3/9/18

Probability

* $P(E) = \dfrac{\text{no of possible events}}{\text{no of sample events}} = \dfrac{O(E)}{O(S)}$.

* $P(E') + P(E) = 1$

* $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

* $P(B/A) = \dfrac{P(B \cap A)}{P(A)}$

Bayer's theorem →

$$P(A/B) = \frac{P(B/A) \cdot P(A)}{\Sigma P(A)}$$

Random Variable →

what it is not? It is not random, Variable.

what it is? Function

Function of what? event → outcomes

Permutation	Combination.
- arrangement	- selection
- $^nP_r = \dfrac{n!}{(n-r)!}$	- $^nC_r = \dfrac{n!}{(n-r)!\,r!}$

Random Variables (RV)

- discrete RV
  - finite
  - countable.
- Countinuous RV
  - infinite
  - uncountable.

Discrete probability distribution →

Let a random variable X assumes values $x_1, x_2, x_3, x_4 \ldots\ldots x_n$ with probabilities, $P_1, P_2, P_3, P_4 \ldots\ldots P_n$ respectively where

$$P(x = x_i) = P_i \geq 0 \text{ for each } x_i \text{ \& }$$

$$P_1 + P_2 + \ldots\ldots P_n = \sum_{i=1}^{n} P_i = 1$$

Then

X	x_1	x_2	x_3	x_4	x_5	$\ldots\ldots x_n$
$P(X)$	P_1	P_2	P_3	P_4	P_5	$\ldots\ldots P_n$

is called discrete probability distribution of X & it spells out how a total probability of '1' is distributed over several values of random variables.

$$\boxed{\Sigma(P(x)) = 1}$$

Mean and Variance of random variables

Let

X	x_1	x_2	x_3	$\ldots\ldots x_n$
$P(X)$	$P_1(x_1)$	$P_2(x_2)$	$P_3(x_3)$	$\ldots\ldots P_n(x_n)$

be a discrete probability distribution we denote the mean by μ and define

$$\mu = \frac{\sum_{i=1}^{n} P_i x_i}{\sum_{i=1}^{n} P_i}$$

but $\Sigma P_i = 1$

$$\therefore \boxed{\mu = \sum_{i=1}^{n} P_i x_i .}$$

Other names of mean are $\longrightarrow$
Average, expected value $E(X)$.

We denote the variance by σ^2 & define

$$\boxed{\sigma^2 = \sum P_i (x_i - \mu)^2}$$

Standard deviation $\longrightarrow$

$$\boxed{\sigma = +\sqrt{\text{Variance}}.}$$

Xamples $\longrightarrow$

1. A fair coin is tossed three times, X denotes the no of heads showing up find the distribution of X. Also find its mean & Variance.

$\longrightarrow$ $S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$.

$s = 8$
X = no of heads show up.
$x = \{HHH\} = 1$
$x = \{HHT, HTH, \cancel{HHT}, THH\} = 3$
$x = \{THT, HTT, TTH\} = 3$
$x = \{TTT\} = 1$

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

$\longrightarrow \sum P(X) = 1$. * should be $= 1$

$$\text{Mean} = \sum_{i=1}^{n} p_i x_i$$

$$\mu = \left[0 \times 1/8 + 1 \times 3/8 + 2 \times 3/8 + 3 \times 1/8\right]$$

$$= 12/8 = 6/4 = 3/2$$

$$\text{mean} = \underline{\underline{1.5}}$$

$$\text{Variance} = \sigma^2 = \sum_{i=1}^{n} p_i (x_i - \mu)^2$$

$$= \left[\frac{1}{8}(0-1.5)^2\right] + \left[3/8\,(1-1.5)^2\right] + \left[3/8\,(2-1.5)^2\right] + \left[3/8\,(3-1.5)^2\right].$$

$$= 3/4$$

$$\sigma^2 = \underline{\underline{0.75}}.$$

(2) A die is tossed thrice a success of getting one or six on a toss then find the mean & variance of the number of success.

→ Let

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$X = \{1, 6\} = 2$$

$$P(X) = \text{no of success} = 2/6 = 1/3$$

$$p'(X) = \text{no of failure} = 1 - 1/3$$

$$= 2/3$$

X	0	1	2	3
P(X)	8/27	4/9	2/9	1/27

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① $P(X=0) = {}^3C_0$ (0 Success, 3 failure)

$= {}^3C_0 \times 2/3 \times 2/3 \times 2/3$

$= 8/27$

② $(P(X=1)) = {}^3C_1$ (1 Success, 2 failure)

$= {}^3C_1 \times 1/3 \times 2/3 \times 2/3$

$= 4/9$

③ $P(X=2) = {}^3C_2$ (2 Success, 1 failure)

$= {}^3C_2 \times 1/3 \times 1/3 \times 2/3$

$= 2/9$

④ $P(X=3) = {}^3C_3$ (3 Success, 0 failure)

$= {}^3C_3 \times 1/3 \times 1/3 \times 1/3$

$= 1/27$

Mean $(\mu) = \sum_{i=1}^{n} p_i x_i$

$= (0 \times 8/27 + 1 \times 4/9 + 2 \times 2/9 + 3 \times 1/27)$

$= 0 + \frac{4}{9} + \frac{4}{9} + \frac{3}{27}$

$= \frac{8}{9} + \frac{1}{9}$

$= \frac{9}{9}$

$\mu = \underline{\underline{1}}.$

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Variance $\longrightarrow \quad \sigma^2 = \sum_{i=1}^{n} P_i (x_i - \mu)^2$

$$= \left[\frac{8}{27}(0-1)^2 + \frac{4}{9}(1-1)^2 + \frac{2}{9}(2-1)^2 + \frac{1}{27}(3-1)^2 \right]$$

$$= \frac{8}{27} + 0 + 2/9 + \frac{4}{27}$$

$$= \frac{12}{27} + \frac{2}{9}$$

$$= \frac{12+6}{27}$$

$$= \frac{18}{27} \longrightarrow \frac{6}{9}$$

$$\sigma^2 = 2/3$$

3) A random variable X has the following probability function

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) find K ii) Evaluate $P(x<6)$, $P(x \geq 6)$

iii) $P(0<x<5)$.

$\longrightarrow$ wkt $\sum P(x) = 1$

$$0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$$

$$9K + 10K^2 = 1$$

$$10K^2 + 9K = 1$$

$$\therefore \quad K = 1/10 \text{ or } K = -1$$

$\boxed{K = 1/10}$

(ii). $P(x<6)$

$\rightarrow P(x<6) = 0+K+2K+2K+3K+K^2$

$= 8K + K^2$

$= \frac{8}{10} + \frac{1}{100}$

$= \frac{81}{100}$

(ii) $P(x \geq 6)$

$\rightarrow P(x \geq 6) = 2K^2 + 7K^2 + K$

$= 9K^2 + K$

$= \frac{9}{100} + \frac{1}{10}$

$= \frac{19}{100}$

(iii) $P(0<x<5)$

$\rightarrow P(0<x<5) = K + 2K + 2K + 3K$

$= 8K$

$= \frac{8}{10}$

$= 4/5$.

(4)

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

(i) find $(P(x<4))$, $P(x \geq 5)$, $P(3<x\leq 6)$

(ii) What will be the minimum value of K so that

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$P(x \leq 2)\ 0.3$

$\rightarrow$ ① $P(x \leq 4) = K + 3K + 5K + 7K + 9K$

$= 18K + 7K$

$= 25K$

$= 25/49$

$\because \Sigma(P_x) = 1$

$1 = K + 3K + 5K + 7K + 9K + 11K + 13K$

$1 = 49K$

$\boxed{K = \frac{1}{49}}$

② $P(x \geq 5)$

$\rightarrow$ $P(x \geq 5) = 11K + 13K$

$= 24K$

$= \frac{24}{49}$

③ $P(3 < x \leq 6)$

$\rightarrow$ $= \cancel{7K} + 9K + 11K + 13K$

$= 20K + 13K$

$= 33K$

$= 33/49$

④ $P(x \leq 2) > 0.3$

$\rightarrow$ $K + 3K + 5K > 0.3$

$9K > 3/10$

$K > 1/30$

4) Four coins are tossed, what is the expectation of the no of heads.

→ S = { HHHH TTTT
HHHT TTTH
HHTT TTHH
HHTH TTHT
HTTT THHH
HTTH THHT
HTHT THTH
HTHH THTT }

S = 16

X = no of heads.

X	0	1	2	3	4
P(X)	1/16	4/16	6/16	4/16	1/16

$$\text{mean } (\mu) = \sum_{i=1}^{n} p_i x_i$$

$$= \left(\frac{1}{16} \times 0 + \frac{4}{16} \times 1 + \frac{6}{16} \times 2 + \frac{4}{16} \times 3 + \frac{1}{16} \times 4\right)$$

$$= 0 + \frac{4}{16} + \frac{12}{16} + \frac{12}{16} + \frac{4}{16}$$

$$= \frac{16}{16} + \frac{16}{16}$$

$$= 32/16$$

$$\mu = \underline{\underline{2}}$$

(5) A random variable has the probability function.

X	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

find K & find mean & Variance.

$\rightarrow$ $0.1 + K + 0.2 + 2K + 0.3 + K = 1$

$4K + 0.6 = 1$

$4K = 1 - 0.6$

$4K = 0.4$

$\boxed{K = 0.1}$

$$\text{Mean}(\mu) = \sum_{i=1}^{n} p_i x_i$$

$= (2 \times -0.1 + (-1) \times 0.1 + 0 + 1 \times 2(0.1) + 2 \times 0.3 + 3 \times 0.1)$

$= -0.2 - 0.1 + 0 + 0.2 + 0.6 + 0.3$

$= -0.3 + 0.11$

$\mu = \underline{\underline{0.8}}$.

$$\text{Variance} = \sigma^2$$

$$= \sum_{i=1}^{n} P_i (x_i - \mu)^2$$

$= (0.1(-2-0.8)^2 + 0.1(-1-0.8)^2 + 0.2(0-0.8)^2 + 0.2(1-0.8)^2 + 0.3(2-0.8)^2 + 0.1(3-0.8)^2)$

$= 0.78 + 0.32 + 0.128 + 0.008 + 0.432 + 0.484$

$= 2.152$

Distribution function →

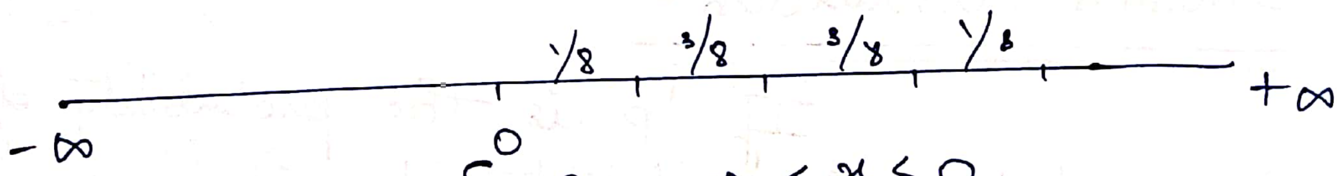
The distribution function of $f(x)$ with the discrete variables X is defined by $f(x) = P(x \le x) = \sum_{i=1}^{n} P(x_i)$

Q) Obtain the distribution function of the total no of heads occuring in 3 tosses of an unbaised coin.

