Computers & Graphics 97 (2021) 258–267

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Special Section on CAD & Graphics 2021

Craniofacial reconstruction based on heat flow geodesic grid regression (HF-GGR) model

Bin Jia[a], Junli Zhao[a,b,*], Shiqing Xin[c], Fuqing Duan[d], Zhenkuan Pan[a], Zhongke Wu[d], Jinhua Li[a], Mingquan Zhou[d]

[a] College of Computer Science and Technology, Qingdao University, Qingdao 266071, China
[b] College of Automation and Electrical Engineering, Qingdao University, Qingdao 266071, China
[c] School of Computer Science and Technology, Shandong University, Qingdao 266237, China
[d] Engineering Research Center of Virtual Reality and Applications, Ministry of Education, Beijing Normal University, Beijing 100875, China

ARTICLE INFO

Article history:
Received 7 February 2021
Revised 7 April 2021
Accepted 13 April 2021
Available online 24 April 2021

Keywords:
Craniofacial reconstruction
Heat flow
Geodesic grid
Partial least squares regression

ABSTRACT

Craniofacial reconstruction is to predict the 3D facial geometry according to the internal relationship between the skull and face, which is widely applied in the field of criminal investigation, archaeology, forensic medicine and so on. In this paper, utilizing the inherent advantage of geodesic to encode craniofacial geometry, we propose a heat flow geodesic grid regression (HF-GGR) model to facilitate craniofacial reconstruction. Our algorithm consists of three steps. In the first step, we extract the nose-tip rooted geodesic distance field and discretize it into a radial grid representation. Then in the second step, we generate geodesic grid of target skull appearance by utilizing the partial least squares regression (PLSR) method. Finally in the third step, we reconstruct the face of target skull according to the geodesic grid and face statistical model. We have conducted experiments on a data set with 213 pairs of craniofacial data. The extensive experimental results show that our algorithm can achieve accurate reconstruction results with faster speed and fewer geodesic grid points than the state-of-the-art method.

1. Introduction

Craniofacial reconstruction is a challenging yet fascinating research task and becomes increasingly important in the field of forensic medicine, archaeology, criminal investigation and so on. In the field of archaeology, reconstructing the remains of mausoleums cannot only satisfy archaeologists desire to explore ancient peoples appearance, but also acquire the analysis of cultural environment of that time by combining the reconstructed appearances and historical facts. In the field of forensic medicine, in order to help patients with large-scale burn or cosmetic surgery to carry out skin transplantation, we can also utilize the craniofacial reconstruction technique to reconstruct patient's original face, which can help doctors carry out the next step of operation. In the field of criminal investigation, craniofacial reconstruction can help policemen to identify the unknown corpse.

However, craniofacial reconstruction is not easy. Traditionally, craniofacial reconstruction, as a professional craft, relies on sculptors' anatomy knowledge and carving skill, which is a tedious and time consuming process. Furthermore, the final reconstruction is inevitably subjective. With the advancement of digital scanning and geometry processing technology, it becomes possible to emancipate sculptors from the hard work. The basic principle to design an automatic craniofacial reconstruction algorithm is based on the observation that there is an interesting coupling relationship between the skull geometry and the facial geometry. Therefore, the algorithm design includes at least two considerations: (1) which kind of information is able to better encode the skull/facial geometry, and (2) how the statistical model is designed to make the reconstruction as fast/accurate as possible.

Geodesic is an important intrinsic measure in differential geometry. Although it can characterize how the surface is curved, the expensive computational cost restricts its use in a wide range of applications. Balance between accuracy and run-time performance has been a central problem of CG (compute geodesic) for decades (since the early beginning). Many excellent geodesic algorithms are proposed for solving the different problems. In this paper, considering the running speed and the smoothness of geodesics, we use the heat flow algorithm to extract the geodesic distances at a group of radially distributed grid points and further encode the geometry of craniofacial training data into an array-like structure. After that, we predict geodesic grid corresponding to the user-

* Corresponding author at: College of Computer Science and Technology, Qingdao University, Qingdao 266071, China.
E-mail address: zhaojl@yeah.net (J. Zhao).

https://doi.org/10.1016/j.cag.2021.04.029
0097-8493/

specified skull data based on the partial least squares regression model. Finally, we perform the real craniofacial reconstruction operation by utilizing the geodesic grid and the face statistical model. Experimental results validate the effectiveness of the novel algorithm framework.

1. We use the angle division map to encode the craniofacial geometry, facilitated by the heat flow method that can report the geodesic distances/paths very quickly.
2. We give a more compact geodesic grid regression model. Rather than use tens of thousands of mesh vertices, we sample the geodesic distances at a group of radially distributed grid points, which is totally independent of the mesh resolution. In this way, the computational cost is greatly reduced.
3. Experiments on 213 pairs of craniofacial data were conducted and the experimental results show that our method can reconstruct facial models more accurate than classic craniofacial reconstruction methods.

The sections of this paper are arranged as follows. Section 2 introduces related works about our method. Our algorithm is elaborated in Section 3, which consists of (a) building craniofacial geodesic grid, (b) computing geodesic distance by heat flow method, (c) generating facial geodesic grid of the user-specified skull by partial least squares regression and (d) reconstructing face by utilizing face statistical model. Next, we present experimental results in Section 4. Finally, we conclude this paper in Section 5.

2. Related works

2.1. Craniofacial reconstruction

Craniofacial reconstruction methods include knowledge-based craniofacial reconstruction and statistical model-based craniofacial reconstruction. Knowledge-based craniofacial reconstruction [1–6] depends on learning of soft tissues location and depth. It involves accurate skull registration and professional knowledge. With the development of science and technology, more and more statistical models are proposed for craniofacial reconstruction.

Statistical model-based craniofacial reconstruction appeal to build statistical model by training skulls and facial data, then predict the face of the target skull by the statistical model. Claes et al. [7] proposed craniofacial statistical model, and optimized performance of statistical model by utilizing facial points sets and sparse feature points set of skull markers for reducing error. Then, statistical model-based craniofacial reconstruction developed rapidly. Paysan et al. [8] utilized statistical model to express skull and facial surface, the model parameters and relation can be acquired by ridge regression learning for craniofacial reconstruction. Claes et al. [9] also proposed fully automatic Bayesian-based statistical frame, which can estimate the most likely facial surface according to known craniofacial data. Xiao et al. [10] presented Gaussian Process Latent Variable Models (GP-LVM), which can express skull and facial surface data in low dimension latent space. In 2010, Zhang et al. [11] proposed regional statistical craniofacial model (RSCM). This method divided whole facial region into eyes, mouth and other several parts, and reconstructed all regions separately. Hu et al. [12] put forward a three dimensional craniofacial reconstruction method based on hierarchical dense deformable model. Madsen et al. [13] carried out joint registration by utilizing two independent statistical model, reasonable face shape distribution was completed with facial shape data, skull shape data and marked tissue depth data. Shui et al. [14] proposed the shape statistical model based on CFRTools(craniofacial reconstruction tools), and constructed the statistical model of skull and face through generalized procrustes analysis (GPA) and principal component analysis (PCA).

Partial least squares (PLS) method is proposed by Wold [15] and developed quickly. As one of geometric prediction methods, partial least squares regression (PLSR) can be utilized for regression of multiple dependent and independent variables. It can modify regression coefficient according to result, it means we can control algorithm process well. Therefore, PLSR has a good performance in many fields. In craniofacial reconstruction, PLSR is usually applied to build model for predicting or reconstructing face. Duan et al. [16,17] represented skull and facial surface by tensor and extract their relation using PLSR in shape parameter space for craniofacial reconstruction. Huang et al. [18] mapped five skull regions into five different regions of facial surface through PLSR, and then completed craniofacial reconstruction by splicing method. He et al. [19] proposed a PLSR coordinate calculation model of facial vertices and built a new local craniofacial reconstruction method. Li et al. [20] utilized least square support vector regression (LSSVR) to finish reconstruction. Deng et al. [21] utilized regional method to adjust coordinates of facial surface and skull, segmented regions, craniofacial reconstruction was completed through calculating their relationship by PLSR and utilizing a new splicing method. PLSR is suitable for the case of few sample data, but multi internal variables and correlation.

