

# The Best Investment Strategies For Gold And Bitcoin

## Summary

With the rapid development of the economy, financial investment has become more and more common. But how to accurately grasp the market trend so as to make reasonable trading choices is a difficult problem. Our team uses historical price data to formulate a portfolio investment strategy of gold and bitcoin to help traders invest safely and make long-term profits.

For question 1, to formulate the best investment strategy, it is necessary to determine the best investment ratio and trading timing. For this reason, we establish an **online portfolio trading model with commissions**. Based on historical price data, risk assessment and expected returns, and introducing **risk preference coefficients**, a **multi-objective planning model** is constructed to obtain the best investment ratio of the day too. We use the "Double Time Period Confirmation Principle" to improve the "**Dow Theory**", and through the monitoring of historical prices, we use the Dow model to analyze the trend of asset prices to determine the long-term and medium-term trends of asset prices. Finally, take the breakout point of the medium-term trend as a suitable date and invest with the best investment ratio. We choose a risk preference coefficient of 0.6. Taking the commission ratio required by the question into the model calculation, we get the result that by the end of the transaction in 2021, the total income will reach \$72,087.61, and the investment rate of return will be 7108.7%.

For question 2, through the comparative analysis of the **profitability, risk, Sharpe coefficient** and **capital liquidity of the investment**, we obtained the conclusion that the model has higher returns, better liquidity, the best performance of risk control and Sharpe coefficient. After comprehensive analysis, it shows that the strategy provided by this model is the best investment strategy.

For question 3, we conducted a **sensitivity analysis** on transaction costs, then came to the conclusion that transaction costs have a greater impact on gold investment, less on Bitcoin investment, and mainly affect the number of transactions and investment ratio. After analysis, transaction fees have a direct impact on transaction frequency, transaction selection and transaction amount, and it affects the benefits and risks in actual transactions by changing the coefficients of the objective function and constraints in the model.

Finally, we analyze the shortcomings of the model and the direction of optimization, and attach a memo.

**Keywords:** Dow Theory; Online Portfolio Investment; risk preference coefficient; Sharpe ratio

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# 1 Introduction

## 1.1 Background and restatement of the problem

In the context of today's rapid financial development, traders can choose from numerous financial investment products and trade the volatile assets they invest in to reap the desired returns. Among them, gold and bitcoin are the financial assets selected in this paper. Bitcoin is traded every day while gold is traded only on the open day, and traders need to pay a proportional commission during the trade. Assume an investment activity with a starting capital of \$1,000 from September 11, 2016 to September 10, 2021. And by trading or holding the assets in the portfolio of gold, bitcoin, and U.S. dollars owned by the trader during this time, ultimately to maximize the final total return. To achieve this goal, we need to do the following:

- Build an investment model for this scenario, providing strategies based only on price data up to that day. And estimate the asset value on September 10, 2021 (investment deadline).
- Prove that the strategy given in the model established above is optimal.
- Find out how sensitive the strategy is to transaction costs and how transaction costs affect the strategy and final outcome.

## 1.2 Our works

1. Analyze the historical price data of the two assets, and design a calculation model for the transaction ratio of the two assets.
2. According to the Dow theory and previous data, determine the price trend and transaction timing.
3. By judging the investment ratio and transaction date, formulate the best trading strategy to achieve higher returns and lower risks
4. Evaluate the investment strategy formulated in terms of profitability, risk, and liquidity , to prove that the model is the best model
5. Analyze the impact of commission on the model, and explore changes in investment strategies and returns under different commission rates.

# 2 Model preparation

## 2.1 Assumptions

Considering the solvability and simplification principle of the model, we have the following assumptions:

**Assumption 1:** All raw data collections are reasonable and objective, and can reflect the real situation.

**Assumption 2:** It is assumed that the transaction time has a good continuity, that is, once a transaction is decided, it can be completed in an instant without interruption or delay.

**Assumption 3:** Individuals or institutions have limited influence on the trading market.

**Assumption 4:** There is no negative equity, that is, no loans.

## 2.2 Abbreviation and definition

Table 1 is abbreviation and definition.

Table 1: Abbreviation and definition	
parameter	description
$r_i$	Asset daily rate of return (i=1 is gold, i=2 is Bitcoin, the same below)
$P_0^j$	The purchase price of asset i
$P_1^i$	The sale price of asset i
$x_i$	investment in asset i
$C_i$	Commission rate for asset i
$D_0$	funds held
$R_p(x_1, x_2)$	Combined Total Return of Gold and Bitcoin
$\sigma_P^2(X_1, X_2)$	Combination Risk of Gold and Bitcoin
SR	sharpe ratio

## 2.3 Data processing

In the process of preprocessing the original data, we found that some data on the opening day of the gold market were missing. The missing date distribution is discretized and the number is small, so we use the Lagrangian interpolation method to fill in the missing data. The calculation formula is as follows:

$$\begin{cases} L_n(x) = \sum_{k=0}^n l_k(x) * f(x_k) \\ l_k(x) = \frac{(x - x_0)(x - x_{k-1})(x - x_{k+1}) \dots (x - x_n)}{(x_k - x_0)(x_k - x_{k-1})(x_k - x_{k+1}) \dots (x_k - x_n)} \end{cases} \quad (1)$$

Where  $l_k(x)$  is the interpolation basis function, and  $f(x_k)$  is the corresponding gold price on date  $x_k$ .

## 3 Online portfolio trading model with commission

Figure 1 is the schematic diagram of portfolio transaction . Before each transaction, we need to determine the ratio of the two transaction shares. Due to the existence of market risks and the dynamic volatility of risks, losses may reach unbearable conditions for traders, so we set the maximum loss tolerance ratio of traders to 10%. Once the loss may exceed this ratio, measures should be taken to minimize the loss as much as possible.

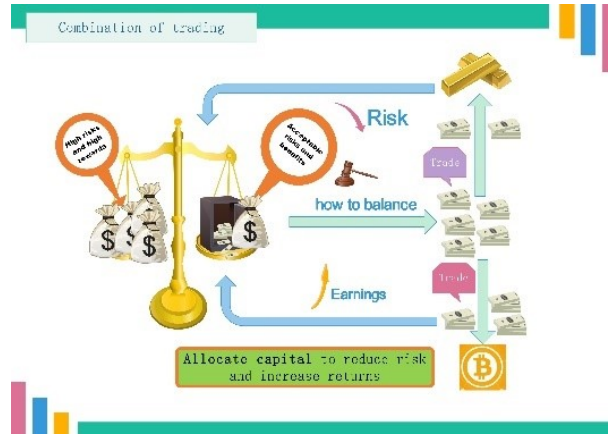


Figure 1: Schematic diagram of portfolio transaction

For both gold and bitcoin in trade, the daily rate of return is:

$$r_i = \frac{p_1^i - p_0^i}{p_0^i} \quad (i = 1 \text{ for gold}, i = 2 \text{ for Bitcoin}) \quad (2)$$

Where  $P_0^i$  is the price when asset  $i$  is purchased, and  $P_1^i$  is the price when asset  $i$  is sold. It can be seen that  $r_i$  is a random variable due to market uncertainty.