→ $S = \{$ HHH TTT
HHT TTH
HTH THT
HTT THH $\}$

$S = 8$

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8



$$F(x) = \begin{cases} 0 & -\infty < x \le 0 \\ 1/8 & 0 \le x \le 1 \\ 1/2 & 1 \le x \le 2 \\ 7/8 & 2 \le x \le 3 \\ 1 & 3 \le x < \infty. \end{cases}$$

Binomial distribution (Bernoulli's distribution)

Bernoulli's trail →

A Random experiment with only two possible outcomes, i.e. success & failure is called Bernoulli's trail, where probability is same for each kind.

Bernoulli's theorem →

The probability of 'x' success in n traile is equal to

$^nC_x \, p^x q^{n-x}$ where

$p \rightarrow$ probability of success

$q \rightarrow$ probability of failure.

Binomial distribution →

If 'p' is the probability of success, 'q' is the probability of failure then the probability of 'x' success out of 'n' traile is given by,

$$P(x) = {}^nC_x \, p^x q^{n-x}$$

X	0	1	2	....	n
P(X)	q^n	$^nC_1 \, p q^{n-1}$	$^nC_2 \, p^2 q^{n-2}$		p^n.

$$P(x) = {}^nC_x\, p^x q^{n-x}$$

$$* \sum P(x) = {}^nC_0\, p^0 q^n + {}^nC_1\, p^1 q^{n-1} + {}^nC_2\, p^2 q^{n-2} + \ldots\ldots + p^n$$
$$= (p+q)^n$$
$$= 1.$$

12/9/19

Mean, Variance and Standard deviation

Note →

Bernoullis's trail

↓

Binomial distribution

↓

Poissons distribution

1) $\text{Mean } (\mu) = \sum_{x=0}^{n} x\, P(x)$

but $P(x) = {}^nC_x \cdot p^x \cdot q^{n-x}$

$$\therefore \mu = \sum_{x=0}^{n} x \cdot {}^nC_x \cdot p^x \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!\,(n-x)!} \cdot p^x \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} \cancel{x} \cdot \frac{n!}{\cancel{x}(x-1)!\,(n-x)!} \cdot p^x \cdot q^{n-x}$$

$$= \sum_{x=\phi}^{n} \frac{n(n-1)!}{(x-1)!\,(n-x)!} \cdot p^x \cdot q^{n-x}.$$

$$= \sum_{x=1}^{n} \frac{n(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!}\, p \cdot p^{x-1}\, q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!}\, p^{x-1}\, q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} {}^{n-1}C_{(x-1)} \cdot p^{(x-1)} \cdot q^{(n-1)-(x-1)}$$

$$= np\,(p+q)^{n-1}$$

$$\mu = np\,(1).$$

$$\therefore \boxed{\text{Mean } (\mu) = np}$$

2) Variance $= \sum_{x=0}^{n} x^2 (p(x)) - \mu^2 \longrightarrow$ 1

$\rightarrow$ Now take $= \sum_{x=0}^{n} x^2 p(x).$

$$= \sum_{x=0}^{n} [x(x-1) + x]\, p(x)$$

$$= \sum_{x=0}^{n} x(x-1)\, \underline{p(x)} + \underline{\sum_{x=0}^{n} x\, p(x)}$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!} \cdot p^{x} \cdot q^{n-x} + np$$

$$= \sum_{x=0}^{n} \frac{\cancel{x(x-1)}\; n!}{\cancel{x(x-1)}(x-2)!\,(n-x)!} \cdot p^{x} \cdot q^{n-x} + np$$

$$= \sum_{x=2}^{n} \frac{n(n-1)(n-2)!}{(x-2)!\,[(n-2)-(x-2)]!} \cdot p^{x-2} p^2 \cdot q^{(n-2)-(x-2)} + np.$$

$$= \sum_{x=2}^{n} \frac{n(n-1)(n-2)!}{(x-2)!\,[(n-2)-(x-2)]!} \cdot p^{x-2} p^2 \cdot q^{(n-2)-(x-2)} + np.$$

$$= n(n-1)p^2 \cdot \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,[(n-2)-(x-2)]!} \cdot p^{x-2} \cdot q^{(n-2)-(x-2)} + np.$$

$$= n(n-1)p^2 \cdot \sum_{x=2}^{n} {}^{n-2}C_{x-2} \cdot p^{x-2} \cdot q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 \cdot (p+q)^{n-2} + np$$

$$= n(n-1)p^2 \cdot 1 + np$$

$$= (n^2-n)p^2 + np$$

$$= n^2p^2 - np^2 + np \longrightarrow (2)$$

Substituting (2) in (1)

$$\text{Variance} = n^2p^2 - np^2 + np - \mu^2$$

$$= n^2p^2 - np^2 + np - (np)^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np - np^2$$

$$= np(1-p)$$

$$\boxed{\text{Variance} = npq}$$

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$SD = \sqrt{\text{Variance}}$

$$\boxed{SD = \sqrt{npq}}$$

Note → Atleast - $x \geq$

Atmost - $x \leq$

Exactly - $x =$

no / not / None - $x = 0$.

Xamples →

① A coin is tossed five times, we have been told to find the probability of getting atleast 3 heads.

→ Atleast means minimum 3 heads & more than that

Atmost means maximum 3 heads & less than that.

② The probability that a pen manufactured by a company will be defective is $1/10$. If 12 such pens are manufactured find the probability that

1) Exactly two will be defective

2) Atleast two will be defective

3) None will be defective.

→ The probability of defective pens, $p = 1/10$

The probability of no defective pens, $q = 1 - p$

$= 1 - \frac{1}{10}$

$= 9/10.$

wkt,

$n = 12$

$^nC_x \cdot p^x \cdot q^{n-x}$

① Exactly $(X = 2)$

→ $n = 12 \quad p = 1/10 \quad q = 9/10$

$$^nC_x \cdot p^x \cdot q^{n-x} = {}^{12}C_2 \cdot p^2 \cdot q^{12-2}$$

$$= {}^{12}C_2 \cdot (1/10)^2 \cdot (9/10)^{10}$$

$$= 0.23013.$$

② Atleast $(x \geq 2)$

→ $n = 12 \quad p = 1/10 \quad q = 9/10$

$$P(x \geq 2) = P(2) + P(3) + \dots\dots P(12).$$

$$P(x \geq 2) = 1 - P(x < 2)$$

$$= 1 - [P(0) + P(1)]$$

$$= 1 - [{}^{12}C_0 \cdot (1/10)^0 \cdot (9/10)^{12} + {}^{12}C_1 \cdot (1/10)^1 \cdot (9/10)^{11}]$$

$$= 1 - [0.28243 + 0.37657]$$

$$= 0.34100.$$

③ $P(X = 0)$

→ $= {}^{12}C_0 (1/10)^0 \cdot (9/10)^{12}$

$= 0.2824.$

③. In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads & 4 tails.

→ When you toss a coin.

$p = 1/2 \quad q = 1/2$

$P(X=8) = {}^{12}C_8 \cdot (1/2)^8 \cdot (1/2)^4$

$= 0.12085.$

for 256 sets, getting 8 heads & 4 tails is $= 256 \times 0.12085$

$= \underline{\underline{30.93}} \approx \underline{\underline{31}}.$

13/9/19. ④. Compute the probability, atleast two 'six' in rolling a fair die four times?

→ Success $p = 1/6$

failure $q = 1-p$

$= 1 - 1/6$

$= 5/6$

since throwing a die is independent

$n=4$

atleast two six $P(2 \le x)$

$= P(X=2) + P(3) + P(4)$

$= {}^{12}C_2 (1/6)^2 \cdot (1/6)^{10} + {}^{12}C_3 (1/6)^3 \cdot (1/6)^7 + {}^{12}C_4 \cdot (1/6)^4 (1/6)^8$

$= \underline{\underline{0.13194}}.$

(5) Determine the binomial distribution for which the mean is 2 & mean plus Variance = 3. Also find ($P(x \leq 3)$).

→ mean = 2

$np = 2$

mean + Variance = 3

$np + npq = 3$

$npq = 3 - 2$

$npq = 1$

$(2)q = 1$

⇒ $\boxed{q = 1/2}$

$p + q = 1$

$p + 1/2 = 1$

$p = 1 - 1/2$

$\boxed{p = 1/2}$

from $np = 2$

$n(1/2) = 2$

$\boxed{n = 4}$

Therefore by binomial distribution.

$P(x \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= {}^4C_0 (1/2)^0 (1/2)^4 + {}^4C_1 (1/2)^1 (1/2)^3 + {}^4C_2 . (1/2)^2 . (1/2)^2 + {}^4C_3 . (1/2)^3 . (1/2)^1$

$= \frac{1}{16} + \frac{1}{4} + 3/8 + 1/4$

$= 15/16$

$= 0.9375$

(6) Compute the probability. (~~atleast two 'six'~~)
where three coin are tossed simultaneously find the probability of the random variable,
X = no of heads & compute the probability of obtaining no heads, precisely one head, atleast one head, not more than three heads.

$\longrightarrow$ (1) $P(X=0)$.

for tossing a coin $p = 1/2$ (success)

$q = 1/2$ (failure)

$n = 3$.

1) $P(X=0) = {}^3C_0 \cdot (1/2)^0 \cdot (1/2)^3$

$= 1/8$

2) $P(X=1) = {}^3C_1 \cdot (1/2)^1 \cdot (1/2)^2$

$= 3/8$

3) $P(X \geq 1) = P(1) + P(2) + P(3)$

$= {}^3C_1 \cdot (1/2)^1 (1/2)^2 + {}^3C_2 (1/2)^2 (1/2)^1 + {}^3C_3 \cdot (1/2)^3 (1/2)^0.$

$= 3/8 + 3/8 + \frac{1}{8}$

$= 7/8.$

A) $P(X \le 3) = P(0) + P(1) + P(2) + P(3)$

$= {}^3C_0 \cdot (1/2)^0 \cdot (1/2)^3 + {}^3C_1 \cdot (1/2)^1 \cdot (1/2)^2 +$

${}^3C_2 \cdot (1/2)^2 \cdot (1/2)^1 + {}^3C_3 \cdot (1/2)^3 \cdot (1/2)^0$.

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + 1/8$

$= 8/8$

$= \underline{\underline{1}}$.

7. Fit a binomial distribution for the following frequency distribution.

X	0	1	2	3	4	5	6
$f(x)$	13	25	52	58	32	16	4

→ $n = 6$

$$\text{mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{0 \times 13 + 25 \times 1 + 52 \times 2 + 58 \times 3 + 32 \times 4 + 16 \times 5 + 4 \times 6}{200}.$$

$$= \frac{0 + 25 + 104 + 174 + 128 + 80 + 24}{200}$$

$$np = \frac{107}{40}$$

$$\boxed{np = 2.6750} \rightarrow (1)$$

from (1)

$p = \frac{2.6750}{6}$

$= \underline{\underline{0.4458}}$.

$q = 1 - p$

$= 1 - 0.4458$

$= \underline{\underline{0.5542}}$.

