Matching Markets with Taxation of Transfers*

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Abstract

We analyze the effects of taxation on outcomes in matching markets. Taxes can make inefficient outcomes stable by causing workers to prefer firms from which they receive high idiosyncratic match utility, but at which they are less productive. In general, efficiency can be non-monotonic in the tax. However, when agents on one side of the market refuse to match without a positive wage, increasing taxes always decreases efficiency. In addition to providing a continuous link between canonical models of matching with and without transfers, our model highlights a cost of taxation that does not appear to have been examined previously.

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1 Introduction

Foundational models of markets with two-sided preference heterogeneity – matching *with* and *without* transfers – have been used to analyze marriage and school choice, as well as some labor markets (see, e.g., Roth (1984); Choo and Siow (2006); Pathak (2011); Galichon and Salanié (2013)). However, because of taxes and other frictions, most labor markets in effect have *imperfect* transfers, placing them outside classical matching settings. The reality of imperfect transfers can be modeled within the framework of matching *with contracts* (Crawford and Knoer (1981); Kelso and Crawford (1982); Quinzii (1984); Hatfield and Milgrom (2005)). However, existing approaches to matching with contracts only provide insight into the structure of stable outcomes for a fixed tax rate; they cannot make comparisons across contract structures, as is necessary when trying to analyze the effect of tax changes on outcomes.[1]

In this paper, we develop a framework for analyzing how the efficiency of (stable) matching market outcomes responds to changes in transfer frictions.[2] Our results highlight that in markets with two-sided preference heterogeneity, a classical economic intuition holds, but with a caveat: In matching markets in which agents on one side of the market do not match unless they receive positive wages, raising taxes always increases equilibrium deadweight loss. In fully general matching markets, however, raising the tax rate can in fact increase equilibrium efficiency.

In our model, agents have heterogeneous rankings of potential match partners and may make transfer payments to their partners. Transfers may be "taxed," causing some of each payment to be taken from the agents. For example, in the case of a proportional tax τ, an agent receives fraction $(1-\tau)$ of the amount his partner gives up (see Section 3). Taxation lowers the value of transfers, causing agents to prefer match partners that provide higher individual-specific match utilities over those offering higher transfers; this may distort away from efficient matching.

The *matching distortion* we identify differs from the well known effects of taxation on intensive (e.g., Blundell et al. (1998); Saez (2004)) and extensive (e.g., Meyer (2002); Saez (2002)) labor supply; it affects the allocation of workers to firms without necessarily changing the provision of labor. This matching distortion arises even in markets with frictionless search, and thus differs from the well known effects of search costs on matching efficiency (e.g., Mortensen and Pissarides (2001); Boone and Bovenberg (2002)) and of taxation on search behavior (e.g., Gentry and Hubbard (2004); Holzner and Launov (2012)).

As in standard taxation models, efficiency is achieved in our framework as the level of taxation approaches 0. However, in general matching markets, taxation can have non-monotonic impact on efficiency: a fully efficient match may be stable in the presence of a high tax, but unstable in the presence of a lower tax, and stable again in the presence of an even lower tax. Nevertheless, monotonicity does obtain in a class of markets particularly important for economic analysis: *wage markets*, in which agents on one side of the market

[1]Legros and Newman (2007) examine how transfer frictions affect the assortativeness of stable match outcomes, but do not investigate the impact on efficiency.

[2]For our discussion, we express frictions in the language of taxation – part of each transfer is lost to a tax authority. Nevertheless, our analysis is not specific to taxation frictions – all that matters is that transfers received are less than transfers paid.

("workers") refuse to match without positive transfers ("wages") from their match partners ("managers"). In wage markets, reducing taxation always improves the efficiency of stable matching outcomes.

Although our results are presented in the language of labor markets, they also have implications for understanding marriage markets. Taxation can be reinterpreted as representing frictions in the (often non-monetary) transfers that occur between partners. For example, it may be the case that the utility a woman gives up by washing the dishes is greater than the utility her husband receives from her doing so.[3] Because marriage markets do not typically behave as wage markets – positive transfers flow in both directions – our non-monotonicity results indicate that it is hard to predict the efficiency response to a reduction in intra-household transfer frictions.

Of the vast literature on taxation, our work is most closely related to the research on the effect of taxation on workers' occupational choices (e.g., Parker (2003); Sheshinski (2003); Powell and Shan (2012); Lockwood et al. (2013)). However, this prior work only reflects part of the matching distortion because it does not model the two-sidedness of the market. If workers and firms *both* have heterogeneous preferences over match partners, then matching distortions can reduce productivity even without causing an aggregate shift in workers from one firm (or industry) to another.

Our approach is also related to the literature on taxation in Roy models. The utility that a manager or firm in our model derives from a worker could be thought of the productivity of the worker in that firm or sector. However, most Roy models assume that workers earn their marginal product (e.g., Rothschild and Scheuer (2012); Boadway et al. (1991)). Explicitly modeling firms allows for the possibility of taxation affecting the share of output that workers receive as wages.

Our work introduces a new link between the literatures on matching with and without transfers: Absent taxation, our framework is equivalent to matching with perfect transfers (e.g., Koopmans and Beckmann (1957); Shapley and Shubik (1971); Becker (1974)); under 100% taxation, it corresponds to the standard model of matching without transfers (e.g., Gale and Shapley (1962); Roth (1982)). Thus, the intermediate tax levels we consider introduce a continuum of models between the two existing, well-studied extremes. While prior work has analyzed frameworks that can embed our intermediate transfer models (Crawford and Knoer (1981); Kelso and Crawford (1982); Quinzii (1984)), it has focused on the structure of the sets of stable outcomes *within* (fixed) models. To the best of our knowledge, our work is the first to show how the efficiency of stable outcomes changes *across* transfer models.

The remainder of the paper is organized as follows: Section 2 introduces our general model. Sections 3 and 4 analyze the cases of proportional and lump sum taxation, respectively. Section 5 discusses structural properties common to both models. Section 6 concludes. All proofs are presented in the Appendix.

2 General Model

Before introducing our models of taxation, we describe our underlying matching framework.

[3] A similar idea is modeled by Arcidiacono et al. (2011), who treat sexual activity as an imperfect transfer from women to men in the context of adolescent relationships.

We study a two-sided, many-to-one matching market with fully heterogeneous preferences. We refer to agents on one side of the market as *managers*, denoted $m \in M$; we refer to agents on the other side *workers*, denoted $w \in W$. Our notation and language are also consistent with modeling marriage markets.

Each agent $i \in M \cup W$ derives utility from being matched to agents on the other side of the market. We denote these *match utilities* by α_Y^m and γ_m^w, with α_Y^m denoting the utility $m \in M$ obtains from matching with the set of workers $Y \subseteq W$ and γ_m^w denoting the utility $w \in W$ obtains from matching with manager $m \in M$. Without loss of generality, we normalize the utility of being unmatched (an agent's reservation value) to 0, setting $\alpha_m^m = \gamma_w^w = 0$ for all $m \in M$ and $w \in W$. In the labor market context, α_Y^m may be the productivity of the set of workers Y when employed by manager m and γ_m^w may be the utility or disutility worker w gets from working for m.[4]

Note that it is possible for workers to disagree about the relative desirabilities of potential managers and for managers to disagree about the relative values of potential workers. We impose no structure on workers' match utilities and only impose enough structure on managers' preferences to ensure the existence of equilibria. Specifically, we assume that managers' preferences satisfy the standard Kelso and Crawford (1982)/Hatfield and Milgrom (2005) *substitutability* condition: the availability of new workers cannot make a manager want to hire a worker he would otherwise reject.[5]

A *matching* μ is an assignment of agents such that each manager is either matched to himself (*unmatched*) or matched to a set of workers who are matched to him. Denoting the power set of W by $\wp(W)$, a matching is then a mapping μ such that

$$\begin{aligned} \mu(m) &\in (\wp(W) \cup \{m\}) \quad \forall m \in M, \\ \mu(w) &\in (M \cup \{w\}) \quad \forall w \in W, \end{aligned}$$

with $w \in \mu(m)$ if and only if $\mu(w) = m$.

We allow for the possibility of (at least partial) transfers between matched agents. We denote the transfer from m to w by $t^{m \to w} \in \mathbb{R}$; if m receives a positive transfer from w, then $t^{m \to w} < 0$. A transfer vector t identifies (prospective) transfers between all manager–worker pairs, not just between those pairs that are matched. We also include in the vector t "transfers" $t^{i \to i}$ for all agents $i \in M \cup W$, with the understanding that $t^{i \to i} = 0$. For notational convenience, we denote by $t^{m \to Y}$ the total transfer from manager m to workers in Y:

$$t^{m \to Y} \equiv \sum_{w \in Y} t^{m \to w}.$$

With these conventions, we have the following lemma.

[4] Although it may seem that α_Y^m should be positive and γ_m^w should be negative, for our general analysis we do not make sign assumptions. That is, we allow for the possibility of highly demanded internships and for counterproductive employees.

[5] Substitutability plays no role in our analysis other than ensuring, through appeal to previous work (Kelso and Crawford (1982)), that equilibria exist. Thus, we leave the formal discussion of the substitutability condition to the Appendix.

Lemma 1. *For a given matching μ and transfer vector t, the sum of transfers managers pay to their match partners equals the sum of the transfers paid by workers' match partners,*

$$\sum_{m \in M} t^{m \to \mu(m)} = \sum_{m \in M} \sum_{w \in \mu(m)} t^{m \to w} = \sum_{w \in W} t^{\mu(w) \to w}. \tag{1}$$

In the presence of taxation, a worker might not receive an amount equal to that which his match partner gives up; in general, a (weakly increasing) *transfer function* $\xi(\cdot)$ converts managers' transfer payments into the amounts that workers receive, post-tax. For all our transfer functions, we use the convention that $\xi(t^{w \to w}) = 0$ for all $w \in W$. Note that if $\xi(t^{m \to w}) \neq t^{m \to w}$, then (1) will generally *not* be equal to the sum of the amounts that the workers receive.

An *arrangement* $[\mu; t]$ consists of a matching and a transfer vector.[6] We assume that agent utility is quasi-linear in transfers and that there are no externalities. With these assumptions, the *utility* values of arrangement $[\mu; t]$ for manager $m \in M$ and worker $w \in W$ are

$$\begin{aligned} u^m([\mu; t]) &\equiv \alpha^m_{\mu(m)} - t^{m \to \mu(m)}, \\ u^w([\mu; t]) &\equiv \gamma^w_{\mu(w)} + \xi(t^{\mu(w) \to w}). \end{aligned}$$

Note that the both the match utilities and the transfers may be either positive or negative. The utility of a worker $w \in W$ depends on the transfer function $\xi(\cdot)$.

Our analysis focuses on the arrangements that are stable, in the sense that no agent wants to deviate.

Definition. An arrangement $[\mu; t]$ is *stable given transfer function* $\xi(\cdot)$ if the following conditions hold:

1. Each agent (weakly) prefers his assigned match partner(s) (with the corresponding transfer(s)) to being unmatched, that is,

$$u^i([\mu; t]) \geq 0 \qquad\qquad \forall i \in M \cup W.$$

2. Each manager (weakly) prefers his assigned match partners (with the corresponding transfers) to any alternative set of workers (with the corresponding transfers), that is,

$$u^m([\mu; t]) = \alpha^m_{\mu(m)} - t^{m \to \mu(m)} \geq \alpha^m_Y - t^{m \to Y}, \qquad \forall m \in M \text{ and } Y \subseteq W;$$

and each worker (weakly) prefers his assigned match partner (with the corresponding transfer) to any alternative manager (with the corresponding transfer), that is,

$$u^w([\mu; t]) = \gamma^w_{\mu(w)} + \xi(t^{\mu(w) \to w}) \geq \gamma^w_m + \xi(t^{m \to w}) \qquad \forall w \in W \text{ and } m \in M.$$

[6] Here were use the term "arrangement" instead of "outcome" for consistency with the matching literature (e.g., Hatfield et al. (2013)), which uses the latter term when the transfer vector only includes transfers between agents who are matched to each other.

