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A simple, analytically solvable, Chamberlinian agglomeration model

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Abstract

This paper presents a simple Chamberlinian agglomeration model which, like the canonical core-periphery (CP) model, contains two agglomerative forces. However, in contrast to that model, the present model is analytically solvable. Moreover, the present model exhibits a 'supercritical pitchfork bifurcation' rather than the 'subcritical pitchfork bifurcation' of the CP model. This may be a better description for some agglomerative processes than the 'catastrophic' emergence of complete agglomeration predicted by the CP model.

JEL classification: F12; F15; F22; R12
Keywords: Economic geography; Agglomeration; Human capital mobility

1. Introduction

The spatial aspects of the economy are at the heart of the 'new economic geography' which was launched with the seminal works by Krugman (1991), Krugman and Venables (1995) and Venables (1996). The analytical essence of the new economic geography is contained in the 'core-periphery (CP) model'.[1] This shows how the interactions among transport costs, increasing returns at the firm level, and supply and demand linkages shape and change the location of economic activity. In order to bring these interactions out clearly, this model is built around a set of simplifying assumptions

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[1] The core-periphery model is conveniently laid out in Fujita et al. (1999), Chapters 4 and 5, and elaborated on and critically assessed in Neary (2001) and Baldwin et al. (2001).

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which have become canonical (e.g. Chamberlinian monopolistic competition with Dixit–Stiglitz preferences, Cobb–Douglas upper-tier utility and iceberg transport costs). It has been perceived as a weakness that, these simplifications notwithstanding, only certain aspects of the CP model are analytically tractable. The model generally has to be solved by numerical simulation. This may be part of the reason why the analysis of the policy implications of the new economic geography, which would necessitate further complications of the model, is yet not fully developed. An analytically solvable case of the CP model has been worked out by Forslid (1999), however.[2] Due to its solvability, this model has proven useful for analyses in the field of capital and income tax competition (Andersson and Forslid, 1999; Baldwin and Krugman, 2000).

This paper presents an alternative model that can be solved analytically and is even simpler than the model of Forslid (1999). The gain in simplicity derives from the substitution of the Cobb–Douglas upper-tier utility with a quasi-linear function. With this modification, the two agglomerative forces contained in his model are still at work but the quasi-linear utility removes all income effects from the manufacturing sector, in which agglomerative forces appear. Moreover, the modified model has the implication that, when transport costs are lowered, a different type of bifurcation arises. The CP model exhibits a 'subcritical pitchfork bifurcation'.[3] When transport costs are at a certain level (the sustain point), two stable fully agglomerative equilibria appear in addition to the initial symmetric equilibrium. At a still lower level of transport costs (the break point), the symmetric equilibrium becomes unstable. The model presented here exhibits a 'supercritical pitchfork bifurcation'. At a certain level of transport costs, the initial symmetric equilibrium becomes unstable and two stable asymmetric equilibria emerge which are increasingly asymmetric as transport costs are lowered.[4] This feature may be a better description for some of the agglomerative processes that are initiated by economic integration (decreasing transport costs) than the 'catastrophic' emergence of complete agglomeration predicted by the CP model.

2. The model

The model builds on Forslids adaptation of the CP model which achieves its solvability by assuming that the fixed cost in the manufacturing sector consists of a separate internationally mobile factor as in Flam and Helpman (1987).[5] Here we combine this

[2] Ottaviano (2001, 1996) has independently developed the dynamic version of this model. A synthesis of Ottaviano (1996) and Forslid (1999) is provided in Forslid and Ottaviano (2002). Solvable models departing from the CP model have also been provided by Baldwin (1999), who stresses factor accumulation and by Ottaviano et al. (2002), who work out a Chamberlinian setting without CES preferences.

[3] A simple introduction to bifurcations is provided in Fujita et al. (1999), appx. to Chapter 3. A technical development can be found in Grandmont (1988).

[4] Supercritical pitchfork bifurcations are also obtained under certain other modifications of the CP model. I return to this below (cf. Section 4).

[5] This assumption is also central in Ottaviano's (2001) dynamic model.

model with the assumption that the upper-tier utility is quasi-linear rather than Cobb–Douglas drawing on a widely used specification (e.g. Dixit, 1990, Chapter 3).

