# Dissecting Different Flavors of Generic Programming in One Haskell Universe

Presented to Galois

Sean Leather

Utrecht University

August 27, 2013

In programming languages, the adjective "generic" is heavily overloaded.

In programming languages, the adjective "generic" is heavily overloaded.

Java/C# generics

In programming languages, the adjective "generic" is heavily overloaded.

- Java/C# generics
- C++ templates

In programming languages, the adjective "generic" is heavily overloaded.

- Java/C# generics
- C++ templates
- Ada generic packages

The goal is often the same.

A higher level of abstraction than "normally" available

The goal is often the same.

A higher level of abstraction than "normally" available

The technique is also often the same.

Some form of parameterization and instantiation

# **Examples of Generic Programming**

```
Java/C#:
public class Stack<T>
{
   public void push(T item) {...}
   public T pop() {...}
}
```

# **Examples of Generic Programming**

```
C++:
template < typename T, typename Compare >
T& min(T& a, T& b, Compare comp) {
  if (comp(b, a))
    return b;
  return a;
}
```

#### In other words:

ullet Java-style generics pprox parametric polymorphism

#### In other words:

- Java-style generics  $\approx$  parametric polymorphism
- ullet C++ templates pprox ad-hoc polymorphism

#### In other words:

- Java-style generics  $\approx$  parametric polymorphism
- ullet C++ templates pprox ad-hoc polymorphism

#### In other words:

- Java-style generics  $\approx$  parametric polymorphism
- ullet C++ templates pprox ad-hoc polymorphism

## In Haskell:

Both forms already exist.

## In other words:

- ullet Java-style generics pprox parametric polymorphism
- ullet C++ templates pprox ad-hoc polymorphism

## In Haskell:

- Both forms already exist.
- We don't call them generics because they're native to the language.

## In other words:

- ullet Java-style generics pprox parametric polymorphism
- ullet C++ templates pprox ad-hoc polymorphism

## In Haskell:

- Both forms already exist.
- We don't call them generics because they're native to the language.

#### In other words:

- Java-style generics  $\approx$  parametric polymorphism
- ullet C++ templates pprox ad-hoc polymorphism

## In Haskell:

- Both forms already exist.
- We don't call them generics because they're native to the language.

## Datatype-generic programming:

• Abstract over the structure of a datatype

#### In other words:

- ullet Java-style generics pprox parametric polymorphism
- ullet C++ templates pprox ad-hoc polymorphism

## In Haskell:

- Both forms already exist.
- We don't call them generics because they're native to the language.

## Datatype-generic programming:

- Abstract over the structure of a datatype
- Also known as "polytypism" and "shape-/structure-polymorphism"

data D  $p = Alt_1 | Alt_2 | Int p$ 

$$\mathbf{data} \; \mathsf{D} \; \mathsf{p} = \mathsf{Alt}_1 \; | \; \mathsf{Alt}_2 \; \mathsf{Int} \; \mathsf{p}$$

## A datatype can have:

• Parameters: type variables (≥ 0)

$$\mathbf{data} \ \mathsf{D} \ \mathsf{p} = \mathsf{Alt}_1 \ | \ \mathsf{Alt}_2 \ \mathsf{Int} \ \mathsf{p}$$

## A datatype can have:

- Parameters: type variables ( $\geqslant 0$ )
- Alternatives: unique constructors (≥ 0)

$$\mathbf{data} \ \mathsf{D} \ \mathsf{p} = \mathsf{Alt}_1 \ | \ \mathsf{Alt}_2 \ \mathsf{Int} \ \mathsf{p}$$

## A datatype can have:

- Parameters: type variables (≥ 0)
- Alternatives: unique constructors (≥ 0)
- Fields: types for each constructor (≥ 0)

$$\mathbf{data} \ \mathsf{D} \ \mathsf{p} = \mathsf{Alt}_1 \ | \ \mathsf{Alt}_2 \ \mathsf{Int} \ \mathsf{p}$$

## A datatype can have:

- Parameters: type variables (≥ 0)
- Alternatives: unique constructors (≥ 0)
- Fields: types for each constructor (≥ 0)

$$\mathbf{data} \ \mathsf{D} \ \mathsf{p} = \mathsf{Alt}_1 \ | \ \mathsf{Alt}_2 \ \mathsf{Int} \ \mathsf{p}$$

## A datatype can have:

- Parameters: type variables (≥ 0)
- Alternatives: unique constructors (≥ 0)
- Fields: types for each constructor (≥ 0)

## Non-syntactic features:

Recursion

$$\mathbf{data} \ \mathsf{D} \ \mathsf{p} = \mathsf{Alt}_1 \ | \ \mathsf{Alt}_2 \ \mathsf{Int} \ \mathsf{p}$$

## A datatype can have:

- Parameters: type variables (≥ 0)
- Alternatives: unique constructors (≥ 0)
- Fields: types for each constructor (≥ 0)

## Non-syntactic features:

- Recursion
- Nesting

$$\mathbf{data} \ \mathsf{D} \ \mathsf{p} = \mathsf{Alt}_1 \ | \ \mathsf{Alt}_2 \ \mathsf{Int} \ \mathsf{p}$$

