Experiment-9

Find a subset of a given set S = {sl,s2,.....,sn} of n positive integers whose sum is equal to a given positive integer d. For example, if S={1, 2, 5, 6, 8} and d = 9 there are two solutions{1,2,6}and{1,8}.A suitable message is to be displayed if the given problem instance doesn't have a solution.

The Backtracking Technique

The procedure whereby, after determining that a node lead to nothing but dead ends, we go back ("backtrack") to the node's parent and proceed with the search on the next child.

- Promising : the node can lead to a solution, otherwise, it is called as nonpromising.
- Pruning : check each node whether it is promising, if not, backtracking to the node's parent.

Backtracking is the procedure to prune state space tree.

5.4 The Sum-of-Subsets Problem

- Find a subset of a given set S={s1,s2,….sn} of n positive integers whose sum is equal to a given positive integer d.
- The state space tree can be constructed as a binary tree. The root of the tree represents the starting point, with no decisions about the given elements made as yet.
- Its left and right child represents respectively inclusion and exclusion of s1 in a set being sought.
- Going to the left from the node of the first level corresponds to inclusion of s2.

Sum of subset Cont..

- Going to the right corresponds to its exclusion.
- A path from the root to a node on the ith level of the tree indicates which of the first i numbers have been included in the subsets represented by that node.
- We record the value of s′, the sum of these numbers in the node. If s′ is equal to d, we have solution to the problem.
- We can either report this result and stop or, if all the solutions need to be found, continue by backtracking to the node's parent.
- problem of determining such sets is called the Sum-of-Subsets Problem.

Example 5.1

Suppose that $n = 3$, $W = 6$, and

$$w_1 = 2,\ w_2 = 4,\ w_3 = 5.$$

Find the solutions.

Sol:

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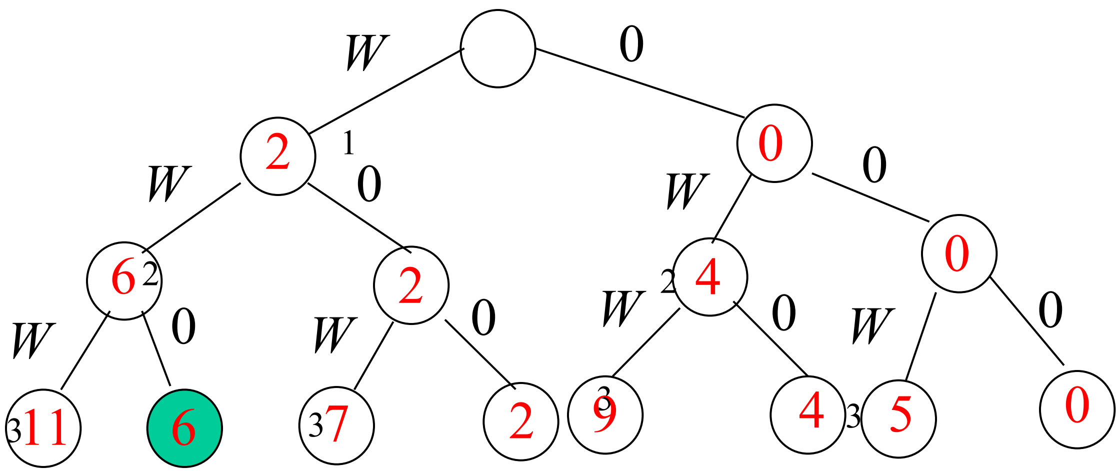
Sol:



Example 5.2

Suppose that $n = 5$, $W = 21$, and

$w_1 = 5$, $w_2 = 6$, $w_3 = 10$, $w_4 = 11$, and $w_5 = 16$.

