Sub Code: 18MATCS41

Sub name:- Graph Theory & Discrete Mathematical Structures

Unit 1 Fundamentals of Logic:

Laws of Logic, Logical implications, Quantifiers, Proof Techniques.

Unit 2 Relations & Functions:

Matrices & directed graphs, Closure & Equivalence relⁿs. Partitions, Partial Orders, Hasse diagrams, Lattices, Prop. of Functions, Composition & invertible fⁿs.

Unit 3 Advanced Counting Techniques:

Sterling no., Pigeonhole principle, Recurrance relⁿs, Solⁿ of linear & non-linear recurrance relⁿs, Merge sort algo.

Unit-4 Basic Graph Theory.

Sub graphs, complement & isomorphism of graphs, Connectivity, Planar graphs, Hamiltonian paths & cycles, Coloring, Matching.

Unit 5 Elementary no. theory & Cryptography.

Fields, Modular Arithmatic, Prime nos, Fermat's & Euler's theorem, Testing of Primality, Chinese Remainder theorem, Encryption & Decryption, Chinese Remainder theorem, Encryption & Decryption, RSA Cryptosystem.

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Unit - 01

Basics of Logic

Sentence $\neq$ Statement

$\longleftrightarrow$

All the sentences need not be statement.

A sentence which can be said as either true or false but not both. is called a Statement / Proposition

* What is your name? — Sentence
* Delhi is capital of Bangalore. — false proposition
* $2+2=4$ — true proposition
* x is an odd no. — sentence

Note: Propositions are always represented by small letters.
ex:- p, q, r, s, t -----

* Truth value:- The truthness or falsity of a proposition is called the Truth value.

If the proposition is true then its truth value is represented by '1' or 'T'. Similarly if it is false then its truth value is represented by '0' or 'F'.

* Logical Connectives:- The propositions are connected or modified using 'Logical Connectives' to obtain the new propositions.

Ex:- 'And', 'Or', 'if - then', 'if and only if' etc....

The new propositions, thus obtained, are called 'Compound propositions'. The original propositions are called 'Component propositions' or 'Primitives'.

The proposition which do not contain any connective

'Component propositions' or 'Primitives'.

The proposition which do not contain any connective is called 'Simple proposition.'

Ex:- 1) 30 is divisible by 2, 5, 3 and 10.

p : 30 is ---- by 2 and 5

q : ——— " — 3 and 10

r : ——— " ——— 2

s : ——— " ——— 3

t : ——— " ——— 5

m : ——— " ——— 10

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1) Negation:- A proposition obtained by inserting the word 'not' in an appropriate place in the given proposition is called 'Negation'.

If 'p' is a given proposition then '$\sim p$' or 'not p' is its negation.

Ex:- p : 2 is a prime no.

$\sim p$: 2 is not a prime no.

Truth table:

p	$\sim p$
0	1
1	0

2) Conjunction:- A compound proposition obtained by inserting the connective 'and' between two propositions is called conjunction.

If 'p' & 'q' are two propositions then '$p \wedge q$' 'p and q' is the conjunction.

Ex:- p : 2 is a prime no.

q : 100 is divisible by 10.

Ex:- p: 2 is a prime no.

q: 100 is divisible by 10.

$p \wedge q$: 2 is a prime no. and 100 is divisible by 10.

Note:- Conjunction will be true only when all the components are true.

Truth table:-

p	q	$p \wedge q$
0	0	0
1	0	0
0	1	0
1	1	1

3) **Disjunction (Inclusive):** A compound proposition obtained by combining two propositions by the connective 'or' is called disjunction.

Note:- The disjunction is denoted by 'p or q'

(inclusive) $p \vee q$

(exclusive) $p \underline{\vee} q$

Truth table:-

1) Inclusive

p	q	$p \vee q$
0	0	0
1	0	1
0	1	1
1	1	1

Ex:- p: 4 is a prime no.

q: $\sqrt{2}$ is an irrational no.

$p \vee q$: 4 is a prime no. or $\sqrt{2}$ is an irrational no.

2) Exclusive

p	q	$p \underline{\vee} q$
0	0	0
1	0	1
0	1	1
1	1	0

Ex:- p: 2 divides 5

q: 6 is divisible by 2

$p \underline{\vee} q$: 2 either divides 5 or 6.

4) **Implication:** The compound proposition obtained by joining two propositions with 'if' & 'then', is called —

4> **Implication:** The compound proposition ... two propositions with 'if' & 'then', is called — implication/conditional.

Notation:- '$p \to q$' or 'If p then q'.

Ex:- p: 16 is divisible by 4.

q: 16 is an even no.

$p \to q$: If 16 is divisible by 4 then 16 is an even no.

or

16 is divisible by 4 implies it is an even no.

Note:- The implication '$p \to q$' is false only when p is true & q is false.

Truth table:-

p	q	$p \to q$
0	0	1
1	0	0
0	1	1
1	1	1

5> **Biconditional:-** The compound proposition obtained by joining two conditionals $p \to q$ & $q \to p$ as 'If $p \to q$ then $q \to p$' is called Biconditional. Biconditional is also written as 'p if and only if q'.

Note:- i> $p \leftrightarrow q$. in notations.

Ex:- $\underbrace{\text{4 is even}}_{p}$ iff $\underbrace{\text{it is divisible by 2}}_{q}$.

ii> The biconditional is true only when both p & q are true or both false.

