The First-Order Approach to Merger Analysis

By Sonia Jaffe and E. Glen Weyl*

Using information local to the pre-merger equilibrium, we derive approximations of the expected changes in prices and welfare generated by a merger. We extend the pricing pressure approach of recent work to allow for non-Bertrand conduct, adjusting the diversion ratio and incorporating the change in anticipated accommodation. To convert pricing pressures into quantitative estimates of price changes, we multiply them by the merger pass-through matrix*, which (under conditions we specify) is approximated by the pre-merger rate at which cost increases are passed through to prices. Weighting the price changes by quantities gives the change in consumer surplus.*

Much recent theoretical and applied antitrust work has focused on using information local to the pre-merger equilibrium to predict the directional price impacts of mergers. This "first-order" approach adopts both the simplicity and transparency of approaches based on market definition and the firm grounding in formal economics of the market simulation approach. Section I gives a more extensive background, but the logic of this approach is intuitive: when firms 1 and 2 merge, firm 1 (and similarly, firm 2) has an incentive to raise its prices because of a new opportunity cost of selling its products – firm 1 now internalizes the profit lost by firm 2 when firm 1 lowers its price. For each extra unit firm 1 sells, the profit lost by firm 2 is the fraction of sales gained by firm 1 that are cannibalized from firm 2 (typically called the *diversion ratio*), multiplied by the profit-value of those sales (firm 2's *mark-up*).

We extend this first-order approach in three ways. First, we develop a model that allows for non-Nash-in-prices oligopoly behavior. Second, we show how information on local pass-through rates can be used to convert directional indicators of "pricing pressure" into quantitative approximations to the price changes caused by a merger. Finally, we describe how these quantitative estimates of price changes can translate into approximations of impacts on consumer and social welfare.

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We show that a merger's impact on consumer surplus is approximately

$$\Delta \mathrm{CS} \approx - \underbrace{\mathbf{g}^{\mathrm{T}}}_{\text{Generalized pricing pressure vector}} \cdot \underbrace{\rho^{\mathrm{T}}}_{\text{Merger pass-through matrix}} \cdot \underbrace{\mathbf{Q}}_{\text{Quantity vector}}, \tag{1}$$

where the approximation error is proportional to the square of the merger's effects on prices and to the curvature of the equilibrium conditions (third derivatives of supply and demand). The first term, $\mathbf{g}$, is the *Generalized Pricing Pressure* (GePP), the change in pricing incentives, allowing for non-Bertrand conduct and non-constant-marginal-cost systems. Developed and explained more fully in Section II, GePP is a vector that has entries of zero for all non-merging firms and – in the case when single-product firms 1 and 2 merge – a first entry of the form

$$g = \underbrace{\overbrace{\tilde{D}_{12}}^{\text{Conjectured diversion ratio}} \cdot \overbrace{(P_2 - mc_2)}^{\text{Mark-up}}}_{\text{Generalized UPP}} - \underbrace{Q_1 \overbrace{\left(\overbrace{\frac{1}{\frac{d^M Q_1}{dP_1}}}^{\text{Post-merger}} - \overbrace{\frac{1}{\frac{dQ_1}{dP_1}}}^{\text{Pre-merger}} \right)}^{\text{Change in inverse slope of demand}}}_{\text{End of (accommodating) reactions}} \tag{2}$$

and an analogous second entry. The first term in equation (2) extends the logic discussed above by replacing the Bertrand diversion ratio, D_{12} with the *anticipated diversion ratio* $\tilde{D}_{12}$. This ratio is the fraction of the units lost by good 1 that are gained by good 2 when firm 1 raises its price, *holding fixed the price of good 2* but *allowing all other prices to adjust* as the merged firm expects.[1] The price of good 2 is held fixed because, as a result of the merger, it is one of the quantities over which the merged firm optimizes. The second term in (2) is the quantity of good 1 multiplied by the (merger-induced) change in the inverse of the derivative of demand: now that the firms are merged, firm 1 no longer anticipates a reaction from firm 2 to a change in its price. If firm 2 were accommodating pre-merger (raising its price in response to a price increase by firm 1) then firm 1's elasticity of demand will be higher post-merger.[2]

The second term in equation (1), ρ, is the *merger pass-through* matrix, the rate at which the changes in opportunity cost created by the merger, the GePP, are passed through to changes in prices. As we show in Section III, this matrix, which is a function of local second-order properties of the demand and cost systems, converts GePP into a quantitative approximation of the price effects of the merger. In Section IV we discuss further the role of pass-through; we argue that in many relevant cases merger pass-through is close to both pre-merger and post-merger pass-through,

[1] We follow the convention in the literature of treating the diversion ratio for substitutes as a positive number – the negative of the ratio of the changes in quantities in the case of single-product firms.

[2] For further discussion of the effects of accommodation see Section II.D.3.

reconciling divergent strains in recent literature on the relevant pass-through rate. In certain cases, exact merger pass-through may be identified from pre-merger pass-through. For other cases, the empirical and numerical results of Cheung (2011) and Miller et al. (2012) provide evidence for the validity of approximating merger pass-through with pre-merger pass-through rates.

The third term in equation (1), Q, is the vector of quantities, which are the correct weights for aggregating the price changes into the change in consumer surplus, as we discuss in Section V. A similar approach may be used to estimate social surplus impacts. This aggregation, which converts price changes into dollar measures of surplus, facilitates comparison with merger effects on consumer welfare not directly mediated by prices, such as changes in network size or product quality.

Section VI discusses extensions and practical implications of our framework, including ways to simplify the formula given time and resource constraints. Our conclusion in Section VII discusses directions for future research. A companion policy piece (Jaffe and Weyl, 2011) proposes a few potential reforms to the merger guidelines based on our analysis. Proofs not in the text are in the appendices.

I. Background on the First-Order Approach

During the 1990s, merger simulation became widespread as a method for predicting the unilateral price effect of mergers. However, Shapiro (1996) and Crooke et al. (1999) argued that the effects of mergers predicted by simulations could differ by an order of magnitude or more based on properties of the curvature of demand not typically measured empirically. To address this concern, Werden (1996) pioneered the first-order approach by arguing that the "compensating marginal cost reductions" necessary to offset the anticompetitive effects of a merger could be calculated from local properties of the demand system. Such cost efficiencies would have to offset the change in first-order conditions created by the new opportunity cost of a sale due to the diversion of sales of a merger partner's product. Shapiro (1996) observed that, regardless of functional form, merger effects appeared to be increasing in this "value of diverted sales," which has come to be known as "Upward Pricing Pressure" (UPP).

Building on this work, antitrust officials in the United Kingdom, led by Peter Davis and Chris Walters, began to use UPP to evaluate mergers (Walters, 2007). Froeb et al. (2005) noted that functional forms which imply higher pass-through rates of cost efficiencies generated by the merger tend to also generate large unilateral merger effects on prices. They proposed an approach, based on Newton's method, for conducting merger simulations in a computationally simpler manner whose first iteration only required information local to pre-merger prices. Building on the practical work in the UK and the theoretical analysis of Froeb et al., Farrell and Shapiro (2010a,b) translated these ideas into intuitive and widely accessible economic terms: they argued that the sign of UPP minus efficiency gains would indicate the direction of merger effects and put forward the measurement of UPP as a practical policy proposal for the evaluation of mergers.

Under the leadership of Farrell and Shapiro, the US incorporated UPP into the

2010 Merger Guidelines (United States Department of Justice and Federal Trade Commission, 2010). The UK followed close behind with an even more explicit incorporation of UPP (Competition Commission and Office of Fair Trading, 2010); the European Union is also considering revising its merger guidelines.

Nevertheless, some objections have been raised against the use of UPP in analyzing mergers:

1) Coate and Simons (2009) object to its near-universal assumption of Nash-in-prices (Bertrand) competition and its reliance, in some settings, on constant marginal costs.
2) Schmalensee (2009) and Hausman et al. (2010) are skeptical of its assumption of default efficiencies and argue that providing only a directional indication of price effects is insufficient.
3) Carlton (2010) emphasizes the difficulty of applying the UPP approach to mergers between multi-product firms.

While these issues also arise to varying degrees in alternative approaches to merger analysis, they are still worth addressing. In this context, our paper makes three contributions that, to the best of our knowledge, have not appeared in previous literature. First, we analyze pricing pressure in a model that is not limited to Nash-in-prices competition or constant marginal cost. Second, we formalize the folk wisdom that pass-through rates can be used to convert pricing pressure into an approximation of quantitative price impacts, thereby forgoing the need for default cost efficiencies. Finally, we present formulae for approximate changes in consumer welfare that allow for the aggregation of multiple price changes for multi-product firms.

Competition agencies' increased openness to a range of simple tools with firm economic grounding (Shapiro, 2010) has sharpened the focus on the appropriateness of the first-order approach for policy and its soundness as a theoretical construct; the agencies' increased interest in broadening the scope of analysis raises the relevance of extending UPP to non-Bertrand settings (see the Office of Fair Trading-comissioned report on the role of conjectural variations in merger policy (Majumdar et al., 2011)).

II. Generalized Pricing Pressure

In this section we adapt the Telser (1972) single-strategic-variable-per-product oligopoly model by formulating it in terms of prices (rather than quantities) and allowing for multi-product firms. As Telser shows, by including non-price behavior in anticipated reactions by other firms, this model encompasses most standard static oligopoly models – including Nash-in-Prices (Betrand), Nash-in-Quantities (Cournot) and most supply function equilibria. In this framework, we derive a formula for the changes in pricing incentives firms face post-merger – the Generalized Pricing Pressure (GePP). In Subsection II.II.C, we show how to incorporate efficiencies from the merger. In Subsection II.II.D, we give two specific examples to illustrate how the formula works in the specific cases of Bertrand and Cournot; we also present an example that explores how the degree of accommodating reaction affects the size of GePP.

A. The model

Consider a market with N firms denoted $i = (1, \ldots, n)$. Firm i produces m_i goods, and chooses a vector of prices $\mathbf{P}_i = (P_{i1}, P_{i2}, \ldots, P_{im_i})$ from $\mathbb{R}^{m_i}$.[3] Following Telser (1972), we permit each firm to conjecture reactions by other firms: changes in those firms' prices in response to changes in its own prices. This formulation is useful for two reasons:

First, it allows us to nest static oligopoly models where firms have a strategic variable other than price, such as quantity or a supply function shifter. If the strategic variable is not price, this is incorporated into a firm's conjectures about other firms' reactions to its price change. For example, as we illustrate in Subsection II.D below, Cournot competition is represented by a firm conjecturing that when it raises its price, other firms will raise their prices so as to hold fixed their quantities. This formulation encompasses many strategic contexts, but does restrict each firm to have a single strategic variable per product (as in Werden and Froeb (2008)).[4]

Second, these conjectured reactions allow for the possibility of non-static Nash behavior in the spirit of the conjectural variations of Bowley (1924). Despite their absence from the mainstream of industrial organization empirics and theory since the 1980s, recently there has been a resurgence in theoretical (Dockner, 1992; Cabral, 1995), empirical (Nevo, 1998; Ciliberto and Williams, 2011) and policy (Majumdar et al., 2011) interest in such non-Nash frameworks as a useful reduced form for the complexities of dynamic models of competition. Conjectured reactions can also result from tacit collusion in the industry pre-merger, which can potentially alter the effects of a merger.

