Lorentz Local Canonicalization: How to Make Any Network Lorentz-Equivariant

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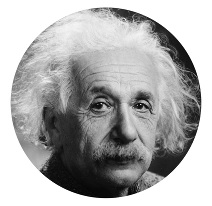
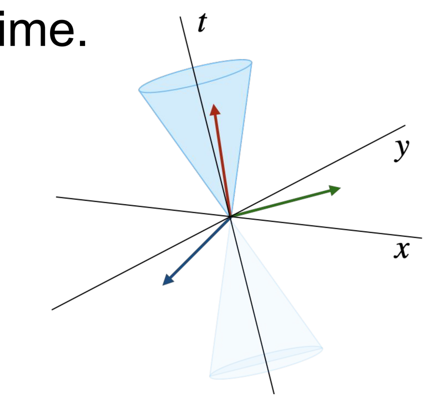
Lorentz group



- **Einstein:** laws of physics take the same form in every *inertial reference frame*.
- Reference frames are related by *Lorentz transformations* Λ.
- Lorentz transformations mix space and time: Rotations × Boosts.
- *Minkowski metric* $g = \mathrm{diag}(1,-1,-1,-1)$ defines the topology of spacetime.

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad x' = \gamma(x - vt) \qquad y' = y \qquad z' = z \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$



Local Canonicalization

- Local reference frames = orthonormal Lorentz-vectors.

$$L = \begin{pmatrix} — u_0^T g — \\ — u_1^T g — \\ — u_2^T g — \\ — u_3^T g — \end{pmatrix} \xrightarrow{\Lambda} L' = \begin{pmatrix} — u_0^T \Lambda^T g — \\ — u_1^T \Lambda^T g — \\ — u_2^T \Lambda^T g — \\ — u_3^T \Lambda^T g — \end{pmatrix} = L\Lambda^{-1}$$

- **Local space-time features are Lorentz-invariant.**
- Transform back in the end for **equivariant outputs**:

$$y \xrightarrow{\Lambda} y' = \rho(L'^{-1}) f'_L = \rho(\Lambda L^{-1}) f_L = \rho(\Lambda)\rho(L^{-1}) f_L = \rho(\Lambda) y$$

- Equivariant **Frames-Net predicts four-vectors**:

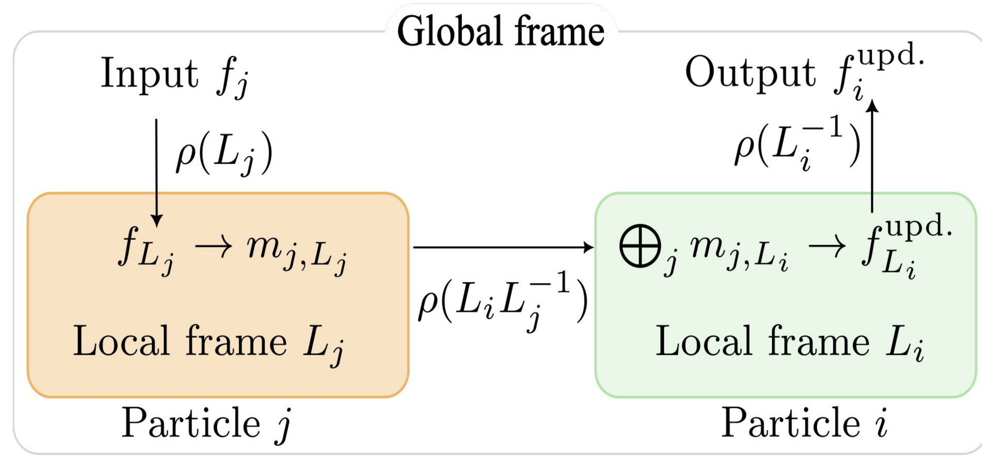
$$v_{i,k} = \sum_{j=1}^{N} \mathrm{softmax}\left(\varphi_k(s_i, s_j, \langle p_i, p_j\rangle)\right) \frac{p_i + p_j}{\|p_i + p_j\| + \epsilon} \qquad \text{for } k = 0,1,2$$

- 4D Gram-Schmidt to obtain the $u_{i,k}$.