Ex:- $\underbrace{\text{4 is odd}}_{p}$ iff $\underbrace{\text{it is not divisible by 2}}_{q}$

F F T

iii) Biconditional is true if both $P \to q$ & $q \to P$ are true.

Truth table:-

P	q	$P \to q$	$q \to P$	$P \leftrightarrow q$
0	0	1	1	1
1	0	0	1	0
0	1	1	0	0
1	1	1	1	1

Ex:- Let P: A circle is a conic

q: $\sqrt{5}$ is a real no.

r: Exponential series is convergent.

Express the following compound propositions in words.

i) $P \wedge (\sim q)$ ii) $(\sim P) \vee q$ iii) $P \underline{\vee} (\sim q)$

iv) $q \to (\sim P)$ v) $P \to (q \underline{\vee} r)$ vi) $\sim P \leftrightarrow q$

sol^n $\rightarrow$

i) $P \wedge (\sim q)$: A circle is a conic and $\sqrt{5}$ is not a real no.

iii) $P \underline{\vee} (\sim q)$: Either a circle is a conic or $\sqrt{5}$ is not a real no.

iv) $q \to \sim P$: If $\sqrt{5}$ is a real no. then a circle is not a conic.

v) $P \to (q \underline{\vee} r)$: If a circle is a conic then either $\sqrt{5}$ is a real no. or exponential series is convergent.

vi) $\sim P \leftrightarrow q$: A circle is not a conic iff $\sqrt{5}$ is a real no.

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* State which of the following are propositions.

i) A triangle contains three sides — true proposition.

ii) $x+2$ is a positive no. — not a proposition

iii) 5 divides x — not a proposition.

iv) Is $\sqrt{2}$ a rational no.? — not a proposition.

v) 21 is an even no. — false proposition

v) 21 is an even no. – false proposition

vi) India had a woman prime minister. – true proposition.

Tautology:– A compound statement which is true for all possible truth values of its components is called a Tautology. It is also called as Logical truth or a Universally valid statement.

Contradiction:– A compound proposition which is false for all possible truth values of its components is called a Contradiction. or Absurdity.

Note:– A compound proposition which is neither a Tautology nor a contradiction is called contingency.

Examples:–

1) S.T. for any proposition 'p', the compound proposition $p \vee \sim p$ is a Tautology and '$p \wedge \sim p$' is a contradiction.

sol^n →

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
0	1	1 ✓	0 ✓
1	0	1 ✓	0 ✓

$\therefore$ $p \vee \sim p$ is Tautology & $p \wedge \sim p$ is contradiction.

2) S.T. for any two propositions p & q

i) $(p \underline{\vee} q) \vee (p \leftrightarrow q)$ is a tautology

ii) $(p \underline{\vee} q) \wedge (p \leftrightarrow q)$ is a contradiction.

iii) $(p \underline{\vee} q) \wedge (p \rightarrow q)$ is a contingency.

→

p	q	$p \underline{\vee} q$	$p \leftrightarrow q$	$(p \underline{\vee} q) \vee (p \leftrightarrow q)$ Tautology	$(p \underline{\vee} q) \wedge (p \leftrightarrow q)$ Contradiction
0	0	0	1	1	0

→

p	q	$p \veebar q$	$p \leftrightarrow q$	$(p \veebar q) \vee (p \leftrightarrow q)$	$(p \veebar q) \wedge (p \leftrightarrow q)$
0	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0
1	1	0	1	1	0

$\therefore$ from the truth table $(p \veebar q) \vee (p \leftrightarrow q)$ is a Tautology

and $(p \veebar q) \wedge (p \leftrightarrow q)$ is a contradiction.

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3) Determine whether the following are tautologies.

i) $p \vee (\sim(p \wedge q))$ ii) $(p \vee q) \vee \sim p$ iii) $p \rightarrow (p \wedge q)$

iv) $p \rightarrow (p \vee q)$ v) $\{\sim(p \Rightarrow q)\} \rightarrow (\sim q)$.

Logical Equivalence and laws of Logic :-

Two propositions p & q are said to be logically equivalent if both p & q have the same truth value.

or

p & q are said to be logically equivalent if $p \leftrightarrow q$ is a Tautology.

Note:- i) If p & q are logically equivalent, then its written as $p \Leftrightarrow q$.

Ex:- S.T. for any two propositions p & q, $(p \rightarrow q) \Leftrightarrow (\sim p) \vee q$.

→

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
0	0	1	1	1
1	0	0	0	0
0	1	1	1	1
1	1	0	1	1

p	q	$\rightarrow$
1	0	0

$\therefore\ (p \to q) \iff (\sim p) \vee q$

HW

(i) $[p \to (\sim q)] \iff [q \to (\sim p)]$

(ii) $\sim(p \to \sim q) \iff (p \wedge q)$

(iii) $\sim(p \Leftrightarrow q) \iff (p \vee q) \wedge (\sim(p \wedge q))$.