These conjectures are modeled by letting a firm believe that when it changes its prices, $\mathbf{P}_i$, its competitors will change their prices, $\mathbf{P}_{-i}$, by $\frac{\partial \mathbf{P}_{-i}}{\partial \mathbf{P}_i}$.[5] Therefore, the total effect of a change in one's own price on a vector of interest is the sum of the direct (partial) effect and the indirect effect working through the effect on others' prices: $\frac{d\mathbf{A}}{d\mathbf{P}_i} \equiv \frac{\partial \mathbf{A}}{\partial \mathbf{P}_i} + \frac{\partial \mathbf{A}}{\partial \mathbf{P}_{-i}} \frac{\partial \mathbf{P}_{-i}}{\partial \mathbf{P}_i}$. In the case of a Bertrand equilibrium, we have $\frac{d\mathbf{A}}{d\mathbf{P}_i} = \frac{\partial \mathbf{A}}{\partial \mathbf{P}_i}$ since $\frac{\partial \mathbf{P}_{-i}}{\partial \mathbf{P}_i} = \mathbf{0}$.

[3] In an earlier version of this paper we considered the analysis for the case where any strategy (such as quantity) is chosen. This more general analysis is available in an online appendix.

[4] We thus rule out changes in the non-price determining characteristics of products as considered in the literature on product repositioning (Mazzeo, 2002; Gandhi et al., 2008). See Section V for a discussion of how merger effects on non-price characteristics can be incorporated into our framework. See Telser (1972) for more details of the range of models that are special cases of this framework.

[5] Throughout we use the notation $\frac{\partial}{\partial}$ to refer to the Jacobian, $\frac{\partial \mathbf{A}}{\partial \mathbf{B}} \equiv \begin{pmatrix} \frac{\partial A_1}{\partial B_1} & \cdots & \frac{\partial A_1}{\partial B_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial A_n}{\partial B_1} & \cdots & \frac{\partial A_n}{\partial B_m} \end{pmatrix}$.

1. PRE-MERGERFirm i's profit π_i depends on both firm its own price vector and its competitors' prices:

$$\pi_i = \mathbf{P}_i^{\mathrm{T}} \mathbf{Q}_i(\mathbf{P}) - \mathbf{C}_i(\mathbf{Q}_i(\mathbf{P})),$$

where C and Q are the cost and demand functions. For brevity we write Q_i for $Q_i(P)$ and $\mathbf{mc}_i$ for the vector of marginal costs. The firm's vector of first-order conditions pre-merger can be written as:

$$\mathbf{0} = \underbrace{-\left(\frac{d\mathbf{Q}_i}{d\mathbf{P}_i}^{-1}\right)^{\mathrm{T}} \mathbf{Q}_i}_{\text{Multiproduct inverse hazard rate/Cournot distortion}} - \underbrace{(\mathbf{P}_i - \mathbf{mc}_i)}_{\text{Mark-up}} \equiv \mathbf{f}_i(\mathbf{P}).$$

This formula is a natural extension of the standard, single-product oligopoly first-order condition: the mark-up on each product is equated to the matrix analog of the partial inverse hazard rate or Cournot distortion.

B. *Incentives created by a merger*

In studying the impact of a merger on firms' incentives, it is useful to define a couple of terms.

DEFINITION 1: *If firms i and j merge, the post-merger* diversion ratio *matrix is*

$$\tilde{\mathbf{D}}_{ij} \equiv - \left(\frac{d^M\mathbf{Q}_i}{d\mathbf{P}_i}^{-1}\right)^T \left(\frac{d^M\mathbf{Q}_j}{d\mathbf{P}_i}\right)^T,$$

where $\frac{d^M\mathbf{Q}_k}{d\mathbf{P}_i} = \frac{\partial\mathbf{Q}_k}{\partial\mathbf{P}_i} + \frac{\partial\mathbf{Q}_k}{\partial\mathbf{P}_{-ij}}\frac{\partial\mathbf{P}_{-ij}}{\partial\mathbf{P}_i}$, which holds fixed the merging partner's prices.[6]

The relevant matrix of diversion ratios is the matrix ratio of the quantity (anticipated to be) gained by the former rival's products to that (anticipated to be) lost by one's own products as a result of an increase in own price, *holding fixed the price of the merger partner* and *allowing all other prices to adjust as they are expected to in equilibrium.*

DEFINITION 2: *Let the pre-merger and post-merger first-order conditions, normalized to be quasi-linear in marginal cost be denoted $\mathbf{f}(\mathbf{P})$ and $\mathbf{h}(\mathbf{P})$, respectively; the pre-merger equilibrium is defined by $\mathbf{f}(\mathbf{P}^0) = \mathbf{0}$; the post-merger equilibrium is defined by $\mathbf{h}(\mathbf{P}^M) = \mathbf{0}$. Then we define the* Generalized Pricing Pressure *(GePP) created by the merger to be*

$$\mathbf{g}(\mathbf{P}^0) \equiv \mathbf{h}(\mathbf{P}^0) - \mathbf{f}(\mathbf{P}^0).$$

Thus GePP is the change in the first-order condition at the pre-merger prices. It captures only the unilateral effects of a merger, *holding fixed the strategies firms use*

[6]If more than 2 firms merge, then Q_j is replaced by $Q_{j,k,\ldots}$.

(price v. quantity, etc.) and conjectures about other firms' reactions. The value of GePP is given in the following proposition.

PROPOSITION 1: *The GePP on firm i generated by a merger between firms i and j is $\mathbf{g}_i(\mathbf{P}^0)$ where $\mathbf{P}^0$ is the pre-merger equilibrium price vector and*

$$\mathbf{g}_i(\mathbf{P}) \equiv \tilde{\mathbf{D}}_{ij}(\mathbf{P}_j - \mathbf{mc}_j) - \Delta\left(\left(\frac{d\mathbf{Q}_i}{d\mathbf{P}_i}^{-1}\right)^T\right)\mathbf{Q}_i. \tag{3}$$

Here $\Delta(\cdot)$ denotes the change from pre- to post-merger value of its argument; the change is due to the merger partner's strategy no longer reacting.[7]

The first term of equation (3) is the change in firm j's profits when firm i increases its price enough to lose one marginal sale: the fraction of a unit gained by firm j for each unit lost by firm i times the value of a unit. The second term is the change in firm i's own marginal profit due to the end of accommodating reactions: once the firms have merged, firm i no longer anticipates an accommodating reaction from its merger partner, so its demand becomes more elastic.

C. *Marginal cost efficiencies*

The GePP formula derived above assumes no cost efficiencies of the merger and as such can be seen as the baseline case. However, if estimates of expected efficiencies are available, they can easily be incorporated. If post-merger firm i's marginal costs are expected to be $\widetilde{\mathbf{mc}}_i$, then the GePP for firm i after a merger of firms i and j is

$$\tilde{\mathbf{g}}_i(\mathbf{P}) = \tilde{\mathbf{D}}_{ij}(\mathbf{P}_j - \widetilde{\mathbf{mc}}_j) - \Delta\left(\left(\frac{d\mathbf{Q}_i}{d\mathbf{P}_i}^{-1}\right)^{\mathrm{T}}\right)\mathbf{Q}_i - (\mathbf{mc}_i - \widetilde{\mathbf{mc}}_i).$$

This adjusted GePP can be used in the calculation of price changes and welfare effects or to calculate "compensating marginal cost reductions" (Werden, 1996). For the marginal cost reductions to counterbalance the other incentive effects and lead to no price change, it must be that

$$\begin{pmatrix} \tilde{\mathbf{g}}_i(\mathbf{P}) \\ \tilde{\mathbf{g}}_j(\mathbf{P}) \end{pmatrix} = 0,$$

which yields compensating cost reductions of

$$\begin{pmatrix} \mathbf{e}_i^\star \\ \mathbf{e}_j^\star \end{pmatrix} \equiv \begin{pmatrix} \mathbf{mc}_i \\ \mathbf{mc}_j \end{pmatrix} - \begin{pmatrix} \widetilde{\mathbf{mc}}_i \\ \widetilde{\mathbf{mc}}_j \end{pmatrix} = \begin{pmatrix} \mathbf{I} & -\tilde{\mathbf{D}}_{ij} \\ -\tilde{\mathbf{D}}_{ji} & \mathbf{I} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{g}_i(\mathbf{P}) \\ \mathbf{g}_j(\mathbf{P}) \end{pmatrix},$$

[7]Note that in the single-product firm case this is exactly equation (2) from the introduction. If more than 2 firms merge, than the quantity and price vectors of firm j are replaced by vectors containing the quantities and prices of all merger partners other than i.

which is equivalent to Werden's formula in the case of single-product Bertrand. Alternatively, if one wishes to apply the more permissive standard of Farrell and Shapiro (2010a) , the off-diagonal terms are ignored and the GePP itself is contrasted to efficiencies.

D. Specific Contexts and Examples

This subsection illustrates the model under a few common equilibrium concepts. The formulae for Bertrand and Cournot are below, followed by a discussion of the effect of accommodating reactions on GePP. For the continuation of the main theory, see Section III.

1. BERTRANDIn the case of Bertrand, the expected accommodating reactions are zero, so GePP equals the (multi-product) UPP formula:

$$\mathbf{g}_i(\mathbf{P}) = -\left(\frac{\partial \mathbf{Q}_i}{\partial \mathbf{P}_i}^{-1}\right)^{\mathrm{T}} \left(\frac{\partial \mathbf{Q}_j}{\partial \mathbf{P}_i}\right)^{\mathrm{T}} (\mathbf{P}_j - \mathbf{mc}_j).$$

To help clarify, we now consider the explicit computation of GePP with two symmetric, multiproduct firms with constant marginal cost, playing Bertrand in a market with linear demand who then merge to (residual) monopoly.[8]

EXAMPLE 1: *Suppose that two symmetric, multiproduct firms* 1 *and* 2 *with symmetric products, all of which are substitutes for one another, and constant marginal cost vector* $\mathbf{c}$ *face the linear demand system*

$$\begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{A} \\ \mathbf{A} \end{pmatrix} - \begin{pmatrix} \mathbf{B}_o & \mathbf{B}_x \\ \mathbf{B}_x & \mathbf{B}_o \end{pmatrix} \begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \end{pmatrix}.$$

Note that $\mathbf{B}_0$ *is positive definite,* $\mathbf{B}_x$ *has all entries negative, and symmetry implies* $\mathbf{B}_0 = \mathbf{B}_0^T$ *and* $\mathbf{B}_x = \mathbf{B}_x^T$*. Profits prior to the merger for either firm* i *are* $(\mathbf{P}_i - \mathbf{c}_i)^T \mathbf{Q}_i = (\mathbf{P}_i - \mathbf{c}_i)^T (\mathbf{A} - \mathbf{B}_o \mathbf{P}_i - \mathbf{B}_x \mathbf{P}_{-i})$ *and thus, by the matrix product rule, the first-order condition is*

$$\mathbf{0} = \mathbf{A} - \mathbf{B}_o \mathbf{P}_i - \mathbf{B}_x \mathbf{P}_{-i} - \mathbf{B}_o (\mathbf{P}_i - \mathbf{c}) \iff 2\mathbf{B}_o \mathbf{P}_i + \mathbf{B}_x \mathbf{P}_{-i} = \mathbf{A} + \mathbf{B}_o \mathbf{c}.$$

If we solve for a symmetric equilibrium, $\mathbf{P}_i = \mathbf{P}_{-i} = \mathbf{P}^0$*, the equation becomes*

$$(2\mathbf{B}_o + \mathbf{B}_x)\mathbf{P}^0 = \mathbf{A} + \mathbf{B}_o \mathbf{c}_i \iff \mathbf{P}^0 = (2\mathbf{B}_o + \mathbf{B}_x)^{-1} (\mathbf{A} + \mathbf{B}_o \mathbf{c}).$$

[8]Note that we have criticized the plausibility of this demand system in other work (Jaffe and Weyl, 2010). We rely on it here not for realism but rather because it nicely illustrates how one may compute our formula in a specific example.