## A datatype can have:

- Parameters: type variables (≥ 0)
- Alternatives: unique constructors (≥ 0)
- Fields: types for each constructor (≥ 0)

## Non-syntactic features:

- Recursion
- Nesting

$$\mathbf{data} \ \mathsf{D} \ \mathsf{p} = \mathsf{Alt}_1 \ | \ \mathsf{Alt}_2 \ \mathsf{Int} \ \mathsf{p}$$

## A datatype can have:

- Parameters: type variables (≥ 0)
- Alternatives: unique constructors (≥ 0)
- Fields: types for each constructor (≥ 0)

## Non-syntactic features:

- Recursion
- Nesting

There are other features of datatypes, but we will consider only the above as a foundation for looking at the structure.

First structural element: alternatives.

data  $AltEx_2 = A_1 Int \mid A_2 Char$ 

First structural element: alternatives.

data 
$$AltEx_2 = A_1 Int \mid A_2 Char$$

Note that the above is similar to a standard type:

```
data Either a b = Left a | Right b
```

First structural element: alternatives.

data 
$$AltEx_2 = A_1 Int \mid A_2 Char$$

Note that the above is similar to a standard type:

And we can, in fact, model  $AltEx_2$  as:

**type** AltEx
$$_2'$$
 = Either Int Char

with the following "smart" constructors:

$$a_1 :: Int \rightarrow AltEx_2'$$
  
 $a_1 = Left$ 

$$\mathsf{a}_2 :: \mathsf{Char} \to \mathsf{AltEx}_2'$$

$$a_2 = Right$$

When talking about alternatives in structural sense, we often call them sums. Either is the basic binary sum type. For conciseness, we use this (identical) binary sum type:

data  $a : +: b = L a \mid R b$ 

When talking about alternatives in structural sense, we often call them sums. Either is the basic binary sum type. For conciseness, we use this (identical) binary sum type:

data 
$$a : +: b = L a \mid R b$$

What about a type with < 2 alternatives?

data 
$$AltEx_3 = B_1 Int \mid B_2 Char \mid B_3 Float$$

When talking about alternatives in structural sense, we often call them sums. Either is the basic binary sum type. For conciseness, we use this (identical) binary sum type:

data 
$$a : +: b = L a \mid R b$$

What about a type with < 2 alternatives?

data 
$$AltEx_3 = B_1 Int \mid B_2 Char \mid B_3 Float$$

The simplest solution is to nest one binary sum inside another:

**type** 
$$AltEx'_3 = Int :+: (Char :+: Float)$$

Note that:

$$\mathsf{b}_3 :: \mathsf{Float} \to \mathsf{AltEx}_3'$$

# Structure of Datatypes: Products

Next: fields.

data  $FldEx_2 = FldEx_2$  Int Char

# Structure of Datatypes: Products

Next: fields.

data  $FldEx_2 = FldEx_2$  Int Char

Again, note the similarity to a standard type, the pair:

data 
$$(,)$$
 a b =  $(,)$  a b

# Structure of Datatypes: Products

Next: fields.

data  $FIdEx_2 = FIdEx_2$  Int Char

Again, note the similarity to a standard type, the pair:

$$data (,) a b = (,) a b$$

And again, we model  $FldEx_2$  similarly:

**type** 
$$FldEx'_2 = (,)$$
 Int Char

with the smart constructor:

$$\mathsf{fldEx}_2' :: \mathsf{Int} \to \mathsf{Char} \to \mathsf{FldEx}_2'$$
$$\mathsf{fldEx}_2' = (,)$$

## Structure of Datatypes: Products

The pair type is the basic binary product type. For symmetry with sums, we will use the following type:

data 
$$a : \times : b = a : \times : b$$

## Structure of Datatypes: Products

The pair type is the basic binary product type. For symmetry with sums, we will use the following type:

data 
$$a : \times : b = a : \times : b$$

And more than two fields...

data  $FldEx_3 = FldEx_3$  Int Char Float

## Structure of Datatypes: Products

The pair type is the basic binary product type. For symmetry with sums, we will use the following type:

data 
$$a : \times : b = a : \times : b$$

And more than two fields...

data 
$$FldEx_3 = FldEx_3$$
 Int Char Float

... are modeled by nested binary products:

**type** 
$$FldEx'_3 = Int : x: (Char : x: Float)$$

with the smart constructor:

$$\begin{array}{l} \mathsf{fldEx}_3' :: \mathsf{Int} \to \mathsf{Char} \to \mathsf{Float} \to \mathsf{FldEx}_3' \\ \mathsf{fldEx}_3' \times \mathsf{y} \ \mathsf{z} = \mathsf{x} : \!\! \times \!\! : (\mathsf{y} : \!\! \times \!\! : \mathsf{z}) \end{array}$$

To "sum" it all up, recall the first datatype example:

data  $D p = Alt_1 | Alt_2 Int p$ 

To "sum" it all up, recall the first datatype example:

data 
$$D p = Alt_1 | Alt_2 Int p$$

We can define an identical type using the sum and product types we have just discussed:

```
type Rep_D p = U :+: Int :\times: p
```

To "sum" it all up, recall the first datatype example:

data 
$$D p = Alt_1 | Alt_2 Int p$$

We can define an identical type using the sum and product types we have just discussed:

**type** 
$$Rep_D p = U :+: Int :\times: p$$

#### Notes:

We use the "unit" type data U = U (identical to the standard type
 () to represent an alternative without fields.

