SPL Fama French

Structure

1. Introduction
2. Data Preparation
3. Simple Regression
4. Replicating the 3-Factor model
5. S&P500 Results
6. Going 5-Factor

1. Introduction

The Fama French model is a model for explaining stock returns. It extends the classical Capital Asset Pricing Model (CAPM) by having additional factors.

$$R_i - R_F = \beta \cdot (R_M - R_F)$$

Fama and French (1993) introduces *SMB* (Small market cap Minus Big / Size) and *HML* (High book-to-market Minus Low / Value) to capture the observation that small capitalization and high book value to market value ("value" in contrast to "growth") stocks tend to outperform the market.

$$R_i - R_F = \beta_M \cdot (R_M - R_F) + \beta_S \cdot SMB + \beta_V \cdot HML$$

Fama and French (2015) adds *RMW* (Robust operating profit Minus Weak / Profitability) and *CMA* (Conservative investment strategy Minus Aggressive / Investment).

$$R_i - R_F = \beta_M \cdot (R_M - R_F) + \beta_S \cdot SMB + \beta_V \cdot HML + \beta_P \cdot RMW + \beta_I \cdot CMA$$

Fama French factors are calculated as return spreads between two portfolios, e.g. SMB is the difference between the return of a small cap portfolio and that of a large cap portfolio.

We choose the Fama French model due to the high quality data available at Kenneth R. French's data library

Refer to Wikepedia for more information.

2. Data Preparation

2.1 Fama French Data

French's data library contains data for the factors, corresponding market returns and risk free rates, as well as the portfolios returns featured in the papers:

- **3 Factors** 1926.07.01 to 2018.03.29 as daily / weekly / monthly data
- **5 Factors** 1963.07.01 to 2018.03.29 as daily / monthly / yearly data
- **25 Portfolios (5x5)** formed on Size and Book-to-Market 1926.07 to 2018.03 corresponding to the Fama and French (1993) 3-factor setup (P24 Table 6).

The downloaded CSV data contains headers and footers that need to be removed before input to R.

Further, the downloaded data is in percentage returns (e.g. 20% return stored as 20). This will not affect replicating the Fama French model, since the portfolio returns are also provided in percentages. However, we need to be careful when regressing stock returns on Fama French factors, as those are calculated from daily prices and 20% will be 0.2. We can always verify the correctness of data magnitude by checking the market beta to be around 1 and not in 0.01s or 100s.

Running the summary statistics of the monthly excess returns on the 25 stock portfolios reveals that they differ from those reported in Fama and French (1993), table 2. Hence we expect that replication of the 3 factors model will generate slightly different regression results.

```r
# resize the output to 5x5 format like Fama French paper
resize <- function(x)
{
  df = data.frame(matrix(x, nrow=5, byrow = TRUE))
  colnames(df) = c("Low", "2", "3", "4", "High")
  rownames(df) = c("Small", "2", "3", "4", "Big")
  return(df)
}

P25.return <- colMeans(P25[,-1]-FF3$RF)
P25.std <- apply(P25[,-1]-FF3$RF, 2, sd)
kable(resize(P25.return), digits = 2)
```

	Low	2	3	4	High
Small	0.44	0.82	0.91	1.08	1.27
2	0.33	0.65	0.88	0.91	0.97
3	0.38	0.70	0.64	0.88	0.97
4	0.43	0.39	0.62	0.78	0.96
Big	0.36	0.43	0.46	0.52	0.63

```r
kable(resize(P25.std), digits = 2)
```

	Low	2	3	4	High
Small	7.93	7.09	6.66	6.31	6.53
2	7.48	6.41	5.85	5.44	6.11
3	6.84	5.82	5.27	4.99	5.78
4	6.01	5.52	5.15	4.96	5.80
Big	5.29	4.94	4.64	4.51	4.85

Dependent variables: Excess returns on 25 stock portfolios formed on ME and BE/ME

Size quintile	Book-to-market equity (BE/ME) quintiles: Means, Low	2	3	4	High	Standard deviations, Low	2	3	4	High
Small	0.39	0.70	0.79	0.88	1.01	7.76	6.84	6.29	5.99	6.27
2	0.44	0.71	0.85	0.84	1.02	7.28	6.42	5.85	5.33	6.06
3	0.43	0.66	0.68	0.81	0.97	6.71	5.71	5.27	4.92	5.69
4	0.48	0.35	0.57	0.77	1.05	5.97	5.44	5.03	4.95	5.75
Big	0.40	0.36	0.32	0.56	0.59	4.95	4.70	4.38	4.27	4.85

E.F. Fama and K.R. French,

2.2 S&P 500 Stock Data

The `BatchGetSymbols` library has a function `BatchGetSymbols()` for downloading S&P500 stock prices and volumes from a cached repository, thus avoiding problems when downloading large amount of data directly from Yahoo or Google (e.g. the `getSymbols` function from the `quantmod` library)

```
library(BatchGetSymbols)

# Get company information incl. tickers for SP500 stocks
Companies <- GetSP500Stocks()
```

```
kable(head(Companies, n=5)[,1:5])
```

tickers	company	SEC.filings	GICS.Sector	GICS.Sub.Industry
MMM	3M Company	reports	Industrials	Industrial Conglomerates
ABT	Abbott Laboratories	reports	Health Care	Health Care Equipment
ABBV	AbbVie Inc.	reports	Health Care	Pharmaceuticals
ABMD	ABIOMED Inc	reports	Health Care	Health Care Equipment
ACN	Accenture plc	reports	Information Technology	IT Consulting & Other Services

```
kable(head(Companies[,6:9], n=5))
```

Address	Date.first.added	CIK	NA
St. Paul, Minnesota		66740	1902
North Chicago, Illinois	1964-03-31	1800	1888
North Chicago, Illinois	2012-12-31	1551152	2013 (1888)
Danvers, Massachusetts	2018-05-31	815094	1981
Dublin, Ireland	2011-07-06	1467373	1989

Function `GetSP500Stocks()` returns S&P500 company information including name, tickers and sectors. For downloading the price data, we only need the tickers.

```
# Batch download data from Yahoo Finance
Stocks<- BatchGetSymbols(tickers = Companies$tickers,
                         first.date = "2017-01-01",
                         last.date = "2017-12-31")
```

The downloaded list contains 2 dataframes:

- **df.control** contains descriptive information like whether the download for the ticker is successful.

- **df.tickers** contains the downloaded price data. Each row is the price data for one ticker at one date, hence we need to process the data into a format easier to work with.

(Use `kable()` function in `Knitr` library to format table output in PDF.)

```
kable(head(Stocks$df.control, n=3))
```

ticker	src	download.status	total.obs	perc.benchmark.dates	threshold.decision
MMM	yahoo	OK	251	1	KEEP
ABT	yahoo	OK	251	1	KEEP
ABBV	yahoo	OK	251	1	KEEP

```
kable(head(Stocks$df.tickers[,1:5], n=3))
```

price.open	price.high	price.low	price.close	volume
178.83	180.00	177.22	178.05	2509300
178.03	178.90	177.61	178.32	1542000
178.26	179.14	176.89	177.71	1447800

```
kable(head(Stocks$df.tickers[,6:10], n=3))
```

price.adjusted	ref.date	ticker	ret.adjusted.prices	ret.closing.prices
171.7699	2017-01-03	MMM	NA	NA
172.0304	2017-01-04	MMM	0.0015164	0.0015165
171.4419	2017-01-05	MMM	-0.0034209	-0.0034208

Below code selects the downloaded tickers (marked by `df.control$threshold.decision=="KEEP"`) and use the dates from 3M as the date column for dataframe `SP500.data`.

It reads stocks ticker by ticker and matches previous price series by date. The unmatched dates will have `NA`s. The new stock price series is merged into the dataframe as a new column with the ticker symbol as the column name.

```
good.tickers <- Stocks$df.control$
        ticker[Stocks$df.control$threshold.decision=="KEEP"]

# Fill dates as the first stock "MMM" happens to have complete dates
# (column name = "date")
SP500.data<-data.frame(date = Stocks$
                         df.tickers$
                         ref.date[1:max(Stocks$df.control$total.obs)])

for(i in 1:length(good.tickers))
{
  # X is a temp dataframe that has 2 columns,
  # 1st is date (for matching), 2nd is the actual data (e.g. closing price)

  # Choose relevant data by matching tickers
  X <- data.frame(date =
        Stocks$df.tickers$ref.date[Stocks$df.tickers$ticker==good.tickers[i]],
        Stocks$df.tickers$price.adjusted[Stocks$df.tickers$ticker==good.tickers[i]])
```

```
  # change the column name of X to be the ticker of the stock
  # colnames(X)[2] = good.tickers[i] # this one don't work
  colnames(X)[2] <- Stocks$df.tickers$
        ticker[Stocks$df.tickers$ticker==good.tickers[i]]

  # merge X as a new column into SP500.data by matching date
  # missing dates will have NA by default
  SP500.data <- merge.data.frame(SP500.data, X, by = "date", all.x = TRUE)
}
```

We write the processed data to CSVs.

3. Simple Regression

`readxl` library for reading Excel data.

The imported data would be stored as `data.frame` and must be `unlist()` into vectors for regression. (data.frame is also a list in R)

```
# unlist: convert the data into vector format
rmrf<-unlist(FF3[,2])
```

OLS regression can be performed with two lines of code:

```
y <- lm(rirf ~ rmrf + smb + hml);
summary(y)
```

```
##
## Call:
## lm(formula = rirf ~ rmrf + smb + hml)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.5622 -1.5796 -0.2347  1.4718 12.2031
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.32141    0.14237  -2.258   0.0246 *
## rmrf         0.97423    0.03493  27.888   <2e-16 ***
## smb          1.55399    0.05201  29.881   <2e-16 ***
## hml         -0.12312    0.05829  -2.112   0.0354 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.568 on 338 degrees of freedom
## Multiple R-squared:  0.896,  Adjusted R-squared:  0.895
## F-statistic: 970.4 on 3 and 338 DF,  p-value: < 2.2e-16
```

`summary(y)` contains the regression results and specific results could be obtained, e.g., via:

```
summary(y)$coefficients
```

which returns the regression betas and their standard errors, t-values and p-values in a matrix.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.32	0.14	-2.26	0.02
rmrf	0.97	0.03	27.89	0.00
smb	1.55	0.05	29.88	0.00
hml	-0.12	0.06	-2.11	0.04

4. Replicating the 3-Factor model

To check that we have implemented the Fama French model correctly, we try to replicate the results of table 6 of Fama and French (1993) which involves monthly return data of 25 value-weighted portfolios from July 1963 to December 1991.