Thus the pre-merger mark-up is

$$(2\mathbf{B}_o + \mathbf{B}_x)^{-1} (\mathbf{A} + \mathbf{B}_o \mathbf{c}) - \mathbf{c} = (2\mathbf{B}_o + \mathbf{B}_x)^{-1} [\mathbf{A} - (\mathbf{B}_o + \mathbf{B}_x) \mathbf{c}] .$$

From the structure of demand, the diversion ratio is $-\mathbf{B_o}^{-1}\mathbf{B_x}$. *Thus the GePP on both firms is*

$$-\mathbf{B}_o^{-1}\mathbf{B}_x (2\mathbf{B}_o + \mathbf{B}_x)^{-1} [\mathbf{A} - (\mathbf{B}_o + \mathbf{B}_x) \mathbf{c}] .$$

2. Cournot The general formula for Cournot appears in the online appendix. Here we focus on a simple example of differentiated Cournot competition which illustrates how the Cournot model fits as a special case of our analysis.

EXAMPLE 2: *Consider two symmetric, single-product firms at a symmetric, Cournot equilibrium. The Slutsky matrix is both symmetric about both diagonals and thus without loss of generality may be specified as*

$$\frac{\partial \mathbf{Q}}{\partial \mathbf{P}} = \begin{pmatrix} -(s+\sigma) & \sigma \\ \sigma & -(s+\sigma) \end{pmatrix},$$

where $s, \sigma > 0$ *as the goods are substitutes and demand is downward sloping when both prices rise by the same amount. To calculate* $\frac{dP_{-i}}{dP_i}$ *we use the chain rule and the fact that under Cournot*

$$0 = \frac{dQ_{-i}}{dP_i} = \frac{\partial Q_{-i}}{\partial P_i} + \frac{\partial Q_{-i}}{\partial P_{-i}} \frac{\partial P_{-i}}{\partial P_i} = \sigma - (s+\sigma)\frac{dP_{-i}}{dP_i} \iff \frac{\partial P_{-i}}{\partial P_i} = \frac{\sigma}{s+\sigma}.$$

This give us $\frac{dQ_i}{dP_i} = \frac{\partial Q_i}{\partial P_i} + \frac{\partial Q_i}{\partial P_{-i}} \frac{\partial P_{-i}}{\partial P_i} = -(s+\sigma) + \sigma \frac{\sigma}{s+\sigma} = \frac{-s(s+2\sigma)}{s+\sigma}$. *Prior to the merger, firms price according to the logic of Subsection II.II.A, so*

$$P_i - mc_i = \frac{-Q_i}{\frac{\partial Q_i}{\partial P_i} + \frac{\partial Q_i}{\partial P_{-i}} \frac{dP_{-i}}{P_i}} = \frac{Q_i(s+\sigma)}{(s+2\sigma)s}.$$

After the merger, there are no reactions, which means $\frac{d^M Q_i}{P_i} = \frac{\partial Q_i}{\partial P_i} = -(s+\sigma)$, *so the end of accommodating reactions term is*

$$Q\left(\frac{-1}{s+\sigma} - \frac{-(s+\sigma)}{(s+2\sigma)s}\right) = Q\frac{(s+\sigma)^2 - s(s+2\sigma)}{s(s+2\sigma)(s+\sigma)} = \frac{Q\sigma^2}{s(s+2\sigma)(s+\sigma)}.$$

The appropriate diversion ratio is the one holding fixed the price of the merger partner; since this is a merger to monopoly, that is just the Bertrand diversion ratio $\frac{\sigma}{s+\sigma}$.

If there are marginal cost efficiencies with a change in marginal costs of $\Delta(mc) <$

0*, then the GePP on each product is*

$$\begin{aligned} g_i(p) =& D(P - mc) - D\Delta(mc) - \Delta\left(\frac{1}{\frac{dQ_i}{dP_i}}\right) Q_i + \Delta(mc) \\ =& \frac{\sigma}{s+\sigma}\frac{Q(s+\sigma)}{(s+2\sigma)s} - \frac{Q\sigma^2}{s(s+2\sigma)(s+\sigma)} + (1-D)\Delta(mc) \\ =& \frac{Q\sigma}{(s+2\sigma)(s+\sigma)} + \frac{s}{s+\sigma}\Delta(mc). \end{aligned}$$

Note that, without marginal cost efficiencies, as the products become undifferentiated ($\sigma \to \infty$) this formula converges to 0*. This seems to indicate that a merger-to-monopoly in undifferentiated Cournot causes no increase in prices, which is clearly absurd. The problem, as we show in Appendix .B, is that as the products become undifferentiated, the pass-through by which the GePP must be multiplied to obtain the price change explodes. This case shows both how our formula works out in a non-Bertrand, but canonical model and illustrates why considering pass-through, and not just pricing pressure, is often crucial.*

3. ACCOMMODATIONA natural concern is that, especially in differentiated product industries, it may be difficult to determine empirically or even grasp intuitively what model of conduct is appropriate (Nevo, 1998). While for many questions this is a serious worry, it may not be a severe problem for merger analysis since anticipated accommodating reactions (arising either from the strategic variable not being price or from non-Nash behavior) have two off-setting effects. If firms 1 and 2 merge, accommodating reactions from the non-merging firms increase the GePP by increasing the (conjectured) diversion ratio: if other firms increase their prices when firm 1 does, that would both reduce the quantity of sales lost by firm 1 when it raises its price and increase the quantity gained by firm 2 (whose price is held fixed). On the other hand, the greater the pre-merger accommodating reactions between the firms 1 and 2, the greater is the increase in firm 1's elasticity of demand due to firm 2 no longer accommodating post-merger; therefore accommodation increases the (subtracted) term for the end of accommodating reactions, thereby decreasing the GePP. Which of these effects dominates will depend on how large the anticipated accommodation between the merging firms and other firms in the industry (first effect) is relative to the accommodation between the merging partners (second effect).

Because of these offsetting effects, the size of GePP may not differ as much across alternative conduct assumptions as it might at first appear. The following example demonstrates this phenomenon and provides another illustration of how GePP can be computed in specific models.

EXAMPLE 3: *Consider a pre-merger symmetric industry with n single-product firms; pre-merger they are in a symmetric equilibrium, earning mark-up m, each* *selling quantity q, with an aggregate (Bertrand) diversion ratio D to the $n-1$ other firms in the industry. On the margin, each firm anticipates an increase in price of λ by all other firms in response to a one unit increase in their own price.*

Prior to the merger, the first-order condition for a single firm requires that

$$m = -\frac{q}{\frac{dQ_i}{dP_i}}.$$

Symmetry implies that $\frac{\partial Q_i}{\partial P_j} = \frac{\partial Q_j}{\partial P_i} = -\frac{\partial Q_i}{\partial P_i}\frac{D}{n-1}$. *This allows us to calculate*

$$\frac{dQ_i}{dP_i} = \frac{\partial Q_i}{\partial P_i} + \sum_{j\neq i}\frac{\partial Q_i}{\partial P_j}\frac{\partial P_j}{\partial P_i} = \frac{\partial Q_i}{\partial P_i} + \frac{\partial Q_i}{\partial P_j}(n-1)\lambda = \frac{\partial Q_i}{\partial P_i}(1-D\lambda),$$

which gives

$$\frac{\partial Q_i}{\partial P_i} = -\frac{q}{m\,(1-D\lambda)}.$$

Post-merger the price of the merger partner is held fixed, rather than increasing by λ in response to an increase in the firm's price. To see how this difference affects the anticipated accommodation, note that the total pre-merger accommodation firm i anticipates from each other firm is the direct effect while holding fixed its merger partner's price plus the indirect effect via the impact on its merger partner. Therefore, post-merger $\tilde{\lambda}$, the symmetric increase in the $n-2$ remaining firms' prices in response to an increase in one of the partners' prices must satisfy

$$\underbrace{\lambda}_{\textit{pre-merger response}} = \underbrace{\tilde{\lambda}}_{\textit{holding fixed partner}} + \underbrace{\lambda}_{\substack{\textit{equilibrium}\\ \textit{partner response}}} \cdot \underbrace{\tilde{\lambda}}_{\substack{\textit{partner's equilibrium effect}\\ \textit{holding i fixed}}},$$

thus $\tilde{\lambda} = \frac{\lambda}{1+\lambda}$.

These quantities allow us to calculate the relevant post-merger derivatives. Call the merging firms, firm 1 and firm 2 and consider $\frac{d^M Q_1}{dP_1}$. *This is composed of the direct effect and the indirect effect from the change in the $n-2$ non-merging firm prices:*

$$\frac{d^M Q_1}{dP_1} = \frac{\partial Q_i}{\partial P_i} + \frac{\partial Q_i}{\partial P_j}(n-2)\tilde{\lambda} = \frac{\partial Q_i}{\partial P_i}\left(1 - \tilde{\lambda}\frac{n-2}{n-1}D\right).$$

For the merger partner, firm 2, the sales gained are the direct diversion plus the indirect diversion from the increase in the $n-2$ non-merging firms' prices:

$$\frac{d^M Q_2}{dP_1} = -\frac{\partial Q_i}{\partial P_i}\frac{D}{n-1}\left(1 + \tilde{\lambda}\,[n-2]\right).$$

Thus,

$$\tilde{D}_{12} \cdot m = \frac{\frac{D}{n-1}\left(1+\tilde{\lambda}\left[n-2\right]\right)}{1-\tilde{\lambda}\frac{n-2}{n-1}D} m = \frac{D\left[1+\tilde{\lambda}(n-2)\right]}{(n-1)\left(1-\tilde{\lambda}D\right)+\tilde{\lambda}D} m,$$

and the change in accommodating reaction term is

$$Q_1\left(\frac{1}{\frac{d^M Q_1}{dP_1}} - \frac{1}{\frac{dQ_1}{dP_1}}\right) = \frac{q}{\frac{q}{m}}\left(1 - \frac{1-D\lambda}{1-\tilde{\lambda}\frac{n-2}{n-1}D}\right) = Dm\frac{\left(\lambda-\tilde{\lambda}\right)(n-1)+\tilde{\lambda}}{(n-1)\left(1-\tilde{\lambda}D\right)+\tilde{\lambda}D}.$$

Subtracting these two terms and using $\lambda - \tilde{\lambda} = \frac{\tilde{\lambda}}{1-\tilde{\lambda}} - \tilde{\lambda} = \frac{\tilde{\lambda}^2}{1-\tilde{\lambda}}$ *yields the GePP,*

$$D \cdot m \frac{1+\tilde{\lambda}(n-3) - D(n-1)\frac{\tilde{\lambda}^2}{1-\tilde{\lambda}}}{\left(1-D\tilde{\lambda}\right)(n-1)+D\tilde{\lambda}}. \tag{4}$$

If we focus on the case where λ *(and thus* $\tilde{\lambda}$*) is small so that we can ignore second-order terms, then* (4) *simplifies to*

$$D \cdot m \frac{1+\tilde{\lambda}(n-3)}{\left(1-D\tilde{\lambda}\right)(n-1)+D\tilde{\lambda}}. \tag{4b}$$

Note that $\tilde{\lambda}$ *is strictly increasing in* λ*. When* $n = 2$*, we are considering a merger to monopoly, equation* (4b) *is proportional to* $1 - \tilde{\lambda}$*, so it is decreasing in* λ*. If accommodation by the merger partner is the only issue, GePP declines with the degree of accommodation as Farrell and Shapiro (2010a) conjecture. However, when* $n = 3$ *equation (4b) is proportional to* $\frac{1}{2-D\tilde{\lambda}}$ *which is increasing in* λ*. This effect gets stronger as* $n \to \infty$*; in the limit the expression is proportional to* $\frac{\tilde{\lambda}}{1-D\tilde{\lambda}}$*, which increases even more quickly in* λ*. Thus, in this basic example, "somewhere between" a merger to a monopoly and a merger by two firms within a triopoly, the effect of accommodation on GePP switches from negative to positive. The exact formula indicates that there is a larger space of parameters for which GePP is decreasing in* λ*; in highly collusive industries it may be that even in the case of 3 or 4 firms, more accommodation will lower GePP.*

The strength of accommodation between each merging firm and the non-merging firms relative to the accommodation between merging firms matters for the effect of accommodation on GePP (See Appendix .C for an example in a non-symmetric industry.) Both of these analyses suggest that while accommodation and the type of oligopoly competition clearly impacts the level of prices, its effect on changes in prices caused by mergers may be smaller and certainly subtler.

