Fast Population Maximization in Designating Areas of Substantial Unemployment (ASUs) in Utah with A* Search and Binary Integer Programming

Joyce Cao (University of Utah)
Advisor: Jeff Phillips (University of Utah)

Designation of ASUs

- **Input**
  - Graph $G = (V, E)$
  - Each vertex has population, employment, unemployment
  - Threshold = 6.5%
- **Problem Definition**
  - Maximize $\sum_{i=1}^{k} p_i$, here p_i is the i-th ASU's population
  - s.t. each ASU is contiguous
  - the unemployment rate of each ASU $\geq 6.5\%$

- **Motivation**
  - Federal Funding is based on the total population of the k ASUs

Dataframe

- **Data**
  - Shape Data (tl_2016_49_tract.shp)
  - Attribute Data (UT_asu_exampleData.csv)
- **Dataframe**

Index	…	Geoid	Geometry	EMP	UNEMP	POP
0		49057210900	<pysal obj>	3638	77	6813
1		49045131200	<pysal obj>	1393	75	3515
2		49045131100	<pysal obj>	4053	205	7987
3		49045130600	<pysal obj>	1241	37	2704

BIP Step 1: Identify Decision Variables

Index	…	Geoid	Geometry	EMP	UNEMP	POP
0		49057210900	<pysal obj>	3638	77	6813
1		49045131200	<pysal obj>	1393	75	3515
2		49045131100	<pysal obj>	4053	205	7987
3		49045130600	<pysal obj>	1241	37	2704

- For each census tract in the dataframe, we need to make a decision.
  - Do we choose census tract 0?
  - Do we choose census tract 1?
  - …
- In Math,

$$x_0 = \begin{cases} 1 & \text{if we decide to choose tract 0} \\ 0 & \text{otherwise} \end{cases}$$

Decision Variable

$$x_1 = \begin{cases} 1 & \text{if we decide to choose tract 1} \\ 0 & \text{otherwise} \end{cases}$$

BIP Step 1: Identify Decision Variables (cont.)

Index	…	Geoid	Geometry	EMP	UNEMP	POP
0		49057210900	<pysal obj>	3638	77	6813
1		49045131200	<pysal obj>	1393	75	3515
2		49045131100	<pysal obj>	4053	205	7987
3		49045130600	<pysal obj>	1241	37	2704

Decision Number	Yes or No (1 or 0)	Decision Variables	EMP	UNEMP	POP
0	Choose 0?	x_0	3638	77	6813
1	Choose 1?	x_1	1393	75	3515
2	Choose 2?	x_2	4053	205	7987
3	Choose 3?	x_3	1241	37	2704

BIP Step 2: Identify Objective

Decision Number	Yes/No (1 or 0)	Decision Variables	EMP	UNEMP	POP
0	Choose 0?	x_0	3638	77	6813
1	Choose 1?	x_1	1393	75	3515
2	Choose 2?	x_2	4053	205	7987
3	Choose 3?	x_3	1241	37	2704

- We want to maximize the total population of these decision variables
- In math, $\text{maximize } 6813x_0 + 3515x_1 + 7987x_2 + 2704x_3$

If this decision variable is 1, then the population is 7,987

If this decision variable is 0, then the population is 0

BIP Step 3: Identify Constraints (1)

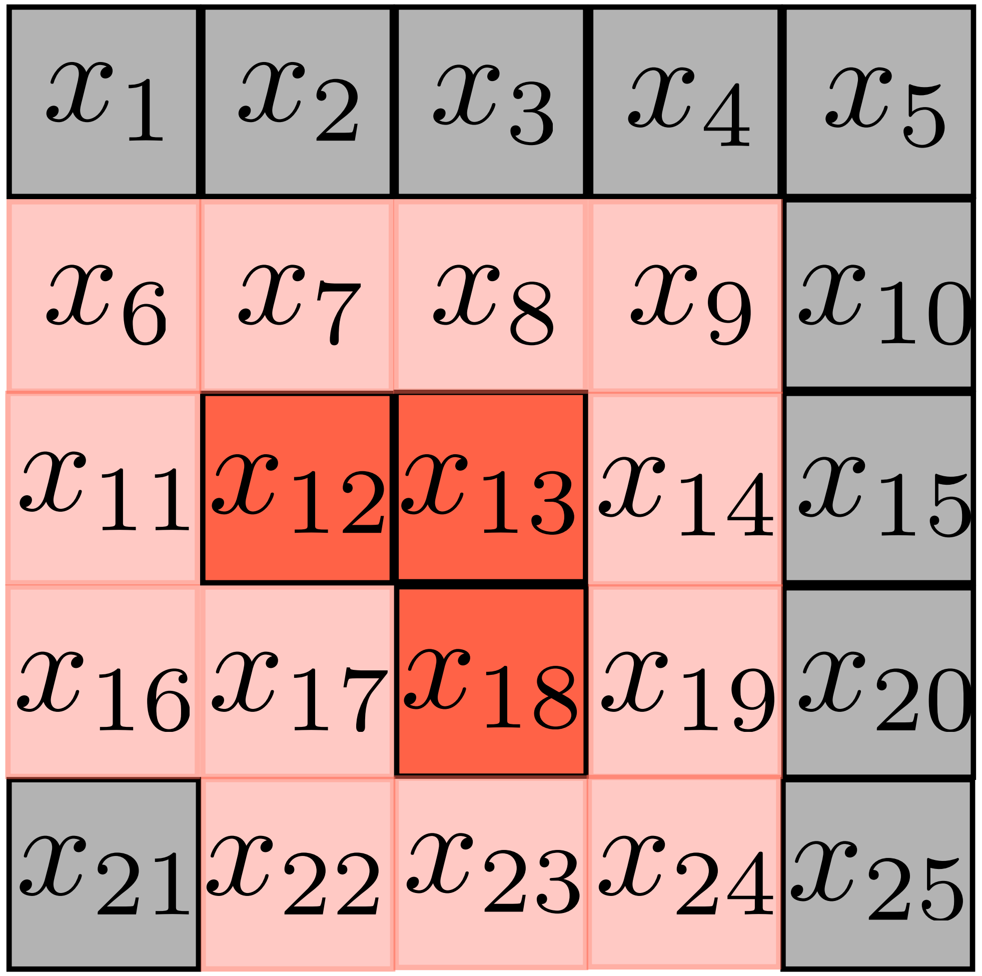
Decision Number	Yes/No (1 or 0)	Decision Variables	EMP	UNEMP	POP
0	Choose 0?	x_0	3638	77	6813
1	Choose 1?	x_1	1393	75	3515
2	Choose 2?	x_2	4053	205	7987
3	Choose 3?	x_3	1241	37	2704