To "sum" it all up, recall the first datatype example:

data D 
$$p = Alt_1 \mid Alt_2 Int p$$

We can define an identical type using the sum and product types we have just discussed:

```
type Rep_D p = U :+: Int :\times: p
```

#### Notes:

- We use the "unit" type data U = U (identical to the standard type
   () ) to represent an alternative without fields.
- :+: is **infixr** 5, and :x: is **infixr** 6, so we can write Rep<sub>D</sub> naturally, without unnecessary parentheses.

So, we think we can model datatypes. But how do we know  $\operatorname{\mathsf{Rep}}_{\mathsf{D}}$  accurately models  $\mathsf{D}$  ?

So, we think we can model datatypes. But how do we know  $\operatorname{\mathsf{Rep}}_\mathsf{D}$  accurately models  $\mathsf{D}$  ?

We define an isomorphism: two total functions that convert between types.

So, we think we can model datatypes. But how do we know  $\operatorname{\mathsf{Rep}}_\mathsf{D}$  accurately models  $\mathsf{D}$  ?

We define an isomorphism: two total functions that convert between types.

```
\begin{array}{lll} \text{from}_D :: D \ p \rightarrow \text{Rep}_D \ p \\ \text{from}_D \ \text{Alt}_1 &= L \ U \\ \text{from}_D \ (\text{Alt}_2 \ i \ p) &= R \ (i : \times : p) \\ \text{to}_D :: \text{Rep}_D \ p \rightarrow D \ p \\ \text{to}_D & (L \ U) &= \text{Alt}_1 \\ \text{to}_D & (R \ (i : \times : p)) = \text{Alt}_2 \ i \ p \end{array}
```

So, we think we can model datatypes. But how do we know  $\operatorname{\mathsf{Rep}}_\mathsf{D}$  accurately models  $\mathsf{D}$  ?

We define an isomorphism: two total functions that convert between types.

```
\begin{array}{ll} \text{from}_D :: D \ p \rightarrow \text{Rep}_D \ p \\ \text{from}_D \ \text{Alt}_1 &= L \ U \\ \text{from}_D \ (\text{Alt}_2 \ i \ p) &= R \ (i : \times : p) \\ \text{to}_D :: \text{Rep}_D \ p \rightarrow D \ p \\ \text{to}_D & (L \ U) &= \text{Alt}_1 \\ \text{to}_D & (R \ (i : \times : p)) = \text{Alt}_2 \ i \ p \end{array}
```

This allows us to convert terms between (1) the familiar datatype and (2) the structure representation used for generic operations.

Oh, but there's one more thing...

Oh, but there's one more thing...

You may have noticed the representation lacked any information about the constructors (e.g. the names).

Oh, but there's one more thing...

You may have noticed the representation lacked any information about the constructors (e.g. the names).

That's easily repaired with another datatype:

data Ca = CStringa

Oh, but there's one more thing...

You may have noticed the representation lacked any information about the constructors (e.g. the names).

That's easily repaired with another datatype:

```
data C a = C String a
```

We modify the representation to store constructor names:

```
\begin{aligned} &\textbf{type} \; \mathsf{Rep}_D \; \textbf{p} = \mathsf{C} \; \mathsf{U} \; \text{:+:} \; \mathsf{C} \; (\mathsf{Int} \; \text{:} \times \text{:} \; \textbf{p}) \\ &\mathsf{from}_D \; \mathsf{Alt}_1 \qquad = \mathsf{L} \; (\mathsf{C} \; \text{"Alt1"} \; \mathsf{U}) \\ &\mathsf{from}_D \; (\mathsf{Alt}_2 \; \mathsf{i} \; \mathsf{p}) = \mathsf{R} \; (\mathsf{C} \; \text{"Alt2"} \; (\mathsf{i} \; \text{:} \times \text{:} \; \mathsf{p})) \end{aligned}
```

Oh, but there's one more thing...

You may have noticed the representation lacked any information about the constructors (e.g. the names).

That's easily repaired with another datatype:

```
data Ca = CStringa
```

We modify the representation to store constructor names:

```
 \begin{aligned} &\textbf{type} \; \mathsf{Rep}_D \; \textbf{p} = \mathsf{C} \; \mathsf{U} \; \text{:+:} \; \mathsf{C} \; (\mathsf{Int} \; \text{:} \times : \; \textbf{p}) \\ &\mathsf{from}_D \; \mathsf{Alt}_1 \qquad = \mathsf{L} \; (\mathsf{C} \; \text{"Alt1"} \; \mathsf{U}) \\ &\mathsf{from}_D \; (\mathsf{Alt}_2 \; \mathsf{i} \; \mathsf{p}) = \mathsf{R} \; (\mathsf{C} \; \text{"Alt2"} \; (\mathsf{i} \; \text{:} \times : \; \mathsf{p})) \end{aligned}
```

We could also put additional metadata (e.g. fixity) into C.