III. Price Changes

GePP measures how much firm incentives shift when firms merge. However, policymakers are typically interested in such shifts in incentives only insofar as they predict changes in prices. We extend the work of Chetty (2009) to show how a comparative static approach without a fully-estimated structural model can be used to analyze structural changes such as mergers. If the change in incentives is small, the effect of a merger can be approximated the same way the effect of a tax would be, despite the fact that unlike a tax we cannot imagine a merger "going to zero" to make our formula exact.

Our approach is to apply the appropriate envelope theorem, viewing the change in incentives created by the merger - the pricing pressure, g, - as a vector of local changes in the equilibrium conditions; we then apply Taylor's Theorem for inverse functions to approximate the post-merger conditions around the pre-merger equilibrium. This allows us to derive an approximation of the effect of the merger based on local properties of demand and to get a bound on the error of the approximation based on the curvature of the first-order conditions and the size of the incentive change.

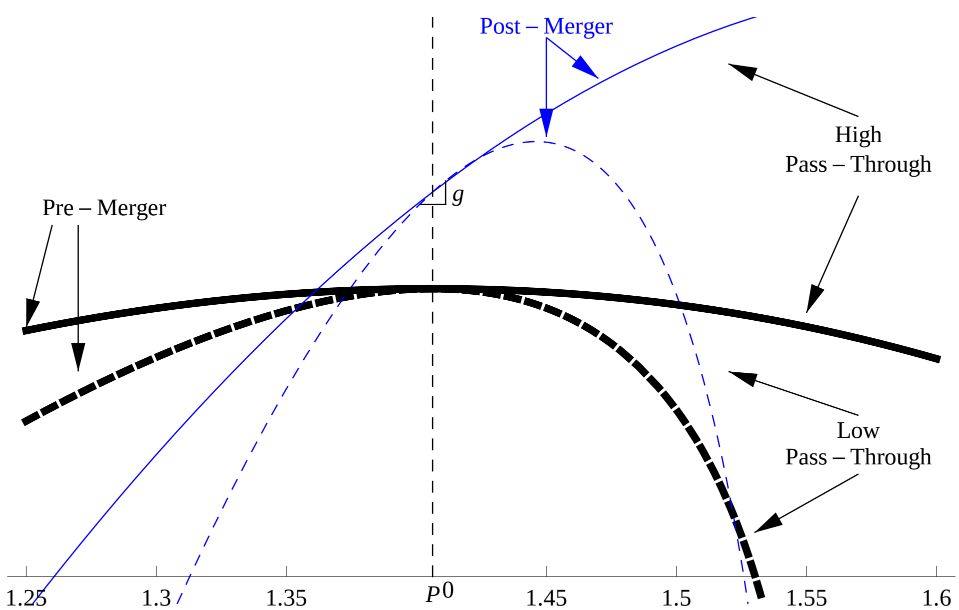

Figure 1. The effect of pricing pressure for low and high pass-through.

For a graphical intuition of why the curvature of profits is important for price changes and GePP is an insufficient indicator, see Figure 1. It shows profits as a function of price for two demand systems, both pre-merger (thick lines) and post-merger (thin, blue lines). The profits are based on a constant pass-through demand system; the dotted lines are low pass-through (.1) and the solid lines are high pass-through (.4). Costs are such that pre-merger both firms' profits are maximized at the same optimal price ($P^0 \approx 1.406$). They also have the same GePP, as evidenced by the tangency of the two post-merger (thin, blue) profit lines at P^0. However the curvature of the profit functions is very different and the post-merger profit-maximizing prices are very different. With low pass-through (dotted line), the post-merger price is approximately 1.445; with high pass-through (solid line), the post-merger price is off the graph to the right. This illustrates that demand systems with the same pre-merger prices and the same pricing pressures can have very different post-merger prices when they have different pass-through rates.

Theorem 1 provides our main result, which formalizes this intuition.

THEOREM 1: *Let $\mathbf{P}^0$ be the pre-merger equilibrium price vector. If $\mathbf{f}$ is the vector of pre-merger first-order conditions and $\mathbf{g}$ is the GePP vector (so $\mathbf{h}(\mathbf{P}) = \mathbf{f}(\mathbf{P})+\mathbf{g}(\mathbf{P})$ is the the post-merger first-order condition) and $(\mathbf{f}+\mathbf{g})$ is invertible, then, to a first-order approximation, the price change induced by the merger is*

$$\Delta\mathbf{P} = -\left(\frac{\partial\mathbf{f}}{\partial\mathbf{P}}(\mathbf{P}^0) + \frac{\partial\mathbf{g}}{\partial\mathbf{P}}(\mathbf{P}^0)\right)^{-1} \cdot \mathbf{g}(\mathbf{P}^0).$$

PROOF:

Let $\mathbf{h}(\mathbf{P}) = \mathbf{f}(\mathbf{P})+\mathbf{g}(\mathbf{P})$. Since $f(\mathbf{P}^0) = \mathbf{0}$, we have $\mathbf{h}(\mathbf{P}^0) = \mathbf{g}(\mathbf{P}^0) \equiv \mathbf{r}$. We want to find $\mathbf{P}^M$ (the post-merger price), such that $\mathbf{h}(\mathbf{P}^M) = \mathbf{0}$. If $\mathbf{h}$ is invertible, then

$$\begin{aligned} \mathbf{P}^M - \mathbf{P}^0 &= \mathbf{h}^{-1}(\mathbf{0}) - \mathbf{h}^{-1}(\mathbf{r}) = \left(\frac{\partial\mathbf{h}^{-1}}{\partial\mathbf{h}}(\mathbf{r})\right)(\mathbf{0}-\mathbf{r}) + O(\|\mathbf{r}\|^2) \\ &\approx -\left(\frac{\partial\mathbf{f}}{\partial\mathbf{P}}(\mathbf{P}^0) + \frac{\partial\mathbf{g}}{\partial\mathbf{P}}(\mathbf{P}^0)\right)^{-1} \cdot \mathbf{g}(\mathbf{P}^0), \end{aligned} \tag{5}$$

which completes the proof.

As we show in Appendix .D, the i-th entry of the error vector in equation (5) takes the form

$$E_i = -\frac{1}{2}\sum_j \left[\left(\frac{\partial\mathbf{h}}{\partial\mathbf{P}}\right)^{-1}\right]_{ij} \mathbf{g}^{\mathrm{T}}\left(\mathbf{P}^0\right)\left(\frac{\partial\mathbf{h}^{\mathrm{T}}}{\partial\mathbf{P}}\right)^{-1}\left(\mathcal{D}^2_{\mathbf{P}}\mathbf{h}_j\right)\left(\frac{\partial\mathbf{h}}{\partial\mathbf{P}}\right)^{-1}\mathbf{g}\left(\mathbf{P}^0\right),$$

where $[\mathbf{A}]_{ij}$ indicates the ij element of matrix $\mathbf{A}$, $\mathcal{D}^2_{\mathbf{P}}\mathbf{h}_j$ indicates the Hessian of $\mathbf{h}_j$, and the derivatives and Hessian are evaluated at some $\mathbf{P} \in [\mathbf{P}^0, \mathbf{P}^M]$. This error is small whenever $\mathbf{g}$ is small and the first-order conditions are not highly curved in the relevant range.[9]

[9]Our approximation is equivalent, in the case of Bertrand conduct with constant marginal cost, to the first step of the Newton's method approach to merger simulation proposed by Froeb et al. (2005), though the justification is different. For example, the second step of their approach *does not* correspond to the second-order term that would be derived from our expansion, as theirs relies on non-local but first-order information while ours uses local, higher-order derivatives.

Willig (1976) famously argued that theoretical approximations of this sort are useful to the extent that the associated errors are small relative to other sources of error such as statistical sampling or mis-specification error involved in accounting for the factors the approximation ignores. Two recent studies have investigated the relative size of the error of our approximation compared to these other sources empirically and numerically.

Cheung (2011) does an empirical analysis of the US Airways-America West merger and found the approximation error of our formula to be small compared to the statistical estimation error, even when the wrong (post-merger) pass-through rates were used. Miller et al. (2012) found larger errors, of up to 30-40%, based on numerical simulations of demand systems commonly used in merger simulation for mergers with true price impact in the 5% range (a cutoff frequently used for merger approval). However these errors were robust across a range of commonly used demand systems and were an order of magnitude smaller than those arising from mis-specifiying the curvature of the demand system within this class (e.g. using standard discrete choice demand systems when the real data came from linear demand).[10]

This suggests that, if demand in potential mergers is in fact generated from the range of demand systems typically used for simulation, the error in our approximation is likely to be smaller than other common sources of error. Two features of our approximation contribute to this relative accuracy:

1) First, It is more accurate when incentive effects are smaller and so seems to do comparatively well for the price changes that are typically the concern of merger analysis – those in the 5%-10% range.

2) Second, almost all functional forms used in demand estimation are very smooth (have sharply bounded curvature of equilibrium conditions) and are therefore well-approximated by our formula. Since any method that tries to estimate merger effects from exclusively pre-merger data will struggle if the true demand system is not smooth, our method may provide robustness over the class of plausibly empirically applicable models, though not outside it.

Miller et al. (2012) find that our approximation is precise in all cases with linear demand. To illustrate theoretically why this is true, we now return to Example 1.
Example 1 Continued

The post-merger equilibirum can be calculated directly by maximizing $2(\mathbf{P} - \mathbf{c})(\mathbf{A} - ($
$Bo +$
$Bx)\mathbf{P})$, which gives a first-order condition

$$\mathbf{0} = -\mathbf{B_o}(\mathbf{P} - \mathbf{c}) + \mathbf{A} - \mathbf{B_o}\mathbf{P} - \mathbf{B_x}\mathbf{P} - \mathbf{B_x}\left(\mathbf{P} - \mathbf{c}\right)$$

[10]Of course, our formula adds little in the case that any one of these particular demand systems applies. However, given that there are many such smooth demand systems, and many others that are not commonly but are equally smooth and have very different demand curvatures and thus pass-through rates, our approach provides a treatment that is robust across this class rather than just being valid for a particular assumed system, as in merger simulation. See Fabinger and Weyl (2012) for a detailed discussion of the restrictions beyond smoothness placed on demand curvature by typical demand systems.

which gives a post-merger price:

$$\mathbf{P}^M = \frac{1}{2}\left(\mathbf{B_o}+\mathbf{B_x}\right)^{-1}\left[\mathbf{A}+\left(\mathbf{B_x}+\mathbf{B_o}\right)\mathbf{c}\right].$$

