lecture

October 14, 2025

1 matlab

solving for modified battle of sexes from lec 4

set A,B matrix of payoffs,

$$Ay = \text{vector of payoffs for each of adrians pure strategies} \tag{1}$$

where y is baileys mix probabilities

for adrian to consider mixing across all strategies, Ay needs to equal a vector where each component is equal.

thus

$$y = \frac{A^{-1}1}{|A^{-1}1|} \tag{2}$$

where 1 represents the ones vector

x^T is adrians mix, x **is transposed since representing the top player on the side requires transpose of the given matrix, thus more accurately it is described as** $B^T x$ where B is the apparent payoff matrix and x is adrians mix as a column

$$x^T B \tag{3}$$

$x^T B$ is vector of expected payoffs of baileys strategies

for partial support strategies, remove the columns/rows that are not being played, and set y to zero on the non supported strategies. alternatively you can also just remove them from the matrix and mix vectors.

checking strategies outside of support is multiply

$$Ay \tag{4}$$

and check if the payoff of the options outside of the support are greater or less than the supposed NE.

note: the payoffs of the mixed strategies of the strategies minus the supposed support must be a linear combination of the pure strategies, thus only checking pure strategies is necessary

question: does negative, or inconsistent equations mean something?

2 systematize

let A be a payoff matrix of a symmetric game.

assume that A is non-negative and $n \times n$. (you can add constant matricies and outcomes dont change proof?)

consider the polytope

$$Az \leq \mathbb{1}, z \geq 0 \tag{5}$$

note both of the inequalities have n entries

the bounds of these equations results in n tight inequalities, draws out a polytope

$$Az \leq 1 \implies A\left(\frac{z}{|z|} \leq \frac{1}{|z|}\right) \tag{6}$$

if this inequality is tight, i.e. the inequality becomes an equality, then it is a best response

Definition 1 (represented strategies)**.** strategy i is *represented* at vertex z if either $z_i = 0$ or $A_i z = 1$

observation: if a vertex has all strategies represented, excluding the origin, then that vertex is a nash equilibrium

start at $\vec{0} = v_0$. pick an arbitrary strategy j. look at vertecies adjacent to v_0. in each of these verticies, exactly one positivity constraint gives slack and one constraint in $Az = 1$ is tight. therefore make positiviy constraint j, get slack and increase until something in $Az \leq 1$ tight. call the component that becomes tight, k.

if $k = j$, then j is the best response to itself and j is a pure equilibrium.

if $k \neq j$, then k is doubly represented. $z_k = 0, [Az]_k = 1$

look at doubily represented strategy k

$$(v_i)_k = 0 \tag{7}$$

and

$$[Av_i]_k = 1 \tag{8}$$

relaxing either of these, gives 2 new vertices

one of these has to be v_{i-1}, go to the other vertex that wasnt just visited

$Ax \leq b$, $x \geq 0$,

$$\begin{pmatrix} A | \mathbb{1} \end{pmatrix} \begin{pmatrix} x \\ s \end{pmatrix} \tag{9}$$

where s is the slack variables. the first matrix is $x \times 2n$ and the second $2n \times 1$.