Laws of Logic:

For any propositions p, q & r

1> Law of double negation: $\sim(\sim p) \iff p$

2> Idempotent laws: $-\ (p \vee p) \iff p \qquad -\ (p \wedge p) \iff p$

3> Identity laws: if T_0 is a tautology & F_0 is a contradiction then,

$$(p \vee F_0) \iff p \qquad (p \wedge T_0) \iff p$$

4> Domination law:

$$(p \vee T_0) \iff T_0 \qquad (p \wedge F_0) \iff F_0$$

5> Inverse law: $(p \vee \sim p) \iff T_0 \qquad (p \wedge \sim p) \iff F_0$

6> Commutative law: $(p \vee q) \iff (q \vee p)\ ,\ (p \wedge q) \iff (q \wedge p)$

7> Absorption law: $[p \vee (p \wedge q)] \iff p\ ,\ [p \wedge (p \vee q)] \iff p$

8> DeMorgan's law: $\sim(p \vee q) \iff (\sim p) \wedge (\sim q)$

$$\sim(p \wedge q) \iff (\sim p) \vee (\sim q)$$

9> Associative law:

$$p \vee (q \vee r) \iff (p \vee q) \vee r$$

$$p \wedge (q \wedge r) \iff (p \wedge q) \wedge r$$

10> Distributive law:- $p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$

$$\dots \iff (p \vee q) \wedge (p \vee r).$$

10> Distributive law:- $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$.

11> Negation of a Conditional:-

$\sim(p \to q) \Leftrightarrow p \wedge (\sim q)$

12> Transitive rule:- If u, v, w are propositions such that $u \Leftrightarrow v$ & $v \Leftrightarrow w$ then $u \Leftrightarrow w$.

13> Substitution rule:- Suppose that a compound proposition 'u' is a tautology & 'p' is a component of 'u'. If we replace each occurance of 'p' in 'u' by a proposition 'q', then the resulting compound proposition 'v' is also a tautology.

Example:

1> Let 'x' be a specified no., write the negation of,

" If x is an integer, then 'x' is a rational no."

$p \to q$

Sol^n → Here p: x is an integer

q: x is a rational no.

Given $p \to q$.

But, wkt, $\sim(p \to q) \Leftrightarrow p \wedge (\sim q)$

∴ x is an integer and x is not a rational no. //

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* " If x is not a real no., then it is not a rational no. and not an irrational no."

Sol^n → Let p: x is a real no.

q: x is a rational no.

r: x is an irrational no.

$\sim(p \to q) \Leftrightarrow p \wedge (\sim q)$

q: x is a rational no.

r: x is an irrational no.

∴ given, $[\sim p \rightarrow (\sim q \wedge \sim r)]$

WKT. $\sim[\sim p \rightarrow (\sim q \wedge \sim r)] \Longleftrightarrow \sim p \wedge (\sim(\sim q \wedge \sim r))$

$\Longleftrightarrow \sim p \wedge (q \vee r)$

∴ The required proposition is,

x is not a real no. and x is a rational no. or irrational no. //

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* If an odd integer is greater than 2 and less than 8 then it is a prime no.

* $(p \wedge r) \rightarrow (q \vee r)$

→ p: an odd integer is greater than 2

q: an odd integer is less than 8

r: an odd integer is prime no.

WKT
$\sim(p \rightarrow q) \Longleftrightarrow p \wedge (\sim q)$

Given $(p \wedge q) \rightarrow r$

Now, $\sim[(p \wedge q) \rightarrow r] \Longleftrightarrow (p \wedge q) \wedge (\sim r)$

∴ required negation is,

an odd integer is greater than 2 and less than 8 and it is not a prime no.

→ $(p \wedge r) \rightarrow (q \vee r)$

$\sim[(p \wedge r) \rightarrow (q \vee r)] \Longleftrightarrow (p \wedge r) \wedge \sim(q \vee r)$

$\Longleftrightarrow (p \wedge r) \wedge [\sim q \wedge \sim r]$ {∵ De Morgan's law

$\Longleftrightarrow p \wedge (r \wedge \sim q \wedge \sim r)$ {Associative law}

$\Longleftrightarrow p \wedge (r \wedge \sim r \wedge \sim q)$ {commutative law}

$\Longleftrightarrow p \wedge (F_0 \wedge \sim q)$ {Inverse law}

$\Leftrightarrow p \wedge (F_0 \wedge \sim q)$ {Inverse law}

$\Leftrightarrow p \wedge F_0$ {Domination law}

$\Leftrightarrow \underline{F_0}$ // {Domination law}

* Simplify the following propositions using the laws of logic.

i) $(p \wedge q) \wedge [\sim\{(\sim p) \wedge q\}]$

sol^n $(p \wedge q) \wedge [\sim\{(\sim p) \wedge q\}] \Leftrightarrow (p \wedge q) \wedge [\sim(\sim p) \vee \sim q]$ {DeMorgan's law

$\Leftrightarrow (p \wedge q) \wedge [p \vee \sim q]$ {double negation law}

$\Leftrightarrow \{(p \wedge q) \wedge p\} \vee \{(p \wedge q) \wedge \sim q\}$ [distribution law]

$\Leftrightarrow [(q \wedge p) \wedge p] \vee [p \wedge (q \wedge \sim q)]$ {Commutative & Associative laws

$\Leftrightarrow [q \wedge (p \wedge p)] \vee [p \wedge F_0]$ {Associative & inverse law}

$\Leftrightarrow [q \wedge p] \vee [F_0]$ {Idempotent & domination laws}

$\Leftrightarrow (q \wedge p)$ {identity law}

$\Leftrightarrow p \wedge q$ // {commutative law}

ii) $(p \vee q) \wedge [\sim\{(\sim p) \vee q\}]$.

