Dissecting Different Flavors of Generic Programming in One Haskell Universe

Presented to Galois

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What is Generic Programming?

In programming languages, the adjective "generic" is heavily overloaded.

- Java/C# generics
- C++ templates
- Ada generic packages

What is Generic Programming?

The goal is often the same.

A higher level of abstraction than "normally" available

The technique is also often similar.

Some form of parameterization and instantiation

Examples of Generic Programming

Java/C#:

```
public class Stack<T>
{
  public void push(T item) {...}
  public T pop() {...}
}
```

In other words:

- Java-style generics $\approx$ parametric polymorphism

Examples of Generic Programming

C++:

```
template<typename T, typename Compare>
T& min(T& a, T& b, Compare comp) {
  if (comp(b, a))
    return b;
  return a;
}
```

In other words:

- C++ templates $\approx$ ad-hoc polymorphism

Generic Programming in Haskell

"Generic programming":

- For other languages, the term tends to be used for late additions.
- Parametric and ad-hoc polymorphism were available in Haskell from the beginning.

In Haskell, we have come to use "generic programming" for datatype-generic programming (a.k.a. "polytypism" or "shape/structure polymorphism").

Datatype-Generic Programming

What is datatype-generic programming?

- Parameterize a function over the *structure* of datatypes
- Instantiate the function with a particular type

The result is a function that

- **works with many types** (polymorphism) but
- **uses knowledge of the type** (unlike parametric) and
- **need not be redefined for every type** (unlike ad-hoc).

Generic Functions

Applications

- Pretty-printing (e.g. `show`), parsing (e.g. `read`)
- Compression, serialization, marshalling (and their inverses)
- Comparison, equality
- (Co-)recursion, map, zip, zippers
- Traversals, queries, updates

Generic Platforms

Many different implementations:

- Preprocessors:
  - PolyP
  - Generic Haskell
- Libraries
  - Scrap Your Boilerplate (SYB) – included with GHC for a long time
  - Extensible and Modular Generics for the Masses (EMGM)
  - Regular – recursion schemes
  - Multirec – mutually recursive datatypes
  - Generic Deriving – available in GHC $\geqslant$ 7.2, similar to Instant Generics
  - (and many, many more)

Generic Flavors

The implementations can be grouped into flavors depending on how they view the structure of a datatype.

Some flavors (or views):

Spine A constructor is a sequence of types.
Example: SYB

Sums-of-products A datatype is a collection of alternative tuples of types.
Example: Generic Deriving

Fixed point A datatype is a sums-of-products with recursive points.
Example: Multirec

Dissecting a Datatype: Sums-of-Products

data $\mathsf{T}_{\mathsf{sum}} = \mathsf{A}_1 \mid \mathsf{A}_2$

A datatype can have:

- Alternatives: unique constructors ($\geqslant 0$)

Dissecting a Datatype: Sums-of-Products

data $\mathsf{T}_{\mathsf{prod}} = \mathsf{P}_2$ Char Int

A datatype can have:

- Fields: types for each constructor ($\geqslant 0$)

Dissecting a Datatype: Sums-of-Products

Other features that are modeled:

- Constant types: each type in a field
- Parameters: type variables ($\geqslant 0$)

Features that are not modeled:

- Recursion
- Nesting (though it can be)

Modeling a Sum

To model (nested) alternatives:

```
data Either a b = Left a | Right b
```

For syntactic elegance:

```
data a :+: b = L a | R b
```

Modeling a Product

To model (nested) fields:

```
data (,) a b = (,) a b
```

For syntactic elegance:

```
data a :×: b = a :×: b
```

Modeling Other Structures

A constructor without fields:

```
data U = U
```

A constructor name:

```
data C a = C String a
```

A field type:

```
data K a = K a
```

Note: There are other features of datatypes, but we consider only the above.

Modeling an Example

An example datatype:

```
data E a = E1 | E2 a (E a) Int
```

The corresponding structure representation type:

```
type Rep_E a = C U :+: C (K a :×: K (E a) :×: K Int)
```

Notes:

- `:+:` is **infixr** 5 and `:×:` is **infixr** 6.
- Operators nest to the right.

Converting Between Types: Isomorphism

- Generic functions work on the sums-of-products model.
- But first we need to convert between the model and the actual value of the datatype.
- We define an isomorphism: two total, dual functions.

```
fromE :: E a → RepE a
fromE E1          = L (C "E1" U)
fromE (E2 x e i) = R (C "E2" ((K x) :×: (K e) :×: (K i)))
```

```
toE :: RepE a → E a
toE (L (C "E1" U))                                  = E1
toE (R (C "E2" ((K x) :×: (K e) :×: (K i)))) = E2 x e i
```

Converting Between Types: Isomorphism

For convenience, we join the representation type and isomorphism in a type class `Generic` with an associated type synonym `Rep`.

```
class Generic a where
  type Rep a
  from :: a → Rep a
  to   :: Rep a → a
```

The instance for `E`:

```
instance Generic (E a) where
  type Rep (E a) = Rep_E a
  from = from_E
  to   = to_E
```

Generic Functions

A generic function

- Is defined on each case of the structure representation and
- Works for every datatype that has a structure representation and isomorphism.

