# R.K. Method.

## [Runge-Kutta Method of Fourth Order]



* Using R.K method solve $\frac{dy}{dx} = 2x - y$ at $x = 1.1$ given that $y = 3$ at $x = 1$.

Soln: We have $\frac{dy}{dx} = 2x - y$, $x_0 = 1$, $y_0 = 3$.

$\therefore f(x, y) = 2x - y$, $h = 0.1$

Compute $k_1, k_2, k_3, k_4$ we get

$k_1 = hf(x_0\, y_0) = 0.1[2x_0 - y_0] = 0.1[2(1) - 3] = -0.1$

$k_2 = hf(x_0 + h/2, y_0 + k_1/2) = 0.1[2(x_0 + h/2) - (y_0 + k_1/2)]$

$= 0.1[2(1 + 0.1/2) - (3 - 0.1/2)] = -0.085$

$k_3 = hf(x_0 + h/2, y_0 + k_2/2) = 0.1[2(x_0 + h/2) - (y_0 + k_2/2)]$

$= 0.1[2(1 + 0.1/2) - (3 + \frac{0.085}{2})]$

$= -0.08575$

$k_4 = hf(x_0 + h, y_0 + k_3) = 0.1[2(x_0 + h) - (y_0 + k_3)]$

$= 0.1[2(1 + 0.1) - (3 \mp 0.08575)]$

$= -0.071425$

$y(1.1) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 2.91451$

* Solve using R.K method. $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ 25

for $x = 0.2, 0.4$ given that $y = 1$ at $x = 0$.

Soln: We have $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ ; $x_0 = 0$, $y_0 = 1$ $h = 0.2$

$\boxed{\text{Step - I}}$

$f(x,y) = \frac{y^2-x^2}{y^2+x^2}$ ; compute $K_1, K_2, K_3, K_4$

$$K_1 = hf(x_0, y_0) = 0.2\left[\frac{y_0^2 - x_0^2}{y_0^2 + x_0^2}\right] = 0.2\left[\frac{1-0}{1+0}\right] = 0.2$$

$$K_2 = hf(x_0 + h/2, y_0 + K_1/2) = 0.2\left[\frac{(y_0 + K_1/2)^2 - (x_0 + h/2)^2}{(y_0 + K_1/2)^2 + (x_0 + h/2)^2}\right]$$

$$= 0.2\left[\frac{(1+0.2/2)^2 - (0+0.2/2)^2}{(1+0.2/2)^2 + (0+0.2/2)^2}\right] = 0.1967$$

$$K_3 = hf(x_0 + h/2, y_0 + K_2/2) = 0.1967$$

$$K_4 = hf(x_0 + h/2, y_0 + K_3/2) = 0.1891$$

$$y(0.2) = y_0 + 1/6(K_1 + 2K_2 + 2K_3 + K_4) = 1.196$$

$\boxed{\text{Step II}}$ $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$, $x_0 = 0.2$, $y_0 = 1.196$, $h = 0.2$

$$K_1 = hf(x_0, y_0) = 0.2\left[\frac{y_0^2 - x_0^2}{y_0^2 + x_0^2}\right] = 0.1891$$

$$K_2 = hf(x_0 + h/2, y_0 + K_1/2) = 0.1795$$

$K_3 = 0.1793$, $K_4 = 0.1688$

$$y(0.4) = 1.3753.$$