Okay, so we have a structure representation. But what can we do with it?

Okay, so we have a structure representation. But what can we do with it?

#### Generic functions

• Defined on each possible case of the structure representation

Okay, so we have a structure representation. But what can we do with it?

#### Generic functions

- Defined on each possible case of the structure representation
- Work for every datatype that has an isomorphism with a structure representation

Okay, so we have a structure representation. But what can we do with it?

#### Generic functions

- Defined on each possible case of the structure representation
- Work for every datatype that has an isomorphism with a structure representation

Okay, so we have a structure representation. But what can we do with it?

#### Generic functions

- Defined on each possible case of the structure representation
- Work for every datatype that has an isomorphism with a structure representation

Example: show ::  $a \rightarrow String$ 

We define a show function for each possible structure case.

We define a show function for each possible structure case.

### Unit:

```
\mathsf{show}_U :: U \to \mathsf{String} \mathsf{show}_U \ U = \text{""}
```

We define a show function for each possible structure case.

#### Unit:

$$\mathsf{show}_U :: \mathsf{U} \to \mathsf{String}$$
 
$$\mathsf{show}_U \ \mathsf{U} = ""$$

### Constructor name:

$$\begin{array}{l} \mathsf{show}_\mathsf{C} :: (\mathsf{a} \to \mathsf{String}) \to \mathsf{C} \; \mathsf{a} \to \mathsf{String} \\ \mathsf{show}_\mathsf{C} \; \mathsf{show}_\mathsf{a} \; (\mathsf{C} \; \mathsf{nm} \; \mathsf{a}) = \\ \text{"("} + \mathsf{nm} + \text{" "} + \mathsf{show}_\mathsf{a} \; \mathsf{a} + \text{")"} \end{array}$$

We define a show function for each possible structure case.

#### Unit:

$$\mathsf{show}_U :: U \to \mathsf{String}$$
 
$$\mathsf{show}_U \ U = \texttt{""}$$

### Constructor name:

$$\begin{array}{l} \mathsf{show}_\mathsf{C} :: (\mathsf{a} \to \mathsf{String}) \to \mathsf{C} \ \mathsf{a} \to \mathsf{String} \\ \mathsf{show}_\mathsf{C} \ \mathsf{show}_\mathsf{a} \ (\mathsf{C} \ \mathsf{nm} \ \mathsf{a}) = \\ \text{"(" #+ nm #+ " " #+ show}_\mathsf{a} \ \mathsf{a} \ \text{#+ ")"} \end{array}$$

### Binary product:

We define a show function for each possible structure case.

#### Unit:

$$\begin{array}{l} \mathsf{show}_U :: \mathsf{U} \to \mathsf{String} \\ \mathsf{show}_U \ \mathsf{U} = "" \end{array}$$

### Constructor name:

$$\begin{array}{l} \mathsf{show}_\mathsf{C} :: (\mathsf{a} \to \mathsf{String}) \to \mathsf{C} \ \mathsf{a} \to \mathsf{String} \\ \mathsf{show}_\mathsf{C} \ \mathsf{show}_\mathsf{a} \ (\mathsf{C} \ \mathsf{nm} \ \mathsf{a}) = \\ \text{"(" #+ nm #+ " " #+ show}_\mathsf{a} \ \mathsf{a} \ \text{#+ ")"} \end{array}$$

### Binary product:

### Binary sum:

$$\begin{array}{l} \mathsf{show}_+ :: (\mathsf{a} \to \mathsf{String}) \to (\mathsf{b} \to \mathsf{String}) \to \mathsf{a} : + : \mathsf{b} \to \mathsf{String} \\ \mathsf{show}_+ \ \mathsf{show}_{\mathsf{a}} \ \_ (\mathsf{L} \ \mathsf{a}) = \mathsf{show}_{\mathsf{a}} \ \mathsf{a} \\ \mathsf{show}_+ \ \_ \mathsf{show}_{\mathsf{b}} \ (\mathsf{R} \ \mathsf{b}) = \mathsf{show}_{\mathsf{b}} \ \mathsf{b} \end{array}$$

We can define a show function for Rep<sub>D</sub> (assuming show<sub>Int</sub>):

```
\begin{split} \mathsf{show}_{\mathsf{Rep}_D} &:: (\mathsf{p} \to \mathsf{String}) \to \mathsf{Rep}_D \; \mathsf{p} \to \mathsf{String} \\ \mathsf{show}_{\mathsf{Rep}_D} &\: \mathsf{show}_{\mathsf{p}} = \\ &\: \mathsf{show}_+ \; \big( \mathsf{show}_\mathsf{C} \; \mathsf{show}_\mathsf{U} \big) \; \big( \mathsf{show}_\mathsf{C} \; \big( \mathsf{show}_\times \; \mathsf{show}_\mathsf{Int} \; \mathsf{show}_\mathsf{p} \big) \big) \end{split}
```