The data set structures the data as 1 column of months ($YYYYDD$ format) plus 25 columns of portfolio monthly returns. The first return column is SMALL (market cap) LoBM (low book-to-market / "growth"). The first 5 return columns are all small cap but with increasing book-to-market ratios. The last 5 return columns are all large cap with the last column being BIG (market cap) HiBM (high book-to-market / "value").

In reporting, results are structured in a matrix with rows representing market cap and columns for book to market ratios.

4.1 Batch regression

With the OLS regression code working, below code runs regression on each portfolio and saves the results in a list `results`.

```
# Store summaries into a results list
results <- list()
# The first column of P25 is dates, not data
for(i in 1:(ncol(P25)-1))
{
  rirf<-unlist(P25[,i+1])-rf # Data starts from the 2nd col of P25
  y<-lm(rirf~rmrf+smb+hml)
  results[[i]]<-summary(y)
}
```

4.2 Formatting the results

We then read out the results, stack them into corresponding vectors, then reshape them into the 5×5 format as in the paper for ease of comparison.

The regression results are highly similar to table 6 in Fama and French (1993) and the differences are due to the data discrepancies in the downloaded portfolio returns (c.f. section 2.1).

```
betas <- vector()
std.errors <- vector()
t.values <- vector()
R.squareds <- vector()
# save all betas
for(i in 1:(ncol(P25)-1))
{
  betas <- cbind(betas,results[[i]]$coefficients[,1])
  std.errors <- cbind(std.errors,results[[i]]$sigma)
  t.values <- cbind(t.values, results[[i]]$coefficients[,3])
  R.squareds <- cbind(R.squareds, results[[i]]$adj.r.squared)
}

# resize alpha
alpha <- resize(betas[1,])
kable(alpha, digits=2)
```

	Low	2	3	4	High
Small	-0.32	-0.01	0.05	0.22	0.31
2	-0.25	-0.06	0.14	0.12	0.00
3	-0.14	0.08	-0.04	0.16	0.06
4	0.05	-0.15	0.01	0.08	0.07
Big	0.14	0.01	-0.04	-0.09	-0.08

```r
# resize beta
market.beta <- resize(betas[2,])
SMB.beta <- resize(betas[3,])
HML.beta <- resize(betas[4,])

# display beta below

kable(market.beta, digits=2)
```

	Low	2	3	4	High
Small	0.97	0.90	0.88	0.83	0.85
2	1.11	1.03	0.98	0.97	1.07
3	1.12	1.03	0.98	0.96	1.07
4	1.07	1.08	1.04	1.03	1.16
Big	1.03	1.05	1.02	1.02	1.05

```r
kable(SMB.beta, digits=2)
```

	Low	2	3	4	High
Small	1.55	1.47	1.38	1.33	1.37
2	1.08	0.97	0.88	0.74	0.88
3	0.77	0.67	0.56	0.49	0.67
4	0.37	0.31	0.28	0.26	0.41
Big	-0.09	-0.05	-0.06	-0.05	0.01

```r
kable(HML.beta, digits=2)
```

	Low	2	3	4	High
Small	-0.12	0.18	0.35	0.42	0.62
2	-0.42	0.07	0.27	0.51	0.74
3	-0.38	0.05	0.33	0.51	0.73
4	-0.41	0.04	0.29	0.54	0.77
Big	-0.45	0.00	0.24	0.52	0.71

Table 6

Regressions of excess stock and bond returns (in percent) on the excess market return ($RM-RF$) and the mimicking returns for the size (SMB) and book-to-market equity (HML) factors: July 1963 to December 1991, 342 months.[a]

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t)$$

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity

Book-to-market equity (BE/ME) quintiles

Size quintile	Low	2	3	4	High	Low	2	3	4	High
			b					$t(b)$		
Small	1.04	1.02	0.95	0.91	0.96	39.37	51.80	60.44	59.73	57.89
2	1.11	1.06	1.00	0.97	1.09	52.49	61.18	55.88	61.54	65.52
3	1.12	1.02	0.98	0.97	1.09	56.88	53.17	50.78	54.38	52.52
4	1.07	1.08	1.04	1.05	1.18	53.94	53.51	51.21	47.09	46.10
Big	0.96	1.02	0.98	0.99	1.06	60.93	56.76	46.57	53.87	38.61
			s					$t(s)$		
Small	1.46	1.26	1.19	1.17	1.23	37.92	44.11	52.03	52.85	50.97
2	1.00	0.98	0.88	0.73	0.89	32.73	38.79	34.03	31.66	36.78
3	0.76	0.65	0.60	0.48	0.66	26.40	23.39	21.23	18.62	21.91
4	0.37	0.33	0.29	0.24	0.41	12.73	11.11	9.81	7.38	11.01
Big	−0.17	−0.12	−0.23	−0.17	−0.05	−7.18	−4.51	−7.58	−6.27	−1.18
			h					$t(h)$		
Small	−0.29	0.08	0.26	0.40	0.62	−6.47	2.35	9.66	15.53	22.24
2	−0.52	0.01	0.26	0.46	0.70	−14.57	0.41	8.56	17.24	24.80
3	−0.38	−0.00	0.32	0.51	0.68	−11.26	0.05	9.75	16.88	19.39
4	−0.42	0.04	0.30	0.56	0.74	−12.51	1.04	8.83	14.84	17.09
Big	−0.46	0.00	0.21	0.57	0.76	−17.03	0.09	5.80	18.34	16.24

Similarly for t-statistics and R^2:

```r
# resize t-stats
market.t <-resize(t.values[2,])
SMB.t <- resize(t.values[3,])
HML.t <- resize(t.values[4,])

kable(market.t, digits=2)
```

	Low	2	3	4	High
Small	27.89	34.27	37.61	39.26	32.63
2	52.14	57.41	59.98	63.20	63.01
3	57.36	56.58	54.54	57.33	51.82
4	55.37	51.93	50.10	52.23	46.23
Big	67.82	63.64	55.67	58.36	41.58

```r
kable(SMB.t, digits=2)
```

	Low	2	3	4	High
Small	29.88	37.34	39.50	42.28	35.27
2	33.93	36.26	36.16	32.21	34.68
3	26.64	24.64	20.94	19.64	21.74
4	12.81	9.89	9.06	8.81	11.08
Big	-4.00	-1.96	-2.15	-1.93	0.15

```r
kable(HML.t, digits=2)
```

	Low	2	3	4	High
Small	-2.11	4.00	8.85	11.88	14.19
2	-11.73	2.45	9.89	19.74	26.02
3	-11.73	1.63	10.88	18.02	21.24
4	-12.79	1.29	8.50	16.51	18.44
Big	-17.60	0.17	7.85	17.69	16.94

```r
# resize R-squareds
kable(resize(R.squareds), digits=2)
```

	Low	2	3	4	High
Small	0.90	0.93	0.93	0.94	0.91
2	0.96	0.96	0.96	0.96	0.96
3	0.96	0.95	0.94	0.94	0.93
4	0.94	0.92	0.91	0.91	0.90
Big	0.96	0.94	0.92	0.92	0.85

```r
kable(resize(std.errors), digits=2)
```

	Low	2	3	4	High
Small	2.57	1.94	1.72	1.56	1.92
2	1.57	1.32	1.20	1.13	1.25
3	1.43	1.33	1.32	1.24	1.52
4	1.42	1.53	1.53	1.45	1.85
Big	1.12	1.22	1.35	1.29	1.85

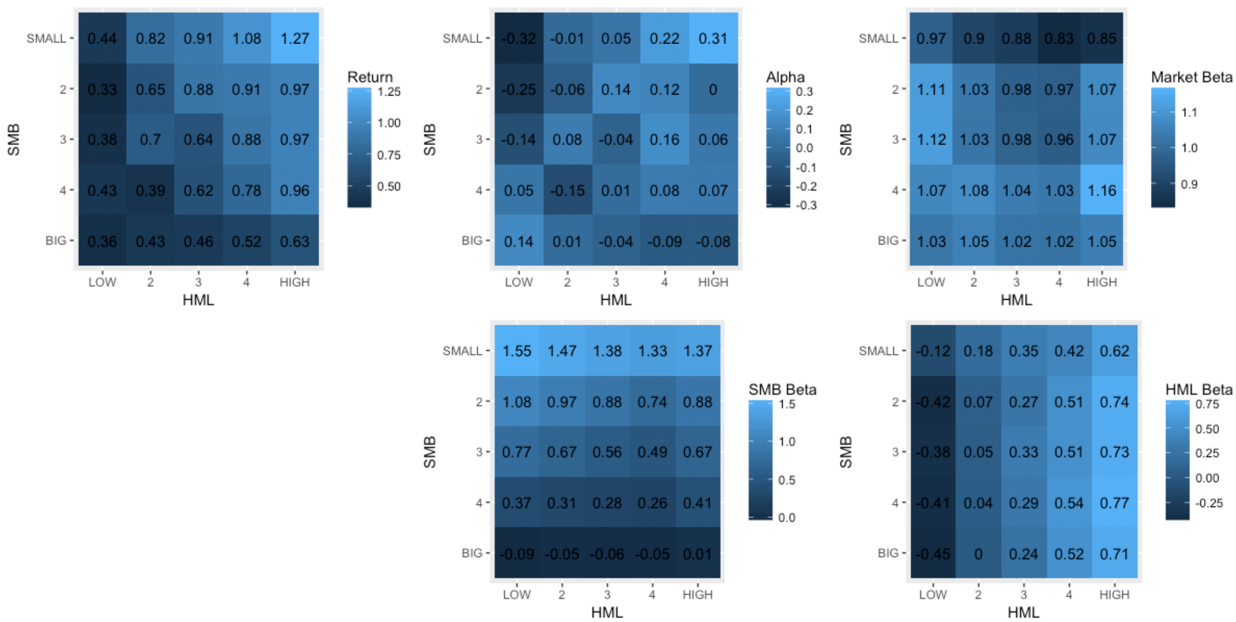
	R^2					$s(e)$				
Small	0.94	0.96	0.97	0.97	0.96	1.94	1.44	1.16	1.12	1.22
2	0.95	0.96	0.95	0.95	0.96	1.55	1.27	1.31	1.16	1.23
3	0.95	0.94	0.93	0.93	0.93	1.45	1.41	1.43	1.32	1.52
4	0.94	0.93	0.91	0.89	0.89	1.46	1.48	1.49	1.63	1.88
Big	0.94	0.92	0.88	0.90	0.83	1.16	1.32	1.55	1.36	2.02

Further, as Fama and French (1993) is mainly about explaining the average returns of the portfolios by the regressed coefficients of the factors, instead of pure statistical significance over the time series. We could visualize the average returns of the portfolios and betas using `ggplot2`'s `geom_tile()`, adding numerical values using `geom_text()`. Aesthetically, `scale_y_discrete()` is used for reversing the default order of the y-axis to match the tables in the paper (SMALL comes on top), and `labs()` for renaming legend titles.