In statistical-based model craniofacial reconstruction methods, establish statistical model through training data is crucial. One kind of method is utilizing all of the points of craniofacial data for constructing statistical model and reconstructing face which will spend a lot of time and computer resources. The other is utilizing manually marked feature points as the statistical model training data for craniofacial reconstruction, which takes a lot of manpower and can not be realized fully automation. To solve these problems, we propose the craniofacial reconstruction method based on heat flow geodesic grid regression model. This method can effectively and automatically encode craniofacial geometric feature. Because of the fewer number of geodesic grid points sampled in our method, the reconstruction time is reduced while remaining comparable accuracy.

2.2. Geodesic

Geodesic is an important concept of differential geometry and an important intrinsic quantity of surface. It is widely utilized in computer vision, computer graphics and image processing. In many applications of 3D models, the introduction of geodesic has achieved better results [22]. So how to solve the discrete geodesic has become a popular topic in computer graphics. Dijkstra [23] proposed the classical Dijkstra algorithm, then the geodesic algorithm is developing rapidly. Geodesic algorithms are studied from two aspects:accurate geodesic algorithm and approximate geodesic algorithm.

Accurate geodesic is calculated by studying the geometric properties of triangular meshes. For example, Sharir and Schorr [24], Mount [25], Mitchell et al. [26], Chen and Han [27], Xin and Wang [28], Xu et al. [29] has proposed some popular and excellent accurate geodesic algorithms. However, the calculation processes of these algorithms are relatively complex and the costs of calculation are huge, which means that they require a lot of professional knowledge, time cost and resource cost. Considering craniofacial reconstruction revolving geodesics and geodesic distances calculation on hundreds of models, and high requirement on speed and robust of geodesic algorithm, we prefer to utilize fast approximation geodesic algorithms.

Approximate geodesic algorithms are simple and run fast. Nowadays, many approximate geodesic algorithms have been proposed. For example, Sethian [30] proposed fast marching method (FMM), geodesic distance can be computed by solving Eikonal equation which is a nonlinear partial differential equation. Keenan

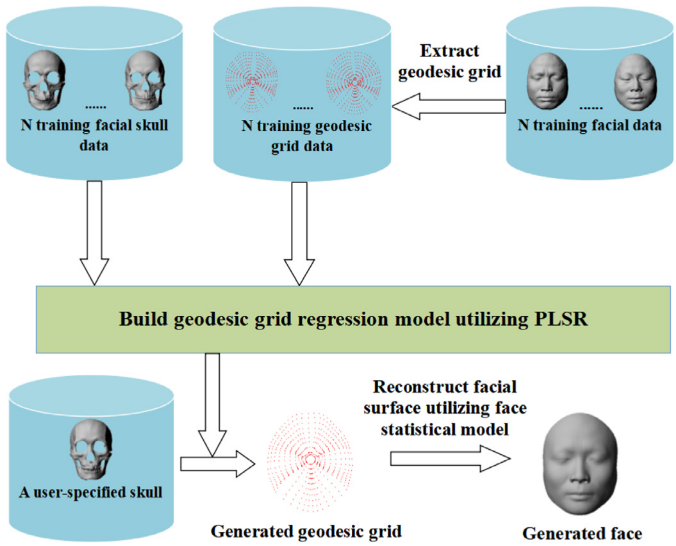


Fig. 1. The flow chart of craniofacial reconstruction method based on heat flow geodesic grid regression model.

Crane et al. [31] put forward heat flow method. They computed discrete parameters (include heat kernels) according to time discretization and space discretization. Finally utilized these discrete parameters to solve Poisson equation to get final geodesic distance. Ying et al. [32] utilized a new Saddle Vertex Graph (SVG) method to solve discrete geodesic problem. The main idea is to decompose the whole discrete geodesic problem into several sub problems by utilizing the local property of the discrete geodesic. Except from this, user can define a parameter to balance accuracy and algorithm complexity. Compared with previous approximate geodesic methods, SVG method is more accurate and efficient. Cao et al. [33] proposed quasi-geodesic distance field (QGDF) method for computing accurate or approximate geodesic distance. This method utilizes standard linear programming to solve geodesic distance by introducing smoothness energy. The smoothness and accuracy of geodesic distance field can be adjusted by the proportion of the smoothness energy. Tao et al. [34] optimized the heat flow method, effectively calculated the geodesic distance on the mesh with 200 million vertices. Although the accuracy of approximate geodesic algorithm is lower than that of accurate geodesic algorithm, the smoothness of geodesic is better and the speed of computation is faster.

In our work, craniofacial reconstruction accuracy and speed are two key issues. The smoother geodesic path will be benefited for improving accuracy of final results of craniofacial reconstruction. Even though its error is near one percent, heat flow algorithm can get smoother geodesic path and run faster than accurate geodesic algorithms. Therefore, heat flow method is utilized to calculate geodesic distance in our geodesic grid regression model for obtaining smooth geodesic quickly and reconstructing more accurate face.

3. Method

In this paper, we encode craniofacial geometry as a geodesic grid, which built on craniofacial training data by heat flow method. The geodesic grid of the face surface corresponding to user-specified skull is established by partial least squares regression model, and then the facial appearance of target skull is estimated utilizing face statistical model. The flow chart of our method is shown by Fig. 1. N is the number of training craniofacial data.

3.1. Constructing craniofacial geodesic grid

Our craniofacial data come from whole head CT scans of volunteers in the North of China with a clinical multislice CT scanner system (Siemens Sensation16). First, we extracted the craniofacial borders from the original CT slice images and reconstructed the 3D craniofacial meshes with a marching cubes algorithm [35]. The back part of the craniofacial model was cut away since the features are mainly concentrated on the front part of the head. One model is randomly chosen as the reference model, and the registration among skulls and skins were respectively conducted according to the reference [12,36,37]. After the above pre-process, all craniofacial surface were aligned and represented by triangular mesh.

A facial surface is considered as a connected 2-manifold in R^3 space. Then we can encode a facial surface by a geodesic grid. Geodesic grid is defined by the intersection sets of iso-geodesics and geodesic paths on a surface. We sample the geodesic distances at a group of radially distributed grid points and use the angle division map to construct geodesic grid.

3.1.1. Locating the source and target points of geodesics

As a most salient and reliable facial feature, the tip point of a nose is selected as the source point o of geodesic grid naturally. The tip point of a nose is located in the middle of the face model and it is easy to be detected. The point with the largest Y-axis value on our pre-processed craniofacial model is the tip point of a nose.

In order to make the calculated geodesics uniformly distributed in the face model as much as possible, the target points of geodesics needs to be defined on the boundary of craniofacial model. Suppose the number of all boundary points is k and we need n target points. Suppose an unit vector $\mathbf{V} = [0, 0, 1]$ aligned with Z-axis and $\mathbf{T} = (\mathbf{T_1}, \mathbf{T_2}, \ldots, \mathbf{T_k})$ is vectors set from source point to each boundary point. Here, we utilize angle division map to get these n target points by the following steps.

Firstly, compute the minimal angle γ between $\mathbf{V}$ and $\mathbf{T_i}$ according to formula (1), where $\mathbf{T_i} \in \mathbf{T}(1 \le i \le k)$. When the minimum γ of $\mathbf{T_i}(1 \le i \le k)$ is obtained, the point i is the first target point we expect. Then we remove $\mathbf{T_i}$ from the boundary points vector set $\mathbf{T}$ and add the first target points q_1 to the target points set $\mathbf{q}$.

$$\gamma = arccos \frac{\mathbf{V} \cdot \mathbf{T_i}}{\|\mathbf{V}\| \|\mathbf{T_i}\|} \tag{1}$$

Secondly, rotate vector $\mathbf{V}$ by $m = 360/n$ degree to compute the minimal angle γ between $\mathbf{V}$ and $\mathbf{T_i}$ and get the second target point q_2 in the same way with the first step.