$\rightarrow (p \vee q) \wedge [\sim\{\sim p \vee q\}] \Leftrightarrow (p \vee q) \wedge [\sim\sim p \wedge \sim q]$ (DeMorgan's law)

$\Leftrightarrow (p \vee q) \wedge (p \wedge \sim q)$ [double negation law]

$\Leftrightarrow (p \wedge (p \wedge \sim q)) \vee (q \wedge (p \wedge \sim q))$ (distribution law)

$\Leftrightarrow [(p \wedge p) \wedge \sim q] \vee [q \wedge (\sim q \wedge p)]$ [Associative & commutative law

$\Leftrightarrow [p \wedge \sim q] \vee [(q \wedge \sim q) \wedge p]$ [idempotent & associative law

$\Leftrightarrow (p \wedge \sim q) \vee (F_0 \wedge p)$ [inverse law]

$\Leftrightarrow (p \wedge \sim q) \vee F_0$ [domination law]

[identity law]

$\Leftrightarrow (p \wedge \sim q) \vee F_0$ [domination law]

$\Leftrightarrow (p \wedge \sim q)$ // [identity law]

iii) $(p \vee q) \wedge [\sim\{(\sim p) \wedge q\}]$ **Ans:** p

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* Prove the following logical equivalence without using truth table.

i) $[p \vee q \vee (\sim p \wedge \sim q \wedge r)] \Leftrightarrow p \vee q \vee r$.

→ LHS $[p \vee q \vee (\sim p \wedge \sim q \wedge r)] \Leftrightarrow (p \vee q) \vee \{\sim(p \vee q) \wedge r\}$ [∵ Associative & D'Morgan's law]

$\Leftrightarrow \{(p \vee q) \vee \sim(p \vee q)\} \wedge \{(p \vee q) \vee r\}$ [distribution law]

$\Leftrightarrow T_0 \wedge \{p \vee q \vee r\}$ [∵ Inverse law]

$\Leftrightarrow p \vee q \vee r$ // RHS [∵ identity law]

ii) $(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r) \Leftrightarrow p \wedge q$.

→ LHS $(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r) \Leftrightarrow \sim(p \wedge q) \rightarrow (p \wedge q \wedge r)$ [D'Morgan's law]

$\Leftrightarrow \sim\sim(p \wedge q) \vee (p \wedge q \wedge r)$ [∵ $p \rightarrow q \Leftrightarrow \sim p \vee q$

$\Leftrightarrow (p \wedge q) \vee \{(p \wedge q) \wedge r\}$ [double negation & associative.

$\Leftrightarrow p \wedge q$ // RHS (Absorption law).

iii) $[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$.

→ LHS $[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow [(\sim p \wedge \sim q) \wedge r] \vee [(q \wedge r) \vee (p \wedge r)]$ [∵ Associative]

$\Leftrightarrow [\sim(p \vee q) \wedge r] \vee [(p \vee q) \wedge r]$ {D'Morgan's law & distribution law}

$\Leftrightarrow [\sim(p \vee q) \vee (p \vee q)] \wedge r$ [distribution law]

$\Leftrightarrow T_0 \wedge r$ [Inverse law]

$\Leftrightarrow r$ // RHS [Identity law]

Duality:- If u is a compound proposition containing $\vee, \wedge, T_0, F_0$ then the compound proposition obtained by interchanging $\vee$ with $\wedge$ & T_0 with F_0 & vice-versa is called Dual of 'u' & it is denoted by 'u^d'.

Ex:- $u : (p \wedge q) \vee (r \wedge T_0)$

$u^d : (p \vee q) \wedge (r \vee F_0)$

Note:- i) dual of dual of u is logically equivalent to u i.e. $(u^d)^d \Leftrightarrow u$

ii> If $u \Leftrightarrow v$ then $u^d \Leftrightarrow v^d$ (Principle of duality)

Ex:- Write the duals of the following

i> $\sim(p \vee q) \wedge [p \vee \sim(q \wedge \sim s)]$

$\longrightarrow$ $u^d : \sim(p \wedge q) \vee [p \wedge \sim(q \vee \sim s)]$.

* Verify the principle of duality for the following logical equivalence.

i> $[\sim(p \wedge q) \longrightarrow \sim p \vee (\sim p \vee q)] \Longleftrightarrow (\sim p \vee q)$ $\qquad p \rightarrow q \Leftrightarrow \sim p \vee q$

$\longrightarrow$ $u : \sim(p \wedge) \rightarrow \sim p \vee (\sim p \vee q)$ $\qquad v : \sim p \vee q$

WKT. $u : (p \wedge q) \vee [\sim p \vee (\sim p \vee q)]$

$\therefore$ $u^d : (p \vee q) \wedge (\sim p \wedge (\sim p \wedge q)) \Longleftrightarrow (p \vee q) \wedge [(\sim p \wedge \sim p) \wedge q]$ (Associative law)

$\Longleftrightarrow (p \vee q) \wedge (\sim p \wedge q)$

$\Longleftrightarrow [(p \wedge \sim p) \wedge q] \vee [q \wedge \sim p \wedge q]$

$\Longleftrightarrow (F_0 \wedge q) \vee (q \wedge \sim p)$ [Idempotent & inverse law

$\Longleftrightarrow F_0 \vee (\sim p \wedge q)$ [domination & commutative]

$u^d \Longleftrightarrow \sim p \wedge q$ (identity law)

$v : \sim p \vee q$ $\quad \therefore v^d : \sim p \wedge q$ $\quad \therefore$ $u^d \Leftrightarrow v^d$ //

Hence the principle of duality //

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* Write the duals of the following (i) $p \rightarrow q$ (ii) $(p \rightarrow q) \rightarrow r$
(iii) $p \rightarrow (q \rightarrow r)$ (iv) $p \veebar q$.

