Third Semester

Statistical - Numerical - Fourier Techniques

Sub. Code: 18MAT31

Unit-1

Numerical solution of algebraic & transcendental equations

Method of False position

1) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places.

Ans: $x = 2.094$ (BSG 920,)

2) Solve $x^4 = 2$, $\cos x = \sqrt{x}$ & $x + \ln x = 2$ with $a = 1, b = 2$ by False position.

(Kreyszig Pg 805)

3) Find a real root of the equation $x \log_{10} x = 1.2$ by regula-falsi method correct to four decimal places

Ans: 2.74063 (BSG Pg - 921)

4) Find the root of the equation $\cos x = xe^x$ using the regula-falsi method correct to four decimal places. (BSG Pg - 921)

Ans: 0.51775

5) Find the fourth root of 12 correct three decimal places using the method of false positio

Ans: ~~0.853~~ 1.861 (BSG Pg 926)

Newton-Raphson method

* Derivation

1). Design a Newton iteration. Compute $\sqrt[3]{7}$, $x_0 = 2$ (Kreyszig Pg. 804)

(Ans :

2) Find by Newton's method, a root of the following equation correct to 3 decimal places in (1.5, 2)

i) $x^3 - 3x + 1 = 0$ (BSG Pg No. 926)

Ans: 1.532

ii) $\sqrt[3]{24}$

Ans : 2.8845 (BSG Pg No 925)

iii) $8x = \cos x + 1$ (BSG Pg No - 923)

Ans : 0.6071

iv) Vibrating beam

Find the solution of $\cos x \cosh x = 1$ near, $x = \frac{3}{2}\pi$ (Kreyszig Pg 805)

Fixed point iteration

1) Find a solution of $f(x) = x^3 + x^2 - 1 = 0$ by fixed point iteration (Kreyszig Pg 798).

Ans : 0.682328

2) $f(x) = x - 0.5\cos x = 0$, $x_0 = 0$. Sketch a fig (Kreyszig Pg 804)

Ans $x = 0.450184$

3) $f(x) = x\cosh x = 1$ (Kreyszig Pg. 804)

4) Find the smallest positive root of $\sin x = e^{-x}$ (Kreyszig Pg 804)

H.W

Numerical Solution of Ordinary differential equation

* Taylor's series method

Find an approximate value of y when $x \neq 0$ using Taylor's series method in the following

i) $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$, find $y(0.1)$ & $y(0.2)$. (BSG Pg 1010)

Ans $y(0.1) = 0.90033$
$y(0.2) = 0.80227$

ii) $\frac{dy}{dx} = \log(xy)$, $y(1) = 2$, find $y(1.1)$ & $y(1.2)$ (BSG Pg 1011)

Ans $y(1.1) = 2.075$
$y(1.2) = 2.1649$

iii) $\frac{dy}{dx} = 3x + y^2$, $y(0) = 1$. find $y(0.1)$ (BSG Pg 1012)

Ans : $y(0.1) = 1.1272$

iv) $\frac{dy}{dx} = e^x - y^2$, $y(0) = 1$ $y(0.1)$ (BSG Pg 1012)

Ans : $y(0.1) = 1.005$

Euler method

* Using Euler's method, find an approximate value of y for the following

i) $\frac{dy}{dx} = x + y$ & $y = 1$ when $x = 0$ $h = 0.2$). (~~BSG Pg 1012~~) (Kreyszig Pg 11)

find $y(1)$

Ans: $y(1) = 0.718$

ii&8) $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0)=1$, find $y(0.1)$ (BSG 1013)

Ans $y(0.1) = 1.0928$

iii) $\frac{dy}{dx} = x+y+xy$, $y(0)=1$, $h=0.025$

find $y(0.1)$ (BSG 1017)

Ans $y(0.1) = 1.1448$

iv) $y' = y$, $y(0)=1$, $h=0.1$ upto 10 steps using Euler's method (Kreyszig Pg 12)

Ans

v) $y' = (y-2)^2$, $y(0)=0$, $h=0.1$ (Kreyszig Pg 12)

Ans $x_1 = 0.0286$ $x_{10} = 0.2196$. upto 10 steps

Modified Euler's method

1) Using Euler's modified method, obtain a solution of the equation $\frac{dy}{dx} = x + |\sqrt{y}|$ with $y=1$ at $x=0$ for the range $0 \le x \le 0.6$ in steps of 0.2 (BSG Pg 1016).

Ans $y(0.2) = 1.2309$
$y(0.4) = 1.5253$
$y(0.6) = 1.8861$

2) Using Euler's modified method find the approximate value of y.

i) $\frac{dy}{dx} = 1-y$, $y(0)=0$, find $y(0.1)$ $y(0.2)$ $y(0.3)$ (BSG 1017)

Ans $y(0.1) = 0.095$
$y(0.2) = 0.181$
$y(0.3) = 0.259$

ii) $\frac{dy}{dx} = \log(x+y)$, $y(0)=2$ $y(0.2)$, $y(0.4)$.
with $h=0.2$ (CBSG Pg 1015).

$y(0.2) = 2.0656$..
$y(0.4) = 2.1416$

iii) $\frac{dy}{dx} = x^2+y$, $y(0)=1$, find $y(0.1)$
taking $h=0.05$ $y(0.05)=1.0534$ (CBSG Pg 1017).
Ans $y(0.1) = 1.1055$

iv) $\frac{dy}{dx} = \frac{4-x}{y+x}$ $y(0)=1$, find $y(0.1)$ (CBSG 1017).
$y(0.1) = 1.0928$

v) $y' = xy^2$ $y(0)=1$, $h=0.1$ upto 10 steps (Kreyszig Pg 907)

Runge - Kutta Method

1) Apply R-K method to find an approximate value of y ~~when~~ for the following

i) $\frac{dy}{dx} = x+y$, $y(1)=0$ find $y(0.2)$ (CBSG Pg 1018)
Ans $y(1.2) = 0.2428$

ii) $\frac{dy}{dx} = x+y^2$, $y(0)=1$ find $y(0.2)$ $h=0.1$ (CBSG Pg 1020).
Ans $y(0.1) = 1.1165$
$y(0.2) = 1.2736$

iii) $\frac{dy}{dx} = 3e^x+2y$, $y(0)=0$ & $h=0.1$ find $y(0.1)$ (CBSG Pg 1021).
Ans $y(0.1) = 0.3487$

iv) $\frac{dy}{dx} = \frac{2y^2}{x-1}$, $h=0.2$, $y(0)=0$.
$y(0.1)$ (Kreyszig 907