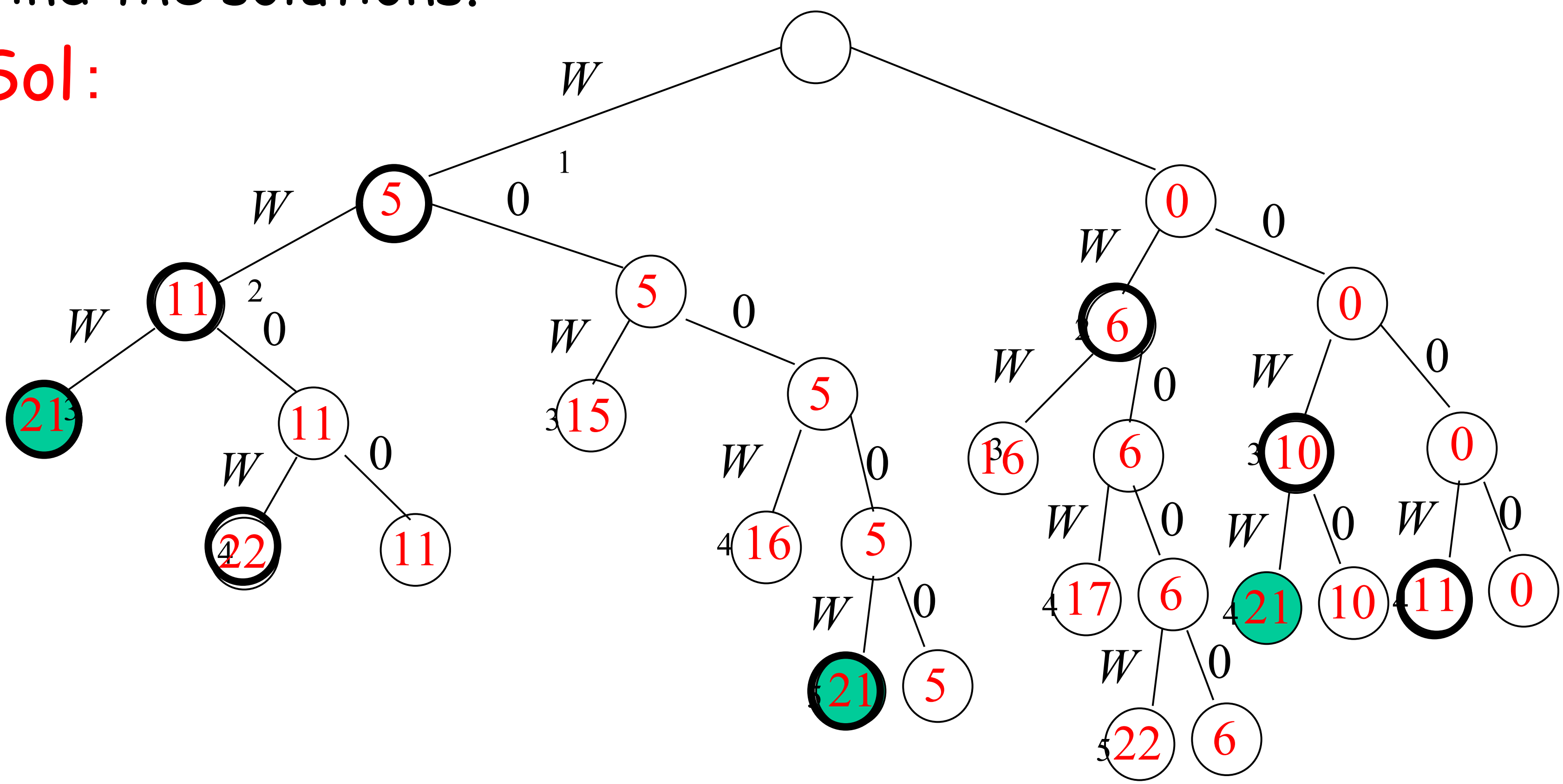
Find the solutions.

Sol:

Example 5.2

Suppose there are $n = 5$, Sum required $W = 21$,
and $n_1 = 5$, $n_2 = 6$, $n_3 = 10$, $n_4 = 11$, $n_5 = 16$.
Find the solutions.