Thus, repeatedly using the Slutsky symmetry of the $\mathbf{B}$ matrices and symmetry of products to commute matrix multiplication,

$$\begin{aligned}\Delta\mathbf{P} =& \frac{1}{2}\left(\mathbf{B_o}+\mathbf{B_x}\right)^{-1}\left[\mathbf{A}+\left(\mathbf{B_x}+\mathbf{B_o}\right)\mathbf{c}\right]-\left(2\mathbf{B_o}+\mathbf{B_x}\right)^{-1}\left(\mathbf{A}+\mathbf{B_o}\mathbf{c}\right)\\ =& \frac{1}{2}\left[\left(\mathbf{B_o}+\mathbf{B_x}\right)\left(2\mathbf{B_o}+\mathbf{B_x}\right)\right]^{-1}\\ &\left[\left(2\mathbf{B_o}+\mathbf{B_x}\right)\left(\mathbf{A}+\left(\mathbf{B_x}+\mathbf{B_o}\right)\mathbf{c}\right)-2\left(\mathbf{B_o}+\mathbf{B_x}\right)\left(\mathbf{A}+\mathbf{B_o}c\right)\right]\\ =& \frac{1}{2}\left[\left(\mathbf{B_o}+\mathbf{B_x}\right)\left(2\mathbf{B_o}+\mathbf{B_x}\right)\right]^{-1}\mathbf{B_x}\left[-\mathbf{A}+\left(\mathbf{B_o}+\mathbf{B_x}\right)\mathbf{c}\right].\end{aligned} \tag{6}$$

On the other hand, we can compute our approximation. The post-merger first-order condition for P_i is

$$\begin{aligned}\mathbf{A}-\mathbf{B_o}\mathbf{P}_i-\mathbf{B_x}\mathbf{P}_{-i}-\mathbf{B_o}\mathbf{P}_i+\mathbf{B_o}\mathbf{c}_i-\mathbf{B_x}(\mathbf{P}_{-i}-\mathbf{c}_{-i})=\mathbf{0}\\ \mathbf{A}-2\mathbf{P}_i-\mathbf{B_o}^{-1}(\mathbf{B_x}(2\mathbf{P}_{-i}-\mathbf{c}_{-i})+\mathbf{c}_i=\mathbf{0}\end{aligned} \tag{7}$$

Taking the derivative of (7), which is linear in own cost, with respect to $\mathbf{P}_i$ yields $-2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix of appropriate size. The derivative with respect to $\mathbf{P}_{-i}$ is $-2\mathbf{B_o}^{-1}\mathbf{B_x}$, so that the merger pass-through is

$$\begin{aligned}\rho =& \frac{1}{2}\begin{bmatrix} I & \mathbf{B_o}^{-1}\mathbf{B_x}\\ \mathbf{B_o}^{-1}\mathbf{B_x} & I\end{bmatrix}^{-1}\\ =& \frac{1}{2}\begin{bmatrix}\left(\mathbf{I}-\left(\mathbf{B_o}^{-1}\mathbf{B_x}\right)^2\right)^{-1} & -\left(I-\left(\mathbf{B_o}^{-1}\mathbf{B_x}\right)^2\right)^{-1}\mathbf{B_o}^{-1}\mathbf{B_x}\\ -\left(I-\left(\mathbf{B_o}^{-1}\mathbf{B_x}\right)^2\right)^{-1}\mathbf{B_o}^{-1}\mathbf{B_x} & \left(\mathbf{I}-\left(\mathbf{B_o}^{-1}\mathbf{B_x}\right)^2\right)^{-1}\end{bmatrix}.\end{aligned}$$

Plugging in the GePP ($\mathbf{g}(\mathbf{P}^0)=-\mathbf{B_o}^{-1}\mathbf{B_x}\left(2\mathbf{B_o}+\mathbf{B_x}\right)^{-1}\left[\mathbf{A}-\left(\mathbf{B_o}+\mathbf{B_x}\right)\mathbf{c}\right]$) (and again heavily using symmetry) gives

$$\begin{aligned}\Delta\mathbf{P}\approx& -\frac{1}{2}\left(I-\left(\mathbf{B_o}^{-1}\mathbf{B_x}\right)^2\right)^{-1}\left(\mathbf{I}-\mathbf{B_o}^{-1}\mathbf{B_x}\right)\cdot\mathbf{B_o}^{-1}\mathbf{B_x}\left(2\mathbf{B_o}+\mathbf{B_x}\right)^{-1}\left[\mathbf{A}-\left(\mathbf{B_o}+\mathbf{B_x}\right)\mathbf{c}\right]\\ =& -\frac{1}{2}\left(\mathbf{I}+\mathbf{B_o}^{-1}\mathbf{B_x}\right)^{-1}\mathbf{B_o}^{-1}\left(2\mathbf{B_o}+\mathbf{B_x}\right)^{-1}\mathbf{B_x}\left[\mathbf{A}-\left(\mathbf{B_o}+\mathbf{B_x}\right)\mathbf{c}\right]\\ =& \frac{1}{2}\left[\left(\mathbf{B_o}+\mathbf{B_x}\right)\left(2\mathbf{B_o}+\mathbf{B_x}\right)\right]^{-1}\mathbf{B_x}\left[-\mathbf{A}+\left(\mathbf{B_o}+\mathbf{B_x}\right)\mathbf{c}\right].\end{aligned}$$

This is identical to the expression in (6) and thus, in this simple case, our approximation is exact.

In order to simplify notation and shorten calculations, this example focused on the case of two symmetric, pre-merger-Bertrand firms with symmetric products merging to monopoly while facing Slutsky symmetric demand. However, our approximation formula remains exact so long as demand is linear and linear conjectures are maintained: Slutsky symmetry, merger to monopoly and symmetry across and within firms are not required.[11]

IV. Role of Pass-Through

Many papers have considered the role of pass-through in determining the price effects of a merger. Shapiro (1996) and Crooke et al. (1999) showed that demand forms with differing curvature but the same elasticities might lead to simulated merger effects differing by an order of magnitude. Froeb et al. (2005) argued that the demand systems that predict large pass-through of efficiencies (which are passed through at *post-merger* rates) also predict large anticompetitive merger effects. Weyl and Fabinger (2009) and Farrell and Shapiro (2010a) argued informally that because UPP is essentially the opportunity cost of sales created by the merger, multiplying it by the *pre-merger* pass-through rates should approximate merger effects. Farrell and Shapiro (2010b) show that in some cases bounds on pre-merger pass-through and UPP can bound merger effects.[12]

We reconcile these views by showing how merger pass-through combines important aspects of both pre-merger and post-merger pass-through. Furthermore, we show that it is pass-through, which is affected by conduct and cost curvature, not demand curvature, that converts GePP into approximate changes in prices.

A. Pre-merger, post-merger and merger pass-through

Marginal costs enter quasi-linearly into $\mathbf{f}_i$, the pre-merger first-order condition for firm i. Thus, if we were to impose a vector of quantity taxes $\mathbf{t}$, the post-tax (but pre-merger) equilibrium would be characterized by

$$\mathbf{f}(\mathbf{P}) + \mathbf{t} = 0,$$

so that by the implicit function theorem

$$\frac{\partial \mathbf{P}}{\partial \mathbf{t}} \frac{\partial \mathbf{f}}{\partial \mathbf{P}} = -\mathbf{I}.$$

The *pre-merger pass-through matrix* is

$$\rho_{\leftarrow} \equiv \frac{\partial \mathbf{P}}{\partial \mathbf{t}} = -\left(\frac{\partial \mathbf{f}}{\partial \mathbf{P}}\right)^{-1}. \tag{8}$$

[11] A proof is available on request.

[12] Since they use a constant marginal cost framework under which pass-through and demand curvature are equivalent, it is not clear which is the relevant quantity.

After the merger between firm i and firm j takes place, the marginal cost of producing firm i's goods enters into h_i (its post-merger first-order conditions) quasi-linearly with a coefficient of 1, but also enters into h_j quasi-linearly with a coefficient of $-\tilde{D}_{ji}$. This follows directly from the fact that the GePP for j includes the mark-up on good i which depends (negatively) on the tax. Thus if we let

$$K = \begin{pmatrix} \mathbf{I} & -\tilde{\mathbf{D}}_{ij} \\ -\tilde{\mathbf{D}}_{ji} & \mathbf{I} \end{pmatrix},$$

then the post-merger and post-tax equilibrium is characterized by

$$\mathbf{h}(\mathbf{P}) = -K\mathbf{t},$$

which implies that the *post-merger pass-through matrix* is[13]

$$\rho_{\rightarrow} \equiv \frac{\partial \mathbf{P}}{\partial \mathbf{t}} = -\left(\frac{\partial \mathbf{h}}{\partial \mathbf{P}}\right)^{-1} K. \tag{9}$$

Our result from the previous section is that

$$\mathbf{P}^M - \mathbf{P}^0 \approx -\left(\frac{\partial \mathbf{h}}{\partial \mathbf{P}}(\mathbf{P}^0)\right)^{-1} \cdot \mathbf{g}\left(\mathbf{P}^0\right).$$

Thus, *merger pass-through*, $-\left(\frac{\partial \mathbf{h}}{\partial \mathbf{P}}\right)^{-1}$, is not equal to pre-merger pass-through, $-\left(\frac{\partial \mathbf{f}}{\partial \mathbf{P}}\right)^{-1}$, or post-merger pass-through, $-\left(\frac{\partial \mathbf{h}}{\partial \mathbf{P}}\right)^{-1} K$; rather it depends on the curvature of the latter and the cost structure of the former. This makes sense since the post-merger first-order conditions are relevant, but the opportunity costs are direct shifts in the first-order conditions, not real (physical) costs that would enter wherever the marginal costs enter the post-merger first-order condition.

B. *Calculation and approximation of merger pass-through*

1. Identification When can we identify the merger pass-through from the pre-merger pass-through? Since $\frac{\partial \mathbf{f}(\mathbf{P})}{\partial \mathbf{P}}$ is equal to the negative inverse of the pass-through matrix, it is clearly calculable. In the case of two firms merging under Bertrand equilibrium, the pass-through matrix, along with the first derivatives of demand, can be used to calculate $\frac{\partial^2 Q_i}{\partial P_i^2}$, $\frac{\partial^2 Q_i}{\partial P_i \partial P_j}$, $\frac{\partial^2 Q_j}{\partial P_j \partial P_i}$, and $\frac{\partial^2 Q_j}{\partial P_j^2}$. If one assumes Slutsky symmetry $\left(\frac{\partial Q_i}{\partial P_j} = \frac{\partial Q_j}{\partial P_i}\right)$, then the other second derivatives are $\frac{\partial^2 Q_j}{\partial P_i^2} = \frac{\partial}{\partial P_i}\frac{\partial Q_j}{\partial P_i} = \frac{\partial}{\partial P_i}\frac{\partial Q_i}{\partial P_j} = \frac{\partial^2 Q_i}{\partial P_i \partial P_j}$ and $\frac{\partial^2 Q_i}{\partial P_j^2} = \frac{\partial^2 Q_j}{\partial P_i \partial P_j}$, which are all that is needed to calculate $\frac{\partial \mathbf{g}(\mathbf{P})}{\partial \mathbf{P}}$.

[13]The term with $\frac{\partial K}{\partial \mathbf{P}} t^0$ drops out because the tax is zero to begin with.

Since there is little intuition to be gained from the form of $\frac{\partial \mathbf{g}(\mathbf{P})}{\partial \mathbf{P}}$, we leave it to Appendix .E. A similar procedure may be applied under Cournot competition.

In the case of more than two merging firms, derivatives of the form $\frac{\partial^2 Q_i}{\partial P_j \partial P_k}$ are needed and cannot be calculated from observed pass-through rates and first derivatives unless one places restriction on the form of demand. A slightly more restrictive version of the horizontality assumption of Weyl and Fabinger (2009),

$$Q_i(P) = h\left(p_i + \sum_{j \neq i} f_j(P_j)\right),$$

is sufficient to calculate the necessary second partials, but it is a rather strong assumption.