A matching μ is *stable given transfer function* $\xi(\cdot)$ if there is some transfer vector t such that the arrangement $[\mu; t]$ is stable given $\xi(\cdot)$; in this case t is said to *support* μ (given $\xi(\cdot)$).

Arguments of Kelso and Crawford (1982) show that the stability concept we use is equivalent to the other standard stability concept of matching theory, which rules out the possibility of "blocks" in which groups of agents jointly deviate from the stable outcome (potentially adjusting transfers).[7] The assumption of substitutable preferences ensures that at least one stable arrangement always exists.[8]

In analyzing stable arrangements we focus on the *total match utility* of the match μ, defined as

$$\mathfrak{M}(\mu) \equiv \sum_{m \in M} \alpha^m_{\mu(m)} + \sum_{w \in W} \gamma^w_{\mu(w)}.$$

We do not model the institution imposing the tax; as we focus on match utility, our analysis is most relevant to the case where the social value of tax revenue equals the private value.

Definition. We say that a matching $\hat{\mu}$ is *efficient* if it maximizes total match utility among all possible matchings, i.e. if $\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\mu)$ for all matchings μ.

Some of our analysis focuses on markets in which workers have nonpositive valuations for matching, so that they will only match if paid positive "wage" transfers. Formally, we say that a market is a *wage market* if

$$\gamma^w_m \leq 0 \tag{2}$$

for all $w \in W$ and $m \in M$; it is a *strictly positive wage market* if the inequality in (2) is *strict* for all $w \in W$ and $m \in M$. The existence of internships notwithstanding, most labor markets can be reasonably modeled as wage markets.

For simplicity, we set our illustrative examples in *one-to-one matching markets*, in which each manager matches to at most one worker. For such markets, we abuse notation slightly by only specifying match utilities for manager–worker pairs and writing w in place of the set $\{w\}$ (e.g., $\alpha^m_{\{w\}}$ is denoted α^m_w).

3 Proportional Taxation

First we analyze proportional (linear) taxation systems, of the type used in some US states and dozens of countries around the world. These taxes take the form of a fixed percentage deduction of each agent's income. Formally, under proportional tax τ, if an agent pays p, then his partner receives $(1-\tau)p$. The associated transfer function $\xi^{\mathsf{prop}}_\tau(\cdot)$ is

$$\xi^{\mathsf{prop}}_\tau(t^{m \to w}) \equiv \begin{cases} (1-\tau)t^{m \to w} & t^{m \to w} \geq 0 \\ \frac{1}{(1-\tau)} t^{m \to w} & t^{m \to w} < 0. \end{cases}$$

Figure 1 illustrates the transfer function $\xi^{\mathsf{prop}}_\tau(\cdot)$ for different tax rates τ.

[7] Our stability concept is defined in terms of arrangements; the block-based definition is defined only in terms of a matching and the transfers *between matched partners*. Kelso and Crawford (1982) used the term *competitive equilibrium* for the former concept and used *the core* to refer to the latter.

[8] Results of Kelso and Crawford (1982) guarantee the existence of a stable arrangement in our framework. Details are provided in the Appendix.

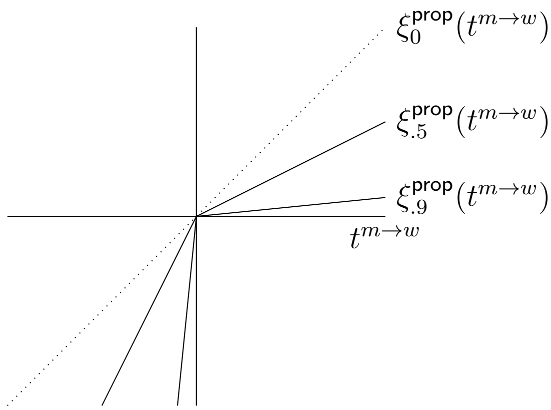

Figure 1: Transfer function $\xi_\tau^{\mathsf{prop}}(\cdot)$.

If an arrangement $[\mu; t]$ or matching μ is stable given $\xi_\tau^{\mathsf{prop}}(\cdot)$, then we say it is *stable under tax* τ. We analyze how the set of stable matchings changes as τ decreases from 1 to 0.

The case $\tau = 1$ corresponds to the standard Gale and Shapley (1962) setting in which transfers are not allowed,[9] so inefficient matchings may be stable. When $\tau = 0$, by contrast, only efficient matchings are stable (see, e.g., Shapley and Shubik (1971); Hatfield et al. (2013)). Given this, one might expect that as the tax rate τ decreases, the match utilities of stable matchings should always (weakly) increase. Unfortunately, a simple example shows that this is not true in general.

3.1 Possible Inefficiencies of Tax-Reduction in General Markets

Example 1. Consider a one-to-one market with one manager, $M = \{m_1\}$, two workers, $W = \{w_1, w_2\}$, and match utilities as pictured in Figure 2a. Worker w_1 receives high utility from matching with m_1. Manager m_1 is indifferent towards worker w_1 and receives moderate utility from matching with w_2. Worker w_2 has a mild preference for being unmatched, rather than matching with m_1.

We can think of w_1 as an intern who would not be very productive in working for m_1, but would learn a lot; w_2 represents a normal worker, who is productive but does not like working. With this interpretation, the tax represents a proportional income tax – which m_1 must also pay if the intern w_1 bribes him in exchange for a job. Alternatively, we may interpret the example in a marriage context: w_1 is an unremarkable woman who really wants to get married; w_2 is a highly desirable woman who prefers to remain single; and m_1 is the last man on Earth. In that case, the tax reflects the extent to which it is difficult to transfer utility between individuals within a couple.

As illustrated in Figure 2b, when $\tau = 1$ (or when transfers are not allowed), the only stable matching $\hat{\mu}$ has $\hat{\mu}(m_1) = w_1$. This happens to be the efficient matching; therefore, it

[9] When $\tau = 1$, the set of stable matchings is the same as in the case that transfers are not allowed. The associated arrangements are not exactly the same, however, because the supporting transfer vectors are not equal to 0. However, if μ is stable when $\tau = 1$, then there is a transfer vector t supporting μ such that $t^{m\to w} = 0$ for all $m \in M$ and $w \in \mu(m)$; the arrangement $[\mu; t]$ therefore replicates the utilities that arise under μ when transfers are not allowed.

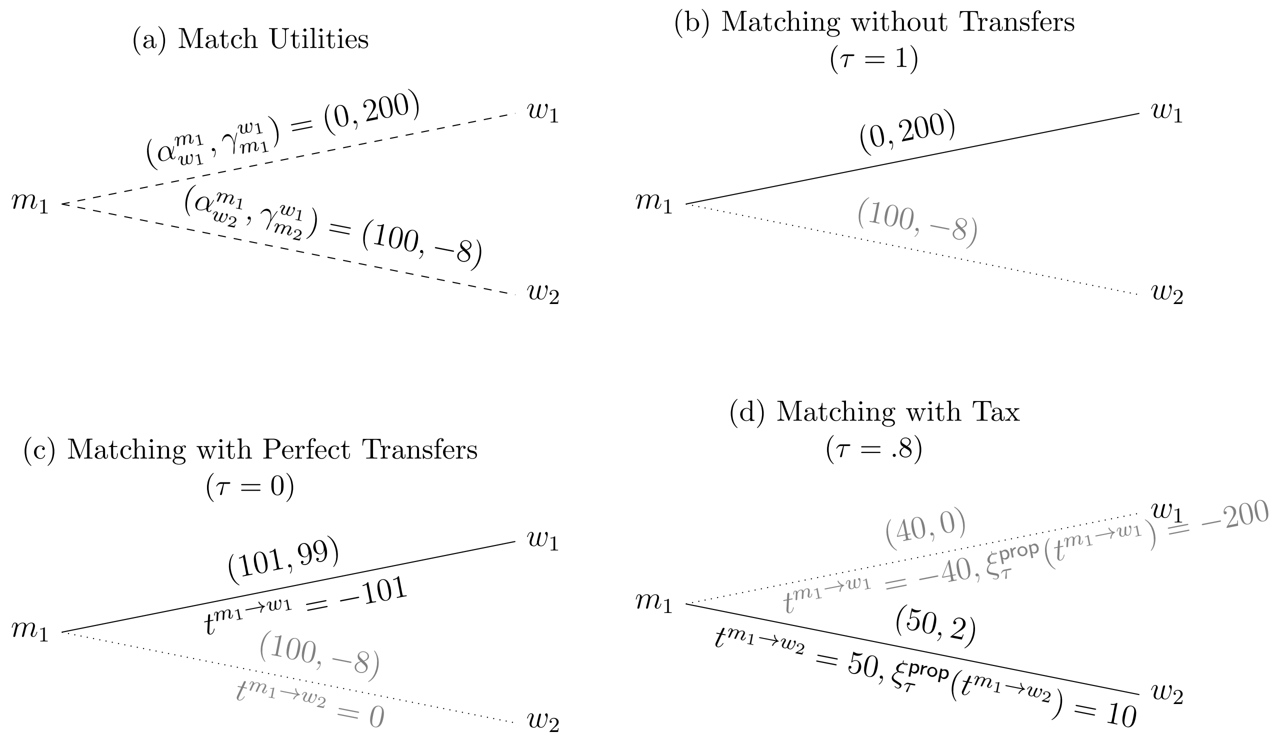

Figure 2: Example 1 – Non-monotonicity under a proportional tax on transfers.

Note: Utilities, net of transfers, are above the lines (manager's, worker's). Possible supporting transfers (when applicable) are below the lines. Solid lines indicate the stable matching.

is also stable when $\tau = 0$, as shown in Figure 2c. This matching yields total match utility $\mathfrak{M}(\hat{\mu}) = 200$.

Figure 2d shows that for $\tau = .8$, an inefficient matching $\tilde{\mu}$, for which $\tilde{\mu}(m_1) = w_2$, is stable. This matching generates a total match utility $\mathfrak{M}(\tilde{\mu}) = 92$. Even if w_1 transfers 200 – his maximal utility of matching – to m_1, there is a transfer m_1 can offer to w_2 that is sufficient to attract w_2, while still providing m_1 more utility than he would obtain from matching with w_1 (and receiving $(1 - .8)(200) = 40$).

Not only is an inefficient matching stable under tax $\tau = .8$, but the efficient matching $\hat{\mu}$ is *not* stable under this tax. Indeed, the efficient matching $\hat{\mu}$ is unstable under any tax $\tau \in (.6, .9)$. For that range, $(100 - 200(1 - \tau))(1 - \tau) - 8 > 0$, so that the maximum m_1 can transfer to w_2 while still preferring w_2 to w_1 is sufficient to outweigh the disutility w_2 gets from matching to m_1.