We can define a show function for  $Rep_D$  (assuming show<sub>Int</sub>):

```
\begin{array}{l} \mathsf{show}_{\mathsf{Rep}_D} :: (\mathsf{p} \to \mathsf{String}) \to \mathsf{Rep}_D \; \mathsf{p} \to \mathsf{String} \\ \mathsf{show}_{\mathsf{Rep}_D} \; \mathsf{show}_{\mathsf{p}} = \\ \; \mathsf{show}_+ \; \big(\mathsf{show}_\mathsf{C} \; \mathsf{show}_\mathsf{U}\big) \; \big(\mathsf{show}_\mathsf{C} \; \big(\mathsf{show}_\times \; \mathsf{show}_\mathsf{Int} \; \mathsf{show}_{\mathsf{p}}\big)\big) \end{array}
```

The show function for D is just a hop away:

```
\begin{aligned} \mathsf{show}_D :: (\mathsf{p} \to \mathsf{String}) \to \mathsf{D} \ \mathsf{p} \to \mathsf{String} \\ \mathsf{show}_D \ \mathsf{show}_{\mathsf{p}} = \mathsf{show}_{\mathsf{Rep}_D} \ \mathsf{show}_{\mathsf{p}} \circ \mathsf{from}_D \end{aligned}
```

```
\begin{array}{l} \mathsf{show}_{\mathsf{Rep}_D} :: (\mathsf{p} \to \mathsf{String}) \to \mathsf{Rep}_D \; \mathsf{p} \to \mathsf{String} \\ \mathsf{show}_{\mathsf{Rep}_D} \; \mathsf{show}_{\mathsf{p}} = \\ \mathsf{show}_+ \; \big(\mathsf{show}_C \; \mathsf{show}_U \big) \; \big(\mathsf{show}_C \; \big(\mathsf{show}_\times \; \mathsf{show}_{\mathsf{Int}} \; \mathsf{show}_{\mathsf{p}} \big) \big) \end{array}
```

#### Some observations:

• This is a sort of predictable pattern (or recipe) for defining show functions on structure representations.

```
\begin{array}{l} \mathsf{show}_{\mathsf{Rep}_D} :: (\mathsf{p} \to \mathsf{String}) \to \mathsf{Rep}_D \; \mathsf{p} \to \mathsf{String} \\ \mathsf{show}_{\mathsf{Rep}_D} \; \mathsf{show}_{\mathsf{p}} = \\ \; \mathsf{show}_+ \; \big(\mathsf{show}_C \; \mathsf{show}_U \big) \; \big(\mathsf{show}_C \; \big(\mathsf{show}_\times \; \mathsf{show}_{\mathsf{Int}} \; \mathsf{show}_{\mathsf{p}} \big) \big) \end{array}
```

#### Some observations:

- This is a sort of predictable pattern (or recipe) for defining show functions on structure representations.
- The functions are recursive but not in the usual way because the argument types differ.

```
\begin{aligned} \mathsf{show}_{\mathsf{Rep}_D} &:: (\mathsf{p} \to \mathsf{String}) \to \mathsf{Rep}_D \; \mathsf{p} \to \mathsf{String} \\ \mathsf{show}_{\mathsf{Rep}_D} &\: \mathsf{show}_{\mathsf{p}} = \\ &\: \mathsf{show}_+ \; \big( \mathsf{show}_\mathsf{C} \; \mathsf{show}_\mathsf{U} \big) \; \big( \mathsf{show}_\mathsf{C} \; \big( \mathsf{show}_\times \; \mathsf{show}_\mathsf{Int} \; \mathsf{show}_\mathsf{p} \big) \big) \end{aligned}
```

#### Some observations:

- This is a sort of predictable pattern (or recipe) for defining show functions on structure representations.
- The functions are recursive but not in the usual way because the argument types differ.
- Each datatype can have a unique structure representation, and we want to support all combinations, *generically*.

In order to jump into "true" genericity (where the structure is a parameter instead of a pattern), we need several additional things:

In order to jump into "true" genericity (where the structure is a parameter instead of a pattern), we need several additional things:

 Polymorphic recursion – functions with a common scheme that reference each other and allow types to change in the calls

In order to jump into "true" genericity (where the structure is a parameter instead of a pattern), we need several additional things:

 Polymorphic recursion – functions with a common scheme that reference each other and allow types to change in the calls

In order to jump into "true" genericity (where the structure is a parameter instead of a pattern), we need several additional things:

 Polymorphic recursion – functions with a common scheme that reference each other and allow types to change in the calls

```
\begin{array}{lll} \mathsf{show}_U :: & \mathsf{U} & \to \mathsf{String} \\ \mathsf{show}_C :: ... \Rightarrow \mathsf{C} \ \mathsf{a} & \to \mathsf{String} \\ \mathsf{show}_+ :: ... \Rightarrow \mathsf{a} : \!\!\! + \!\!\! : \mathsf{b} \to \mathsf{String} \\ ... \end{array}
```