### 3.1 Model introduction

The online portfolio trading model of commissions, selects different capitals for trading, and maximizes the expected return of the investment portfolio through online portfolio investment and reduces unsystematic trading risks. Based on real-time price information, investors dynamically adjust the proportion of the investment portfolio according to the selected investment strategy, while taking into account the commission expenses. This investment method utilizes computer programs for dynamic position investment adjustment, which simplifies the difficulty of strategy implementation.

Markowitz pointed out that a trader's propensity to trade is determined by his attitude towards risky trades, the average return of the project and the risk of the trade, that is, a function of the average return and standard deviation of the trade portfolio. The average rate of return and risk for class  $i$  capital is:

$$R_i = E(r_i) \quad (3)$$

$$\sigma_i = E(r_i - R_i)^2 \quad (4)$$

### 3.2 Establishment of the portfolio trading model

Let  $x_i$  be the investment ratio of traders investing in assets of type  $i$  ( $i=1$  represents gold,  $i=2$  represents Bitcoin, the following definitions are the same as here), and  $C_i$  is the transaction commission ratio of assets of type  $i$ . We set the observation period of risk and return as 60 days,

then the 60-day return on the combined trading of gold and Bitcoin is:

$$\begin{cases} R_t(x_i) = \frac{\sum_0^t R_i x_i - C_i x_i}{C_i x_i} & (t < 60) \\ R_t(x_i) = \frac{\sum_{t-60}^t R_i x_i - C_i x_i}{C_i x_i} & (t \geq 60) \end{cases} \quad (5)$$

Their 60-day trading risks are:

$$\begin{cases} \sigma(R_t) = \sqrt{\frac{\sum_0^t (r_i - r)^2}{t - 1}} & (t < 60) \\ \sigma(R_t) = \sqrt{\frac{\sum_{t-60}^t (r_i - r)^2}{59}} & (t \geq 60) \end{cases} \quad (6)$$

Of course, traders' betting fees on gold and bitcoin should not exceed their own dollar holdings, namely:

$$x_{0j} + x_{ij} \leq D_j \quad (7)$$

where  $D_0$  is the current trader's dollar holdings. Then, we initially established the following combined trading model considering commissions:

$$\max(R_j - C_0)x_0 + (R_j - C_1)x_1 \quad (8)$$

$$\min \varsigma_{i=1}^2 \sigma_t(R_t)x_i \quad (9)$$

$$\sum_{i=1}^2 x_i + \sum_{i=1}^2 x_i C_i \leq D_0 \quad (10)$$

$$x_i \geq 0 \quad i = 1, 2 \quad (11)$$

In order to fit the reality and simplify the calculation and solution, we introduce the risk preference coefficient  $m$  to unify the benefits and risks, and use the weight distribution method to convert the multi-objective planning into a single-objective planning. Then, the above model can be optimized into the following form:

$$\max[m(R_j - C_0) - (1 - m)\sigma_0]x_0 + [m(R_j - C_1) - (1 - m)\sigma_1]x_1 \quad (12)$$

$$\sum_{i=1}^2 x_i + \sum_{i=1}^2 x_i C_i \leq D_0 \quad (13)$$

$$x_i \geq 0 \quad i = 1, 2 \quad (14)$$

### 3.3 The solution of portfolio transaction planning model

As mentioned above, the volatility of the price market is actually a reflection of the traders' past, present, and future will, which leads to a positive trend between returns and risks. Therefore, it is difficult to achieve the maximum benefit and the minimum risk at the same time. In order to fully consider the situation of traders' assessment of market risks and comprehensive judgment of returns, we adopt the weight coefficient conversion method and introduce the risk preference

coefficient  $m$  to measure traders' comprehensive consideration of returns and risks. The weighted objective function is as follows:

$$\max[m(R_j - C_0) - (1 - m)\sigma_0]x_0 + [m(R_j - C_1) - (1 - m)\sigma_1]x_1 \quad (15)$$

The multi-objective programming model is transformed into a single-objective programming model. Obviously, there are quadratic terms in the planning at this time, so it is a quadratic programming model. The objective function is a strictly convex function (it can be judged by calculating its Hessian matrix positive definite), and the feasible region (the range of constraints) is a convex set (it can be judged according to the definition of a convex set). Therefore, the solution of the objective belongs to the convex programming problem, and we use the simplex method to solve it, so that the objective function can be solved optimally in the corresponding time. (respectively, the capital investment for the purchase of gold and Bitcoin obtained at this time)

## 4 Trading strategies based on Dow Theory

### 4.1 Overview of theoretical principles

Dow Theory is a trend theory that predicts future price movements based on past price trends. The length of the reference period can be divided into long-term, medium-term and short-term trends. If in the trend of the trading price, each period of increase can push the price to break through the previous high, but the lowest point of the downward trend between these two highs is still higher than the previous low, that is, the high and lows are higher than the previous ones, a movement known as an uptrend. Similarly, a price action in which both the high and the low are lower than the previous one is called a downtrend.

Due to the continuity of trends, trend reversals may occur on trends of smaller time lengths, and Dow Theory does not deny that a mid-term trend change may signal a longer-term trend change. However, if traders only make decisions based on short-term trends, it is easy to fall into the predicament of contrarian trading, and the risk of loss is even greater. The core of Dow Theory is that traders should trade when the medium-term trend and the long-term trend are consistent, that is, when they are in the same trend. At other times, they should wait more patiently and look for opportunities.

Because the conclusions drawn by Dow Theory lag behind changes in market prices, the signals sent by the theory have a lag. Only choosing to enter and exit the market according to Dow Theory will reduce a lot of profits and trading space, which may cause the loss of trading opportunities. If the judgment interval is shortened and the timing of entry and exit is judged through multiple criteria, this problem will be solved to a great extent.

### 4.2 Formation of trading strategy

#### 4.2.1 Principle of "Dual Time Period Confirmation"

In order to deal with the shortcomings of the hysteresis of Dow Theory and increase the investment profit range, we have improved the Dow Theory. Adopt the "Dual Time Period Confirmation Principle" to judge the long-term and mid-cycle price trends by analyzing the previous prices. Take the long-term trend as the macro judgment direction and the medium-term trend as the trading

basis. When the long-term trend is the same as the medium-term trend, grasp the best time to buy and sell.

Due to the existence of commissions, the transactions of gold and bitcoin should not be too frequent. The change of gold transaction price is relatively stable, which is suitable for long-term investment. The price of bitcoin changes drastically, which is suitable for short- and medium-term investment. In view of the differences in capital attributes, trading trends, and price peaks of gold and bitcoin, we select different observation periods for gold and bitcoin to judge the price trend, so as to choose the best trading period.

#### 4.2.2 Trend judgment

##### 1. Selection of observation period

For gold, we use 20 days as the long-term trend observation period and 5 days as the medium-term trend observation period. For Bitcoin, we use 10 days as the long-term trend observation period and 4 days as the medium-term trend observation period.

