

Generic Programming in Haskell

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What is Generic Programming?

The adjective “generic” is heavily overloaded.

- Java/C# generics
- C++ templates
- Ada generic packages

What is Generic Programming?

The goal is often the same.

A higher level of abstraction than “normally” available

The technique is also often the same.

Some form of parameterization and instantiation

Examples of Generic Programming

Java/C#:

```
public class Stack<T>
{
    public void push(T item) {...}
    public T pop() {...}
}
```

Examples of Generic Programming

C++:

```
template<typename T, typename Compare>
T& min(T& a, T& b, Compare comp) {
    if (comp(b, a))
        return b;
    return a;
}
```

Generic Programming in Haskell

In other words:

- Java-style generics \approx parametric polymorphism
- C++ templates \approx ad-hoc polymorphism

In Haskell:

- Both forms already exist.
- We don't call them generics because they're native to the language.

Datatype-generic programming:

- Abstract over the *structure of a datatype*
- Also known as “polytypism” and “shape-/structure-polymorphism”

Datatypes

```
data D p = Alt1 | Alt2 Int p
```

A datatype can have:

- **Parameters**: type variables (≥ 0)
- **Alternatives**: unique constructors (≥ 0)
- **Fields**: types for each constructor (≥ 0)

Non-syntactic features:

- Recursion
- Nesting

There are other features of datatypes, but we will consider only the above as a foundation for looking at the structure.

Structure of Datatypes: Sums

First structural element: alternatives.

```
data AltEx2 = A1 Int | A2 Char
```

Note that the above is similar to a standard type:

```
data Either a b = Left a | Right b
```

And we can, in fact, model `AltEx2` as:

```
type AltEx'2 = Either Int Char
```

with the following “smart” constructors:

```
a1 :: Int → AltEx'2  
a1 = Left
```

```
a2 :: Char → AltEx'2  
a2 = Right
```


Structure of Datatypes: Sums

When talking about alternatives in structural sense, we often call them **sums**. `Either` is the basic binary sum type. For conciseness, we use this (identical) binary sum type:

```
data a :+: b = L a | R b
```

What about a type with < 2 alternatives?

```
data AltEx3 = B1 Int | B2 Char | B3 Float
```

The simplest solution is to nest one binary sum inside another:

```
type AltEx'3 = Int :+: (Char :+: Float)
```

Note that:

$$b_3 :: \text{Float} \rightarrow \text{AltEx}'_3$$
$$b_3 = R \circ R$$

Structure of Datatypes: Products

Next: fields.

```
data FldEx2 = FldEx2 Int Char
```

Again, note the similarity to a standard type, the pair:

```
data (,) a b = (,) a b
```

And again, we model `FldEx2` similarly:

```
type FldEx'2 = (,) Int Char
```

with the smart constructor:

```
fldEx'2 :: Int → Char → FldEx'2  
fldEx'2 = (,)
```

Structure of Datatypes: Products

The pair type is the basic binary **product** type. For symmetry with sums, we will use the following type:

```
data a × b = a × b
```

And more than two fields...

```
data FldEx3 = FldEx3 Int Char Float
```

... are modeled by nested binary products:

```
type FldEx'3 = Int × (Char × Float)
```

with the smart constructor:

```
fldEx'3 :: Int → Char → Float → FldEx'3  
fldEx'3 x y z = x × (y × z)
```

Structure of Datatypes: Sums of Products

To “sum” it all up, recall the first datatype example:

```
data D p = Alt1 | Alt2 Int p
```

We can define an identical type using the sum and product types we have just discussed:

```
type RepD p = U :+: Int :×: p
```

Notes:

- We use the “unit” type `data U = U` (identical to the standard type `()`) to represent an alternative without fields.
- `:+:` is **infix** 5, and `:×:` is **infix** 6, so we can write `RepD` naturally, without unnecessary parentheses.

Structure of Datatypes: Isomorphism

So, we think we can model datatypes. But how do we know Rep_D accurately models D ?