• A common encoding for isomorphisms

In order to jump into "true" genericity (where the structure is a parameter instead of a pattern), we need several additional things:

 Polymorphic recursion – functions with a common scheme that reference each other and allow types to change in the calls

```
\begin{array}{lll} \mathsf{show}_U :: & \mathsf{U} & \to \mathsf{String} \\ \mathsf{show}_C :: ... \Rightarrow \mathsf{C} \ \mathsf{a} & \to \mathsf{String} \\ \mathsf{show}_+ :: ... \Rightarrow \mathsf{a} : \!\!\! + \!\!\! : \mathsf{b} \to \mathsf{String} \\ ... \end{array}
```

• A common encoding for isomorphisms

# Generic Functions, Generically

In order to jump into "true" genericity (where the structure is a parameter instead of a pattern), we need several additional things:

 Polymorphic recursion – functions with a common scheme that reference each other and allow types to change in the calls

```
\begin{array}{lll} \mathsf{show}_{\mathsf{U}} :: & \mathsf{U} & \to \mathsf{String} \\ \mathsf{show}_{\mathsf{C}} :: ... \Rightarrow \mathsf{C} \ \mathsf{a} & \to \mathsf{String} \\ \mathsf{show}_{+} :: ... \Rightarrow \mathsf{a} : \!\!\! + \!\!\! : \mathsf{b} \to \mathsf{String} \\ ... \end{array}
```

A common encoding for isomorphisms

```
\begin{tabular}{lll} \mbox{\bf data $T$} &= ... & -- & \mbox{User-defined datatype} \\ \mbox{\bf type } & \mbox{Rep}_T = ... & -- & \mbox{Structure representation} \\ \mbox{from } & :: & T \rightarrow \mbox{Rep}_T \\ \mbox{to } & :: & \mbox{Rep}_T \rightarrow T \\ \end{tabular}
```

There are several ways to encode polymorphic recursion. We will use type classes.

 Standard classes already use polymorphic recursion for deriving instances: Show, Eq, etc.

- Standard classes already use polymorphic recursion for deriving instances: Show, Eq, etc.
- The class declaration specifies the type signature.

- Standard classes already use polymorphic recursion for deriving instances: Show, Eq, etc.
- The class declaration specifies the type signature.
- Each recursive case is specified by an instance of the class.

- Standard classes already use polymorphic recursion for deriving instances: Show, Eq, etc.
- The class declaration specifies the type signature.
- Each recursive case is specified by an instance of the class.

There are several ways to encode polymorphic recursion. We will use type classes.

- Standard classes already use polymorphic recursion for deriving instances: Show, Eq, etc.
- The class declaration specifies the type signature.
- Each recursive case is specified by an instance of the class.

A simplified definition of the Show class:

#### class Show a where

show ::  $a \rightarrow String$ 

The instances for each structure representation case:

The instances for each structure representation case:

Unit:

instance Show U where  $show = show_U$ 

The instances for each structure representation case:

Unit:

instance Show U where  $show = show_U$ 

Constructor name:

instance Show a  $\Rightarrow$  Show (C a) where show = show<sub>C</sub> show

The instances for each structure representation case:

Unit:

Constructor name:

instance Show U where  $show = show_U$ 

instance Show  $a \Rightarrow Show (C a)$  where  $show = show_C show$ 

Binary product:

**instance** (Show a, Show b)  $\Rightarrow$  Show (a :×: b) **where** show = show<sub>×</sub> show show

The instances for each structure representation case:

Unit:

Constructor name:

instance Show U where  $show = show_U$ 

instance Show  $a \Rightarrow Show (C a)$  where  $show = show_C show$ 

Binary product:

**instance** (Show a, Show b) 
$$\Rightarrow$$
 Show (a :×: b) **where** show = show<sub>×</sub> show show

Binary sum:

instance (Show a, Show b) 
$$\Rightarrow$$
 Show (a :+: b) where show = show<sub>+</sub> show show

```
Now, recall \mathsf{show}_{\mathsf{Rep}_D}: (\mathsf{p} \to \mathsf{String}) \to \mathsf{Rep}_D \; \mathsf{p} \to \mathsf{String} \mathsf{show}_{\mathsf{Rep}_D} \; \mathsf{show}_{\mathsf{p}} = \mathsf{show}_+ \; (\mathsf{show}_C \; \mathsf{show}_U) \; (\mathsf{show}_C \; (\mathsf{show}_\times \; \mathsf{show}_{\mathsf{Int}} \; \mathsf{show}_{\mathsf{p}}))
```

```
Now, recall \mathsf{show}_{\mathsf{Rep}_D}: \mathsf{show}_{\mathsf{Rep}_D}: \mathsf{show}_{\mathsf{Rep}_D}:: (\mathsf{p} \to \mathsf{String}) \to \mathsf{Rep}_D \mathsf{p} \to \mathsf{String} \mathsf{show}_{\mathsf{Rep}_D} \mathsf{show}_{\mathsf{p}} \mathsf{show}_{\mathsf{p}}
```