Repeat the second step for n times until vector $\mathbf{V}$ rotated by 360 degree, we can get the target points set $\mathbf{q} = (\mathbf{q_1}, \mathbf{q_2}, \ldots, \mathbf{q_n})$ with n target points by equal angle interval m.

3.1.2. Constructing the geodesic grid

Next, the geodesic distance from the tip point of a nose is calculated by heat flow algorithm (The details of calculate geodesic distance are provided in Section 3.2). And then n geodesic paths from the nose-tip point to the n target boundary points can be calculated by utilizing inverse gradient method. Inverse gradient method is a famous and effective method for geodesic path calculation. It starts from the user-specified target point, finding next point on adjacency point or edge of adjacency mesh according to the direction of gradient descent. This algorithm would stop until the next point is the source point. All geodesic paths from the nose-tip point o to the n target points can be represented as the vector $\mathbf{g} = (\mathbf{g_1}, \mathbf{g_2}, \ldots, \mathbf{g_n})$, where in $\mathbf{g_i}(i \in [1, n])$ represents the geodesic from o to an target point q_i.

Geodesic grid is the intersection points set of geodesic paths and iso-geodesics. Then the geodesic grid points can be calculated

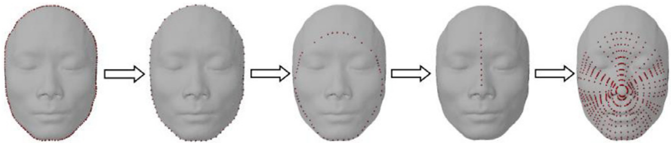


Fig. 2. The calculation process of geodesic grid on the face.

by the iso-geodesics and geodesic paths. Iso-geodesics can be extracted by setting a standard distance parameter on geodesic paths. Firstly, find the minimum geodesic distance value d_{min} between the tip point of a nose and n target boundary points. If we want to get h iso-geodesics points on each geodesic path, the standard distance interval value can be calculated by the formula $d = d_{min}/h$. Then we can compute h iso-geodesics on the geodesics according to find all the points where the geodesic distances from the nose-tip point to the points are equal to $d, 2 \times d, \ldots, h \times d$ respectively corresponding to the first iso-geodesic, second iso-geodesic,..., the h-th iso-geodesic. Thus, all geodesic grid point set y on a 3D facial model can be denoted by the following equation:

$$y = \left\{ y_{i,j} | d_g(o, y_{i,j}) = j \times d, y_{i,j} \in \mathbf{g_i} \right\}, \quad i \in [1, n], j \in [1, h] \tag{2}$$

where $y_{i,j}$ is the geodesic grid point with the distance $j \times d$ from source point o on the i-th geodesic g_i, $d_g(o, y_{i,j})$ is the geodesic distance from source point o to $y_{i,j}$.

The calculation of all fundamental parameters is complete. So the $h \times n$ geodesic grid points on each skin data can be calculated by utilizing basic mathematical formula. The process of calculation of geodesic grid on the skin model is shown in Fig. 2 and Algorithm 1 as follows.

Algorithm 1: Calculation of geodesic grid

Input: Craniofacial model
Output: Geodesics mesh
1. Locate the tip point of a nose and find k boundary points;
2. Calculate unit vector **V**=[0,0,1] and Vector set **T**;
3. Calculate minimum included angle for finding n target boundary points;
4. Calculate geodesic distances and geodesics from the tip point of a nose to n boundary points;
5. Search the minimum geodesic distance d_{min} among n points and compute standard distance d;
6. Calculate h iso-geodesics according to d and calcluate corresponding geodesic grid.
7. **return** *Geodesic grid.*

3.2. Computing geodesic distance by heat flow method

After the source point and target points are determined, heat flow method can be utilized for solving geodesic distance between two vertices on 3D craniofacial surface. The general idea is to simulate the diffusion of heat from the source point to the surrounding points. Firstly, we compute heat value by utilizing time discretization and space discretization. Then solve laplacian matrix, gradient and divergence step by step. Finally solve Poisson equation for getting geodesic distance.

In the first step, time discretization is utilized to change formula (3) into formula (4):

$$\dot{u} = \Delta u \tag{3}$$

$$(id - h_t \Delta) u_t = u_0 \tag{4}$$

id can be considered as a smooth manifold, h_t is heat flow diffusion time, u_0 is an integer value of one or zero, u_t is unknown parameter obviously.

Suppose there are v vertices on a skin model, the tip point of a nose is source vertex o. Then, we should do space discretization. Here we calculate discrete Laplace operator by formula (5):

$$(Lu)_i = \frac{1}{2A_i} \sum_j (cot\alpha_{ij} + cot\beta_{ij})(u_j - u_i) \tag{5}$$

$u \in R^{|V|}$ is a piece-wise linear function of a designated triangular surface, $i(1 \le i \le v)$ is a start vertex on triangular mesh of face, $j(1 \le j \le v)$ is the adjacency vertex of i, A_i is the mass of vertex i based on barycentric method, α and β are relative angles of the edge formed by i and j.

The matrix L can be obtained by combining the discrete Laplacian formulas at v different vertices.

$$L = A^{-1} L_C \tag{6}$$

A is a mass matrix composed by A_i, L_C is cotangent weight matrix. Substitute formula (6) into formula (4) to get formula (7).

$$(A - h_t L_C) H = \delta \tag{7}$$

Heat flow diffusion time h_t we use is five times the mean value of the sum of double areas of all meshes, δ is the column vector which is one at the source vertex o and zero at the other vertices. Parameter H can be acquired by solving formula (7), it can be taken as heat value on every vertex of facial model.

Then the gradient of scalar field of facial surface is computed according to formula (8).

$$\nabla H = \frac{1}{2A_f} \sum_i H_i (\mathbf{N_i} \times \mathbf{e_i}) \tag{8}$$

where A_f is the area of the face, $\mathbf{N_i}$ is the unit normal vector of the adjacency mesh of vertex i. $\mathbf{e_i}$ is a vector composed of two vertices except vertex i. Then the divergence of scalar field of facial model is solved according to formula (9).

$$\nabla \cdot X = \frac{1}{2} \sum_j cot\theta_1 (\mathbf{e_1} \cdot \mathbf{X_j}) + cot\theta_2 (\mathbf{e_2} \cdot \mathbf{X_j}) \tag{9}$$

Vector $\mathbf{e_1}$ and $\mathbf{e_2}$ is from vertex i to other two vertices in triangular mesh respectively. θ_1 and θ_2 are the angles of two vertices except vertex i, $\mathbf{X_j}$ is the gradient of the adjacent triangular mesh of vertex i. The definitions of parameters in cotangent weight matrix, gradient and divergence are shown in Fig. 3. The final geodesic distance can be obtained by solving Poisson equation:

$$L_C \phi = b \tag{10}$$

b is divergence solved by formula (9), $\phi - \phi_o$ is geodesic distance.

Geodesic paths from the source point o to the n target boundary points on 3D facial surface can be obtained by computed geodesic distance utilizing inverse gradient method. Then the geodesic grid on 3D craniofacial surface is established by the geodesics and iso-geodesics according to the method in Section 3.1.

Considering that geodesic is intrinsic and geodesic distance is preserved under isometric deformation, the geodesic grid points calculated by geodesics and iso-geodesics can encode the face model effectively. Through the angle division map to encode the craniofacial geometry, the feature points can be marked automatically without too much manpower. And the geodesic grid points can encode facial geometry fully.