##### 2. The method of judging the trend

Taking the long-term trend judgment of gold trading as an example, the observation period of the long-term trend is 20 days, and the period of medium-term trend observation is 5 days. Take historical transaction price data for analysis, and take 5 days as a small set. Take the 5-day average transaction price as the price reference data, and draw a line chart. Every 4 points is a large set, and the highest and lowest points are taken to form Dow highs and Dow lows, and rising highs and lows are determined as long-term uptrends, and falling highs and lows are to determine that the long-term period is a downward trend. The trend diagram is shown in Figure 2. If the highs and lows change irregularly, the price changes drastically during this period, the risk is high, and it is not suitable to invest. The judgment of the mid-cycle trend disk is similar to that of the long-term cycle.

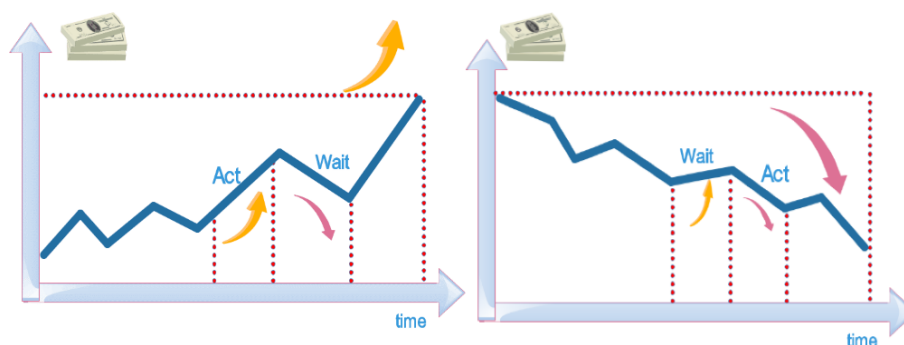


Figure 2: Dow Theory schematic

#### 4.2.3 Selection of transaction time

After consolidating and evaluating the current phase of the mid- to long-term trend, all a trader has to do is time the trade. We refer to buying gold or bitcoin as entering the market and the opposite as leaving the market.



First of all, for the entry into the market, it is required that both the long-term trend and the medium-term trend are downtrends, at which time the price continues to move lower. In a medium-term trend, the first reversal point from a downtrend to an uptrend is the time to enter the market. For example, we assume that the long-term trend at this time is declining, and the medium-term trend is also declining. The Dow high point of the previous mid-term trend is a, then when the price exceeds a, it means that the situation has reversed, and it is suitable to buy the asset at this time.

For the case of leaving the market, it is also judged by the reversal point in the medium-term trend. For example, we assume that the mid-term trend and the long-term trend are both uptrends at this time, the Dow high of the mid-term trend gradually rises, and then the most recent Dow high is broken, indicating that the situation has reversed, and the asset is sold at this time.

For cases where gold is not bought and sold on Saturdays and Sundays but Bitcoin can be bought and sold on Saturdays and Sundays, we choose to round up the gold transaction price on those days to the price of the previous trading day. In the actual calculation, we use the uncompleted raw data of gold to calculate the medium and long period, monitor the price trend of the long period and the medium period, and avoid trading on Saturday and Sunday. The schematic diagram of transaction timing is shown in Figure 3.

When determining the investment ratio of the two assets, the gold transaction data of gold on the day before Saturday and Sunday can be completed, and it can be regarded that the price does not rise or fall on Saturday and Sunday, and the asset does not trade. Use the gold-completion price table and the bitcoin historical price table as data reference, and use the online portfolio trading model to determine the best investment ratio for each day.

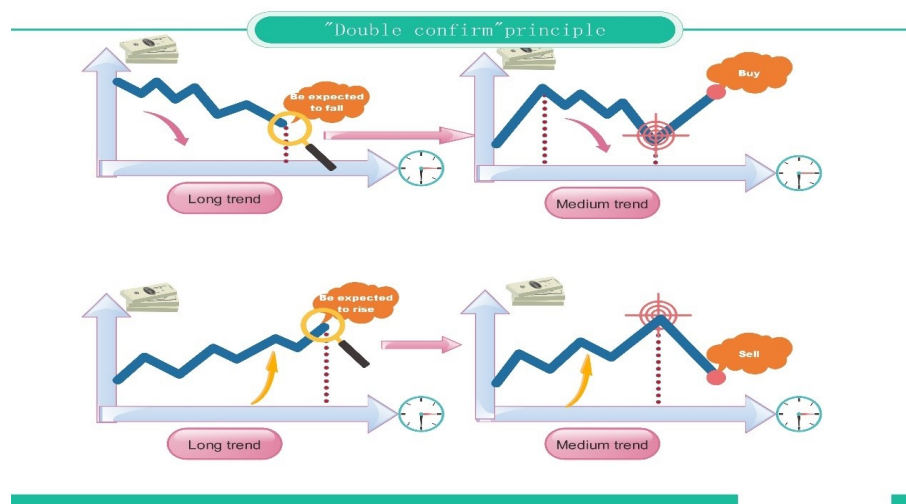


Figure 3: Schematic diagram of transaction timing

#### 4.2.4 Dow Theory Algorithm Structure

Timing judgment process for exit and entry is shown in Table 2.

Algorithm Principle of Dow Theory is shown in Table 4.

Table 2: Timing judgment process for exit and entry

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Algorithm: Loss control algorithms

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**Input:** Historical transaction data, intraday price, medium or long period length (multiple relationship between them)

**Output:** Judging the timing of entry and exit

1. Calculate the average transaction price for each mid-period
2. Mark the highest and lowest points of the daily trading price in each mid-cycle, and mark the rise and fall of the mid-cycle of the corresponding point. (marked since the second mid-cycle)

3. **If** both the high and the low are marked as an increase or both are marked as a decline: Mark the mid-cycle as a rise or fall (same trend as the marked point)

**Elif** The highest and lowest points are different marks:

4. Take the medium cycle from the first long cycle

**If** this medium period is the second medium period in the long period:

Mark the ups and downs of the previous mid-cycle as the state of this mid-cycle

Else

**For:** one day in a mid-cycle

**If** the transaction record of the previous mid-period is empty:

**if** the ups and downs of the last mid-cycle are up: skip today

**elif** the ups and downs of the last mid-cycle are down And yesterday's ups and downs were up:

Buy, and no longer trade in this mid-cycle

**Elif** the transaction record of the previous period is not empty:

Operates according to the Dow Theory algorithm

**For** mid-cycle in other long cycle:

**If** no previous transaction And The ups and downs of the last medium cycle are down And yesterday's ups and downs were up:

Buy and out of this long cycle

**Else**

The operation is performed according to the Dow Theory algorithm. If there is an operation that jumps out of this long cycle, if there is no operation, it will enter the next medium cycle.