The world is composed of two countries, home and foreign (denoted by an asterisk (*)), two factors of production, labor (L) and human capital (K), and two sectors, manufacturing (X) and agriculture (A). Labor is intersectorally mobile. Countries are assumed to have identical preferences, technology and trade costs. In the long-run, human capital is assumed to be mobile internationally, while labor is not. The agricultural good is homogeneous, traded without costs and produced perfectly competitively under constant returns with labor as the only input. This good is the numéraire and assumed to be produced in both countries after trade. The monopolistically competitive X sector employs both factors to produce differentiated goods with a linear cost function. Labor is the only variable input. Human capital enters only the fixed cost. One unit of it is needed (for R&D or headquarter services) to produce at all. Trade in X is inhibited by iceberg costs.

There are $L+K$ households, L laborers and K human capital owners each of whom supplies one unit of labor and human capital, respectively. Their wages are denoted by W and R, respectively. Each household's preferences are characterised by:[6]

$$U=\alpha\ln C_X+C_A,\quad C_X=\left(\int_0^N x_i^{\frac{\sigma-1}{\sigma}}+\int_N^{N*} x_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},\quad \alpha>0,\quad \sigma>1 \tag{1}$$

where C_X is the manufacturing aggregate, C_A is the consumption of the agricultural good, x_i (x_j) is the quantity consumed of a domestic variety i (foreign variety j), N and $N*$ are the number of varieties produced in home and foreign and σ is the elasticity of substitution between manufacturing varieties. The budget constraint is given by

$$PC_X+C_A=Y,\quad P=\left[NP_i^{1-\sigma}+N*\left(\tau P_j\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},\quad \tau>1 \tag{2}$$

where Y denotes the household's income, P is the perfect CES-price index, P_i (P_j) is the price set by a domestic (foreign) firm. Iceberg transport costs are formalised by the constant τ. These imply that only $1/\tau$ of a unit of a foreign variety arrives for consumption and that the consumer price of an imported variety is τP_j. Utility maximisation yields the demand functions and indirect utility, V:[7]

$$C_X=\alpha P^{-1},\quad C_A=Y-\alpha,\quad x_i=\alpha P_i^{-\sigma}P^{\sigma-1},\quad x_j=\alpha\left(\tau P_j\right)^{-\sigma}P^{\sigma-1} \tag{3}$$

$$V=-\alpha\ln P+Y+[\alpha(\ln\alpha-1)]. \tag{4}$$

Choosing units and letting L_A denote labour input, the production function of the agricultural good is $X_A=L_A$. Perfect competition ensures that this good is priced at marginal (which is also average) cost. Since this good is the numéraire, the wage rate is unity, $W=1$.

[6] Martin and Rogers (1995) and Pflüger (2001) use this preference specification in non-agglomeration contexts. A similar specification is used in Helpman and Krugman (1989), Chapter 7.

[7] We assume that $\alpha<Y$ in order to assure that both types of goods are consumed (cf. Dixit, 1990).

Market clearing for domestic variety i is expressed by $X_i = (L + K)x_i + (L^* + K^*)\tau x_i^*$, where X_i is production and x_i^* is the demand of the foreign representative household. Part of demand is indirect, caused by transport losses. Each product type is supplied by a single firm. With $W = 1$ and the technology $L_i = cX_i$ ($c > 0$, a constant), the marginal cost is given by c. The fixed cost due to the requirement of one unit of human capital is given by R. Let the producer prices charged to domestic (foreign) households be denoted P_i (P_i^*). Profits of the representative firm in the home region, Π_i, are then given by:

$$\Pi_i = (P_i - c)(L + K)x_i + (P_i^* - c)(L^* + K^*)\tau x_i^* - R \tag{5}$$

With the Chamberlinian large group assumption, profit maximising prices are constant markups on marginal costs:

$$P_i = P_i^* = c\sigma/(\sigma - 1) \tag{6}$$

The compensation of human capital adjusts so as to ensure zero profit equilibrium. Using the market clearing condition, a relationship between firm scale X_i and fixed costs R obtains:

$$X_i = R(\sigma - 1)/c. \tag{7}$$

3. Short-run equilibrium

In the short-run, human capital is immobile between countries so that $N = K$ and $N^* = K^*$. With free trade in goods, and using Eqs. (2), (3), (5), (6) and their foreign counterparts, the zero profit conditions in home and foreign are given by:

$$\sigma R = \frac{\alpha(L + K)}{K + \phi K^*} + \frac{\phi\alpha(L^* + K^*)}{\phi K + K^*}; \quad \sigma R^* = \frac{\phi\alpha(L + K)}{K + \phi K^*} + \frac{\alpha(L^* + K^*)}{\phi K + K^*} \tag{8}$$

where $0 \leq \phi \equiv \tau^{1-\sigma} \leq 1$. Human capital's compensation in home and foreign in the short-run equilibrium, R and R^*, can directly be read of Eq. (8). This is the simplification obtained by the quasi-linear upper tier utility which eliminates all income effects from the manufacturing sector. With a Cobb–Douglas, domestic and foreign incomes enter in the numerators of Eq. (8) which then have to be solved simultaneously. Once R is derived, the firm scale X_i follows directly from Eq. (7) and all other endogenous variables can be derived in a straightforward way. The X sector employs $NcX_i = NR(\sigma - 1)$ units of labor (using Eq. (7)) which we assume to be less than L in order to fulfill the assumption that both sectors are active after trade. Formally, this leads to the condition $\alpha < \rho\sigma/(2\rho + 1)(\sigma - 1)$ where $\rho \equiv L/(K + K^*)$.[8]

[8] It can be straightforwardly shown that the product NR achieves its maximum at $N = K + K*$. In that case it follows that $R = \alpha(2\rho + 1)/\sigma$ and the formal condition stated above is then obtained immediately.

4. Long-run equilibrium

In the long run, human capital owners are internationally mobile and will move to the region where their indirect utility is higher. The utility differential, $V - V^* = \alpha \ln(P^*/P) + (R - R^*)$, can easily be derived analytically for general trade costs in this model:

$$V - V^* = \frac{\alpha}{1-\sigma}\ln\left[\frac{\lambda\phi + (1-\lambda)}{\lambda + (1-\lambda)\phi}\right] + \frac{\alpha(1-\phi)}{\sigma} \times \left[\frac{\rho + \lambda}{\lambda + (1-\lambda)\phi} - \frac{\rho * + (1-\lambda)}{\lambda\phi + (1-\lambda)}\right] \tag{9}$$

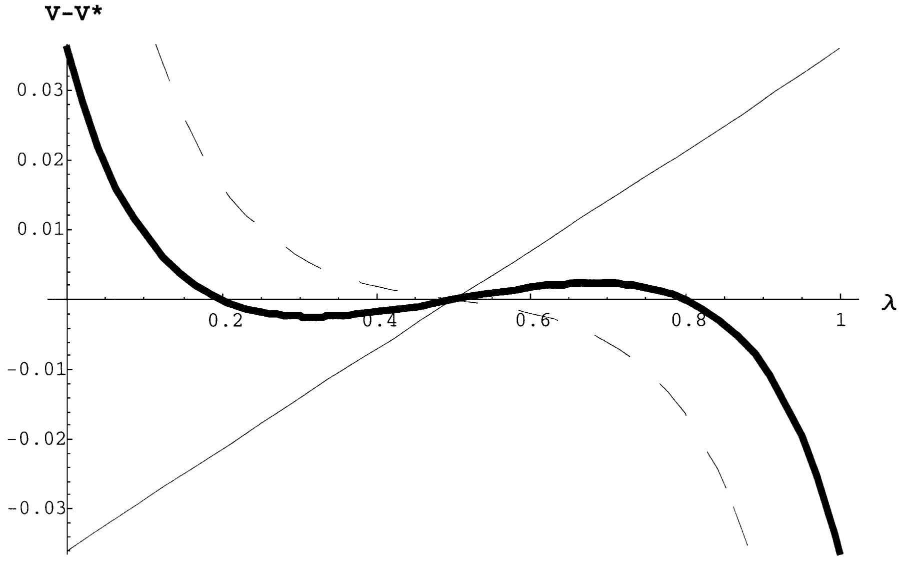
where ρ is as defined before, $\lambda \equiv K/(K + K^*)$ and $\rho^* \equiv L^*/(K + K^*)$. A long-run equilibrium in which both regions produce manufactures is given when $V - V^* = 0$. It is easily verified that $\lambda = 1/2$ is always such an equilibrium with identical countries. However, this equilibrium is not necessarily stable because the model contains two agglomerative forces. There is a supply linkage as the region with the higher share of human capital has a larger manufacturing sector and therefore a lower price index. This is captured in the first term in Eq. (9). There is also a demand linkage since increasing the share of human capital in one region implies a larger market. This raises the profitability of firms as expressed by the differential $(R - R^*)$ and thus attracts more human capital. The demand linkage is captured in the second term in Eq. (9). Transport costs act as a stabilising force in this model, i.e. tend to disperse production. When transport costs are increased it becomes more profitable for firms to locate near the immobile customers.[9]