We define an **isomorphism**: two total functions that convert between types.

```
fromD :: D p → RepD p
fromD Alt1          = L U
fromD (Alt2 i p)    = R (i :: p)

toD :: RepD p → D p
toD (L U)           = Alt1
toD (R (i :: p))   = Alt2 i p
```

This allows us to convert terms between (1) the familiar datatype and (2) the **structure representation** used for generic operations.

Structure of Datatypes: Constructors

Oh, but there's one more thing...

You may have noticed the representation lacked any information about the constructors (e.g. the names).

That's easily repaired with another datatype:

```
data C a = C String a
```

We modify the representation to store constructor names:

```
type RepD p = C U :+: C (Int ×: p)  
fromD Alt1      = L (C "Alt1" U)  
fromD (Alt2 i p) = R (C "Alt2" (i ×: p))
```

We could also put additional metadata (e.g. fixity) into C.

Generic Functions

Okay, so we have a structure representation. But what can we *do* with it?

Generic functions

- Defined on each possible case of the structure representation
- Work for every datatype that has an isomorphism with a structure representation

Example: `show :: a → String`

Generic Functions: `show`

We define a `show` function for each possible structure case.

Unit:

```
showU :: U → String  
showU U = ""
```

Constructor name:

```
showC :: (a → String) → C a → String  
showC showa (C nm a) =  
  "(" ++ nm ++ " " ++ showa a ++ ")"
```

Binary product:

```
show× :: (a → String) → (b → String) → a :× b → String  
show× showa showb (a :× b) = showa a ++ " " ++ showb b
```

Binary sum:

```
show+ :: (a → String) → (b → String) → a :+ b → String  
show+ showa _ (L a) = showa a  
show+ _ showb (R b) = showb b
```


Generic Functions: `show`

We can define a `show` function for `RepD` (assuming `showInt`):

```
showRepD :: (p → String) → RepD p → String
showRepD showp =
  show+ (showC showU) (showC (show× showInt showp))
```

The `show` function for `D` is just a hop away:

```
showD :: (p → String) → D p → String
showD showp = showRepD showp ∘ fromD
```

Generic Functions: `show`

```
showRepD :: (p → String) → RepD p → String  
showRepD showp =  
  show+ (showC showU) (showC (show× showInt showp))
```

Some observations:

- This is a sort of predictable pattern (or recipe) for defining `show` functions on structure representations.
- The functions are recursive but not in the usual way because the argument types differ.
- Each datatype can have a unique structure representation, and we want to support all combinations, *generically*.

Generic Functions, Generically

In order to jump into “true” genericity (where the structure is a parameter instead of a pattern), we need several additional things:

- **Polymorphic recursion** – functions with a common scheme that reference each other and allow types to change in the calls

```
showU ::      U      → String
showC :: ... ⇒ C a    → String
show+ :: ... ⇒ a :+: b → String
...
```

- A common encoding for isomorphisms

```
data T      = ...  -- User-defined datatype
type RepT = ...  -- Structure representation
from :: T → RepT
to    :: RepT → T
```

Polymorphic Recursion

There are several ways to encode polymorphic recursion. We will use type classes.

- Standard classes already use polymorphic recursion for deriving instances: `Show` , `Eq` , etc.
- The class declaration specifies the type signature.
- Each recursive case is specified by an instance of the class.

A simplified definition of the `Show` class:

```
class Show a where  
  show :: a → String
```

Polymorphic Recursion

The instances for each structure representation case:

Unit:

```
instance Show U where  
  show = showU
```

Constructor name:

```
instance Show a  $\Rightarrow$  Show (C a) where  
  show = showC show
```

Binary product:

```
instance (Show a, Show b)  $\Rightarrow$  Show (a  $\times$  b) where  
  show = show $\times$  show show
```

Binary sum:

```
instance (Show a, Show b)  $\Rightarrow$  Show (a  $+$  b) where  
  show = show+ show show
```

Polymorphic Recursion

Now, recall `showRepD` :

```
showRepD :: (p → String) → RepD p → String
showRepD showp =
  show+ (showC showU) (showC (show× showInt showp))
```

Compare to the new version that is now possible:

```
show'RepD :: Show p ⇒ RepD p → String
show'RepD = show
```

Encoding Isomorphisms

To define the `show` function for `D`, we still need to define another function:

$$\begin{aligned} \text{show}'_D &:: \text{Show } p \Rightarrow D \text{ } p \rightarrow \text{String} \\ \text{show}'_D &= \text{show}'_{\text{Rep}_D} \circ \text{from}_D \end{aligned}$$

Next goal:

- Define one `show` function that knows how to convert any type `T` to its structure representation type `RepT`, given an isomorphism between `T` and `RepT`.

