UNIT- 4
PROBABILITY DISTRIBUTIONS

Let ‘S‘ be a sample space of a random experiment.
Suppose to each
element s of ‘S’ ,a unique real number X is associated according to some rule.
Then X, is called a random variable on S.

Example:

Consider a random experiment of tossing three coins together.

The corresponding sample space is
S = {HHH,HHT,HTH,THH,HTT,THT,TTH,TTT}
which has 8 possible outcomes.

Suppose we define the mapping $f: S \rightarrow R$ by $f(s)=$ number of heads in an outcome s i.e.,

$f(HHH)=3, f(HHT)=2, f(HTH)=2, f(THH)=2, f(HTT)=1, f(TTH)=1, f(TTT)=0$

As s varies over the set S ,X varies over the set $\{0,1,2,3\}$ belongs to R.

Note: One can define infinitely many random variable on a given sample space.

Discrete Random Variables:

A random variable which can take some specified values only is called as Discrete Random Variables.
(Varying only over integral values)

Ex: Tossing a coin and observing the number of heads turning up.

Continuous Random Variables:

A random variable which can take any value in a specified range is called Continuous Random Variable.

(can assume any value in the interval of real numbers)

Example: Speed ,time etc…..

Discrete Probability Distributions:

If for each value x_i of a discrete random variable X, a real number p(x_i) is assigned such that a)p(x_i)≥ 0

b)$\sum_i p(x_i) = 1$

Then the function p(x) is called Probabilty Function

The set of values $[x_i, p(x_i)]$ is called a discrete probability distribution of discrete random variable X.
The function p(x) is called the probability density function(pdf).

The distribution function f(x) is defined by $f(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$, x being an integer is called the cumulative distribution function(cdf).

Note:

$$Mean(\mu) = \sum_i x_i . p(x)_i$$

$$Variance\ (V) = \sum_i (x_i - \mu)^2 . p(x_i)$$

$$= \sum x_i{}^2 . p(x_i) - \mu^2$$

$$Standard\ deviation(\sigma) = \sqrt{V}$$