Compare to the new version that is now possible:

```
\begin{array}{l} \mathsf{show}'_{\mathsf{Rep}_D} :: \mathsf{Show} \; \mathsf{p} \Rightarrow \mathsf{Rep}_D \; \mathsf{p} \to \mathsf{String} \\ \mathsf{show}'_{\mathsf{Rep}_D} = \mathsf{show} \end{array}
```

```
\begin{array}{l} \mathsf{show}_D' :: \mathsf{Show} \ \mathsf{p} \Rightarrow \mathsf{D} \ \mathsf{p} \to \mathsf{String} \\ \mathsf{show}_D' = \mathsf{show}_{\mathsf{Rep}_D}' \circ \mathsf{from}_D \end{array}
```

To define the show function for D , we still need to define another function:

```
\begin{array}{l} \mathsf{show}_\mathsf{D}' :: \mathsf{Show} \ \mathsf{p} \Rightarrow \mathsf{D} \ \mathsf{p} \to \mathsf{String} \\ \mathsf{show}_\mathsf{D}' = \mathsf{show}_\mathsf{Rep}' \circ \mathsf{from}_\mathsf{D} \end{array}
```

#### Next goal:

• Define one show function that knows how to convert any type T to its structure representation type  $Rep_T$ , given an isomorphism between T and  $Rep_T$ .

We define a class of function pairs.

• We again use a type class, but with the addition of a type family.

- We again use a type class, but with the addition of a type family.
- Each function pair implements an isomorphism between a datatype  $\mathsf{T}$  and its structure representation  $\mathsf{Rep}_\mathsf{T}$ :

- We again use a type class, but with the addition of a type family.
- Each function pair implements an isomorphism between a datatype  $\mathsf{T}$  and its structure representation  $\mathsf{Rep}_\mathsf{T}$ :

- We again use a type class, but with the addition of a type family.
- Each function pair implements an isomorphism between a datatype  $\mathsf{T}$  and its structure representation  $\mathsf{Rep}_\mathsf{T}$ :

```
from :: T \to Rep_T
```

- We again use a type class, but with the addition of a type family.
- Each function pair implements an isomorphism between a datatype
   T and its structure representation Rep<sub>T</sub>:

$$\mathsf{from} :: \mathsf{T} \to \mathsf{Rep}_\mathsf{T}$$

to :: 
$$Rep_T \rightarrow T$$

We define a class of function pairs.

- We again use a type class, but with the addition of a type family.
- Each function pair implements an isomorphism between a datatype  $\mathsf{T}$  and its structure representation  $\mathsf{Rep}_\mathsf{T}$ :

$$\mathsf{from} :: \mathsf{T} \to \mathsf{Rep}_\mathsf{T} \qquad \qquad \mathsf{to} :: \mathsf{Rep}_\mathsf{T} \to \mathsf{T}$$

• Each requires two types, so each instance must have two types (unlike the Show instances which needed only the structure representation type).

- We again use a type class, but with the addition of a type family.
- Each function pair implements an isomorphism between a datatype  $\mathsf{T}$  and its structure representation  $\mathsf{Rep}_\mathsf{T}$ :

$$\mathsf{from} :: \mathsf{T} \to \mathsf{Rep}_\mathsf{T} \qquad \qquad \mathsf{to} :: \mathsf{Rep}_\mathsf{T} \to \mathsf{T}$$

- Each requires two types, so each instance must have two types (unlike the Show instances which needed only the structure representation type).
- Rep<sub>T</sub> is precisely determined by T, so really we only need one unique type and a second type derivable from the first.

- We again use a type class, but with the addition of a type family.
- Each function pair implements an isomorphism between a datatype  $\mathsf{T}$  and its structure representation  $\mathsf{Rep}_\mathsf{T}$ :

$$\mathsf{from} :: \mathsf{T} \to \mathsf{Rep}_\mathsf{T} \qquad \qquad \mathsf{to} :: \mathsf{Rep}_\mathsf{T} \to \mathsf{T}$$

- Each requires two types, so each instance must have two types (unlike the Show instances which needed only the structure representation type).
- Rep<sub>T</sub> is precisely determined by T, so really we only need one unique type and a second type derivable from the first.
- In this case, a (1) multiparameter type class with a functional dependency and a (2) type class with a type family are equally expressive. (It's a matter of taste, really.)

The type class:

class Generic a where

type Rep a

 $from:: a \to Rep \ a$ 

to :: Rep  $a \rightarrow a$ 

#### The type class:

```
class Generic a where
```

#### type Rep a

```
from :: a \rightarrow Rep a to :: Rep a \rightarrow a
```

Rep is a type family or, more precisely, an associated type synonym.

The type class:

```
class Generic a where
```

```
type Rep a
```

```
from :: a \rightarrow Rep a to :: Rep a \rightarrow a
```

- Rep is a type family or, more precisely, an associated type synonym.
- Think of Rep as a function on types. Given a unique type (index)
   T, you get a type (synonym) Rep T.

#### The type class:

```
class Generic a where type Rep a
```

```
from :: a \rightarrow Rep a to :: Rep a \rightarrow a
```

- Rep is a type family or, more precisely, an associated type synonym.
- Think of Rep as a function on types. Given a unique type (index)
   T, you get a type (synonym) Rep T.
- Note that Rep T need not be different from Rep U even though T and U are different.