Tensorial Message Passing

- Works with **any backbone architecture.**
- BUT using space-time **tensor messages** is crucial:

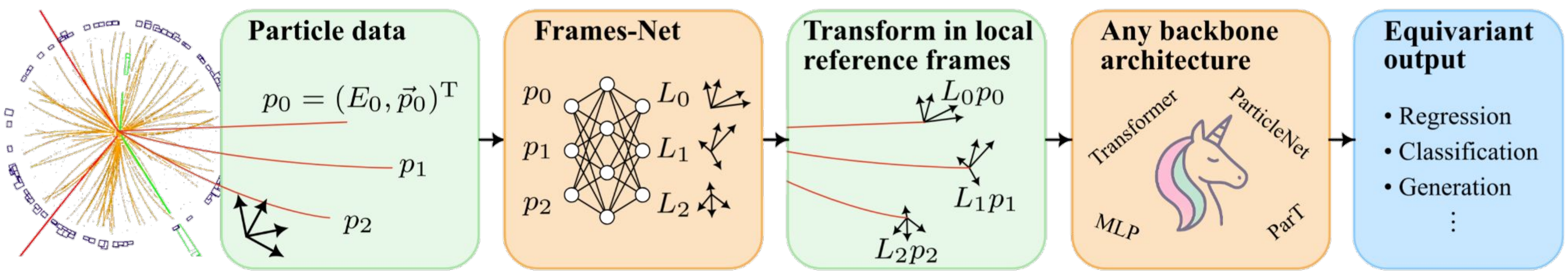
$$f_{L_i}^{\text{upd.}} = \psi\left(f_{L_i}, \bigoplus_{j=1}^{N} \phi\left(\rho(L_i\, L_j^{-1})\, m_{j,L_j}\right)\right)$$



➔ **Freely choose the message's geometric structure.**

Highlights

➔ **LLoCa** is the first local canonicalization framework for the Lorentz group.
➔ **LLoCa** trains 4x faster and uses 10x less FLOPs compared to other SOTA architectures.
➔ **LLoCa** makes any architecture Lorentz-equivariant.
➔ **LLoCa** works with any input, output, and latent representations.

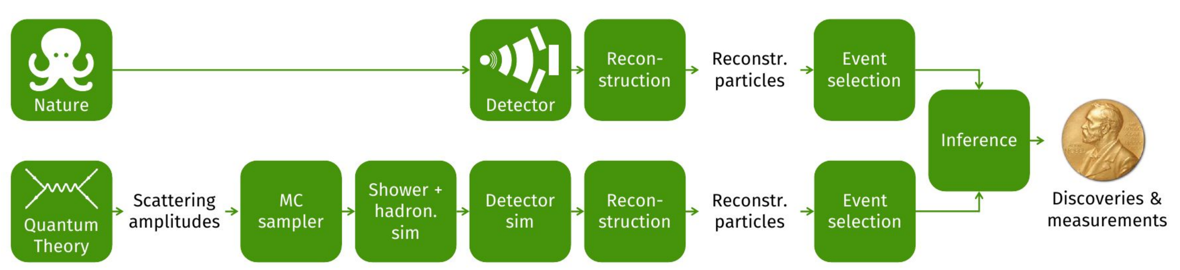


LLoCa

Paper · Code · Package

High-Energy Physics

- LHC studies the fundamental laws of nature.
- **Petabyte-scale dataset**: one collision every **25 ns**
- Complex, high-dimensional data is compared to simulations ➔ new physics!



- Fundamental objects at the LHC: **Particles** with four-momentum $(E, \vec{p})$, plus additional scalar information.