3.3. Generating facial geodesic grid of the user-specified skull by PLSR

As a popular method for multivariate statistical data analysis, partial least squares regression (PLSR) is a regression method of

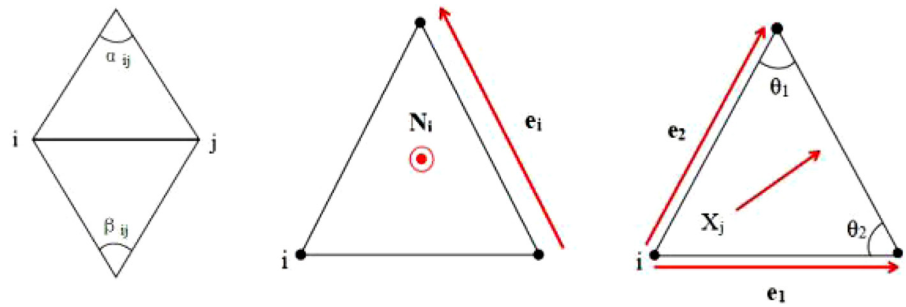


Fig. 3. The definition of parameters in cotangent weight matrix, the gradient and the divergence are illustrated from left to right.

multiple dependent variables to multiple independent variables. It combines principal component analysis (PCA), multivariate linear regression and canonical correlation analysis. PLSR is an improvement of the least square method, it can obtain relations between multiple independent variables and multiple dependent variables and make an effective prediction.

We apply PLSR for generating facial geodesic grid of the user-specified skull. Suppose the data of skull is $X = (x_1, x_2, ..., x_w)$, the data of geodesic grid is $Y = (y_1, y_2, ..., y_w)$, w is the number of training skull data or training geodesic grid data. Dimension-reduced principal component f_1 and g_1 corresponding to X and Y can be obtained by PCA.

$$f_1 = Xr_1 \tag{11}$$

$$g_1 = Ys_1 \tag{12}$$

r_1 and s_1 are eigenvectors corresponding to max eigenvalues of X^TYY^TX and Y^TXX^TY respectively. Update regression equation and continue the calculation after the first principal components corresponding to X and Y are obtained.

$$X = f_1 d_1^T + X_1 \tag{13}$$

$$Y = f_1 t_1^T + Y_1 \tag{14}$$

d_1 and t_1 are regression coefficients, the calculation formula is as follows.

$$d_1 = \frac{X^T f_1}{||f_1||^2} \tag{15}$$

$$t_1 = \frac{Y^T f_1}{||f_1||^2} \tag{16}$$

Repeat several times, the regression coefficient R can be solved.

$$R = X \sum_{i=1}^{e} r_i t_i^T \tag{17}$$

The parameter e can be determined by cross validity analysis. Then the intermediate variable O can be calculated as follows:

$$O = R(X_t - \overline{X})P_s \tag{18}$$

X_t is the data of target skull, $\overline{X}$ is the average of all training skull data, P_s is the principal component of training skull data. Finally, we can generate the facial geodesic grid of the target skull G_t as follows:

$$G_t = P_g O + \overline{G} \tag{19}$$

P_g is the principal component of training geodesic grid data, $\overline{G}$ is the average of all training geodesic grid data. Now the facial geodesic grid prediction of the target skull is finished by PLSR.

3.4. Reconstructing face by utilizing face statistical model

Based on the generated facial geodesic grid of the target skull, we reconstruct target face according to combine face statistical model. The first step is to build face statistical model and the second step is to estimate the face of the target skull by the generated geodesic grid and face statistical model. Finally, craniofacial reconstruction will be finished.

Face statistical model is constructed by the training facial data utilizing principal component analysis (PCA). Suppose there are p training facial data $F = (F_1, F_2, ..., F_p)$, F is the set of training facial data. First, we calculate covariance matrix C_M.

$$C_M = \frac{1}{p} \sum_{s=1}^{p} (F_s - \overline{F})(F_s - \overline{F})^T \tag{20}$$

$$\overline{F} = \frac{1}{p} \sum_{s=1}^{p} F_s \tag{21}$$

$\overline{F}$ is the average value of p training facial data.

Then compute the first g eigenvalues $\lambda = (\lambda_1, \lambda_2, ..., \lambda_g)$ and eigenvectors $\mathbf{U} = (\mathbf{U_1}, \mathbf{U_2}, ..., \mathbf{U_g})$ of C_M to compose corresponding matrix respectively. So the face statistical model F_μ can be computed by utilizing of principal component P.

$$P = (F_1 - \overline{F}, F_2 - \overline{F}, ..., F_p - \overline{F})\mathbf{U_i}, 1 \le i \le g \tag{22}$$

$$F_\mu = \overline{F} + \sum_{i=1}^{g} \mu_i P_i \tag{23}$$

$\mu = (\mu_1, \mu_2, ..., \mu_g)$ is model parameter which obeys Gaussian distribution.

The second step is to estimate the face of the target skull by utilizing the generated geodesic grid. In fact, it is to estimate the parameter of face statistical model by the generated geodesic grid. Firstly, the reconstructed face model is initialized as an average face. Then, the new geodesic grid G_n on the face can be calculated with G_t by iterative closest point method. Next, the coefficient μ of face statistical model can be calculate by the formula (24).

$$\mu = (P_f^T P_f)^{-1} P_f^T (G_t - G_n) \tag{24}$$

P_f is the corresponding part of geodesic points from all principal components of training facial data. Finally, the reconstruction face $\acute{f}$ can be computed according to formula (25) by introducing formula (24) into formula (23).

$$\acute{f} = \overline{F} + P^T \mu \tag{25}$$

4. Experimental results and analysis

In experiments, we firstly compute geodesic distance, extract geodesic path based on heat flow method for establishing geodesic

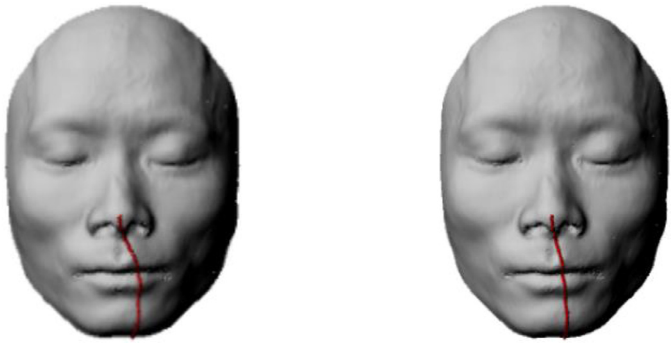


Fig. 4. Comparison of geodesic smoothness calculated by FMM (left) and heat flow method (right).

Table 1
The average errors of 213 craniofacial data by three craniofacial reconstruction methods: HF-GGR method, FMM-GR method and classical PCA method.

Geometry error	HF-GGR	FMM-GR	PCA
Min error	0.00000058	0.00000079	0.00254907
Mean error	0.01420848	0.01474215	0.05528382
Max error	0.05656939	0.06039514	0.15897597

grid. Then we establish face statistical model and utilize generated geodesic grid for craniofacial reconstruction. PLSR is not utilized for reconstructing face directly, but utilized for generate facial geodesic grid of target skull. The final craniofacial reconstruction results can be obtained by utilizing face statistical model and generated geodesic grid. Parameters of PLSR are adjusted for getting better geodesic grid. In our experiment, the number of training craniofacial data $N = 212$, the facial surface of each skull is reconstructed by Leave-one-out method, the total number of craniofacial data used is 213. The geometric errors are calculated by comparing reconstruction results with the original data. Our method is compared with classical PCA [38] craniofacial reconstruction and FMM geodesic regression (FMM-GR) model [39] craniofacial reconstruction.

The reconstruction experiments were performed on HP OMEN 5 air notebook, Intel(R) Core(TM) i7-9750H CPU @ 2.60GHz and 16G memory platform, and the system is Windows 10.