Example: $\text{show}_{\text{Rep a}} :: a \rightarrow \text{String}$

- We will define a `show` function for each case.

Defining a Generic Function: show

Unit:

$$\text{show}_U :: U \to \text{String}$$
$$\text{show}_U\ U = ""$$

Constructor name:

$$\text{show}_C :: (a \to \text{String}) \to C\ a \to \text{String}$$
$$\text{show}_C\ \text{show}_a\ (C\ nm\ a) = "("\ +\!\!+\ nm\ +\!\!+\ "\ \ "\ +\!\!+\ \text{show}_a\ a\ +\!\!+\ ")"$$

Field:

$$\text{show}_K :: (a \to \text{String}) \to K\ a \to \text{String}$$
$$\text{show}_K\ \text{show}_a\ (K\ a) = \text{show}_a\ a$$

Defining a Generic Function: show

Binary sum:

```
show+ :: (a → String) → (b → String) → a :+: b → String
show+ showa _ (L a) = showa a
show+ _ showb (R b) = showb b
```

Binary product:

```
show× :: (a → String) → (b → String) → a :×: b → String
show× showa showb (a :×: b) = showa a ++ "  " ++ showb b
```

Defining a Generic Function: show

Recall:

```
type Rep_E a = C U :+: C (K a :×: K (E a) :×: K Int)
```

We can define a show function (assuming show_{Int}):

```
show_RepE :: (a → String) → ((a → String) → E a → String)
          → Rep_E a → String

show_RepE show_a show_E =
  show_+ (show_C show_U)
         (show_C (show_× (show_K show_a)
                         (show_× (show_K (show_E show_a)) (show_K show_Int))))
```

Defining a Generic Function: show

$$
\begin{aligned}
&\mathsf{show}_{\mathsf{Rep_E}} :: (\mathsf{a} \to \mathsf{String}) \to ((\mathsf{a} \to \mathsf{String}) \to \mathsf{E\ a} \to \mathsf{String}) \\
&\qquad \to \mathsf{Rep_E\ a} \to \mathsf{String} \\
&\mathsf{show}_{\mathsf{Rep_E}}\ \mathsf{show_a}\ \mathsf{show_E} = \\
&\quad \mathsf{show}_{+}\ (\mathsf{show_C}\ \mathsf{show_U}) \\
&\qquad (\mathsf{show_C}\ (\mathsf{show}_{\times}\ (\mathsf{show_K}\ \mathsf{show_a}) \\
&\qquad\qquad (\mathsf{show}_{\times}\ (\mathsf{show_K}\ (\mathsf{show_E}\ \mathsf{show_a}))\ (\mathsf{show_K}\ \mathsf{show_{Int}}))))
\end{aligned}
$$

The $\mathsf{show_E}$ function itself is just an isomorphism away:

$$
\begin{aligned}
&\mathsf{show_E} :: (\mathsf{a} \to \mathsf{String}) \to \mathsf{E\ a} \to \mathsf{String} \\
&\mathsf{show_E}\ \mathsf{show_a} = \mathsf{show}_{\mathsf{Rep_E}}\ \mathsf{show_a}\ \mathsf{show_E} \circ \mathsf{from_E}
\end{aligned}
$$

Defining a Generic Function: show

$$
\begin{aligned}
&\mathsf{show}_{\mathsf{Rep_E}} :: (\mathsf{a} \to \mathsf{String}) \to ((\mathsf{a} \to \mathsf{String}) \to \mathsf{E\ a} \to \mathsf{String}) \\
&\qquad \to \mathsf{Rep_E\ a} \to \mathsf{String} \\
&\mathsf{show}_{\mathsf{Rep_E}}\ \mathsf{show_a}\ \mathsf{show_E} = \\
&\quad \mathsf{show}_{+}\ (\mathsf{show_C}\ \mathsf{show_U}) \\
&\qquad (\mathsf{show_C}\ (\mathsf{show}_{\times}\ (\mathsf{show_K}\ \mathsf{show_a}) \\
&\qquad\qquad (\mathsf{show}_{\times}\ (\mathsf{show_K}\ (\mathsf{show_E}\ \mathsf{show_a}))\ (\mathsf{show_K}\ \mathsf{show_{Int}}))))
\end{aligned}
$$

Some observations:

- This is **not** a generic function.
- It is defined on the structure of E, not on datatypes in general.
- It demonstrates a predictable pattern for defining the generic function.

