Interobject Spacing Control and Controllability of a Manufacturing Transportation System [1]

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Abstract

This paper presents a control and controllability approach for a general manufacturing transportation system. The system transports objects from a production unit to a stamping/packaging section. At arrival time, the position and velocity of every object must synchronize with the stamping device. The system constraints and a simplified model of the system dynamics are developed. The hybrid character of the system makes classical control approaches difficult. A control scheme that controls interobject spacings and satisfies the constraints is proposed. A controllability analysis, independent from the control scheme and satisfying all constraints, shows the influence of system parameters on system controllability. Simulations verify the control scheme and controllability analysis.

1. Introduction

Some production systems require timely arrival of produced objects at various stages of the production system. This paper describes a possible control approach to the timed transfer of products from a production unit to a stamping and packaging unit. Section 2 discusses the general system setup and how objects are being transported in the production system.

The need for controlled object transportation originates from fluctuations in production rates and disturbances along the transportation path. The control system specifications are given in section 3.

Next, section 4 discusses the system model. The hybrid character of the system and the specific constraints involved, result in a hierarchical control algorithm, as illustrated in section 5. Controlling a general transportation system with transport elements coupled into sections has been shown to be more difficult than the case with independently controlled transport elements [1]. This is mainly due to the hybrid character of the system which also makes classical linear controllability analysis impossible to use. An alternative method for the purpose of controllability analysis is given in section 7. The results can be used to develop a design strategy for the transportation system. Simulations in section 8 finally verify the developed theory. An overview of all symbols used in this paper can be found in appendix A.

2. System Overview

The system considered in this paper is a transportation system, part of a general manufacturing system, as depicted in figure 1. The transportation system connects the production unit with the stamping/packaging section. The production unit produces objects at regular intervals and places them at the beginning of the transportation system. The objects are then transported by several conveyor belts to the stamping/packaging section, where they are stamped and packaged. The objects are finally transported to the boxing section where they are put into boxes.

The transportation system usually consists of a set of rollers spaced along the trajectory at small distances compared to the object length. In order for the system to be economically feasible, it is not possible to drive each roller independently. Instead, several rollers are tied together into longer *sections* using a conveyor belt. Each section is then driven independently from other sections by its own separate motor.

3. Control System Specifications

Fluctuations in production rate and various disturbances along the trajectory can throw off objects from their desired trajectories. The overall goal of the control system is to control the velocities of the different sections such that each object arrives at the stamping section, at position x_f, at the same instant and with the same velocity, v_d, as the stamping device. The time at which stamping occurs for object i is defined as $t_{f,i}$. For various mechanical reasons, the stamping/packaging section is driven at *constant speed*, v_d. Therefore, objects should arrive at a fixed frequency. If a position error exists at time $t_{f,i}$, $\tilde{x}_i \triangleq x_i(t_{f,i}) - x_f$, the image will not be stamped onto the correct location on the object. If a velocity error exists, $\tilde{v}_i \triangleq v_i(t_{f,i}) - v_d$, the image will be smeared on the object.

Furthermore, the control system has to ensure that sections that contact the same object run at equal velocities, $s_j = s_{j+1}$. This is mostly a concern when handling fragile objects, such as soap bars that have just been pressed. In case of a speed mismatch between section j and section $j+1$, the object will be stretched or compressed during

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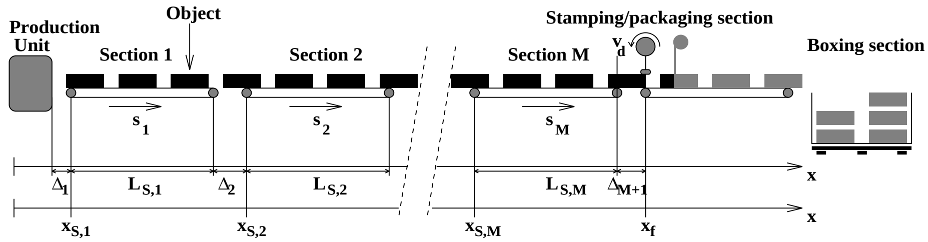

Figure 1: Overview of manufacturing system

transfer between the sections. This could consequently result in damage to the object.

