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Euler's method :
For $dy/dx = f(x)$ with initial condition $y(x_0) = x_0$.

Formula:

$$\boxed{y_{n+1} = y_n + hf(x_n, y_n)}$$

1) Using Euler's method find an appropriat value of y corresponding to x=1 given that $dy/dx = x+y$, $y(0) = 1$ taking step size h=1.

→ $\frac{dy}{dx} = x+y$, $y(0) = 1$

and $h = 0.1$.

Let us take $n = 10$ & $h = 0.1$
(sufficiently small)

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x	y	$\frac{dy}{dx} = x+y$	$y_{n+1} = y_n + h f(x_n, y_n)$ $old\,y + 0.1\left(\frac{dy}{dx}\right) = new\,y$
0	1	1	$1 + 0.1(1) = 1.1$
0.1	1.1	1.2	$1.1 + 0.1(1.2) = 1.22$
0.2	1.22	1.42	$1.22 + 0.1(1.42) = 1.36$
0.3	1.36	1.66	$1.36 + 0.1(1.66) = 1.53$
0.4	1.53	1.93	$1.53 + 0.1(1.93) = 1.72$
0.5	1.72	2.22	$1.72 + 0.1(2.22) = 1.94$
0.6	1.94	2.54	$1.94 + 0.1(2.54) = 2.19$
0.7	2.19	2.89	$2.19 + 0.1(2.89) = 2.48$
0.8	2.48	3.29	$2.48 + 0.1(3.29) = 2.81$
0.9	2.81	3.71	$2.81 + 0.1(3.71) = 3.18$
1.0	3.18	4.18	$3.18 + 0.1(4.18) = 3.59$

$$\boxed{y(1) = 3.18}$$

2) Given $dy/dx = \frac{y-x}{y+x}$ with initial condition $y = 1$ at $x = 0$ find y for $x = 0.1$ by Euler's method.

→ Let $n = 5$ $h = 0.02$

x	y	$\frac{dy}{dx} = \frac{y-x}{y+x}$	$new\,y = old\,y + h\left(\frac{dy}{dx}\right)$
0	1	$\frac{1-0}{1+0} = 1$	$1 + 0.02(1) = 1.02$
0.02	1.02	$\frac{1.02-0.2}{1.02+0.2} = 0.9615$	1.0392
0.04	1.0392	0.9258	1.0577
0.06	1.0577	0.8926	1.0755
0.08	1.0755	0.8615	1.0927
0.1	1.0927	0.8323	1.0935

$$\boxed{y(0.1) = 1.0928}$$

Modified Euler's method:

- This is a predictor corrector method.

Let us first predict the value of 'y' by using Euler's method

$$\boxed{y_1^{(0)} = y_0 + h f(x_0, y_0)}$$

Now improve the values of 'y' of using

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

1) Using modified Euler's method find $y(20.2)$ given $dy/dx = \log_{10}(x/y)$, $y(20) = 5$.

→ By data, $x_0 = 20$, $y_0 = 5$, $h = 0.2$

$x_1 = x_0 + h = 20 + 0.2 = 20.2$

$f(x, y) = \log_{10}(x/y)$

$f(x_0, y_0) = f(20, 5)$

$= \log_{10}\left(\frac{20}{5}\right)$

$= 0.6021$

Euler's formula:

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 5 + 0.2(0.6021)$

$= \underline{5.1204}$

Modified Euler's method:

$y_1^{(1)} = y_0 + \frac{h}{2}\left(f(x_0, y_0) + f(x_1, y_1^{(0)})\right)$

$= 5 + \frac{0.2}{2}\left(0.6021 + \log_{10}\left(\frac{20.2}{5.1204}\right)\right)$

$y_1^{(1)} = 5.1198$

$$y_1^{(2)} = 5 + \frac{0.2}{2}\left[0.6021 + \log_{10}\left(\frac{20.2}{5.1198}\right)\right]$$

$$y_1^{(2)} = 5.1198$$

$$\boxed{\therefore y(0.2) = 5.1198}$$

2) Use modified Euler's method to compute $y(0.1)$ given that $dy/dx = x^2 + y$, $y(0) = 1$ by taking $h = 0.05$ considering the accuracy up to two approximation in each step.

→

Given: $\frac{dy}{dx} = x^2 + y$

(I) $x_0 = 0 \quad y_0 = 1 \quad h = 0.05$

$x_1 = x_0 + h = 0 + 0.05$

$x_1 = 0.05$

$f(x_0, y_0) = f(0,1)$

$= 0^2 + 1$

$= 1$

Euler's method

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 1 + 0.05 (1)$

$y_1^{(0)} = 1.05$

Modified Euler's method

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 1 + \frac{0.05}{2}\left[1 + (0.05)^2 + (1.05)\right]$$

$y_1^{(1)} = 1.0526$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1 + \frac{0.05}{2}\left[1 + (0.05)^2 + (1.0526)\right]$$

$y_1^{(2)} = 1.0527 \qquad \therefore y(0.05) = 1.0527$

(II) $x_0 = 0.05$, $y_0 = 1.0527$, $h = 0.05$

$x_1 = x_0 + h = 0.1$

$f(x_0, y_0) = f(0.05, 1.0527)$

$= 1.0552$

Euler's method :

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 1.0527 + 0.05(1.0552)$

$y_1^{(1)} = 1.0553$

Modified Euler's method :