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### 4.3 Analysis of the effect and rationality of the strategy

1. The judgment result of the trend: The Figure 4 and Figure 5 show the long-term and medium-term trends of gold and Bitcoin (1 means up, -1 means down, 0 means irregular price changes)  
Taking the trend chart of gold as an example, in the two long cycles of A and B, the price trend of gold is rising. Therefore, we judge that the long-term C is also an upward trend. The D interval corresponds to the long period C, and it can be seen that the trend in the D interval is a continuous rise. Next, we take the gold price in the long period C and draw the Figure 6 as follows: Some of the Dow highs and Dow lows in this period are marked in the figure, and it is not difficult to see that the trend in this long period is upward. Therefore, our prediction

Table 3: Algorithm Principle of Dow Theory

## Algorithm: Dow Theory Algorithm

**Input:** The ups and downs of the last mid-cycle, yesterday's ups and downs, whether the last trade was entry or exit

**Output:** Whether or not to operate today and what to do

**If:** the ups and downs of the last medium cycle are down And yesterday's ups and downs were up and the last transaction was an entry

Buy, no more trading in this mid-cycle **Elif:** the ups and downs of the last medium cycle are up And yesterday's ups and downs were down and the last transaction was an exit

Sell today, no more trading in this mid-cycle

**Else::**

No operation today

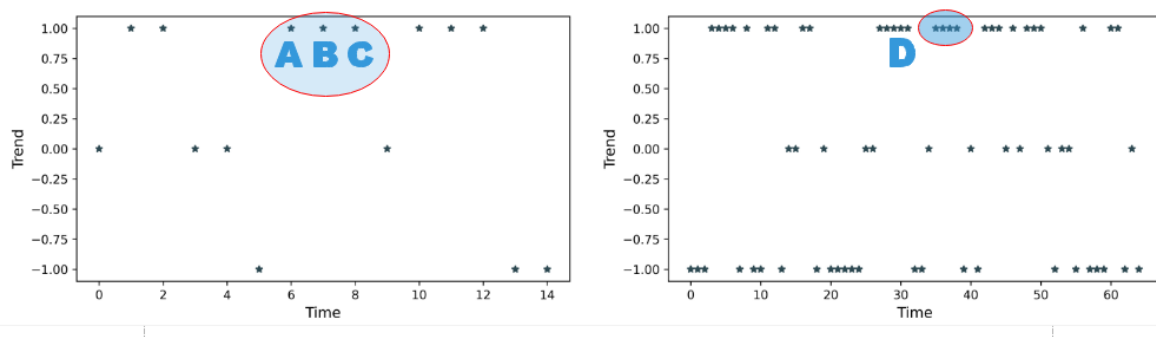


Figure 4: Gold's mid-cycle, long-term trend point chart

of the trend is reasonable. Of course, there will be instances of misjudgment, which are normal, as no prediction theory is 100% perfect. Even if the judgment is biased, losses may occur in the short term, but long-term investment has a great probability of achieving better profits.

## 2. rationality Analysis

Judging from the price trends of gold and bitcoin, whether from a long-term or short-term perspective, the distribution of gold is relatively uniform, while the distribution of bitcoin is very dense, basically showing a cluster distribution. This shows that the price volatility of gold is more stable than that of Bitcoin. Therefore, gold is more suitable for long-term trading, and bitcoin is more suitable for medium-term trading.

Table 4 lists some trading time judgment tables. According to the data in the table, it can be seen that according to this investment strategy, it is generally a good trading plan to buy at low points and sell at high points. Even on non-trading days for gold (gold is not open on the day corresponding to #), the combination trade is proceeding normally.

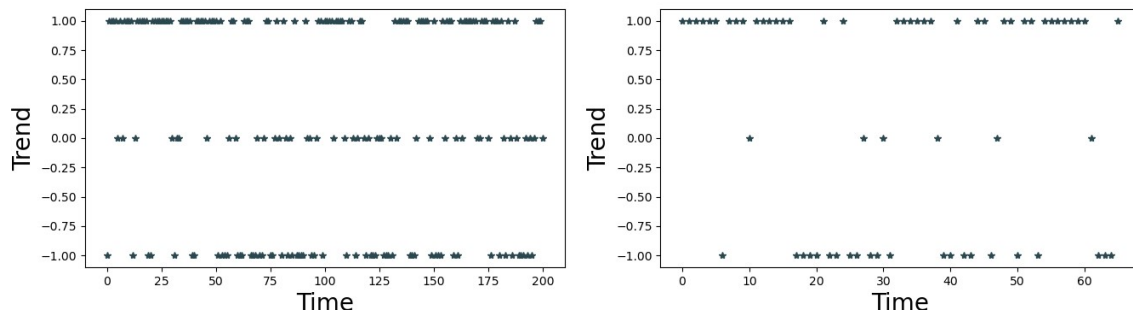


Figure 5: Bitcoin's mid-cycle, long-term trend chart

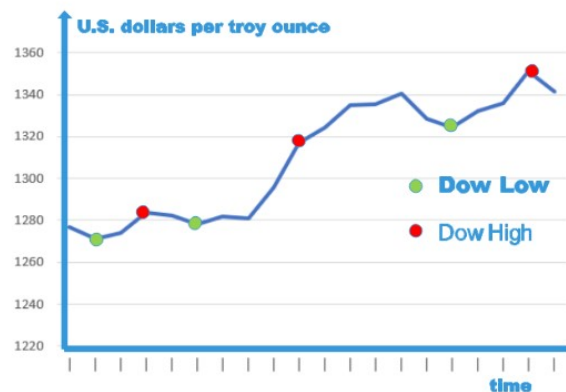


Figure 6: Line chart of gold price in long period C

## 5 Overall trading plan

### 5.1 Division of trading periods

A schematic diagram of the transaction date division is shown in Figure 7.

- Observation period - accumulating data

At the beginning of the transaction, the lack of historical price data, the long-term trend and the medium-term trend cannot be judged. In order to avoid the blindness of transactions, we choose not to conduct transactions for the time being and see the direction of market transactions. The observation time is 20 days, so the market transaction prices of the previous 20 days are accumulated as data.

- Entire capital period - test the market

After data accumulation, we can already predict the medium-term trend based on the obtained data. Only relying on the mid-cycle trend to guide the investment is risky, and it is easy to miss the best trading time. Although it will not achieve the maximum profit, it has a high probability of making a profit. At this point, we start investing (belonging to testing the market), predicting the mid-cycle Dow trend, monitoring the daily market prices of Bitcoin

Table 4: Judgment table for the best trading time

Data	Gold	Gold price of the day	Bitcoin	Bitcoin price of the day
8/27/19	0	1532.95	-1	10360.28
8/28/19	0	1537.15	0	10360.28
8/29/19	0	1540.2	0	9717.82
...	...	...	...	...
9/13/19	1	1503.1	0	10420.16
9/14/19	#	#	0	10363.9
9/15/19	#	#	0	10361.33
...	...	...	...	...
9/16/19	0	1497.2	0	10310.43

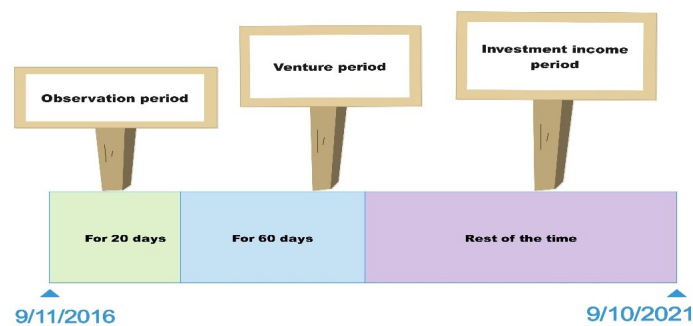


Figure 7: Diagram of the division of transaction time

and gold, and using the Dow theory trading strategy to capture the right time to buy and sell. At the same time, monitor the long-term cycle. When the long-term market trend can be judged, the investment plan will gradually mature and enter the investment income period.