While Example 1 may appear quite specialized, simulations suggest that non-monotonicities in the total match utility of stable outcomes as a function of τ can be relatively common. We examine simulations of a one-to-one market with twenty agents on each side of the market and match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$. We vary the tax rate, τ, from 0 to .99 in increments of .01. Non-monotonicities in the total match utility of stable matchings appear in over half of the markets (55%).[10]

[10] There may be additional non-monotonicities that we do not observe because we cannot vary τ continu-

Figure 3 plots the total match utility as a function of the tax rate in ten randomly-selected simulation markets with non-monotonicities. These ten markets are fairly representative, in that they have relatively small losses from non-monotonicity, mostly occuring at high tax rates. Nevertheless the non-monotonicities in our simulation markets can be dramatic. Figure 4 presents a simulation market in which, just as in Example 1, the efficient matching is stable under full taxation ($\tau = 1$) but is unstable under a range of tax rates between 0 and 1.

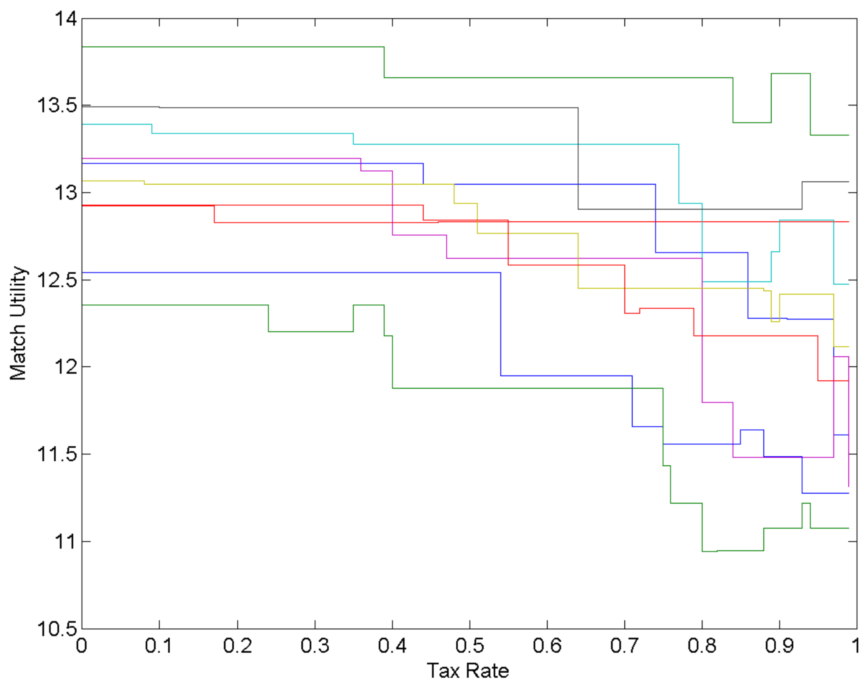

Figure 3: Total match utility of stable outcomes in ten simulated markets as the tax rate τ ranges from 0 to 1.

Note: The markets presented were randomly-selected from the set of simulated markets exhibiting non-monotonicities. Each market is one-to-one and has 20 agents on each side of the market, with match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$.

Table 1 summarizes the non-monotonocities arising in our simulations. Row 1 shows the fraction of markets that have non-monotonicities in a given tax rate range. While the majority of non-monotonicities occur at very high tax rates, 10% of our simulation markets have non-monotonicities at tax rates below 50%. Row 2 gives the (normalized) average size of the non-monotonicities in each tax rate range. Again, we see that non-monotonicities are most significant for high tax rates. Row 3 incorporates information on the persistence of non-monotonicity by computing the fraction of the deadweight loss from taxation that is due to non-monotonicity. This is relatively high for lower tax rates because there is less total deadweight loss at those tax rates.

ously. However, the non-monotonicities we fail to observe necessarily occur over very small ranges of τ, as we observe all non-monotonicities that occur following changes in τ of size at least .01.

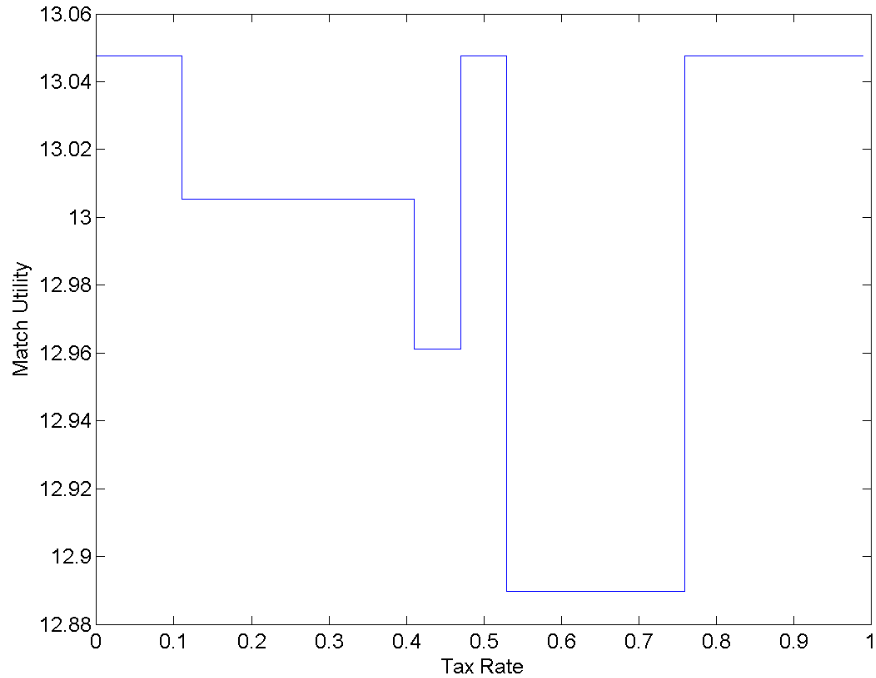

Figure 4: Total match utility of stable outcomes in a selected market as the tax rate τ ranges from 0 to 1.

Note: The market pictured is one-to-one and has 20 agents on each side of the market, with match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$.

Overall, our simulations suggest non-monotonicities in the tax rate are not just artifacts of example selection. However, they also suggest that non-monotonicities are relatively rare at more realistic tax rates ($\tau \in [0, .5)$) and tend not to persist over large ranges of τ.

Although Example 1 and the simulations show that total match utility of stable matchings may decrease when the tax rate falls, an arrangement that is stable under a tax rate $\hat{\tau}$ must improve the utility of at least one agent, relative to an arrangement that is stable under a tax rate $\tilde{\tau} > \hat{\tau}$.

Proposition 1. *Suppose that $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under tax $\tilde{\tau}$, with $\tilde{\tau} > \hat{\tau}$. Then, $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau}$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$).*[11]

To see the intuition behind Proposition 1, we consider the case in which $\tilde{\tau} = 1$: If $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau} = 1$) Pareto dominates $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$), then every manager $m \in M$ (weakly)

[11] We say that an arrangement $[\tilde{\mu}; \tilde{t}]$ under tax $\tilde{\tau}$ *Pareto dominates* arrangement $[\hat{\mu}; \hat{t}]$ under tax $\hat{\tau}$ if

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \qquad \forall m \in M,$$
$$\gamma^w_{\tilde{\mu}(w)} + \xi^{\mathsf{prop}}_{\tilde{\tau}}(\tilde{t}^{\tilde{\mu}(w) \to w}) \geq \gamma^w_{\hat{\mu}(w)} + \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\hat{\mu}(w) \to w}) \qquad \forall w \in W,$$

with strict inequality for some $i \in M \cup W$.

Table 1: Summary of the non-monotonicities arising in simulated markets.

	Range of τ				
	$[0, .25)$	$[.25, .5)$	$[.5, .75)$	$[.75, 1)$	All τ
Fraction of markets with non-monotonicity	0.006	0.088	0.190	0.394	0.548
Avg size of non-monotonicity, as fraction of range	0.021	0.066	0.111	0.140	0.120
Fraction of deadweight loss from taxation due to non-monotonicity	0.076	0.070	0.051	0.027	0.037

Note: The table summarizes 500 simulations of one-to-one matching markets with 20 agents on each side of the market. All agents' match utilities are independently and identically distributed according to a uniform distribution on $[-.5, .5]$. We vary the tax rate, τ, from 0 to .99 in increments of .01. Row 1 presents the fraction of markets that have non-monotonicities in a given tax rate range. Row 2 presents the average size of non-monotonicities within each range, normalized as a fraction of the (within-market) gap between the highest and lowest total stable match utilities that arise under any tax rate. Row 3 presents the average fraction of taxation deadweight loss that is due to non-monotonicity, across all markets. The deadweight loss from non-monotonicity for each tax rate τ is computed as the difference between the highest total match utility achieved at a stable matching for a tax rate $\tilde{\tau} \geq \tau$ and the total match utility of stable matchings under tax rate τ; the total deadweight loss from taxation at tax rate τ is computed as the difference in total match utility between the efficient matching and the total match utility of the matchings stable under tax rate τ.

prefers $\tilde{\mu}(m)$ to $\hat{\mu}(m)$ with the transfer $\hat{t}^{m \to \hat{\mu}(m)}$.[12] But then, because $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$,

$$\alpha^m_{\tilde{\mu}(m)} \overbrace{\geq}^{\text{Pareto}} \alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \overbrace{\geq}^{\text{Stability}} \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)},$$

so m must be offering a weakly positive transfer to $\tilde{\mu}(m)$ under $\hat{t}$ (that is, $\hat{t}^{m \to \tilde{\mu}(m)} \geq 0$). An analogous argument shows that each worker $w \in W$ must be offering a weakly positive transfer to $\tilde{\mu}(w)$ under $\hat{t}$ (that is, $\xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\tilde{\mu}(w) \to w}) \leq 0$). Moreover, Pareto dominance implies that at least one manager or worker must be paying a *strictly* positive transfer. But then, that agent must pay a strictly positive transfer and receive a weakly positive transfer – impossible.

[12]To see this, we first note that under tax $\tilde{\tau} = 1$, an arrangement with transfers of 0 among match partners Pareto dominates any other arrangement associated to the same matching. Thus, the transfers between match partners under $[\tilde{\mu}; \tilde{t}]$ can be assumed to be 0. Then, the comparison between $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau} = 1$) and $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$) amounts to a comparison of agents' match utilities under $\tilde{\mu}$ and their total utilities under $[\hat{\mu}; \hat{t}]$.

3.2 Efficiency of Tax-Reduction in Wage Markets

The preceding discussion shows that in general markets, decreasing the tax rate on transfers may *decrease* the total match utility of stable matchings. Our next result shows that in wage markets, these non-monotonicities do not arise – decreasing the tax rate in a wage market always makes (weakly) more efficient matchings stable.[13]

In wage markets, payments flow from managers to workers; hence, any stable matching can be supported by a non-negative transfer vector.[14] Thus, the transfer function $\xi_\tau^{\mathsf{prop}}(\cdot)$ takes the simpler form

$$\xi_\tau^{\mathsf{prop}}(t^{m\to w}) = (1-\tau)t^{m\to w} \geq 0.$$

As all positive transfers are paid from managers to workers, there cannot be a scenario in which, as in Example 1, when the tax is reduced, a manager m_1 can transfer enough to get a worker w_2 he prefers, but when the tax falls more, a different worker w_1 can "buy m_1 back." Our next result shows that this intuition extends to wage markets more generally.