Defining a Generic Function: show

Consider these typical expressions and their types:

$$\begin{aligned} &\mathsf{show}_{\mathsf{C}}\ \mathsf{show}_{\mathsf{U}} &&:: \mathsf{C\ U} &&\to \mathsf{String} \\ &\mathsf{show}_{\times}\ (\mathsf{show}_{\mathsf{K}}\ \mathsf{show}_{\mathsf{Int}})\ (\mathsf{show}_{\mathsf{K}}\ \mathsf{show}_{\mathsf{Char}}) &&:: (\mathsf{K\ Int} :\times: \mathsf{K\ Char}) &&\to \mathsf{String} \end{aligned}$$

- $\mathsf{show}_?$ functions call other $\mathsf{show}_?$ functions.
- They can be considered recursive but not in the usual way.
- Polymorphic recursion – functions with different types that have a common scheme that reference each other

Defining a Generic Function: show

There are several ways to encode polymorphic recursion. We use type classes.

- The class declaration specifies the type signature.
- Each recursive (type) case is specified by an instance of the class.

A simplified definition of the Show class:

```
class Show a where
  show :: a → String
```

Defining a Generic Function: show

Some of the instances for each structure representation case:

Constructor name:

$$\textbf{instance}\ \text{Show a} \Rightarrow \text{Show (C a)}\ \textbf{where}$$
$$\quad \text{show} = \text{show}_{\text{C}}\ \text{show}$$

Binary sum:

$$\textbf{instance}\ (\text{Show a}, \text{Show b}) \Rightarrow \text{Show (a :+: b)}\ \textbf{where}$$
$$\quad \text{show} = \text{show}_{+}\ \text{show show}$$

The remaining instances are straightforward.

Defining a Generic Function: show

Now, compare:

$$
\begin{aligned}
&\mathsf{show}_{\mathsf{Rep_E}} :: (\mathsf{a} \to \mathsf{String}) \to ((\mathsf{a} \to \mathsf{String}) \to \mathsf{E\ a} \to \mathsf{String}) \\
&\qquad \to \mathsf{Rep_E\ a} \to \mathsf{String} \\
&\mathsf{show}_{\mathsf{Rep_E}}\ \mathsf{show_a}\ \mathsf{show_E} = \\
&\quad \mathsf{show}_{+}\ (\mathsf{show_C}\ \mathsf{show_U}) \\
&\qquad (\mathsf{show_C}\ (\mathsf{show}_{\times}\ (\mathsf{show_K}\ \mathsf{show_a}) \\
&\qquad\qquad (\mathsf{show}_{\times}\ (\mathsf{show_K}\ (\mathsf{show_E}\ \mathsf{show_a}))\ (\mathsf{show_K}\ \mathsf{show_{Int}}))))
\end{aligned}
$$

To:

$$
\begin{aligned}
&\mathsf{show}_{\mathsf{Rep_E}} :: (\mathsf{Show\ a}, \mathsf{Show\ (E\ a)}) \Rightarrow \mathsf{Rep_E\ a} \to \mathsf{String} \\
&\mathsf{show}_{\mathsf{Rep_E}} = \mathsf{show}
\end{aligned}
$$

Defining a Generic Function: `show`

Finally, we can use a slightly different `Show` class to support generic functions for any type that has a representation.

```
class Show a where
  show :: a → String
  default show :: (Show (Rep a), Generic a) ⇒ a → String
  show = show ∘ from
```

- This uses default signatures: if type `a` has the instances `Show (Rep a)` and `Generic a`, then the given definition is used.

To define the instance for `E`:

```
instance Show a ⇒ Show (E a)
```

GP in General

- Datatype-generic programming:
  - Datatype is the parameter
  - Instantiation gives you a large class of generic functions
- Many generic functions:
  - Pretty-printing (`show`) and parsing (`read`)
  - Compression, serialization, and the reverse
  - Comparison, equality
  - Folds (catamorphisms), unfolds (anamorphisms), maps, zips, zippers
  - Traversals, updates, queries
- Many different libraries:
  - Instant Generics – presented here
  - Generic Deriving – GHC $\geqslant$ 7.2, similar to Instant Generics
  - EMGM – maintained by me
  - Regular – folds, etc.
  - Multirec – mutually recursive datatypes, folds, etc.
  - Scrap Your Boilerplate (SYB) – GHC, traversals, queries
  - ...

References

Generic Programming in Haskell:

- Johan Jeuring, Sean Leather, José Pedro Magalhães, Alexey Rodriguez Yakushev. *Libraries for Generic Programming in Haskell*. AFP 2008. pp. 165-229, 2009.
- Generic Deriving: `http://www.haskell.org/haskellwiki/GHC.Generics`