Fig. 1 depicts (9) for different levels of trade costs and for the parameter values $\rho = \rho^* = 1; \sigma = 6; \alpha = 0.3$. For high trade costs, the utility differential is monotonically falling and the symmetric equilibrium is stable. It becomes unstable for lower trade costs and two stable asymmetric equilibria emerge which become increasingly asymmetric when trade costs are continuously reduced. For still lower trade costs, the utility differential slopes up monotonically implying that these asymmetric equilibria involve full specialisation in one of the countries. When trade costs are nil, difference doesn't matter in the model and human capital owners are indifferent where to locate.

Fig. 2 depicts the stable equilibria in the corresponding bifurcation diagram. The model exhibits a 'supercritical pitchfork bifurcation'. An analytical expression for the level of transport costs at which the symmetric equilibrium becomes unstable and the two arms of the fork appear can be obtained by using Eq. (9) and the condition $\partial(V - V^*)/\partial\lambda = 0$ at $\lambda = 1/2$. This yields:[10] $\phi_c \equiv \tau_c^{1-\sigma} = [\sigma(2\rho - 1) - 2\rho]/[\sigma(3 + 2\rho) - 2(1 + \rho)]$. By differentiation, $\delta\tau_c/\delta\rho < 0$. Intuitively, the critical level of transport costs at which the

[9] As in the CP model we want to rule out that the agglomerative forces are so strong that the symmetric equilibrium is unstable even at high (infinite) trade costs (Fujita et al., 1999). In the present model this 'no-black-hole-condition' commands: $\sigma/(\sigma - 1) < 2\rho$.

[10] By straightforward calculations it can be verified that, at $\lambda = 1/2$ and the critical $\phi_b \equiv \tau_b^{1-\sigma}$ the following holds: $\partial^2(V - V*)/\partial\lambda^2 = 0$ and $\partial^3(V - V*)/\partial\lambda^3 = -64\alpha(2\sigma - 1)^3/(2\rho + 1)^3(\sigma - 1)^4 < 0$. The formal conditions characterising a supercritical pitchfork bifurcation are therefore fulfilled (Grandmont, 1988: 49f.).



Trade costs:		
High:	Dashed Line:	$\tau = 1.5$
Intermediate:	Thick Line:	$\tau = 1.4$
Low:	Regular Line:	$\tau = 1.1$

Fig. 1. International utility differential, human capital. Trade costs: High: Dashed Line: $\tau = 1.5$; Intermediate: Thick Line: $\tau = 1.4$; Low: Regular Line: $\tau = 1.1$.

symmetric equilibrium becomes unstable has to be lower, when the number of immobile workers relative to mobile human capital owners is higher. Although it is not possible to explicitly derive the level of transport costs, τ_f, at which full agglomeration in one of the two countries obtains in this model, an implicit condition can be obtained by setting the utility differential Eq. (9) equal to zero and setting $\lambda = 1$. This condition reads: $\ln\tau_f = [(\rho + 1)\tau_f^{1-\sigma} + \rho\tau_f^{-(1-\sigma)} - (1 + 2\rho)]/\sigma$.