2. APPROXIMATIONThe difference between pre-merger and merger pass-through (and post-merger pass-through) may in fact be small. For our approximation to be valid, $\mathbf{g}(\mathbf{P}^0)$ and the curvature of the equilibrium conditions need to be jointly sufficiently "small." If $\mathbf{g}(\mathbf{P}^0)$ is small, then it seems likely that $\frac{\partial \mathbf{g}}{\partial \mathbf{P}}$ would also be small at $\mathbf{P}^0$and thus $\left(\frac{\partial \mathbf{h}}{\partial \mathbf{P}}\right)^{-1} \approx \left(\frac{\partial f}{\partial \mathbf{P}}\right)^{-1}$ at P^0. If this were not the case, then while $\mathbf{g}(\mathbf{P}^0)$ is small, if $\mathbf{g}(\mathbf{P})$ were evaluated at a relatively close price in the direction of maximal gradient rather than at $\mathbf{P}^0$ it would then no longer be small. To the extent that the smallness of $\mathbf{g}$ is "fragile" in this sense, it is unlikely to form a solid basis for using first-order approximations.

Thus, in many cases when the first-order approximation would be valid, the merger pass-through is approximately equal to pre-merger pass-through. Furthermore, if small diversion ratios, rather than other factors, cause $\mathbf{g}(\mathbf{P}^0)$ to be small, then post-merger pass-through will also be close to merger pass-through since K will be close to the identity matrix. If a merger is likely to have a small impact on prices, then it is likely to have a small impact on pass-through rates and thus both pre- and post-merger pass-through rates will approximate merger pass-through. Miller et al. (2012) have an even more surprising finding in the numerical simulations they consider: results are typically more accurate when pre-merger pass-through is substituted for the theoretically-desired merger pass-through. While these results of course depend on the particular functional forms used in the simulations, they suggest that substituting pre-merger pass-through for merger pass-through may not cause large errors. Similarly, Cheung (2011) shows that using post-merger pass-through in place of merger pass-through leads to total approximation error smaller than statical estimation error in her application.

This interpretation, which views pre-, post- and merger pass-through as close to one another, has a number of benefits. First, it is consistent with the apparent coincidence (Froeb et al., 2005) that demand forms with high pre-merger pass-through rates have been found to generate high pass-through of merger efficiencies (which are driven by post-merger pass-through) and large anti-competitive effects (which are proportional to merger pass-through). Second, it shows that the Froeb et al. and the Shapiro et al. logic are on some level consistent: when pre-merger and post-merger pass-through are good estimates of merger pass-through, they are also similar to each other. Finally, it shows that using intuitions about pre-merger pass-through rates to approximate the rate at which GePP is passed through to prices may be reasonable.

V. Welfare Changes

In this section we show how changes in prices calculated in Section III can be converted into estimates of changes in consumer or social surplus in a market (ignoring externalities and potential cross-market effects of the price changes). Dividing by the value of the market, $P^{\mathrm{T}}Q$, converts these into unit-free indices.

3. Consumer SurplusTo a first-order, the change in consumer surplus in the evaluated market is just the sum across goods of the change in price times the quantity:

$$\Delta CS \approx -\Delta P^{\mathrm{T}} Q.$$ [14]

4. Social SurplusThe predicted change in social surplus, again ignoring externalities and out-of-market effects, is the sum of the change in quantities, approximated by $\Delta Q \approx \frac{\partial Q}{\partial P}\Delta P$, multiplied by the absolute mark-ups:

$$\Delta SS \approx \Delta Q^{\mathrm{T}}(P - \mathbf{mc}) \approx \left(\frac{\partial Q}{\partial P}\Delta P\right)^{\mathrm{T}}(P - \mathbf{mc}).$$

The mark-ups can be pre-merger, post-merger or some combination of the two.[15] It would also be natural to include (as an additional term) an expected change in fixed (or infra-marginal) costs due to the merger as in Williamson (1968).[16]

5. ProfitsWhile changes to profits are not typically an object of regulatory concern, an assumption that these must be positive (by the firms' revealed preference for merging) may provide some information.[17] If ΔF_i is the (presumably negative) change in firm i's fixed costs and $\boldsymbol{\Delta}\mathbf{mc}_i$ is the (uniform) change in inframarginal

[14] Since we have calculated the first and second derivatives of Q, we could add higher order terms to this approximation, but, since ΔP itself is an approximation, that would be adding some second order terms and not others. The formula may be evaluated at pre-merger (in the spirit of Laspeyres) or post-merger (Paasche) quantities or an arithmetic (Marshall-Edgeworth) or geometric (Fisher) average of the two.

[15] Using the tax inclusive price would include tax revenue in social surplus in the spirit of Kaplow (2004).

[16] See Section II.II.C above for a discussion of changes in marginal cost.

[17] A natural direction for future research would be to use such a formula in extending Deneckere and Davidson (1985)'s analysis of the incentives for a merger.

costs then

$$\Delta \pi_i \approx (\Delta \mathbf{P}_i - \Delta \mathbf{mc}_i)^{\mathrm{T}} Q_i + (\mathbf{P} - \mathbf{mc}_i)^{\mathrm{T}} \left(\frac{\partial Q}{\partial P} \widehat{\Delta P} \right) - \Delta F_i.$$

The incentive for firms i and j to merge is just $\Delta \pi_i + \Delta \pi_j$.

6. ADVANTAGES OF NORMATIVE ANALYSISEstimating a unified, normatively significant quantity, such as the impact on consumer welfare, offers several potential benefits over estimating a group of price effects. First, while in some cases it is possible to find remedies addressing particular areas of concern without impacting others, often a package of impacts are inherently tied to one another and must be evaluated as a whole. It may frequently be the case that some of the prices of a firm's products in a market are predicted to rise (or rise by a large amount) and others to fall (or rise only slightly) after a merger. When making a decision in such a case, it is necessary to aggregate the relevant information. Such an aggregation requires some implicit or explicit normative standard; welfare criteria are the natural choice, intuitively putting the greatest weight on the products with the largest markets.

Additionally, many of the potential benefits and harms of a merger may arise through channels different from or only indirectly related to a change in price. One example is consumption externalities (e.g. network or platform effects): in an industry with advertising-funded media, a primary harm from elevated prices to readers may be the reduction in the readership accessible to advertisers. A welfare standard facilitates accounting for such harms by making them comparable to price harms, as illustrated by White and Weyl (2012), who provide an extension of our formula to allow for benefits and harms from network externalities. These effects and others – such as from innovation or quality adjustment – are typically considered separately from price effects; a social welfare framework can include such effects whenever estimates or guesses as to their welfare effects are available.

VI. Applying the formula

In this section we discuss implications of our results for applied merger analysis. The advantages of our approach, relative to UPP, come at a computational cost: direct use of the formulae we derive requires many more inputs than the calculation of UPP. In this section, we illustrate how one might go about applying our approach in practice.

A. Simplifying the formula

While it seems that UPP is, in some sense, a simpler calculation than those we suggest, this is because a UPP-based calculation imposes simplifying assumptions. For example, if we were to assume that all firms produced a single product, that conduct were Bertrand, that all cross-product pass-through rates were zero, and that own pass-through rates were symmetric (ρ), then our formula would simplify to $\rho \sum_i Q_i UPP_i$.

This is a somewhat extreme example, but it illustrates that beginning with our formula there are numerous simplifying assumptions one might make to reduce the complexity of the analysis. A few categories of assumptions one might consider are:

1) Pass-through: assuming zero cross-pass-through, either across firms or across products in a firm, would simplify the calculations. Alternatively, one could assume symmetry of pass-through rates or a demand structure, such as horizontality discussed in Subsection IV.IV.B, that assumes a relationship between elasticities and relative pass-through rates.

2) Heterogeneity: assuming some form of symmetry across all firms or for non-merging firms can be a reasonable simplification. Grouping all non-merging firms into one can sometimes be an easy way to capture the important relationships.[18] Slutsky symmetry also reduces the number of parameters one needs to estimate.

3) Conduct: different conduct assumptions may be easier to work with depending on what data is available. For example, when information on diversion ratios comes from survey data on "next favorite alternative," assuming Nash conduct may be easiest because that data directly reflects demand patterns. If internal firm documents are used, then a conjectures model that fits how firms discuss the reactions of competitors may be appropriate. Adjustments for biases introduced by using an incorrect conduct model could then be made using heuristics as in Subsection II.D.3.

Given time limitations and other constraints of the judicial process, some potentially unattractive assumptions will inevitably be imposed. The full force of our formula is likely to be used in only rare cases, though Miller et al. (2012) show how it can be applied empirically with scanner data with identifying assumptions, data requirements, and time demands that do not greatly exceed those of standard empirical methods. When the data and time demands are excessive, our formulation allows easy selection and application of any combination of assumptions – it does not force all industries into one mold. Furthermore, it is easy to conduct GePP analysis under several combinations of assumptions, facilitating the comparison of the resulting conclusions and thus clarifying the exact role each of these assumptions plays. For example, simplifications could be made more extreme on mark-ups, which are often nearly as difficult to estimate as pass-through or conduct, while allowing greater flexibility or robustness on diversion ratios and conduct. Such variations can clarify the robustness (or weakness) of results in any specific case.

B. Approximateness

While our approach only *approximates* the effects of mergers, we want to emphasize that this is a direct result of the sparsity of assumptions we make. If one were to

[18]See Appendix .C for an example.

assume a functional form for demand, that assumption would define all the higher order terms for the Taylor expansion and yield a precise result. However, in practice, such assumptions typically restrict important quantities, such as pass-through rates and elasticities (Crooke et al., 1999; Weyl and Fabinger, 2009). We understand that it is typically difficult to estimate pass-through rates precisely, but we believe it is preferable to use any information available on these, even if it is informal, or be explicit about assuming their values rather than indirectly constraining them via functional form assumptions. The findings of Miller et al. (2012) seem to confirm this, as they show that the error created from mis-specifying pass-through rates is an order of magnitude greater than that arising from our approximation. Another advantage of our approach is that the robustness of results to differing functional form, cost-side, or conduct assumptions can be tested by adjusting some of the numbers in the relevant matrices, without building additional computational models.

C. At which stage should our tools apply?

Merger review typically proceeds in stages, beginning with an initial screen, proceeding through a more thorough investigation if the screen indicate the the possibility of large anti-competitive effects and, if no settlement can be reached, proceeding to a full court case. As Werden and Froeb (2011) emphasize, the first-order approach is usually promoted as appropriate for an initial screen, with some value during an investigation, but inadequate for a thorough investigation or in-court proceedings where a detailed merger simulation will typically be more compelling.

An advantage of our approach is that it avoids a sharp distinction between these different phases. A version of the formula with many assumptions imposed may be used initially to accommodate limited time and data. As more time and data become available these assumptions can gradually be relaxed and replaced with estimates from data or detailed intuitions. If network effects, product repositioning or other factors are thought to be important, they may be incorporated into the analysis (using extensions of our formula as described in Section V), initially in a highly restricted way and, again, more comprehensively as the analysis progresses. Thus our approach aims to provide a framework that can be used at multiple stages of analysis.

D. Other applications

While our focus has been almost exclusively on merger analysis, our approach and some of our results may apply to problems beyond this narrow context. Our approach to first-order approximation illustrates how local approximations may be used even in analyzing interventions that are in some sense discrete or discontinuous. Of course, this is valid only when the intervention is in some relevant sense small. However, there are many cases of interest, at least in industrial economics, when an intervention (such as the introduction of a new product or the entry of a new firm) may have only a small impact on the prices of other products and on consumer welfare even though it may constitute a discrete change. In these cases, our technique allows the sufficient statistics or first-order identification approach advocated by

Chetty (2009) and Weyl (2009) to be applied more broadly than was originally envisioned.

VII. Conclusion

We extend the modeling framework of Telser (1972) to incorporate price as firms' choice variable and to allow for multi-product firms. We then quantify, in this general setting, the change in pricing incentives created by a merger. Next, we illustrate how first-order approximations may be applied to a discontinuous event such as a merger: using pass-through rates we derive formulae to approximate quantitative effects of mergers on prices and welfare.