$\longrightarrow$ (i) $u : p \rightarrow q \Leftrightarrow \sim p \vee q$

$\therefore u^d : \sim p \wedge q$

(ii) $(p \rightarrow q) \rightarrow r \Leftrightarrow \sim(p \rightarrow q) \vee r \Leftrightarrow (p \wedge \sim q) \vee r$

$\therefore u : (p \wedge \sim q) \vee r$ $\quad \therefore u^d : (p \vee \sim q) \wedge r$

(iii) $p \rightarrow (q \rightarrow r) \Leftrightarrow \sim p \vee (q \rightarrow r)$

$\Leftrightarrow \sim p \vee (\sim q \vee r)$

$\therefore u : \sim p \vee (\sim q \vee r)$

$u^d = \sim p \wedge (\sim q \wedge r)$.

(iv) $p \veebar q \Leftrightarrow (p \wedge \sim q) \vee (\sim p \wedge q)$

$u : (p \wedge \sim q) \vee (\sim p \wedge q)$ $\quad \therefore u^d : (p \vee \sim q) \wedge (\sim p \vee q)$. //

The connectives NAND & NOR :-

The connectives NAND & NOR are represented by $(p \uparrow q)$ & $(p \downarrow q)$ respectively They are given by,

$p \uparrow q : \sim(p \wedge q) \Longleftrightarrow \sim p \vee \sim q$

$p \downarrow q : \sim(p \vee q) \Longleftrightarrow \sim p \wedge \sim q$.

Ex:- For any propositions p & q prove that (i) $\sim(p \uparrow q) \Leftrightarrow \sim p \downarrow \sim q$

(ii) $\sim(p \downarrow q) \Leftrightarrow \sim p \uparrow \sim q$.

→ (i) LHS $\sim(p \uparrow q) \Leftrightarrow \sim(\sim(p \wedge q))$

$\Leftrightarrow \sim(\underbrace{\sim p}_{u} \vee \underbrace{\sim q}_{v})$ $\quad \{\sim(u \vee v) \Leftrightarrow u \downarrow v\}$

$\Leftrightarrow \sim p \downarrow \sim q$ // RHS.

* For any propositions p, q & r prove the following

(i) $p \uparrow (q \uparrow r) \Leftrightarrow \sim p \vee (q \wedge r)$ $\quad$ (ii) $p \downarrow (q \downarrow r) \Leftrightarrow (p \vee q) \wedge \sim r$.

→ (i) LHS $p \uparrow (q \uparrow r) \Leftrightarrow \sim(p \wedge (q \uparrow r))$

$\Leftrightarrow \sim(p \wedge \sim(q \wedge r))$

$\Leftrightarrow \sim p \vee (q \wedge r)$ // RHS.

(ii) $(p \downarrow q) \downarrow r \Leftrightarrow \sim((p \downarrow q) \vee r)$

$\Leftrightarrow \sim(\sim(p \vee q) \vee r)$

$\Leftrightarrow (p \vee q) \wedge \sim r$ // RHS

* Express the following propositions interms of only NAND & NOR.

i> $\sim p$

→ (i) $\sim p \Leftrightarrow \sim(p \wedge p) \Leftrightarrow p \uparrow p$

Also, $\sim p \Leftrightarrow \sim(p \vee p) \Leftrightarrow p \downarrow p$

(ii) $p \wedge q$

→ $p \wedge q \Leftrightarrow \sim(\sim(p \wedge q)) \Leftrightarrow \sim(\sim p \vee \sim q) \Leftrightarrow (\sim p) \downarrow (\sim q)$

$\Leftrightarrow (p \downarrow p) \downarrow (q \downarrow q)$

Also, $p \wedge q \Leftrightarrow \sim(\sim(p \wedge q)) \Leftrightarrow \sim(\sim p \vee \sim q) \Leftrightarrow (\sim p \vee \sim q) \uparrow (\sim p \vee \sim q)$

$\Leftrightarrow (p \uparrow q) \uparrow (p \uparrow q)$ //

H.W.

(iii) $p \vee q$

Converse, Inverse & Contrapositive:-

Consider the implication $p \to q$ for any propositions $p \to q$, then

i> $q \to p$ is called Converse of $p \to q$

ii> $\sim p \to \sim q$ is called Inverse of $p \to q$

iii> $\sim q \to \sim p$ is called Contrapositive of $p \to q$

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	1	0	0	1	1	1	1

Note: From the above truth table we can see that
① $p \to q \iff \sim q \to \sim p$ i.e. Implication $\iff$ contrapositive
② $q \to p \iff \sim p \to \sim q$ i.e. Converse $\iff$ inverse.

Ex:- Let p: 2 is an integer & q: 9 is a multiple of 3. then write the conditional, converse, inverse & contrapositive of these statements.

$\longrightarrow$ $p \to q$: If 2 is an integer then 9 is a multiple of 3

$q \to p$: If 9 is a multiple of 3 then 2 is an integer

$\sim p \to \sim q$: If 2 is not an integer then 9 is not a multiple of 3.

$\sim q \to \sim p$: If 9 is not a multiple of 3 then 2 is not an integer.

Logical Implication:-

$\Rightarrow$ $\to$

Hypothetical (Implicative) statements.

