Combinatorics HW 1.1

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1. A large tournament has 569 entrants in total. If it is a single elimination tournament, how many matches have to be played out before the champion can be decided? (Please calculate the precise value)

   One person is knocked out of the tournament every match played. 568 people need to be knocked out in order for there to be a single winner. Therefore, 568 matches need to be played.

2. The figure below shows a partial 4X4 matrix，is there some way of filling up the rest of the omitted entries to produce a magic square of size 4？

$$\begin{bmatrix} 2 & 3 & w & x \\ 4 & & & \\ y & & & \\ z & & & \end{bmatrix}$$

   Consider the sum of the top row and the sum of the left-most column. Since $M = \frac{4(4^2+1)}{2} = 34$, this gives us two equations

$$w + x = 29$$
$$y + z = 28$$

   All numbers in the magic square must be between 1 and 16, so we must have

$$w, x \in \{13, \dots, 16\}, \ \ y, z \in \{12, \dots, 16\}$$

   Note that w, x, y, z are unique. Now consider the two pairs (w, x), (y, z). Clearly $(w, x) = (x, w)$ and $(y, z) = (z, y)$, so it must be that

$$(w, x) \in \{(13,16), (14,15)\}$$
$$(y, z) \in \{(12,16), (13,15)\}$$

   This quickly shows that the above magic square is ***not possible*** as choosing either option for (w, x) results in one of (y, z) being equal to one of (w, x). That is, if we have $w = 13, x = 16, y = 12$ we get $z = 16 = x$. Similarly, $w = 14, x = 15, y = 12$ means $z = 16 = x$. Clearly, the final two options lead to $z = y$. Hence it is not possible to create the desired magic square.