- Investment income period - the model guides the investment and realizes the income

With the further deepening of investment, historical transaction data gradually increased, and the grasp and judgment of long-term trends became more accurate.

## 5.2 Confirmation of the transaction plan

The investment strategy process is shown in Figure 8. The details are as follows:

1. According to the degree of personal risk preference, select the risk preference coefficient 0-1. Here, we take the risk preference degree of 0.6 as an example. The larger the value of the risk preference coefficient, the greater the investment risk value.
2. Monitor prices in real time, make a judgment every day based on the improved Dow investment strategy, and make a decision whether it is suitable to buy or sell gold or Bitcoin

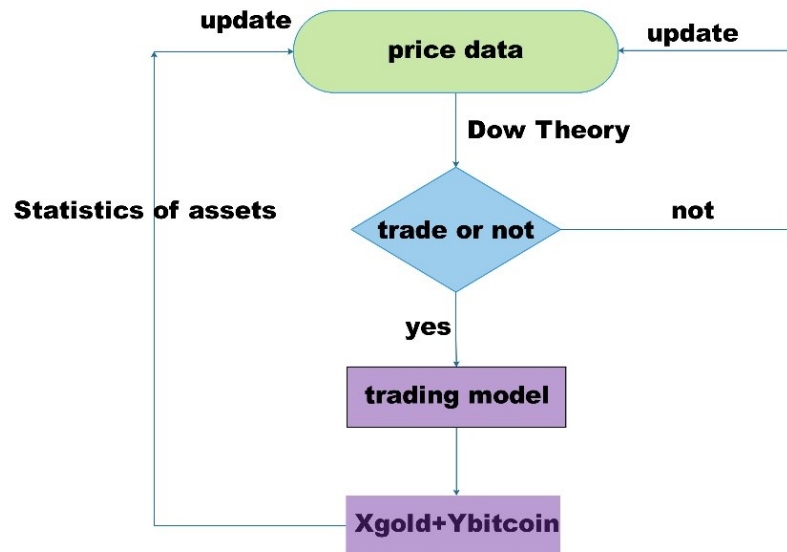


Figure 8: Investment strategy process

3. When a suitable transaction date is found, it will be based on the online portfolio investment strategy with transaction fees. then calculate the investment ratio, and get the optimal investment ratio  $(x_1, x_2)$ .
4. Invest according to the investment ratio, assuming that it is suitable to buy gold today. At this time, the dollar amount is  $D$ , then buy  $D \times x_1$  gold, The next amount is used to prepare for investing in Bitcoin and risk control .
5. Today's trading ends, enter the next day's observation.

### 5.3 Analysis of transaction results

According to this investment strategy, an investment plan for the five-year trading period from September 11, 2016 to September 10, 2021 is formulated. Figure 9 and Figure 10 is its schematic diagram .(Gold is bought in troy ounces ,the unit of bitcoin buying is bitcoin, and the selling proceeds are all converted into US dollars)

In the portfolio investment strategy, observe Figure 10 and compare the purchase and sale records of gold and Bitcoin. It can be seen that before 2018 , there were many transactions in gold, with gold as the main investment object. After 2018 , for the increase in bitcoin transactions,

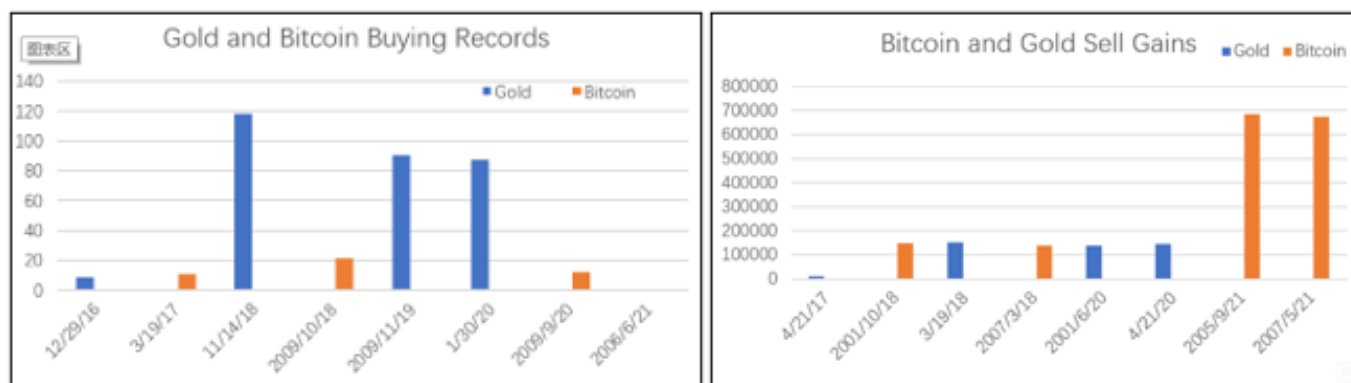


Figure 9: Gold (left), Bitcoin (right) purchase and sale records

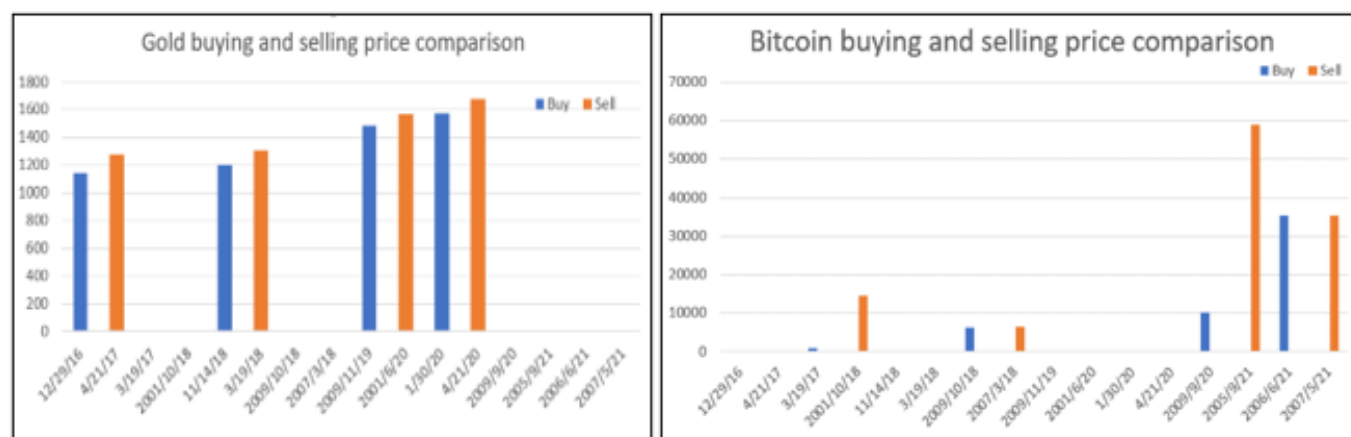


Figure 10: Comparison of changes in purchase and sale prices of gold (left) and Bitcoin (right)

bitcoin has become the main investment object. As can be seen from Figure 10, before 2018 , by investing in gold, the price difference between gold purchases and sales was small, and stable income was obtained. After 2018, The price difference between when Bitcoin is bought in 2020 and when it is sold in 2021 is huge. Assets will soar at the intersection of 2020 and 2021. The portfolio investment strategy has seized the moment of the Bitcoin price surge while constantly trading gold to ensure investment stability, achieving huge gains at a small cost.

