lecture

September 30, 2025

payoff functions

$$u_i : S \to \mathbb{R} \tag{1}$$

- I is the set of all players
- i is a player
- S_i is the set of all possible strategies for player i
- S is **space** strategy of profiles
- s is a specific strategy profile
- $-i$ is defined as $\{1, \ldots, I\} \setminus i$
  - $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots s_I)$

Definition 1 (Nash Equilibrium)**.** A strategy profile s from which no player has an incentive to deviate unilaterally

ie for player i

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \forall s_i' \in S_i \tag{2}$$

it is generally not true that games have unique nash equilibria

r_i is the "Best response multifunction/correposndence"?

r_i tells what the best response for a player is based on what they believe other players will do

note that given an opponent's strategy, the players strategy output of r_i might be multiple strategies, thus it is a multifunction rather than a function [1]

$$r_i : S_{-i} \rightrightarrows s_i \tag{3}$$

note: $\rightrightarrows$ means that r_i is a *multifunction*

σ_i is the probability distribution on S_i

$$\sigma = \bigtimes_{i \in I} \sigma_i \tag{4}$$

[1] a multifunction outputs a subset of the codomain. a correspondence is a function that outputs in 2^D where D is the codomain (powerset of the codomiain)

Σ_i is the space of all possible mixed strategies for player i

$$\Sigma = \bigtimes_{i \in I} \Sigma_i \tag{5}$$

$|s_i| - 1$ simplex ??
σ is a mixed nash equilibrium is missing $\forall$delimiter

$$u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i}), \forall \sigma_i \in \sigma \tag{6}$$

$$r_i : \Sigma_{-i} \rightrightarrows \Sigma_i \tag{7}$$

$$r_i(\sigma_-1) \mapsto \text{argmax}_{\sigma_i} u(\sigma_i, \sigma{-}i) \tag{8}$$

$$r : \Sigma \rightrightarrows \Sigma \tag{9}$$

Theorem 1 (nash's theorem)**.** In every finite game with a finite number of players and a finite set of pure strategies per player, there exists at least one Nash equilibrium in mixed strategies.

Definition 2 (symmetric game)**.** all players have same strategy space. a payoff to any player is the same for any other player i.e. permutations of any players strategies results in such that player i's strategy is played by player j results in the payoff of i and j being identical.

$S_i = S_j$, $u_i(s_i, \ldots, s_n) = u_j(s_{\pi(1)}, \ldots, s_{\pi(n)})$

$$\begin{aligned} S_i = S \quad & \text{for all } i \\ u_i(s) = u_j(\pi(s)) \quad & \text{for any permutation } \pi \text{ of players} \end{aligned} \tag{10}$$