

lecture

October 6, 2025

semi definite programming (?)

often try to take discrete problem, use continuous relaxation then approximate back into the discrete problem

continuous relaxation

1 max cut algorithm

max cut - maximize number of edges that cross between the two "sets"

$G(V, E, W)$, $w \in W > 0$

an algorithm $A: V \rightarrow \{0, 1\}$

objective: maximize

$$\sum_{uv \in E} w_{uv} \mathbb{1}\{A(u) \neq A(v)\}$$

bipartite graph can be cut perfectly in half

new idea: for every vertex $v \in V$, variable $x_v \in \{0, 1\}$ tells whether the vertex v goes left or right. for every edge $e \in E$, $z_e \in \{0, 1\}$ tells whether the edge is cut or not.

objective function becomes:

$$\max \sum_{uv \in E} w_{uv} z_e \quad (1)$$

w constraint:

$$\begin{aligned} z_{uv} &\leq x_u + x_v \\ z_{uv} &\leq 2 - (x_u + x_v) \end{aligned} \quad (2)$$

this is extremely cursed, use $z_{uv} = x_u \oplus x_v$? ¹

¹ \oplus is xor

nah

approx is to use interval $[0, 1]$ for both x and z .²

Definition 1 (LP). shorthand for linear program

problem for using the interval, using 2, 1, setting all x to 0.5, we can set z to 1 for everything and all programs give the max value again.

try another formulation since the last one doesn't work

$\forall v \in V, y \in \{-1, 1\}$. use

$$\max \sum_{uv \in E} w_{uv} \frac{1 - y_v y_u}{2} \quad (3)$$

try allow $y_v \in \mathbb{R}^n$ where $|v| = n, |y_v| = 1$

after placing the vertices of y_u in space, taking random hyperplane to divide the set of vectors, we get something that is 0.878 optimal³

finding another algorithm that is better shows $P=NP$?????

²this breaks the xor notation
³???