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$$P(X) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

$$P(X) = {}^{6}C_{x} \cdot p^{x} \cdot q^{6-x}$$

for now wkt,

$$\boxed{F(x) = N\,P(X)}$$

$$= 200 \times {}^{6}C_{x} \cdot p^{x} \cdot q^{6-x}$$

X	0	1	2	3	4	5	6
P(X)	5.7946	27.969	56.2504	60.3347	36.4025	11.7137	1.5705

$\boxed{200.3054}$

$= 199.98$

$\approx$

$= 200$

$N = 200$

$\therefore P(X) = 1$

Poissons distribution →

Here $n \to \infty$, $p \to 0$

$m = np$

Poissons distribution is regarded as the limiting form of Binomial distribution when 'n' is large ($n \to \infty$) & p is the probability of success is very small ($p \to 0$) so that np tends to a finite fixed constant.

$$p(x) = \frac{e^{-m}\, m^{x}}{x!} \quad \text{or} \quad \frac{\mu^{x}\, e^{-\mu}}{x!}$$

this is called poisson's probability function where x is poisson's variate.

This distribution of probabilities is $x = 0, 1, 2, 3, \ldots\ldots$

x	0	1	2	3	....
$P(x)$	e^{-m}	$e^{-m} m$	$\frac{e^{-m} m^2}{2!}$	$\frac{e^{-m} m^3}{3!}$	

We have $P(x) \geq 0$ &

$$\sum_{x=0}^{\infty} P(x) = e^{-m} + e^{-m} m + e^{-m} \frac{m^2}{2!} + e^{-m} \frac{m^3}{3!} + \ldots\ldots$$

$$= e^{-m} \left[1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \ldots\ldots \right]$$

$$= e^{-m} . e^{m}$$

$$\left[\because e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots \right]$$

$$P(x) = e^{0}$$

$$\therefore P(x) = \underline{\underline{1}}.$$

Hence $P(x)$ is probability function.

Mean & Variance $\rightarrow$

$$\text{Mean}(\mu) = \sum_{x=0}^{\infty} x \, P(x)$$

$$= \sum_{x=0}^{\infty} x . \frac{e^{-m} m^{x}}{x!}$$

$$= \sum_{x=0}^{\infty} \cancel{x} . \frac{e^{-m} m^{x}}{\cancel{x}(x-1)!}$$

$$= \sum_{x=0}^{\infty} e^{-m} m^{x} / (x-1)!$$

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$$= \sum_{x=0}^{\infty} \frac{e^{-m} m^{x}}{(x-1)!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m} m^{x-1} . m}{(x-1)!}$$

$$= e^{-m} . m \sum_{x=0}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$= e^{-m} . m \left[1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots \right]$$

$$= e^{-m} . m . e^{m}$$

$$= e^{0} . m$$

$$\boxed{\text{Mean}(\mu) = m}$$

$$\text{Variance} = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2 \rightarrow (1)$$

Now,

$$= \sum_{x=0}^{\infty} x^2 p(x)$$

$$= \sum_{x=0}^{\infty} [x(x-1) + x] P(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) P(x) + \sum_{x=0}^{\infty} P(x) x$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-m} m^{x}}{x!} + m$$

$$= \sum_{x=0}^{\infty} \cancel{x(x-1)} \frac{e^{-m} m^{x}}{\cancel{x(x-1)}(x-2)!} + m$$

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$$= \sum_{x=0}^{\infty} \frac{e^{-m} m^{x}}{(x-2)!} + m.$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m} m^{x-2} m^{2}}{(x-2)!} + m$$

$$= e^{-m} m^{2} \left[\sum_{x=0}^{\infty} \frac{m^{x-2}}{(x-2)!} + m \right]$$

$$= e^{-m} m^{2} \cancel{+} \left[1 + m + \frac{m^2}{2!} + \ldots , \right] + m$$

$$= e^{-m} \cdot m^{2} \cdot e^{m} + m$$

$$= m^{2} + m$$

from ① $= \cancel{m^2} + m - \cancel{m^2}$

Variance $= m$

$$\boxed{\text{Variance} = (\sigma^2) = m}$$

$$\boxed{S.D = \sqrt{m}}$$

Solve.

① If the probability of a bad reaction from a certain infection is 0.001, determine the chance that out of 2000 individuals more than 2 will get bad reaction?

$\longrightarrow$ $n = 2000$

$p = 0.001$

$q = 1 - p = 0.999$

$m = np = 2$

The probability that more than two will get bad reaction.

$P(x>2)$

$= 1 - P(x \leq 2)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2}(2)^2}{2!}\right]$

$= 0.323$

(2) If a random variable has poissons distribution such that $P(1) = P(2)$ find i) mean ii) $P(4)$.

$\rightarrow$ $P(1) = P(2)$

$$\frac{\cancel{e^{-m}} m^1}{1!} = \frac{\cancel{e^{-m}} m^2}{2!}$$

$$\cancel{2m} = m^{\cancel{2}}$$

$$\boxed{m = 2}$$

$$P(4) = \frac{e^{-m} m^4}{4!}$$

$$= \frac{e^{-2} (2)^4}{4!}$$

$$= 0.09022$$

③ If the probability of producing a defective screw is $p = 0.01$, what is the probability a lot of 100 screws will contain more than 2 defectives.

→ $n = 100$

$p = 0.01$

$m = np = 1$

$P(X > 2)$

$1 - P(X \le 2)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{e^{-m} m^0}{0!} + \frac{e^{-m} m^1}{1!} + \frac{e^{-m} m^2}{2!}\right]$

$= 1 - \left[\frac{e^{-1}(1)^0}{0} + \frac{e^{-1}(1)^1}{1} + \frac{e^{-1}(1)^2}{2}\right]$

$= \underline{\underline{0.080}}$.

④ Fit a poissons distribution of the following

X	0	1	2	3	4
$f(x)$	46	38	22	9	1

→ $\text{mean} = \frac{\Sigma x_i P_i}{\Sigma f_i \rightarrow N}$

$= \frac{0 \times 46 + 1 \times 38 + 2 \times 22 + 3 \times 9 \times 4 \times 1}{46 + 38 + 22 + 9 + 1}$

$= 0 + 38 + 44 + 27 + 4 / 116.$

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mean (μ) = $\underline{0.974143}$.

$F(x) = N\,P(x)$

$= 116 \times \frac{e^{-m} \cdot m^{x}}{x}$

X	0	1	2	3	4
F(X)	43.79	42.65	20.77	6.74	1.64

$F(X) = N : P(X)$

$116 = 115.59 \times P(X)$

$\Rightarrow 115.59 \approx 116$

$\boxed{P(X) = 1}$

20/9/19.

Countinous propability distribution.

Countinous Random Variable →

Countinuous probability distribution :

for every x belonging to the of a countinuous random variable X, i.e we assign a real number f(x) satisfying the condition.

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(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x) = 1$

$f(x)$ is called countinous probability function or probability density function.

Now if (a, b) is a subinterval of the X, then the probability that the X lies in (a, b) is defined to be the integrated / integral part of $f(x)$ between (a, b).

$$P(a \leq x \leq b) = \int_a^b f(x) dx.$$

Comulative distribution function.

Exponential distribution function.

24/9/19 Note: If r is a real number then probability of $P(x \geq r) = \int_r^\infty f(x).dx$

$$p(x < r) = 1 - P(x \geq r)$$

$$P(x > r) = 1 - \int_r^\infty f(x).dx$$

$\int_{-\infty}^{\infty} f(x) dx = 1$	$\sum_{x=0}^{n} p(x) = 1$
$\mu = \int_{-\infty}^{\infty} x . f(x) dx$	$\mu = \sum x_i P(x_i)$
$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$	$\sigma^2 = \sum (x-\mu)^2 P(x_i)$

Problems $\longrightarrow$

① A random variable x has the following density function

$$p(x) = \begin{cases} kx^2 & -3 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Evaluate K & find

i) $P(1 \le x \le 2)$ ii) $P(x \le 2)$

iii) $P(x > 1)$

$\longrightarrow$ Since $p(x) \ge 0$ &

$$\int_{-\infty}^{\infty} p(x).dx = 1$$

$$\Rightarrow \int_{-3}^{3} Kx^2 dx = 1$$

$$K \int_{-3}^{3} x^2 dx = 1$$

$$1 = K \left[\frac{x^3}{3}\right]_{-3}^{3} \Longrightarrow \boxed{K = 1/18}$$

i) $P(1 \le x \le 2) = \int_{1}^{2} \frac{1}{18} x^2 dx = 7/54$

ii) $P(x \le 2) = \int_{-3}^{2} \frac{1}{18} x^2 dx = 35/54$

iii) $P(x > 1) = \int_{1}^{3} \frac{1}{18} x^2 dx = 13/27.$

(2) The time 't' years required to complete a software project has a probability density function of the form.

$$f(t) = \begin{cases} Kt(1-t) & 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

find K & also that project will be completed in less than 4 monthes.

$\rightarrow$ since $f(t) \ge 0$

$$\Rightarrow \int_{-\infty}^{\infty} f(t)\,dt = 1$$

$$\int_0^1 Kt(\cancel{1}-t)\,dt = 1$$

$$K\int_0^1 (t-t^2)\,dt = 1$$

$$K\left[\frac{t^2}{2} - \frac{t^3}{3}\right]_0^1 = 1$$

$$K\left[\left(\frac{1}{2}-0\right) - \left(\frac{1}{3}-0\right)\right] = 1$$

$$K\left[\frac{1}{2} - \frac{1}{3}\right] = 1$$

$$K\left(\frac{1}{6}\right) = 1$$

$$\Longrightarrow \boxed{K = 6}$$

$P(x < \frac{4}{12}) \approx P(x < \frac{1}{3})$ in moths

$$= \int_0^{1/3} 6t(1-t)\,dt$$

$$= \frac{7}{27}$$

(3) The kilometers run (in thousands of km) without any sort of problem in respect of a certain Vehicle is a random variable having

P.d.f $f(x) = \begin{cases} \frac{1}{40} e^{-x/40} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$

find the probability that the vehicle is trouble free.

1) atleast for 25000 km

2) atmost for 25000 km

3) between 16000 km to 3200 km

→ Let x be the kilometer respectively in 1000

$$P(x \geq 25) = 1 - \int_{25}^{0} (25 > x)$$

$$= 1 - \int_{25}^{0} \frac{1}{40} e^{-x/40} \cdot dx$$

$$= 0.5353$$

$$P(x \leq 25) = \int_{0}^{25} \frac{1}{40} e^{-x/40} \cdot dx$$

$$= 0.4647$$

$$\int_{16}^{32} \frac{1}{40} e^{-x/40} dx$$

$$= 0.2210$$

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(4) find K such that

$$f(x) = \begin{cases} Kxe^{-x} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a p.d.f, find mean.