Theorem 1. *In a wage market with proportional taxation, a decrease in taxation (weakly) increases the total match utility of stable matchings. That is, if in a wage market, matching $\tilde{\mu}$ is stable under tax $\tilde{\tau}$, matching $\hat{\mu}$ is stable under tax $\hat{\tau}$, and $\hat{\tau} < \tilde{\tau}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

To prove Theorem 1, we let $\hat{t} \geq 0$ and $\tilde{t} \geq 0$ be transfer vectors supporting $\hat{\mu}$ and $\tilde{\mu}$ respectively. The stability of $[\hat{\mu}; \hat{t}]$ under tax $\hat{\tau}$ implies that

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}, \tag{3}$$

$$\gamma^w_{\hat{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\hat{\mu}(w)\to w} \geq \gamma^w_{\tilde{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\tilde{\mu}(w)\to w}. \tag{4}$$

Summing (3) and (4) across agents, applying Lemma 1, and regrouping terms, we find that

$$\begin{aligned}\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) &= \sum_{m\in M}(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}) + \sum_{w\in W}(\gamma^w_{\hat{\mu}(w)} - \gamma^w_{\tilde{\mu}(w)}) \\ &\geq \hat{\tau}\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right). \end{aligned} \tag{5}$$

Intuitively, if we had $\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) < 0$, then lowering the tax from $\tilde{\tau}$ to $\hat{\tau}$ would increase workers' relative preference for $\tilde{\mu}$ over $\hat{\mu}$, as the tax change has a larger effect

[13]The non-monotonicities described in Section 3.1 arise from transfers flowing in both directions, either simultaneously or across equilibria. As transfers are an equilibrium phenomenon, requiring that transfers flow in one direction does not directly correspond to conditions on the primitives of the market. However, the wage market condition we use in Theorem 1 is a *sufficient* condition on primitives to guarantee that transfers to flow in one direction, and thus is sufficient to rule out non-monotonicity.

All the results in this section hold in any market where transfers always (across matches and tax rates) flow in one direction.

[14]There may be a supporting transfer vector where some off-path transfers (transfers between unmatched agents) are negative, but in that case there is always another supporting transfer vector that replaces those negative transfers with 0s. Our results only require the existence of a non-negative supporting transfer vector.

on larger transfers. Since $\hat{\mu}$ is stable under the lower tax $\hat{\tau}$, the difference in (5) must thus be positive; this implies Theorem 1.

Theorem 1 shows that the non-monotonicities observed in fully general markets in Section 3.1 do not arise in wage markets. To gain insight into how quickly non-monotonicity disappears as a market's structure becomes closer to that of a wage market, we return to our simulation environment. We begin with simulations in a setting identical to that used in Section 3.1: one-to-one markets with all match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$. We next consider one-to-one markets with match utilities slightly imbalanced across the market: managers' match utilities are independently and identically distributed according to a uniform distribution on $[-.45, .55]$, while workers' match utilities are independently and identically distributed according to a uniform distribution on $[-.55, .45]$. We repeat this process, adjusting the match utility means by .05 each time, to generate a series of markets ranging from our original symmetric markets to wage markets with manager utilities distributed uniformly on $[0, 1]$ and worker utilities distributed uniformly on $[-1, 0]$.[15] Figure 5 shows the how the fraction of markets with non-monotonocities changes as the mean manager utility varies.

Although total match utility in wage markets increases as the tax is reduced, *individual* utility may be non-monotonic. For example, pursuant to a tax decrease, a manager m may be made worse off because his match partner is now able to receive more from some other manger: In this circumstance, m might lose his match partner to his competitor; but even if m's match is unchanged, his total utility may decrease because he is forced to increase his transfer to compensate for a competitor's increased offer.

Individual managers' match utilities *may* decrease with a decrease in the tax rate, but the sum of workers' match utilities *must* decrease.

Proposition 2. *In a wage market with proportional taxation, if a matching $\tilde{\mu}$ is stable under tax $\tilde{\tau}$, and a matching $\hat{\mu}$ is stable under tax $\hat{\tau} < \tilde{\tau}$, then workers' aggregate match utility must be (weakly) higher under $\tilde{\mu}$ than under $\hat{\mu}$. That is,*

$$\sum_{w \in W} \gamma^w_{\tilde{\mu}(w)} \geq \sum_{w \in W} \gamma^w_{\hat{\mu}(w)}.$$

The logic is that in order for a less efficient match to be stable at the higher tax rate, it must be that workers prefer that match – and managers cannot lure workers to a more efficient match because of the high tax. Note that Proposition 2 does not imply that lower taxes necessarily make workers worse off: workers might receive higher transfers to compensate for their lower match utilities.

Finally, we show that if two distinct matchings $\hat{\mu}$ and $\tilde{\mu}$ are both stable under tax τ, then either managers and workers must disagree as to which matching is preferred, or both groups must be indifferent between the two matchings. This is a consequence of the following more general result.

[15] To reduce noise in the simulation process, we use a single set of baseline markets and repeatedly shift each match utility by .05 in the appropriate direction.

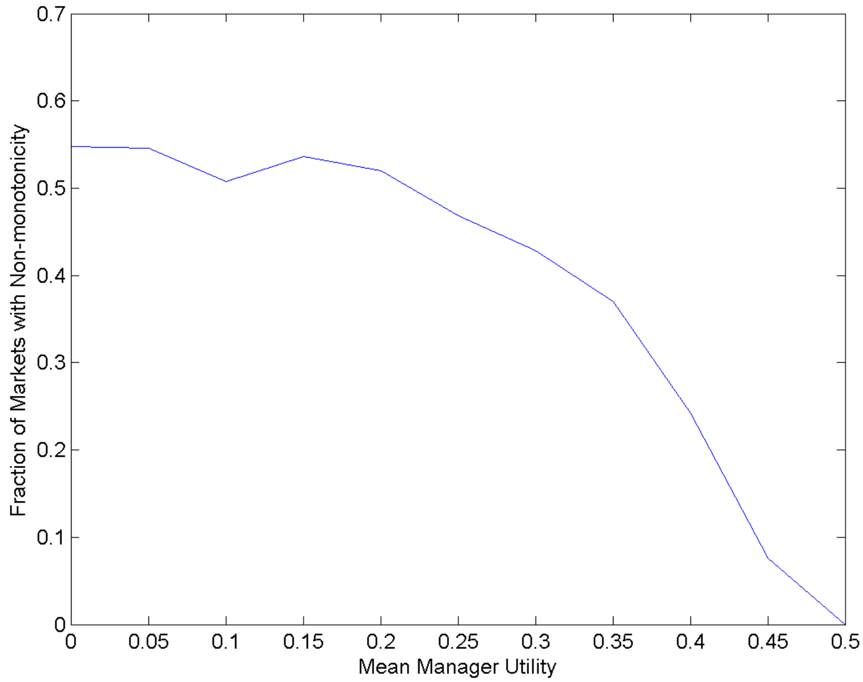

Figure 5: Fraction of simulated markets with non-monotonicities in the total match utility of stable outcomes (as the tax rate τ is increased from 0 to 1), as the mean manager utility ranges from 0 to .5.

Note: For each mean manger utility level, we report the fraction of the 500 simulated markets that have a non-monotonicity in total match utility as the tax rate increases from 0 to 1. All simulated markets are one-to-one and have 20 agents on each side of the market.

Proposition 3. *In a wage market with proportional taxation, if two distinct matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable under tax τ, then*

$$\sum_{w \in W} \left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)} \right) = (1-\tau) \sum_{m \in M} \left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)} \right). \tag{6}$$

Thus, if the managers are not indifferent in aggregate between $\tilde{\mu}$ and $\hat{\mu}$, then the only tax rate τ under which both $\tilde{\mu}$ and $\hat{\mu}$ can be stable is

$$\tau = 1 + \frac{\sum_{w \in W} \left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)} \right)}{\sum_{m \in M} \left(\alpha^m_{\tilde{\mu}(m)} - \alpha^m_{\hat{\mu}(m)} \right)}. \tag{7}$$

For τ as defined in (7) to be less than 1, the fraction in (7) must be negative, so that that managers and workers in aggregate disagree about which matching they prefer.

In order for there to be multiple values of τ at which two given matchings are both stable, it must be that both managers and (following (6)) workers are indifferent between those two matchings.

Corollary 1. *In a wage market with proportional taxation, if there is more than one tax under which two distinct matchings $\tilde{\mu}$ and $\hat{\mu}$ both are stable, then $\mathfrak{M}(\tilde{\mu}) = \mathfrak{M}(\hat{\mu})$.*

Corollary 1 implies that for generic match utilities, there is at most one value of τ at which two matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable; in this case, since there are finitely many matchings, there is a unique stable matching under almost every tax τ.

4 Lump Sum Taxation

While not typically phrased in the exact language of taxation, lump sum taxes are present throughout labor markets. They might take the form of costs for hiring (e.g., employee health care costs) or for entering employment (e.g., licensing requirements). In the marriage market context, lump sum taxes can take the form of marriage license fees or tax penalties for marriage.

4.1 Lump Sum Taxation of Transfers

We first consider a lump sum tax that is levied only on (nonzero) transfers between match partners.[16] Such a *lump sum tax on transfers*, f, corresponds to the transfer function

$$\xi_f^{\mathsf{lump}}(t^{m \to w}) \equiv \begin{cases} t^{m \to w} - f & t^{m \to w} \neq 0 \\ t^{m \to w} & t^{m \to w} = 0. \end{cases}$$

Figure 6 shows this transfer function. Under this tax structure, the case $f = 0$ corresponds to the standard (Shapley and Shubik (1971)) model of matching with transfers and the case $f = \infty$ corresponds to (Gale and Shapley (1962)) matching without transfers.

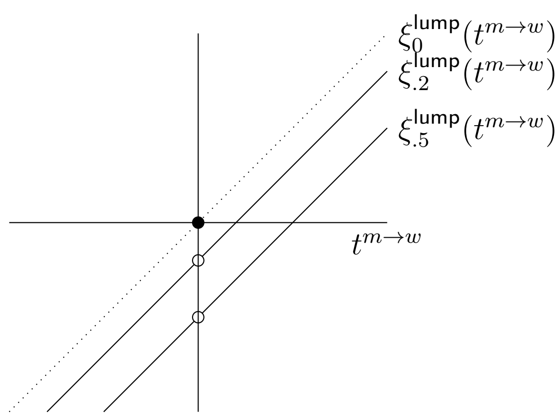

Figure 6: Transfer function $\xi_f^{\mathsf{lump}}(\cdot)$.

We say that an arrangement or matching is *stable under lump sum tax f* if it is stable given transfer function $\xi_f^{\mathsf{lump}}(\cdot)$.

[16] An alternative approach to lump sum taxation, which we discuss in the next section, imposes a flat fee on all matches.

A lump sum tax on transfers has an extensive margin effect that makes being unmatched more attractive relative to matching with a transfer. In non-wage markets,[17] a lump sum tax on transfers can also encourage matchings in which transfers are unnecessary.[18] As our next example illustrates, this second distortion can cause the total match utility of stable matchings to be non-monotonic in the size of the lump sum tax.

Example 2. Consider a one-to-one market with two managers – $M = \{m_1, m_2\}$ – three workers – $W = \{w_1, w_2, w_3\}$ – and match utilities as pictured in Figure 7a. Worker w_1 likes m_1 – who has a slight preference for w_2 – but w_2 strongly prefers m_2. Manager m_2, meanwhile, has a strong preference for w_3, who slightly dislikes him. When transfers are not allowed (or when there is a high lump sum tax on transfers, $f \geq 195$), the only stable matching is the matching μ_1 in which $\mu_1(m_1) = w_1$ and $\mu_1(m_2) = w_2$, as shown in Figure 7b. This matching yields total match utility of $\mathfrak{M}(\mu_1) = 360$.