## 6 Analysis of model advantages

In trading of the financial markets, there may be a variety of situations that need to be taken into account. We comprehensively expound its rationality and superiority from the perspective of model solving process and results of profitability, risk, Sharpe ratio and liquidity.

### 6.1 Profitability perspective

We get the trading strategy and income of only trading gold are Figure 12: (buy at the red dot, sell at the blue dot)

The trading strategy and income of only investing in Bitcoin are Figure 13: (red dots buy, blue

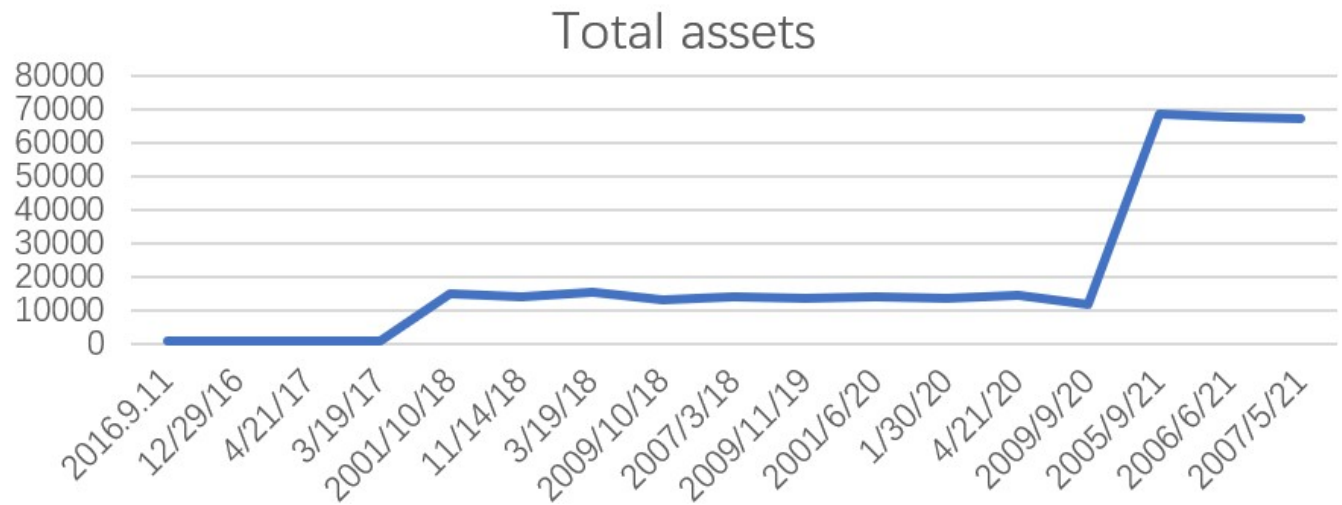


Figure 11: Total asset returns

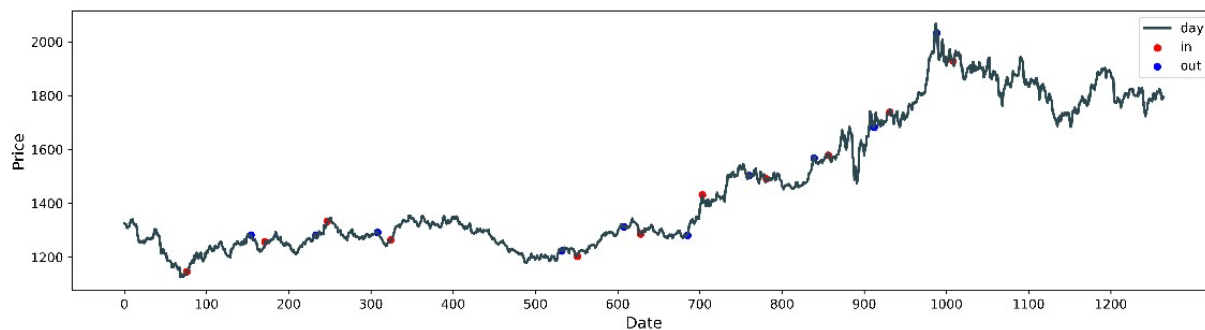


Figure 12: Gold Trading Chart

dots sell):

The Figure 13 shows that only buying Bitcoin has the highest return, only gold has the lowest return, and the case of combination trading is in the middle. Therefore, purely from the perspective of profitability, the portfolio trading model is only better. However, the combination trading model ensures the stability of the transaction. In the early stage of Bitcoin trading, there is a large decline in the early stage, and there is also a large decline in the later stage of the transaction. At this time, the price of gold rose steadily. It is a good time to enter gold, and the combination trading model has seized this opportunity to ensure the controllable risks while maintaining high returns.

## 6.2 Risk perspective

### 6.2.1 Control of the maximum loss amount

In the process of formulating the trading strategy, the maximum loss limit is introduced, and we set the maximum loss ratio that investors can bear for a single investment to be 10%. In each day's price monitoring, the price of the day is compared with the purchase price of the asset. When the decline exceeds 10%, in order to avoid the abnormal collapse of the asset system, the asset will



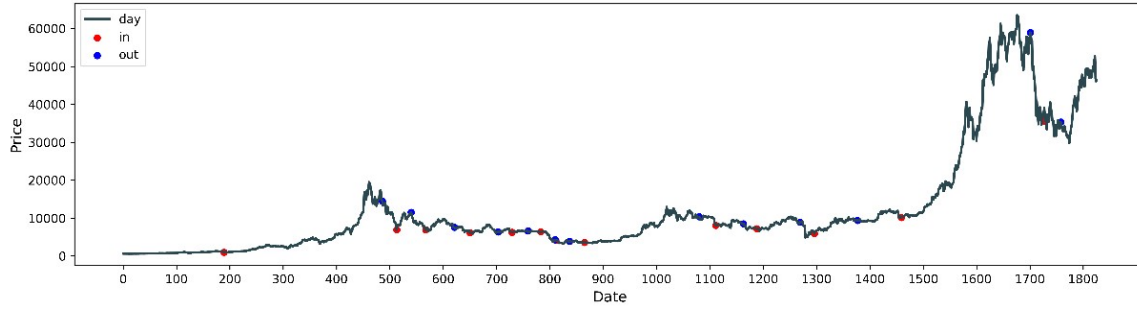


Figure 13: Bitcoin transaction diagram

be sold immediately and the loss will be stopped in time. The Figure 14 shows the change curve of return with risk appetite coefficient.