4.1. Comparison of geodesic smoothness

In the process of establishing geodesic grid for craniofacial reconstruction, the smoothness of geodesics is more important than the accuracy. In the approximate geodesic algorithm, we have conducted experiments on comparing the smoothness of geodesic based on heat flow and FMM. The experimental results show that compared with FMM, the extracted geodesic by heat flow method is smoother than FMM (Fig. 4). This smoother result will be benefit for representing facial features and improving the similarity of reconstruction results, so it is more suitable for craniofacial reconstruction.

4.2. Comparison of reconstruction accuracy

We compare craniofacial reconstruction accuracy based on HF-GRM, FMM-GR and classical PCA method. Euclidean distance is utilized to compute the maximum, minimum and mean geometric errors between the reconstructed face and the original face. The average reconstruction errors on 213 reconstruction faces of three methods by Leave-one-out method are listed in Table 1.

It can be seen from Table 1 that in terms of reconstruction errors, the craniofacial reconstruction results based on HF-GGR is dramatically improved compared with classical PCA reconstruction method. Craniofacial reconstruction methods based on HF-GGR and FMM-GR can both get better reconstruction results than PCA method. It means that the application of geodesics will be benefited for improving the reconstruction accuracy. Compared with craniofacial reconstruction based on FMM-GR, the errors of HF-GGR reconstruction results are close. However, in our geodesic grid regression model of heat flow method, fewer geodesic grid points (600 points in our experiments) are utilized and 12000 geodesic points are utilized in FMM-GR, that is to say, our method can get similar(even better) reconstruction results by fewer points of geodesic grid.

Mean error can represent the overall level of reconstruction results. So take Fig. 5 and Fig. 6 for example, the mean error of each data in our method is smaller than that of the classical PCA craniofacial reconstruction method. On the one hand, the accuracy of our method is 3.89 times higher than that of classical PCA reconstruction method. It reflects that our method is better than the classical PCA method because geodesics are used. On the other hand, the craniofacial reconstruction mean error based on HF-GGR is close to FMM-GR method, even be lower. HF-GGR method utilizes far less points than FMM-GR, it means using fewer points does not increase the mean error, on the contrary, we can get the lower mean error results through our HF-GGR method. It not only shows the advantage (utilize less points but can get better results) of our method, but also shows good performance of applying geodesic in craniofacial reconstruction.

In addition, we conducted the experiments on HF-GGR method utilizing 12000 geodesic grid points to explore the influence of different number grid points. We show the reconstruction results and the mean error under different number of points in supplementary material. Increase of geodesic grid points as feature points will not necessarily improve the accuracy of the reconstruction results.

The partial reconstruction results and corresponding error charts of the three methods are shown in Table 2. The craniofacial reconstruction results based on HF-GGR and FMM-GR are close, and the facial boundary is similar to original data. But some details of the face are not fully reconstructed, especially the mouth. The mouth part of some reconstruction results is blurry. The results of classical PCA reconstruction method is not good, although error chart shows good, many reconstruction results are not similar to the original face. Obvious differences can be seen in the boundary part of results compared with original face. Therefore, from the subjective perspective of human beings, the reconstruction results of our method are better, such as the ninth and tenth reconstruction results in Table 2.

4.3. Comparison of reconstruction time

Compared with PCA method (Fig. 9, Table 5), whether it's single time of each data or the average time of all data, the running time of craniofacial reconstruction method based on HF-GGR is less than classical PCA craniofacial reconstruction method. Even though HF-GGR method is complicated, it requires far less points than classical PCA method, classical PCA reconstruction method requires to input all the points of training craniofacial data. Therefore, HF-GGR method needs less time. Compared with reconstruction method based on FMM-GR (Table 4, Table 5, Fig. 7, Fig. 8), the running time is also less whatever reconstruction face time or building geodesic model time because of fewer points we utilized in our method. In craniofacial reconstruction method based on FMM-GR, we input all the points of facial model but only utilize 12,000 points. In reconstruction method based on HF-GGR, we utilize fewer points (600 points). So the running time of craniofacial reconstruction utilizing HF-GGR is much less than that of other two reconstruction methods. In general, our