4. System Model

The current model only includes the object dynamics $\dot{\boldsymbol{x}} = \boldsymbol{v}$. Actuator dynamics are neglected in order to simplify the analysis. It is assumed that section velocities are directly controllable. An overview of the system model is presented below:

4.1. Constraints

Several constraints need to be considered when modeling a general transportation system:

- *System imposed constraints*
  Velocity constraints on objects:
  Objects in the same section move at the same velocity (no slipping assumed).

- *Controller imposed constraints*
  Some constraints are not imposed by the machine configuration, but have to be handled by the control system.

  - Velocity constraints on sections:
    When one object is in contact with several sections, either because it is being transferred from one section to the next or because the length of the object spans several sections, the velocities of these sections need to be synchronized to avoid potential damage to the objects.
  - Collision avoidance:
    Objects should not collide during transportation [1].

4.2. Object Dynamics

The transportation system is assumed to consist of M sections with a total of N objects being produced. Furthermore, the object dynamics is considered to be a single integrator. The object velocities, $\boldsymbol{v}$, are obtained from section surface velocities, $\boldsymbol{s}$, through a linear mapping with matrix representation Q. It is assumed that the surface velocities are directly controllable with no actuator dynamics.

$$\boldsymbol{x} = [x_1 \; x_2 \; \dots \; x_N]^T \tag{1}$$

$$\dot{\boldsymbol{x}} \triangleq \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{s}, t) = \boldsymbol{v} \tag{2}$$

$$\boldsymbol{v} = Q(\boldsymbol{x}, t)\boldsymbol{s} \qquad \boldsymbol{s} \in \Re^M \qquad Q \in \Re^{N \times M} \tag{3}$$

All objects start at the beginning of the first section, $x_{0,i} = x_{S,1}$. When object i exits the production unit, at $t = t_{0,i}$, the i:th row of Q, corresponding to object i changes from $[0 \; 0 \; \dots \; 0]$ to $[1 \; 0 \; \dots \; 0]$, such that section 1 *starts* to drive the object. As a result, the Q-matrix has an explicit time-dependency. When the time becomes equal to the desired stamping time for object i, at $t = t_{f,i}$, the corresponding row i changes from $[0 \; 0 \; \dots \; 1]$ to $[0 \; 0 \; \dots \; 0]$. This effectively *stops* an object at its desired stamping time. When object i reaches the starting position of section j, $x_i(t) = x_{S,j}$, the 1 in row i corresponding to section $j-1$, which is currently driving the object, shifts one position to the right.

Defining $t_0 \triangleq t_{0,1}$ as the starting time of a production job and $t_f \triangleq t_{f,N}$ as the ending time of a production job, the errors at the respective stamping times of *all* objects are given by

$$\tilde{\boldsymbol{x}}(t_f) \triangleq \boldsymbol{x}(t_f) - x_f \tag{4}$$

$$\tilde{\boldsymbol{v}}(t_f) \triangleq \boldsymbol{v}(t_f) - v_d \tag{5}$$

As discussed in section 3, the goal of the control system is to minimize these errors.

The matrix $Q(\boldsymbol{x}, t)$ has an explicit state and time dependency. Compared with a standard linear, time-invariant system

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \tag{6}$$

one can see that the object dynamics can be described as a time-varying system with $B = Q(\boldsymbol{x}, t)$ and $A = 0$.

As a result, the system is of a *hybrid* nature. The discrete changes in Q can be described by an automaton for which every state corresponds to a different Q. In every state, the continuous system dynamics is given by a linear differential equation containing the corresponding value for Q. State transitions in the automaton are triggered by the continuous state $\boldsymbol{x}$ as illustrated in the example above. The combination of the linear dynamics and the switched dynamics results in a hybrid system that has been shown difficult to analyze and control using regular linear-system methods.