When the lump sum tax is lowered to $f = 185$, only the matching μ_2 is stable, where $\mu_2(m_1) = w_2$ and $\mu_2(m_2) = w_3$; this matching gives a total match utility $\mathfrak{M}(\mu_2) = 280$, as shown in Figure 7d. When $f = 185$, the tax is low enough that m_2 can convince w_3 to match with him, but not low enough for w_1 to hold onto m_1 when he has the option of matching with w_2 (which he has once m_2 does not want to match with w_2). The first change relative to μ_1 (m_2 switching to w_3) progresses towards the efficient matching, μ_3, shown in Figure 7c, but the second change relative to μ_1 (m_1 switching to w_2) does not. Lowering the lump sum tax from 200 to 185 decreases the total match utility of the stable matching.

Just as in Section 3, we use simulations to confirm that Example 2 is not an exceptional case. We return to the 500 randomly drawn one-to-one markets presented in Section 3, and consider lump sum taxes varying from 0 to 1 in increments of .01. We find that match utility is non-monotonic in the lump sum tax in 61% of our simulated markets.

Figure 8 plots the total match utility as a function of the tax rate in ten randomly-selected simulation markets with non-monotonicities under lump-sum taxation. In all markets, the total match utility is unchanged for lump sum taxes above .5 because this is the maximum individual match utility. (In equilibrium there are no transfers paid when the tax is .5, so increasing the lump sum tax on transfers above .5 has no effect.)

In strictly positive wage markets, all matchings require a transfer, so a lump sum tax on transfers does not distort agents' preferences among match partners – for a given transfer vector, if a worker prefers manager m_1 to m_2 without a tax, then that worker also prefers m_1 to m_2 under a lump sum tax. Thus, in strictly positive wage markets, the matching distortion of the lump sum tax is only on the extensive margin – the decision of *whether* to match – under a higher lump sum tax, fewer agents find matching desirable. This intuition is captured in the following lemma.

[17]Since it is difficult to observe transfers in non-wage markets, such as marriage markets, it is somewhat hard to imagine taxing them. Nevertheless, lump-sum taxes on transfers could correspond to instituting a lump sum tax on gifts between spouses, and flat fees for matching could correspond to requiring marriage license fees.

[18]To see this, consider the case of one-to-one matching markets. In such markets, lump sum taxes on transfers promotes pairing (m, w) in which the match utility $\alpha_w^m + \gamma_m^w$ is evenly distributed between the two partners ($\alpha_w^m \approx \gamma_m^w$), so that transfers are unnecessary.

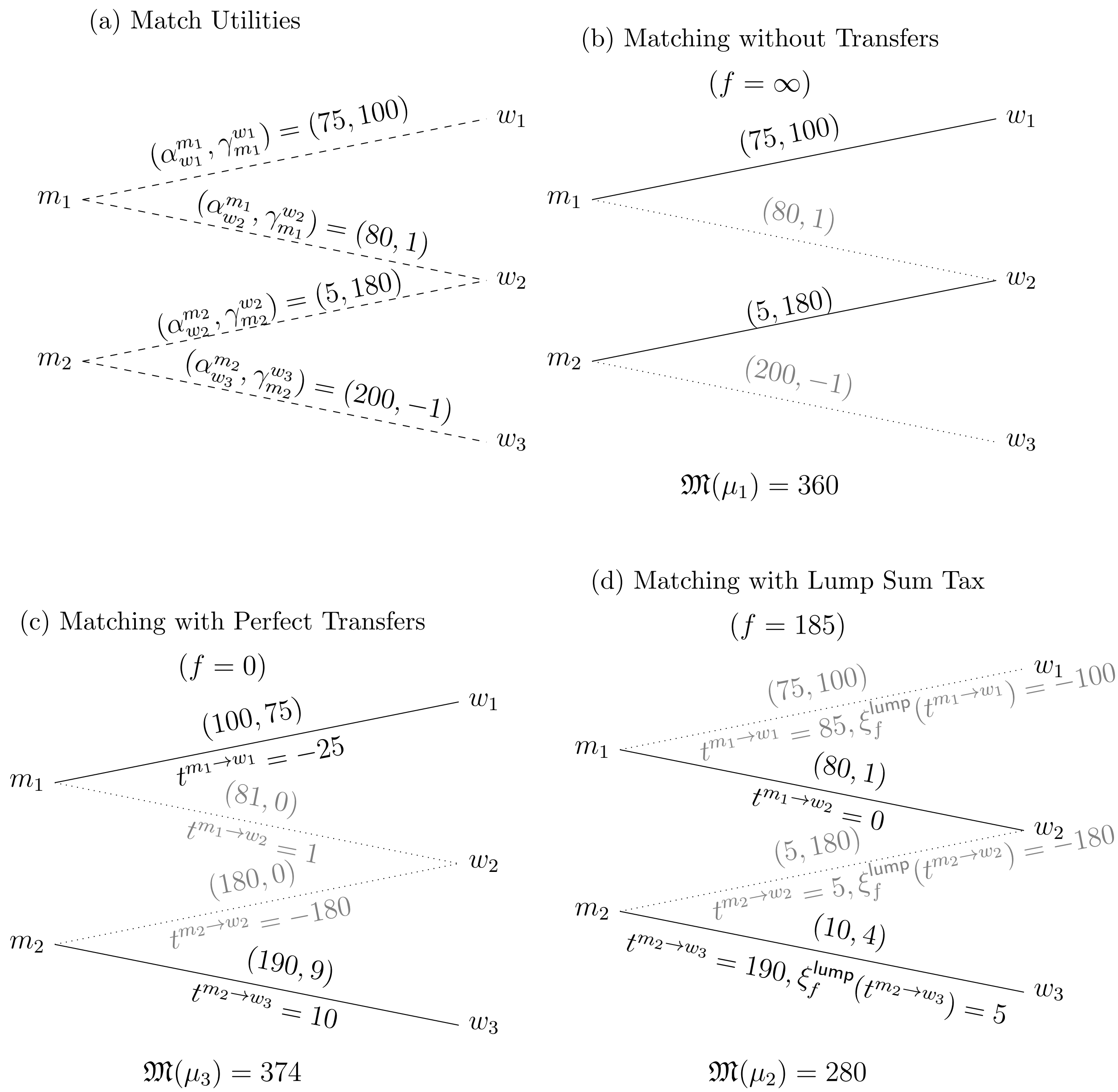

Figure 7: Example 2 – Non-monotonicity under a lump sum tax on transfers.

Note: Utilities, net of transfers, are above the lines (manager's, worker's). Possible supporting transfers (when applicable) are below the lines. Solid lines indicate the stable matching.

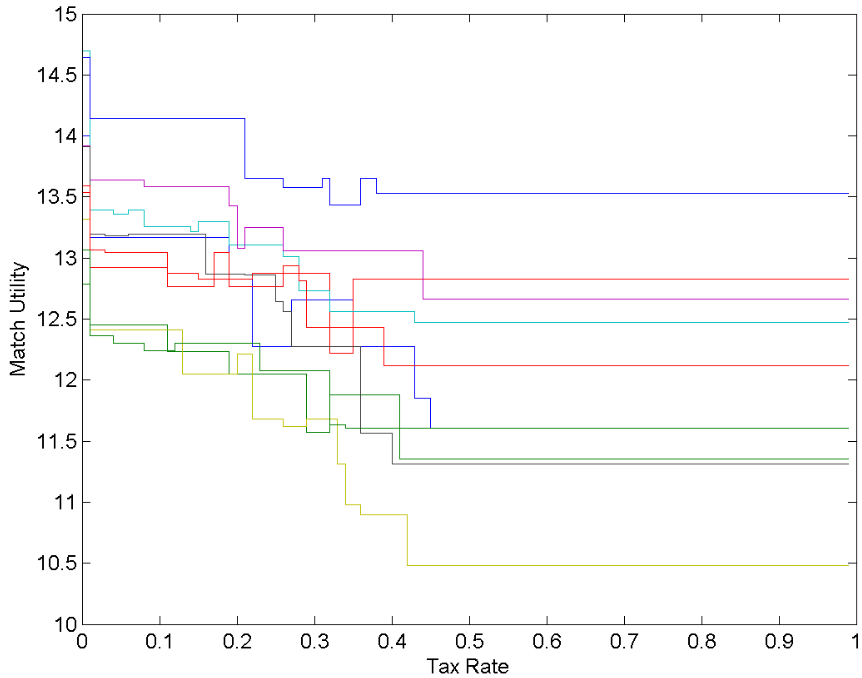

Figure 8: Total match utility of stable outcomes in ten simulated markets as the lump sum tax f ranges from 0 to 1.

Note: The markets presented were randomly-selected from the set of simulated markets exhibiting non-monotonicities. Each market is one-to-one and has 20 agents on each side of the market, with match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$.

Lemma 2. *In strictly positive wage markets, reduction in a lump sum tax on transfers (weakly) increases the number of workers matched in stable matchings. That is, if matching $\tilde{\mu}$ is stable under lump sum tax $\tilde{f}$, matching $\hat{\mu}$ is stable under lump sum tax $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\#(\hat{\mu}) \geq \#(\tilde{\mu}),$$

where $\#(\mu)$ denotes the number of workers matched in matching μ.

In non-wage markets, the conclusion of Lemma 2 is not true, in general, because distortion among match partners can dominate the extensive margin effect.[19]

As lump sum taxes do not distort among match partners in strictly positive wage markets, they can only reduce the efficiency of stable matchings in such markets by reducing the number of workers matched. This observation, when combined with Lemma 2, gives the following result.

[19]To see this in the context of Example 2, consider adding a manager m_3 who receives match utility ε when matched with worker w_3, and gets negative match utility from all other partners. If w_1, w_2, and w_3 respectively receive 0, 0, and ε from matching with m_3 and the lump sum tax is $f \geq 195$, then w_3 will match with m_3 – so that all three workers are matched. When $f = 185$, meanwhile, the matching is the same as without m_3 – so worker w_1 ends up unmatched.

Theorem 2. *In strictly positive wage markets, a reduction in a lump sum tax on transfers (weakly) increases the total match utility of stable matchings. That is, if $\tilde{\mu}$ is stable under lump sum tax $\tilde{f}$, $\hat{\mu}$ is stable under lump sum tax $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

Theorem 2 indicates that in strictly positive wage markets, match utility increases monotonically as lump sum taxation decreases.

Just as in the case of proportional taxation, non-monotonicity disappears as a market's structure becomes closer to that of a wage market. Using the same set of simulation markets described in Section 3.2, we analyze how the fraction of markets with non-monotonicities changes as we move from symmetric markets to wage markets. Figure 9 shows the results. We see that a substantial amount of market asymmetry is needed before the fraction of markets with non-monotonicities drops below 50%.

