hw1

October 10, 2025

1 1

for defenders there are no dominated strategies. note for offense that turnover is totally dominated by field goal.

also note that the field goal strategy is dominated by a 5050 split between touchdown N and S.

$$\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 7 = 3.5 \tag{1}$$

which is greater than the 3 given by field goal.

let us find the nash equilibria given by the supports touchdown N, S and defend N, S

assume that the probability of defend n is given by p and the probability of touchdown n is given by q.

to make the offense indifferent,

$$0 \cdot p + 7 \cdot (1 - p) = 0 \cdot (1 - p) + 7 \cdot p \tag{2}$$

we find $p = 0.5$.

similarly for defense,

$$0 \cdot q - 7 \cdot (1 - q) = 0 \cdot (1 - q) - 7 \cdot q \tag{3}$$

we find that $q = 0.5$

2 2

joe vote 0 table

	S0	S1
L0	(-1,0,0)	(1,-1,0)
L1	(0,0,0)	(0,1,0)

joe vote 1 table

	S0	S1
L0	(1,0,-1)	(1,1,1)
L1	(0,0,1)	(0,1,1)

the pure nash equilibrium sits on L0, S1, J1

3 3

1. for player 1,

$$\alpha = xya + (1-y)xc \tag{4}$$

,

$$\beta = (1-x)ye + (1-x)(1-y)g \tag{5}$$

. solving whether α or β is larger determines whether the player should go pure up or down(x=1,x=0). if they are equal, they they can choose to go any value x.

for player 2,

$$\alpha = xyb + (1-x)yf \tag{6}$$

,

$$\beta = x(1-y)d + (1-x)(1-y)h \tag{7}$$

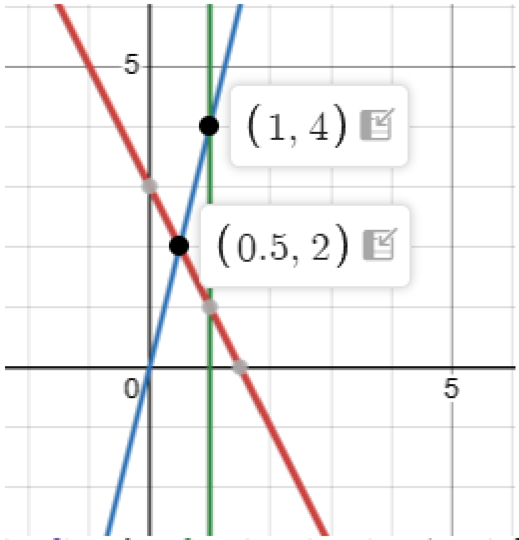
solving whether α or β is larger determines whether the player should go pure L or R (y=1, y=0). if they are equal, they they can choose to go any value y.

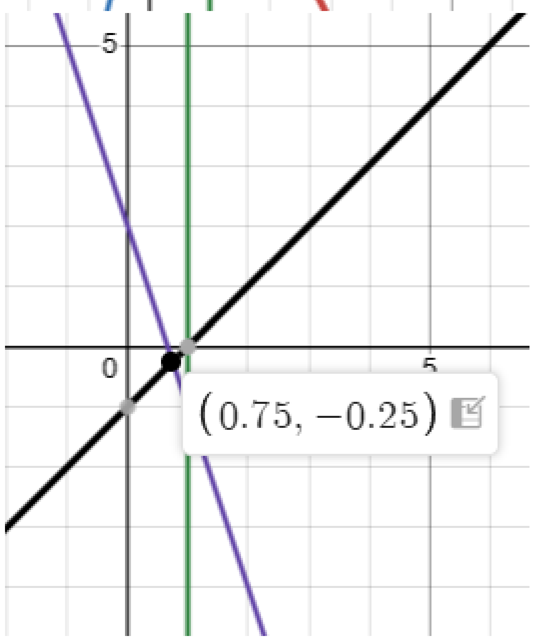
2. for player 1, if a=e and c=g, then they can play anything regardless of opponents strategy. similarly for p2, if b=d and f=h they can play anything

3. for p1 there are two scenerios. if $a > e$ and $c > g$, then U is strictly dominant, but if $a < e$ and $c < g$, then D is strictly dominant. for p2, similarly there are two scenerios. if $b > d$ and $f > h$, L is s. dominant but if $b < d$ and $f < h$, R is s. dominant

4. if we assume that there exists a pure NE, say UL, then $a \geq e$ and $b \geq d$. but no strategy is strictly dominant which means that $g > c$ and $h > f$. if this is the case, then DR must also be a pure NE, which is a contradiction since we assumed there was a unique NE.

5. •





note the first one image is player 1's payoffs and the second is p2's payoffs

the red line represents going up and the blue line represents going down. the purple line represents left and the black, right.

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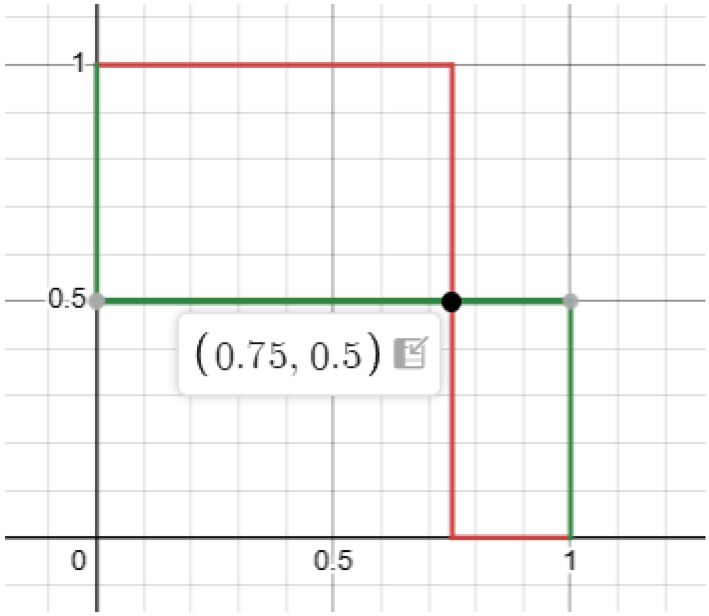

the x axis represents x, the probability of going up, and the y axis represents the probability of going Left. there are three NE. pure down left, pure up right, and the mix option shown by p1 75 25 between up and down, and p2 going 50 50 between left and right. $x = .75, y = .5$

4 4

1. if 1 person taking the bus generates enough shame to make the other 5 take the bus, then any additional bus takers also results in the plane takers switching. thus we need to find S such that

$$-S \cdot 5 < -1 \tag{8}$$

   note that $\frac{1}{5} \leq \frac{6-n}{n}, n \in \{1, 2, 3, 4\}$

   we find that if $S > 1/5$, then everyone takes the bus.

2. for any value $S > 0$, there always exists a NE where everyone takes the plane, since no shame is generated, everyone gets 0 utility which is better than -1.

3. when $S = \frac{N}{8-N}$, there exists a NE where more than 1 person takes the bus and more than 1 person takes the plane. this is because the shame in taking the plane is equal to -1, thus they ahve no reason to deviate.