In addition to proposing tools for applied merger analysis, we also hope to stimulate further work in this area. Perhaps the most natural extension of our analysis is the work started by Miller et al. (2012) to analyze the accuracy of the first-order approximation for various demand and cost systems. Another step would be to consider an actual second-order approximation to the merger effect, with a focus on what variation would be needed to identify such an approximation and its intuitive interpretation. In a similar spirit, it would be natural to add coordinated effects – changes in the strategy space and conjectures – using a more explicit model of dynamic coordination. The incorporation into our model of non-Jevons effects on consumer welfare, such as those arising when prices affect network size, is an active area of research being pursued by White and Weyl (2012) .

Empirical work oriented towards measuring pass-through rates and how they vary across markets will be crucial in helping to calibrate policymakers' intuitions about these important, but often difficult-to-measure parameters. Similarly, work on understanding the empirical relationship between pre-merger, post-merger and merger pass-through rates will be important. Such work will help policymakers determine reasonable simplifying assumptions that can safely be made without sacrificing too much accuracy. The formulation of such simplifications is central to making the work here directly relevant to the often severely time-constrained analysis of particular mergers.

A. Deriving GePP

PROOF OF PROPOSITION 1:

Writing Q_i for $Q_i(P)$ for conciseness, the firm's first order conditions are

$$\mathbf{Q}_i + \left(\frac{\partial \mathbf{Q}_i}{\partial \sigma_i} + \frac{\partial \mathbf{Q}_i}{\partial P_i}^{\mathrm{T}} \frac{\partial P_i}{\partial P_i}\right)(\mathbf{P}_i - \mathbf{mc}_i(Q_i)) = 0.$$

Remembering that $\frac{dQ}{dP_i} = \frac{\partial Q}{\partial P_i} + \left(\frac{\partial Q}{\partial P_{-i}}\right)^{\mathrm{T}} \frac{\partial P_{-i}}{\partial P_i}$, and then multiplying by $-\left(\frac{d\mathbf{Q}_i}{dP_i}^{\mathrm{T}}\right)^{-1}$ the firm's first-order conditions can be rewritten as:

$$0 = -\left(\frac{d\mathbf{Q}_i}{d\mathbf{P}_i}^{\mathrm{T}}\right)^{-1} \mathbf{Q}_i - (\mathbf{P}_i - \mathbf{mc}_i(\mathbf{Q}_i)) \equiv f_i(P).$$

After a merger of firms i and j, the newly formed firm takes into account the effect of $\mathbf{P}_i$ on π_j and no longer expects $\mathbf{P}_j$ to react to $\mathbf{P}_i$ since the two are chosen jointly. The merged firm's first-order derivatives with respect to $\mathbf{P}_i$ can be written:

$$\begin{aligned} h(\mathbf{P}) = &-(\mathbf{P}_i - \mathbf{mc}_i(\mathbf{Q}_i)) - \left(\frac{d\mathbf{Q}_i}{d\mathbf{P}_i}^{\mathrm{T}} - \frac{\partial \mathbf{Q}_i}{\partial \mathbf{P}_j}\frac{\partial \mathbf{P}_j}{\partial \mathbf{P}_i}\right)^{-1} \mathbf{Q}_i \\ &- \left(\frac{d\mathbf{Q}_i}{d\mathbf{P}_i}^{\mathrm{T}} - \frac{\partial \mathbf{Q}_i}{\partial \mathbf{P}_j}\frac{\partial \mathbf{P}_j}{\partial \mathbf{P}_i}\right)^{-1} \left(\frac{d\mathbf{Q}_j}{d\mathbf{P}_i} - \frac{\partial \mathbf{Q}_j}{\partial \mathbf{P}_j}\frac{\partial \mathbf{P}_j}{\partial \mathbf{P}_i}\right)^{\mathrm{T}} (\mathbf{P}_j - \mathbf{mc}_j(\mathbf{Q}_j)), \end{aligned}$$

where the last term equals $\tilde{D}_{ij}(\mathbf{P}_j - \mathbf{mc}_j(Q_j))$. Using the definition $g(\mathbf{P}) = h(\mathbf{P}) - f(\mathbf{P})$, we have:

$$\mathbf{g}_i(\mathbf{P}) = \tilde{D}_{ij}(\mathbf{P}_j - \mathbf{mc}_j(Q_j)) - \left(\left(\frac{d\mathbf{Q}_i}{d\mathbf{P}_i}^{\mathrm{T}} - \frac{\partial \mathbf{Q}_i}{\partial \mathbf{P}_j}\frac{\partial \mathbf{P}_j}{\partial \mathbf{P}_i}\right)^{-1} - \left(\frac{d\mathbf{Q}_i}{d\mathbf{P}_i}^{\mathrm{T}}\right)^{-1}\right)\mathbf{Q}_i.$$

B. Cournot

EXAMPLE 2 CONTINUED: Note that post-merger-to-monopoly, the firm is just a multiproduct monopoly which we can think of as choosing prices. Beginning from symmetry, in the limit as the products become undifferentiated, a unit tax on each of the two goods will then increase prices according to the pass-through rate facing the monopolist. Yet note that $\tilde{D}$, the diversion ratio, is always 1 since the goods are undifferentiated and thus any sales lost by one on the margin are picked up by the other as we saw above. Let the actual pass-through rate for this common cost shock (which must be the same across products as within, given that products are homogeneous) post-merger be ρ; then by equation (9), which is valid so long as $\left(\frac{\partial h}{\partial P}\right)^{-1}$ is strictly negative definite

$$0 \neq \rho = -\left[\left(\frac{\partial h}{\partial P}\right)^{-1}_{11} - \left(\frac{\partial h}{\partial P}\right)^{-1}_{21}\right] + \left[\left(\frac{\partial h}{\partial P}\right)^{-1}_{11} - \left(\frac{\partial h}{\partial P}\right)^{-1}_{21}\right] = 0,$$

a contradiction. Thus it must be that $\left(\frac{\partial h}{\partial P}\right)^{-1}$ is *not* in fact strictly negative definite at symmetric prices and thus pass-through must locally be infinite. Thus, in the undifferentiated limit, the 0 value of GePP is misleading: the net effect any common cost shock will have on incentives will apparently be 0, but because pass-through is so large, this cancels out. Away from the limit in the symmetric case, exact calculations of merger pass-through are a simplification of the formula in Appendix .E.

C. Conjectural variations examples

Often, the two merging firms are closer competitors (and potential accommodators) with each other than with other firms in the industry. Therefore, we now consider a three firm model, with the two merging firms being symmetric but the third-firm being asymmetric, representing a reduced form for the rest of the industry. To keep things simple, though, we assume that the quantity of all firms (q) and all firms' (Bertrand) demand slopes are the same, but now we have two diversion ratios: d, the (Bertrand) diversion to and from the third firm from and to each of the two merger partners and δ, the diversion from each merger partner to the other. The mark-ups of the two merger partners are m. We assume that conjectures are in proportion to diversion: each merger partner anticipates an accommodating reaction of $\lambda\delta$ from its partner and λd from the third firm, while the third firm expects λd from each of the merger partners.

PROPOSITION 2: *In the three-firm example, GePP from a merger of the two close firms is*

$$m\frac{\delta+\tilde{\lambda}\left(d^2-\delta^2\right)-\left(d^2+\delta^2\right)\frac{\delta\tilde{\lambda}^2}{1-\delta\tilde{\lambda}}}{1-d^2\tilde{\lambda}}\approx m\frac{\delta+\tilde{\lambda}\left(d^2-\delta^2\right)}{1-d^2\tilde{\lambda}}, \tag{1}$$

where $\tilde{\lambda}\equiv\frac{\lambda}{1+\delta\lambda}$ and again the approximation is valid for small λ. Approximate GePP is thus increasing (decreasing) in λ if and only if d is greater (less) than $\frac{\delta}{\sqrt{1+\delta}}$. Approximate GePP is constant in λ if and only if $d=\frac{\delta}{\sqrt{1+\delta}}$. Precise GePP decreases in strictly more cases than does approximate GePP.

If the strength of the within-merger interaction is small compared to that outside the merger, GePP increases with anticipated accommodation. Conversely, if the strength of within-merger interaction is sufficiently greater than the total outside interaction then accommodation decreases GePP. Some relevant cases may be close to the point where the degree of accommodation anticipated has little effect. It is reassuring for the theory of oligopoly that, even if the levels of prices may be quite sensitive to conduct, their comparative statics under interventions of interest may be less so.

PROOF:

Our proof here is almost entirely analogous to that of Example 3. The first-order condition now requires that for the merging firms

$$m=-\frac{q}{\frac{\partial Q^1}{\partial P^1}\left(1-\left[d^2+\delta^2\right]\lambda\right)},\quad\implies\frac{\partial Q^1}{\partial P^1}=-\frac{q}{m\left(1-\left[d^2+\delta^2\right]\lambda\right)}.$$

On the other hand by the logic of conjectures discussed in the proof of Example 3, if l represents the pre-merger merging-firm-to-non-merging-firm conjecture, L represents the same between the merging firms, and $\tilde{l}$ represents the post-merger version of l then

$$l=\tilde{l}\left(1+L\right)\iff\tilde{l}=\frac{l}{1+L}.$$

Plugging in our definitions of $l=d\lambda$ and $L=\delta\lambda$ we obtain $\tilde{l}=\frac{d\lambda}{1+\delta\lambda}\equiv\delta\tilde{\lambda}$. Now we can compute

$$\begin{aligned}\frac{d^MQ^1}{dP^1}&=\frac{\partial Q^1}{\partial P^1}\left(1-\tilde{l}d\right)=-\frac{q\left(1-d^2\tilde{\lambda}\right)}{m\left(1-\left[d^2+\delta^2\right]\lambda\right)},\\ \frac{d^MQ^2}{dP^1}&=-\frac{\partial Q^1}{\partial P^1}\left(\delta+\tilde{l}d\right)=\frac{q\left(\delta+d^2\tilde{\lambda}\right)}{m\left(1-\left[d^2+\delta^2\right]\lambda\right)},\end{aligned}\quad\implies m\tilde{D}_{12}=m\frac{\delta+d^2\tilde{\lambda}}{1-d^2\tilde{\lambda}}, \tag{2}$$

and

$$Q_1\left(\frac{1}{\frac{d^MQ^1}{dP^1}}-\frac{1}{\frac{dQ^1}{dP^1}}\right)=m\left(-\frac{1-\left[d^2+\delta^2\right]\lambda}{1-d^2\tilde{\lambda}}+1\right)=-m\frac{d^2\tilde{\lambda}-\left(d^2+\delta^2\right)\lambda}{1-d^2\tilde{\lambda}}. \tag{3}$$

With a little algebra, subtracting (3) from (2) yields the formula in the text, given that $\tilde{\lambda}-\lambda=\frac{\delta\tilde{\lambda}^2}{1-\delta\tilde{\lambda}}$. As before, the more sophisticated formula is decreasing in λ whenever the simpler version is, but also decreases in some cases (for larger λ) when the simpler version does not.

Returning to the simpler formula and taking the derivative with respect to λ yields an expression proportional to $d^2(1+\delta)-\delta^2$, which is clearly positive or negative depending on the sign of the inequality in the proposition.