- The average unemployment rate of an ASU should be >= 6.5%.
- In Math,

$$\frac{x_0 77 + x_1 75 + x_2 205 + x_3 37}{x_0(3638+77) + x_1(75+1393) + x_2(205+4053) + x_3(37+1241)} \geq 6.5\%$$

threshold = 0.0645 for rounding to thousandths

BIP Step 3: Identify Constraints (2)



Seed (tract added)

1st-order Neighbor

2nd-order Neighbor

Radius = 2

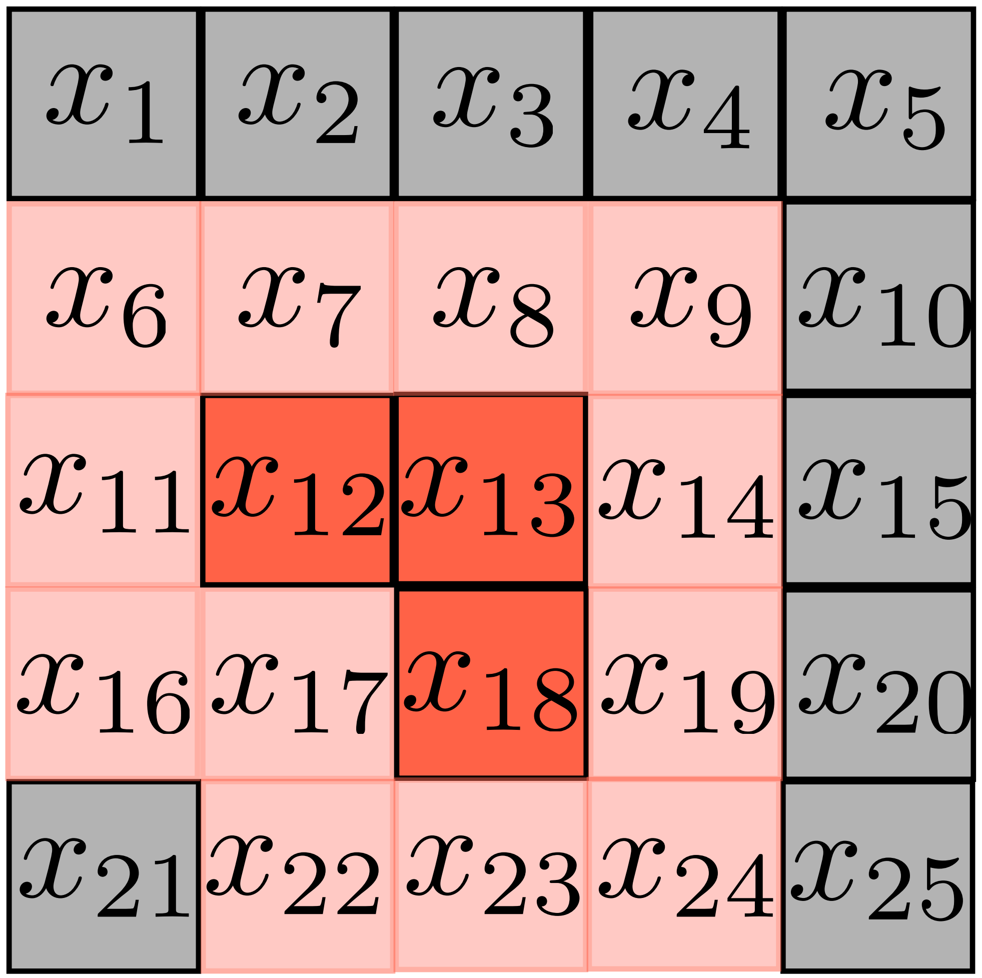
- An ASU should be a contiguous area.
- The way we use: if we decide to choose a tract only if at least one of its previous order neighbors has been chosen.
- Take x_{21} for example
- In Math, $x_{21} \leq x_{16} + x_{17} + x_{22}$