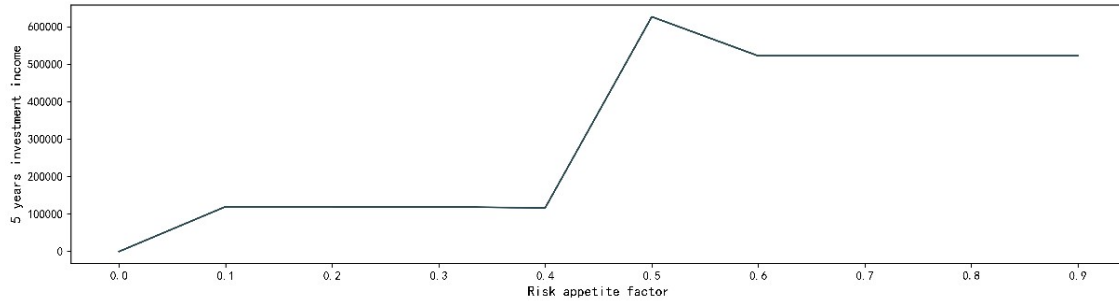


Figure 14: Risk-Reward Diagram

### 6.2.2 Free risk control

By comparison, when  $m < 0.4$ , the investment is relatively conservative and the benefit is small; when  $0.4 < m < 0.6$ , the risk is moderate and the benefit is the largest; when  $m > 0.6$ , the risk increases, the probability of investment loss increases, and the benefit decreases. Due to the continuous soaring price of Bitcoin in recent years, risk lovers are more inclined to invest in Bitcoin in order to obtain greater returns.

### 6.3 Risk-return composite indicator - Sharpe ratio

The Sharpe ratio is a classic indicator that takes both return and risk into account. It means that investors can get a little bit of excess return for every extra risk they take; if it is positive, it means that the underlying return is higher than the risk of volatility; if it is negative value, which means that the underlying operational risk is higher than the rate of return. It represents the ratio of investment return to risk taking, the higher the ratio, the better the portfolio.

The formula for calculating the Sharpe coefficient is as follows:

$$SR = \frac{r_p - r_f}{\sigma_q} \quad (16)$$

In the formula,  $r_p$  is the expected annualized rate of return of the portfolio,  $r_f$  is the annualized return of risk-free assets (there is no risk-free asset in this paper), and  $\sigma_q$  is the standard deviation of the annualized rate of return of the investment portfolio. In our investment selection, risk-free investment is not included, that is,  $r_f = 0$ .

By calculating the annual Sharpe coefficients of the four investment strategies of only investing in gold, only investing in Bitcoin, gold and Bitcoin with a constant proportion of investment, and online portfolio trading investment, the advantages and disadvantages of the four trading strategies are compared and analyzed.

The Table 5 is the Sharpe coefficients of the four portfolios: According to the table below, among the four investment strategies, online portfolio investment has the best performance in terms of Sharpe coefficients, all of which are positive, and its Sharpe coefficient values are higher than those of the other three. This kind of investment strategy is relatively large. In the case of considering both the return and the risk, the online portfolio investment strategy can effectively control the risk and improve the return. The table below shows the calculated Sharpe coefficients:

Table 5: Annualized Sharpe Coefficient

Investment Strategy	2016	2017	2018	2019	2020	2021
Gold	-0.2128	0.0752	-0.0085	0.0978	0.0784	-0.0362
Bitcoin	0.2436	0.1725	-0.0691	0.0663	0.1173	0.0638
Constant ratio investment	-0.2128	0.0752	-0.0085	0.0978	0.0784	-0.0362
Online Portfolio Investing	0.3035	0.1753	0.0069	0.1189	0.1576	0.1505

## 6.4 Liquidity perspective

Table 6: transaction records of different strategies

Investment Strategy	Earnings (USD)	Number of transactions
Gold	8327	21
Bitcoin	387823.31	26
Portfolio investment	72087.61	16

According to the Table 6, the number of transactions of portfolio investment is 5 less than that of only investing in gold, but the income is 9 times that of only investing in gold. Although the return of portfolio investment is no better than that of only investing in gold, the number of transactions is less than 10 times, which effectively avoids some risks in the process of Bitcoin investment, so that the return has been in a relatively stable upward state.

By analyzing the investment results under the guidance of the online portfolio investment model, this investment strategy has high profitability, and the risk preference coefficient is selected as 0.6, which is a risk preference investment type. At the same time, the maximum loss amount can be effectively controlled, the Sharpe coefficient has the best performance, and the capital liquidity is relatively high. The above investment indicators are considered in a comprehensive comparison, and the investment model is the optimal model.

## 7 Sensitivity of the strategy to transaction costs

1. Sensitivity of the strategy to transaction costs In a transaction, whether it is a profit or a loss, a corresponding proportion of the commission needs to be paid. Therefore, the commission value is a data that affects the transaction for a long time, and the higher the commission is, the profit of the trader will definitely decrease. However, due to the differences in the price fluctuations of gold and bitcoin markets, different commission ratios have different effects on them. In the title, the commissions for gold and bitcoin are 1% and 2% of the transaction amount, respectively. In order to analyze the influence of commission on the model, we take the commission ratio of gold to be approximately 0, 1%, 5%, 10%, and the commission ratio of Bitcoin to be approximately 0, 1%, 5%, 10%. According to the model established above, we solve these different commission ratios respectively, and obtain the corresponding final income values under the combination of these commission ratios, as shown in the Table 6 (the horizontal axis represents the change of gold commission, and the vertical axis represents the bit Coin commission changes): According to the analysis in 7, when the commission

Table 7: Optimal income statement of portfolio transactions with different commission ratios

Commission Rate	0	0.01	0.05	0.1
0	77768.6	54489	41627.4	21703.6
0.01	77768.6	46708	32899.6	17153
0.05	77768.6	46708	28329.4	13618
0.1	77768.6	46708	28329.4	8451.8

ratio of Bitcoin remains unchanged and the commission ratio of gold increases, the final income will drop rapidly; but when the commission ratio of gold remains unchanged and the commission ratio of Bitcoin increases, the final income changes are not as drastic as in the previous case. This makes perfect sense, since Bitcoin sometimes trades at extremely high prices and the market price fluctuates so much that commissions affect it less than gold. When the commissions of both are close to 0, the final profit price is nearly five thousand dollars higher than the scenario set, and when the commissions of both are close to 10%, the final profit will become very small, which also conforms to the general law that the income changes with the commission ratio.

In conclusion, the commission ratio has a far greater impact on gold trading than on Bitcoin, and has a huge impact on the returns of the portfolio investment model.

2. The impact of transaction costs on the model

Figure 15 is a schematic diagram of the impact of transaction costs on investment strategies. In the model we established, transaction cost mainly affects the investment ratio and the actual amount of investment. In the determination of the investment ratio, an online portfolio investment model with transaction costs is used to construct a multi-objective programming algorithm. The transaction costs affect the constraints and the coefficients of the objective function. When the transaction cost changes, the rate of return changes. In order to achieve the maximum return and the minimum risk, the investment ratio is adjusted to find a new optimal solution.