$\longrightarrow$ $f(x) \geq 0 = 1 \implies \int_{-\infty}^{\infty} Kxe^{-x} = 1$

$$\int_0^1 K x e^{-x} . dx = 1$$

$$K \int_0^1 xe^{-x} . dx = 1$$

$$K (0.26424) = 1$$

$$K = 1/0.26424 \implies \boxed{K = 3.78444}.$$

Exponential distribution →

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{\infty} \alpha e^{-\alpha x}\,dx$$

Mean → $$\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{\infty} x.f(x)\,dx$$

$$\int_{0}^{\infty} x f(x)\,dx = \int_{0}^{\infty} x.\alpha e^{-\alpha x}\,dx$$

$$= \alpha \int_{0}^{\infty} x e^{-\alpha x}\,dx$$

$$= \alpha \left[x\left(-\frac{e^{-\alpha x}}{\alpha}\right) - \frac{e^{-\alpha x}}{\alpha^2} \right]_0^{\infty}$$

$$= \alpha \left[1/\alpha^2 \right]$$

$$\text{Mean} = \frac{1}{\alpha}$$

Variance → $$\sigma^2 = \int_{0}^{\infty} (x-\mu)^2 \alpha e^{-\alpha x}\,dx$$

$$= \alpha \left[(x-\mu)^2 \left(\frac{e^{-\alpha x}}{-\alpha}\right) - 2(x-\mu)\left(\frac{e^{-\alpha x}}{\alpha^2}\right) + 2\left(\frac{e^{-\alpha x}}{-\alpha^3}\right) \right]_0^{\infty}$$

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$$= \alpha \left[-\frac{1}{\alpha} \{ 0 - \mu^2 \} - \frac{2}{\alpha^2} \{ 0 - (-\mu) \} \times (\cancel{x - \mu}) - \frac{2}{\alpha^3} \{ 0 - 1 \} \right]$$

$$= \alpha \left[\frac{\mu^2}{\alpha} - \frac{2\mu}{\alpha^2} + \frac{2}{\alpha^3} \right]$$

but $\mu = 1/\alpha$

$$= \alpha \left[\frac{\alpha}{\alpha^3} - \cancel{\frac{2}{\alpha^3}} + \cancel{\frac{2}{\alpha^3}} \right]$$

$$= \alpha \left(1/\alpha^3 \right)$$

$$\sigma^2 = 1/\alpha^2$$

24/9/19. ① If x is an exponential variate with mean as 3 find i) $P(x>1)$ ii) $P(x<3)$.

$\Rightarrow$ The p.d.f exponential distribution

probability distribution function

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 \le x \le \infty \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Mean} = 1/\alpha$$

$$3 = 1/\alpha$$

$$\boxed{\alpha = 1/3}$$

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i) $P(X>1) = 1 - P(X \le 1)$

$$= 1 - \int_0^1 \alpha e^{-\alpha x}\, dx$$

$$= 1 - \int_0^1 \tfrac{1}{3} e^{-x/3}\, dx$$

$$= 1 - 0.2834$$

$P(X>1) = 0.7165$

ii) $P(X<3) = \int \alpha e^{-\alpha x}\, dx$

$$= \int_0^3 \left(\frac{1}{3}\right) e^{-x/3}\, dx$$

$P(X<3) = 0.6321$

2) The length of telephone conservation in a booth has been an exponential distribution & found on an average to be 5 minutes. Find the probability that a random call made from this booth

i) ends less than 5 minutes

ii) between 5 & 10 minutes.

→ The p.d.f exponential distribution

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 \le x \le \infty \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Mean} = 1/\alpha$$

$$5 = 1/\alpha$$

$$\boxed{\alpha = 1/5}$$

i) $P(x < 5) = \int_0^5 \alpha e^{-\alpha x}\, dx$

$$= \int_0^5 (1/5) e^{-x/5}\, dx$$

$$= 0.6321$$

ii) $P(5 < x < 10) = \int_5^{10} \alpha e^{-\alpha x}\, dx$

$$= \int_5^{10} (1/5)\, e^{-x/5}\, dx$$

$$= 0.2325$$

(8) The duration of showerin certain town during the period of depression is exponentially distributed with mean 5 min. what is the probability that the duration of a down pour is

i) 10 min or more.

ii) less than 10 min.

→ $$f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 \le x \le \infty \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Mean} = 1/\alpha$$

$$5 = 1/\alpha \implies \boxed{\alpha = 1/5}$$

i) $P(X>10) = 1 - P(X \le 10)$

$$= 1 - \int_0^{10} (1/5)\, e^{-x/5}\, dx$$

$$= 1 - 0.8647$$

$$= \underline{0.1353}$$

ii) $P(X<10) = \int_0^{10} \alpha e^{-\alpha x}\, dx$

$$= \int_0^{10} (1/5)\, e^{-x/5}\, dx$$

$$= \underline{0.8647}$$

④ Life of an battery is a random variable which has an exponential distribution with mean 200 hrs. Find the probability that the life of the battery is

1) less than 100 hrs

2) between 400 to 600 hrs.

$\longrightarrow$

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & 0 \le x \le \infty \\ 0 & \text{otherwise} \end{cases}$$

$$\text{mean} = 1/\alpha$$

$$200 = \frac{1}{\alpha} \Rightarrow \boxed{\alpha = 1/200}$$

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i) $P(X<100) = \int_0^{100} \alpha e^{-x} . dx$

$= \int_0^{100} (1/200) . e^{-x/200} . dx$

$= \underline{0.3935}$.

ii) $\int_{400}^{600} \alpha e^{-x} dx = P(400 < x < 600)$

$= \int_{400}^{600} (1/200) e^{-x/200} dx$

$= \underline{0.0855}$.

<u>Normal distribution (continuous random variable)</u> $\rightarrow$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The continuous probability distribution having the p.d.f (probability distribution function) $f(x)$ is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$ where

σ = deviation $\quad x$ = variate.

μ = mean

& $-\infty \le x \le \infty$, $-\infty \le \mu \le \infty$ & $\sigma > 0$

is known as Normal distribution, evidently $f(x) \ge 0$

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2}\,dx.$$

put $t = \frac{x-\mu}{\sqrt{2}\sigma}$ or $x = \mu + \sqrt{2}\sigma t$

$dx = \sqrt{2}\sigma\,dt$

$t \to -\infty$ to ∞

$$= \frac{1}{\cancel{\sigma}\sqrt{\cancel{2}\pi}} \int_{-\infty}^{\infty} e^{-t^2}\, \cancel{\sqrt{2}\sigma}\,dt$$

$$= \frac{1}{\sqrt{\pi}}\, 2 \int_{0}^{\infty} e^{-t^2}\,dt$$

$$= \frac{\cancel{2}}{\cancel{\sqrt{\pi}}} \times \frac{\cancel{\sqrt{\pi}}}{\cancel{2}} \qquad \left(\because \int_0^\infty e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2} \text{ by standard gamma function}\right)$$

$$= \underline{\underline{1}}.$$

27/9/19.

Mean & standard deviation of Normal distribution →

$$\text{Mean}(\mu) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x \cdot e^{-(x-\mu)^2}$$

put $t = \frac{x-\mu}{\sqrt{2}\sigma}$ i.e $x = \mu + \sqrt{2}\sigma t$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sqrt{2}\sigma t)\, e^{-t^2} \cdot \sqrt{2}\sigma\, dt.$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mu e^{-t^2} + \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} \cdot dt$$

$$= \frac{\mu}{\sqrt{\pi}} \cdot 2\int_{0}^{\infty} e^{-t^2} dt + \cancel{\frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_{-\infty}^{\infty} t e^{-t} dt} \to 0$$

① $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$

if $f(x)$ is an even function $f(-x) = f(x)$

② $\int_{-a}^{a} f(x)\,dx = 0$ if $f(x)$ is odd function

$f(-x) = -f(x)$

Also $\int_{0}^{\infty} e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$ (by gamma function)

$\therefore$ Mean $(\mu) = \frac{\cancel{2}\mu}{\cancel{\sqrt{\pi}}} \times \frac{\cancel{\sqrt{\pi}}}{\cancel{2}}$

$$\boxed{\text{Mean} = \mu}$$

Therefore, we say that the mean of normal distribution is equal to the mean of given distribution.

Variance $\rightarrow (\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-(x-\mu)^2/2\sigma^2} dx$$

put $t = \frac{x-\mu}{\sqrt{2}\sigma} \Rightarrow x = \mu + \sqrt{2}\sigma t$

$dx = \sqrt{2}\sigma\, dt$

$t \rightarrow -\infty$ to ∞

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sqrt{2}\sigma t - \mu)^2 e^{-t^2} \sqrt{2}\sigma\, dt$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} t^2 \sigma^2 . 2 . e^{-t^2} \sqrt{2}\sigma\, dt$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^2 (e^{-t^2} . 2)\, dt$$

$$= \frac{1}{\sqrt{\pi}} \sigma^2 \int_{-\infty}^{\infty} t\, (e^{-t^2} 2t)\, dt$$

$$= \frac{1}{\sqrt{\pi}} \sigma^2\, 2 \int_{0}^{\infty} t\, (e^{-t^2} . 2t)\, dt$$

let $u = t$ $\quad u = e^{-2t} . 2t$

wkt $\int uv\, dt = u \int v\, dt - \int\int v\, dt . u'\, dt$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left\{ \left[t(-e^{-t^2}) \right]_0^{\infty} - \int_0^{\infty} -e^{-t^2} . 1\, dt \right.$$

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$$= \frac{2\sigma^2}{\sqrt{\pi}} \left\{ 0 + \int_0^\infty e^{-t^2}\,dt \right\}$$

gamma

$$= \frac{2\sigma^2}{\sqrt{\pi}} \times \frac{\sqrt{\pi}}{2}$$

$$= \sigma^2$$

Note :

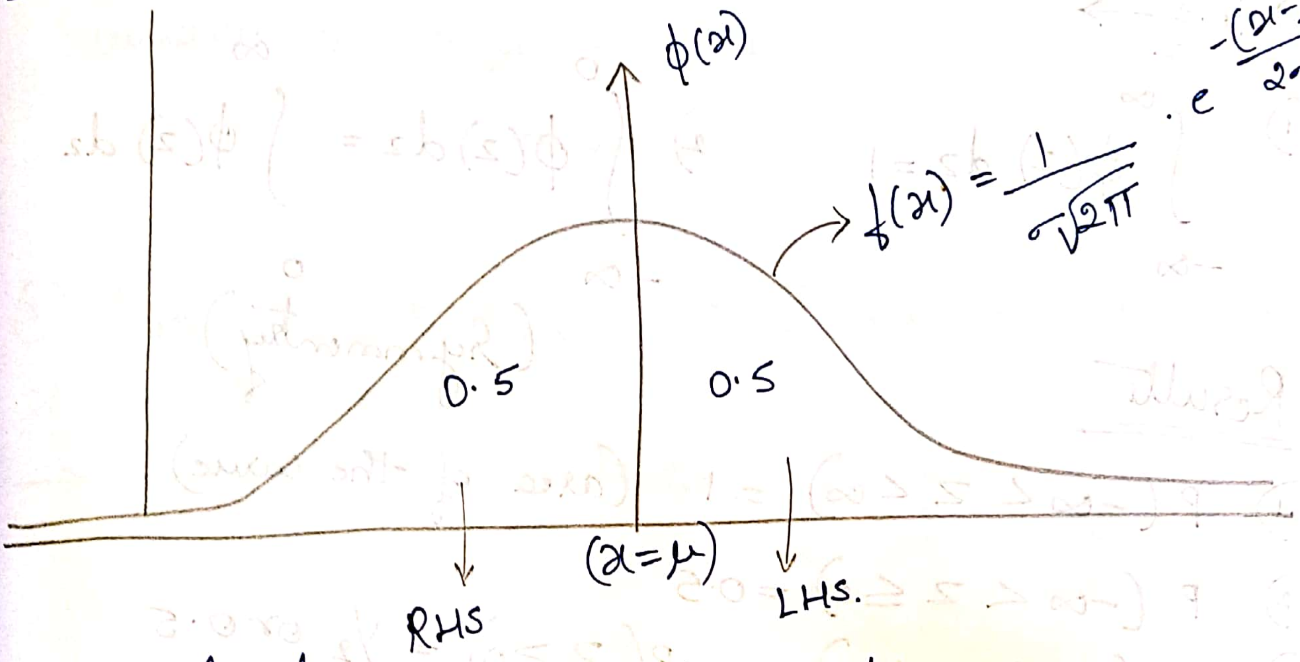
$e^{\log x}(1/x) = e^{\log x}$

$e^{-t^2}\cdot(2t) = -e^{t^2}$

$$\boxed{\text{Variance} = \sigma^2}$$

Hence variance and standard deviation of normal distribution is same.

