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OBA - 2

3) Given: $a_{n+2} - 5a_{n+1} + 6a_n = 0$, $\forall n \geq 0 \rightarrow (1)$

$a_0 = 1$, $a_1 = 2$

Solving by characteristic equatⁿ method

degree = 1

order = $n+2-n = 2$

Put $a_{n+2} = \alpha^2$, $a_{n+1} = \alpha^1$, $a_n = \alpha^0 = 1$

From (1)

AE : $\alpha^2 - 5\alpha + 6(1) = 0$

$\alpha^2 - 5\alpha + 6 = 0$

$\alpha^2 - 3\alpha - 2\alpha + 6 = 0$

$\alpha(\alpha-3) - 2(\alpha-3) = 0$

$(\alpha-3)(\alpha-2) = 0$

$\therefore \alpha_1 = 3$, $\alpha_2 = 2$ (Roots are real & distinct)

General solⁿ is,

$a_n = A_1 \alpha_1^n + A_2 \alpha_2^n$

$a_n = A_1 3^n + A_2 2^n \rightarrow (2)$

Given, $a_0 = 1$

Put $n=0$ in (2)

$1 = A_1 + A_2$

$a_1 = 2$

Put 1 n in (2)

$2 = A_1(3) + A_2(2)$

$2 = 3A_1 + 2A_2 \rightarrow (4)$

Solving (3) & (4), we get

3) $2 = 2A_1 + 2A_2$

$(-)\ 2 = (-)3A_1 \pm 2A_2$

$0 = -A_1$

$\therefore A_1 = 0$

Put $A_1 = 0$ in (3)

$1 = A_1 + A_2$

$1 = 0 + A_2$

$\therefore A_2 = 1$

$\therefore$ Sol^n is

$a_n = 0 + 2^n$

$\boxed{a_n = 2^n}$

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1) $A = \{0, 2, 5, 10, 11\}$

$R = \{(0,0), (0,2), (0,5), (0,10), (0,11), (2,2), (2,5), (2,10), (2,11), (5,5), (5,10), (10,10), (10,11), (11,11), (5,11)\}$

i) Reflexive : $aRa\ \forall a \in A$

$(0,0), (2,2), (5,5), (10,10), (11,11) \in R$

$\therefore R$ is reflexive

ii) Anti Symmetric : If $a \neq b$ then $(a,b) \in R$, $(b,a) \notin R$

$2 \neq 5 \rightarrow (5,2) \notin R$	$10 \neq 11 \rightarrow (11,10) \notin R$
$2 \neq 10 \rightarrow (10,2) \notin R$	$0 \neq 2 \rightarrow (2,0) \notin R$
$2 \neq 11 \rightarrow (11,2) \notin R$	$0 \neq 5 \rightarrow (5,0) \notin R$
$5 \neq 10 \rightarrow (10,5) \notin R$	$0 \neq 10 \rightarrow (10,0) \notin R$
$5 \neq 11 \rightarrow (11,5) \notin R$	$0 \neq 11 \rightarrow (11,0) \notin R$

$\therefore R$ is anti-symmetric

1) iii) Transitive : $(a,b) \in R$ & $(b,c) \in R$ then $(a,c) \in R$

$(0,0)$, $(0,2) \in R \rightarrow (0,2) \in R$

$(2,5)$, $(5,10) \in R \rightarrow (2,10) \in R$

$(5,10)$, $(10,11) \in R \rightarrow (5,11) \in R$

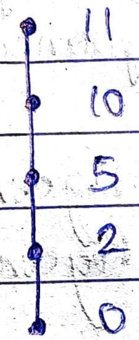
$(2,5)$, $(5,11) \in R \rightarrow (2,11) \in R$

$(2,11)$, $(11,11) \in R \rightarrow (2,11) \in R$

$\therefore$ R is transitive

R is Partial Order Relation

* Hasse Diagram,



2) i) Given $A = \{1, b, c, d, e\}$ & $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

then, $n(A) = 5$ & $n(B) = 8$

WKT, number of onto functions

$$P(m, n) = n!\, S(m, n)$$

$$= \sum_{K=0}^{n} (-1)^K \, {}^nC_K (n-K)^m$$

$$P(8, 5) = \sum_{K=0}^{5} (-1)^K \, {}^5C_K (5-K)^8$$

$$= 126000$$

ii) Let books denote pigeonholes & pages denote pigeons

∴ By generalized pigeonhole principle,

WKT, $K = \left[\frac{m-1}{n}\right] + 1$, m - Pigeons, n - Pigeonholes, $m > 2$

$$K = \left[\frac{27551 - 1}{50}\right] + 1$$

$$= 552$$

∴ At least one book contains 552 pages at least