Combinatorics HW 2.1

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1. How many different permutations for word"Combinatorics"? (Case sensitive)

   11 distinct elements, with o and i repeated twice. Thus the number of unique permutations is $\frac{13!}{2! \times 2!}$.

2. The coefficient number of $a^2 b^2 c^2$ in the expanded equation of $(2a + b + c)^6$ is 360.

   The coefficient of $4a^2 (b + c)^4$ is $C(6,4)$, and the coefficient of $b^2 c^2$ is $C(4,2)$. Hence the coefficient of $a^2 b^2 c^2$ is $4 \times C(6,4) \times C(4,2) = 360$.

3. For the case of giving fruits to 3 kids, in total there are 12 identical apples, each child may at least have one apple, how many different ways to give apples to 3 kids?

   Line up the 12 apples and take two separators. There are 11 gaps between the apples, and we must choose two of the gaps to put the separators in. Thus the number of different ways is $C(11,2) = 55$.

4. What is the number of integral solutions of the equation $x_1+x_2+x_3=30$, in which $x_1 \geqslant 5$, $x_2 \geqslant -8$, $x_3 \geqslant 5$.

   Let $y_1 = x_1 - 5$, $y_2 = x_2 + 8$, $y_3 = x_3 - 5$. Then the equation becomes $y_1 + y_2 + y_3 = 28$, which has $C(28 + 3 - 1,2) = C(30,2)$ solutions.