

April 10

Reading J Tolar, "On Clifford groups in quantum computing"

An N state system corresponds to a hilbert space \mathbb{C}^N .

"Generalized Pauli Matrices" generate a group, "Weyl-Heisenberg group", semantics.

The "normalizer" of this is called the Clifford group. So I guess WH is not normal in $U(N)$, but in Clifford group it is. That's surprising to me, but I guess it makes sense given the normal property becomes weaker the less group elements you are conjugating against. So Clifford group is the set $\{g \mid g^{-1}Xg \in WH, \forall x \in WH\}$.

"Clifford quotient group" sounds like Clifford group without scalar multiplication, which sounds good to me. $U(N)$ seems so redundant/free I will take every quotient I can get.

"Symmetries of Pauli gradings" of an algebra apparently describe some detail of clifford quotient groups, and this paper will describe something more detailed than that? No idea what a Pauli grading is.

$Q_N|j\rangle = \omega_N^j|j\rangle$, $P_N|j\rangle = |j+1\rangle$, so in 2d:

$$Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z$$

$$P_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X$$

These elements along with ω_N are order N , and are nearly commutative.

$$\Pi_N = \{\omega_N^i P_N^j Q_N^k\}$$

This is not $H(N)$ apparently? Do we need a generalized version of Y before this becomes the Weyl Heisenberg group? Or am I missing something.

ω_N and Q_N clearly have computational basis as their eigenvectors, being diagonal, and P_N will have $|v_i\rangle = \sum_j \omega_N^{ij}|j\rangle$ as eigenvectors, eigenvalues ω_N^i transforming into this basis is the discrete fourier transform! Aha! Ok back to the text. I don't know what a configuration space is or what "eigenvector of position means".

Ah yes $\tau_N = \omega_N^{\frac{1}{2}}$ lets us define Y .

$$\tau_2 P_2 Q_2 = i \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = Y$$

Then $H(N) = \{\tau_N^h \omega_N^i Q_N^j P_N^k\}$, good. $|H(N)| = 2 |\Pi_N| = 2N^3$.

Oh this phase factor is just for even N . Fascinating. Naively that sounds like a novel thing to attack in a mixed level system?

Apparently $\tau_2 = -i$, so the equation is actually $Y = \tau_2 Q_2 P_2$

Then the centralizer is just the set of scalars $\{\tau^i\}$, and since Q_N and P_N commute, up to phase shift ω_N , quotienting by the centralizer gives the abelian group \mathbb{Z}_N^2 . Easy.