Dissecting Different Flavors of Generic Programming in One Haskell Universe

Presented to Galois

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What is Generic Programming?

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The technique is also often similar.

Some form of parameterization and instantiation

Examples of Generic Programming

Java/C#:

```
public class Stack<T>
{
  public void push(T item) {...}
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In other words:

- Java-style generics $\approx$ parametric polymorphism

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template<typename T, typename Compare>
T& min(T& a, T& b, Compare comp) {
  if (comp(b, a))
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- C++ templates $\approx$ ad-hoc polymorphism

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In Haskell, we have come to use "generic programming" for datatype-generic programming (a.k.a. "polytypism" or "shape/structure polymorphism").

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- **need not be redefined for every type** (unlike ad-hoc).

Generic Functions

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- Pretty-printing (e.g. `show`), parsing (e.g. `read`)

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- Compression, serialization, marshalling (and their inverses)
- Comparison, equality
- (Co-)recursion, map, zip, zippers
- Traversals, queries, updates

Generic Platforms

Many different implementations:

- Preprocessors:

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  - (and many, many more)

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We will look at depth into sums-of-products.

Dissecting a Datatype: Sums-of-Products

data $\mathsf{T}_{\mathsf{sum}} = \mathsf{A}_1 \mid \mathsf{A}_2$

A datatype can have:

- Alternatives: unique constructors ($\geqslant 0$)

Dissecting a Datatype: Sums-of-Products

data T_{prod} = P Char Int

A datatype can have:

- Fields: types for each constructor ($\geqslant 0$)

Dissecting a Datatype: Sums-of-Products

Other features that are modeled:

- Constant types: each type in a field
- Parameters: type variables ($\geqslant 0$)

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Other features that are modeled:

- Constant types: each type in a field
- Parameters: type variables ($\geqslant 0$)

Features that are not modeled:

- Recursion
- Nesting (though it can be)

Modeling a Sum

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data a :+: b = L a | R b
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A constructor without fields:

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A constructor name:

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data C a = C String a
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A field type:

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data K a = K a
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Note: There are other features of datatypes, but we consider only the above.

Modeling an Example

An example datatype:

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data E a = E1 | E2 a (E a) Int
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type Rep_E a = C U :+: C (K a :×: K (E a) :×: K Int)
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```

Notes:

- `:+:` is **infixr** 5 and `:×:` is **infixr** 6.
- Operators nest to the right.

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$$
\begin{aligned}
&\mathsf{from_E} :: \mathsf{E\ a} \to \mathsf{Rep_E\ a} \\
&\mathsf{from_E\ E_1} \qquad\quad = \mathsf{L\ (C\ "E1"\ U)} \\
&\mathsf{from_E\ (E_2\ x\ e\ i)} = \mathsf{R\ (C\ "E2"\ ((K\ x) :\times: (K\ e) :\times: (K\ i)))}
\end{aligned}
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Converting Between Types: Isomorphism

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```
fromE :: E a → RepE a
fromE E1          = L (C "E1" U)
fromE (E2 x e i) = R (C "E2" ((K x) :×: (K e) :×: (K i)))
```

```
toE :: RepE a → E a
toE (L (C "E1" U))                          = E1
toE (R (C "E2" ((K x) :×: (K e) :×: (K i)))) = E2 x e i
```

Converting Between Types: Isomorphism

For convenience, we join the representation type and isomorphism in a type class `Generic` with an associated type synonym `Rep`.

```
class Generic a where
  type Rep a
  from :: a → Rep a
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Converting Between Types: Isomorphism

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class Generic a where
  type Rep a
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```

The instance for `E`:

```
instance Generic (E a) where
  type Rep (E a) = Rep_E a
  from = from_E
  to   = to_E
```

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A generic function

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Example: $\text{show}_{\text{Rep a}} :: a \to \text{String}$

- We define a `show` function for each case.

Defining show

Unit:

$\text{show}_{\mathsf{U}} :: \mathsf{U} \rightarrow \text{String}$
$\text{show}_{\mathsf{U}}\ \mathsf{U} = \text{""}$

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$$\text{show}_U :: U \to \text{String}$$
$$\text{show}_U\ U = ""$$

Constructor name:

$$\text{show}_C :: (a \to \text{String}) \to C\ a \to \text{String}$$
$$\text{show}_C\ \text{show}_a\ (C\ nm\ a) = "("\ +\!\!+\ nm\ +\!\!+\ "\ \ "\ +\!\!+\ \text{show}_a\ a\ +\!\!+\ ")"$$

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Field:

$$\text{show}_K :: (a \to \text{String}) \to K\ a \to \text{String}$$
$$\text{show}_K\ \text{show}_a\ (K\ a) = \text{show}_a\ a$$

Defining show

Binary sum:

$$\begin{aligned}
&\text{show}_+ :: (a \to \text{String}) \to (b \to \text{String}) \to a :+: b \to \text{String} \\
&\text{show}_+\ \text{show}_a\ _\ (L\ a) = \text{show}_a\ a \\
&\text{show}_+\ _\ \text{show}_b\ (R\ b) = \text{show}_b\ b
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Binary product:

$$\begin{aligned}&\text{show}_\times :: (a \to \text{String}) \to (b \to \text{String}) \to a :\times: b \to \text{String}\\&\text{show}_\times\ \text{show}_a\ \text{show}_b\ (a :\times: b) = \text{show}_a\ a +\!\!+ "\ \ " +\!\!+ \text{show}_b\ b\end{aligned}$$

Defining show

Recall:

```
type Rep_E a = C U :+: C (K a :×: K (E a) :×: K Int)
```

We can define a show function (assuming show_{Int}):

$$\begin{aligned}
&\text{show}_{\text{Rep}_E} :: (a \to \text{String}) \to ((a \to \text{String}) \to E\ a \to \text{String}) \\
&\qquad \to \text{Rep}_E\ a \to \text{String} \\
&\text{show}_{\text{Rep}_E}\ \text{show}_a\ \text{show}_E = \\
&\quad \text{show}_+\ (\text{show}_C\ \text{show}_U) \\
&\qquad (\text{show}_C\ (\text{show}_\times\ (\text{show}_K\ \text{show}_a) \\
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\end{aligned}
$$

The $\mathsf{show_E}$ function itself is just an isomorphism away:

$$
\begin{aligned}
&\mathsf{show_E} :: (\mathsf{a} \to \mathsf{String}) \to \mathsf{E\ a} \to \mathsf{String} \\
&\mathsf{show_E}\ \mathsf{show_a} = \mathsf{show}_{\mathsf{Rep_E}}\ \mathsf{show_a}\ \mathsf{show_E} \circ \mathsf{from_E}
\end{aligned}
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- This is **not** a generic function.

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- This is **not** a generic function.
- It is defined on the structure of E, not on datatypes in general.
- It demonstrates a predictable pattern for defining the generic function.

Defining show

Consider these typical expressions and their types:

$$\begin{aligned} &\mathsf{show}_{\mathsf{C}}\ \mathsf{show}_{\mathsf{U}} &&:: \mathsf{C\ U} &&\to \mathsf{String} \\ &\mathsf{show}_{\times}\ (\mathsf{show}_{\mathsf{K}}\ \mathsf{show}_{\mathsf{Int}})\ (\mathsf{show}_{\mathsf{K}}\ \mathsf{show}_{\mathsf{Char}}) &&:: (\mathsf{K\ Int} :\times: \mathsf{K\ Char}) &&\to \mathsf{String} \end{aligned}$$

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- $\text{show}_?$ functions call other $\text{show}_?$ functions.
- They can be considered recursive but not in the usual way.
- Polymorphic recursion – functions with different types that have a common scheme that reference each other

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A simplified definition of the Show class:

```
class Show a where
  show :: a → String
```

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The remaining instances are straightforward.

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Now, compare:

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&\quad \mathsf{show}_{+}\ (\mathsf{show_C}\ \mathsf{show_U}) \\
&\qquad (\mathsf{show_C}\ (\mathsf{show}_{\times}\ (\mathsf{show_K}\ \mathsf{show_a}) \\
&\qquad\qquad (\mathsf{show}_{\times}\ (\mathsf{show_K}\ (\mathsf{show_E}\ \mathsf{show_a}))\ (\mathsf{show_K}\ \mathsf{show_{Int}}))))
\end{aligned}$$

To:

$$\begin{aligned}
&\mathsf{show}_{\mathsf{Rep_E}} :: (\mathsf{Show\ a}, \mathsf{Show\ (E\ a)}) \Rightarrow \mathsf{Rep_E\ a} \to \mathsf{String} \\
&\mathsf{show}_{\mathsf{Rep_E}} = \mathsf{show}
\end{aligned}$$

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Finally, we can use a slightly different `Show` class to support generic functions for any type that has a representation.

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class Show a **where**

show :: a → String

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- This uses the `DefaultSignatures` language extension: if type `a` has the instances `Show (Rep a)` and `Generic a`, then the given definition is used.

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The instance for `E`:

```
instance Show a ⇒ Show (E a)
```

Sums-of-Products and Beyond

We presented a sums-of-products view.

- We used Haskell2010 plus a few GHC language extensions:

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```

- Typically, a GP library does not support another view.
- But we can, with a few more extensions:

```
{-# LANGUAGE FlexibleInstances #-}
{-# LANGUAGE MultiParamTypeClasses #-}
{-# LANGUAGE OverlappingInstances #-}
```

The Uniplate View

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- The spine is the sequence of fields in a constructor.
- SYB models the "full" spine, i.e. all fields (which can naturally have different types).
- Uniplate models only a list of the (recursive) children (which have the same type).

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  - Polymorphic recursion on the structure (as usual) *and*
  - A function parameter whose type matches only some of the fields.

Defining descend′

Consequently, we use a signature with different types for the function parameter and the structure representation:

```
class Uniplate' a r where
  descend' :: (r → r) → a → a
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- We will come back to Uniplate later.