5. Object Spacing Control

The central idea behind the object spacing control is the observation that in a general transportation system, the spacing between objects can be controlled only if they are in *different* sections. The spacing between objects in the *same* section cannot be controlled. When an object becomes the most downstream object in a section, there exists a time window during which its spacing to the object in front of it can be adjusted. When the object reaches the next section, the velocity of the upstream section synchronizes with the velocity of the downstream section. The spacing to the object in front can no longer be adjusted until the object once again becomes the most downstream object in its section. In the meantime, the velocity of the object is determined by the control action for the most downstream object in its section.

For sections $j = 1 \dots (M-1)$, there exist two different modes of control. In the *object spacing control* mode, the velocity of section j is controlled in such a way that the interobject spacing between the most downstream object in section j, object $l \triangleq obj(j)$, to the object in front of it, object $l-1$, is kept as close to $d_{s,j+1}$ as possible. $d_{s,j+1}$ is the nominal interobject spacing in the next downstream section $j+1$. It is assumed here that $d_{s,j+1}$ is short compared to the length of section $j+1$, such that object $l-1$ is located in section $j+1$ while object l is being transferred to that section. The interobject spacing for object l is defined as $d_l(t) \triangleq x_{l-1}(t) - x_l(t) - L$. The controller attempts to make $d_l = d_{s,j+1}$ before object l reaches section $j+1$. Once object l and $l-1$ are both in section $j+1$, their relative spacing becomes fixed. One advantage of tracking a desired interobject spacing instead of an absolute reference position is that the controller is in a better position to avoid collisions.

Section j stays in the object spacing control mode until object l arrives at section $j+1$. The controller then switches to *section tracking control*. The velocity of section j, s_j, is controlled to track the velocity of the downstream section $j+1$, s_{j+1}. $s_{j+1}(t)$ can vary between $s_{min,j+1}$ and $s_{max,j+1}$. Hence, a master/slave relationship is imposed between any two neighboring sections. During synchronization, the upstream section becomes the slave and the downstream section the master. This control scheme fulfills the section velocity constraints given in section 4.1. To use the same control strategy between the production unit and section 1, it is assumed that the velocity with which the production unit places objects on the transportation system is controllable. In the extreme case that objects are being transferred between all sections in the machine simultaneously, the stamping unit dictates the speed and all sections are required to run at $s_j(t) = v_d$. The likelihood at which this situation occurs is a function of the transportation system geometry and disturbances.

When object l leaves section j, the process repeats itself with the next following object becoming object l, etc.

For section M, the tracking control mode remains identical. However, instead of interobject spacing control, *target tracking control* is used. In this mode, section M tracks a reference position derived from the position of the stamping device. By no longer using the interobject spacing controller, this strategy ensures that position errors at arrival time of an object are not transferred to its upstream object. This prevents errors from accumulating in case of a systematic error affecting all objects right before the stamping section.

6. Control algorithm

This section presents a more formal description of the control algorithm, better suited for implementation.

1. for j=M:-1:1
2. if no objects in contact with section j, $s_j(t) = v_d$
3. else ($*$ some objects are in contact with section j $*$)
   (a) let l = most downstream object in section j
   (b) if ($x_l(t) < x_{S,j+1}$) then
      i. if ($j = M$), ($*$ target tracking control $*$) control $s_M(t)$ such that object position error $\tilde{x}_l(t) \triangleq x_l(t) - x_{d,l}(t)$ approaches zero
      ii. else ($*$ $j \neq M$, object spacing control $*$) control $s_j(t)$ such that object spacing, $d_l(t) \triangleq x_{l-1}(t) - x_l(t) - L$, approaches desired interobject spacing $d_{s,j+1}$.
   (c) else ($*$ $x_l(t) \geq x_{S,j+1}$, section tracking control $*$) set $s_j(t) = s_{j+1}(t)$ to track the velocity of downstream section.
4. end (for j=M:-1:1)

This loop is continuously repeated. Note that different types of controllers can be used to do the object spacing and tracking control in step 3. A minimum-time controller is used in the simulations.

The interobject spacing algorithm generalizes easily to handle long objects that span more than 2 sections. All sections for which the index of the most downstream object is the same should be synchronized. This means that all sections touching the same object are considered to be one long section whose size changes dynamically depending on where the object is located along the trajectory.