In a hypothetical statement $p \to q$, q is true whenever p is true, such propositions are called Logical implication. & we say that p logically implies q. Symbolically '$p \Rightarrow q$'.

If q is not neccessarily true when p is true then we say p doesn't imply q i.e. '$p \not\Rightarrow q$'

Whenever we say $p \Rightarrow q$ we mean that ① p is sufficient for q ② q is neccessary for p.

Ex:- ① If a Quadrilateral is a square then it is a rectangle.

p: A Quadrilateral is a square.

q: A Quadrilateral is a rectangle. $p \Rightarrow q$.

② If Belagavi is in Karnataka then it is in India.

Special cases:- Some times it so happens that

i) $p \Rightarrow q$ but $q \not\Rightarrow p$

ii) $p \not\Rightarrow q$ but $q \Rightarrow p$

But when $p \Rightarrow q$ & also $q \Rightarrow p$ then we say that "p is necessary & sufficient condition for q & vice-versa". And we denote it as '$p \Leftrightarrow q$'.

p iff q. $p \Leftrightarrow q$.

Example:-

1) Prove the following

i) $[p \wedge (p \to q)] \Rightarrow q$ ② $[(p \to q) \wedge \sim q] \Rightarrow \sim p$

iii) $[(p \vee q) \wedge \sim p] \Rightarrow q$

$\longrightarrow$ p q $\sim p$ $\sim q$ $p \to q$ $p \vee q$ | i) $[p \wedge (p \to q)] \Rightarrow q$

iii> $[(p \vee q) \wedge \sim p] \Rightarrow q$

$\longrightarrow$

p	q	$\sim p$	$\sim q$	$p \to q$	$p \vee q$
0	0	1	1	1	0
0	1	1	0	1	1
1	0	0	1	0	1
1	1	0	0	1	1

i> $[p \wedge (p \to q)] \Rightarrow q$

ii> $[(p \to q) \wedge \sim q] \Rightarrow \sim p$

* Prove the following (i) $p \wedge q \Rightarrow p \vee q$ (ii) $\sim p \Rightarrow (p \to q)$ (iii) $q \Rightarrow (p \to q)$ (iv) $\{p \wedge (q \vee r)\} \Rightarrow (p \wedge q) \vee r$ (v) $(p \to q) \Rightarrow p \to (p \wedge q)$.

Sol^n:

p	q	r	$p \wedge q$	$p \vee q$	$\sim p$	$p \to q$	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee r$	$p \to (p \wedge q)$
0	0	0	0	0	1	1	0	0	0	1
0	0	1	0	0	1	1	1	0	1	1
0	1	0	0	1	1	1	1	0	0	1
0	1	1	0	1	1	1	1	0	1	1
1	0	0	0	1	0	0	0	0	0	0
1	0	1	0	1	0	0	1	1	1	0
1	1	0	1	1	0	1	1	1	1	1
1	1	1	1	1	0	1	1	1	1	1

---

* State the converse, inverse & contrapositive of the following.

i> If a quadrilateral is a parallelogram then its diagonals bisect each other.

$\longrightarrow$ $p \to q$

Converse: $q \to p$: If the diagonals of the quadrilateral bisect each other then it is a parallelogram.

Inverse :- $\sim p \to \sim q$: If a quadrilateral is not a parallelogram then its diagonals donot bisect each other.

Contrapositive :- $\sim q \to \sim p$: If the diagonals of the quadrilateral don't bisect each other then it is not a parallelogram.

* If a real no. x^2 is greater than zero, then x is not equal to zero.

* If a triangle is not isosceles then it is not equilateral

* Write the 'necessary condition' language & 'sufficient condition' language of the following.

i>

Rules of Inference :- Let q & $P_1, P_2, P_3 --- P_n$ be the ~~set~~ set of given propositions then $(P_1 \wedge P_2 \wedge P_3 \wedge --- \wedge P_n) \longrightarrow q$ is called Argument. Here $P_1, P_2, P_3 --- P_n$ are called Premises & q is called Conclusion.

It is to be noticed that in an argument, the premises are always taken to be true where as the conclusion may be true or false. The conclusion is true only when the argument is valid. The rules which talk about the validity of the argument are called Rules of Inference. They are listed as below,

i> Rule of Conjuctive Simplification: For any two propositions p & q, if $p \wedge q$ is true then p is true.

i.e. $(p \wedge q) \Rightarrow p$

ii> Rule of disjunctive amplification: For any two propositions p & q, if p is true then $p \vee q$ is true i.e. $p \Rightarrow p \vee q$

iii> Rule of Syllogism: For any three propositions p, q & r, if $p \to q$ & $q \to r$ are true then $p \to r$ is also true.

i.e. $\{(p \to q) \wedge (q \to r)\} \Rightarrow (p \to r)$

In otherwords, Tabular form.

$$\begin{array}{c} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

iv> Modus Pones (Rule of detachment): For any two propositions p & q, if p is true & $p \to q$ is true then q is true.

i.e. $\{p \wedge (p \to q)\} \Rightarrow q$ in otherwords

$$\begin{array}{c} p \\ p \to q \\ \hline \therefore q \end{array}$$

v> Modus Tollens: For any two propositions p & q, if $p \to q$ is true & q is false then p is false.

i.e. $\{(p \to q) \wedge \sim q\} \Rightarrow \sim p$ | i.e.

$$\begin{array}{c} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

vi> Rule of disjunctive syllogism:-

If $p \vee q$ is true & p is false then q is true.

i.e. $\{(p \vee q) \wedge \sim p\} \Rightarrow q$ | i.e

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$

vii> Rule of Contradiction:-

If $(\sim p \to F_0)$ is true then p is true.

i.e. $(\sim p \to F_0) \Rightarrow p$.