Encoding Isomorphisms

We define a class of function pairs.

- We again use a type class, but with the addition of a *type family*.
- Each function pair implements an isomorphism between a datatype T and its structure representation Rep_T :

$\text{from} :: T \rightarrow \text{Rep}_T$

$\text{to} :: \text{Rep}_T \rightarrow T$

- Each requires two types, so each instance must have two types (unlike the `Show` instances which needed only the structure representation type).
- Rep_T is precisely determined by T , so really we only need one unique type and a second type derivable from the first.
- In this case, a (1) multiparameter type class with a functional dependency and a (2) type class with a type family are equally expressive. (It's a matter of taste, really.)

Encoding Isomorphisms

The type class:

```
class Generic a where  
  type Rep a  
  from :: a → Rep a  
  to   :: Rep a → a
```

- `Rep` is a type family or, more precisely, an associated type synonym.
- Think of `Rep` as a function on types. Given a unique type (index) `T`, you get a type (synonym) `Rep T`.
- Note that `Rep T` need not be different from `Rep U` even though `T` and `U` are different.
- Concretely: two datatypes may have the same representation.

Encoding Isomorphisms

We need `Generic` instances for every datatype that we want to use with generic functions.

The instance for `D` uses definitions that we've already seen:

```
instance Generic (D p) where  
  type Rep (D p) = RepD p  
  from = fromD  
  to   = toD
```

- Other instances are defined similarly.
- In fact, `Rep T`, `from`, and `to` are precisely determined by the definition of `T`, so these instances can be automatically generated (e.g. using Template Haskell or a preprocessor).

The Generic `show` Function

Finally:

```
gshow :: (Show (Rep a), Generic a) => a -> String
gshow = show o from
```

GP in General

- Datatype-generic programming:
 - ▶ Datatype is the parameter
 - ▶ Instantiation gives you a large class of generic functions
- Many generic functions:
 - ▶ Pretty-printing (`show`) and parsing (`read`)
 - ▶ Compression, serialization, and the reverse
 - ▶ Comparison, equality
 - ▶ Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
 - ▶ Traversals, updates, queries
- Many different libraries:
 - ▶ Instant Generics – presented here
 - ▶ Generic Deriving – GHC \geq 7.2, similar to Instant Generics
 - ▶ EMGM – maintained by me
 - ▶ Regular – folds, etc.
 - ▶ Multirec – mutually recursive datatypes, folds, etc.
 - ▶ Scrap Your Boilerplate (SYB) – GHC, traversals, queries
 - ▶ ...

A Different Application

- Now, I want to present a different approach to using the structure of datatypes.
- This applies to the `HollingBerries` specification.
- The problem we look at is `printf`.

Presenting `printf`

In C and related languages, we have a function such as:

```
int printf(const char *format, ...)
```

The following code works:

```
printf(" %s W%drld!\n", "Hello", 0); // Hello W0rld!
```

But so does the following:

```
printf(" %s W%drld!\n", 0, "Hello"); // (null) W134514152rld!
```

As does this:

```
printf(" %s W%drld!\n", "Hello"); // Hello W134514152rld!
```

And we can't do this (where `Y` is a new format descriptor):

```
printf(" %Y\n", x); // Y
```

Problems With `printf`

- No type-checking of arguments
- No arity check
- Restricted set of format descriptors
 - ▶ Not extensible

Text.Printf

Is this the solution in Haskell?

```
printf :: PrintfType r => String -> r
```

- From `Text.Printf` in base
- Abstract type class `PrintfType`
- `String` format descriptor

