Honours Diary 2020

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Set up TeXstudio and basic document structure.

Exercise 2.1

Linear Dependence, show that $(1, -1)$, $(1, 2)$ and $(2, 1)$ are linearly dependent.

$$\begin{aligned} &(1,-1)+(1,2)-(2,1) \\ =\ &(1+1-2, -1+2-1) \\ =\ &(0,0) \end{aligned}$$

Exercise 2.2

Matrix representations: Suppose V is a vector space with basis vectors $|0\rangle$ and $|1\rangle$, and A is a linear operator from V to V such that $A|0\rangle = |1\rangle$ and $A|1\rangle = |0\rangle$. Give a matrix representation for A, with respect to the input basis $|0\rangle$, $|1\rangle$, and the output basis $|0\rangle$, $|1\rangle$. Find input and output bases which give rise to a different matrix representation of A.

Equation 2.12 gives us the defining property of matrix representations:

$$A|v_j\rangle = \sum_i A_{ij}|w_i\rangle$$

This gives us a pair of vector equations:

$$|1\rangle = A|0\rangle = A_{00}|0\rangle + A_{10}|1\rangle$$
$$|0\rangle = A|1\rangle = A_{01}|0\rangle + A_{11}|1\rangle$$

By linear independence of $|0\rangle$, $|1\rangle$, it follows that

$$A_{00} = 0 \qquad A_{01} = 1$$
$$A_{10} = 1 \qquad A_{11} = 0$$

i.e. A has the matrix representation:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$