Problems:-

* Test whether the following statement is valid.

(i) If Sachin hits a century then he gets a free car $\leftarrow$ $p \to q$

Sachin hits a century

∴ Sachin gets a free car

$\therefore$ Sachin gets a free car

$\longrightarrow$ Here p: Sachin hits a century & q: Sachin gets a free car.

$\therefore$ given statement is,

$$\begin{array}{l} p \to q \\ \quad\ p \\ \hline \therefore\ q \end{array} \implies \{(p \to q) \wedge p\} \Rightarrow q$$

W.K.T. from rule of detachment $\{p \wedge (p \to q)\} \Rightarrow q$

$\therefore$ Given statement is valid.

ii) If Sachin hits a century then he gets a free car
Sachin gets a free car.

$\therefore$ Sachin has hit a century.

Sol^n $\rightarrow$ p: Sachin hits a century q: Sachin gets a free car.

$\therefore$ given argument is,

$$\begin{array}{l} p \to q \\ \quad\ q \\ \hline \therefore\ p \end{array} \implies \{(p \to q) \wedge q\} \Rightarrow p$$

p	q	$p \to q$	$(p \to q) \wedge q$
0	1	1	1

But from the truth table, we can see that p need not be true for $((p \to q) \wedge q)$ is to be true $\therefore$ given argument is not valid.

* If I drive to work then I will arrive tired.
I am not tired (when I arrive at work)

$\therefore$ I do not drive to work.

$\longrightarrow$ p: I drive to work q: I will arrive tired.

$\therefore$ given argument,

$$\begin{array}{l} p \to q \\ \ \sim q \\ \hline \therefore\ \sim p \end{array}$$

, W.K.T. from Modus Tollens $\{(p \to q) \wedge \sim q\} \Rightarrow \sim p$

$\therefore$ Given argument is valid.

$\sim p \vee q \Leftrightarrow (p \to q)$

$p \vee \sim q \Leftrightarrow \sim q \vee p \Leftrightarrow q \to p$

* I will become famous or I will not become a musician
I will become a musician.

$\therefore$ I will become famous.

$\longrightarrow$ p: I will become famous q: I will become a musician.

$\therefore$ given argument,

$$\begin{array}{l} p \vee \sim q \\ \quad\ q \\ \hline \therefore\ p \end{array} \Longleftrightarrow \begin{array}{l} q \to p \\ \quad\ q \\ \hline \therefore\ p \end{array} \Longleftrightarrow \{(q \to p) \wedge q\} \Rightarrow p$$

W.K.T. Modus Pones $\{(p \to q) \wedge p\} \Rightarrow q$ $\therefore$ given argument is valid.

(or)

p: I will become famous q: I will not become a musician.

$\therefore$ given argument,

$$\begin{array}{l} p \vee q \\ \ \sim q \\ \hline \end{array} \Longleftrightarrow \{(p \vee q) \wedge \sim q\} \Rightarrow p$$

... from rule of disjunctive syllogism given

$\therefore$ given argument, $\quad \dfrac{p \vee q \quad \sim q}{\therefore \; p} \iff \{(p\vee q) \wedge \sim q\} \Rightarrow p$

$\therefore$ from rule of disjunctive syllogism given argument is valid.

* If Ravi studies then he will pass in Discrete Mathematics
  If Ravi doesn't play cricket then he will study.
  Ravi failed in Discrete Mathematics

$\therefore$ Ravi played cricket.

$\longrightarrow$ p: Ravi studies $\quad q$: Ravi passes in D.M. $\quad r$: Ravi plays cricket.

$\therefore$ given argument,

$$\begin{array}{c} p \to q \\ \sim r \to p \\ \sim q \\ \hline \therefore \; r \end{array} \iff \begin{array}{c} \sim r \to p \\ p \to q \\ \sim q \\ \hline \therefore \; r \end{array} \iff \begin{array}{c} \sim r \to q \\ \sim q \\ \hline \therefore \; r \end{array} \quad \{\because \text{rule of syllogism}\}$$

WKT, from Modus Tollens $\{(p\to q) \wedge \sim q\} \Rightarrow \sim p \quad \therefore \{(\sim r \to q) \wedge \sim q\} \Rightarrow r$

$\therefore$ given argument is valid.

* If there is a strike by students then exam will be postponed
  There was no strike by strike by students

$\therefore$ The exam was not postponed.

Sol $\to$ p: There is a strike by students $\quad q$: exam gets postponed.

$\therefore$ given argument, $\quad \dfrac{p \to q \quad \sim p}{\therefore \; \sim q}$

WKT, when $p\to q$ is true & p is false it is not neccesary that q must be false. Because

p	q	$p\to q$	$\sim p$	$(p\to q) \wedge \sim p$
0	1	1	1	1

$\therefore$ given argument is not valid.

* If I drive to work then I will arrive tired
  I do not drive to work

$\therefore$ I will not arrive tired.

Quantifiers:- The declarative sentences, such as (i) $x+3=6$, (ii) $x^2<10$ (iii) x is divisible by 5 are called Open Statements/Open Sentences. Here 'x' is called free variable. The set to which this x belongs is called Universe.

