2 Let $A = \{1, 2, 3, 4\}$ $B = \{a, b, c, d\}$ detamine whether the function from $A \to B$ are inveatible or not

$f \cdot \{(1, a)\ (2, a)\ (3, c)\ (4, d)\}$

$g : \{(1, a)\ (2, c)\ (3, d)\ (4 \cdot d)\}$

Sol^n $fog(1) = f\{g(1)\} = f\{a\}$

insufficient data

7/2/2020.

3. Let $A = B = R$, the set of all real no.s and the $f : A \to B$ and $g : B \to A$ be defined by $f(x) = 2x^3 - 1\ \forall\ x \in A$ $g(y) = \left(\frac{y+1}{2}\right)^{1/3}\ \forall\ y \in B$

ST f and g are the inverses of each other

Sol^n: $fog(y) = f\{g(y)\} = f\left\{\frac{y+1}{2}\right\}^{1/3}$

$= \cancel{2\left(\frac{y+1}{2}\right)^{1/3}} = \cancel{2}\left\{\left(\frac{y+1}{\cancel{2}}\right)^{1/3}\right\}^3 - 1$

$= y = I_B.$

$gof(x) = g\{f(x)\} = g\{2x^3 - 1\}$

$= \left\{\frac{\cancel{2}x^3 \cancel{-1} \cancel{+1}}{\cancel{2}}\right\}^{1/3} = x = I_A$

Hence they are inverse of each other

This shows f and g are invertible function.

Theorem : 1

If a function $f: A \to B$ is invertible then it has unique inverse, further if $f(a) = b$ then $f^{-1}(b) = a$

Theorem : 2

A function $f: A \to B$ is invertible iff it is one to one and onto

Theorem.

Let A and B be two finite sets with $|A| = |B|$ and f be the function from A to B then the following statements are equivalent

i. f is one to one.

ii. f is onto

iii. f is invertible

Theorem.

If $f: A \to B$ and $g: B \to C$ are invertible functio then $g \circ f: A \to C$ is an invertible function. and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

1. Let $A = B = C = R$ and $f: A \to B$ and $g: B \to C$ are defined by $f(a) = 2a+1$ and $g(b) = \frac{1}{3}b$ $\forall\ a \in A,\ b \in B$ then compute gof and show that gof is invertible. Also find $(gof)^{-1}$

Sol^n: $gof(a) = g\{f(a)\}$

$= g\{2a+1\}$

$= \frac{1}{3}(2a+1)$

We have $f(a) = b \Rightarrow f^{-1}(b) = a$

$\Rightarrow 2a+1 = b$ $\quad f^{-1}(b) = \frac{b-1}{2}$

$a = \frac{b-1}{2}$

III^{ly} $g(b) = c \Rightarrow g^{-1}(c) = b$

$\frac{1}{3}b = c$ $\quad g^{-1}(c) = 3c$

$b = 3c$

Now let us find $(gof)^{-1}c = f^{-1}og^{-1}(c)$

$= f^{-1}\{g^{-1}(c)\}$

$= f^{-1}\{3c\}$

$= \frac{3c-1}{2}$

2 Consider a function $f: A \to B$ defined by $f(a) = \sqrt{a+1}$ $\forall a \in A$ show that f is invertible and determine f^{-1}

Soln: Let us consider $a_1, a_2 \in A$

$f(a_1) = \sqrt{a_1+1}$ and $f(a_2) = \sqrt{a_2+1}$

$\Rightarrow f(a_1) = f(a_2)$

$\sqrt{a_1+1} = \sqrt{a_2+1}$

$\Rightarrow a_1 = a_2$

f is one to one

Take any $b \in B$ then $b = f(a)$

$\Rightarrow b = \sqrt{a+1}$

$b^2 = a+1$

$b^2 - 1 = a$

Since $b \geq 0$ and $b^2 - 1 \geq -1$

Q → $A = \{x \mid x$ is real $x \geq -1\}$

$B = \{x \mid x$ is real and $x \geq 0\}$.

Thus every $b \in B$ has $a = b^2 - 1$ as its preimage in A. under f. $\therefore$ f is onto.

This proves f is invertible

$\therefore a = f^{-1}(b) = b^2 - 1$ $\quad$ $b = f(a)$

$f^{-1}(b) = a$.

10/2/2020

UNIT-1

MATHEMATICAL LOGIC

$(p \vee q) \equiv (q \vee p)$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$p \vee (q \vee r) \equiv (p \vee q) \vee r$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

$\sim(p \to q) \equiv \cancel{p} \; p \wedge \sim q$

Suppose $p \to q$ is given preposition

i. $q \to p$ → converse

ii. $\sim p \to \sim q$ → inverse

iii. $\sim q \to \sim p$ → Contrapositive.

Conjuctive simplification:

$(p \wedge q) \Rightarrow p$

Disjunctive Amplification

$p \Rightarrow (p \vee q)$

p, q, r

$$\begin{array}{c} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

$p \wedge (p \to q) \Rightarrow q$

$(p \to q) \wedge \sim q \Rightarrow \sim p$

1. Let p and q be primitive stmts for which $p \to q$ is false. Determine the truth values of the foll. compound proposition.

i. $p \wedge q$ ii. $\sim p \vee q$ iii. $\sim q \to \sim p$

P	q	$p \to q$	$p \wedge q$	$\sim p \vee q$	$\sim q \to \sim p$
T	F	F	F	F	F

2. PT $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology

P	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	Given P
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

3. $[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$ using truth table

Sol^n.

p	q	r	$q \wedge r$	$p \rightarrow q \wedge r$	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Hence verified