In the standard CP model with a Cobb–Douglas upper tier utility two qualitatively identical linkages are at work. However, the CP model exhibits a ‘subcritical pitchfork bifurcation’ so that internal asymmetric equilibria do not exist: only the symmetric equilibrium and/or full agglomeration in one of the regions are stable equilibria. Intuitively, the bifurcations obtained in new economic geography models are the result of a tension between centripetal and centrifugal forces, in general. A subcritical pitchfork (supercritical pitchfork) results, when, as an increasing amount of mobile factors relocates from the symmetric equilibrium, the incentive for further relocation increases (decreases) (see Fujita et al., 1999: 34f. and Puga, 1999: 324). The difference between the present model and the standard CP model can now be rationalised by noting that the quasi-linear upper tier utility, by channelling all income effects to the numéraire sector, reduces the strength of the demand linkage compared to the Cobb–

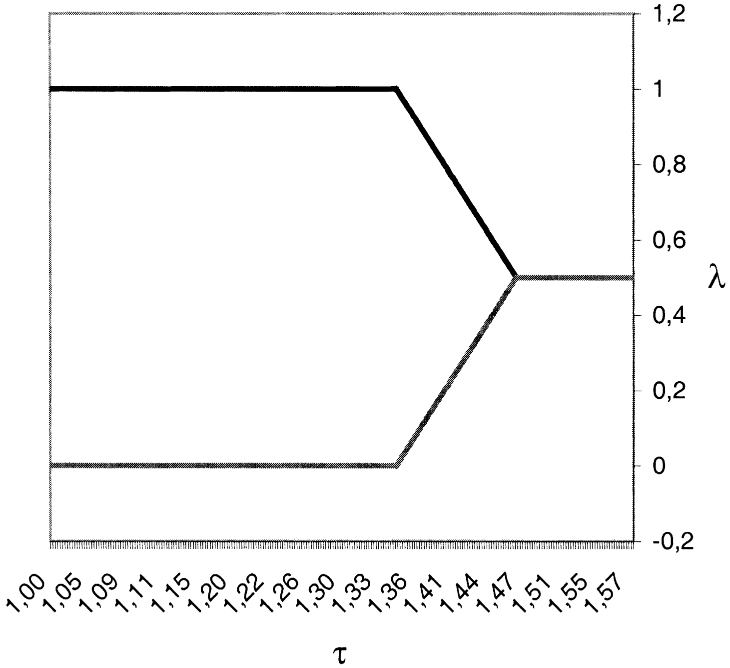


Fig. 2. Bifurcation diagram.

Douglas case.'[11] This can be seen most clearly with the help of Eq. (8). As noted before, the only difference with a Cobb–Douglas upper tier utility is that the domestic and the foreign aggregate incomes enter in the numerators of the two terms on the right-hand sides. In comparing the two cases we therefore hold the denominators of Eq. (8) fixed and focus on these numerators. Take the first equation of Eq. (8) and perform the thought experiment of relocating one unit of human capital from the foreign to the domestic region starting from the symmetric equilibrium. With the quasi-linear utility, the compensation of human capital owners in the domestic region (R on the left hand side) will then surely increase when transport costs are positive, and hence, $\phi < 1$. With the Cobb–Douglas utility function this happens too, but this effect is stronger because R feeds back on domestic income (on the right hand side) and thus reinforces the rise in R. Thus, the demand linkage is stronger in the Cobb–Douglas case. The incentives for further relocation are therefore reinforced in the Cobb–Douglas case compared to the case of the quasi-linear utility. This weakening of the demand linkage is sufficient to generate the possibility of internal asymmetric equilibria. It should be noted that such internal asymmetric equilibria are also found

[11] The supply linkage in the two models is determined by the way in which price level reductions affect the indirect utilities: in the present model, the price levels enter in logarithmic form; in the CP model they enter with an exponential weight of less than one (the Cobb–Douglas budget share spent on manufacturing goods). Hence, in contrast to the demand linkages, the supply linkages are very similar in the two models.

in other recent papers. I interpret these findings as the result of modifications of the standard CP model which incorporate centrifugal forces or reduce centripetal forces: Helpman (1998) includes non-traded goods, Puga (1999) and Fujita et al. (1999, Chapter 14) allow for decreasing rather than constant returns to labour in the production of the agricultural good and Ludema and Wooton (1999) assume limited factor mobility. Arguably, the supercritical pitchfork is a more convincing description of some of the concentration processes triggered by falling transport costs than the 'catastrophe' implied by the subcritical pitchfork bifurcation.

