CSE 206G: Game Theory and Applications in Computer Science and Engineering

Assignment 1
DUE October 9, 2025
30 points

1. Consider the following strategic game.

	Defend North	**Defend South**
Touchdown North	0, 0	7, -7
Touchdown South	7, -7	0,0
Field Goal	3,-3	3, -3
Turnover	0,0	0,0

   Find all Nash Equilibria in both pure and mixed strategies. Hint: eliminate any dominated pure strategies and look for mixed strategy equilibria with support on the remaining strategies. [5 points]

2. Several years ago there was a vote in the Senate to confirm now Justice Kavanuagh to the Supreme Court. We recreate this contest as a three player game between the three Senators : Lisa Murkowski (R-AK), Susan Collin (R-ME), and Joe Manchin (D-WV). Besides these three, there are 48 Democratic votes fimly against confirmation and 50 Republican votes (49 senators + VP Pence) fimly for confirmation. Thus, all three players here would have to vote no to block confirmation. Otherwise, it passes. Let x_L = 1 (yes) or 0 (no) be Lisa Murkowski's vote. Similarly define x_S and x_J. Also, Let x_{-i} bet the sum of the votes of the Senators besides Senator i.
   - Lisa Murkowski "wants" to vote against, so she can appear to be independent from her party. However, she doesn't want to do that if her vote is pivotal since her party would punish her for that. Thus

$$u_L(x_L, x_{-L}) = \begin{cases} 1 & if\ x_L = 0\ and\ x_{-L} \geq 1 \\ -1 & if\ x_L = 0\ and\ x_{-L} = 0 \\ 0 & otherwise. \end{cases}$$

- Susan Colins wants to vote for, to get support from the party in her reelection campaign. However, she needs to be able to tell Maine voters that her yes vote wasn't pivotal. So if it were pivotal, she'd want to vote no. Thus,
$$u_S(x_S, x_{-S}) = \begin{cases} 1 & if\ x_S = 1\ and\ x_{-S} \geq 1 \\ -1 & if\ x_S = 1\ and\ x_{-S} = 0 \\ 0 & otherwise. \end{cases}$$
- Joe Manchin wants to vote for confirmation, so he can tell West Virginia voters he is not a typical Democrat. However, if his vote is pivotal, he wants to vote no, so that he doesn't frustrate his party too much.
$$u_J(x_J, x_{-J}) = \begin{cases} 1 & if\ x_J = 1\ and\ x_{-J} \geq 1 \\ -1 & if\ x_J = 1\ and\ x_{-J} = 0 \\ 0 & otherwise. \end{cases}$$

Analyze this game and find all Nash equilibria. It may help to put the game in matrix form in the following way:

Joe Votes 0 Table:

Susan

		0	1
Lisa	0	--,--,--	
	1		

Joe Votes 1 Table:

Susan

		0	1
Lisa	0	--,--,--	
	1		

For each strategy profile (table location) put the payoff in the order
Lisa's payoff, Susan's payoff, Joe's payoff.

Once you've made the table, circle payoff numbers in each profile entry that constitute a best response for that player. See where all players play best responses. [10 points]

3. Fudenberg and Tirole, exercise 1.1 (pg 36) [10 points]

4. Six friends that knew each other in college a decade ago decide to have a reunion in Las Vegas. The friends need to decide whether to fly or take the bus to Las Vegas, and all of them are aware that even one flight can put more CO_2 in the air than many put in the air in an entire year. The friends are also aware that taking a bus to Las Vegas is a miserable experience. Let 1 be the misery cost of taking the bus. Suppose

that each friend that takes the bus will generate *S* shaming insults to distribute to the friends that do not fly. Assume *S*>0, and S can be non-integer. Thus if *N* is the number of friends that fly, then the payoff for flying is

$$-S\,\frac{6-N}{N}.$$

This is because 6 - *N* non-flying friends are dishing out insults, and the total insults get split among the *N* friends that fly. The payoff for taking the bus is simply -1. Assume *S*>0. [5 points]

a) Under what conditions would it be a Nash equilibrium for everyone to take the Bus?
b) Under what conditions would it be a Nash equilibrium for everyone to fly?
c) Can there be an equilibrium in which the number of people that fly, N, is more than 0 but less than 6?