One detail in formatting is that the "+" sign between blocks of ggplot functions cannot be at the beginning of the line, and only works at the end of the line or between two blocks.

```r
heat.prep   = function(df)  {
  df.return = expand.grid(
        HML = c("LOW", "2", "3", "4", "HIGH"),
        SMB = c("SMALL", "2", "3", "4", "BIG"))
  df.return$Return  = df
```

```r
  return(df.return)
}

heat.plot   = function(df, legend.label = "Return") {
    ggplot(data = heat.prep(df), aes(x = HML, y = SMB, fill = Return)) +
    geom_tile() + geom_text(aes(label=round(Return, digits = 2))) +
    scale_y_discrete(limits = rev(levels(heat.prep(P25.return)$SMB))) +
    labs(fill = legend.label)
}
```



From the heat maps we can clearly see that there is no clear trend in Alpha (intercept) or Market Beta, with Alphas close to 0 and Market Betas close to 1, consistent with CAPM. The regressed SMB Betas increase monotonically going small caps, while the regressed HML Betas increase monotonically going high book-to-market values (growth stocks). These corresponds to the general increase of portfolio returns from the lowest at bottom left (large cap and value stocks) to the highest at top right (small cap and growth stocks).

5. S&P500 Results

We first apply the above methods on the downloaded S&P 500 stocks' price returns to see if there is any pattern with the regression results. Also to test out the code for handling hundreds of stocks.

Then we separate the data by 5-year periods and loop over both years and stocks to see if patterns change over time.

5.1 Running the model for S&P 500 stocks

Below code works as follows:

1. Read-in price data and do the necessary formatting.
2. Frame the data to the desired time period.
3. Convert the price data series into XTS series as required by 5.
4. Remove stocks with `NA`s in the series.
5. Use `quantmod` library's `monthlyReturn()` function to batch convert the whole price matrix into a monthly return matrix.

We need to remove `NA`s for using the `monthlyReturn()` function. Most `NA`s are due to data not available on the starting date of the series, e.g. the company has not IPO yet.

Here we face choices:

- Remove all columns with `NA`s, then all remaining stocks could have the regression in the same period, i.e. with the same number of observations. (This section)
- Dynamically frame the data based on the available non-`NA` data points, but then some stocks in the regression analysis will have fewer observations. (Tested in Section 5.2)

```
library(quantmod)

# Read SP500 daily data and convert date column to date format
SP500.data <- read.csv("Data/SP500_price.adjusted_2010-2017.csv")
SP500.data$date <- as.Date(SP500.data$date)

# Select 2010 - 2017 range
Stock.Prices.Daily <- SP500.data[SP500.data$date>="2010-01-01" &
                                   SP500.data$date<="2017-12-31",-1]

# Current FF3 till 201803, monthly
FF3 <- read.csv("Data/original/FF3.csv")
FF <- FF3[FF3$X >= 201001 & FF3$X <= 201712,]
FF3[,-1] <- FF3[,-1]/100.00

# Convert series to XTS for using quantmod's monthlyReturn function
Stock.Prices.Daily <- xts(Stock.Prices.Daily[,-1],
                          order.by = as.POSIXct(Stock.Prices.Daily$date))

# Number of stocks to start with
ncol(Stock.Prices.Daily)
```

```
## [1] 465
```

```r
# Remove stocks with NAs in the series, otherwise monthly Return will not work properly
Stock.Prices.Daily <- Stock.Prices.Daily[,colSums(is.na(Stock.Prices.Daily)) == 0]

# Apply monthlyReturn function to each column (it seems it converts only one column at a time)
Stock.Prices.Monthly <- do.call(cbind, lapply(Stock.Prices.Daily, monthlyReturn))
# Stock.Prices.Monthly <- na.omit(Stock.Prices.Monthly)
colnames(Stock.Prices.Monthly) <- colnames(Stock.Prices.Daily)

# Number of stocks left
ncol(Stock.Prices.Monthly)
```

```
## [1] 442
```

As in this example, we start with 465 stocks and remove 23 stocks with incomplete data (95% preserved).

Then the regression part is similar to Section 4.1, except that we need to transpose the coefficients to get the dimensions right before stacking them together column by column, with each column representing one stock.

```r
Results <- list()
for(i in 1:ncol(Stock.Prices.Monthly))
{
  RiRF <- Stock.Prices.Monthly[,i] - FF$RF
  Regression <- lm(RiRF ~ FF$Mkt.RF + FF$SMB + FF$HML)
  Results[[i]] <- summary(Regression)
}

# Results!
betas <- vector()
std.errors <- vector()
t.values <- vector()
p.values <- vector()
r.squareds <- vector()
adj.r.squareds <- vector()

for(i in 1:ncol(Stock.Prices.Monthly))
{
  betas <- cbind(betas,Results[[i]]$coefficients[,1])
  std.errors <- cbind(std.errors,Results[[i]]$sigma)
  t.values <- cbind(t.values, Results[[i]]$coefficients[,3])
  p.values <- cbind(p.values, Results[[i]]$coefficients[,4])

  r.squareds <- cbind(r.squareds, Results[[i]]$r.squared)
  adj.r.squareds <- cbind(adj.r.squareds, Results[[i]]$adj.r.squared)

}

Regression.results <- cbind(data.frame(colnames(Stock.Prices.Monthly)),
                    t(r.squareds), t(adj.r.squareds),
                    t(betas), t(p.values))

colnames(Regression.results) = c("Ticker", "R.Squared", "Adj.R.Squared",
                         "Intercept", "Mkt.Rf", "SMB", "HML",
                         "P(Intercept)", "P(Mkt.Rf)", "P(SMB)", "P(HML)")
```

We add company information like name and sector to make the results easier to understand. The constituent data is from a downloaded CSV file, which can also be found in the downloaded data introduced in Section 2.2.

We use a left join (`merge()` function with parameter all.x = TRUE) to add company name and sector to our regression results.

```
# Read in SP500 company ticker information
Mapping <- read.csv("Data/constituents.csv")
colnames(Mapping)[1] <- "Ticker"
Regression.results <- merge(x = Regression.results, y = Mapping, by = "Ticker", all.x = TRUE)
```

Then we can easily filter out specific companies, e.g. companies and sectors whose returns have the highest Rˆ2 in the Fama French model. Interesting to see Financials come on top:

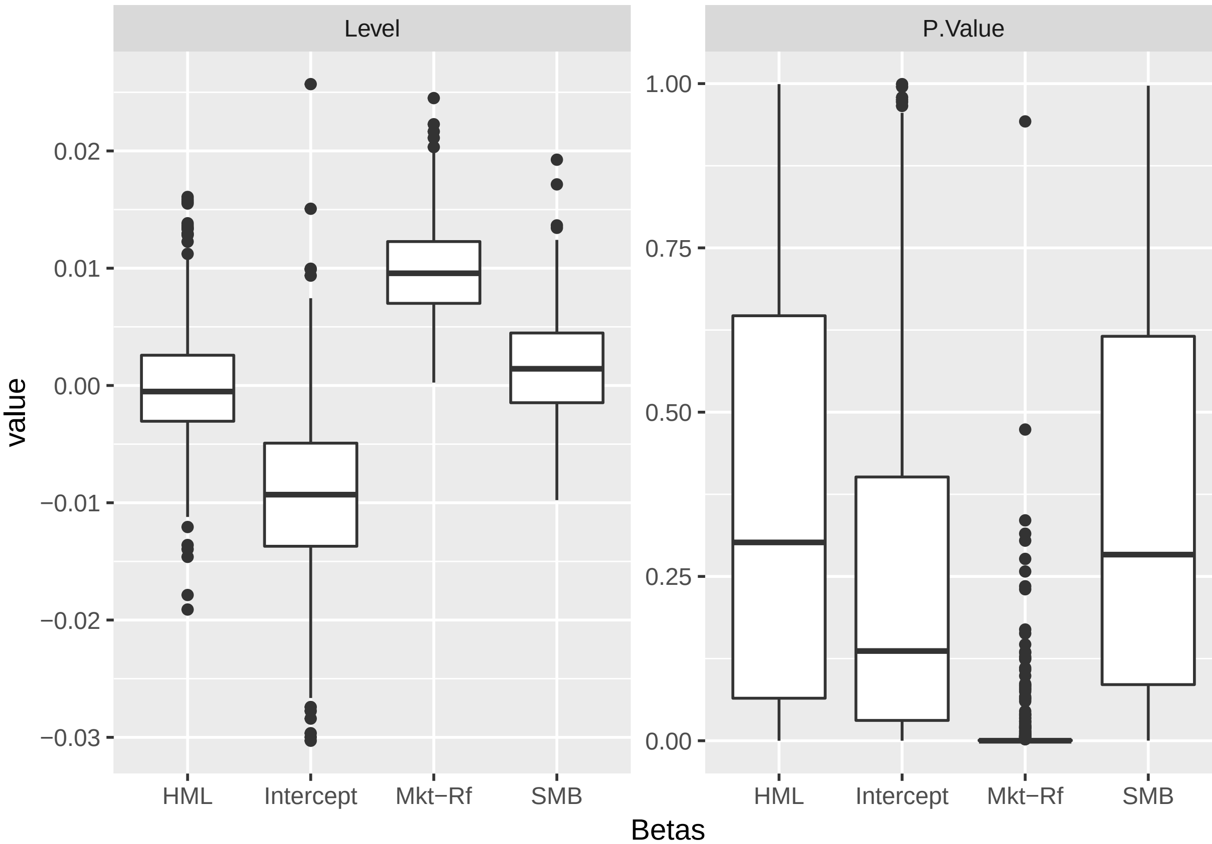
```
# select stocks with R2>=0.08
R2 <- Regression.results[Regression.results$R.Squared>=0.08,
                         c("Ticker","Name","Sector","R.Squared")]

# sort with R2 from largest to smallest, get top 10
kable(head(R2[order(R2$R.Squared, decreasing = T),], n=10), digits = 4)
```