5. Conclusions

This paper presents a simple, analytically solvable, Chamberlinian agglomeration model which departs from the canonical CP model in two respects. First, the fixed cost in the manufacturing sector is assumed to consist of a separate internationally mobile factor (human capital). Second, preferences of households are characterised by a quasi-linear upper tier utility which eliminates income effects from the sector which has the potential to agglomerate. Like the CP model, the present model contains two agglomerative forces, a supply and a demand linkage. However, the model has the attractive feature to exhibit a 'supercritical pitchfork bifurcation' rather than the catastrophic 'subcritical pitchfork bifurcation' of the CP model. Moreover, due to its simplicity and analytical solvability, the model should be useful in policy applications, notably in the fields of labor economics and public economics. This is an important area of future work.

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References

Andersson, F., Forslid, R., 1999. Tax Competition and Economic Geography, CEPR Discussion Paper 2220.

Baldwin, R.E., 1999. Agglomeration and endogenous capital. European Economic Review 43, 253–280.

Baldwin, R.E., Krugman, P., 2000. Agglomeration, Integration, and Tax Harmonization, CEPR Discussion Paper 2630.

Baldwin, R.E., Forslid, R., Ottaviano, G., Robert-Nicoud, F., 2001. The Core-Periphery Model: Key Features and Effects, Manuscript, University of Geneva.

Dixit, A., 1990. Optimization in Economic Theory, 2nd Edition. Oxford University Press.

Flam, H., Helpman, E., 1987. Industrial policy under monopolistic competition. Journal of International Economics 22, 79–102.

Forslid, R., 1999. Agglomeration with Human and Physical Capital: An Analytically Solvable Case, CEPR Discussion Paper 2102.

Forslid, R., Ottaviano, G.I.P., 2002. An Analytically Solvable Core-Periphery Model. Journal of Economic Geography, forthcoming.

Fujita, M., Krugman, P., Venables, A.J., 1999. The Spatial Economy. Cities, Regions, and International Trade. MIT Press, Cambridge, MA.
Grandmont, J.M., 1988. Nonlinear Difference Equations, Bifurcations and Chaos: An Introduction. Lecture Notes No. 5, May, IMSSS—Economics Lecture Notes Series, Institute for Mathematical Studies in the Social Sciences, Stanford, CA.
Helpman, E., 1998. The size of regions. In: Pines, D., Sadka, E., Zilcha, I. (Eds.), Topics in Public Economics. Theoretical and Empirical Analysis Cambridge University Press, pp. 33–54.
Helpman, E., Krugman, P.R., 1989. Trade Policy and Market Structure. MIT Press, Cambridge, MA.
Krugman, P., 1991. Increasing returns and economic geography. Journal of Political Economy 99, 483–499.
Krugman, P., Venables, A.J., 1995. Globalization and the inequality of nations. Quarterly Journal of Economics 60, 857–880.
Ludema, R.D., Wooton, I., 1999. Regional integration, trade and migration: are demand linkages relevant in Europa. In: Faini, R., De Melo, J., Zimmermann, K.F. (Eds.), Migration. The Controversies and the Evidence. CEPR and Cambridge University Press, pp. 51–75.
Martin, P.J., Rogers, C.A., 1995. Trade effects of regional aid. In: Baldwin, R., Haaparanta, P., Kiander, J. (Eds.), Expanding Membership of the European Union. Cambridge University Press, pp. 166–188.
Neary, P.J., 2001. Of hype and hyperbolas: introducing the new economic geography. Journal of Economic Literature 39, 536–561.
Ottaviano, G.I.P., 1996. Monopolistic Competition, Trade, and Endogenous Spatial Fluctuations. CEPR Discussion Paper 1327.
Ottaviano, G.I.P., 2001. Monopolistic competition, trade, and endogenous spatial fluctuations. Regional Science and Urban Economics 31, 51–77.
Ottaviano, G.I.P., Tabuchi, T., Thisse, J.-F., 2002. Agglomeration and trade revisited. International Economic Review 43 (2), 409–435.
Pflüger, M., 2001. Ecological dumping under monopolistic competition. Scandinavian Journal of Economics 103 (4), 689–706.
Puga, D., 1999. The rise and fall of regional inequalities. European Economic Review 43, 303–334.
Venables, A.J., 1996. Equilibrium location of vertically linked industries. International Economic Review 37, 341–359.