1. 分别用单纯形法中的大M法和两阶段法求解下列线性规划问题。

解：

1. 大M法：将原问题化为标准形，并加入人工变量如下：

利用单纯形法进行迭代：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 段 |  | | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  | 1 | 2 | 3 | 4 | 1 | 0 | 1.75 |
|  |  |  | 2 | 1 | 1 | [2] | 0 | 1 | 1.5 |
|  | | | 5-3M | -2-3M | 3-4M | -6-6M | 0 | 0 |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 2 |  |  |  |  |  |  |  |  |  |  |
| M |  | 1 | -3 | 0 | [1] | 0 | 1 | -2 | 1 |
| -6 |  | 1.5 | 1 | 0.5 | 0.5 | 1 | 0 | 0.5 | 3 |
|  | | | 11+3M | 1 | 6-M | 0 | 0 | 3M+3 |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 3 |  |  |  |  |  |  |  |  |  |  |
| 3 |  | 1 | -3 | 0 | 1 | 0 | 1 | -2 | 1 |
| -6 |  | 1 | 2.5 | 0.5 | 0 | 1 | -0.5 | 1.5 | 3 |
|  | | | 29 | 1 | 0 | 0 | M-6 | M+15 |  |

因为是一个很大的正数，此时均为正，因此得到最优解，最优值为

2)两阶段法：第一阶段，构造辅助问题

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 段 |  |  |  | 0 | 0 | 0 | 0 | 1 | 1 |  |
|  |  | b |  |  |  |  |  |  |
| 1 | 1 |  | 7 | 1 | 2 | 3 | 4 | 1 | 0 | 1.75 |
| 1 |  | 3 | 2 | 1 | 1 | [2] | 0 | 1 | 1.5 |
|  |  |  | -3 | -3 | -4 | -6 | 0 | 0 |  |

利用单纯形表求解：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 2 | 1 |  | 1 | -3 | 0 | [1] | 0 | 1 | -2 | 1 |
| 0 |  | 1.5 | 1 | 0.5 | 0.5 | 1 | 0 | 0.5 | 3 |
|  |  |  | 4 | 0 | -1 | 0 | 0 | 3 |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 3 | 0 |  | 1 | -3 | 0 | 1 | 0 | 1 | -2 |  |
| 0 |  | 1 | 2.5 | 0.5 | 0 | 1 | -0.5 | 1.5 |  |
|  | | | 0 | 0 | 0 | 0 | 1 | 1 |  |

所以，最优解为：最优值：

因人工变量，则原问题的基可行解为：,进入第二阶段，计算如下表所示：

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 段 |  | | | 5 | -2 | 3 | -6 |  |
|  |  | b |  |  |  |  |
| 1 | 3 |  | 1 | -3 | 0 | 1 | 0 | 0 |
| -6 |  | 1 | 2.5 | 0.5 | 0 | 1 | 0 |
|  | | | 29 | 1 | 0 | 0 |  |

由上表可知，检验数均大于等于0，所以得到最优解：，最优值为

1. 用单纯形法中的大M法求解下列线性规划问题。

1. 大M法：将原问题化为标准形，并加入人工变量

利用单纯形表求解：

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 段 |  | | |  |  |  |  |  |  | -M | -M |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | 1 | -1 | -1 | -1 | 1 | 0 | 0 | 0 | / |
| - |  |  | 0 | 1 | 1 | -1 | 0 | -1 | 1 | 0 | / |
| -M |  | 1 | 1 | [1] | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
|  | | | 15+M | 10+2M | 8+2M | 12 | 0 | -M | 0 | 0 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | / |
|  |  |  | 1 | 0 | 0 | [2] | 0 | 1 | -1 | 1 | 1/2 |
| 10 |  | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
|  | | | 5-M | 0 | -2 | 2-2M | 0 | -M | 0 | -10-2M |  |