7. Controllability Analysis

This section of the report discusses the controllability of the transportation system. For every section in the machine, a controllable region is determined around the nominal inter-object spacing. When an object is outside the specified controllable region of its section, there is a possibility that its position error cannot be reduced to zero before it reaches the stamping/packaging section. The controllable regions are based on a game-theoretic worst case analysis, where it is assumed that the downstream section is the adversary. Only when objects are within their controllable region, zero final position error can be guaranteed. Note that these controllable regions are conservative, since the worst case situation is rather unlikely, but they do guarantee zero final errors. The size of the controllable regions will depend on actuator limitations and transportation system geometry.

In order to reduce the complexity of the problem, actuator dynamics are neglected. Therefore, the object dynamics are modeled as a pure integrator, i.e. $\dot{\boldsymbol{x}} = \boldsymbol{v}$. The actuator limitations for section j are given by $s_j(t) \in \mathcal{S}_j = [s_{min,j}, s_{max,j}]$. Note that the controllability analysis is independent from controller design. It offers the limits of performance. To achieve these limits, a proper controller has to be chosen.

First, a formal definition of controllability is given. Then, the controllable region for section M is determined, the section right before the stamping/packaging section. Next, the upstream sections are considered. The controllable region for section j is given by $\mathcal{D}_j \triangleq [d_{min,j}, d_{max,j}]$.

Theorem 1 (Controllability Theorem)

For an object i in the transportation system, if the interobject spacings $d_k(t) \in \mathcal{D}_{sec(k)}$, where $k = i, i+1, \ldots, obj(M)$, then $\exists \boldsymbol{s}(\tau)$, $\tau \in [t, t_{f,i}]$ such that $\tilde{x}(t_{f,i}) = 0$.

In other words, if object i and all its downstream objects are in their respective controllable regions, then it is possible to control object i such that its final position error will be 0.

Proof: The boundaries of the controllable regions are defined as the minimum and maximum reachable interobject spacings for sections $sec(k)$ when going backwards in time from an initial zero position error at the stamping/packaging section, observing actuator limitations and system constraints. The controllable regions only change from one section to another. This corresponds to the only opportunity to change interobject spacings. During the time-window when the controllable region changes, the matrix Q is constant. Therefore, the system is linear in $\boldsymbol{s}$ so that any initial conditions *between* the two controllability region boundary points will result in zero position error. ∎

7.1. The controllable region for section M: $\mathcal{D}_M$

The controllable region for an object in section M is given by $\mathcal{D}_M \triangleq [d_{min,M}, d_{max,M}]$. $d_{min,M}$ and $d_{max,M}$ represent the minimum resp. maximum distance to the next downstream object in section M. The nominal spacing satisfies $d_{s,M} \in \mathcal{D}_M$. This is illustrated in figure 2.

Assume object 1, which arrived at the stamping/packaging section with zero position error, leaves section M. At that point section M switches to target tracking

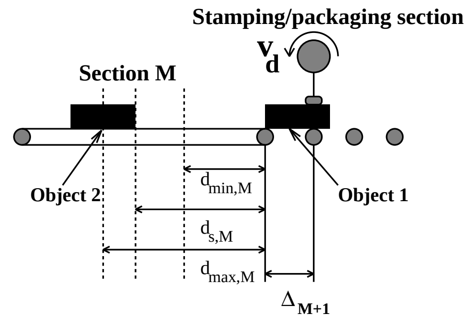

Figure 2: Transportation system configuration for derivation of $\mathcal{D}_M$

control. Object 2 has to arrive at the stamping/packaging section Δt seconds later, where Δt is given by

$$\Delta t = \frac{d_s + \Delta_{M+1}}{v_d}$$

$d_{max,M}$ is determined by the maximum distance that object 2 can travel at $s_{max,M}$ and still arrive at the stamping/packaging section in time, i.e. Δt seconds later. Since

$$\frac{d_{max,M} + \Delta_{M+1}}{s_{max,M}} = \Delta t$$

the following expression is obtained for $d_{max,M}$:

$$d_{max,M} = s_{max,M}\left(\frac{d_s + \Delta_{M+1}}{v_d}\right) - \Delta_{M+1} \tag{7}$$