LLoCa = Efficient + Accurate

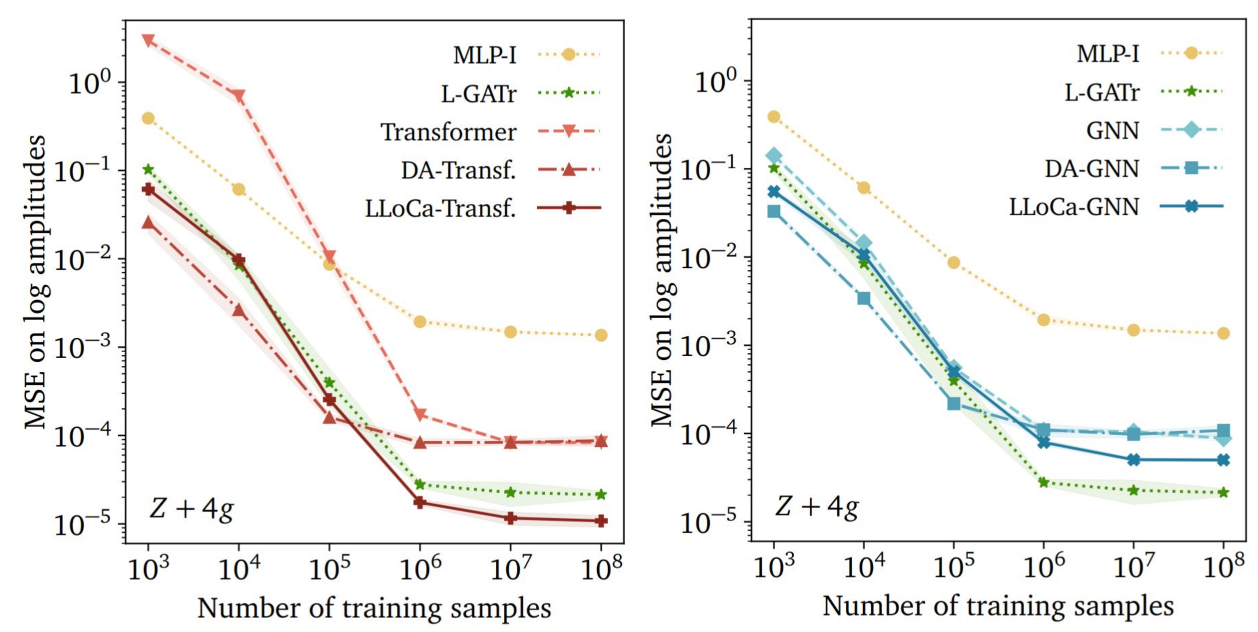
Jet tagging: **multiclass classification at the LHC**

- LLoCA + Vanilla transformer 4 times faster than SOTA.
- Successfully handels symmetry breaking effects.
- Plus, LLoCa is SOTA on prediction of scattering probabilities.

Model	Accuracy (↑)	AUC (↑)	Time	FLOPs
PFN [26]	0.772	0.9714	3h	3M
P-CNN [41]	0.809	0.9789	3h	12M
LorentzNet [21]	0.847	0.9856	64h	676M
MIParT-L [48]	0.861	0.9878	43h	225M
L-GATr* [9]	**0.866**	**0.9885**	166h	2060M
ParticleNet [37]	0.844	0.9849	25h	413M
LLoCa-ParticleNet*	0.845	0.9852	43h	517M
ParT [38]	0.861	0.9877	33h	211M
LLoCa-ParT*	0.864	0.9882	66h	315M
Transformer	0.855	0.9867	15h	210M
LLoCa-Transformer*	0.864	0.9882	31h	301M

Equivariance at scale

- Data augmentation as special case of LLoCa, with random global frames.
- Data augmentation wins for little data.
- Lorentz-equivariance wins at scale.



Message representation

- What is the best latent space-time representation? **Our answer**: equal mix of scalar and vector reps.
- With tensorial messages: Local canonicalization $\gg$ global canonicalization.

Transformer $Z + 4g$	MSE ($\times 10^{-5}$)
Non-equivariant	8.3 ± 0.5
Global canonical.	4.4 ± 1.0
LLoCa (16 scalars)	40 ± 4
LLoCa (single 2-tensor)	2.0 ± 0.4
LLoCa (4 vectors)	1.4 ± 0.2
LLoCa (8 scalars, 2 vectors)	**1.0** ± 0.1