variables A & B.

Eliminate ϵ-productions
Unit productions
Useless symbols
} order to eliminate

Eliminating useless symbols :

We say a symbol X is useful for a grammar G if there is some derivation of the form

$$S \overset{*}{\Rightarrow} \alpha X \beta \overset{*}{\Rightarrow} w$$, where w is in T^*

Note : X may be variable or terminal.

The approach to eliminate useless symbols begins by identifying two things :

i) Generating - We say X is generating if $X \overset{*}{\Rightarrow} w$

ii) We say X is reachable if there is a derivation

$$S \overset{*}{\Rightarrow} \alpha X \beta \text{ for some } \alpha \,\&\, \beta$$

Eliminate useless symbols from the following grammar :

1) $S \to AB \mid a$

$A \to b$

→ Symbols → S, A, B, a, b

- Generating → S, A, a, b (atleast one terminal state has to be generated)

$S \to$ ~~AB~~ a

$A \to b$

- Reachable

S, a are reachable

$S \to a$

2) $S \to AB \mid CA$

$A \to a$

$B \to BC / AB$

$C \to aB / b$

→ Symbols → S, A, B, C, a, b

- Generating → S, A ~~B~~, C, a, b

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$S \rightarrow CA$

$A \rightarrow a$

$C \rightarrow b$

- Reachable → S, C, A, a, b

$S \rightarrow CA$

$A \rightarrow a$

$C \rightarrow b$

3) $S \rightarrow ASB / \epsilon$

$A \rightarrow aAS / a$

$B \rightarrow SbS / A / bb$

→ Symbols → S, A, B, a, b

- Generating → S, A, B, a, b
- Reachable → S, A, B, a, b

Grammar remains same

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Eliminating ϵ-productions:

Nullable symbol: These symbols derive epsilon.

A variable 'A' is nullable if $A \overset{*}{\Rightarrow} \epsilon$. If 'A' is nullable whenever 'A' appears in a production body say $S \rightarrow BAC$, 'A' might (might not) derive epsilon.

BASIS: If $A \rightarrow \epsilon$ is a production of G then 'A' is nullable

INDUCTION: If there is a production $B \rightarrow C_1C_2 \ldots C_k$ where each C_i is nullable then B is also nullable.

Eliminate ϵ productions from the following grammar:

$S \to AB$
$A \to aAA | \epsilon$
$B \to bBB | \epsilon$

→ S, A, B are all nullable

$S \to AB | A | B |$ (null)
$A \to aAA | a | aA | aA$
$\to aAA | a | aA$
$B \to bBB | b | bB | bB$
$\to bBB | b | bB$

$S \to ASB | \epsilon$
$A \to aAS | a$
$B \to SbS | A | bb$

→ S is nullable $\quad S \to ASB | AB$
$A \to aAS | aA | a$
$B \to SbS | b | bS | Sb | A | bb$

Eliminate unit productions.

Unit pairs - A pair (A,B) is said to be unit pair such that $A \overset{*}{\Rightarrow} B$ using only unit productions.
Unit production is production of the form $A \to B$ where both A & B are variables.

BASIS: (A,A) is unit pair for any variable A
ie $A \overset{*}{\Rightarrow} A$ in '0' steps.
INDUCTION: Suppose we have determined that (A, B) is a unit pair and $B \to C$ is a production where C is a variable then (A, C) is a unit pair

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Eliminate unit productions from the following grammar

$S \rightarrow ASB \mid \epsilon$

$A \rightarrow aAS \mid a$

$B \rightarrow SbS \mid A \mid bb$

→ Variables – S, A, B

Unit pairs – (S,S) (A,A) (B,B) from BASIS step

(B,B) & B→A (B,A) is unit pair

Unit pair	non unit production body
(S,S)	$S \rightarrow ASB \mid \epsilon$
(A,A)	$A \rightarrow aAS \mid a$
(B,B)	$B \rightarrow SbS \mid bb$
(B,A) Body of A without unit production	$B \rightarrow aAS \mid a$