In the same way, $d_{min,M}$ is given by

$$d_{min,M} = s_{min,M}\left(\frac{d_s + \Delta_{M+1}}{v_d}\right) - \Delta_{M+1} \tag{8}$$

Since $d_{min,M} \geq 0$,

$$s_{min,M} \geq \frac{\Delta_{M+1}}{\Delta_{M+1} + d_s} v_d$$

7.2. The controllable region for all upstream sections: $\mathcal{D}_j$

For sections $j = 1, \ldots, M-1$, the velocity of the downstream section is no longer constant, but can vary between $s_{min,j+1}$ and $s_{max,j+1}$. In order to find the controllable region for this case, a worst case scenario is assumed. Figure 3 illustrates the initial conditions.

Assume section M just switched to target tracking control and object 1 is at a distance $d_{max,M}$ from the downstream object. Also, assume object 2 just left section $M-1$, so section $M-1$ switched to spacing control. Object 3 is at a distance $d_{max,M-1}$ from object 2. Section M runs at $s_{max,M}$ until object 1 reaches the stamping/packaging section. This will occur as late as possible, since object 1 is at the edge of its controllable region. Object 2 also travels

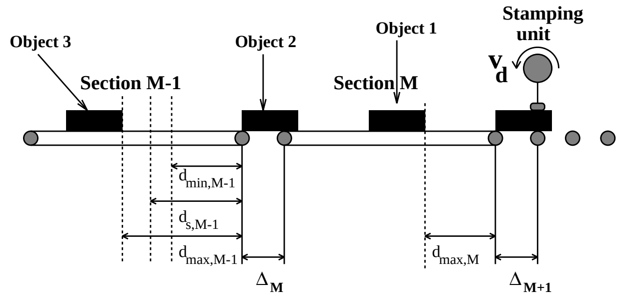

Figure 3: Transportation system configuration for derivation of $\mathcal{D}_j$

at $s_{max,M}$, since it resides in section M. Object 3 has to enter section M before object 2 has traveled $d_{max,M} + \Delta_M$. These are the worst possible conditions for section $M-1$. Section $M-1$ has to change $d_{max,M-1}$ to $d_{max,M}$ before object 3 enters section M. At the same time, object 2 is moving as fast as possible, for as long as possible.

Note that object 1 has to be as far as possible from the stamping/packaging section. If object 1 were closer, object 2 would travel the distance $d_{max,M} + \Delta_M$ at a lower average speed. This would offer more time to section $M-1$ to control object 3, since object 3 would be able to enter section M at a later point in time. Therefore, that scenario cannot be the worst case.

7.2.1 Case 1: $\Delta_M \leq \Delta_{M+1}$ Object 3 has to arrive at section M before object 2 has travelled a distance $d_{max,M}+\Delta_M$. If this condition cannot be satisfied, the spacing between object 2 and object 3 will be larger than the allowable $d_{max,M}$ once they are both in section M. Since

$$d_{max,M} + \Delta_M \leq d_{max,M} + \Delta_{M+1},$$

section M will constantly run at $s_{max,M}$ before object 3 reaches section M. Object 3 will arrive at section M before object 1 is transferred to the stamping/packaging section. $d_{max,M-1}$ can now be determined from

$$\frac{d_{max,M-1} + \Delta_M}{s_{max,M-1}} = \frac{d_{max,M} + \Delta_M}{s_{max,M}}$$

which results in

$$d_{max,M-1} = \frac{s_{max,M-1}}{s_{max,M}}(d_{max,M} + \Delta_M) - \Delta_M \quad (9)$$

7.2.2 Case 2: $L \geq \Delta_M > \Delta_{M+1}$ In this case, section M will synchronize with the stamping/packaging section before object 3 reaches section M. Therefore, during the time period that section $M-1$ tries to change $d_{max,M-1}$ to $d_{max,M}$, section M will first run at $s_{max,M}$ and then switch to v_d. The initial situation, as depicted in figure 3, remains the worst case scenario. The time at which object 3 has to reach the stamping/packaging section is now given by

$$\Delta t = \frac{d_{max,M} + \Delta_{M+1}}{s_{max,M}} + \frac{\Delta_M - \Delta_{M+1}}{v_d}$$

which results in the following expression for $d_{max,M-1}$:

$$d_{max,M-1} = s_{max,M-1}\Big(\frac{d_{max,M} + \Delta_{M+1}}{s_{max,M}} + \frac{\Delta_M - \Delta_{M+1}}{v_d}\Big) - \Delta_M \quad (10)$$

