[Exercise 4.28] Let $G = Q_8 \times A \times B$, where A is a (necessarily abelian) group of exponent 2 and B is an abelian group in which every element has odd order. We are tasked with proving that every subgroup of G is normal.

Since Q_8 is Hamiltonian, all its subgroups are normal. Both A and B are abelian, so all their subgroups are also normal.

Moreover, since G is a direct product, every subgroup $H \leq G$ is contained in some internal direct product $Q_8 \times A' \times B'$, where $A' \leq A$, $B' \leq B$, and the projections of H into each factor yield subgroups. Since A has exponent 2, it is an elementary abelian 2-group, so $A \times B$ is abelian. Thus, the commutator subgroup $[G, G] \subseteq Q_8$. Since Q_8 is normal in G, and the only nonabelian factor, any conjugation acts trivially on A and B. Therefore, for any $g \in G$, conjugation by g preserves H, so $H \trianglelefteq G$. Hence, every subgroup of G is normal.