Growth Accounting with Constant Returns to Scale

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Introduction

- Growth accounting is a method for decomposing economic growth into its contributing factors.
- We will use a constant returns to scale production function and derive the growth accounting equation using the time derivative of $\log X(t)$.
- We will also interpret the cost share as the production function parameter using the Euler theorem and the assumption of perfect competition for factor markets.

Constant Returns to Scale Production Function

Consider a constant returns to scale production function:

$$Y(t) = A(t)F(K(t), L(t))$$

where:

- $Y(t)$ is output
- $A(t)$ is total factor productivity
- $K(t)$ is capital
- $L(t)$ is labor

Time Derivative of Log X(t)

The time derivative of log X(t) leads to the rate of growth of X(t):

$$\begin{aligned}
\frac{d}{dt}\log X(t) &= \frac{d\log X(t)}{dX(t)}\frac{dX(t)}{dt} \\
&= \frac{1}{X(t)}\dot{X}(t) \\
&= \frac{\dot{X}(t)}{X(t)}
\end{aligned}$$

where $\dot{X}(t)$ is the time derivative of $X(t)$.

Growth Accounting Equation

Taking the logarithm of the production function and differentiating with respect to time yields the growth accounting equation:

$$
\begin{aligned}
Y(t) &= A(t)K(t)^{\alpha}L(t)^{1-\alpha} \\
\log Y(t) &= \log A(t) + \alpha \log K(t) + (1-\alpha)\log L(t) \\
\frac{\dot{Y}(t)}{Y(t)} &= \frac{\dot{A}(t)}{A(t)} + \alpha\frac{\dot{K}(t)}{K(t)} + (1-\alpha)\frac{\dot{L}(t)}{L(t)}
\end{aligned}
$$

where α is the output elasticity of capital.

Euler Theorem and Perfect Competition

The Euler theorem states that for a homogeneous function of degree one (constant returns to scale):

$$F(K(t), L(t)) = \frac{\partial F}{\partial K} K(t) + \frac{\partial F}{\partial L} L(t)$$

$$Y(t) = A(t)F(K(t), L(t)) = A(t)\frac{\partial F}{\partial K} K(t) + A(t)\frac{\partial F}{\partial L} L(t)$$

Under perfect competition, factors are paid their marginal products:

$$\frac{\partial Y(t)}{\partial K(t)} = A(t)\frac{\partial F}{\partial K} = r(t)$$

$$\frac{\partial Y(t)}{\partial L(t)} = A(t)\frac{\partial F}{\partial L} = w(t)$$

Cost Share and Production Function Parameter

$$
\begin{aligned}
Y(t) = A(t)F(K(t), L(t)) &= A(t)\frac{\partial F}{\partial K}K(t) + A(t)\frac{\partial F}{\partial L}L(t) \\
&= r(t)K(t) + w(t)L(t)
\end{aligned}
$$

where $r(t)$ is the rental rate of capital and $w(t)$ is the wage rate.

$$
\begin{aligned}
1 &= \frac{r(t)K(t)}{Y(t)} + \frac{w(t)L(t)}{Y(t)} \\
&= \quad \alpha \quad + \ (1-\alpha)
\end{aligned}
$$

where the cost share of capital and labor re given by:

$$
\alpha = \frac{r(t)K(t)}{Y(t)}, \quad 1-\alpha = \frac{w(t)L(t)}{Y(t)}
$$

Conclusion

$$
\begin{aligned}
\frac{\dot{Y}(t)}{Y(t)} &= \frac{\dot{A}(t)}{A(t)} + \alpha\frac{\dot{K}(t)}{K(t)} + (1-\alpha)\frac{\dot{L}(t)}{L(t)} \\
&= \frac{\dot{A}(t)}{A(t)} + \frac{r(t)K(t)}{Y(t)}\frac{\dot{K}(t)}{K(t)} + \frac{w(t)L(t)}{Y(t)}\frac{\dot{L}(t)}{L(t)} \\
&= \frac{\dot{A}(t)}{A(t)} + \frac{A(t)F_K K(t)}{Y(t)}\frac{\dot{K}(t)}{K(t)} + \frac{A(t)F_L L(t)}{Y(t)}\frac{\dot{L}(t)}{L(t)}
\end{aligned}
$$

- Growth accounting decomposes economic growth into the contributions of total factor productivity, capital, and labor.
- The time derivative of log X(t) leads to the rate of growth of X(t).
- The Euler theorem and the assumption of perfect competition allow us to interpret the cost share as the production function parameter.

Growth Accounting Equation

The growth accounting equation can be written as:

$$g_Y = g_A + \alpha_K g_K + \alpha_L g_L \tag{7.2}$$

where:

- g_X is the growth rate of variable X
- $\alpha_X \equiv \frac{AF_X X}{Y}$ is the elasticity of output with respect to factor X

Competitive Factor Markets

Under the assumption of competitive factor markets (i.e., factors are paid their marginal productivity):

- α_X is also the share of output that factor X obtains as payment for its services
- Equation (7.2) enables us to estimate the contributions of factor accumulation and technological progress (often referred to as total factor productivity (TFP)) to economic growth

Estimating the Solow Residual

In practice:

- From national accounts and other data sources, one can estimate the values of g_Y, g_K, g_L, α_K, and α_L
- From (7.2), one can then back out the estimate for g_A (widely referred to as the Solow residual)

Solow's computation for the U.S. economy:

- The bulk of economic growth, about $2/3$, could be attributed to the residual
- Technological progress, and not factor accumulation, seems to be the key to economic growth

Caveats and Limitations

- g_A is calculated as a residual, not directly from measures of technological progress (it is the measure of our ignorance!)
- Underestimating the increase in K or L will result in an overestimate of g_A
- Much effort has been devoted to better measure the contribution of the different factors of production

Alwyn Young's Research on East Asian "Tigers"

- Alwyn Young (1995) studied the sources of growth in Hong Kong, Singapore, South Korea, and Taiwan
- Most observers thought their fantastic growth performance implied amazing rates of technological progress
- Young showed that their pace of factor accumulation had been astonishing:
  - Rising rates of labour force participation (increasing L)
  - Skyrocketing rises in investment rates (increasing K)
  - Increasing educational achievement (increasing H)
- Once accounted for, their Solow residuals were not particularly outliers compared to the rest of the world

Implications of Young's Findings

- Factor accumulation cannot sustain growth in the long run (decreasing returns)
- Young's findings seemed to predict that the tigers' performance would soon hit the snag of decreasing returns
- Paul Krugman (1994) explicitly predicted this, which was interpreted by many as having predicted the 1997 East Asian crisis