Equations (9) and (10) define a continuous function with respect to Δ_M. Note that it is impossible for section M, after running at v_d, to switch back to spacing control before object 3 reaches section M. This can be seen from the fact that object 2 covers the same total distance as object 1. The maximum distance that object 2 can travel before object 3 reaches section M equals $\Delta_M + d_{max,M}$. At the time at which section M would switch back to $s_{max,M}$, i.e. when object 1 leaves section M, object 1 and object 2 would have traveled $d_{max,M} + L$. Therefore, since $\Delta_M \leq L$, object 3 has to have reached section M before this section can switch back to $s_{max,M}$.

7.2.3 Generalization The previous discussion can be generalized for any two neighboring sections.

$$d_{max,j-1} = \frac{s_{max,j-1}}{s_{max,j}}(d_{max,j} + \Delta_j) - \Delta_j \quad (11)$$

in case $\Delta_j \leq \Delta_{j+1}$ or

$$d_{max,j-1} = s_{max,j-1}\Big(\frac{d_{max,j} + \Delta_{j+1}}{s_{max,j}} + \frac{\Delta_j - \Delta_{j+1}}{s_{synch}}\Big) - \Delta_j \quad (12)$$

in case $L \geq \Delta_j > \Delta_{j+1}$.

s_{synch} represents the velocity of section j when it synchronizes with section j+1. The safe value for $d_{max,j-1}$ is found by assuming $s_{synch} = s_{max,j+1}$.

The values for $d_{min,j-1}$ are determined in an analoguous way.

Note that equation (12) is only valid when section j is filled with objects. If this requirement were not satisfied, section j would not necessarily synchronize with section $j+1$ and therefore equation (11) should be used. It is also assumed that objects will not leave their respective controllable regions. The controllable regions calculated for the upstream objects would otherwise not correspond to their true controllable regions.

One further exception is the first object. It does not use interobject spacing control since there is no object in front of it. Instead, it is tracking a reference position, linearly increasing with velocity v_d, that will arrive at the stamping section, x_f at the desired stamping time for object 1, $t_{f,1}$. This is equivalent to having all downstream sections running at velocity v_d. $d_{max,j}$ as object 1 is moving through section j, is given by (13) and (14).

$$d_{max,M} = \frac{L_{S,M} + \Delta_{M+1}}{s_{max,M}}(s_{max,M} - v_d) \quad (13)$$

Figure 4: Transportation system configuration nr. 1 used in simulations

$$d_{max,j} = d_{max,j+1} + \frac{L_{S,j} + \Delta_{j+1}}{s_{max,j}}(s_{max,j} - v_d), \qquad (14)$$

$$j = (M-1), \ldots, 1 \qquad (15)$$

7.3. Choice of controllable regions

The desired size of the controllable regions will determine the geometry of the transportation system and the required actuator specifications. A first option is to let them decrease gradually towards the stamping/packaging section. This case is to be preferred when the position errors introduced along the transportation system are small compared to the timing errors of the production unit. On the other hand, for the case where the disturbances are larger than the timing errors of the production unit, it is better to keep the controllable regions as large as possible along the transportation system. One could also vary the size of the controllable regions according to the size of the disturbances at that location. For all cases, the desired object spacing can be chosen anywhere inside the controllable regions, depending on the mean and distribution of the position disturbances along the transportation system.

8. Simulations

This section presents some of the simulation results when using interobject spacing control.

The geometry of the transportation system is depicted in figure 4. There are $M = 4$ sections and $N = 3$ objects will run through the system. The spacing between each section is 0.1 m.

