

Ex: 2 Witness Reliability

In the city of Cambridge, there are 2 taxi companies. One taxi company uses yellow taxis & the other uses white taxis. The yellow taxi company has 90 cars, & the white taxi company has just 10 cars.

A hit & run incident has been reported, & an eye witness has stated that she is certain that the car was a white taxi. The experts have asserted that given the foggy weather at the time of the incident, the witness had 75% chance of correctly identifying the taxi.

Given that the lady has said that the taxi was white, what is the likelihood that she is right?

Solⁿ:- $P(Y) = 0.9 \rightarrow$ Since total ~~no. cars~~ ^{taxis} is 100. out of which No. of yellow taxi is 90.
 $P(W) = 0.1 \rightarrow$ 10 taxis out of 100

$P(C_y) \rightarrow$ Probability of culprit driving yellow taxi

$$P(C_y) = 0.9.$$

$P(C_w) \rightarrow$ Probability of culprit driving white taxi

$$P(C_w) = 0.1$$

$P(W_y|C_w) = 0.75$ (According to the expert she can be correct only 75% of the time)
 \rightarrow Probability of witness saying she saw ~~yellow~~ ^{white} car, when she really saw a ~~yellow~~ ^{white} car.

$P(W_y|C_y) = 0.75$ (For same reason as above)

\rightarrow Probability of witness saying she saw a yellow car, when she really saw a yellow car

According to the problem statement, we need to find the probability of car being white, given that the witness saying that the car is white. i.e. $P(C_w|W_w)$

$$P(C_w|W_w) = \frac{P(W_w|C_w) \cdot P(C_w)}{P(W_w)}$$
$$= \frac{0.75 \times 0.1}{P(W_w)}$$

We need to calculate that the Witness would say she saw a white car (Prior probability)

For calculating $P(W_w)$ ~~Refer the text book for this~~

~~$P(W_w)$~~

Out of 90 yellow cars, she will incorrectly classify 22.5 cars as white cars (Because she can be correct only 75% of the time. So 25% of the time she will be wrong)

$$\therefore \frac{25}{100} \times 90 \text{ will be perceived as white cars.}$$
$$= 22.5 \text{ as white cars}$$

Similarly, out of 10 white cars, 2.5 will be misidentified as yellow & only 7.5 will be correctly identified as white.

$$\therefore \frac{75}{100} \times 10 = 7.5 \text{ will be white cars.}$$

\therefore Out of 100 cars $(22.5 + 7.5) = 30 \text{ cars}$ will be identified by the witness as white cars.

$$\therefore P(W_w) = \frac{30}{100} = 0.3$$

$$\therefore P(C_w | W_w) = \frac{0.75 \times 0.1}{0.3} = 0.25$$

The likelihood of the lady (Witness) saying the car was white, that it was in fact white is only 0.25, whereas the likelihood of it was ~~not~~ actually yellow is 3 times more than it was white.