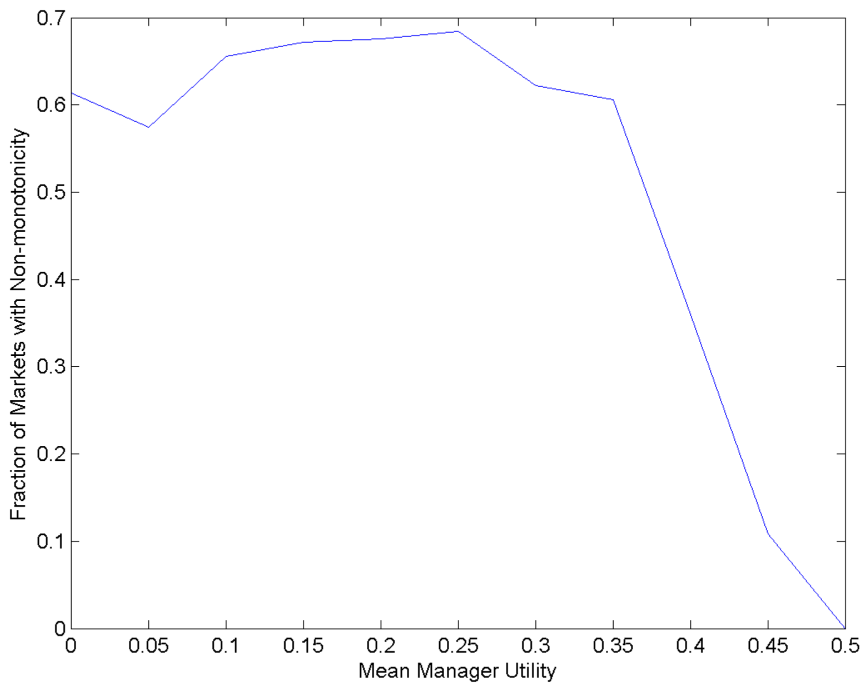

Figure 9: Fraction of simulated markets with non-monotonicities in the total match utility of stable outcomes (as the lump sum tax f is increased from 0 to 1), as the mean manger utility ranges from 0 to .5.

Note: For each mean manger utility level, we report the fraction of the 500 simulated markets that have a non-monotonicity in total match utility as the lump sum increases from 0 to 1. All simulated markets are one-to-one and have 20 agents on each side of the market.

In strictly positive wage markets, we can also bound the total match utility loss from a given lump sum tax.

Proposition 4. *In a strictly positive wage market, let $\hat{\mu}$ be an efficient matching, and let $\tilde{\mu}$ be stable under lump sum tax on transfers $\tilde{f}$. Then,*

$$0 \leq \mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) \leq \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})).$$

The intuition for Proposition 4 is that since the workers unmatched under a lump sum tax of $\tilde{f}$ have negative surplus from matching under that lump sum tax, their surplus from matching could not be more than $\tilde{f}$. So the change in total utility is less than the change in the number of unmatched workers times a maximum surplus of $\tilde{f}$ per worker.

Finally, we can show that, for a *fixed* limit on the number of workers matched in the presence of a lump sum tax, stable matchings in strictly positive wage markets must generate the maximal match utility possible.

Proposition 5. *In a strictly positive wage market, a matching $\tilde{\mu}$ can be stable under a lump sum tax on transfers only if*

$$\tilde{\mu} \in \underset{\{\mu : \#(\mu) \leq \#(\tilde{\mu})\}}{\arg\max} \left[\mathfrak{M}(\mu)\right].$$

Proposition 5 shows that a lump sum tax is an efficient way for a market designer to limit the number of matches (in strictly positive wage markets): the matchings stable under lump sum taxation have maximal utility, given the tax's implied limit on the number of agents matched. Analogously, if a market designer wants to encourage matches, a lump-sum subsidy will maximize total match utility for a given (subsidy-induced) lower bound on the number of agents matched. This suggests that if a government wants to use tuition subsidies to encourage people to go to school, then uniform tuition subsidies are more efficient than subsidies proportional to the cost of tuition.

4.2 Lump Sum Taxation of Matches

Some fee structures tax *all* pairings, rather than just those that include nonzero transfers. Such *flat fees for matching* can also be interpreted in the language of taxation: they correspond to the transfer function

$$\xi_f^{\mathsf{fee}}(t^{m \to w}) \equiv t^{m \to w} - f.$$

Figure 10 shows this transfer function for different levels of f.

Unlike lump sum taxes on transfers, flat fees for matching never distort among match partners – even in non-wage markets. Flat fees for matching only have extensive margin effects, and thus markets with such fees are similar to strictly positive wage markets with lump sum taxes on transfers.[20] As we show in the Appendix, the conclusions of Lemma 2, Theorem 2, and Propositions 4 and 5 *always* hold in markets with flat fees for matching.

5 Discussion

Before concluding, we briefly remark on structural properties common to both models of taxation.

[20] Indeed, in strictly positive wage markets, lump sum taxation of transfers is equivalent to lump sum taxation of matchings because workers never match without receiving a strictly positive transfer.

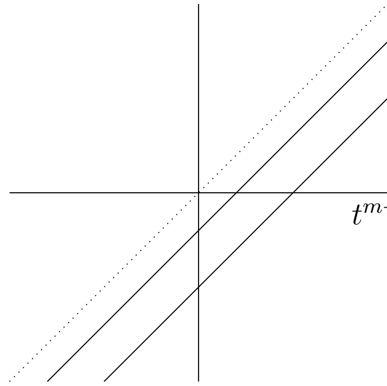

Figure 10: Transfer function $\xi_f^{\mathsf{fee}}(\cdot)$.

The Effect of Very Small Taxes

Unlike in non-matching models of taxation, in our setting there is always a non-zero tax that does not generate distortions. To see this in the proportional tax setting, let $\hat{\mu}$ be an efficient matching. Our results show that if $\tilde{\mu}$ is stable under $\tilde{\tau}$, then[21]

$$\tilde{\tau} \geq \frac{\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu})}{\sum_{m\in M}(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)})}. \tag{8}$$

For any inefficient matching $\tilde{\mu}$, there is a strictly positive minimum tax $\underline{\tau}(\tilde{\mu})$ at which $\tilde{\mu}$ could possibly be stable. Since there are finitely many possible matchings, we can just take the minimum of this threshold across inefficient matchings,

$$\underline{\tau}^* = \min_{\{\mu:\mathfrak{M}(\mu)<\mathfrak{M}(\hat{\mu})\}} [\underline{\tau}(\mu)] .$$

For $\tau < \underline{\tau}^*$ only an efficient matching can be stable.[22] The argument for the case of lump sum taxation is similar.

Structure of the Set of Stable Arrangements

Results of Kelso and Crawford (1982) and Hatfield and Milgrom (2005) imply that for any fixed τ, or f, if there are multiple stable arrangements, then workers' and managers' interests are opposed. If all managers prefer $[\mu; t]$ to $[\hat{\mu}; \hat{t}]$, then all workers prefer $[\hat{\mu}; \hat{t}]$ to $[\mu; t]$. Moreover, there exists a manager-optimal (worker-pessimal) stable arrangement that the managers weakly prefer to all other stable arrangements and a worker-optimal (manager-pessimal) stable arrangement that all workers weakly prefer. In wage markets with proportional taxation, where there is generically a unique stable matching, this opposition of interests carries over to the set of supporting transfer vectors.

[21]See Equation (38) of the Appendix.

[22]One caveat is that if there are multiple efficient matchings (all of which are stable when $\tau = 0$), some of them may *not* be stable in the limit as $\tau \to 0$ or $f \to 0$.

6 Conclusion

We analyze the matching distortion that arises when taxes on transfers affect the matching of workers to managers. This matching distortion is not necessarily monotonic in the tax level. In wage markets, however, matching distortions always decrease as taxes are reduced. The matching distortion we identify affects the "allocative margin" and is distinct from distortions on the intensive or extensive margins.

An extension of our work would examine the interaction of allocative and intensive margin effects in a model that allows for non-binary labor supply decisions. It would also be valuable to analyze how the magnitude of the matching distortion depends on the variance and heterogeneity of agents' preferences. Such work might inform the estimation of the losses due to matching distortions in real-world labor markets.

It is also natural to ask about revenue: How much revenue do different tax structures generate in matching markets? For a given revenue requirement, does a proportional tax generate more or less distortion than a lump sum tax?

The first challenge in answering questions about revenue is that for any stable match, there may be an infinite lattice of supporting transfer vectors. For proportional taxation, revenue depends on the choice of supporting transfer vector. The easiest transfer vectors to think about are the maximal (worker-optimal) supporting transfer vectors and the minimal (manager-optimal) supporting transfer vectors, which, in wage markets, correspond to maximal and minimal revenue (given the match and tax rate).

Even after focusing on the extremal supporting transfers, addressing revenue questions requires adding structure on agents' match utilities. Unfortunately, it is not clear whether there is a natural structure to impose. Most papers in the matching literature that (unlike our work) do not allow completely arbitrary match utilities both (1) model match surplus as a function of one-dimensional agent types and (2) assume positive assortative matching. Assuming one-dimensional agent types implies that all agents on a given side of the market agree on the ordinal rankings of agents on the other side of the market. This shifts much of the distortion to the extensive margin – at any tax rate or level, the most desirable agents on each side will be matched. Moreover, in our framework, just measuring match surplus is insufficient because the *ex ante* (i.e. pre-transfer) split of match utility, rather than just the total surplus, affects match outcomes in the presence of taxation. Assuming positive assortative matching, meanwhile, reduces our setting to a discrete version of the traditional supply and demand framework: deadweight loss just comes from agents choosing not to transact, and the associated welfare losses are equal to the taxes those agents pay right before exiting the market.

A tractable model that enables revenue analysis while still allowing for substantial preference heterogeneity is beyond the scope of this paper. But such a model would an interesting goal for future work – it would likely be useful for realistic applied matching analysis more generally.

Appendix

Existence of Stable Arrangements

In this section, we use results from the literature on matching with contracts to show the existence of stable arrangements in our framework. For a given transfer vector t, the *demand of manager $m \in M$*, denoted $D^m(t)$, is

$$D^m(t) \equiv \arg\max_{Y \subseteq W} \left\{ \alpha_Y^m - t^{m \to Y} \right\}.$$

Definition (Kelso and Crawford (1982))**.** The preferences of manager $m \in M$ are *substitutable* if for any transfer vectors t and $\tilde{t}$ with $\tilde{t} \geq t$, there exists, for each $Y \in D^m(t)$, some $\tilde{Y} \in D^m(\tilde{t})$ such that

$$\tilde{Y} \supseteq \{w \in Y : t^{m \to w} = \tilde{t}^{m \to w}\}.$$

That is, the preferences of $m \in M$ are substitutable if an increase in the "prices" of some workers cannot decrease demand for the workers whose prices remain unchanged.[23]

Theorem 2 of Kelso and Crawford (1982) shows that under the assumption that all managers' preferences are substitutable, there is an arrangement $[\mu; t]$ that is *strict core*, in the sense that:[24]

- Each agent (weakly) prefers his assigned match partner(s) (with the corresponding transfer(s)) to being unmatched, that is,

$$u^i([\mu; t]) \geq 0 \qquad\qquad \forall i \in M \cup W.$$

- There does not exist a manager $m \in M$, a set of workers $Y \subseteq W$, and a transfer vector $\tilde{t}$ such that

$$\begin{aligned} \alpha_Y^m - \tilde{t}^{m \to Y} &\geq \alpha_{\mu(m)}^m - t^{m \to \mu(m)}, && \text{and} \\ \gamma_m^w + \xi(\tilde{t}^{m \to w}) &\geq \gamma_{\mu(w)}^w + \xi(t^{\mu(w) \to w}) && \forall w \in Y, \end{aligned}$$

  with strict inequality for at least one $i \in (\{m\} \cup Y)$.

The Kelso and Crawford (1982) (p. 1487) construction of competitive equilibria from strict core allocations then implies that there is some transfer vector $\hat{t}$, having $\hat{t}^{\mu(w) \to w} = t^{\mu(w) \to w}$ (for each $w \in W$), such that $[\mu; \hat{t}]$ is stable in our sense.