Object 1 is produced so late that all the sections it passes through have to run at ($s_{max,j}$) in order for the object to arrive at the stamping/packaging section on time. Objects 2 and 3 are also produced late. Object 2 is produced at an interobject spacing almost corresponding to $d_{max,1}$, the edge of the controllable region for section 1. Since object 1 is going at its maximum speed all the time, producing object 2 this late represents the worst case for object 2 since $d_{max,1}$ only represents the worst case if object 1 is going at its maximum speed!

The trajectories for all objects are shown in figure 5. The solid lines correspond to the positions of the leading edges, the dotted lines correspond to the trailing edges. The dash-dotted lines correspond to the start and end of each section and the top dotted line corresponds to the start of the stamping/packaging section. The crosses correspond to the nominal feeding times.

The velocities for all sections are shown in figure 6. An offset has been added such that the velocity profiles do not overlap. The dotted lines correspond to the stamping/packaging section velocity $v_d = 1.3$ m/s. The solid lines

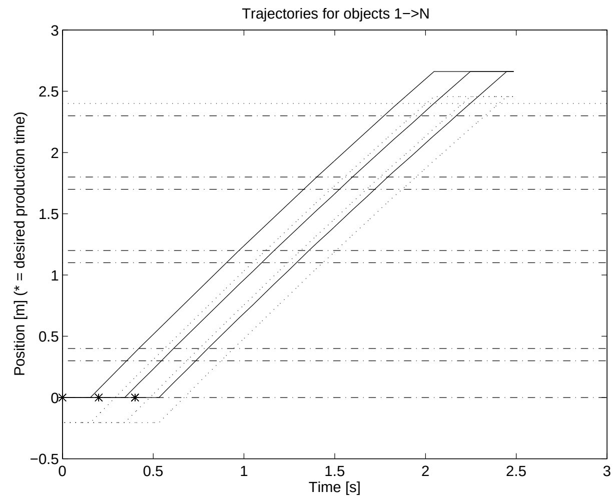

Figure 5: Simulation: Object trajectories

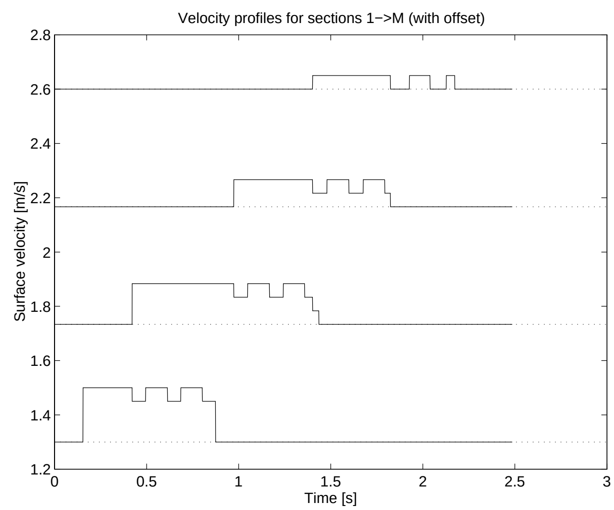

Figure 6: Simulation: Section velocities

correspond to each section velocity $s_j(t)$. It can be seen that $s_{max,j}$ is largest for the first section (the lower plot) and smallest for section 4 (the upper plot). Notice how the section velocities synchronize when an object is transferred between sections. For example, around $t = 1.4$ s, notice how section 2 has to synchronize with section 3 and just a short time later, with section 4 also (section 3 synchronizes with section 4 at this time).

Figure 7 shows the error evolution, i.e. $\tilde{d}_i(t) \triangleq d_i(t) - d_{s,sec(i)}$ for object $i = 2$. The dotted lines correspond to the start and end of each section and the dash-dotted line corresponds to the desired interobject spacing at the stamping/packaging section, d_s. The dashed lines correspond to $d_{max,j}$ for every section. The object starts out close to the edge of its controllable region. The interobject spacing

