lecture

October 2, 2025

a 2-player game is *symmetric* when the outcome matrix for p1 A and p2 B is such that

$$A = B^T \tag{1}$$

the vector of payoffs for p1's pure strategies is

$$Ay \tag{2}$$

x is the probability matrix of p1, y is the probability matrix for p2

these determine the probability of p1 or p2 playing each option

ex:

$$A = \begin{pmatrix} -1 & -4 \\ 0 & -3 \end{pmatrix} \tag{3}$$

$$B = \begin{pmatrix} -1 & 0 \\ -4 & -3 \end{pmatrix} \tag{4}$$

$$A \begin{pmatrix} .8 \\ .2 \end{pmatrix} \tag{5}$$

p1 gets $x^T Ay$ p2 gets $x^T By$

p1's best response correspondence is

$$r_1(y) = \underset{||x||_1=1, x\geq 0}{\operatorname{argmax}} x^T Ay \tag{6}$$

best response can only have a mix when there are "ties" in Ay

$$r_2(x) = \underset{||y||_1=1, y\geq 0}{\operatorname{argmax}} x^T By \tag{7}$$

a nash equilibrium is a pair (x^*, y^*) with $x^* \in r_1(y * 2)$, $y^* \in r(x^*)$

for any 2 player game given by (A, B) we can define a related symmetric game $(\tilde{A}, \tilde{B})$ with

$$\tilde{A} = \begin{pmatrix} 0 & A \\ B^T & 0 \end{pmatrix} \tag{8}$$

and

$$\tilde{B} = \tilde{A}^T \tag{9}$$

then if

$$I \in \begin{pmatrix} x^* \\ y^* \end{pmatrix} \tag{10}$$

is a symmetric NE of $(\tilde{A}, \tilde{B})$ then $\left(\frac{x^*}{||x^*||}, \frac{y^*}{||y^*||}\right)$ is a NE of the game (A, B)

proof sketch

$$|x| \tag{11}$$

$$\underset{|x|=|x^*|,|y|=|y^*|}{\operatorname{argmax}} \quad x^T A y^* + y^T B^T x^* \tag{12}$$

1 rock paper scissors

$$A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \tag{13}$$

does there eixst x^*

$$x^* \in \underset{x:|x|=1,x\geq 0}{\operatorname{argmax}} x^T A x^* \tag{14}$$

node Ax^* is the payoff column of the pure strategies