Try it in GHCi:

```
ghci> printf "%s W%drld!\n" "Hello" 0
```

```
ghci> printf "%s W%drld!\n" 0 "Hello"
```

```
ghci> printf "%s W%drld!\n" "Hello"
```

Apparently, `Text.Printf` is not the solution.

xformat

The solution:

```
printf :: Format f  $\Rightarrow$  f  $\rightarrow$  A (F f) (IO ())
```

- From `Text.XFormat.Show` in `xformat`
- `f` is the format descriptor
- `A` and `F` are type families
- Also: `showf :: Format f \Rightarrow f \rightarrow A (F f) String`
- `IO ()` (or `String`) is the result type

Try it in GHCi:

```
ghci> printf (String, " W", Int, "rld!\n") "Hello" 0
```

```
ghci> printf (String, " W", Int, "rld!\n") 0 "Hello"
```

```
ghci> printf (String, " W", Int, "rld!\n") "Hello"
```

Application to HollingBerries

Text.Printf :

```
printf "R%8.2f%d/%02d/%02d%-31s\n" price year month day desc
```

Text.XFormat.Show :

```
priceF = ("R", fillL 8 (Prec 2))
```

```
dateF = Int % "/" % zero 2 Int % "/" % zero 2 Int
```

```
descF = fillR' 31 String
```

```
printf (priceF, dateF, descF, "\n") price year month day desc
```

Advantages of XFormat

Just from this example, there are several obvious advantages:

- Type-safe: see types in GHCi
- Computable
 - ▶ Compute numbers (e.g. column widths) instead of strings
- Modular and composable
 - ▶ Reuse formats in different combinations

There is also a non-obvious advantage:

- Flexible
 - ▶ Use any `Real` for `Prec` (e.g. `Int`, `Float`, or `Double`)

Behind the Scenes

How does `xformat` work?

- Polyvariadic functions
 - ▶ Variable number of arguments (of different types)
- Format descriptors may...
 - ▶ Require an argument:

```
printf String "Sharp!"
```

- ▶ Not allow any argument:

```
printf "Yebo."
```

- ▶ Compose for multiple arguments (or not):

```
printf (String, " yebo ", String) "Ja" "yes!"
```

Polyvariadic Functions

- In the end, we construct a function, $T_1 \rightarrow \dots \rightarrow T_n$, given a composition, $f_1 \diamond \dots \diamond f_n$, of format descriptors.
- We track the argument (and result) type expected by each descriptor using functors.
- We combine multiple descriptors using functor composition.

Polyvariadic Functions

- Identity – no argument

```
newtype Id a = Id a
```

- Arrow – one argument

```
newtype Arr a b = Arr (a → b)
```

- Composition – combining

```
newtype (∴) f g a = Comp (f (g a))  
infixr 8 ∴
```

Polyvariadic Functions

The `Functor` instances:

instance Functor Id **where**

$\text{fmap } f (\text{Id } x) = \text{Id } (f x)$

instance Functor (Arr a) **where**

$\text{fmap } f (\text{Arr } g) = \text{Arr } (f \circ g)$

instance (Functor f, Functor g) \Rightarrow Functor (f \circ g) **where**

$\text{fmap } f (\text{Comp } fga) = \text{Comp } (\text{fmap } (\text{fmap } f) fga)$

Polyvariadic Functions

- The functors give us terms for the components of a function composition.
- We need a way to “lift” the functor to get the actual function type.

```
class Functor f  $\Rightarrow$  Apply f where  
  type A f a  
  apply :: f a  $\rightarrow$  A f a
```

- `A` is the polyvariadic function type derived from the functor `f`.