	Ticker	Name	Sector	R.Squared
388	TROW	T. Rowe Price Group	Financials	0.6806
227	IVZ	Invesco Ltd.	Financials	0.6777
31	AMG	Affiliated Managers Group Inc	Financials	0.6646
283	MS	Morgan Stanley	Financials	0.6334
334	PRU	Prudential Financial	Financials	0.6308
201	HON	Honeywell Int'l Inc.	Industrials	0.6299
270	MET	MetLife Inc.	Financials	0.6279
321	PFG	Principal Financial Group	Financials	0.6262
60	BEN	Franklin Resources	Financials	0.6225
232	JPM	JPMorgan Chase & Co.	Financials	0.6027

We could also box-plot the distribution of the betas and their p-values. A new column is needed for using the `melt()` function (**reshape2** library) for the convenience of box-plot. In general, each column in the dataframe will be plotted into a separated graph, while data within each column is grouped by the value in the added column. Hence in the below code, the original data frame contains two columns: the estimated β's and their p-values. The added column in the dataframe marks which rows are the estimated coefficients for intercept, which rows are the estimated β_M, etc.

```
# boxplot of regression results
library(ggplot2)
library(reshape2)
num.stocks <- dim(Regression.results)[1]

plot.data <- data.frame(Betas = rep(c("Intercept", "Mkt-Rf", "SMB", "HML"),
                            rep(num.stocks, 4)))

plot.data$Level <- as.vector(cbind(
                     Regression.results$Intercept,
                     Regression.results$Mkt.Rf,
                     Regression.results$SMB,
                     Regression.results$HML))

plot.data$P.Value<- as.vector(cbind(
```

```
                         Regression.results$`P(Intercept)`,
                         Regression.results$`P(Mkt.Rf)`,
                         Regression.results$`P(SMB)`,
                         Regression.results$`P(HML)`))

plot.melt <- melt(plot.data, "Betas")
ggplot(plot.melt, aes(x=Betas, y=value)) + geom_boxplot() +
                         facet_wrap(~ variable, scales='free')
```



From the p-values, *SMB* and *HML* are not significant for many stocks.

5.2 Running the model for each 5-year period from 1980 to 2015

Data downloaded with `BatchGetSymbols` has an issue that the earlier the series (e.g. in the 1980s), the less stocks are available, most probably due to stocks being replaced in the S&P 500 index. To fix this issue, we could either:

1. Get the constituents for S&P 500 for each period and download those exact tickers, which may not work due to data availability. Even if it worked, we might be comparing apples to oranges, if the set of companies change over time.
2. Limit the data set to companies that survive over time. But then we have a much smaller set and miss out large names like Google or Facebook since they IPO in the 2000s.

Currently we simply take all the data available for each period for the regression, thus the results should be interpreted with a grain of salt.

Code is built based on Section 5.1, except that we stored only the results needed for plotting. Here in the document the `print()` and `cat()` functions are muted as they were merely for displaying the progress of the code in run time. Library `lubridate` provides some nice functions like `year()` for handling dates.

```r
# loop over above codes to regress data from 1980 - 2015, group every 5 yrs.
library(lubridate)
```

```
##
## Attaching package: 'lubridate'

## The following object is masked from 'package:base':
##
##     date
```

```r
List.of.start.date <- seq(as.Date("1980/1/1"), as.Date("2016/1/1"), "years")
List.of.start.date <- List.of.start.date[year(List.of.start.date)%%5==0]

# FF3: 192607 - 201803, monthly
FF3 <- read.csv("Data/original/FF3.csv")

# Each batch stores results for a 5yr group
Batch <- list()
Descriptions <- list()

Beta.batch <- list()

for(i in 1:(length(List.of.start.date)-1))
{
  start.date <- as.Date(List.of.start.date[i])
  end.date <- as.Date(List.of.start.date[i+1])-1
  # print(paste(start.date, end.date,sep=" - "))

  # read data
  file.name <- paste("Data/SP500_price.adjusted_",
                     paste(year(start.date), year(end.date), sep="-"), ".csv", sep="")
  SP500.data <- read.csv(file.name)
  SP500.data$date <- as.Date(SP500.data$date)

  # remove first column "X" created due to importing
  Stock.Prices.Daily <- SP500.data[SP500.data$date>= start.date &
                                   SP500.data$date<= end.date,-1]

  # Convert series to XTS for using quantmod's monthlyReturn function
  Stock.Prices.Daily <- xts(Stock.Prices.Daily[,-1],
                            order.by = as.POSIXct(Stock.Prices.Daily$date))

  # try a diff approach: loop over stocks and convert to monthly for each stock

  # initialize
  Results <- list()
  Description <- data.frame()

  betas <- data.frame()

  # loop through stocks
  for(j in 1:ncol(Stock.Prices.Daily))
  {
```

```r
    # The j-th stock
    Rj <- Stock.Prices.Daily[,j]

    # cat(colnames(Stock.Prices.Daily[,j]), " ")
    # non-NA entries
    Rj <- Rj[!is.na(Rj),]
    Rj <- monthlyReturn(Rj)

    # matching FF data
    FF <- FF3[FF3$X >= format(index(head(Rj, n=1)), "%Y%m") &
              FF3$X <= format(index(tail(Rj, n=1)), "%Y%m"), ]

    # Rj is now RjRF
    Rj <- Rj-FF$RF
    Regression <- lm(Rj ~ FF$Mkt.RF + FF$SMB + FF$HML)
    Results[[j]] <- summary(Regression)
    Description <- rbind(Description,
                         data.frame(colnames(Stock.Prices.Daily[,j]),
                                    format(index(head(Rj, n=1)), "%Y%m"),
                                    format(index(tail(Rj, n=1)), "%Y%m"),
                                    length(Rj)))

    # try read-out results at regression time
    # betas, p-values, r-squareds
    betas <- rbind(betas, cbind(data.frame(t(Results[[j]]$coefficients[,1])),
                                data.frame(t(Results[[j]]$coefficients[,4])),
                                data.frame(t(Results[[j]]$r.squared))))
  }

  # Save all regression summaries
  Batch[[i]] <- Results

  # Save the ticker / dates for ease of tracking the regression summary
  colnames(Description) = c("Ticker", "Start.Month", "End.Month", "Number.of.Months")
  Descriptions[[i]] <- Description

  # Save the regression results for plotting
  colnames(betas) <- c("Intercept", "Mkt-Rf", "SMB", "HML",
                       "P(Intercept)", "P(Mkt-Rf)", "P(SMB)", "P(HML)",
                       "R-squared")

  # Try rbind here instead of list for convenience of melt.
  Beta.batch[[i]] <- betas

  # remove temp variables
  rm(Description, Results, Regression, Rj, betas)
}
```

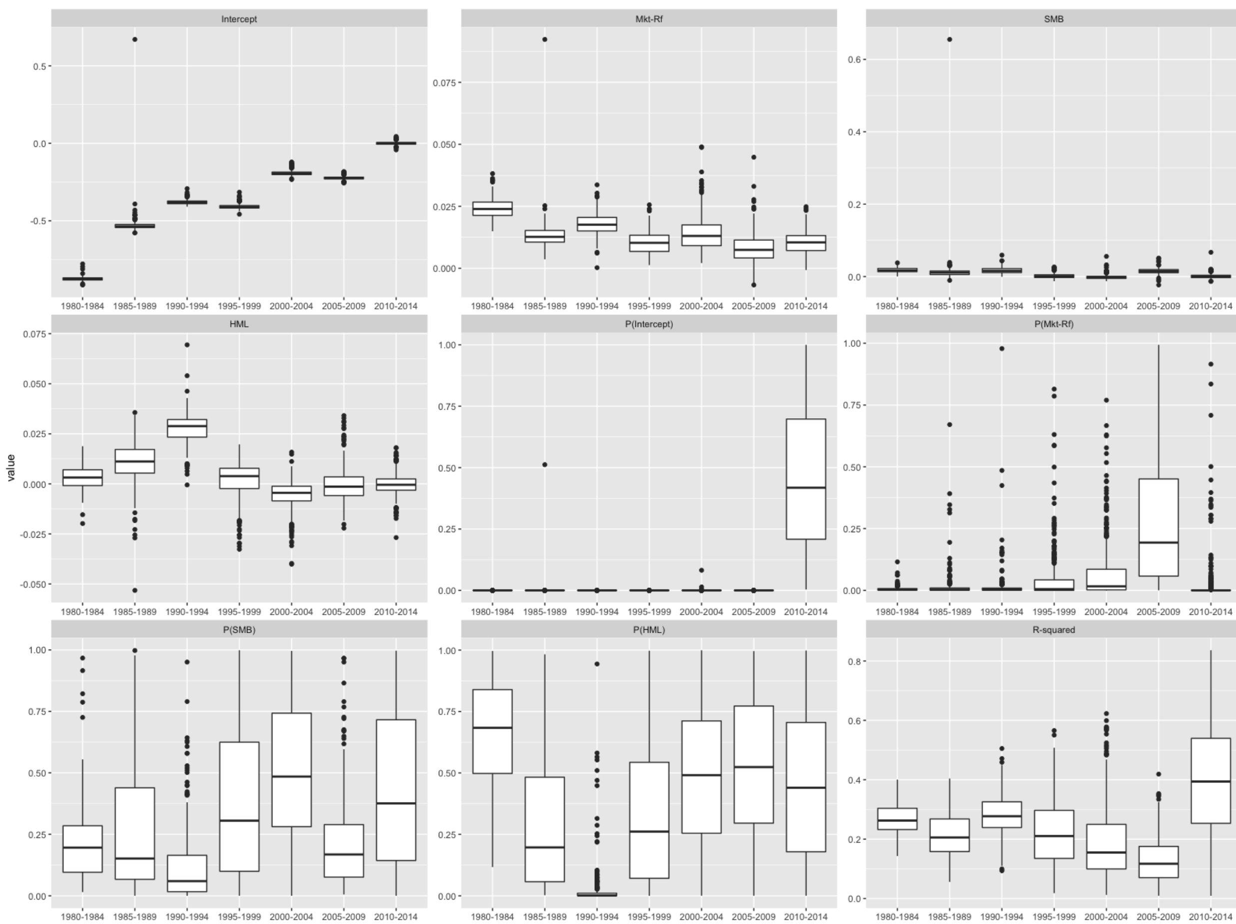
Similar to Section 5.1, we use `melt()` function and `ggplot()` for visualizing the results:

```r
df <- data.frame()
Num.Obs <- data.frame()
for(i in 1:(length(List.of.start.date)-1))
{
```

```r
  start.date <- as.Date(List.of.start.date[i])
  end.date <- as.Date(List.of.start.date[i+1])-1

  label <- paste(year(start.date), year(end.date),sep="-")
  df <- rbind(df, cbind(rep(label, dim(Beta.batch[[i]])[1]), Beta.batch[[i]]))
  Num.Obs <- rbind(Num.Obs,
                   cbind(  paste(year(start.date), year(end.date),sep="-"),
                           dim(Beta.batch[[i]])[1]))
}

colnames(df) <- c("Year",
                  "Intercept", "Mkt-Rf", "SMB", "HML",
                  "P(Intercept)", "P(Mkt-Rf)", "P(SMB)", "P(HML)",
                  "R-squared")
```

```r
df.melt <- melt(df, "Year")
ggplot(df.melt, aes(x=Year, y=value)) + geom_boxplot()
              + facet_wrap(~ variable, scales='free')
```