#### The type class:

#### class Generic a where

#### type Rep a

```
from :: a \rightarrow Rep a to :: Rep a \rightarrow a
```

- Rep is a type family or, more precisely, an associated type synonym.
- Think of Rep as a function on types. Given a unique type (index)
   T, you get a type (synonym) Rep T.
- Note that Rep T need not be different from Rep U even though T and U are different.
- Concretely: two datatypes may have the same representation.

We need Generic instances for every datatype that we want to use with generic functions.

We need Generic instances for every datatype that we want to use with generic functions.

The instance for D uses definitions that we've already seen:

```
\label{eq:continuous_problem} \begin{split} & \textbf{instance} \; \mathsf{Generic} \; (\mathsf{D} \; \mathsf{p}) \; \textbf{where} \\ & \textbf{type} \; \mathsf{Rep} \; (\mathsf{D} \; \mathsf{p}) = \mathsf{Rep}_{\mathsf{D}} \; \mathsf{p} \\ & \mathsf{from} = \mathsf{from}_{\mathsf{D}} \\ & \mathsf{to} \; = \mathsf{to}_{\mathsf{D}} \end{split}
```

We need Generic instances for every datatype that we want to use with generic functions.

The instance for D uses definitions that we've already seen:

```
\label{eq:constraints} \begin{split} & \textbf{instance} \  \, \textbf{Generic} \  \, (D \ p) \  \, \textbf{where} \\ & \textbf{type} \  \, \textbf{Rep} \  \, (D \ p) = \textbf{Rep}_D \  \, p \\ & \textbf{from} = \textbf{from}_D \\ & \textbf{to} = \textbf{to}_D \end{split}
```

• Other instances are defined similarly.

We need Generic instances for every datatype that we want to use with generic functions.

The instance for D uses definitions that we've already seen:

```
\label{eq:constraints} \begin{split} & \textbf{instance} \  \, \textbf{Generic} \  \, (D \ p) \  \, \textbf{where} \\ & \textbf{type} \  \, \textbf{Rep} \  \, (D \ p) = \textbf{Rep}_D \  \, p \\ & \textbf{from} = \textbf{from}_D \\ & \textbf{to} = \textbf{to}_D \end{split}
```

- Other instances are defined similarly.
- In fact, Rep T, from, and to are precisely determined by the definition of T, so these instances can be automatically generated (e.g. using Template Haskell or a preprocessor).

## The Generic show Function

#### Finally:

```
gshow :: (Show (Rep a), Generic a) \Rightarrow a \rightarrow String gshow = show \circ from
```

#### GP in General

• Datatype-generic programming:

#### GP in General

- Datatype-generic programming:
  - Datatype is the parameter

- Datatype-generic programming:
  - Datatype is the parameter
  - ▶ Instantiation gives you a large class of generic functions

- Datatype-generic programming:
  - Datatype is the parameter
  - ▶ Instantiation gives you a large class of generic functions
- Many generic functions:

- Datatype-generic programming:
  - Datatype is the parameter
  - ▶ Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:
  - Instant Generics presented here

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:
  - Instant Generics presented here
  - Generic Deriving GHC ≥ 7.2, similar to Instant Generics

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:
  - Instant Generics presented here
  - Generic Deriving GHC ≥ 7.2, similar to Instant Generics
  - ► EMGM maintained by me

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:
  - Instant Generics presented here
  - Generic Deriving GHC ≥ 7.2, similar to Instant Generics
  - ▶ EMGM maintained by me
  - Regular folds, etc.

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:
  - Instant Generics presented here
  - Generic Deriving GHC ≥ 7.2, similar to Instant Generics
  - ▶ EMGM maintained by me
  - Regular folds, etc.
  - Multirec mutually recursive datatypes, folds, etc.

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - ▶ Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:
  - ▶ Instant Generics presented here
  - Generic Deriving GHC ≥ 7.2, similar to Instant Generics
  - ▶ EMGM maintained by me
  - Regular folds, etc.
  - ▶ Multirec mutually recursive datatypes, folds, etc.
  - ▶ Scrap Your Boilerplate (SYB) GHC, traversals, queries

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing ( show ) and parsing ( read )
  - Compression, serialization, and the reverse
  - Comparison, equality
  - ► Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:
  - ▶ Instant Generics presented here
  - Generic Deriving GHC ≥ 7.2, similar to Instant Generics
  - ▶ EMGM maintained by me
  - Regular folds, etc.
  - Multirec mutually recursive datatypes, folds, etc.
  - Scrap Your Boilerplate (SYB) GHC, traversals, queries
  - **.**..

#### References

#### Generic Programming in Haskell:

- Johan Jeuring, Sean Leather, José Pedro Magalhães, Alexey Rodriguez Yakushev. Libraries for Generic Programming in Haskell. AFP 2008. pp. 165-229, 2009.
- Generic Deriving: http://www.haskell.org/haskellwiki/GHC.Generics