Normal distribution curve →



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)}{2\sigma^2}}$$

Standard Normal distribution →

We have $P(a \le x \le b) = \int^b f(x)\,dx$

for normal distribution.

We have $P(a \le x \le b) = \frac{1}{\sigma\sqrt{2\pi}} \int_a^b e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$ → ①

The integral, on the RHS cannot be evaluated by known method.

We think of same standardization put

$$z = \frac{x-\mu}{\sigma} \quad \text{or} \quad x = \mu + \sigma z$$

(standard normal variate)

Let $z_1 = \frac{a-\mu}{\sigma}$ & $z_2 = \frac{b-\mu}{\sigma}$ be the values of z corresponding to $x=a$ & $x=b$.

$\therefore$ Eq ① becomes

$$P(z_1 \le z \le z_2) = \int_{z_1}^{z_2} e^{-z^2/2} \cdot dz$$

9/10/19

Normal distribution →

Note →

1) $\int_{-\infty}^{\infty} \phi(z)\, dz = 1$

2) $\int_{-\infty}^{0} \phi(z)\, dz = \int_{0}^{\infty} \phi(z)\, dz$

(Symmentry)

Results

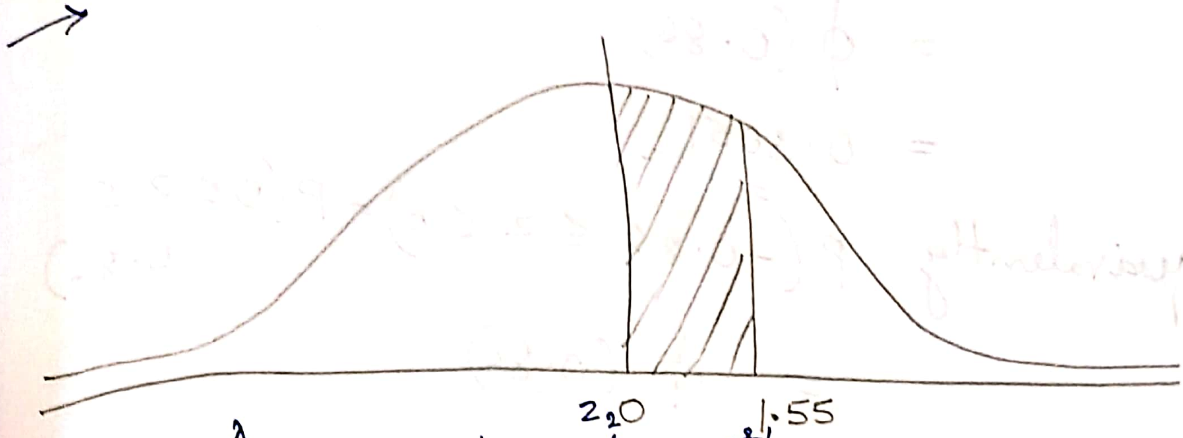
1) $P(-\infty < z < \infty) = 1$ (area of the curve)

2) $P(-\infty < z \le 0) = 0.5$

3) $P(0 \le z < \infty)$ or $P(z \ge 0) = 1/2$ or 0.5

Xamples →

① To find area under the standard normal curve between $z=0$ & 1.55



$$\text{Area} = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2} \, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{1.55} e^{-z^2/2} \, dz$$

$$= \phi(1.55)$$

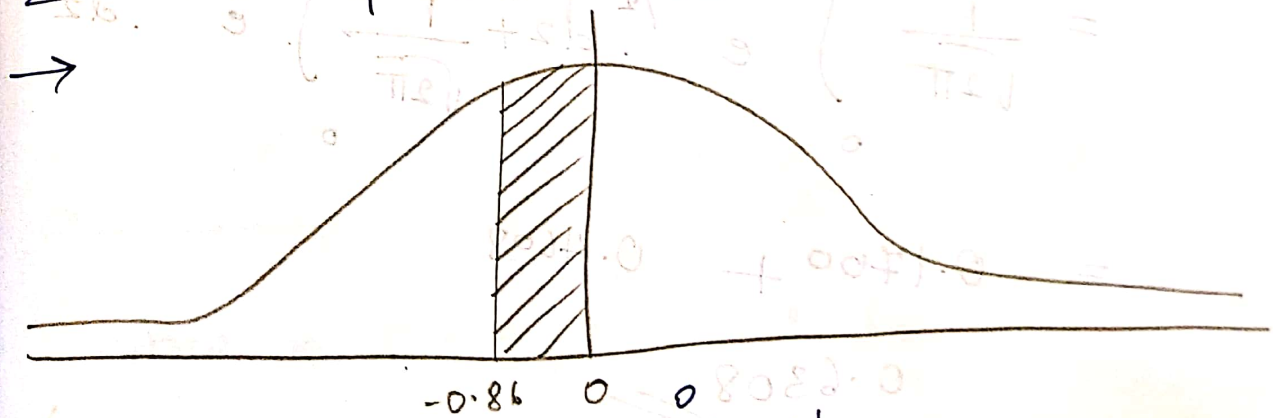
$$= \underline{\underline{0.4394}}.$$

Equivalently $P(z_1 \le z \le z_2) = P(0 \le z \le 1.55)$

$$= \phi(1.55)$$

$$= \underline{\underline{0.4394}}.$$

(2) Area under the (curve) standard normal curve $z = -0.86$ & $z = 0$



$$\text{Area} = \frac{1}{\sqrt{2\pi}} \int_{-0.86}^{0} e^{-z^2/2} \, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{0.86} e^{-z^2/2} \, dz$$

$$= \phi(0.86)$$

$$= 0.3051$$

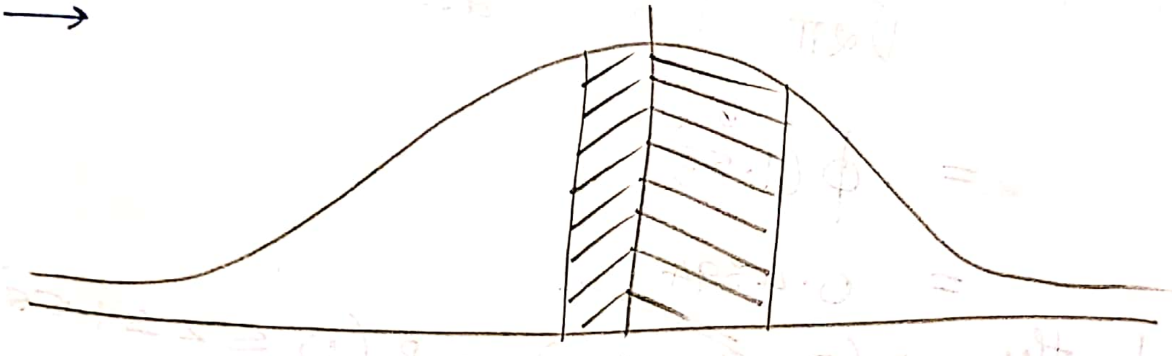
Equivalently $P(-0.86 \le z \le 0) = P(0 \le z \le 0.86)$

$$= \phi(0.86)$$

$$= 0.3051$$

③ Area under the curve $z = -0.44$ & $z = 1.76$

$\rightarrow$



$$\text{Area} = \frac{1}{\sqrt{2\pi}} \int_{-0.44}^{0} e^{-z^2/2} \, dz + \frac{1}{\sqrt{2\pi}} \int_{0}^{1.76} e^{-z^2/2} \, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{0.44} e^{-z^2/2} \, dz + \frac{1}{\sqrt{2\pi}} \int_{0}^{1.76} e^{-z^2/2} \, dz$$

$$= 0.1700 + 0.4608$$

$$= 0.6308$$

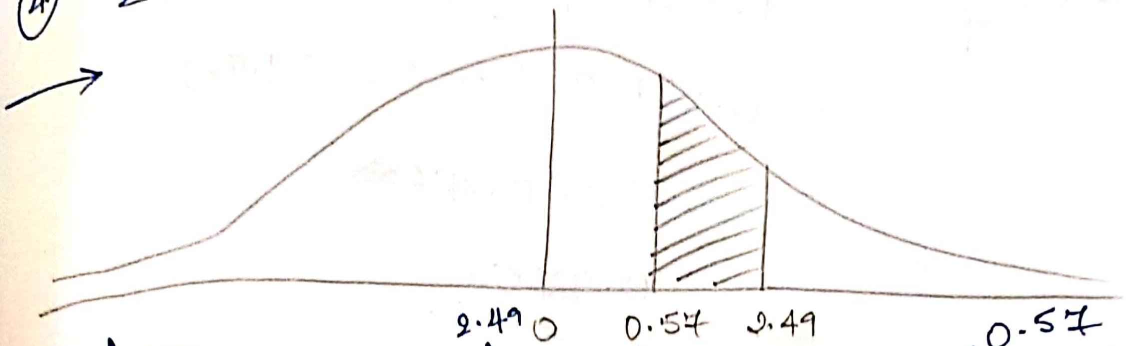
Equivalently $\rightarrow$ $P(-0.44 \le z \le 0) + P(0 \le z \le 1.76)$

$$= P(0 \le z \le 0.44) + P(0 \le z \le 1.76)$$

$$= 0.1700 + 0.4608$$

$$= 0.6308$$

(4) $z = 0.57$ to $z = 2.49$



$$\text{Area} = \frac{1}{\sqrt{2\pi}}\int_0^{2.49} e^{-z^2/2}\,dz - \frac{1}{\sqrt{2\pi}}\int_0^{0.57} e^{-z^2/2}\,dz$$

$= 0.4936 - 0.2156$

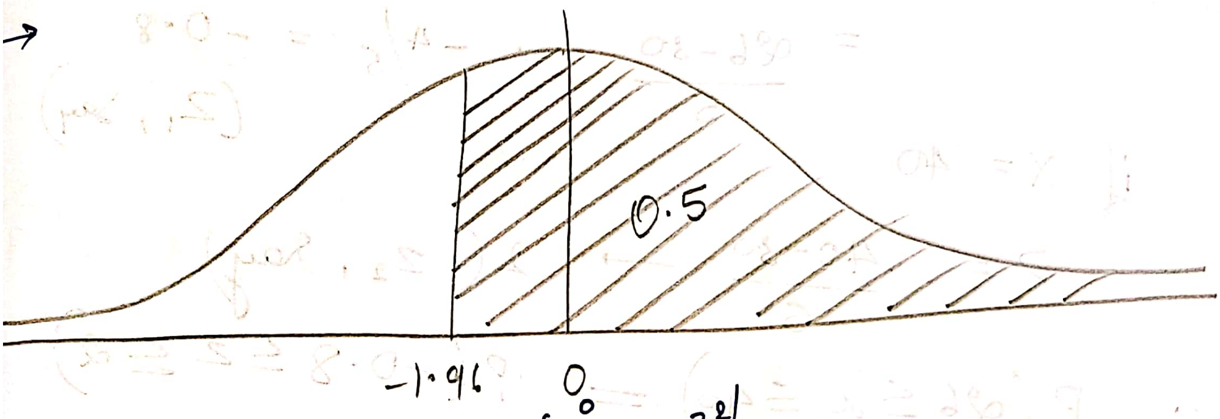
$= \underline{0.2780}$.