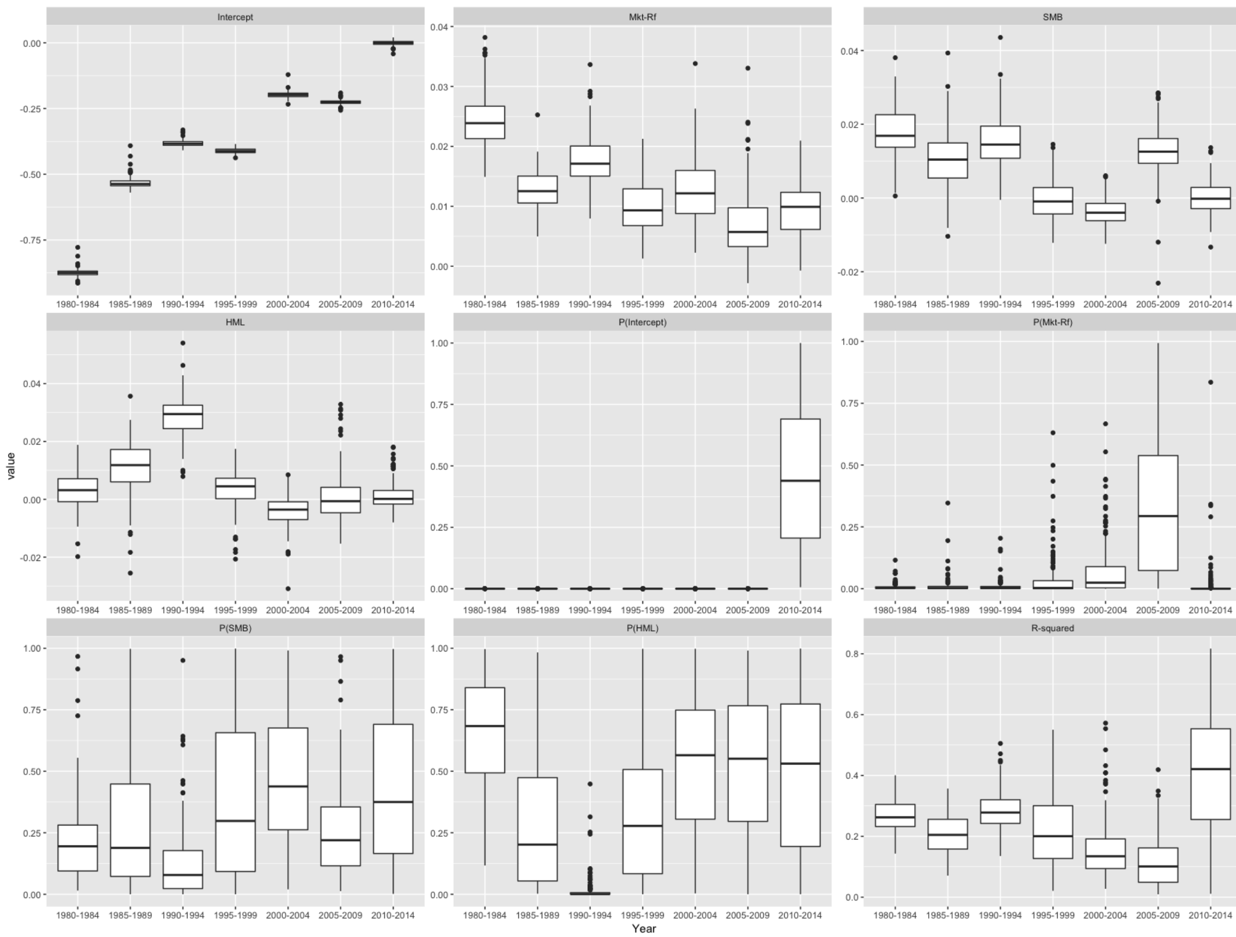
Regressing the data in different periods tells that the regression coefficients have changed over time. The explanatory power of the Fama French does not stay constant. Interestingly during 1990 to 1994 when Fama and French (1993) was published, *SMB* is most significant from p-values.



As emphasized earlier, the results are probably due to having varying stocks in each period:

Time Period	Number of Stocks
1980-1984	170
1985-1989	229
1990-1994	271
1995-1999	345
2000-2004	394
2005-2009	432
2010-2014	459

If we filter out stocks surviving all periods, we get 168 tickers. Surprisingly, results has only minor changes.



5.3 Stock Selection

A natural question arised from the study is whether we could use the model for stock selection. We can easily calculate period returns by $R_i = P_T/P_0 - 1$ from the first and last adjusted closing prices of each stock.

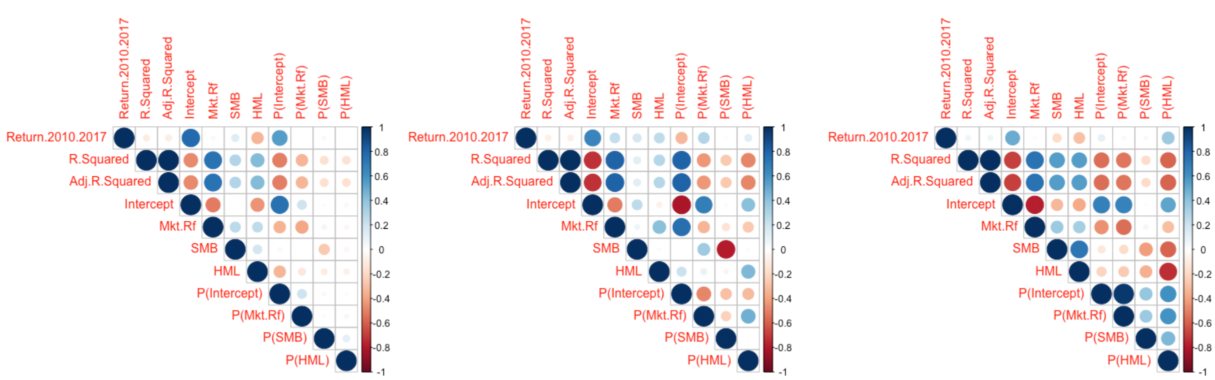
The top and bottom 10 stocks in terms of gross returns from January 2010 to December 2017 are:

Ticker	Name	Sec	Ri	R2	a	Mkt	SMB	HML	P(a)	P(M)	P(S)	P(H)
NFLX	Netflix Inc.	IT	24.13	0.04	0.03	0.01	0.00	0.00	0.20	0.11	0.62	0.60
URI	United Rentals, Inc.	I	16.12	0.55	0.00	0.02	0.01	0.01	1.00	0.00	0.02	0.01
REGN	Regeneron	H	14.26	0.15	0.01	0.01	0.01	-0.01	0.45	0.01	0.28	0.01
STZ	Constellation Brands	CS	13.58	0.10	0.01	0.01	0.00	0.00	0.25	0.00	0.89	0.60
AVGO	Broadcom	IT	12.52	0.20	0.01	0.01	0.00	-0.01	0.48	0.00	0.80	0.05
IPGP	IPG Photonics Corp.	IT	11.33	0.19	0.00	0.02	0.00	-0.01	0.83	0.00	0.63	0.32
ALGN	Align Technology	H	11.01	0.40	0.00	0.02	0.00	-0.01	0.72	0.00	0.91	0.06
ULTA	Ulta Beauty	CD	11.00	0.10	0.01	0.01	0.01	-0.01	0.34	0.03	0.21	0.16
AOS	A.O. Smith Corp	I	10.47	0.43	0.00	0.01	0.00	0.00	0.86	0.00	0.08	0.18
NVDA	Nvidia Corporation	IT	10.29	0.20	0.00	0.02	0.00	0.00	0.86	0.00	0.53	0.80

Ticker	Name	Sec	Ri	R2	a	Mkt	SMB	HML	P(a)	P(M)	P(S)	P(H)
RRC	Range Resources Corp.	E	-0.67	0.14	-0.03	0.01	0.01	0.01	0.03	0.06	0.22	0.04
APA	Apache Corporation	E	-0.56	0.32	-0.03	0.01	0.01	0.01	0.00	0.00	0.12	0.02
MOS	The Mosaic Company	M	-0.52	0.26	-0.03	0.01	0.00	0.01	0.00	0.00	0.85	0.12
FCX	Freeport-McMoRan Inc.	M	-0.42	0.27	-0.03	0.02	0.00	0.01	0.06	0.00	0.72	0.19
DVN	Devon Energy Corp.	E	-0.40	0.39	-0.03	0.01	0.01	0.01	0.00	0.00	0.16	0.00
NFX	Newfield Exploration Co	E	-0.37	0.28	-0.03	0.01	0.01	0.01	0.01	0.00	0.04	0.18
ARNC	Arconic Inc.	I	-0.21	0.29	-0.03	0.02	0.00	0.00	0.00	0.00	0.43	0.38
HES	Hess Corporation	E	-0.17	0.48	-0.03	0.01	0.00	0.01	0.00	0.00	0.29	0.00
CTL	CenturyLink Inc	T	-0.17	0.11	-0.02	0.01	-0.01	0.00	0.01	0.00	0.09	0.69
NEM	Newmont Mining Corporation	M	-0.13	0.00	-0.01	0.00	0.00	0.00	0.47	0.94	0.91	1.00

We can use the `cor()` function to calculate the correlation matrix of data series and the `corrplot` library for plotting.