此时检验数全为负值，选择绝对值最大的进基。

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | [2] | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1/2 |
|  |  |  | 1/2 | 0 | 0 | 1 | 0 | 1/2 | -1/2 | 1/2 | 1 |
| 10 |  | 1/2 | 1/2 | 1 | 1 | 0 | 0 | -1/2 | -1/2 | 1/2 | 1 |
|  | | | 4 | 0 | -2 | 0 | 0 | -1 | 1-  M | -M-11 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | 1 | 0 | 0 | 0 | 1/2 | 0 | 0 | 1/2 |  |
|  |  |  | 0 | 0 | 0 | 1 | -1/4 | 1/2 | -1/2 | 1/4 |  |
| 10 |  | 1/4 | 0 | 1 | 1 | 0 | -1/4 | -1/2 | 1/2 | 1/4 |  |
|  | | | 0 | 0 | -2 | 0 | -2 | -1 | 1-M | -M-13 |  |

因此，最优解为,最优值为

3、分别用两阶段法和大M法求解：

解：化为标准形，并加入人工变量如下：

利用单纯形法进行迭代：

第一次迭代：

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | -3 | -2 | -1 | 1 | M | M |  |
|  |  |  |  |  |  |  |  | 右端项 |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | |  |  |  |  |  |  |  |

入基，出基

第二次迭代：

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | -3 |  |  | 1 | M | M |  |
|  |  |  |  |  |  |  |  | 右端项 |
|  |  | 0 |  |  |  |  |  | 6 |
|  |  | 0 |  |  |  |  |  | 3 |
|  |  | 1 |  |  |  |  |  | 4 |
|  | | 0 |  |  |  |  |  |  |

入基，出基

第三次迭代：

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | -3 | -2 | -1 | 1 | M | M |  |
|  |  |  |  |  |  |  |  | 右端项 |
|  |  | 0 | 0 |  |  |  |  |  |
| -2 |  | 0 | 1 |  |  |  |  |  |
|  |  | 1 | 0 |  |  |  |  |  |
|  | |  | 0 |  |  |  |  |  |

入基，出基

第四次迭代：

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | -3 | -2 | -1 | 1 | M | M |  |
|  |  |  |  |  |  |  |  | 右端项 |
|  |  | 0 | 0 | 1 |  |  |  |  |
|  |  | 0 | 1 | 0 |  |  |  |  |
|  |  | 1 | 0 |  |  |  |  |  |
|  | |  | 0 | 0 |  |  |  |  |

因为M是一个很大的正数，此时的 均为正，因此得到最优解

,最优值为15.

②解：

因为约束条件的系数矩阵中不含有3阶单位矩阵，所以需引入人工变量。

**将原问题化为标准型：**

min

s.t.

**第一阶段问题：**

Min

s.t.

下面以表格形式给出迭代过程：

(1)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
|  |  |  |  |  |  |  |  | 右端项 |
| 1 |  | 3 | 2 | 1 | 0 | 1 | 0 | 15 |
| 1 |  | 5 | 1 | 2 | 0 | 0 | 1 | 20 |
| 0 |  | 1 | 2 | 1 | 1 | 0 | 0 | 10 |
|  | | -8 | -3 | -3 | 0 | 0 | 0 | -35 |

入基，出基

(2)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
|  |  |  |  |  |  |  |  | 右端项 |
| 1 |  | 0 | 7/5 | -1/5 | 0 | 1 | -3/5 | 3 |
| 0 |  | 1 | 1/5 | 2/5 | 0 | 0 | 1/5 | 4 |
| 0 |  | 0 | 9/5 | 3/5 | 1 | 0 | -1/5 | 6 |
|  | | 0 | -7/5 | 1/5 | 0 | 0 | 8/5 | -3 |

入基，出基

(3)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
|  |  |  |  |  |  |  |  | 右端项 |
| 0 |  | 0 | 1 | -1/7 | 0 | 5/7 | -3/7 | 15/7 |
| 0 |  | 1 | 0 | 3/7 | 0 | -1/7 | 2/7 | 25/7 |
| 0 |  | 0 | 0 | 6/7 | 1 | -9/7 | 4/7 | 15/7 |
|  | | 0 | 0 | 0 | 0 | 1 | 1 | 0 |

此时人工变量都是非基变量，这样就得到了初始基本可行解

最优值

**第二阶段问题为：**

min

s.t.