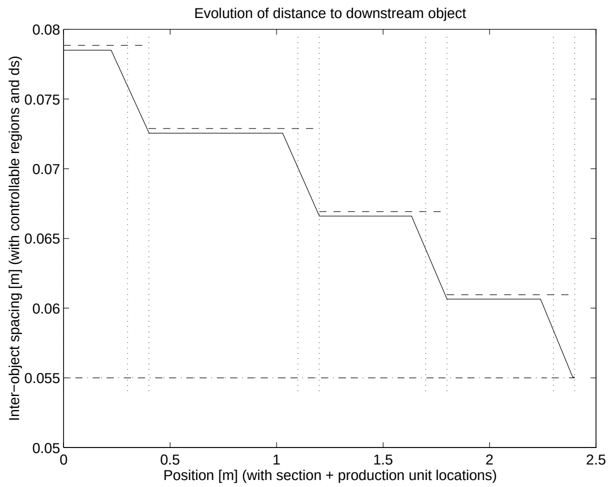

Figure 7: Simulation: Error evolution for object $i = 2$

decreases when the object becomes the most downstream object in its section. As expected, the error becomes zero right before entering the stamping/packaging section.

9. Conclusions

This paper presents a possible control approach to ensure timely arrival of objects at a stamping and packaging unit in a manufacturing system. The proposed hierarchical control algorithm satisfies the constraints imposed by the system and the transportation problem itself. Actuator dynamics are neglected. The algorithm, which is scalable, has shown to perform well in simulations.

Due to the hybrid nature of the system involved, the system controllability analysis differs from standard linear control techniques. The technique used in this paper, identifies controllable regions for the object positions along the transportation system. From the derived expressions for these regions, the influence of the system architecture on the controllability of the system can be analyzed.

10. Future Work

The current analysis neglects any actuator dynamics. A first extension to the theory developed in this paper is to include these dynamics and analyze its effect on the size of the controllable regions. Furthermore, specific disturbances have to be identified on a real system to develop the desired evolution of the controllable regions. The identification of the disturbances and according sizes of controllable regions should eventually lead to a set of optimal design rules for a transportation system.

11. Acknowledgements

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A. Symbol Sheet

N	-	number of objects in copy job
M	-	number of sections
i	-	object index, $i = 1, \ldots, N$
j	-	section index, $j = 1, \ldots, M$
T	-	time period between consecutive stamping instances
$obj(j)$	$\triangleq$	index of most downstream object in section j
l	$\triangleq$	$obj(j)$
$sec(i)$	$\triangleq$	index j of the section that drives object i
$t_{0,i}$	-	production unit exit time for object i
$t_{f,i}$	-	desired stamping time for object i
L	-	object length
x_f	-	starting position of stamping section
$x_{S,j}$	-	starting position of section j
$x_i(t)$	-	leading edge position of object i
$v_i(t)$	-	leading edge velocity of object i
$\boldsymbol{x}(t)$	$\triangleq$	$[x_1(t) \ldots x_N(t)]^T$
$\boldsymbol{v}(t)$	$\triangleq$	$[v_1(t) \ldots v_N(t)]^T$
$L_{S,j}$	-	length of section j
Δ_j	-	distance between section $(j-1)$ and j
$d_{s,j}$	-	desired interobject spacing in section j
d_s	$\triangleq$	$T\, v_d$, nominal spacing between stamping impacts
$d_i(t)$	$\triangleq$	$x_{i-1}(t) - x_i(t) - L$ interobject spacing between object i and its downstream neighbor, object $i-1$
$d_{min,j}$	-	minimum interobject spacing allowed in section j
$d_{max,j}$	-	maximum interobject spacing allowed in section j
$\mathcal{D}_j$	$\triangleq$	$[d_{min,j}, d_{max,j}]$ controllable region for section j
$s_j(t)$	-	surface velocity of section j
$s_{min,j}$	-	minimum surface velocity of section j
$s_{max,j}$	-	maximum surface velocity of section j
$\boldsymbol{s}(t)$	$\triangleq$	$[s_1(t) \ldots s_M(t)]^T$
$\mathcal{S}_j$	$\triangleq$	$[s_{min,j}, s_{max,j}]$ allowable velocities for section j
v_d	-	stamping unit velocity
$Q(\boldsymbol{x}, t)$	-	linear mapping matrix from section velocities $\boldsymbol{s}$ to object velocities $\boldsymbol{v}$

References

[1] Per Tunestål, Martin Kruciński
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