D. Taylor Series Error Term

For notational convenience let $x=h^{-1}$. The error term is

$$\begin{aligned}\frac{1}{2}\begin{pmatrix}\sum_i\sum_j\frac{\partial^2x_1}{\partial h_i\partial h_j}\mathbf{g}_i(P^0)\mathbf{g}_j(P^0)\\ \vdots\\ \sum_i\sum_j\frac{\partial^2x_n}{\partial h_i\partial h_j}\mathbf{g}_i(P^0)\mathbf{g}_j(P^0)\end{pmatrix}&=\frac{1}{2}\left[\sum_i\begin{pmatrix}\frac{\partial^2x_1}{\partial h_i\partial h_1}&\cdots&\frac{\partial^2x_1}{\partial h_i\partial h_n}\\ \vdots&\ddots&\vdots\\ \frac{\partial^2x_n}{\partial h_i\partial h_1}&\cdots&\frac{\partial^2x_n}{\partial h_i\partial h_n}\end{pmatrix}g_i(P^0)\right]g(P^0)\\ &\equiv\frac{1}{2}\left[\sum_i\left(\frac{\partial^2x}{\partial h_i\partial h}g_i(P^0)\right)\right]g(P^0).\end{aligned} \tag{4}$$

We know $\frac{\partial x}{\partial h}\frac{\partial h}{\partial x}=I$. Differentiating with respect to h_i gives

$$\frac{\partial^2x}{\partial h_i\partial h}\frac{\partial h}{\partial x}+\frac{\partial x}{\partial h}\begin{pmatrix}\sum\limits_k\frac{\partial^2h_1}{\partial x_i\partial x_k}\frac{\partial x_k}{\partial h_i}&\cdots&\sum\limits_k\frac{\partial^2h_1}{\partial x_n\partial x_k}\frac{\partial x_k}{\partial h_i}\\ \vdots&\ddots&\vdots\\ \sum\limits_k\frac{\partial^2h_n}{\partial x_i\partial x_k}\frac{\partial x_k}{\partial h_i}&\cdots&\sum\limits_k\frac{\partial^2h_n}{\partial x_n\partial x_k}\frac{\partial x_k}{\partial h_i}\end{pmatrix}=0.$$

Solving for $\frac{\partial^2x}{\partial h_i\partial h}$, using $\frac{\partial x}{\partial h}=\left(\frac{\partial h}{\partial x}\right)^{-1}$ and substituting into (4) gives

$$E=-\frac{1}{2}\sum_i\frac{\partial x}{\partial h}\begin{pmatrix}\sum_k\frac{\partial^2h_1}{\partial x_1\partial x_k}\frac{\partial x_k}{\partial h_i}&\cdots&\sum_k\frac{\partial^2h_1}{\partial x_n\partial x_k}\frac{\partial x_k}{\partial h_i}\\ \vdots&\ddots&\vdots\\ \sum_k\frac{\partial^2h_n}{\partial x_1\partial x_k}\frac{\partial x_k}{\partial h_i}&\cdots&\sum_k\frac{\partial^2h_n}{\partial x_n\partial x_k}\frac{\partial x_k}{\partial h_i}\end{pmatrix}\frac{\partial x}{\partial h}g_i\left(\mathbf{P}^0\right)g\left(\mathbf{P}^0\right).$$

If we look at just the ath entry of the vector, we have

$$
\begin{aligned}
E_a &= -\frac{1}{2}\sum_i\sum_j \frac{\partial x_a}{\partial h_j}\begin{pmatrix} \sum_k \frac{\partial^2 h_j}{\partial x_1 \partial x_k}\frac{\partial x_k}{\partial h_i} & \cdots & \sum_k \frac{\partial^2 h_j}{\partial x_n \partial x_k}\frac{\partial x_k}{\partial h_i} \end{pmatrix}\frac{\partial x}{\partial h} g_i\left(\mathbf{P}^0\right) g\left(\mathbf{P}^0\right) \\
&= -\frac{1}{2}\sum_i\sum_j \frac{\partial x_a}{\partial h_j}\sum_k\sum_l \frac{\partial^2 h_j}{\partial x_l \partial x_k}\frac{\partial x_k}{\partial h_i}\frac{\partial x_l}{\partial h} g_i\left(\mathbf{P}^0\right) g\left(\mathbf{P}^0\right) \\
&= -\frac{1}{2}\sum_i\sum_j\sum_m\sum_k\sum_l \frac{\partial x_a}{\partial h_j}\left(\frac{\partial x_k}{\partial h_i}\frac{\partial^2 h_j}{\partial x_l \partial x_k}\frac{\partial x_l}{\partial h_m}\right) g_i\left(\mathbf{P}^0\right) g_m\left(\mathbf{P}^0\right) \\
&= -\frac{1}{2}\sum_i\sum_j\sum_m \frac{\partial x_a}{\partial h_j}\left(\frac{\partial x^T}{\partial h_i}\mathcal{D}_x^2 h_j \frac{\partial x}{\partial h_m}\right) g_i\left(\mathbf{P}^0\right) g_m\left(\mathbf{P}^0\right) \\
&= -\frac{1}{2}\sum_j\sum_m \frac{\partial x_a}{\partial h_j}\left(\sum_i \frac{\partial x^T}{\partial h_i} g_i\left(\mathbf{P}^0\right)\right)\mathcal{D}_x^2 h_j \frac{\partial x}{\partial h_m} g_m\left(\mathbf{P}^0\right) \\
&= -\frac{1}{2}\sum_j\sum_m \frac{\partial x_a}{\partial h_j} g^T\left(\mathbf{P}^0\right)\left(\frac{\partial x}{\partial h}\right)^T \mathcal{D}_x^2 h_j \frac{\partial x}{\partial h_m} g_m\left(\mathbf{P}^0\right) \\
&= -\frac{1}{2}\sum_j \frac{\partial x_a}{\partial h_j} g^T\left(\mathbf{P}^0\right)\left(\frac{\partial x}{\partial h}\right)^T \mathcal{D}_x^2 h_j \frac{\partial x}{\partial h} g\left(\mathbf{P}^0\right),
\end{aligned}
$$

where $\mathcal{D}_x^2 h_j$ denotes the Hessian. Letting $[A]_{ij}$ indicate the ij element of matrix A,

$$
E_a = -\frac{1}{2}\sum_j \left[\left(\frac{\partial h}{\partial x}\right)^{-1}\right]_{aj} g^T\left(\mathbf{P}^0\right)\left(\left(\frac{\partial h}{\partial x}\right)^{\mathrm{T}}\right)^{-1}\left(\mathcal{D}_x^2 h_j\right)\left(\frac{\partial h}{\partial x}\right)^{-1} g\left(\mathbf{P}^0\right),
$$

where the Hessian and derivatives are evaluated at some price in $[P^0, P^M]$.

E. Calculating $\frac{\partial \mathbf{g}}{\partial \mathbf{P}}$

In the case of single-product firms in a Bertrand equilibrium, we know that

$$
-\frac{\partial \mathbf{f}(\mathbf{P})}{\partial \mathbf{P}} = -\begin{pmatrix} 2 - \frac{Q_i \frac{\partial^2 Q_i}{\partial P_i^2}}{(\frac{\partial Q_i}{\partial P_i})^2} & \frac{\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_i} - Q_i \frac{\partial^2 Q_i}{\partial P_i \partial P_j}}{(\frac{\partial Q_i}{\partial P_i})^2} \\ \frac{\frac{\partial Q_j}{\partial P_i}\frac{\partial Q_j}{\partial P_j} - Q_j \frac{\partial^2 Q_j}{\partial P_j \partial P_i}}{(\frac{\partial Q_j}{\partial P_j})^2} & 2 - \frac{Q_j \frac{\partial^2 Q_j}{\partial P_j^2}}{(\frac{\partial Q_j}{\partial P_j})^2} \end{pmatrix} = \rho^{-1} \equiv -\begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}.
$$

Also,

$$
\frac{\partial \mathbf{g}(\mathbf{P})}{\partial \mathbf{P}} = \begin{pmatrix} -(P_j - C_j)\frac{\frac{\partial^2 Q_j}{\partial P_i^2}\frac{\partial Q_i}{\partial P_i} - \frac{\partial^2 Q_i}{\partial P_i^2}\frac{\partial Q_j}{\partial P_i}}{(\frac{\partial Q_i}{\partial P_i})^2} & -\frac{\frac{\partial Q_j}{\partial P_i}}{\frac{\partial Q_i}{\partial P_i}} - (P_j - C_j)\frac{\frac{\partial^2 Q_j}{\partial P_i \partial P_j}\frac{\partial Q_i}{\partial P_i} - \frac{\partial^2 Q_i}{\partial P_i \partial P_j}\frac{\partial Q_j}{\partial P_i}}{(\frac{\partial Q_i}{\partial P_i})^2} \\ -\frac{\frac{\partial Q_i}{\partial P_j}}{\frac{\partial Q_j}{\partial P_j}} - (P_i - C_i)\frac{\frac{\partial^2 Q_i}{\partial P_j \partial P_i}\frac{\partial Q_j}{\partial P_j} - \frac{\partial^2 Q_j}{\partial P_j \partial P_i}\frac{\partial Q_i}{\partial P_j}}{(\frac{\partial Q_j}{\partial P_j})^2} & -(P_i - C_i)\frac{\frac{\partial^2 Q_i}{\partial P_j^2}\frac{\partial Q_j}{\partial P_j} - \frac{\partial^2 Q_j}{\partial P_j^2}\frac{\partial Q_i}{\partial P_j}}{(\frac{\partial Q_j}{\partial P_j})^2} \end{pmatrix},
$$

so, using Slutsky symmetry of $\frac{\partial^2 Q_j}{\partial P_i^2} = \frac{\partial^2 Q_i}{\partial P_i \partial P_j}$ and $\frac{\partial^2 Q_i}{\partial P_j^2} = \frac{\partial^2 Q_j}{\partial P_i \partial P_j}$, we have

$$\frac{\partial \mathbf{g}(\mathbf{P})}{\partial \mathbf{P}} = \begin{pmatrix} v_i & v_j \end{pmatrix},$$

where

$$v_i = \begin{pmatrix} -\frac{(P_j - C_j)(\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_i} - m_2(\frac{\partial Q_i}{\partial P_i})^2)\frac{\partial Q_i}{\partial P_i} - (\frac{\partial Q_i}{\partial P_i})^2(2-m_1)\frac{\partial Q_j}{\partial P_i}}{Q_i(\frac{\partial Q_i}{\partial P_i})^2} \\ -\frac{\frac{\partial Q_i}{\partial P_j}}{\frac{\partial Q_j}{\partial P_j}} - \frac{(P_i - C_i)((\frac{\partial Q_i}{\partial P_i} - m_2(\frac{\partial Q_i}{\partial P_i})^2)\frac{1}{Q_i}\frac{\partial Q_j}{\partial P_j} - (\frac{\partial Q_j}{\partial P_i}\frac{\partial Q_j}{\partial P_j} - m_3(\frac{\partial Q_j}{\partial P_j})^2)\frac{1}{Q_j}\frac{\partial Q_i}{\partial P_j}}{(\frac{\partial Q_j}{\partial P_j})^2} \end{pmatrix},$$

$$v_j = \begin{pmatrix} -\frac{\frac{\partial Q_j}{\partial P_i}}{\frac{\partial Q_i}{\partial P_i}} - \frac{(P_j - C_j)((\frac{\partial Q_j}{\partial P_j} - m_3(\frac{\partial Q_j}{\partial P_j})^2)\frac{1}{Q_j}\frac{\partial Q_i}{\partial P_i} - (\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_i} - m_2(\frac{\partial Q_i}{\partial P_i})^2)\frac{1}{Q_i}\frac{\partial Q_j}{\partial P_i}}{(\frac{\partial Q_i}{\partial P_i})^2} \\ -\frac{(P_i - C_i)(\frac{\partial Q_j}{\partial P_i}\frac{\partial Q_j}{\partial P_j} - m_3(\frac{\partial Q_j}{\partial P_j})^2)\frac{\partial Q_j}{\partial P_j} - (\frac{\partial Q_j}{\partial P_j})^2(2-m_4)\frac{\partial Q_i}{\partial P_j}}{Q_j(\frac{\partial Q_j}{\partial P_j})^2} \end{pmatrix}.$$

*

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