$S \rightarrow ASB \mid \epsilon$

$A \rightarrow aAS \mid a$

$B \rightarrow Sbs \mid bb \mid aAS \mid a$

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$F \rightarrow I \mid (E)$

$T \rightarrow F \mid T*F$

$E \rightarrow T \mid E+T$

→ Variables – I, F, T, E

E is start symbol

Unit pairs – (E,E) (T,T) (F,F) (I,I)

(E,E) E→T (E,T)

(E,T) T→F (E,F)

(E,F) F→I (E,I)

(T,T) T→F (T,F)

(T,F) F→I (T,I)

~~(F,F)~~ F→I (F,I)

(

unit pair	non unit pair productions
(E,E)	E → E+T
(E,T)	E → T*F
(E,F)	E → (E)
(E,I)	E → a|b| Ia | Ib | I0 | I1
(T,T)	T → T*F
(T,F)	T → (E)
(T,I)	T → a|b| Ia | Ib | I0 | I1
(F,F)	F → (E)
(F,I)	F → a|b| Ia | Ib | I0 | I1
(I,I)	I → a|b| Ia | Ib | I0 | I1

E → E+T | T*F | (E) | a|b|Ia|Ib|I0|I1

T → T*F | (E) | a|b|Ia|Ib|I0|I1

F → (E) | a|b| Ia|Ib| I0 | I1

I → a|b| Ia|Ib|I0|I1

7/1/22 Converting CFG to CNF

Chomsky ~~Normal~~ form

A → BC

A → a

Eliminate ε productions

Eliminate unit productions

Eliminate useless symbols

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Task1: Arrange that all bodies of length 2 or more consists of only variables.

Task 2: Break bodies of length 3 or more into a cascade of productions each with a body consisting of two variables.

1. (a) Begin with grammar

$S \to ASB \mid \epsilon$

$A \to aAS \mid a$

$B \to SbS \mid A \mid bb$ put the grammar into CNF.

→ Eliminate ϵ productions

$S \to ASB \mid AB$

$A \to aAS \mid aA \mid a$

$B \to SbS \mid Sb \mid bS \mid b \mid A \mid bb$

Eliminate unit productions.

(S, S) $S \to ASB \mid AB$

(A, A) $A \to aAS \mid aA \mid a$

(B, B) $B \to SbS \mid Sb \mid bS \mid b \mid \cancel{aAS}\ bb$

(B, A) $B \to aAS \mid aA \mid a$

$S \to ASB \mid AB$

$A \to aAS \mid aA \mid a$

$B \to SbS \mid Sb \mid bS \mid b \mid aAS \mid aA \mid a \mid bb$

Eliminate useless symbols

Symbols → S, A, B, a, b

All are generating and all are reachable from the start state so resulting grammar is same.

$S \to ASB \mid AB$

$A \to PAS \mid PA \mid a$ $\quad P \to a$

$B \to SQS \mid SQ \mid QS \mid b \mid PAS \mid PA \mid a \mid QQ$ $\quad Q \to b$

$S \to AC_1 \mid AB$ $\quad C_1 \to SB$

$A \to PC_2 \mid PA \mid a$ $\quad C_2 \to AS$

$B \to SC_3 \mid SQ \mid QS \mid b \mid PC_2 \mid PA \mid a \mid QQ$ $\quad C_3 \to QS$

$P \to a$

$Q \to b$

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Convert $S \to 0A0 / 1B1 / BB$

$A \to C$

$B \to S \mid A$

$C \to S \mid \epsilon$ to CNF

→ Eliminate ϵ productions – All are nullable

~~$S \to 0A0 / 1B1 / BB$~~

~~$A \to C$~~

~~$B \to S/A$~~

$S \to 0A0 / 00 / 1B1 / 11 / BB \mid B$

$A \to C$

$B \to S / A$

$C \to S$

Eliminate unit productions

(S,S)	(S,S) & S→B (S,B)
(A,A)	(B,B) & B→A (S,A)
(B,B)	(S,A) & A→C (S,C)
(C,C)	(A,A) & A→C (A,C)
(S,B)	(A,C) & C→S (A,S)
(A,C)	(A,S) & S→B (A,B)
(B,A)	(B,B) & B→S (B,S)
(S,S)	(B,B) & B→A (B,A)
(S,B)	(B,A) & A→C (B,C)
(S,A)	(C,C) & C→S (C,S)
(A,A)	(C,S) & S→B (C,B)
(A,C)	(C,B) & B→S (C,S)
(A,S)	(C,S) & S→B (C,B)
(B,B)	(C,B) & C→A (C,A)
(B,B)	
(B,A)	
(B,C)	
(C,S)	
(C,B)	
(C,S)	
(C,B)	

