Warm Starting Series of Mixed-Integer Linear Programs

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Overview

1. Why care about series of MILP's?
2. What opportunity is there to more efficiently solve series of MILP's?
3. How to demonstrate efficiency is improved?

Motivation

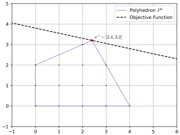
- Mixed-Integer Programming has created great value in industry.
- One important source comes from problems solved by a series of Mixed-Integer Linear Programs (MILPs).
  - Electric Grid Production Planning (Stochastic Dual Decomposition)
  - Vehicle Routing (Branch and Price)
- Many MILP instances in such series differ only by objective coefficients or bounds on their constraints.
- Solvers can leverage what they discover solving one MILP to more quickly solve a similar MILP (a.k.a. "Warm Start").
- Warm starting would enable greater performance and expand the space of tractable problems for those solved as a series of MILPs.

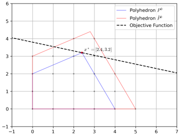
More problems are solved as series of MILP's than meet the eye. Given their economic importance, it is worth understanding how to solve them efficiently.

Simple Example

$$\max\{x + 4y : -\frac{x}{2} + y \leq 2, \quad x + \frac{y}{2} \leq 4, (x, y) \in \mathbb{Z}_+^2\} \tag{1}$$

$$\max\{x + 4y : -\frac{x}{2} + y \leq 3, \quad x + \frac{y}{2} \leq 5, (x, y) \in \mathbb{Z}_+^2\} \tag{2}$$





Update bounds $\Longrightarrow$

Branch on $x \leq 2$ or $x \geq 3$

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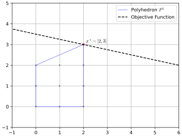


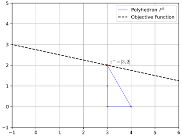
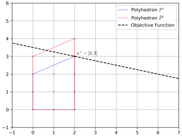


Figure 1: (1)'s Branch and Bound tree



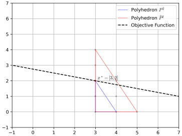


Figure 2: (2)'s Branch and Bound tree

After solving (1), we can get a tree for (2) without another solve by substituting (2)'s bounds in each subproblem of (1).

A Better Warm-Starting Approach

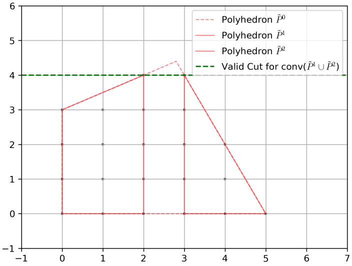


Figure 3: A cut valid for both of (2)'s subproblems

- When warm-starting, we would like to avoid:
  - Reprocessing each terminal node.
  - Refining the feasible region far from the solution.
- We want to find cuts such that:
  - No terminal subproblem is violated.
  - Some estimate of the solution is maximally violated.
- [1] provides an algorithm to find such cuts.

My first paper will augment this algorithm to become a cutting plane technique tightening the initial LP relaxation of MILP's in a series.

Branch and Price

- Branch and Price algorithms solve a Dantzig-Wolfe reformulation of the LP relaxation in each node.
- Dantzig-Wolfe relies on column generation to produce partially integer feasible solutions.
- The column generation subproblem is a series of MILP's.
- One of the most important applications is solving the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW).
- The CVRPTW could benefit from warm-starting by:
    - Outperfoming current dynamic programming subproblem formulations.
    - Providing an efficient way to model side constraints on routes.

My second paper will demonstrate my methodology's effectiveness at improving run time and expanding modeling richness for the CVRPTW.

Dual Decomposition for Stochastic MILP

- Dual Decomposition algorithms break stochastic MILP's into multiple independent deterministic MILP's.
- Each MILP is solved repeatedly with perturbed objective and constraint bounds
- One of the most important applications is solving the Two Stage Stochastic Unit Commitment (TSSUC).
- The TSSUC could benefit from warm-starting by outperfoming cold-started Lagrangian Dual formulations.

My third paper will demonstrate my methodology's effectiveness at improving run time for the TSSUC.

Conclusion

My dissertation will address the following research gaps:

- Overcoming current obstacles when warm-starting series of MILP's in general.
- The lack of warm-starting applied problems.

My dissertation will yield:

- A new methodology for warm-starting series of MILP's.
- Demonstrations of how to apply it to important real-world problems.
- Proof of its effectiveness in improving how such problems are solved.

Status

Current Items Completed:

- Theory for disjunctive cuts
- Plan for future research
- Disjunctive cuts implemented
- Numerical experiments implemented

Items to Complete This Semester

- Clarify disjunctive cut theory
- Add first round of numerical experiments to paper
- Convert paper to poster

These deliverables will frame my remaining research work at Lehigh and enable me to quickly assemble presentation materials for conferences.