All Stocks / Top 20 / Bottom 20



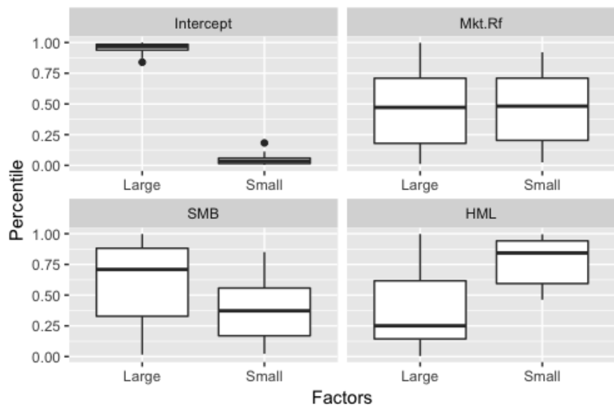
Plotting the correlations between regression results and stock returns reveal no particular pattern except for the intercept term in general. Top 20 stock returns do show positive correlations of the 3 factors, while the bottom 20 show negative correlations, which is consistent with the rationale behind the factors.

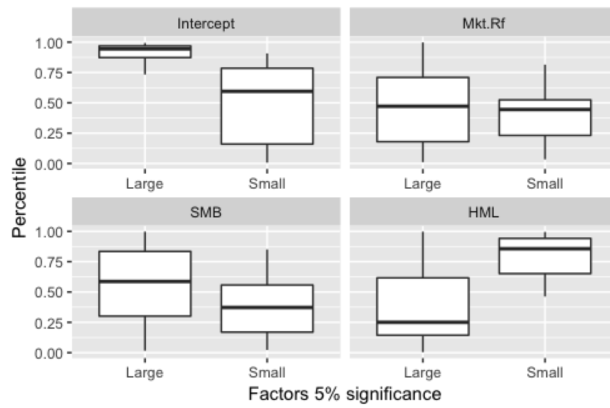
Another perspective is we could look at the regressed factor values and see whether we can select stock based on these values. We first calculate the percentile for each stock return using the `ecdf()` function: below code first defines our percentile function by supplying the all stock returns as a vector, then the `ecdf_percentile()` function can return a vector of percentiles, given a vector of returns.

```
# Define function
ecdf_percentile <- ecdf(Results$Return)
# Apply function.
ecdf_percentile(Results$Return)
```

Among 442 stocks with regression results, we take top 20 and bottom 20 stocks for the estimated coefficient of each factor. We then boxplot their return percentiles. Stock with greatest return from 2010 to 2017 will have a return percentile close to 1, stocks with poor returns will have a percentile close to 0.

We perform the selection with and without filtering for significance of the estimated coefficient.





From the results, *HML* shows strong separation power that **Growth** stocks with low book-to-market ratio outperform in this period, while **Value** stocks perform below average. Looking into the 20 stocks with the lowest estimated *HML* coefficient reveals that they are mostly Health Care or Consumer Discretionary. Limiting statistical significance to 5% does not alter the results much.

Ticker	Name	Sector	Factor	Est.Beta	P-Value	Return	Percentile
INCY	Incyte	Health Care	HML	-1.94	0.00	8.97	0.97
ILMN	Illumina Inc	Health Care	HML	-1.81	0.00	6.15	0.94
REGN	Regeneron	Health Care	HML	-1.49	0.01	14.26	1.00
NKTR	Nektar Therapeutics	Health Care	HML	-1.42	0.13	5.17	0.91
MNST	Monster Beverage	Consumer Staples	HML	-1.39	0.00	8.67	0.97
AMZN	Amazon.com Inc.	Consumer Discretionary	HML	-1.23	0.00	7.73	0.96
EXPE	Expedia Inc.	Consumer Discretionary	HML	-1.15	0.01	2.39	0.57
RHT	Red Hat Inc.	Information Technology	HML	-1.11	0.00	2.86	0.67
WYNN	Wynn Resorts Ltd	Consumer Discretionary	HML	-1.11	0.01	2.53	0.60
WAT	Waters Corporation	Health Care	HML	-1.08	0.00	2.13	0.46
CNC	Centene Corporation	Health Care	HML	-1.06	0.01	8.30	0.96
VRTX	Vertex Pharmaceuticals Inc	Health Care	HML	-0.96	0.10	2.39	0.56
CRM	Salesforce.com	Information Technology	HML	-0.95	0.01	4.47	0.86
AGN	Allergan, Plc	Health Care	HML	-0.95	0.00	3.17	0.74
ATVI	Activision Blizzard	Information Technology	HML	-0.95	0.00	5.13	0.90
CMG	Chipotle Mexican Grill	Consumer Discretionary	HML	-0.92	0.04	2.29	0.53
CERN	Cerner	Health Care	HML	-0.87	0.00	2.20	0.49
EW	Edwards Lifesciences	Health Care	HML	-0.86	0.03	4.15	0.83
ALXN	Alexion Pharmaceuticals	Health Care	HML	-0.85	0.04	3.96	0.81

Ticker	Name	Sector	Factor	Est.Beta	P-Value	Return	Percentile
VRSN	Verisign Inc.	Information Technology	HML	-0.83	0.00	4.45	0.86

6. Going 5-Factor

Fama and French (2015) adds two additional factors *RMW* and *CMA*:

- **RMW**: Profitability factor: the return of **R**obust (profitability) stocks **M**inus **W**eak ones.
- **CMA**: Investment factor: the return of **C**onservative (low investment) firms **M**inus the **A**ggressive (high investment) ones.

The process is mostly identical to section 5.1 except for adding the two factors into regression. We tested on 2010-2017 data and identify a data issue with the downloaded S&P500 data: Ticker "BHY" *Brighthouse Financial Inc.* which has a large gap of `NA`s in 2016. It was not revealed in section 5, as 5.1 removed all stocks with `NA`s while 5.2 was tested with 1980 to 2015 data.

We have the following code to address this problem in the beginning, but decided to drop the BHY due to seemingly wrong results. Hence the actual code only needs to handle `NA`s at the beginning and at the end of the series, but not abnormalities in between.

```
# Pick out non-NA entries and convert to monthly return
Ri <- Stock.Prices.Daily[, i]
Ri <- Ri[!is.na(Ri),]
Ri <- monthlyReturn(Ri)

# Convert the existing row index to YYYYMM format to match the Fama French data
Ri <- data.frame(date = format(index(Ri), "%Y%m"), Ri)

# Select the matching FF periods
# Actually we do not need this one as
# the next code will match the relevant periods anyway
FF <- FF5[FF5$X >= format(index(head(Ri, n=1)), "%Y%m") &
          FF5$X <= format(index(tail(Ri, n=1)), "%Y%m"), ]

# New matching: dropped and revert to old matching because we exclude BHY
FF <- FF5[Ri$date,]

# Change due to Ri is now dataframe with two columns (date, return)
RiRF <- Ri$monthly.returns - FF$RF
```

We create short codes for sectors for ease of plotting.

Sec	Sector	Number of Companies
CD	Consumer Discretionary	82
CS	Consumer Staples	34
E	Energy	31
FI	Financials	69
H	Health Care	61
I	Industrials	67
IT	Information Technology	72
M	Materials	25
RE	Real Estate	33
T	Telecommunication Services	3

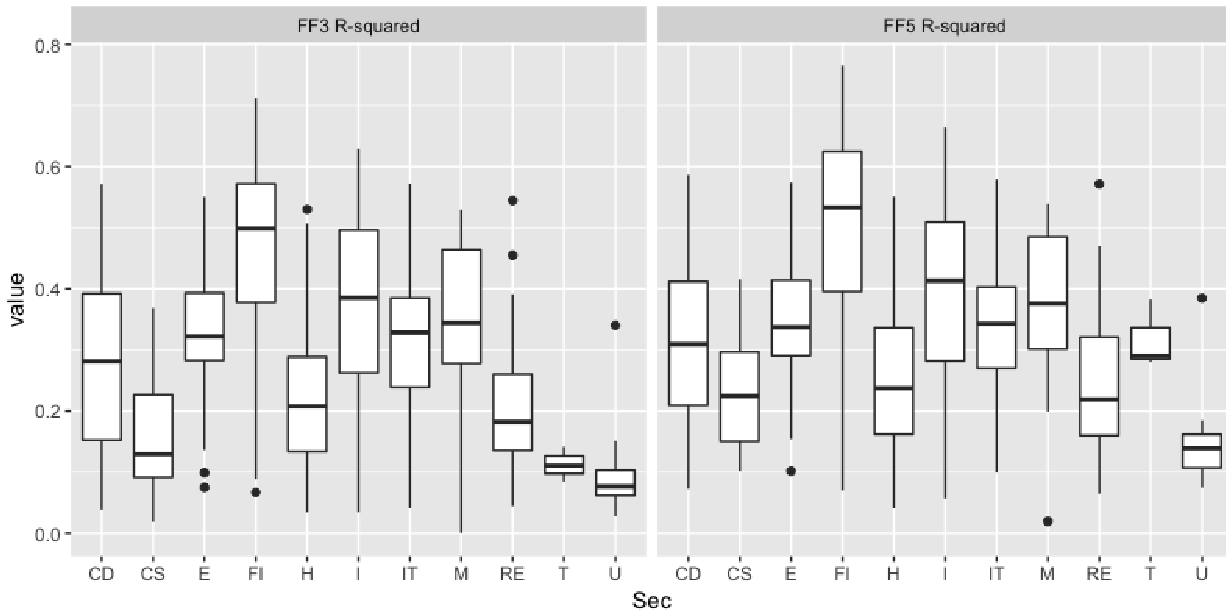
Sec	Sector	Number of Companies
U	Utilities	28