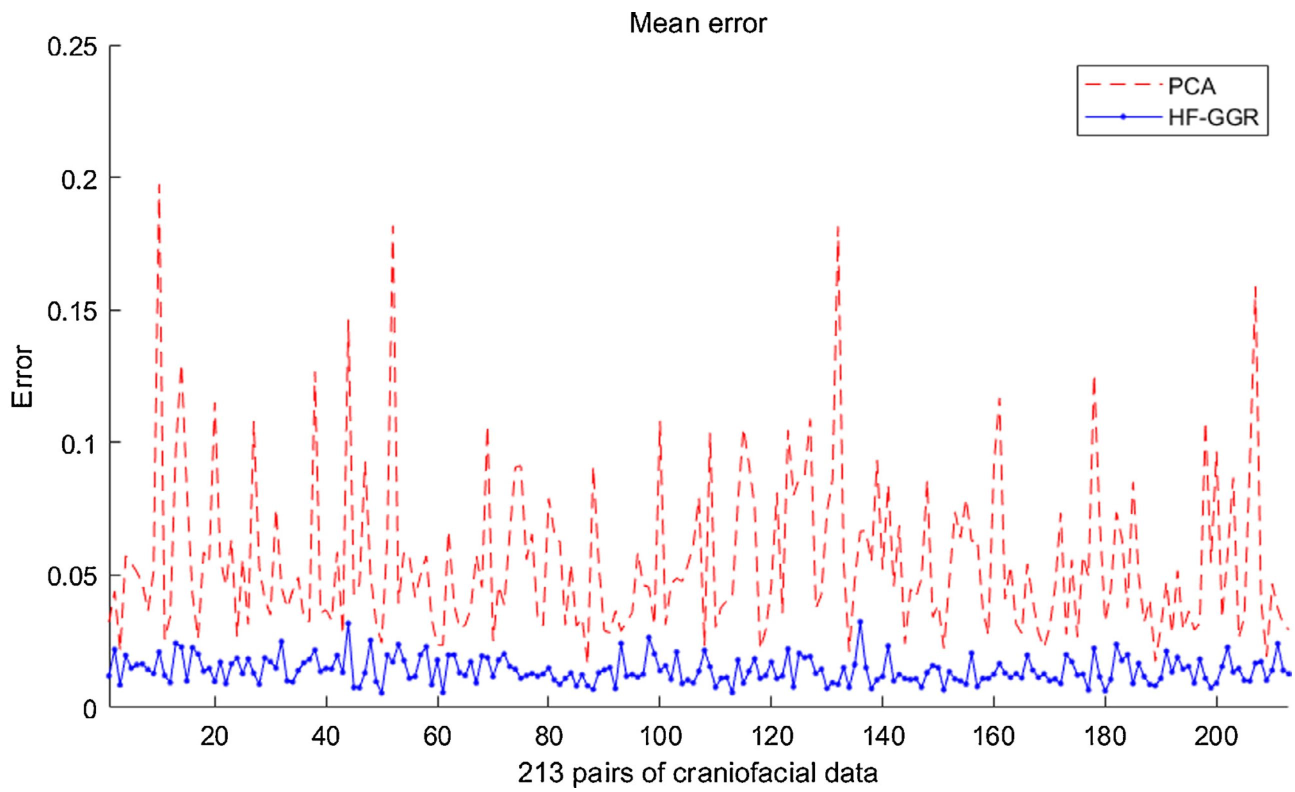


Fig. 5. Mean error of HF-GGR and PCA

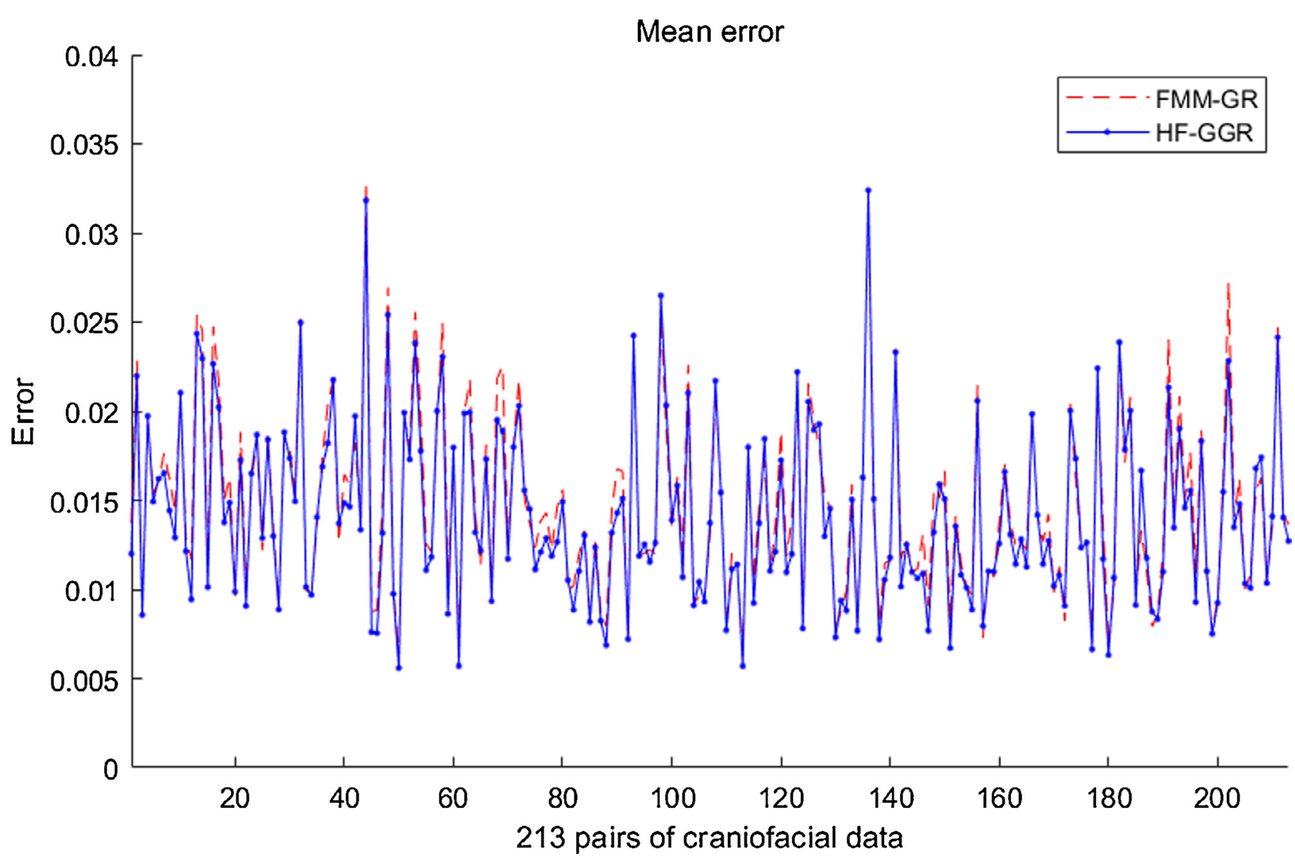
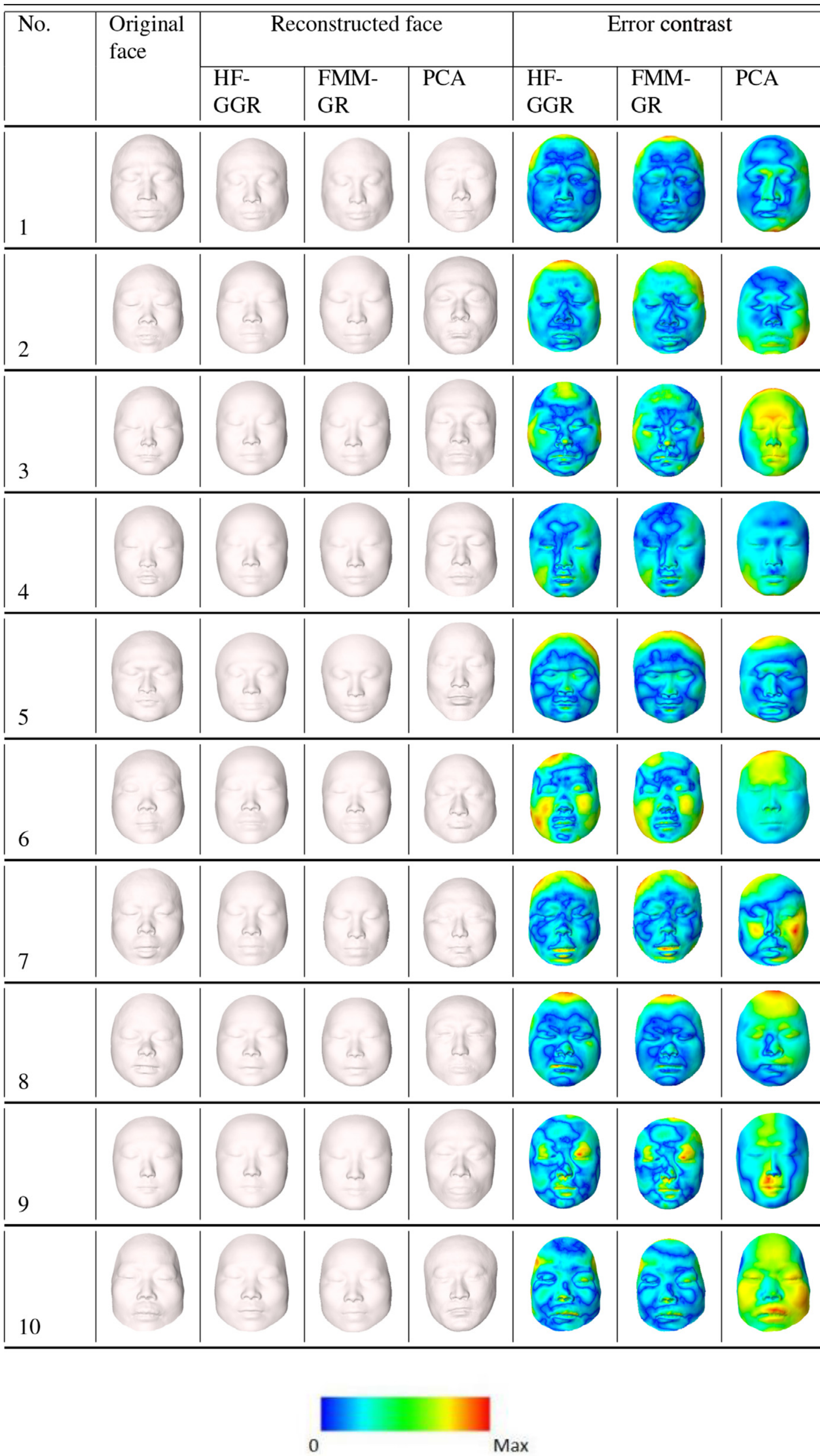


Fig. 6. Mean error of HF-GGR and FMM-GR

Table 2
Craniofacial reconstruction results and error comparisons by HF-GGR method, FMM-GR method and classical PCA method.



method is 1.30 times faster than the classical PCA reconstruction method and 1.33 times faster than FMM-GR reconstruction method.

Compared with classic craniofacial reconstruction method, we utilize less number of geodesic grid points for reconstructing, the reconstructed speed is improved. The fewer geodesic grid points does not lead to the increase of error, but can get reconstruction results remaining comparable (even better) accuracy.

Table 3
The different training facial data in HF-GGR method, FMM-GR method and classical PCA method.