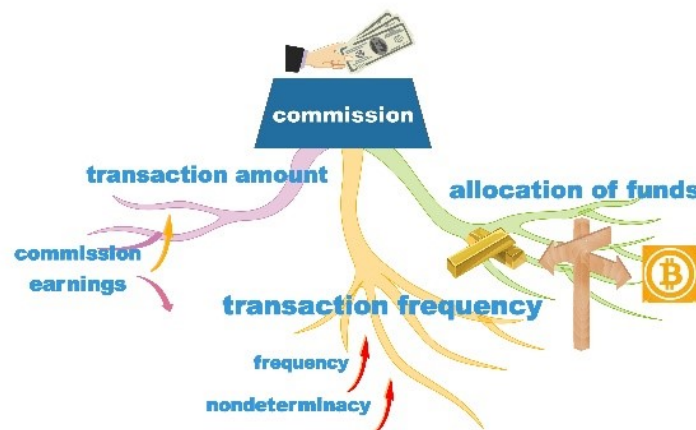


Figure 15: The impact of commissions on strategies

As transaction fees increase, the amount actually used to purchase investment products will decrease, and the return on a single investment will also decrease. When transaction costs increase, the number of transactions will decrease accordingly. The decrease in the number of transactions will have an uncertain impact on profitability, and investment risks will also increase.

## 8 Model advantages and disadvantages and optimization solutions

1. **Model Advantages** The model makes full use of historical transaction data. First, the investor chooses the risk preference coefficient ( $0 < m < 1$ ), and obtains the optimal solution of the investment ratio  $(x_1, x_2)$  through the online portfolio investment strategy including transaction costs. Combined with the classic Dow Theory in the investment market, this paper analyzes the shortcomings and deficiencies of the Dow Theory, and adopts the principle of "Dual Time Period Confirmation" to improve the Dow Theory. Successfully seized the best time to buy or sell an asset.
2. **Disadvantages of the model** The early investment judgment of the model is only based on the medium-term trend, and the Dow theory is used to monitor the timing of buying or selling assets. There is a certain lag in the grasp of the transaction date, resulting in less profit from the early investment. Only when the time exceeds 60 days, the long-term trend is gradually clear, and the investment enters the investment income period, it will have good returns.
3. **Model optimization direction**
  - Select a more suitable observation period. The difference of observation period has certain influence on the selection of transaction date. The shorter the observation period, the more sensitive it is to market price changes, the more transactions are

recommended, and the greater the risk. The longer the observation period, the more likely it is to miss many suitable trading dates, resulting in reduced returns. Therefore, according to different assets, exploring an observation period that conforms to the law of market changes will allow you to have a better grasp of the timing of transactions and achieve greater profits and less risk.

- Selection of risk preference coefficient. The risk preference coefficient directly determines the choice of investment ratio. Risk-loving people are keen to invest in Bitcoin, and risk-averse people are keen to invest in gold. Therefore, the choice of risk appetite coefficient plays a key role in investment decisions. In different trading periods, there may be different optimal risk factors. Therefore, the dynamic selection of risk coefficients according to market laws will help to achieve stable returns.
- Optimization algorithm structure. By optimizing the algorithm structure, conducting more index analysis on asset transaction prices, conducting joint analysis of different assets, and exploring their internal correlations, we will be able to grasp the market trend better, make more accurate investments, and obtain greater returns.

## 9 Memorandum

The formulation of our investment strategy is mainly the selection of the best investment ratio and the grasp of the best trading date. The investment ratio and transaction date are based on the analysis of historical transaction data. With the passage of time, the model is in a dynamic change and has strong universality.

The models used in our investment strategies are mainly: online portfolio investment models with transaction fees and trading strategies based on Dow Theory.

The online portfolio investment model is used to find the best investment ratio allocation in real time. By analyzing the income and risk of the transaction data of the first 60 days, the multi-objective planning model is used to pursue the maximum return and the minimum risk, and the risk preference coefficient is introduced to integrate the risk and return according to the unification of weights. It is transformed into a single-objective programming problem, and the simplex method is used to find the optimal solution of the investment ratio.

Trading strategies based on Dow Theory are used to capture the best trading opportunities. Through the detection and trend analysis of historical transaction data, as well as the monitoring of the long-term and medium-term trends of Bitcoin and gold, the Dow highs, Dow lows and trend trends can be judged, and combined with "dual time cycle confirmation" principles to determine the best date to buy or sell.

The investment strategy is: using the transaction model based on Dow Theory, if a suitable purchase opportunity is found, the online portfolio transaction model with transaction fees is used to calculate the optimal investment allocation ratio at this time, and purchase the corresponding assets proportionally. If it is judged that it is suitable to buy gold on that day, the investment ratio is calculated as  $\alpha$ , and the dollar holdings are  $D$ . So  $D\alpha$  is used to purchase gold, and the remaining amount will not be invested for the time being to prepare for Bitcoin investment and risk control. Using our investment model to guide traders to invest, traders only need to choose their preferred

risk preference coefficient ( $0 < m < 1$ ). The larger the risk preference coefficient, the greater the risk. We choose the risk preference degree as 0.6 to guided investments. After that, traders only need to input the price data of gold and bitcoin in the past, and the model will output whether it is recommended to trade today, the trading behavior, the type of trading assets, and the best trading ratio. Traders make buy, hold, and sell operations based on model recommendations.

On Saturdays and Sundays, the price of gold can be regarded as unchanged and without profit. Based on our Dow Theory model, we will not choose a day without profit for trading, but continue to monitor the price trend of Bitcoin to determine whether it is suitable for trading. It effectively solves the problem that the gold market does not open on Saturdays and Sundays, and Bitcoin can be traded every day.

Using our investment model to guide traders to invest, traders only need to choose their preferred risk preference coefficient ( $0 < m < 1$ ). The larger the risk preference coefficient, the greater the risk. We choose the risk preference degree as 0.6 to guided investments. After that, traders only need to input the price data of gold and bitcoin in the past, and the model will output whether it is recommended to trade today, the trading behavior, the type of trading assets, and the best trading ratio. Traders make buy, hold, and sell operations based on model recommendations.

Guided by our model, the asset increased from \$1,000 on September 11, 2016 to \$72,087.61 on September 10, 2021. The investment rate of return is 7108.7%, the total number of transactions is 16, the maximum loss ratio of a single investment does not exceed 10%, the average Sharpe coefficient is 0.152, and the Sharpe coefficient is positive every year. The investment has good profitability, and at the same time, it realizes a good control of risks, realizes the optimal combination of income and risk, and achieves good investment results.

By calculating the five-year investment returns under different commission ratios, and comparing the return and risk data of the investment, we analyze the sensitivity of the investment model to transaction costs. Transaction fees have a large impact on the expected return of the model. Transaction costs mainly affect the investment ratio. With the increase of transaction costs, due to the relatively stable change of gold price, the income of investing in gold decreases rapidly, so the proportion of gold investment drops sharply. Due to the rapid price rise of Bitcoin in recent years, the profitability is high. So the investment ratio is less affected by transaction fees.

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