(1)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | | -3 | -2 | -1 | 1 | 0 |
|  |  |  |  |  |  | 右端项 |
| -2 |  | 0 | 1 | -1/7 | 0 | 15/7 |
| -3 |  | 1 | 0 | 3/7 | 0 | 25/7 |
| 1 |  | 0 | 0 | 6/7 | 1 | 15/7 |
|  | | 0 | 0 | -6/7 | 0 | 90/7 |

入基，出基

(2)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | | -3 | -2 | -1 | 1 | 0 |
|  |  |  |  |  |  | 右端项 |
| -2 |  | 0 | 1 | 0 | 1/6 | 5/2 |
| -3 |  | 1 | 0 | 0 | -1/2 | 5/2 |
| 1 |  | 0 | 0 | 1 | 7/6 | 5/2 |
|  | | 0 | 0 | 0 | 1 | 15 |

由上表可知，检验数均大于等于0，所以得到最优解

最优值为

因此得到原问题目标函数最大值为15。

4、

解：先化标准形

两阶段法：

第一阶段：引入人工变量，并求解以下最优化问题

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 迭代次数 |  | | 0 | 0 | 0 | 0 | 1 | 1 | b |  |
|  |  |  |  |  |  |
| 1 | 1 |  | 1 | 1 | 1 | 0 | 1 | 0 | 7 | 7 |
| 1 |  | 2 | -5 | 1 | -1 | 0 | 1 | 10 | 5 |
|  | | -3 | 4 | -2 | 1 | 0 | 0 | -17 |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 迭代次数 |  | | 0 | 0 | 0 | 0 | 1 | 1 | b |  |
|  |  |  |  |  |  |
| 2 | 1 |  | 0 | 7/2 | 1/2 | 1/2 | 1 | -1/2 | 2 | 4/7 |
| 0 |  | 1 | -5/2 | 1/2 | -1/2 | 0 | 1/2 | 5 | / |
|  | | 0 | -7/2 | -1/2 | -1/2 | 0 | 3/2 | -2 |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 迭代次数 |  | | 0 | 0 | 0 | 0 | 1 | 1 | b |  |
|  |  |  |  |  |  |
| 3 | 0 |  | 0 | 1 | 1/7 | 1/7 | 2/7 | -1/7 | 4/7 |  |
| 0 |  | 1 | 0 | 6/7 | -1/7 | 5/7 | 1/7 | 45/7 |  |
|  | | 0 | 0 | 0 | 0 | 1 | 1 | 0 |  |

第一阶段求得的最优解为

第二阶段：从第一阶段的最终单纯形表中取消人工变量并且填入原问题的目标函数的系数开始

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 迭代次数 |  | | 2 | 3 | -5 | 0 | b |  |
|  |  |  |  |
| 1 | 3 |  | 0 | 1 | 1/7 | 1/7 | 4/7 |  |
| 2 |  | 1 | 0 | 6/7 | -1/7 | 45/7 |  |
|  | | 0 | 0 | -50/7 | -1/7 | -102/7 |  |

最优解为，最大值

大M法

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 迭代次数 |  | | 2 | 3 | -5 | 0 | -M | -M | b |  |
|  |  |  |  |  |  |
| 1 | -M |  | 1 | 1 | 1 | 0 | 1 | 0 | 7 | 7 |
| -M |  | 2 | -5 | 1 | -1 | 0 | 1 | 10 | 5 |
|  | | 2+3M | 3-4M | -5+2M | -M | 0 | 0 | 17M |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 迭代次数 |  | | 2 | 3 | -5 | 0 | -M | -M | b |  |
|  |  |  |  |  |  |
| 2 | -M |  | 0 | 7/2 | 1/2 | 1/2 | 1 | -1/2 | 2 | 4/7 |
| 2 |  | 1 | -5/2 | 1/2 | -1/2 | 0 | 1/2 | 5 | / |
|  | | 0 | 8+7M/2 | -6+M/2 | -1+M/2 | 0 | -1-3M/2 | 2M-10 |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 迭代次数 |  | | 2 | 3 | -5 | 0 | -M | -M | b |  |
|  |  |  |  |  |  |
| 3 | 3 |  | 0 | 1 | 1/7 | 1/7 | 2/7 | -1/7 | 4/7 | 0 |
| 2 |  | 1 | 0 | 6/7 | -1/7 | 5/7 | 1/7 | 45/7 | 0 |
|  | | 0 | 0 | -50/7 | -1/7 | -M-16/7 | -M+1/7 | -102/7 |  |

最优解为，最大值