Method	Training data	Number of the data
HF-GGR	Geodesic grid points	600 points
FMM-GR	Geodesics	60 geodesics
PCA	Whole face	40,969 points

Table 4
The average time of generating geodesic.

Method	HF-GGR	FMM-GR
Average generating time	0.6028 s	0.9490 s

Table 5
The average reconstruction time.

Method	HF-GGR	FMM-GR	PCA
Reconstruction time	1.4247 s	1.8881 s	1.8541 s

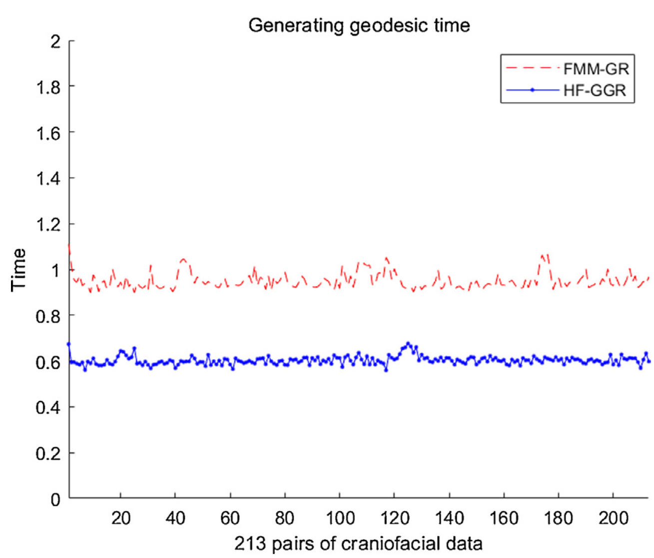


Fig. 7. Generating geodesic time of HF-GGR and FMM-GR.

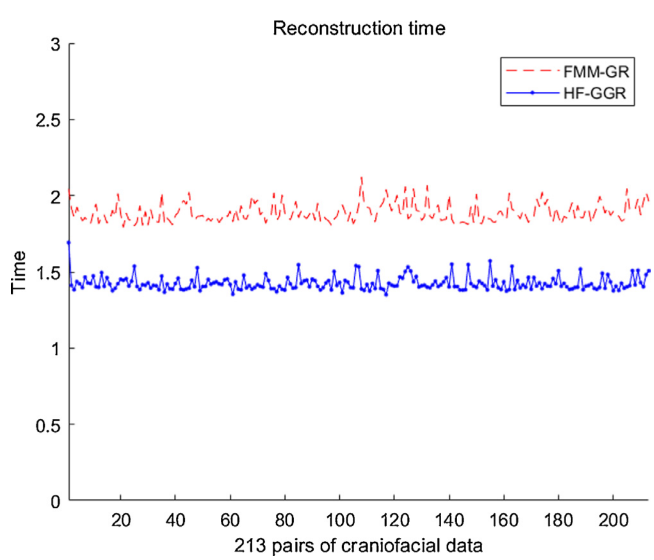


Fig. 8. Reconstruction time of HF-GGR and FMM-GR.

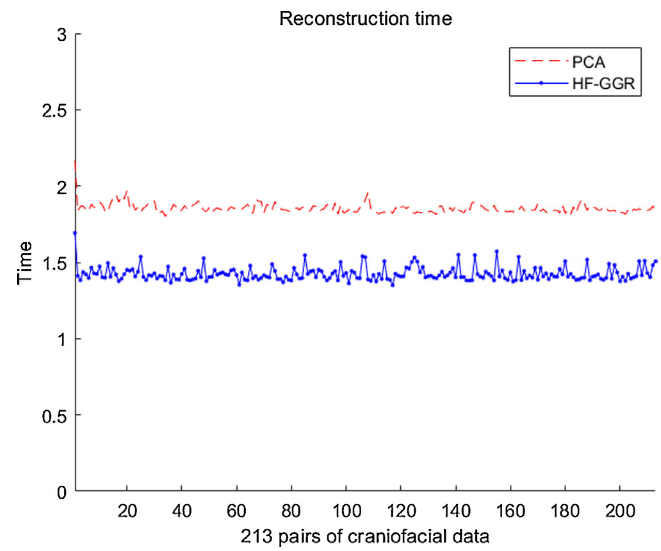


Fig. 9. Reconstruction time of HF-GGR and PCA.

4.4. Comparison of reconstruction data

HF-GGR method, FMM-GR method and classical PCA method are statistical-based craniofacial reconstruction methods, training data is crucial. Therefore, we compare three methods from the view of training data.All these three reconstruction methods use 212 pairs craniofacial data for training,and Leave-one-out method for testing. But in the algorithm level, the form of training facial data used in the three methods are different. Classical PCA method utilizes the whole faces for training data, each face sample 40,969 points in experiments. FMM-GR method uses geodesics of facial surfaces as training data, each geodesic sample 200 points and totally 12,000 points on 60 geodesics of each face in experiments. HF-GGR method use geodesic grid on a face as training data, only 600 geodesic grid points are sampled in experiments. It can be seen in Table 3.

The final experimental results show that, compared with FMM-GR and classical PCA method, HF-GGR method used fewer points and can get even better results. The more detailed experimental results are shown in Section 4.2 and Section 4.3.

Overall analysis shows that the application of geodesics in craniofacial reconstruction improves the accuracy of craniofacial reconstruction results. Whether FMM-GR or our HF-GGR can obtain better reconstructed face model than PCA method. What's more, in our HF-GGR method, we utilize less number of geodesic grid points to obtain comparable reconstruction accuracy. It benefits from applying heat flow algorithm for calculating geodesics in our method, which can calculate smooth geodesics and geodesic grid. At the same time, due to less number of geodesic grid points used in HF-GGR, our method run faster than the other two methods.

5. Conclusion

In this paper, a craniofacial reconstruction based on heat flow geodesic grid regression model is proposed. We utilize the inherent advantage of geodesic to encode craniofacial geometry automatically. And facilitated by the heat flow method, the calculated geodesic distance, geodesic path for establishing geodesic grid are smoother than calculated by FMM. PLSR is not utilized for reconstructing facial surface directly, but be utilized for generating geodesic grid. With the help of geodesic grid regression model, the facial surfaces of target skulls are reconstructed by Leave-one-out method, and the geometric error and running time is analyzed. We utilize less number points of geodesic grid for regression model, so there are less time cost while not increase the error. Our method effectively encode craniofacial geometric feature automatically and improves the accuracy. Compared with other methods, HF-GGR method can reduce the time cost appropriately.

In the future, we consider to utilize QGDF method for building new geodesic grid regression model, which can allow user to adjust between accuracy and smoothness for achieving the desirable craniofacial reconstruction results by user-specified requires. In addition, we will optimize our method by the parallel method [34] to calculate the geodesic distance for improving running speed.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Bin Jia: Methodology, Software, Validation, Writing - original draft. **Junli Zhao:** Methodology, Conceptualization, Investigation, Writing - review & editing. **Shiqing Xin:** Methodology, Writing - review & editing. **Fuqing Duan:** Methodology, Supervision, Writing - review & editing. **Zhongke Wu:** Methodology, Supervision. **Jinhua Li:** Methodology, Supervision. **Mingquan Zhou:** Data curation.

Acknowledgments

The authors gratefully appreciated the anonymous reviewers for all of their helpful comments. This work was supported by the National Key R&D Program of China under Grant No. 2019YFC1521103, the National Natural Science Foundation of China under Grant Nos. 61702293, 11472144, 61772016 61772294, 61572078, the National Statistical Science Research Project (No.2020355) and Development Plan - Major Scientific and Technological Innovation Projects of Shandong Province (No.2019JZZY02010), and the Smart Computing Joint Foundation - National Natural Science Foundation of Shandong Province (No.ZR2019LZH002). We thank the support of Xianyang Hospital for providing CT images.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.cag.2021.04.029.

