Combinatorics HW 1.2

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1. How many odd numbers between 1000 and 9999 whose digits are distinct with each other?

   Represent the number as $abcd$ where $a \in \{1, \dots, 9\}$, $b \in \{0, \dots 9\}$, $c \in \{0, \dots, 9\}$, $d \in \{1,3,5,7,9\}$. There are $\boldsymbol{5}$ options for d. This means there are then $\boldsymbol{8}$ options for a, since one of the digits is already used in d. There are then $10 - 2 = \mathbf{8}$ options for b, and $10 - 3 = \mathbf{7}$ options for c. Thus the number of distinct odd numbers is $7 \times 8 \times 8 \times 5 = 2240$.

2. How many 7-digit numbers are there such that the digits are distinct integers taken from {1, 2, …, 9} and such that the digits 5 and 6 do not appear consecutively in either order?

   Number of ways to choose a 7-digit number with distinct digits is $P(9,7)$. The number of ways to choose a seven digit number where 5 and 6 are adjacent is $6 \times P(7,5)$. We can also have 65 rather than 56, so we multiply this by two. Thus the result is $P(9,7) - P(7,5) \times 6 \times 2 = 151200$.

3. How many different lattice paths from (-1,1) to (5,4)?

   A total of $5 - (-1) = 6$ steps need to be taken in the x direction, and $4 - 1 = 3$ steps need to be taken in the y direction. Thus the number of different lattice paths is $C(6 + 3,6) = 84$.

4. How many non-repeating 8-strings such that *a* and *b* are not adjacent can be formed with 26 English letters?
   Please explain the calculation in detail.

   Let ζ be the letters a and b next to one another, in that order. ζ can go in 7 positions in the 8 string. Since the string is nonrepeating, there are $P(24,6)$ ways the other letters could fill out the string. We multiply the result by two to take into account a and b next to one another, but in the opposite order. This gives us $P(24,6) \times 7 \times 2$ 8-strings where a and b ***are*** adjacent to one another. The total number of non-repeating 8-strings is just $P(26,8)$, hence our result is $P(26,8) - P(24,6) \times 7 \times 2 = 61634200320$.