

lecture

October 13, 2025

1 Random Hyperplane Rounding (RHR)

$$\sum_{ij \in E} w_{ij} \frac{\theta_{ij}}{\pi} \quad (1)$$

optimal sdp value is

$$\max \sum_{ij \in E} w_{ij} \left(\frac{1 - \vec{y}_i \vec{y}_j}{2} \right) \quad (2)$$

SDP value \geq OPT maxcut

2 hierarchical clustering HC

$$\max \sum_{ij \in E} w_{ij} (n - |T_{ij}|) \quad (3)$$

note $n - |T_{ij}|$ is the # of non leaves

thus eq ?? can also be written as idk

levels of HC: level t is a partition of V , the vertices, in $1, \dots, n$ into maximal clusters that have at most t nodes.

if there is a $t = 0$, it is the same as $t = 1$.

Definition 1 (maximal cluster). maximal clusters are such that merging any two clusters does not result in a valid clustering

every t is considered a graph partitioning problem.

$$x_{ij}^t = \begin{cases} 0, & \text{at level } t, i, j \text{ are not separated} \\ 1, & \text{else} \end{cases} \quad (4)$$

note eq ?? is equal to

$$\max \sum_{ij \in E} w_{ij} \sum_{t=0}^n (1 - x_{ij}^t) \quad (5)$$

with constraints

$$x_{ij}^{t+1} \leq x_{ij}^t \quad (6)$$

and

$$\sum_{j \in V \setminus i} x_{ij}^t \leq n - t \quad (7)$$

this condition is called the spreading condition

for all $i \in V$ assign $\vec{v}_i^t \in \mathbb{R}^n$

relaxing x into not just 0 and 1,

$$x_{ij}^t = 1 - v_i^t v_j^t \quad (8)$$