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Unit productions

(S,S)	S→ 0A0/ 00 / 1B1 / 11 / BB
(S,A)	S→ ~~0A0/00/1B1/11/BB~~
(S,B)	S→ ~~0A0/00/1B1/11/BB~~
(S,C)	S→ ~~0A0/00/1B1/11/BB~~
(A,A)	A→
(A,S)	A→ 0A0/ 00/ 1B1/ 11/ BB
(A,C)	A→ ~~0A0/00/1B1/11/BB~~
(A,B)	A→ ~~0A0/00/1B1/11/BB~~
(B,B)	B→
(B,S)	B→ 0A0 / 00/ 1B1 / 11 / BB
(B,A)	B→
(B,C)	B→
(C,C)	C→
(C,S)	C→ 0A0/ 00 / 1B1 / 11 / BB
(C,A)	C→
(C,B)	C→

S→ 0A0/00/1B1/11/BB

A→ 0A0/00/1B1/11/BB

B→ 0A0/00/1B1/11/BB

C→ 0A0/00/1B1/11/BB

Eliminate unit productions

C is not reachable

So

S→ 0A0 / 00/ 1B1 / 11 / BB

A → 0A0 / 00 / 1B1 / 11 / BB

B→ 0A0 / 00 / 1B1 / 11 / BB

S→ PAP | PP | QBQ | QQ | BB

A→ PAP / PP / QBQ | QQ | BB

B→ PAP | PP | QBQ | QQ | BB

P→0

Q→1

S→ PC_1 | PP | QC_2 | QQ | BB

A → PC_1 / PP | QC_2 | QQ | BB

B → PC_1 | PP | QC_2 | QQ | BB

C_1→AP

C_2→ BQ

P→0 Q→1

S → aAa / bBb / ε

A → C / a

B → C / b

C → CDE / ε

D → A / B / ab

→ Eliminate ε productions: [S, A, B, C are nullable]

S → aAa / aa / bBb / bb / aA / Aa / bB, Bb

A → C / a

B → C / b

C → CDE / ~~CE~~ / DE / ~~E~~

D → A / B / ab

Eliminate unit productions

(S,S)	S → aAa / aa / bBb / bb / aA / Aa / bB / Bb
(A,A)	A → a
(A,C)	A → CDE / DE
(B,B)	B → b
(B,C)	B → CDE / DE
(C,C)	C → CDE / DE
(D,D)	D → ab
(D,A)	D → a
(D,B)	D → b
(D,C)	D → CDE / DE

S → aAa / aa / bBb / bb / aA / Aa / bB / Bb

A → a / CDE / DE

B → b / CDE / DE

C → CDE / DE

D → ab / a / b / CDE / DE

Eliminate useless symbols.

S → aAa / aa / bBb / bb / aA / Aa / bB / Bb

A → a / CDE / DE

D → ab / a / b / CDE / DE

B → b / CDE / DE

$S \rightarrow aAa \mid aa \mid bBb \mid bb \mid aA \mid Aa \mid bB \mid Bb$

$A \rightarrow a \mid$

$B \rightarrow b$

$D \rightarrow ab \mid a \mid b$ D is not reachable

$S \rightarrow aAa \mid aa \mid bBb \mid bb \mid aA \mid Aa \mid bB \mid Bb$

$A \rightarrow a$

$B \rightarrow b$

$a \rightarrow P$

$b \rightarrow Q$

$S \rightarrow PAP \mid PP \mid QBQ \mid QQ \mid PA \mid AP \mid QB \mid BQ$

$A \rightarrow a$

$B \rightarrow b$