[23] Theorem A.1 of Hatfield et al. (2013) shows that in our setting the Kelso and Crawford (1982) substitutability condition is equivalent to the choice-based substitutability condition of Hatfield and Milgrom (2005), that we describe in the main text: *the availability of new workers cannot make a manager want to hire a worker he would otherwise reject.*

[24] Strictly speaking, Kelso and Crawford (1982) have one technical assumption not present in our framework: they assume that $\alpha_w^m + \gamma_m^w \geq 0$, in order to ensure that all workers are matched. However, examining the Kelso and Crawford (1982) arguments reveals that this extra assumption is not necessary to ensure that a strict core arrangement exists – the Kelso and Crawford (1982) salary adjustment processes can be started at some arbitrarily low (negative) salary offer and all of the steps and results of Kelso and Crawford (1982) remain valid, with the caveat that some workers may be unmatched at core outcomes.

Proof of Lemma 1

We let $\mathfrak{B}$ be the set of managers who are matched at μ and let $\mathscr{B}$ be the set of workers who are matched at μ. This means that

$$\begin{aligned} \mu(m) &\subseteq \mathscr{B} && \forall m \in \mathfrak{B}, \\ \mu(w) &\in \mathfrak{B} && \forall w \in \mathscr{B}. \end{aligned}$$

These observations, combined with the fact that $t^{m\to m} = t^{w\to w} = 0$, enable us to show that

$$\begin{aligned} \sum_{m\in M} t^{m\to\mu(m)} &= \sum_{m\in\mathfrak{B}} t^{m\to\mu(m)} + \sum_{m\in M\setminus\mathfrak{B}} t^{m\to\mu(m)}, \\ &= \sum_{m\in\mathfrak{B}} t^{m\to\mu(m)}, \\ &= \sum_{m\in\mathfrak{B}} \sum_{w\in\mu(m)} t^{m\to w}, \\ &= \sum_{w\in\mathscr{B}} t^{\mu(w)\to w}, \\ &= \sum_{w\in\mathscr{B}} t^{\mu(w)\to w} + \sum_{w\in W\setminus\mathscr{B}} t^{\mu(w)\to w}, \\ &= \sum_{w\in W} t^{\mu(w)\to w}. \end{aligned}$$

Proof of Proposition 1

First, we show that the outcomes stable under full taxation ($\tilde{\tau} = 1$) cannot Pareto dominate those stable under tax $\hat{\tau} < 1$.

Claim. *Suppose that $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau} < 1$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under tax $\tilde{\tau} = 1$. Then, $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau} = 1$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau} < 1$).*

Proof. As no transfers get through under full taxation, an arrangement stable under full taxation is most likely to Pareto dominate some other arrangement when all transfers between match partners are 0. Thus, we assume that $\tilde{t}^{\tilde{\mu}(w)\to w} = 0$ for each $w \in W$, and suppose that $[\tilde{\mu}; \tilde{t}]$ (under full taxation) Pareto dominates $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$). This would imply that

$$\alpha^m_{\tilde{\mu}(m)} = \alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)}, \tag{9}$$

$$\gamma^w_{\tilde{\mu}(w)} = \gamma^w_{\tilde{\mu}(w)} + \xi^{\mathsf{prop}}_{\tilde{\tau}}(\tilde{t}^{\tilde{\mu}(w)\to w}) \geq \gamma^w_{\hat{\mu}(w)} + \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\hat{\mu}(w)\to w}), \tag{10}$$

with strict inequality for some m or w. However, stability of $[\hat{\mu}; \hat{t}]$ under tax $\hat{\tau}$ implies that

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}, \tag{11}$$

$$\gamma^w_{\hat{\mu}(w)} + \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\hat{\mu}(w)\to w}) \geq \gamma^w_{\tilde{\mu}(w)} + \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\tilde{\mu}(w)\to w}). \tag{12}$$

Combining (9) and (11) gives

$$0 \geq -\hat{t}^{m\to\tilde{\mu}(m)}, \tag{13}$$

for each $m \in M$, while combining (10) and (12) gives

$$0 \geq \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\tilde{\mu}(w) \to w}), \tag{14}$$

for each $w \in W$. Strict inequality must hold in (13) or (14) for some m or w.

In the first of these cases, we have

$$\hat{t}^{m' \to \tilde{\mu}(m')} > 0$$

for some $m' \in M$; hence, there exists at least one $w \in \hat{\mu}(m')$ for whom

$$\hat{t}^{\tilde{\mu}(w) \to w} > 0. \tag{15}$$

But (15) contradicts (14).

In the second case, we have

$$0 > \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\tilde{\mu}(w') \to w'}), \tag{16}$$

for some $w' \in W$. If we take $m = \tilde{\mu}(w')$, then (16) and (14) together imply that

$$0 > \sum_{w \in \tilde{\mu}(m)} \hat{t}^{\tilde{\mu}(w) \to w} = \hat{t}^{m \to \tilde{\mu}(m)},$$

contradicting (13). $\square$

For $\tilde{\tau} < 1$, $\xi^{\mathsf{prop}}_{\tilde{\tau}}(\cdot)$ is strictly increasing and the conclusion of the proposition follows from the following more general result.

Proposition 1'. *Suppose that $\tilde{\xi}(\cdot)$ is strictly increasing, that $[\hat{\mu}; \hat{t}]$ is stable under $\hat{\xi}(\cdot)$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under $\tilde{\xi}(\cdot)$, with $\tilde{\xi}(\cdot) \leq \hat{\xi}(\cdot)$. Then, $[\tilde{\mu}; \tilde{t}]$ (under $\tilde{\xi}(\cdot)$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under $\hat{\xi}(\cdot)$).*[25]

Proof. The case for $\tilde{\tau}$ Pareto dominance of $[\tilde{\mu}; \tilde{t}]$ (under $\tilde{\xi}(\cdot)$) over $[\hat{\mu}; \hat{t}]$ (under $\hat{\xi}(\cdot)$) would imply that

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)}, \tag{17}$$

$$\gamma^w_{\tilde{\mu}(w)} + \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w) \to w}) \geq \gamma^w_{\hat{\mu}(w)} + \hat{\xi}(\hat{t}^{\hat{\mu}(w) \to w}), \tag{18}$$

with strict inequality for some m or w. However, stability of $[\hat{\mu}; \hat{t}]$ under $\hat{\xi}(\cdot)$ implies that

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)}, \tag{19}$$

$$\gamma^w_{\hat{\mu}(w)} + \hat{\xi}(\hat{t}^{\hat{\mu}(w) \to w}) \geq \gamma^w_{\tilde{\mu}(w)} + \hat{\xi}(\tilde{t}^{\tilde{\mu}(w) \to w}) \geq \gamma^w_{\tilde{\mu}(w)} + \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w) \to w}), \tag{20}$$

[25] We say that an arrangement $[\tilde{\mu}; \tilde{t}]$ (under $\tilde{\xi}(\cdot)$) *Pareto dominates* arrangement $[\hat{\mu}; \hat{t}]$ under (under $\hat{\xi}(\cdot)$) if

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \qquad \forall m \in M,$$

$$\gamma^w_{\tilde{\mu}(w)} + \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w) \to w}) \geq \gamma^w_{\hat{\mu}(w)} + \hat{\xi}(\hat{t}^{\hat{\mu}(w) \to w}) \qquad \forall w \in W,$$

with strict inequality for some $i \in M \cup W$.

where the second inequality in (20) follows from the fact that $\hat{\xi}(\cdot) \geq \tilde{\xi}(\cdot)$.

Combining (17) and (19) gives

$$\hat{t}^{m \to \tilde{\mu}(m)} \geq \tilde{t}^{m \to \tilde{\mu}(m)}, \tag{21}$$

for each $m \in M$, while combining (18) and (20) gives

$$\begin{aligned} \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w) \to w}) &\geq \tilde{\xi}(\hat{t}^{\tilde{\mu}(w) \to w}) \\ \tilde{t}^{\tilde{\mu}(w) \to w} &\geq \hat{t}^{\tilde{\mu}(w) \to w} \end{aligned} \tag{22}$$

for each $w \in W$, where the second line of (22) follows from the fact that $\tilde{\xi}(\cdot)$ is strictly increasing. Strict inequality must hold in (21) or (22) for some m or w.

In the first of these cases, we have

$$\hat{t}^{m' \to \tilde{\mu}(m')} > \tilde{t}^{m' \to \tilde{\mu}(m')}$$

for some $m' \in M$; hence, there exists at least one $w \in \hat{\mu}(m')$ for whom

$$\hat{t}^{\tilde{\mu}(w) \to w} > \tilde{t}^{\tilde{\mu}(w) \to w}. \tag{23}$$

But (23) contradicts (22).

In the second case, we have

$$\tilde{t}^{\tilde{\mu}(w') \to w'} > \hat{t}^{\tilde{\mu}(w') \to w'} \tag{24}$$

for some $w' \in W$. If we take $m = \tilde{\mu}(w')$, then (24) and (22) together imply that

$$\sum_{w \in \tilde{\mu}(m)} \tilde{t}^{\tilde{\mu}(w) \to w} > \sum_{w \in \tilde{\mu}(m)} \hat{t}^{\tilde{\mu}(w) \to w};$$

hence, we find that

$$\tilde{t}^{m \to \tilde{\mu}(m)} > \hat{t}^{m \to \tilde{\mu}(m)},$$

contradicting (21). □

Proof of Theorem 1

If $\hat{\mu} = \tilde{\mu}$, then the theorem is trivially true. Thus, we consider a wage market in which $[\tilde{\mu}; \tilde{t}]$ is stable under tax $\tilde{\tau}$, $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$, $\tilde{\tau} > \hat{\tau}$, and $\tilde{\mu} \neq \hat{\mu}$.

The stability conditions for the managers imply that

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \tilde{t}^{m \to \hat{\mu}(m)}, \tag{25}$$

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)}; \tag{26}$$

these inequalities together imply that

$$\sum_{m \in M} \left(\tilde{t}^{m \to \hat{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \right) \geq \sum_{m \in M} \left(\hat{t}^{m \to \hat{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)} \right). \tag{27}$$

As the market is a wage market, we have

$$\xi_\tau^{\mathsf{prop}}(\tilde{t}^{\tilde{\mu}(w)\to w}) = (1-\tilde{\tau})\tilde{t}^{\tilde{\mu}(w)\to w} \quad \text{and} \quad \xi_\tau^{\mathsf{prop}}(\hat{t}^{\tilde{\mu}(w)\to w}) = (1-\hat{\tau})\hat{t}^{\tilde{\mu}(w)\to w};$$

hence, the stability conditions for the workers imply that

$$\gamma^w_{\tilde{\mu}(w)} + (1-\tilde{\tau})\tilde{t}^{\tilde{\mu}(w)\to w} \geq \gamma^w_{\hat{\mu}(w)} + (1-\tilde{\tau})\tilde{t}^{\hat{\mu}(w)\to w}, \tag{28}$$
$$\gamma^w_{\hat{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\hat{\mu}(w)\to w} \geq \gamma^w_{\tilde{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\tilde{\mu}(w)\to w}. \tag{29}$$

Summing these inequalities and applying Lemma 1, we obtain

$$(1-\hat{\tau})\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq (1-\tilde{\tau})\sum_{m\in M}\left(\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)}\right). \tag{30}$$

Combining (27) and (30), we find that

$$(1-\hat{\tau})\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq (1-\tilde{\tau})\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right). \tag{31}$$

Since $\hat{\tau} < \tilde{\tau}$, (31) implies that

$$\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq 0. \tag{32}$$

Next, using (26) and (29), we find that

$$\begin{aligned}
\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) &= \sum_{m\in M}(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}) + \sum_{w\in W}(\gamma^w_{\hat{\mu}(w)} - \gamma^w_{\tilde{\mu}(w)}) \\
&\geq \sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) - (1-\hat{\tau})\sum_{w\in W}\left(\hat{t}^{\hat{\mu}(w)\to w} - \hat{t}^{\tilde{\mu}(w)\to w}\right), \\
&= \hat{\tau}\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq 0,
\end{aligned}$$

where the final inequality follows from (32).