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Most of the instances are straightforward:

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  descend' f (R b) = R (descend' f b)
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  descend' f (R b) = R (descend' f b)
```

```
instance (Uniplate' a r, Uniplate' b r) ⇒ Uniplate' (a :×: b) r where
  descend' f (a :×: b) = descend' f a :×: descend' f b
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And an instance where we apply the function parameter:

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- Overlapping instances implies type equality.
- This is the "trick" that allows us to determine when to choose this instance.

Defining descend

Coming back to an improved `Uniplate` class:

```
class Uniplate a where
  descend :: (a → a) → a → a
  default descend :: (Generic a, Uniplate′ (Rep a) a) ⇒ (a → a) → a → a
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- We again use DefaultSignatures to simplify instantiation.
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- They only differ "behind the scenes."

Uniplate View and Beyond

We presented a traversal function

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With a bit more work, we can also define functions that work on all fields and not just the recursive children, e.g.:

```
topDown :: C b a ⇒ (a → a) → b → b
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```
topDown :: C b a ⇒ (a → a) → b → b
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- For a class `C` that supports matching on any type `T` for which there is an instance `C T T`
- Similar to the function `everywhere'` in SYB

The Fixed-Point View

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- In the basic fixed-point view, we define one case of a generic function on the recursive points structural element.
- In our library, we pass the top-level type `T` through the type cases.
- The case at which we can match on `T` is the recursive point.

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class Fold a where
  fold :: Alg (Rep a) r → a → r
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  fold :: Alg (Rep a) r → a → r
```

- Given an algebra and a value, compute the result of applying the algebra to the structure of the value.

Defining Alg

The algebra of the fold is a type family:

```
type family Alg a r

type instance Alg U           r = r
type instance Alg (K a)       r = Either a r → r
type instance Alg (C a)       r = Alg a r
type instance Alg (a :+: b)   r = (Alg a r, Alg b r)
type instance Alg (K a :×: b) r = Either a r → Alg b r
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- `Alg` is indexed on the representation type of the input type `a`.
- The type `r` is the result of the fold.
- `K` types can be either non-recursive (`a`) or recursive (`r`) points.

Defining Alg

For the example type:

```
type Rep_E a = C U :+: C (K a :×: K (E a) :×: K Int)
```

The algebra type is:

```
type instance Alg (Rep (E a)) r =
  (r, Either a r → Either (E a) r → Either Int r → r)
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```

- `E a` is the recursive point, even though it does not appear so in the type.
- The instances of the generic function ensure the separation of non-recursive and recursive `K` cases.

Defining Fold′

We again define a helper generic function:

```
class Fold′ a t where
  fold′ :: proxy t → Alg (Rep t) r → Alg a r → a → r
```

- `a` is the structure type.

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```

- `a` is the structure type.
- `t` is the recursive type.
- The "proxy" provides proof of `t` while preventing the instances of `Fold′` from using it.

Defining Fold′

The instances that do not have recursion:

```
instance Fold′ U t where
  fold′ _ _ alg U = alg
```

```
instance Fold′ a t ⇒ Fold′ (C a) t where
  fold′ p palg alg (C _ a) = fold′ p palg alg a
```

```
instance (Fold′ a t, Fold′ b t) ⇒ Fold′ (a :+: b) t where
  fold′ p palg (alg, _) (L a) = fold′ p palg alg a
  fold′ p palg (_, alg) (R b) = fold′ p palg alg b
```

Defining Fold′

The fall-back K instance:

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Defining Fold′

The fall-back K instance:

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instance Fold′ (K a) t where
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The recursive K instance:

```
instance Fold t ⇒ Fold′ (K t) t where
  fold′ p palg alg (K t) = alg (Right (fold palg t))
```

Defining Fold′

The fall-back :×: instance:

```
instance Fold′ b t ⇒ Fold′ (K a :×: b) t where
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Defining Fold′

The fall-back :×: instance:

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instance Fold′ b t ⇒ Fold′ (K a :×: b) t where
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The recursive :×: instance:

```
instance (Fold t, Fold′ b t) ⇒ Fold′ (K t :×: b) t where
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Defining `fold`

The improved `Fold` class:

```
class Fold a where
  fold :: Alg (Rep a) r → a → r
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- We use `Maybe` as a simple proxy.

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- We use `Maybe` as a simple proxy.
- The algebra is needed twice: the second argument is pattern-matched by the instances of `Fold′`.

Fold and Beyond

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We presented a generic recursive pattern in a library that would not typically have it.

- We can also define many other (co-)recursive functions, including the generic zipper.
- The unfortunate aspect of `Alg` is that we must use `Either` since, in the type family, we cannot distinguish overlapping instances.
- We believe this can be fixed with the new ordered overlapping instances in GHC.

Conclusions

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- We are still searching for that optimal view.
- The library presented here is quite simple.
- Yet, with a few tricks, it is also quite powerful.
- We have also done this work in the more complicated Generic Deriving library.

References

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