Equivalently → $P(0 \le z \le 2.49) - P(0 \le z \le 0.57)$

$= 0.4936 - 0.2156$

$= \underline{0.2780}$.

(5) $z = -1.96$ & $z =$ extrem right.



$$\text{Area} = 0.5 + \frac{1}{\sqrt{2\pi}}\int_{-1.96}^{0} e^{-z^2/2}\,dz$$

$$= 0.5 + \frac{1}{\sqrt{2\pi}}\int_0^{1.96} e^{-z^2/2}\,dz$$

$= 0.5 + 0.4750 \rightarrow \underline{0.9750}$.

Equivalently $\rightarrow$ $0.5 + P(-1.96 \le z \le 0)$

$= 0.5 + P(0 \le z \le 1.96)$

$= 0.5 + 0.4750$

$= \underline{\underline{0.9750}}$.

<u>Xample</u> $\longrightarrow$

① X is a normal variate with mean 30 and S.D has 5, find the probability that

1) $26 \le x \le 40$

2) $x \ge 45$

3) $|X - 30| > 5$

$\rightarrow$ 1) Given, $P(26 \le x \le 40)$

if $X = 26$

wkt $z = \dfrac{X - \mu}{\sigma}$

$= \dfrac{26 - 30}{5} \rightarrow -4/5 = -0.8$

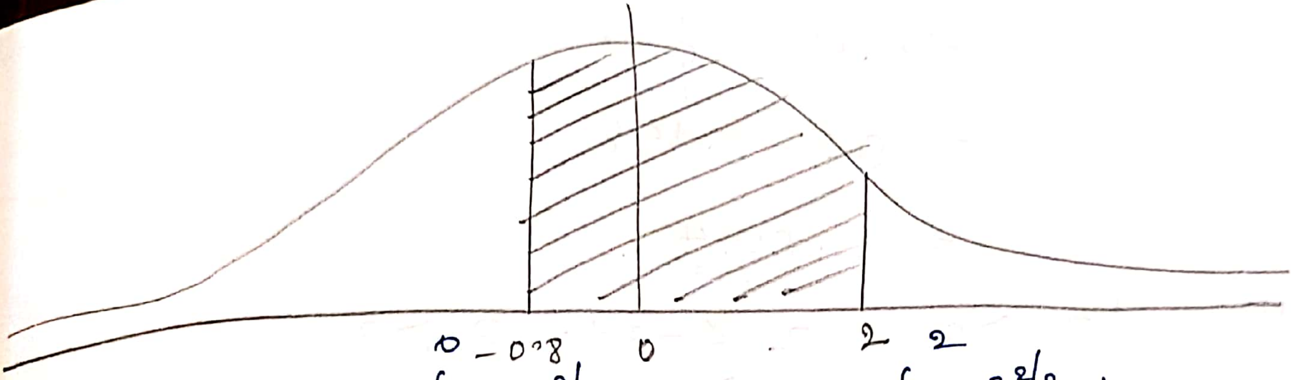
$(z_1, \text{ say})$

if $X = 40$

$z = \dfrac{40 - 30}{5} \longrightarrow 2\ (z_2, \text{ say})$

$\therefore P(26 \le x \le 40) = P(-0.8 \le z \le 2)$

$P(-0.8 \le z \le 2)$ standardizing



$$\text{Area} = \frac{1}{\sqrt{2\pi}} \int_{-0.8}^{0} e^{-z^2/2}\,.dz + \frac{1}{\sqrt{2\pi}} \int_{0}^{2} e^{-z^2/2}\,.dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{0.8} e^{-z^2/2}\,.dz + \frac{1}{\sqrt{2\pi}} \int_{0}^{2} e^{-z^2/2}\,.dz$$

$$= \phi(0.8) + \phi(2)$$

$$= 0.2881 + 0.4772$$

$$= \underline{0.7653}.$$

ii) if $X = 45$

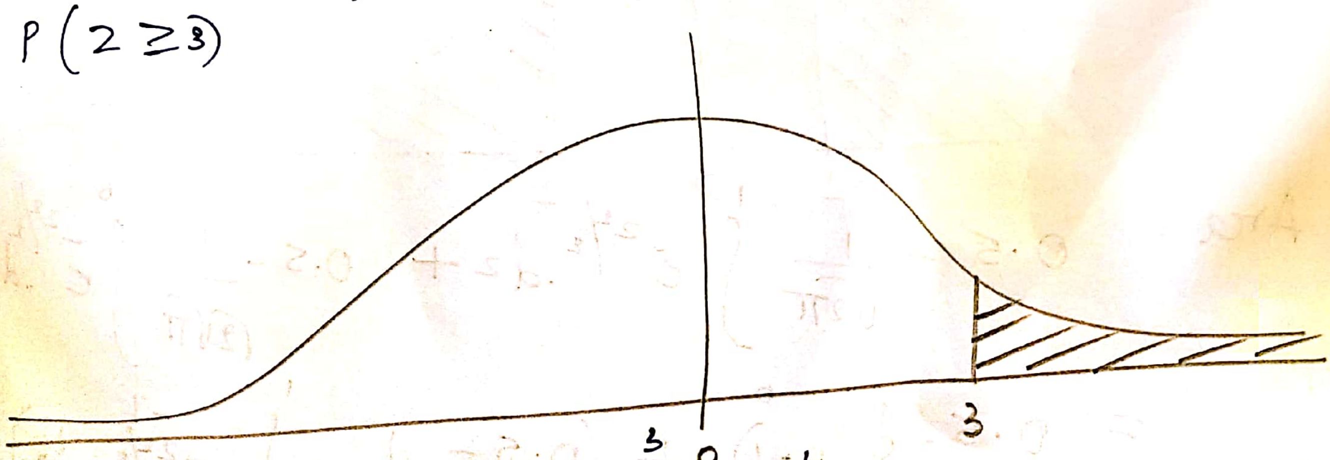
wkt $z = \dfrac{X-\mu}{\sigma}$

$$z = \frac{45-30}{5} = 3$$

$$P(X \geq 45) = P(z \geq 3)$$

standardizing

$P(z \geq 3)$



$$\text{Area} = 0.5 - \frac{1}{\sqrt{2}\sqrt{\pi}} \int_{0}^{3} e^{-z^2/2}\,.dz$$

$$= 0.5 - \phi(3)$$

$$= 0.5 - 0.4986$$

$$= \underline{\underline{0.0014}}.$$

iii) $|z| > a = z < -a + z > a$

iii^{ly} $p(|X-30| > 5) =$

$$= X-30 < -5 + X-30 > 5$$

$$= (X < -5+30) + (X > 5+30)$$

$$= (X < 25) + (X > 35)$$

if $X = 25$

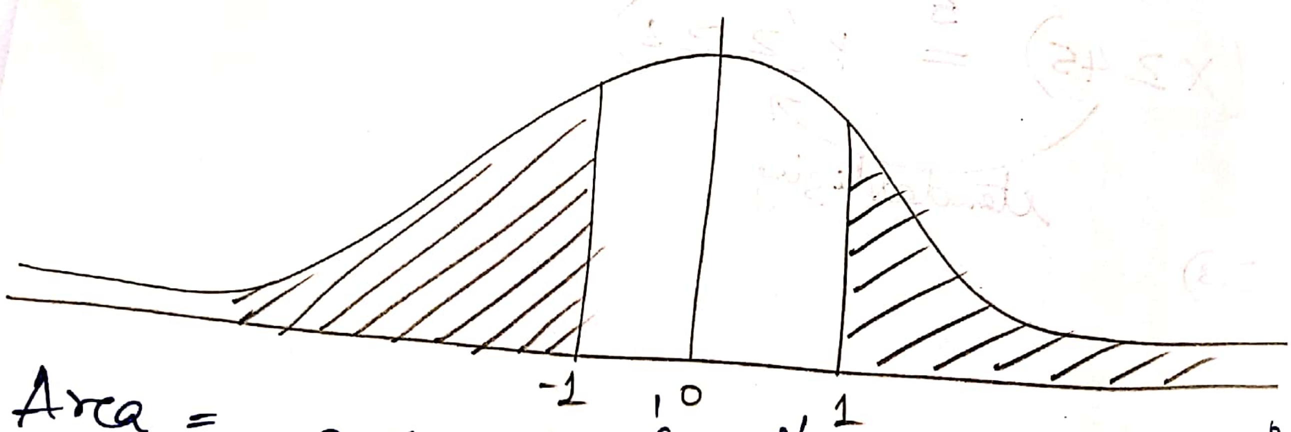
$$z = \frac{25-30}{5} = -1$$

if $X = 35$

$$z = \frac{35-30}{5} = 1$$

$$P(X < 25) + P(X > 35) = P(z < -1) + P(z > 1)$$

$P(z < -1) + P(z > 1)$



$$\text{Area} = 0.5 - \frac{1}{\sqrt{2\pi}} \int_{0}^{1} e^{-z^2/2}.dz + 0.5 - \frac{1}{2\sqrt{\pi}} \int_{0} e^{-z^2/2}$$

$$= 0.5 - \phi(0.1) + \left(0.5 - \frac{1}{\sqrt{2\pi}} \int_{0}^{1} e^{-z^2/2}.dz\right)$$

$= 0.5 - \phi(1) + (0.5 - \phi(1))$

$= 0.5 - 0.3413 + (0.5 - 0.3413)$

$= 0.1587 + 0.1587$

$= 0.3174$.

Equivalently, $\rightarrow$ $P(-1 \leq z \leq \infty) + P(1 \leq z < \infty)$

$= P(\infty \leq z \leq 1) + P(1 \leq z \leq \infty)$

$= 0.1587 + 0.1587$

$= 0.3174$.

10/10/19

② In a test on 2000 electric bulb, it was found that the life of a particular make, was normally distributed with an average life of 2040 hrs & S.D of 60 hrs. Estimate the number of bulbs, likely to burn for.

1) more than 2150 hrs.
2) less than 1950 hrs.
3) more than 1920 hrs & but less than 2160 hrs.

$\rightarrow$ given $n = 2000$ hrs

mean, $\mu = 2040$ hrs

$S.D = \sigma = 60$ hrs.

