UNIT 5: Joint Probability Distributions and Markov Chain

If X and Y are two discrete random variables, the joint probability function of X and Y is defined as,

$$P(X = x, Y = y) = f(x, y)$$

Where $f(x, y)$ satisfy the conditions

$i)\ f(x, y) \geq 0\ and \quad ii)\ \sum_x \sum_y f(x, y) = 1$

Note:

Suppose $X = \{ x_1, x_2, x_3 \ldots\ldots x_m\}\ and\ Y = \{y_1, y_2, y_3 \ldots.\ y_n\}$ then, $P\left(X = x_i, Y = y_j\right) = f(x_i, y_j)$ denoted by J_{ij}.

The set of values of the function $f\left(x_i, y_j\right)\ for\ i = 1,2,3 \ldots.\ m$; $j = 1,2,3 \ldots.\ n$ is called the joint probability distribution of X and Y.

TWO WAY TABLE: JOINT PROBABILITY TABLE:

$X \backslash Y$	y_1	y_2	y_3	$\cdots$	y_n	Sum
x_1	J_{11}	J_{12}	J_{13}	$\cdots$	J_{1n}	$f(x_1)$
x_2	J_{21}	J_{22}	J_{23}	$\cdots$	J_{2n}	$f(x_2)$
x_3	J_{21}	J_{32}	J_{33}	$\cdots$	J_{2n}	$f(x_3)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
x_m	J_{m1}	J_{m2}	J_{m3}	$\cdots$	J_{mn}	$f(x_m)$
Sum	$g(y_1)$	$g(y_2)$	$g(y_3)$	$\cdots$	$g(y_n)$	$\underline{1}$

<u>Note</u>:

$$f(x_1) = J_{11} + J_{12} + J_{13} + \cdots + J_{1n};$$
$$f(x_2) = J_{21} + J_{22} + J_{23} + \cdots + J_{2n};$$
$$\vdots$$
$$f(x_m) = J_{m1} + J_{m2} + J_{m3} + \cdots + J_{mn};$$
$$g(y_1) = J_{11} + J_{21} + J_{31} + \cdots + J_{m1}$$
$$g(y_2) = J_{12} + J_{22} + J_{32} + \cdots + J_{m2}$$
$$\vdots$$
$$g(y_n) = J_{1n} + J_{2n} + J_{3n} + \cdots + J_{mn}.$$

Also,

$$f(x_1) + f(x_2) + f(x_3) + \dots + f(x_m) = 1$$

$$g(y_1) + g(y_2) + g(y_3) + \quad g(x_n) = 1$$

* **Marginal Probability Distribution:**

$\{f(x_1), f(x_2), f(x_3) \dots f(x_m)\}$ &

$\{g(y_1), g(y_2), g(y_3) \dots g(x_n)\}$

are called the Marginal Probability Distribution of X and Y respectively.

Independent random variables

The random variables X and Y are said to be random variables if,

$$P(X = x_i, Y = y_j) = P(X = x_i).P(Y = y_j)$$

That is., If each entry J_{ij} is equal to the product of its marginal entries.

Expectation, Variance, Covariance and Correlation:

If X and Y are two discrete random variables having the joint probability function *f(x, y)* then,

$$\mu_X = E(X) = \sum_i x_i f(x_i)$$

$$\mu_y = E(Y) = \sum y_j g(y_j)$$

$$E(XY) = \sum_{i,j} x_i y_j J_{ij}$$

Covariance of X and Y

$$\begin{aligned} Cov(X,Y) &= E[(X-\mu_X)(Y-\mu_Y)] \\ &= E(XY) - \mu_X\mu_Y \end{aligned}$$

Correlation of X and Y

$$\rho(X,Y) = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

Note:

If X and Y are independent random variables then

$1) E(XY) = E(X).E(Y)$

2) $Cov(X,Y)$ = 0 hence $\rho(X,Y) = 0$