Sol:



Example 5.4

Suppose that $n = 4$, $W = 13$, and

$w_1 = 3,\ w_2 = 4,\ w_3 = 5,\ w_4 = 6.$

Find the solutions.

Sol: For the weights sorting in nondecreasing order,

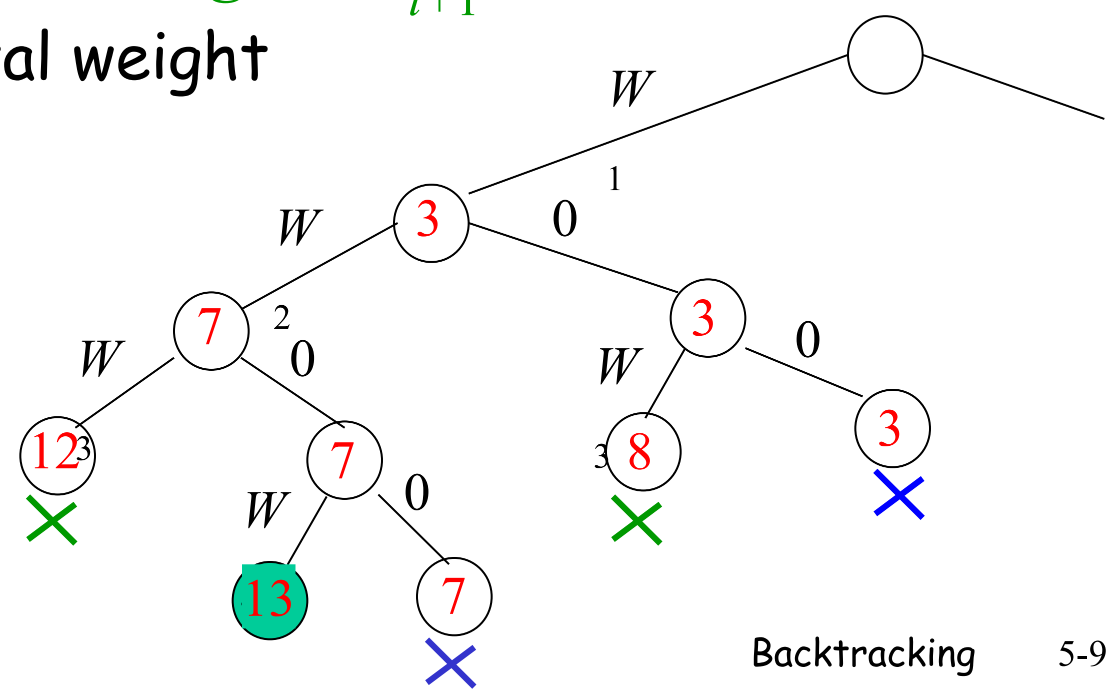
a node is nonpromising if $weight + w_{i+1} > W$

where $weight$ is the total weight

up to a node at level i.

$weight + total_r < W$

is also nonpromising.



Algorithm 5.4

- Problem: Given n positive weights and a positive integer W, find all combinations of the weights that sum to W.

  Inputs: positive integer n, sorted array a index from 1 to n, and a positive integer W.

  Output: all combinations of the weights that sum to W.

```
void sum_of_subsets (index i, int weight, int total)
{  if (promising(i))
      if (weight = W)
         cout << include[1] through include[i];
      else {  include[i+1] = "yes";
              sum_of_subsets(i+1, weight+w[i+1], total−w[i+1]);
              include[i+1] = "no";
              sum_of_subsets(i+1, weight, total−w[i+1]); }
}
      bool promising (index i)
      {  return (weight+total >= W) && (weight =W || weight+w[i+1]<=W); }
```

Analysis of Algorithm 5.4

Worst-Case Time Complexity:

Number of nodes in the state space tree searched

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$$

If $\sum_{i=1}^{n-1} w_i < W$ and $w_n = W$,

it needs an exponentially large number of nodes to be visited.