1) $P(2150 < X)$

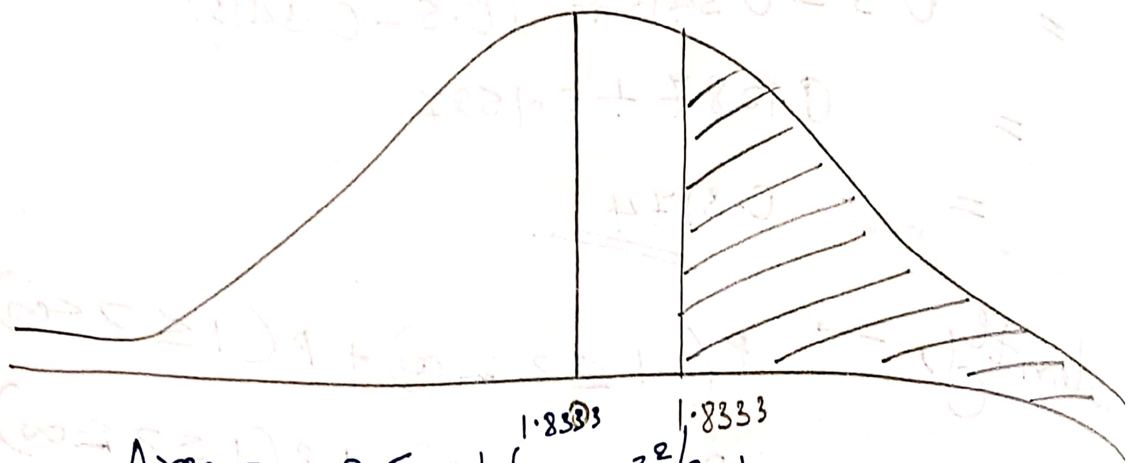
if $X = 2150$

$z = \frac{X - \mu}{\sigma}$

$= \frac{2150 - 2040}{60}$

$= 1.8333$

$P(2150 < X) = P(1.8333 < Z)$



$$\text{Area} = 0.5 - \frac{1}{\sqrt{2\pi}}\int_0^{1.8333} e^{-z^2/2}\,dz$$

$= 0.5 - \phi(1.8333)$

$= 0.5 - 0.4666$

$= 0.0334$

Equivalently $\rightarrow P(0 \le Z < \infty) - P(0 \le Z \le 1.8333)$

$= 0.5 - 0.4666$

$= 0.0334$

$\therefore$ The estimate number of bulbs to burn for more than 2150 hrs is

$= 2000 \times 0.0334 \rightarrow 66.8 \approx 67$

$= 67$

ii) $P(X < 1950)$

if $X = 1950$

$$Z = \frac{X - \mu}{\sigma} \rightarrow \frac{1950 - 2040}{60}$$

$= -1.5$

$P(X < 1950) = P(Z < -1.5)$

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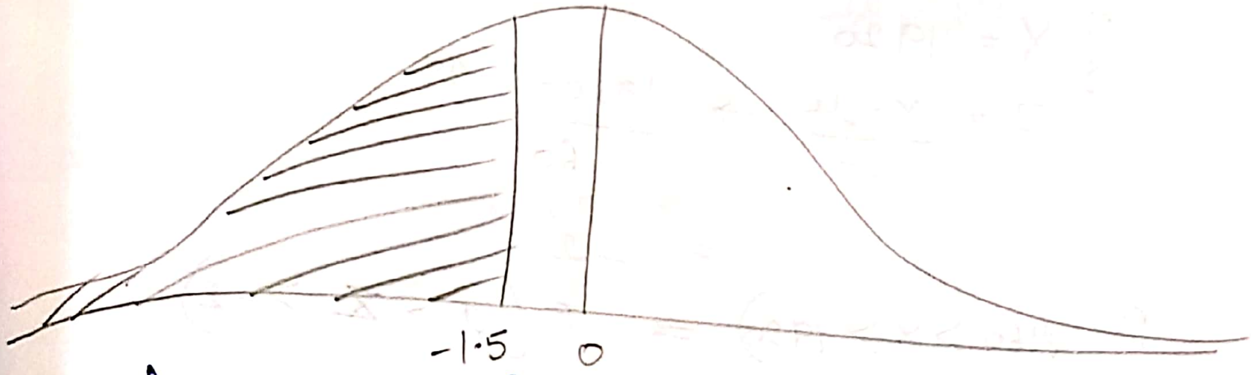


$$\text{Area} = \frac{1}{\sqrt{2\pi}} \int_{-1.5}^{0} e^{-z^2/2} \, dz + 0.5$$

$$= 0.5 - \frac{1}{\sqrt{2\pi}} \int_{0}^{1.5} e^{-z^2/2} \, dz$$

$$= 0.5 - \phi(1.5)$$

$$= 0.5 - 0.4332$$

$$= 0.0668$$

Equivalently $\rightarrow$ $P(-\infty < z < 0) - P(-1.5 \leq z \leq 0)$

$$= 0.5 - 0.4332$$

$$= 0.0688$$

$\therefore$ The estimate number of bulbs to burn for less than 1950 hrs is =

$$2000 \times 0.0668$$

$$= 133.6$$

$$\approx$$

$$134$$

(iii) $P(2160 > X > 1920)$

$\rightarrow$ if $X = 2160$

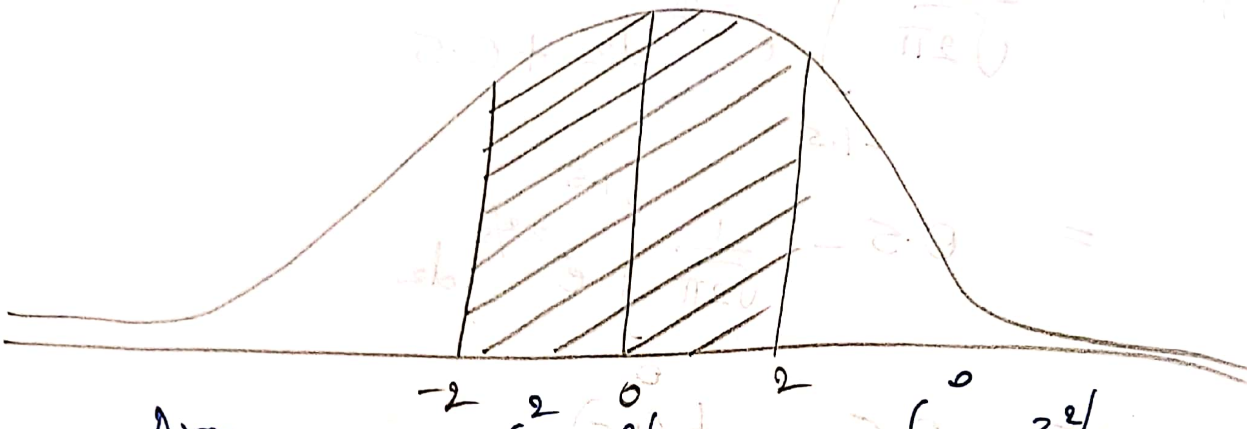
$z = \frac{X - \mu}{\sigma} \rightarrow \frac{2160 - 2040}{60} = 2$

if $X = 1920$

$$Z = \frac{X-\mu}{\sigma} \rightarrow \frac{1920-2040}{60}$$

$$= -2$$

$$P(2160 > X > 1920) = P(2 > Z > -2)$$



$$\text{Area} = \frac{1}{\sqrt{2\pi}} \int_{-2}^{0} e^{-z^2/2} . dz + \frac{1}{\sqrt{2\pi}} \int_{0}^{2} e^{-z^2/2} . dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{2} e^{-z^2/2} . dz + \frac{1}{\sqrt{2\pi}} \int_{0}^{2} e^{-z^2/2} . dz$$

$$= \phi(2) + \phi(2)$$

$$= 2\phi(2)$$

$$= 2(0.4772)$$

$$= \underline{\underline{0.9544}}$$

Equivalently $\rightarrow$ $P(-2 \leq z \leq 0) + P(0 \leq z \leq 2)$

$$= 0.4772 + 0.4772$$

$$= \underline{\underline{0.9544}}$$

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∴ The estimate number of bulbs, likely to burn for more than 1920 hrs & less than 2160 hrs is

$$2000 \times 0.9544$$
$$= 1908.8$$
$$\approx$$
$$\underline{\underline{1909}}.$$

(3) The mean height of 500 students is 151 cm & SD is 15 cm. Assuming that the height are normally distributed, find how many students height lie between 120 & 155 cm.

→ $P(155 \geq X \geq 120)$

given, $\mu = 151$
$\sigma = 15$
$n = 500$

if $X = 155$

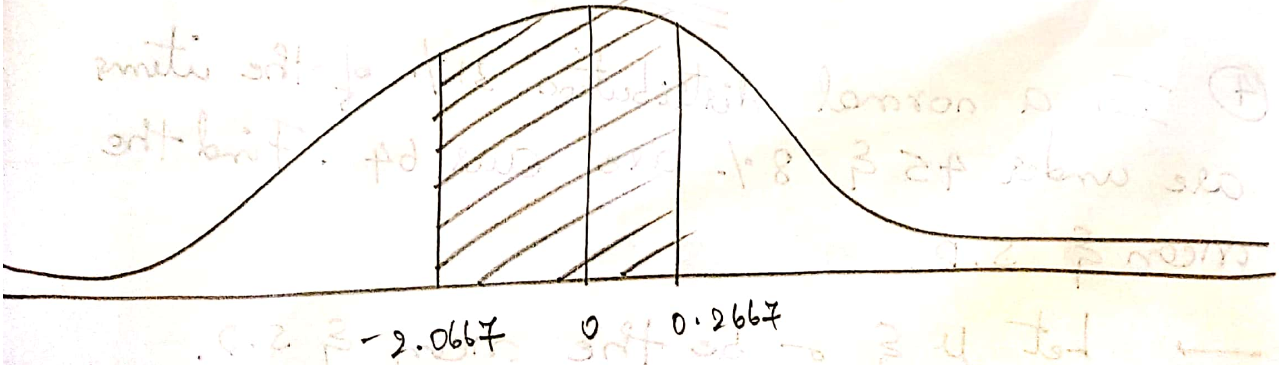
$$z = \frac{X-\mu}{\sigma}$$

$$= \frac{155-151}{15} = 0.2667$$

if $X = 120$

$$z = \frac{X-\mu}{\sigma} \rightarrow \frac{120-151}{15} = -2.0667$$

∴ $P(155 \geq X \geq 120) = P(0.2667 \geq z \geq -2.0667)$



$$\text{Area} = \frac{1}{\sqrt{2\pi}} \int_0^{0.2667} e^{-z^2/2} \cdot dz + \frac{1}{\sqrt{2\pi}} \int_{-2.0667}^{0} e^{-z^2/2} \cdot dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{0.2667} e^{-z^2/2} \cdot dz + \frac{1}{\sqrt{2\pi}} \int_0^{+2.0667} e^{-z^2/2} \cdot dz$$

$$= \phi(0.2667) + \phi(2.0667)$$

$$= 0.1051 + 0.4806$$

$$= \underline{\underline{0.5857}}$$

Equivalently → $P(-2.0667 < z < 0) + P(0 < z < 0.2667)$

$= 0.1051 + 0.4806$

$= \underline{\underline{0.5857}}$

∴ The number of students height that lie between 120cm to 155cm is

500×0.5857

$= 292.8$

$\approx$

$\underline{\underline{293}}$

(4) In a normal distribution 31% of the items are under 45 & 8% are over 64. Find the mean & S.D.