Proof of Proposition 2 and Derivation of Equation (8)

Summing (28) across women, we find that

$$\begin{aligned}
\sum_{w\in W}\left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)}\right) &\geq (1-\tilde{\tau})\sum_{w\in W}\left(\tilde{t}^{\hat{\mu}(w)\to w} - \tilde{t}^{\tilde{\mu}(w)\to w}\right) && (33)\\
&\geq (1-\tilde{\tau})\sum_{w\in W}\left(\hat{t}^{\hat{\mu}(w)\to w} - \hat{t}^{\tilde{\mu}(w)\to w}\right) && (34)\\
&\geq 0, && (35)
\end{aligned}$$

where the inequality (34) follows from (27), and the inequality (35) follows from (32). Thus, we see Proposition 2 – the workers receive higher match utility under $\tilde{\mu}$ than under $\hat{\mu}$.

Furthermore, this implies that

$$\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right) \geq 0, \tag{36}$$

so that we may calculate the lowest tax under which a given inefficient match $\tilde{\mu}$ can be stable. Combining (25) and (33), we find that

$$\sum_{w\in W}\left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)}\right) \geq (1-\tilde{\tau})\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right). \tag{37}$$

The inequality in (36) allows us to rearrange (37) to obtain

$$\frac{\sum_{w\in W}\left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)}\right)}{\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right)} \geq (1-\tilde{\tau}),$$

so that we find

$$\begin{aligned}\tilde{\tau} &\geq \frac{\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right)}{\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right)} + \frac{\sum_{w\in W}\left(\gamma^w_{\hat{\mu}(w)} - \gamma^w_{\tilde{\mu}(w)}\right)}{\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right)} \\ &= \frac{\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu})}{\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right)}.\end{aligned} \tag{38}$$

Proofs of Proposition 3 and Corollary 1

Suppose that in a wage market, both $[\tilde{\mu};\tilde{t}]$ and $[\hat{\mu};\hat{t}]$ are stable under tax τ. The stability conditions for the managers imply that

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \tilde{t}^{m\to\hat{\mu}(m)}, \tag{39}$$
$$\alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)} \leq \alpha^m_{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)}, \tag{40}$$

so that

$$\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)} \geq \hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}. \tag{41}$$

Meanwhile, the stability conditions for the workers imply that

$$\gamma^w_{\tilde{\mu}(w)} + (1-\tau)\tilde{t}^{\tilde{\mu}(w)\to w} \geq \gamma^w_{\hat{\mu}(w)} + (1-\tau)\tilde{t}^{\hat{\mu}(w)\to w}, \tag{42}$$
$$\gamma^w_{\tilde{\mu}(w)} + (1-\tau)\hat{t}^{\tilde{\mu}(w)\to w} \leq \gamma^m_{\hat{\mu}(w)} + (1-\tau)\hat{t}^{\hat{\mu}(w)\to w}, \tag{43}$$

so that

$$(1-\tau)(\tilde{t}^{\hat{\mu}(w)\to w} - \tilde{t}^{\tilde{\mu}(w)\to w}) \leq (1-\tau)(\hat{t}^{\hat{\mu}(m)\to w} - \hat{t}^{\tilde{\mu}(m)\to w}). \tag{44}$$

Summing (41) and (44) across agents and using Lemma 1, we find that

$$\sum_{m\in M}\left(\tilde{t}^{m\to\hat{\mu}(m)}-\tilde{t}^{m\to\tilde{\mu}(m)}\right)=\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)}-\hat{t}^{m\to\tilde{\mu}(m)}\right).$$

For this to hold, we must have equality in (41) for each $m\in M$. But this implies equality in (39) and (40), for each $m\in M$. Similarly, it requires that (44) hold with equality for each $w\in W$, which implies equality in (42) and (43), for each $w\in W$. Combining these equalities, and summing across workers $w\in W$, shows that

$$\begin{aligned}\sum_{w\in W}\left(\gamma^w_{\tilde{\mu}(w)}-\gamma^w_{\hat{\mu}(w)}\right)&=(1-\tau)\sum_{m\in M}\left(\tilde{t}^{m\to\hat{\mu}(m)}-\tilde{t}^{m\to\tilde{\mu}(m)}\right),\\&=(1-\tau)\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)}-\alpha^m_{\tilde{\mu}(m)}\right).\end{aligned}\tag{45}$$

If the managers are not indifferent in aggregate between $\tilde{\mu}$ and $\hat{\mu}$, so that

$$\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)}-\alpha^m_{\tilde{\mu}(m)}\right)\neq 0,\tag{46}$$

we have,

$$\tau=1+\frac{\sum_{w\in W}\left(\gamma^w_{\tilde{\mu}(w)}-\gamma^w_{\hat{\mu}(w)}\right)}{\sum_{m\in M}\left(\alpha^m_{\tilde{\mu}(m)}-\alpha^m_{\hat{\mu}(m)}\right)}.\tag{47}$$

This shows Proposition 3.

To see Corollary 1, it suffices to observe that (47) pins down a unique tax rate in the case that (46) holds. Thus, if there are two tax rates under which matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable, then we must have

$$\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)}-\alpha^m_{\tilde{\mu}(m)}\right)=0.\tag{48}$$

But then, we also have

$$\sum_{w\in W}\left(\gamma^w_{\hat{\mu}(w)}-\gamma^w_{\tilde{\mu}(w)}\right)=0,\tag{49}$$

by (45). Combining (48) and (49), we find that

$$\mathfrak{M}(\hat{\mu})-\mathfrak{M}(\tilde{\mu})=\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)}-\alpha^m_{\tilde{\mu}(m)}\right)+\sum_{w\in W}\left(\gamma^w_{\hat{\mu}(w)}-\gamma^w_{\tilde{\mu}(w)}\right)=0,$$

as desired.

Proof of Lemma 2

In a strictly positive wage market, all matches are accompanied by a strictly positive transfer; hence, a lump sum tax on transfers is equivalent to a flat fee for matching. Thus, Lemma 2 follows from the following slightly more general result.

Here and hereafter, we say that an arrangement or matching is *stable under flat fee* f if it is stable given transfer function $\xi_f^{\mathsf{fee}}(\cdot)$.

Lemma 2'. *Reduction of a flat fee for matching (weakly) increases the number of workers matched in stable matchings. That is, if matching $\tilde{\mu}$ is stable under flat fee $\tilde{f}$, matching $\hat{\mu}$ is stable under flat fee $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\#(\hat{\mu}) \geq \#(\tilde{\mu}),$$

where $\#(\mu)$ denotes the number of workers matched in matching μ.

Proof. As $[\tilde{\mu}; \tilde{t}]$ is stable under flat fee $\tilde{f}$, we have

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \tilde{t}^{m \to \hat{\mu}(m)}$$
$$\gamma^w_{\tilde{\mu}(w)} + \tilde{t}^{\tilde{\mu}(w) \to w} - \tilde{f} \cdot \{1_{\tilde{\mu}(w) \neq w}\} \geq \gamma^w_{\hat{\mu}(w)} + \tilde{t}^{\hat{\mu}(w) \to w} - \tilde{f} \cdot \{1_{\hat{\mu}(w) \neq w}\}$$

where $\{1_{\mu(w) \neq w}\}$ is an indicator function that equals 1 if w is matched in matching μ and 0 if w is unmatched in matching μ. Summing these inequalities across agents, and using Lemma 1, we find that

$$\sum_{m \in M} \left(\alpha^m_{\tilde{\mu}(m)} - \alpha^m_{\hat{\mu}(m)}\right) + \sum_{w \in W} \left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)}\right) + \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0. \tag{50}$$

Similarly, as $[\hat{\mu}; \hat{t}]$ is stable under flat fee $\hat{f}$,

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)}$$
$$\gamma^w_{\hat{\mu}(w)} + \hat{t}^{\hat{\mu}(w) \to w} - \hat{f} \cdot \{1_{\hat{\mu}(w) \neq w}\} \geq \gamma^w_{\tilde{\mu}(w)} + \hat{t}^{\tilde{\mu}(w) \to w} - \hat{f} \cdot \{1_{\tilde{\mu}(w) \neq w}\};$$

these inequalities yield

$$\sum_{m \in M} \left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right) + \sum_{w \in W} \left(\gamma^w_{\hat{\mu}(w)} - \gamma^w_{\tilde{\mu}(w)}\right) + \hat{f} \cdot (\#(\tilde{\mu}) - \#(\hat{\mu})) \geq 0. \tag{51}$$

upon summation.

Adding (50) and (51) shows that

$$(\tilde{f} - \hat{f})(\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0.$$

Thus, if $\tilde{f} > \hat{f}$, we must have $\#(\hat{\mu}) \geq \#(\tilde{\mu})$; this proves the result. □

Proof of Theorem 2

As in the proof of Lemma 2, Theorem 2 follows from the following slightly more general result.

Theorem 2'. *A reduction in a flat fee for matching (weakly) increases the total match utility of stable matchings. That is, if $\tilde{\mu}$ is stable under flat fee $\tilde{f}$, $\hat{\mu}$ is stable under flat fee $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

Proof. Using (51) and Lemma 2', we find that

$$\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) = \sum_{m \in M} \left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right) + \sum_{w \in W} \left(\gamma^w_{\hat{\mu}(w)} - \gamma^w_{\tilde{\mu}(w)}\right) \geq \hat{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0; \tag{52}$$

this proves Theorem 2'. □

Proof of Proposition 4

As in the proof of Lemma 2, Proposition 4 follows from the following slightly more general result.

Proposition 4'. *Let $\hat{\mu}$ be an efficient matching, and let $\tilde{\mu}$ be stable under flat fee $\tilde{f}$. Then,*

$$0 \leq \mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) \leq \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})).$$

Proof. This is immediate from (50). □

Proof of Proposition 5

As in the proof of Lemma 2, Proposition 5 follows from the following slightly more general result.

Proposition 5'. *A matching $\tilde{\mu}$ can be stable under a flat fee only if*

$$\tilde{\mu} \in \underset{\{\mu \,:\, \#(\mu) \leq \#(\tilde{\mu})\}}{\arg\max} \left\{\mathfrak{M}(\mu)\right\}.$$

Proof. From (50), we see that if $[\tilde{\mu}; \tilde{t}]$ is stable under flat fee $\tilde{f}$, then for any matching $\hat{\mu} \neq \tilde{\mu}$,

$$\mathfrak{M}(\tilde{\mu}) - \mathfrak{M}(\hat{\mu}) + \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0. \tag{53}$$

If fewer workers are matched in $\hat{\mu}$ than in $\tilde{\mu}$ (i.e. $\#(\tilde{\mu}) \geq \#(\hat{\mu})$), (53) implies that

$$\mathfrak{M}(\tilde{\mu}) - \mathfrak{M}(\hat{\mu}) \geq \hat{f} \cdot (\#(\tilde{\mu}) - \#(\hat{\mu})) \geq 0,$$

so that $\tilde{\mu}$ must have higher total match utility than $\hat{\mu}$. □

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