Results show a large jump in R^2 for Telecommunication sector but then it contains only 3 companies

Symbol	Name	Sector	Sec
T	AT&T Inc.	Telecommunication Services	T
CTL	CenturyLink Inc	Telecommunication Services	T
VZ	Verizon Communications	Telecommunication Services	T

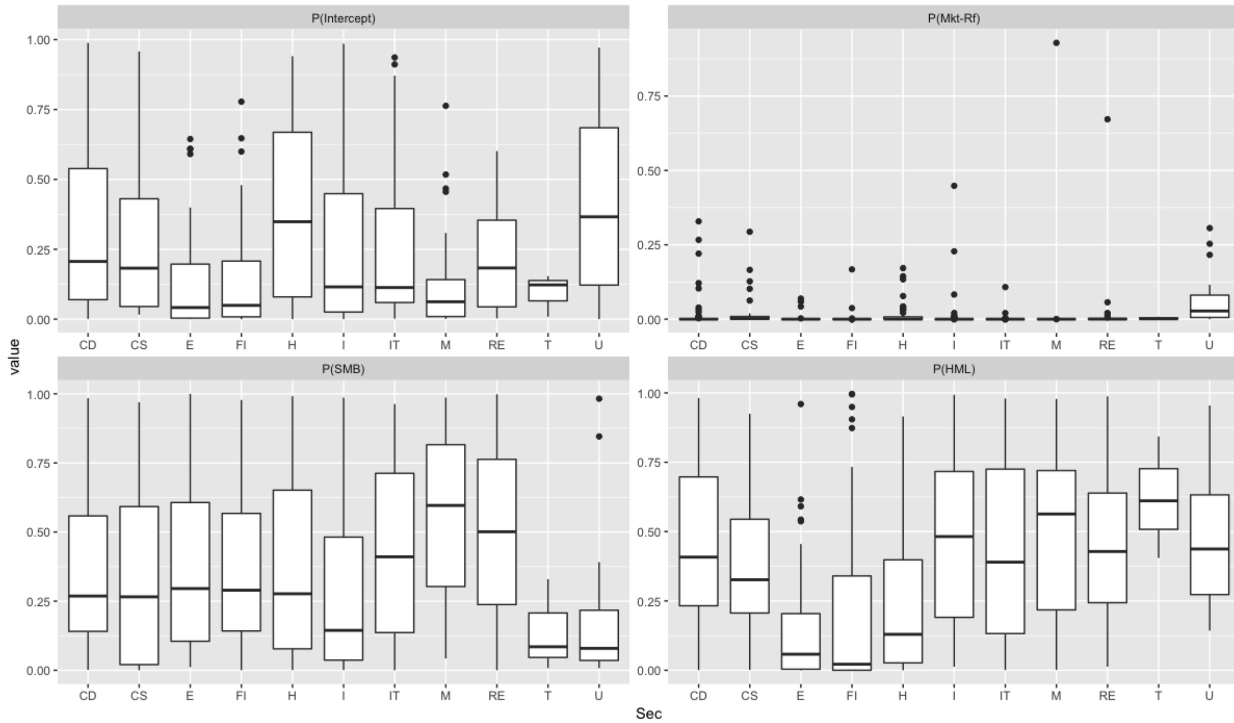
Rˆ2 comparison: Fama French 3 Factors vs. 5 Factors.

Rˆ2's are generally higher with the 5 Factor models.

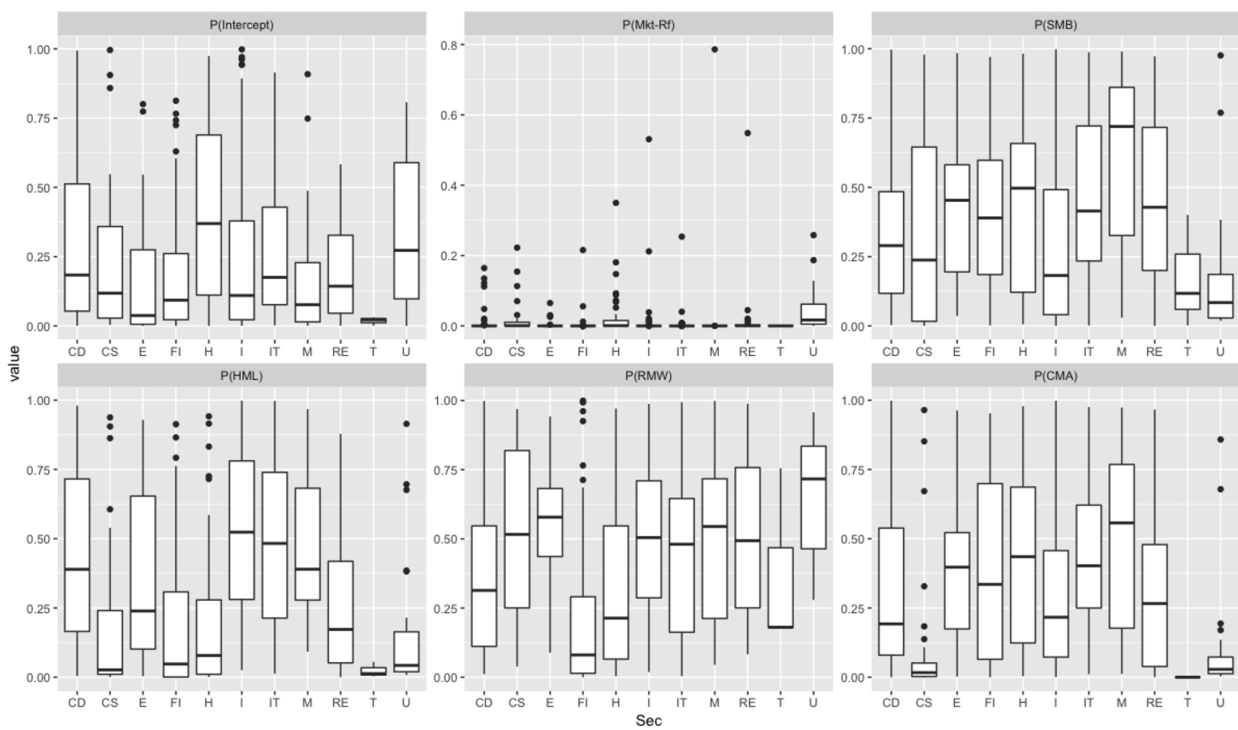


The most noticeable difference between 3 factors and 5 factors is also with Telecommunication sector, with p-values for *HML* (value) and *CMA* (investment) being much smaller (significant) in the 5-factor model.

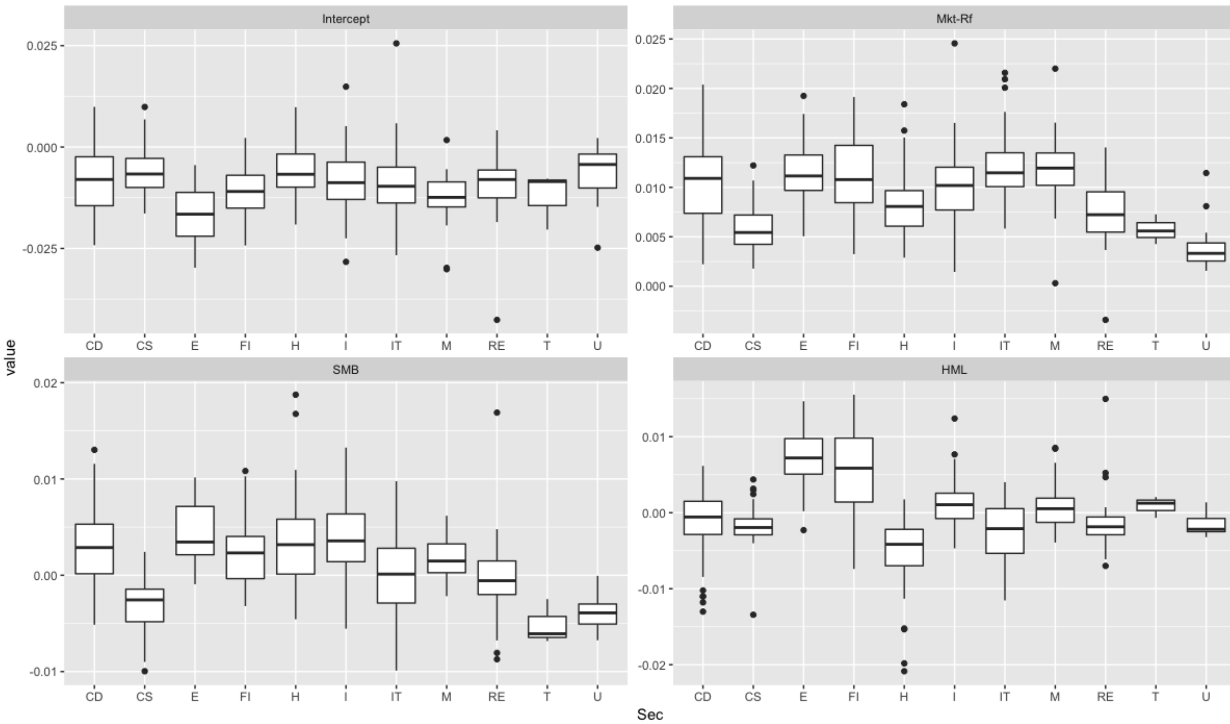
Fama French 3 Factors:



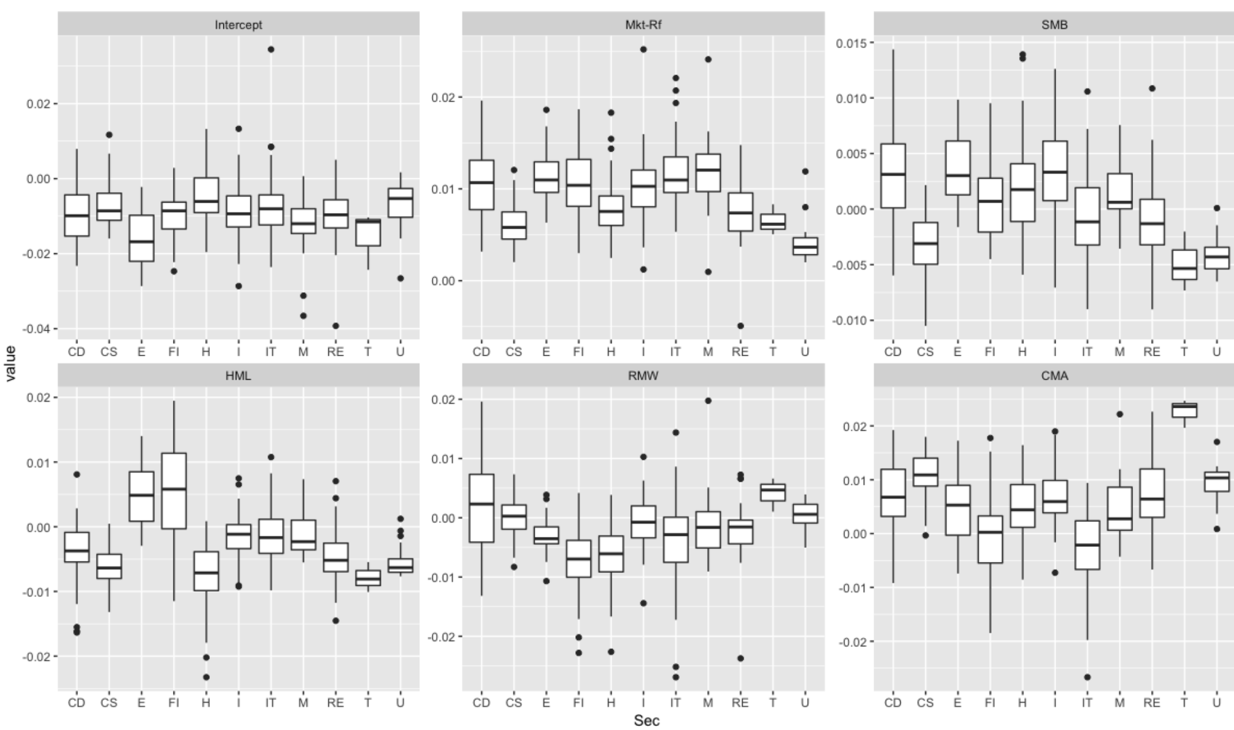
Fama French 5 Factors:



The regressed coefficients from the 3 Factors model:

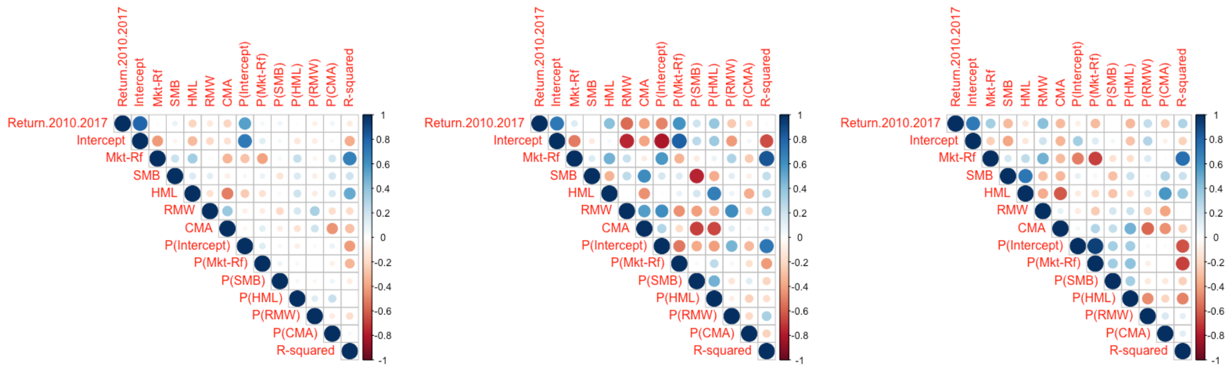


and 5 Factors model:



We can use the same method as section 5.3 to visualize the correlation between stock returns and the regression results of the Fama French 5 Factors model. Here the top 20 are the 20 stocks with the greatest returns from January 2010 to December 2017, same as in section 5.3.

All Stocks / Top 20 / Bottom 20



A large portion of the return is still captured by *alpha* the intercept. Surprisingly, the top 20 stock returns show negative correlations with the added *RMW* and *CMA* factor, while the bottom 20 stock returns show positive correlations with *RMW* and still negative correlations with *CMA*. This proves that one cannot predict the future from the past:

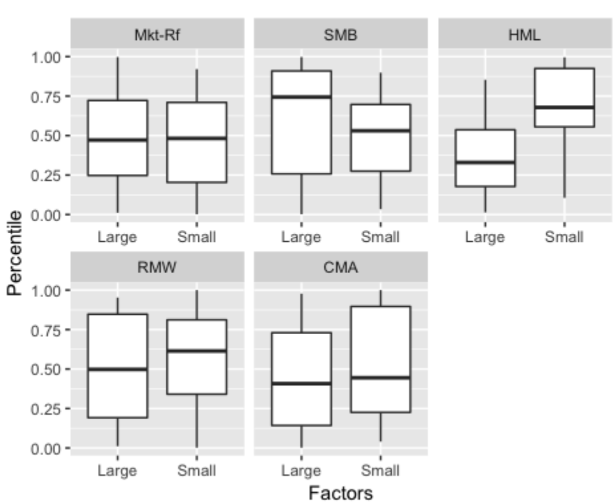
1. Winning stocks might not have robust operating profitability: a large portion of earnings is invested.
2. Winning stocks seem to benefit from past aggressive investments.

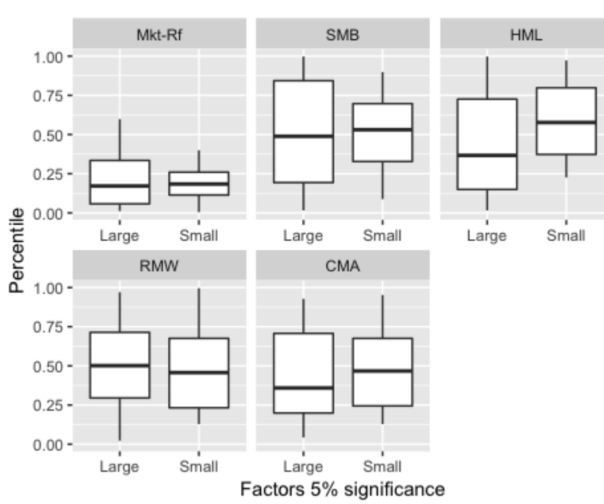
Another perspective is we could look at the regressed factor values and see whether we can select stock based on these values. We first calculate the percentile for each stock return using the `ecdf()` function: below code first defines our percentile function by supplying the all stock returns as a vector, then the `ecdf_percentile()` function can return a vector of percentiles, given a vector of returns.

```
# Define function
ecdf_percentile <- ecdf(Results$Return)
# Apply function.
ecdf_percentile(Results$Return)
```

Among 443 stocks with regression results, we take top 20 and bottom 20 stocks for the estimated coefficient of each factor. We then boxplot their return percentiles. Stock with greatest return from 2010 to 2017 will have a return percentile close to 1, stocks with poor returns will have a percentile close to 0.

We perform the selection with and without filtering for significance of the estimated coefficient.





From the results, if we only consider value without significance, *HML* shows strong separation power that

Growth stocks with low book-to-market ratio outperform in this period, while **Value** stocks perform below average. Looking into the 20 stocks with the lowest estimated *HML* coefficient reveals that they are mostly Health Care or Consumer Discretionary. When limiting scope to coefficients that are significant at 5% level, however, shows no particular separation power of all Fama French factors. Interestingly, at 5% significance, stocks with largest market exposure and smallest market exposure (CAPM beta / beta for Mkt-Rf) all perform below average.

Ticker	Name	Sector	Factor	Est.Beta	P-Value	Return	Percentile
INCY	Incyte	Health Care	HML	-0.02	0.04	8.97	0.97
REGN	Regeneron	Health Care	HML	-0.02	0.56	14.26	1.00
ILMN	Illumina Inc	Health Care	HML	-0.02	0.03	6.15	0.94
NKTR	Nektar Therapeutics	Health Care	HML	-0.02	0.34	5.17	0.91
UAA	Under Armour Class A	Consumer Discretionary	HML	-0.02	0.09	3.11	0.72
WYNN	Wynn Resorts Ltd	Consumer Discretionary	HML	-0.02	0.74	2.53	0.60
EW	Edwards Lifesciences	Health Care	HML	-0.01	0.10	4.15	0.83
CNC	Centene Corporation	Health Care	HML	-0.01	0.52	8.30	0.96
O	Realty Income Corporation	Real Estate	HML	-0.01	0.70	2.21	0.50
MNST	Monster Beverage	Consumer Staples	HML	-0.01	0.78	8.67	0.97
CELG	Celgene Corp.	Health Care	HML	-0.01	0.52	2.74	0.64
VRTX	Vertex Pharmaceuticals Inc	Health Care	HML	-0.01	0.58	2.39	0.56
CMG	Chipotle Mexican Grill	Consumer Discretionary	HML	-0.01	0.01	2.29	0.53
VTR	Ventas Inc	Real Estate	HML	-0.01	0.10	0.69	0.14
CERN	Cerner	Health Care	HML	-0.01	0.53	2.20	0.49
HRB	Block H&R	Financials	HML	-0.01	0.59	0.54	0.11
DLTR	Dollar Tree	Consumer Discretionary	HML	-0.01	0.00	5.68	0.92
EXPE	Expedia Inc.	Consumer Discretionary	HML	-0.01	0.92	2.39	0.57
ALXN	Alexion Pharmaceuticals	Health Care	HML	-0.01	0.84	3.96	0.81
AMGN	Amgen Inc.	Health Care	HML	-0.01	0.37	2.47	0.58

References

Fama, Eugene F., and Kenneth R. French. 1993. “Common risk factors in the returns on stocks and bonds.” *Journal of Financial Economics* 33 (1): 3–56. doi:10.1016/0304-405X(93)90023-5.

———. 2015. “A five-factor asset pricing model.” *Journal of Financial Economics* 116 (1). Elsevier: 1–22. doi:10.1016/j.jfineco.2014.10.010.