For Exercise 4.9 (i) and (ii):

Let $x \in X$. Since H acts transitively on X, for any $g \in G$, we can find $h \in H$ such that:

$$h \cdot x = g \cdot x \quad \Rightarrow \quad h^{-1}g \cdot x = x \quad \Rightarrow \quad h^{-1}g \in G_x.$$

So $g = h \cdot (h^{-1}g) \in HG_x$. Hence:

$$G = HG_x.$$

The Frattini argument follows directly from the general fact that **if a subgroup $H \leq G$ acts transitively on a finite G-set X**, then $G = HG_x$ for any $x \in X$. In the Frattini setting, G **acts on the set of Sylow p-subgroups of a normal subgroup K by conjugation, and K acts transitively on this set by Sylow's theorem.** Fixing a Sylow p-subgroup $P \leq K$, its stabilizer under the action is $N_G(P)$. Thus, applying the transitivity result yields $G = KN_G(P)$, which is precisely the Frattini argument.