hw1

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1 1

for defenders there are no dominated strategies. note for offense that turnover is totally dominated by field goal.

also note that the field goal strategy is dominated by a 5050 split between touchdown N and S.

$$\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 7 = 3.5 \tag{1}$$

which is greater than the 3 given by field goal.

let us find the nash equilibria given by the supports touchdown N, S and defend N, S

assume that the probability of defend n is given by p and the probability of touchdown n is given by q.

to make the offense indifferent,

$$0 \cdot p + 7 \cdot (1 - p) = 0 \cdot (1 - p) + 7 \cdot p \tag{2}$$

we find $p = 0.5$.

similarly for defense,

$$0 \cdot q - 7 \cdot (1 - q) = 0 \cdot (1 - q) - 7 \cdot q \tag{3}$$

we find that $q = 0.5$

2 2

joe vote 0 table

	S0	S1
L0	(-1,0,0)	(1,-1,0)
L1	(0,0,0)	(0,1,0)

joe vote 1 table

	S0	S1
L0	(1,0,-1)	(1,1,1)
L1	(0,0,1)	(0,1,1)

no person has any dominated strategies.

3 3

let the remaining options for player one be the set A and the set for player two to be B. loop over the powerset of $P = (\mathcal{P}(A) \setminus \emptyset \times \mathcal{P}(B) \setminus \emptyset)$. wlog let $i \in P$

$i[a]$ represents the support for which the iteration in the loop tries strategies for p1, and $i[b]$ represents the strategies for p2.

evaluate for mixed strategies by setting probabilities a_i for each option in $i[a]$ and b_i in $i[b]$. consider the set of equations over m

$$\sum_{n \in i[x_{-1}]} (x_{-1})_n \cdot f_x(n, m) \tag{4}$$

where x denotes player 1 or 2 (stands in for a and b), x_{-1} represents the other player, and $f_x(n, m)$ represents the payoff for player x given player x does action m and player x_{-1} does action n.

setting the set of equations over m of eq 4 equal and solving for $(x_{-1})_n$ results in either

- a consistent set of equations
- an inconsistent set of equations

if a consistent set of equations is found for both players, check if switching to any other pure strategy not in $A \setminus i[x]$ is unilaterally better, for both players. if not, then a NE is found. otherwise, continue with a new element in the powerset P

4 4

1. if 1 person taking the bus generates enough shame to make the other 5 take the bus, then any additional bus takers also results in the plane takers switching. thus we need to find S such that

$$-S \cdot \frac{1}{5} \leq -1 \tag{5}$$

note that $\frac{1}{5} \leq \frac{6-n}{n}, n \in \{1, 2, 3, 4\}$
we find that if $S > 5$, then everyone takes the bus.

2. if 5 people taking the bus generates less shame for 1 person than -1, each person taking the bus would switch to taking the plane.

$$-1 \leq -5S\frac{6-1}{1} \tag{6}$$

if S