These open statements are represented by $p(x)$, $q(x)$, $r(x)$ ----

If 'x' is given some particular value 'a' then $p(a)$ is called proposition.

The phrases of the form 'all', 'for all', 'there exist', 'every', 'some' etc are called Quantifiers.

Ex: $\exists x \in R,\ x+3=6$, $\forall x \in R,\ x^2<10$

$\exists$ ↩

$\forall$ ↩

i) All squares are rectangles.

ii) For every integer x, x^2 is a non-negative integer.

iii) Some determinants are equal to zero.

iv) There exists a real no., whose square is equal to itself.

$\forall$	$\exists$
for all	there exists
for every	for some
for each	for atleast one.
for any	
Universal quantifiers	Existential quantifier.

The proposition involving the quantifiers is called Quantified Statement.

It will be of the form,

$\forall x \in S,\ p(x)$ or $\exists x \in S,\ p(x)$.

where $p(x)$ is open statement, 'S' is the universe, 'x' free variable.

Truth value of the Quantified Statement:-

To determine the truth value of the quantified statement we have the following rules.

i) The statement, "$\forall x, p(x)$" is true only when $p(x)$ is true for all $x \in S$.

ii) The statement, "$\exists x, p(x)$", is false only when $p(x)$ is false for all $x \in S$.

A point $x = a \in S$ when the universally quantified statement is false or the existentially quantified statement is true, is called Counter example.

Two rules of inference:-

As a consiquence of the rules mentioned above, we obtain the following rules of inference.

→ If an open statement $p(x)$ is known to be true for all $x \in S$ & if $a \in S$ then $p(a)$ is true (Rule of Universal specification).

→ If an open statement $p(x)$ is proved to be true for arbitrary $x \in S$ then the quantified statement "$\forall x \in S, p(x)$" is true (Rule of Universal generalisation).

Logical Equivalence:- Two quantified statements are said to be logically equivalent if they have same truth values. in all possible ways.

Few results:

i) $\forall x, [p(x) \wedge q(x)] \iff [\forall x, p(x)] \wedge [\forall x, q(x)]$

ii) $\exists x, [p(x) \vee q(x)] \iff [\exists x, p(x)] \vee [\exists x, q(x)]$

iii) $\exists x, [p(x) \rightarrow q(x)] \iff \exists x, [\sim p(x) \vee q(x)]$.

iv) $\forall x, \sim p(x) \iff$ For no x, $p(x)$.

Ex:- For every integer x, x^2 is non-negative

$\iff$ For no integer x, x^2 is negative.

Ex: For every integer x, x^2 is non-negative

$\Longleftrightarrow$ For no integer x, x^2 is negative.

Rule of negation of a quantified statement:-

$\sim[\forall x, p(x)] \Longleftrightarrow \exists x, \sim p(x)$

$\sim[\exists x, p(x)] \Longleftrightarrow \forall x, \sim p(x)$.

Problems:-

* For all integers 'x'. $p(x)$: $x>0$, $q(x)$: x is even, $r(x)$: x is perfect square, $s(x)$: x is divisible by 3, $t(x)$: x is divisible by 7.

i> Write down the following quantified statements in symbolic form.

i> Atleast one integer in even.

$\longrightarrow \exists x, q(x)$

ii> There exists a positive integer that is even.

$\longrightarrow \exists x, [p(x) \wedge q(x)]$

iii> Some even integers are divisible by 3.

$\longrightarrow \exists x, \{q(x) \wedge s(x)\}$

iv> If x is even & perfect square then x is not divisible by 3.

$\longrightarrow \forall x, [\{q(x) \wedge r(x)\} \longrightarrow \sim s(x)]$

v> Every integer is either even or odd.

$\longrightarrow \forall x, [q(x) \veebar \sim q(x)]$

vi> If x is odd or is not divisible by 7 then x is divisible by 3.

$\longrightarrow \forall x, [\{\sim q(x) \vee \sim t(x)\} \longrightarrow s(x)]$

* Consider the above open statements $p(x), q(x), r(x), s(x), t(x)$. Express the following symbolic statements in words & indicate its truth value.

i> $\forall x, [r(x) \longrightarrow p(x)]$

$\longrightarrow$ If x is a perfect square then x is positive integer — False ($\because x=0$)

ii> $\exists x, [s(x) \wedge \sim q(x)]$

$\longrightarrow$ There exists some integer x divisible by 3 which is odd. — True. ($\because x=9$)

iii> $\forall x, \sim r(x)$

$\longrightarrow$ For all integers x, x is not a perfect square — False ($\because x=4$)

iv> $\forall x, [r(x) \vee t(x)]$

$\longrightarrow$ For all integers x, x is a perfect square or divisible by 7. — False $(\because x=8)$

* Consider the following open statements with the set of all real no-s as universe,

  $p(x): |x|>3$, $q(x): x>3$. Find the truth value of the statement,

  $\forall x, [p(x) \rightarrow q(x)]$. Also write its converse, inverse & contrapositive.

* Negate & Simplify the following.

  i> $\exists x, [p(x) \vee q(x)]$ ii> $\forall x, [p(x) \wedge \sim q(x)]$

  iii> $\forall x, [p(x) \rightarrow q(x)]$ iv> $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$

* Write down the following statements in symbolic form & their negation

  i> All integers are rational nos & some rational no-s are not integers.

  ii> If all triangles are right angled then no triangle is equilateral.