Polyvariadic Functions

The `Apply` instances:

instance `Apply Id` **where**

type `A Id a = a`

`apply (Id x) = x`

instance `Apply (Arr a)` **where**

type `A (Arr a) b = a → b`

`apply (Arr f) = f`

instance `(Apply f, Apply g) ⇒ Apply (f ∘ g)` **where**

type `A (f ∘ g) a = A f (A g a)`

`apply (Comp fg) = apply (fmap apply fg)`

Polyvariadic Functions

Resolve these types in GHCi (using `kind!` from ≥ 7.4):

```
apply (Id T1) :: A Id T1
```

```
apply (Arr ( $\lambda T1 \rightarrow T2$ )) :: A (Arr T1) T2
```

```
apply (Comp (Arr ( $\lambda T1 \rightarrow$  Arr ( $\lambda T2 \rightarrow T3$ )))) :: A (Arr T1  $\circ$  Arr T2) T3
```

```
apply (Comp (Arr ( $\lambda T1 \rightarrow$  Id T2))) :: A (Arr T1  $\circ$  Id) T2
```

```
apply (Comp (Id (Arr ( $\lambda T1 \rightarrow T2$ )))) :: A (Id  $\circ$  Arr T1) T2
```

Format Functors

With the help of...

```
(◇) :: (Functor f, Functor g) ⇒ f String → g String → (∴) f g String  
f ◇ g = Comp (fmap (λs → fmap (λt → s ++ t) g) f)  
infixr 8 ◇
```

... we can easily compose functors, ...

```
wrldF :: Show a ⇒ (Arr String ∴ Id ∴ Arr a ∴ Id) String  
wrldF = Arr id ◇ Id " W" ◇ Arr show ◇ Id "rld!\n"
```

... lift the resulting functor to a function type, ...

```
wrld :: Show a ⇒ String → a → String  
wrld = apply wrldF
```

... and greet the w0rld.

```
ghci> putStr $ wrld "Hello" 0  
Hello W0rld!
```

Format Descriptors

But this isn't quite good enough.

We can go a step further by using meaningful descriptors instead of `Id`, `Arr`, and `Comp`.

Introducing the `Format` class:

```
class Apply (F f)  $\Rightarrow$  Format f where  
  type F f :: *  $\rightarrow$  *  
  showf' :: f  $\rightarrow$  F f String
```

And the `showf` function:

```
showf :: Format f  $\Rightarrow$  f  $\rightarrow$  A (F f) String  
showf = apply  $\circ$  showf'
```

Format Descriptors

```
class Apply (F f)  $\Rightarrow$  Format f where
```

```
  type F f :: *  $\rightarrow$  *
```

```
  showf' :: f  $\rightarrow$  F f String
```

- Descriptors are instances of `Format`
- Some instances define primitives:

```
instance Format String where
```

```
  type F String = Id
```

```
  showf' s = Id s
```

- Some instances define argument types:

```
data IntF = Int
```

```
instance Format IntF where
```

```
  type F IntF = Arr Int
```

```
  showf' Int = Arr show
```

Format Descriptors

```
class Apply (F f)  $\Rightarrow$  Format f where  
  type F f :: *  $\rightarrow$  *  
  showf' :: f  $\rightarrow$  F f String
```

- Some instances define recursive descriptors:

```
instance (Format f1, Format f2)  $\Rightarrow$  Format (f1, f2) where  
  type F (f1, f2) = F f1  $\bowtie$  F f2  
  showf' (f1, f2) = showf' f1  $\diamond$  showf' f2
```

- And yet other instances are even more interesting. See code!

Format Descriptors

- Recall `showf` :

```
showf :: Format f => f -> A (F f) String
showf = apply ∘ showf'
```

- How do we define `printf` ?
- Note that we cannot simply do `putStr ∘ showf` :

```
ghci> :t putStr . showf
putStr . showf
  :: (Format a, A (F a) String ~ String) => a -> IO ()
```

- But `F f` is a `Functor` :

```
ghci> :t fmap putStr . showf'
fmap putStr . showf' :: Format a => a -> F a (IO ())
```

- So, we can now define `printf` :

```
printf :: Format f => f -> A (F f) (IO ())
printf = apply ∘ fmap putStr ∘ showf'
```

Summary of xformat

- This description is based on `Text.XFormat.Show` :
 - ▶ One difference: package uses the more efficient `String → String` instead of `String` .

- Also `Text.XFormat.Read` :

`readf :: Format f ⇒ f → String → Maybe (R f)`

- ▶ `R` is a type family that determines the structure of the result from the format descriptor `f` .
- ▶ No functors involved. Simpler.

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