$y_1^{(1)} = y_0 + \frac{h}{2} \{ f(x_0, y_0) + f(x_1, y_1^{(0)}) \}$

$y_1^{(1)} = 1.1061$

$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$y_1^{(2)} = 1.106$

$\therefore \boxed{y(0.05) = 1.106}$

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3) Solve using Euler's modified method to obtain a solution of the eqn $dy/dx = x + |\sqrt{y}|$ which initial conditions $y = 1$ at $x = 0$ for the range $0 \leq x \leq 0.4$ in steps of 0.2

→ $\frac{dy}{dx} = x + |\sqrt{y}|$

$x_0 = 0, \; y_0 = 1. \qquad h = 0.2.$

$x_1 = x_0 + h$

$= 0 + 0.2$

$x_1 = 0.2.$

$f(x_0, y_0) = f(0, 1)$

$= x + |\sqrt{y}|$

$= 0 + 1$

$= 1$

By Euler's method,

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 1 + 0.2(1)$

$y_1^{(0)} = 1.2$

Modified Euler's method

$y_1^{(1)} = y_0 + h/2 [f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1 + 0.2/2 [1 + (0.2 + |\sqrt{1.2}|)]$.

$y_1^{(1)} = 1.2295$

$y_1^{(2)} = y_0 + h/2 \left[f(x_0, y_0) + f(x_1 + y_1^{(1)}) \right]$

$= 1 + 0.2/2 \left[1 + 0.2 + \sqrt{1.2295} \right]$

$y_1^{(2)} = 1.2308$

$\boxed{y_1(0.2) = 1.2309}$

$x_0 = 0.2 \quad y_0 = 1.2309$

$x_1 = x_0 + h = 0.2 + 0.2 = 0.4.$

By Euler's method

$y_1^{(0)} = y_0 + hf(x_0, y_0)$

$= 1.2309 + 0.2\,(0.2 + \sqrt{1.2309})$

$y_1^{(0)} = 1.4927$

By modified Euler's method

$y_1^{(1)} = y_0 + h/2 \left[f(x_0, y_0) + f(x_1, y_1^{(0)}) \right]$

$y_1^{(1)} = 1.5240$

$y_1^{(2)} = y_0 + h/2 \left[f(x_0, y_0) + f(x_1, y_1^{(1)}) \right]$

$y_1^{(2)} = 1.5253$

$\therefore \boxed{y(0.4) = 1.5253}$

Range - Kutta method:

Formula:

$$\boxed{y_1 = y_0 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)}$$

where,

$k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + 1/2 h,\ y_0 + 1/2 k_1)$

$k_3 = hf(x_0 + 1/2 h,\ y + 1/2 k_1)$

$k_4 = hf(x_0 + h,\ y_0 + k_3).$

1) Use R-K method to find approximat value of y when $x = 0.2$ given that $dy/dx = x + y^2$ and $y = 1$ when $x = 0$ & $h = 0.1$

→ $f(x, y) = \frac{dy}{dx} = x + y^2 \qquad x_0 = 0,\ y_0 = 1$

$h = 0.1$

(step I)

$k_1 = hf(x_0, y_0)$

$= 0.1\,(0 + 1)$

$k_1 = 0.100$

$k_2 = hf(x_0 + 1/2 h,\ y_0 + 1/2 k_1)$

$= 0.1\left(\left(0 + \frac{1}{2}(0.1)\right) + \left(1 + \frac{1}{2}(0.1)\right)^2 \right)$

$= 0.1\left(\frac{0.1}{2} + \left(1 + \frac{0.1}{2}\right)^2 \right) = 0.1152$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$k_3 = 0.1168$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1347$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$.

$y_1 = 1.1165 \quad y_1(0.1) = 1.1165$

(Step II) $x_1 = x_0 + h$

$= 0 + 0.1 = 0.1$.

$y_1 = 1.1165, \quad h = 0.1$

$k_1 = hf(x_1, y_1)$

$= 0.1(0.1 + 1.1165^2)$.

$= 0.1346$

$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$.

$= 0.1\left(\left(0.1 + \frac{0.1}{2}\right) + \left(1.1165 + \frac{0.134}{2}\right)^2\right)$.

$k_2 = 0.1550$

$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$.

$= 0.1575$

$k_4 = hf(x_1 + h, y_1 + k_3)$

$k_4 = 0.1823$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$y_2 = 1.2736$.

$y(0.2) = 1.2736$.

2). Using R-K method to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y$ & $y = 1$ when $x = 0$

$\rightarrow \quad x_0 = 0, \quad y_0 = 1 \quad \frac{dy}{dx} = x + y \quad h = 0.2$

$k_1 = hf(x_0, y_0) \qquad x = x_0 + h$

$= 0.2(0 + 1) \qquad 0.2 = 0 + h$

$k_1 = 0.200 \qquad \boxed{h = 0.2}$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.2(0.1 + 1.1)$

1.2
0.2
0.24

$k_2 = 0.24$

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$k_3 = h f(x_0 + h/2, y_0 + k_2/2)$

$= 0.244$

$k_4 = h f(x_0 + h, y_0 + k_3)$

$= 0.2 (0.2 \cancel{+} 1.244)$

$k_4 = 0.2888$

By R-K method,

$y(x_0 + h) = y_0 + 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

$= 1 + \frac{1}{6} (0.2 + 2(0.24) + 2(0.244) + 0.288)$

$y(0.2) = 1.2428$ ##

HW

3) Use R-K method to find an approximate value of y when $x = 1.1$ given that $y = 1.2$ when $x = 1$ &

$\frac{dy}{dx} = 3x + y^2.$