lecture

October 8, 2025

1 Random Hyperplane Rounding (RHR)

$$\sum_{ij \in E} w_{ij} \frac{\theta_i j}{\pi} \tag{1}$$

optimal sdp value is

$$\max \sum_{ij \in E} w_{ij} \left(\frac{1 - \vec{y}_i \vec{y}_j}{2} \right) \tag{2}$$

SDP value ≥ OPT maxcut

2 hierarchical clustering HC

$$\max \sum_{ij \in E} w_{ij} \left(n - |T_{ij}| \right) \tag{3}$$

note $n - |T_{ij}|$ is the # of non leaves

thus eq 3 can also be written as idk

levels of HC: level t is a partition of V, the verticies, in $1, \ldots, n$ into maximal clusters that have at most t nodes.

if there is a $t = 0$, it is the same as $t = 1$.

Definition 1 (maximal cluster)**.** maximal clusters are such that merging any two clusters does not result in a valid clustering

every t is considered a graph partitioning problem.

$$x_{ij}^t = \begin{cases} 0, & \text{at level } t,\ i, j \text{ are not separated} \\ 1, & \text{else} \end{cases} \tag{4}$$

note eq 3 is equzl to

$$\max \sum_{ij \in E} w_{ij} \sum_{t=0}^{n} (1 - x_{ij}^t) \tag{5}$$

with constraints

$$x_{ij}^{t+1} \leq x_{ij}^t \tag{6}$$

and

$$\sum_{j \in V \setminus i} x_{ij}^t \leq n - t \tag{7}$$

this condition is called the spreading condition

for all $i \in V$ assign $\vec{v}_i^t \in \mathbb{R}^n$

relaxing x into not just 0 and 1,

$$x_{ij}^t = 1 - v_i^t v_j^t \tag{8}$$