

11 Jan '21

1/24) INTRODUCTION - ANALYSIS OF ALGORITHMS (70 min) [25 min]

Asymptotic analysis is a theoretical approach where we measure the order of growth in terms of input size.

θ	Theta	\equiv
O	Big O	\leq
Ω	Omega	\geq

$$\begin{aligned}\theta(n^2) &= 100n^2 + 2n \\ &= 10n^2 + 3000 \\ &= n^2 + 10^8n\end{aligned}$$

$$\begin{aligned}O(n^2) &= 100n^2 + 2n \\ &= 2n + 5 \\ &= 3\end{aligned}$$

$$\begin{aligned}\Omega(n^2) &= 100n^2 + 2n \\ &= 5n^3 + 24 \\ &= 10000n^8 + 8n^4 + 3\end{aligned}$$

Linear Search

Time Complexity : $O(n)$
Worst Case Time Complexity : $\theta(n)$

Q1) $TC = \lceil n/c \rceil = \theta(n)$

Q2) $TC = \lceil n/c \rceil = \theta(n)$

Q3) $TC = 1, c, c^2, \dots, c^K = \theta(\log_c n)$

Q4) $TC = n/c, n/c^2, \dots = \theta(\log_c n)$

Q5) $TC = \log_c [\log_c(n)]$

Q6) $TC = O(n)$

TC in best case = $\theta(1)$

TC in worst case = $\theta(n)$

Q7) $TC = C_1.m + C_2.n = O(m+n)$

Q8) $TC = \frac{n(n+1)}{2} = n^2/2 + n/2 = \theta(n^2)$

* [20 min] Analysis of Recursion

Recursion Tree Method says that we write non-recursive part as root of tree and recursive part as children. Then we keep expanding child until we see a pattern. Other methods of finding time complexity is master method.

Q1) $TC = O(n \cdot \log n) \leftarrow \text{Merge Sort}$

Q2) $TC = \theta(2^n)$

$$\Rightarrow AP = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow GP = a \left[\frac{r^n - 1}{r - 1} \right]$$

$$\Rightarrow y = x^{\log_x(y)}$$

Q3) $TC = \theta(\log n)$

Q4) $TC = \theta(n)$

In some cases, the recursion tree method can not give exact answer but can still give us a good upper bound.

Q1) $TC = O(n)$

Q2) $TC = O(2^n) \leftarrow \text{Factorial}$

Hence, in relationships when the tree is dividing in un-equal time. Just consider the equation that takes slowest time to fill. Hence take the worst case scenario.