References

[1] Bulut O, Sipahioglu S, Hekimoglu B. Facial soft tissue thickness database for craniofacial reconstruction in the turkish adult population. Forensic Sci Int 2014;242:44–61.
[2] Shui W, Zhou M, Deng Q, Wu Z, Ji Y, Li K, et al. Densely calculated facial soft tissue thickness for craniofacial reconstruction in chinese adults. Forensic Sci Int 2016;266:573.e1–573.e12.
[3] Shrimpton S, Daniels K, De Greef S, Tilotta F, Willems G, Vandermeulen D, et al. A spatially-dense regression study of facial form and tissue depth: towards an interactive tool for craniofacial reconstruction. Forensic Sci Int 2014;234:103–110.
[4] Tilotta F, Richard F, Glaunès J, Berar M, Gey S, Verdeille S, et al. Construction and analysis of a head ct-scan database for craniofacial reconstruction. Forensic Sci Int 2009;191:112.e1–112.e12.
[5] De Donno A, Sablone S, Lauretti C, Mele F, Martini A, Introna F, et al. Facial approximation: soft tissue thickness values for caucasian males using cone beam computer tomography. Legal Med 2019;37:49–53.
[6] Gietzen T, Brylka R, Achenbach J, Zum Hebel K, Schömer E, Botsch M, et al. A method for automatic forensic facial reconstruction based on dense statistics of soft tissue thickness. PloS One 2019;14:e0210257.
[7] Claes P, Vandermeulen D, De Greef S, Willems G, Suetens P. Craniofacial reconstruction using a combined statistical model of face shape and soft tissue depths: methodology and validation. Forensic Sci Int 2006;159:S147–S158.
[8] Paysan P, Lüthi M, Albrecht T, Lerch A, Amberg B, Santini F, et al. Face reconstruction from skull shapes and physical attributes. In: Joint pattern recognition symposium; 2009. p. 232–241.
[9] Claes P, Vandermeulen D, De Greef S, Willems G, Clement JG, Suetens P. Bayesian estimation of optimal craniofacial reconstructions. Forensic Sci Int 2010;201:146–152.
[10] Xiao Z, Zhao J, Qiao X, Duan F. Craniofacial reconstruction using gaussian process latent variable models. In: International conference on computer analysis of images and patterns; 2015. p. 456–464.
[11] Zhang Y-f, Zhou M-q, Geng G-h, Feng J. Face appearance reconstruction based on a regional statistical craniofacial model (rcsm). In: 2010 20th international conference on pattern recognition; 2010. p. 1670–1673.
[12] Hu Y, Duan F, Yin B, Zhou M, Sun Y, Wu Z, et al. A hierarchical dense deformable model for 3d face reconstruction from skull. Multim Tools Appl 2013;64:345–364.
[13] Madsen D, Lüthi M, Schneider A, Vetter T. Probabilistic joint face-skull modelling for facial reconstruction. In: Proceedings of the IEEE conference on computer vision and pattern recognition; 2018. p. 5295–5303.
[14] Shui W, Zhou M, Maddock S, Ji Y, Deng Q, Li K, et al. A computerized craniofacial reconstruction method for an unidentified skull based on statistical shape models. Multim Tools Appl 2020;79:25589–25611.
[15] Wold S, Martens H, Wold H. The multivariate calibration problem in chemistry solved by the pls method. In: Matrix pencils; 1983. p. 286–293.
[16] Duan F, Yang S, Huang D, Hu Y, Wu Z, Zhou M. Craniofacial reconstruction based on multi-linear subspace analysis. Multim Tools Appl 2014;73:809–823.
[17] Duan F, Huang D, Tian Y, Lu K, Wu Z, Zhou M. 3d face reconstruction from skull by regression modeling in shape parameter spaces. Neurocomputing 2015;151:674–682.
[18] Huang D, Duan F, Deng Q, Wu Z, Zhou M. Face reconstruction from skull based on partial least squares regression. In: 2011 seventh international conference on computational intelligence and security; 2011. p. 1118–1121.
[19] He Y, Gao N, Ru S, Yang M. A local modeling approach of craniofacial shape correlation based on plsr. In: 2017 2nd international conference on image, vision and computing (ICIVC); 2017. p. 545–549.
[20] Li Y, Chang L, Qiao X, Liu R, Duan F. Craniofacial reconstruction based on least square support vector regression. In: 2014 IEEE international conference on systems, man, and cybernetics (SMC); 2014. p. 1147–1151.
[21] Deng Q, Zhou M, Wu Z, Shui W, Ji Y, Wang X, et al. A regional method for craniofacial reconstruction based on coordinate adjustments and a new fusion strategy. Forensic Sci Int 2016;259:19–31.
[22] Zhao J, Pan Z, Duan F, Lv Z, Li J, Zhou Q, et al. A survey on 3d face recognition based on geodesics. J Inf Hiding Multim Signal Process 2019a:368–383.
[23] Dijkstra EW, et al. A note on two problems in connexion with graphs. Numer Math 1959;1:269–271.
[24] Sharir M, Schorr A. On shortest paths in polyhedral spaces. SIAM J Comput 1986;15:193–215.
[25] Mount DM. On finding shortest paths on convex polyhedra. MARYLAND UNIV COLLEGE PARK CENTER FOR AUTOMATION RESEARCH; 1985.
[26] Mitchell JS, Mount DM, Papadimitriou CH. The discrete geodesic problem. SIAM J Comput 1987;16:647–668.
[27] Chen J, Han Y. Shortest paths on a polyhedron. In: Proceedings of the sixth annual symposium on computational geometry; 1990. p. 360–369.
[28] Xin S-Q, Wang G-J. Applying the improved chen and hans algorithm to different versions of shortest path problems on a polyhedral surface. Comput-Aided Des 2010;42:942–951.
[29] Xu C, Wang TY, Liu Y-J, Liu L, He Y. Fast wavefront propagation (fwp) for computing exact geodesic distances on meshes. IEEE Trans Vis Comput Graph 2015;21:822–834.
[30] Sethian JA. A fast marching level set method for monotonically advancing fronts. Proc Natl Acad Sci 1996;93:1591–1595.
[31] Crane K, Weischedel C, Wardetzky M. Geodesics in heat: a new approach to computing distance based on heat flow. ACM Trans Graph (TOG) 2013;32:1–11.
[32] Ying X, Wang X, He Y. Saddle vertex graph (svg) a novel solution to the discrete geodesic problem. ACM Trans Graph (TOG) 2013;32:1–12.
[33] Cao L, Zhao J, Xu J, Chen S, Liu G, Xin S, et al. Computing smooth quasi–geodesic distance field (qgdf) with quadratic programming. Comput-Aided Des 2020;127:102879.
[34] Tao J, Zhang J, Deng B, Fang Z, Peng Y, He Y. Parallel and scalable heat methods for geodesic distance computation. IEEE Trans Pattern Anal Mach Intell 2021;43:579–594.
[35] Lorensen WE, Cline HE. Marching cubes: A high resolution 3d surface construction algorithm. ACM Siggraph Comput Graph 1987;21:163–169.
[36] Zhao J, Qi X, Wen C, Lei N, Gu X. Automatic and robust skull registration based on discrete uniformization. In: Proceedings of the IEEE international conference on computer vision; 2019b. p. 431–440.
[37] Huang R, Zhao J, Duan F, Li X, Liu C, Deng X, et al. Automatic craniofacial registration based on radial curves. Comput Graph 2019;82:264–274.
[38] Desvignes M, Bailly G, Payan Y, et al. 3D semi-landmarks based statistical face reconstruction. J Comput Inf Tech 2006;14(1):31–43.
[39] Chen Z, Zhao J, Yu H, Pan Z, Jia B, Li J. Craniofacial Reconstruction Based on Geodesic Regression Model. J Comput Aided Des Comput Graph 2021;33(3):395–404.