→ Let μ & σ be the mean & S.D.

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By data

1. $P(X<45) = 31\%$

wkt $z = \dfrac{X-\mu}{\sigma} \longrightarrow z = \dfrac{45-\mu}{\sigma}$ (z_1, say)

$P(X<45) = 0.31$

$P(z < z_1) = 0.31.$

2. $P(64<X) = 8\%$

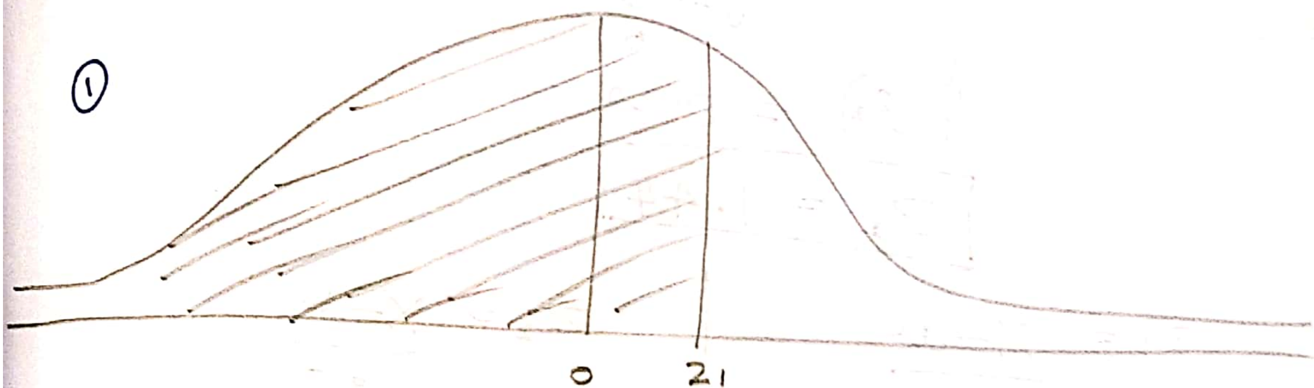
$P(64<X) = 0.08$

wkt $z = \dfrac{X-\mu}{\sigma} \longrightarrow z = \dfrac{64-\mu}{\sigma}$ (z_2, say)

$P(64<X) = 0.8$

$P(z_2 < z) = 0.8.$

1.

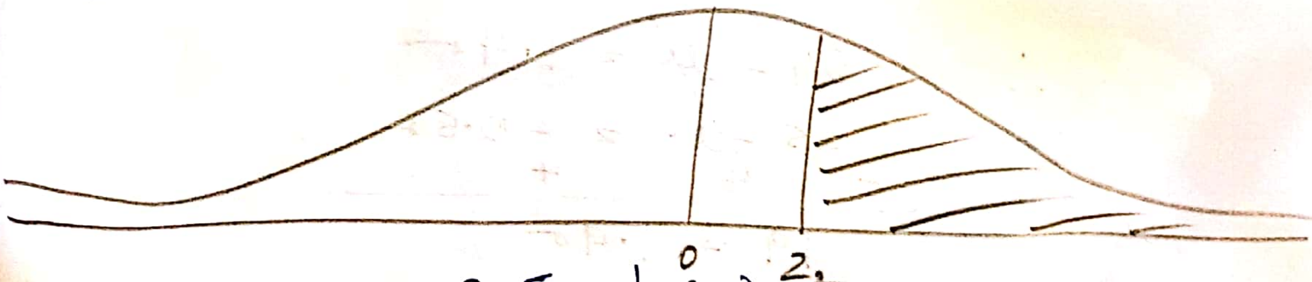


$0.5 + \phi(z_1) = 0.31$

$\phi(z_1) = 0.31 - 0.5$

$\boxed{\phi(z_1) = -0.19.}$

2.



$0.5 - \phi(z_2) = 0.08$

$\phi(z_2) = 0.5 - 0.08$

$\boxed{\phi(z_2) = 0.42}$

Reffering to normal probability table we have

$$\phi(z_1) = -\phi(0.5) \quad (\text{from log. ta}$$

$$= -\frac{1}{\sqrt{2\pi}}\int_0^{0.5} e^{-z^2/2}\,dz$$

$$= -0.19$$

$$\boxed{z_1 = -0.5}$$

$$\phi(z_2) = \phi(1.44)$$

$$= \frac{1}{\sqrt{2\pi}}\int_0^{1.44} e^{-z^2/2}\,dz$$

$$\phi(z_2) = 0.42$$

$$\boxed{z_2 = 1.44}$$

$$z_1 = \frac{X-\mu}{\sigma} \qquad z_2 = \frac{X-\mu}{\sigma}$$

$$z_1 = \frac{45-\mu}{\sigma} \qquad z_2 = \frac{64-\mu}{\sigma}$$

$$-0.5\sigma = 45-\mu \qquad 1.4\sigma = 64-\mu$$

$\therefore$

$$64 - \cancel{\mu} = 1.4\sigma$$
$$-\;45 - \cancel{\mu} = -0.5\sigma$$
$$19 = 1.9\sigma$$

$$\Rightarrow \sigma = \frac{19}{1.9}$$

$$\boxed{\sigma = 10}$$

$$1.4 = \frac{64-\mu}{10}$$

$$14 = 64-\mu$$

$$\Rightarrow \mu = 64-14$$

$$\boxed{\mu = 50}$$

(5) In a normal distribution 7% of the item are under 35 & 89% are under 63 find the mean & S.D

→ Let μ & σ be the mean and S.D

By data

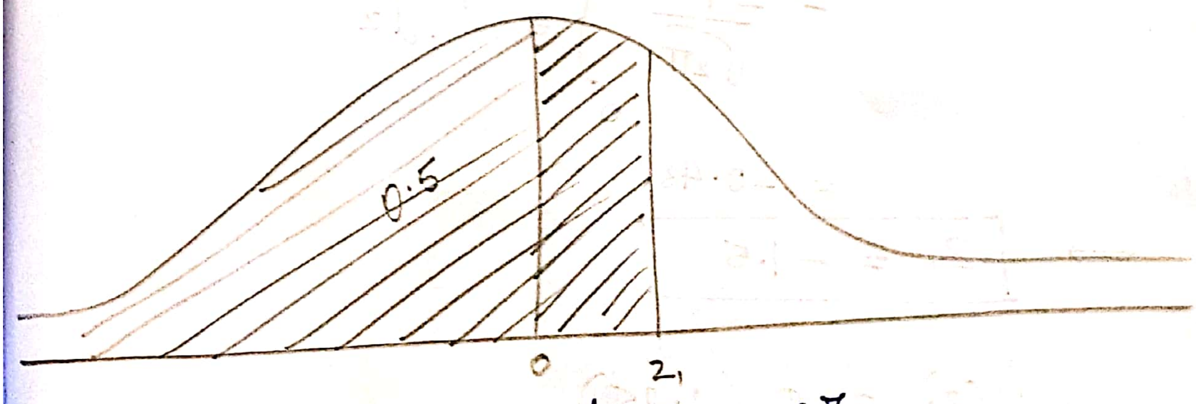
$$P(X<35) = 7\%$$

$$P(X<35) = 0.07$$

$$z = \frac{X-\mu}{\sigma}$$

$$z = \frac{35-\mu}{\sigma} \quad (z_1, \text{ say})$$

$$P(X<35) = 0.07, \; P(z<z_1) = 0.07$$



$$0.5 + \phi(z_1) = 0.07$$

$$\phi(z_1) = 0.07 - 0.5$$

$$\boxed{\phi(z_1) = -0.43}$$

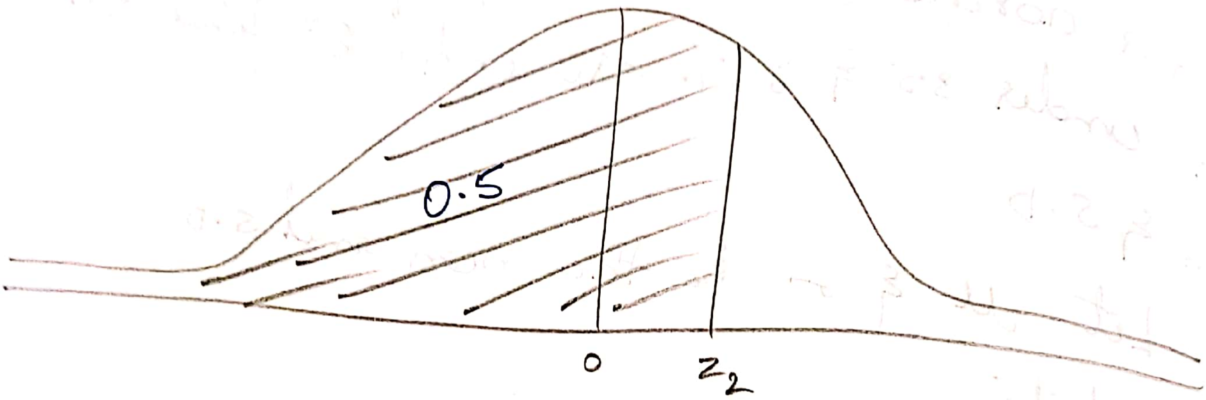
$$P(X < 63) = 89\%$$

$$P(X < 63) = 0.89$$

$$z = \frac{X-\mu}{\sigma}$$

$$z = \frac{63-\mu}{\sigma} \quad (z_2, \text{say})$$

$$P(X<63) = 0.89 \Rightarrow P(z < z_2) = 0.89$$



$$0.5 + \phi(z_2) = 0.89$$

$$\phi(z_2) = 0.89 - 0.5$$

$$\boxed{\phi(z_2) = 0.39}$$

Reffering to normal probability table we have

$$\phi(z_1) = -\phi(1.5)$$

$$= -\frac{1}{\sqrt{2\pi}} \int_0 e^{-z^2/2}\, dz$$

$$= -0.43$$

$$\Rightarrow \boxed{z_1 = -1.5}$$

$$\phi(z_2) = \phi(1.25)$$

$$= +\frac{1}{\sqrt{2\pi}} \int_0 e^{-z^2/2}\, dz$$

$$= 0.39$$

$$\boxed{z_2 = 1.25}$$

$$z_1 = \frac{x-\mu}{\sigma}$$

$$z_1 = \frac{35-\mu}{\sigma}$$

$$-1.5\sigma = 35-\mu \longrightarrow (1)$$

$$z_2 = \frac{x-\mu}{\sigma}$$

$$z_2 = \frac{63-\mu}{\sigma}$$

$$1.25\sigma = 63-\mu \longrightarrow (2)$$

$\therefore$

$$\left.\begin{aligned} 63-\cancel{\mu} &= 1.25\sigma \\ 35-\cancel{\mu} &= -1.5\sigma \\ - \quad + \quad & \quad\;\; + \end{aligned}\right\} \text{from (1) \& (2)}$$

$$28 = 2.75\sigma$$

$$\Rightarrow \sigma = \frac{28}{2.75}$$

$$\boxed{\sigma = 10.18}$$

In (2),

$$1.25(10.18) = 63-\mu$$

$$12.7250 = 63-\mu$$

$$\mu = 63 - 12.7250$$

$$\boxed{\mu = 50.27}$$

———— x ————

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