TP - Inferencia

$X_1, \dots X_n \overset{iid}{\sim} \mathcal{E}(\lambda)$

a) $H_0: \lambda \leq 0.5$ vs. $H_1: \lambda > 0.5$

b) 1. Test de nivel exacto α a partir de $T_1 = \sum_{i=1}^{n} x_i$

$X_1, \dots, X_n \sim \mathcal{E}(\lambda) \to f_X(x) = \lambda e^{-\lambda x} I_{\{x \geq 0\}}$

• Veamos que es flia. exponencial:

$$f_\lambda(\underline{x}) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} I_{\{x_i \geq 0\}} = \underbrace{\lambda^n}_{A(\lambda)} e^{\overbrace{-\lambda}^{c(\lambda)} \overbrace{\sum_{i=1}^{n} x_i}^{r(\underline{x})}} \underbrace{\prod_{i=1}^{n} I_{\{x_i \geq 0\}}}_{h(\underline{x})}$$

↓

flia. exp.

• $c(\lambda)$ es decreciente

• $r(\underline{x}) = \sum_{i=1}^{n} x_i$

$\Rightarrow T = r(\underline{x}) = \sum_{i=1}^{n} x_i$

$\Rightarrow$ El test UMP queda:

$$\delta(\underline{x}) = \begin{cases} 1 & \text{si } -\sum_{i=1}^{n} x_i > k_\alpha \\ 0 & \text{si no} \end{cases}$$

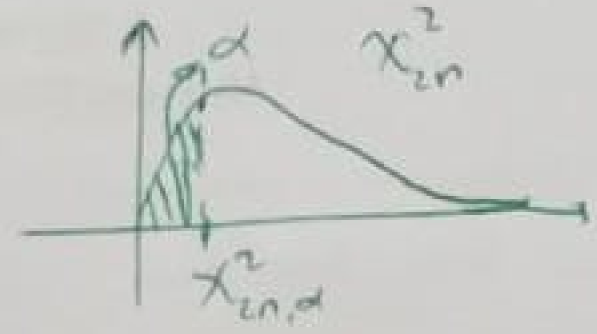
con k_α tal que $P_{\lambda=0.5}(\delta(\underline{x}) = 1) = \alpha$

~~Para~~ Bajo H_0, considero, $\lambda = 0.5$: $X_i \overset{iid}{\sim} \mathcal{E}(1/2)$

$$\Rightarrow \sum_{i=1}^{n} X_i \sim \Gamma(n, 1/2) = \chi^2_{2n}$$

$$\Rightarrow \alpha = P_{\lambda=0.5}\left(\delta(\underline{X}) = 1\right) = P_{\lambda=0.5}\left(-\sum_{i=1}^{n} X_i > k_\alpha\right)$$

$$= P_{\lambda=0.5}\left(\sum_{i=1}^{n} X_i < \underbrace{-k_\alpha}_{= \chi^2_{2n,\alpha}}\right)$$



Luego, el test resulta:

$$\delta(\underline{X}) = \begin{cases} 1 & \text{si } \sum_{i=1}^{n} X_i < \chi^2_{2n,\alpha} \\ 0 & \text{si no} \end{cases}$$

2. Test de nivel asintótico α a partir de $T_2 = \sqrt{n}\,\dfrac{\bar{X} - 2}{2}$

$X_1, \dots, X_n \sim \mathcal{E}(\lambda)$, $E(X_i) = \dfrac{1}{\lambda}$, $V(X_i) = \dfrac{1}{\lambda^2}$ $\begin{cases} E(\bar{X}) = \frac{1}{\lambda} \\ V(\bar{X}) = \frac{1}{n} \cdot \frac{1}{\lambda^2} \end{cases}$

Por TCL : $\dfrac{\bar{X} - \frac{1}{\lambda}}{\sqrt{\frac{1/\lambda^2}{n}}} \xrightarrow{\mathcal{D}} N(0,1)$

$$\Leftrightarrow \sqrt{n}\,\frac{\bar{X} - \frac{1}{\lambda}}{\frac{1}{\lambda}} \xrightarrow{\mathcal{D}} N(0,1)$$

Luego, bajo H_0 considerando $\lambda = 0.5$: $\sqrt{n}\,\dfrac{\bar{X} - \frac{1}{0.5}}{\frac{1}{0.5}} \xrightarrow{\mathcal{D}} N(0,1)$

$$\sqrt{n}\,\frac{\bar{X} - 2}{2} \xrightarrow{\mathcal{D}} N(0,1)$$

↪ ESTADÍSTICO PROPUESTO.

Luego, quiero k'_α tal que

$$P_{\lambda=0.5}\Big(\underbrace{\sqrt{n}\,\frac{\bar{X} - 2}{2}}_{\text{Aprox } N(0,1)} < k'_\alpha\Big) \xrightarrow{n \to \infty} \alpha$$

$k'_\alpha = z_\alpha$

⊛ ACLARACIÓN →

De esa manera, el test:

$$\delta(\underline{X}) = \begin{cases} 1 & \text{si } \sqrt{n}\,\dfrac{\bar{X}-2}{2} < z_\alpha \\ 0 & \text{si no} \end{cases}$$

Tiene nivel asintótico α.

(*) ACLARACIÓN: El test exacto era

$$\delta(\underline{X}) = \begin{cases} 1 & \text{si } -\sum_{i=1}^{n} X_i > k_\alpha \leftrightarrow \sum_{i=1}^{n} X_i < -k_\alpha \\ 0 & \text{si no} \end{cases}$$

$$P_{\lambda=0.5}\left(\sum_{i=1}^{n} X_i < -k_\alpha\right) = P_{\lambda=0.5}\left(\sqrt{n}\,\frac{\bar{X}-2}{2} < k'_\alpha\right)$$

↓

Busco que sea aprox. α