

By Lagrange's theorem, the order of a subgroup of S_4 divides $|S_4| = 4! = 24$, so we are looking for subgroups of orders 1, 2, 3, 4, 6, 8, 12, and 24. We go through the list, often using Sylow's theorems:

- **Order 1:** Trivial subgroup $\{1\}$ — 1 subgroup.
- **Order 2:** Subgroups generated by:
 - 6 transpositions: $\langle (i\ j) \rangle$
 - 3 double transpositions: $\langle (i\ j)(k\ l) \rangle$
 Total: $6 + 3 = 9$, all cyclic.
- **Order 3:** Subgroups $\langle (i\ j\ k) \rangle$, all cyclic, all conjugate — $\frac{4!}{3 \cdot 2} = 4$ subgroups.
- **Order 4:**
 - 3 Klein 4-subgroups: isomorphic to $V_4 = \{1, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$
 - 3 cyclic subgroups generated by 4-cycles: $\langle (1\ 2\ 3\ 4) \rangle$, $\langle (1\ 2\ 4\ 3) \rangle$, $\langle (1\ 3\ 2\ 4) \rangle$ (note: $\langle (1\ 3\ 4\ 2) \rangle = \langle (1\ 2\ 4\ 3) \rangle$, $\langle (1\ 4\ 2\ 3) \rangle = \langle (1\ 3\ 2\ 4) \rangle$, $\langle (1\ 4\ 3\ 2) \rangle = \langle (1\ 2\ 3\ 4) \rangle$)
 Total: $3 + 3 = 6$ subgroups, split into 2 conjugacy classes (the three V_4 are conjugate, and the three cyclic groups of order 4 are conjugate).
- **Order 6:** Each isomorphic to $S_3 = \langle (i\ j), (i\ j\ k) \rangle$, where $\{i, j, k\}$ runs over 3-element subsets — $\binom{4}{3} = 4$ subgroups.
- **Order 8:** Sylow 2-subgroups. The order is 2^3 , and $n_2 \equiv 1 \pmod{2}$ and n_2 divides $24/8 = 3$, so $n_2 = 1$ or $n_2 = 3$. These are isomorphic to the dihedral group D_4 — 3 subgroups.
- **Order 12:** Unique subgroup isomorphic to A_4 — 1 subgroup.
- **Order 24:** The whole group S_4 — 1 subgroup.

Grand Total: $1 + 9 + 4 + 6 + 4 + 3 + 1 + 1 = \boxed{29 \text{ subgroups}}$

Excluding $\{1\}$ and S_4 , you get 27 proper subgroups, and 26 nontrivial proper ones.