The real contiguous constraint is as long as there is a path from the tract to the seed

For example, when x_{21} = 1, at least one of x_{16}, x_{17}, x_{22} =1 such that the inequality is valid

BIP Put Everything Together

- First, we construct a region around the seed with a specified radius



Seed (tract added)

1st-order Neighbor

2nd-order Neighbor

Radius $= 2$

- Second, we formulate the BIP model

$$\text{maximize } \sum_{i=1}^{25} p_i x_i$$

$$\text{s.t.} \frac{\sum_{i=1}^{25} u_i x_i}{\sum_{i=1}^{25} (u_i + e_i) x_i} \geq 0.0645$$

$$x_{12} = 1$$

$$x_{13} = 1$$

$$x_{18} = 1$$

$$x_i^c \leq \sum_q x_j^{c-1}$$

$$x_i = \text{binary}$$

p_i : pop of the i-th tract

u_i : unemp of the i-th tract

e_i : emp of the i-th tract

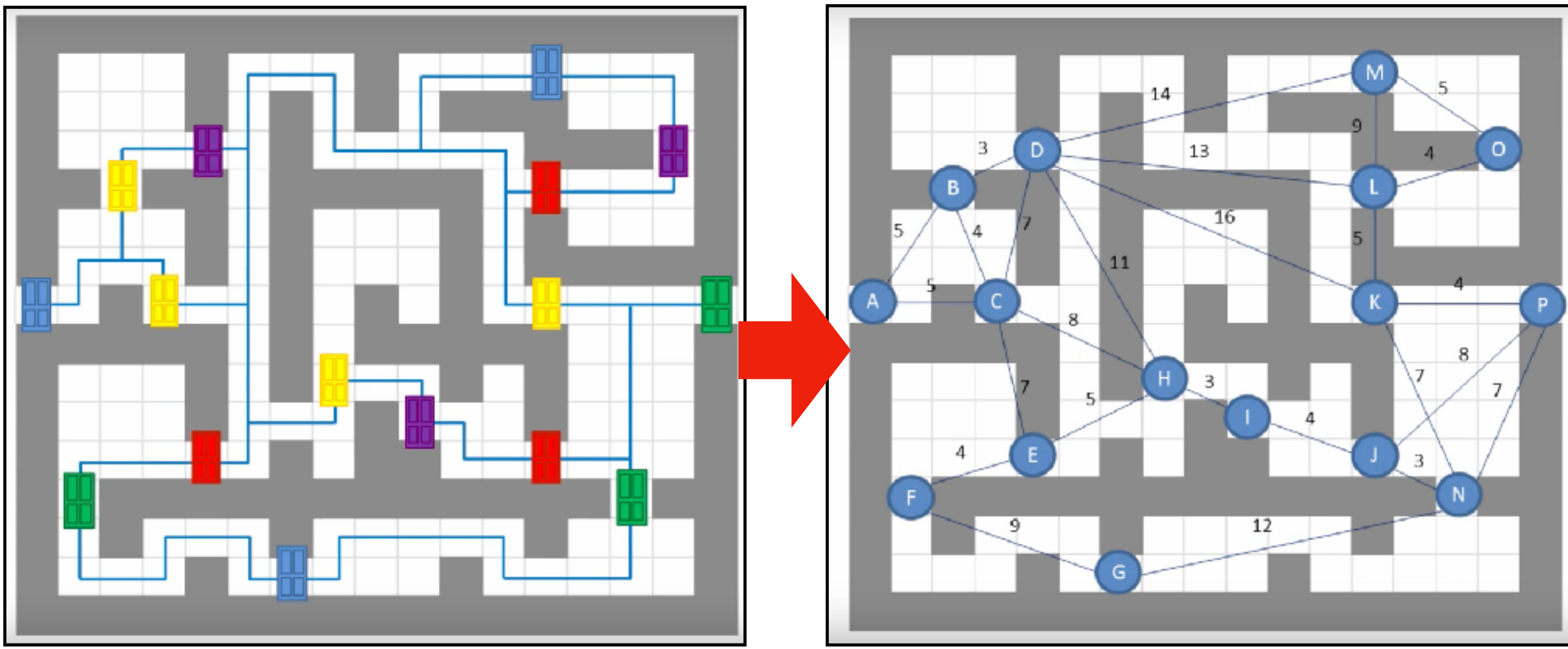
c : order of neighborhood

q : queen neighbors

- Third, we use Python and PuLP to solve it.

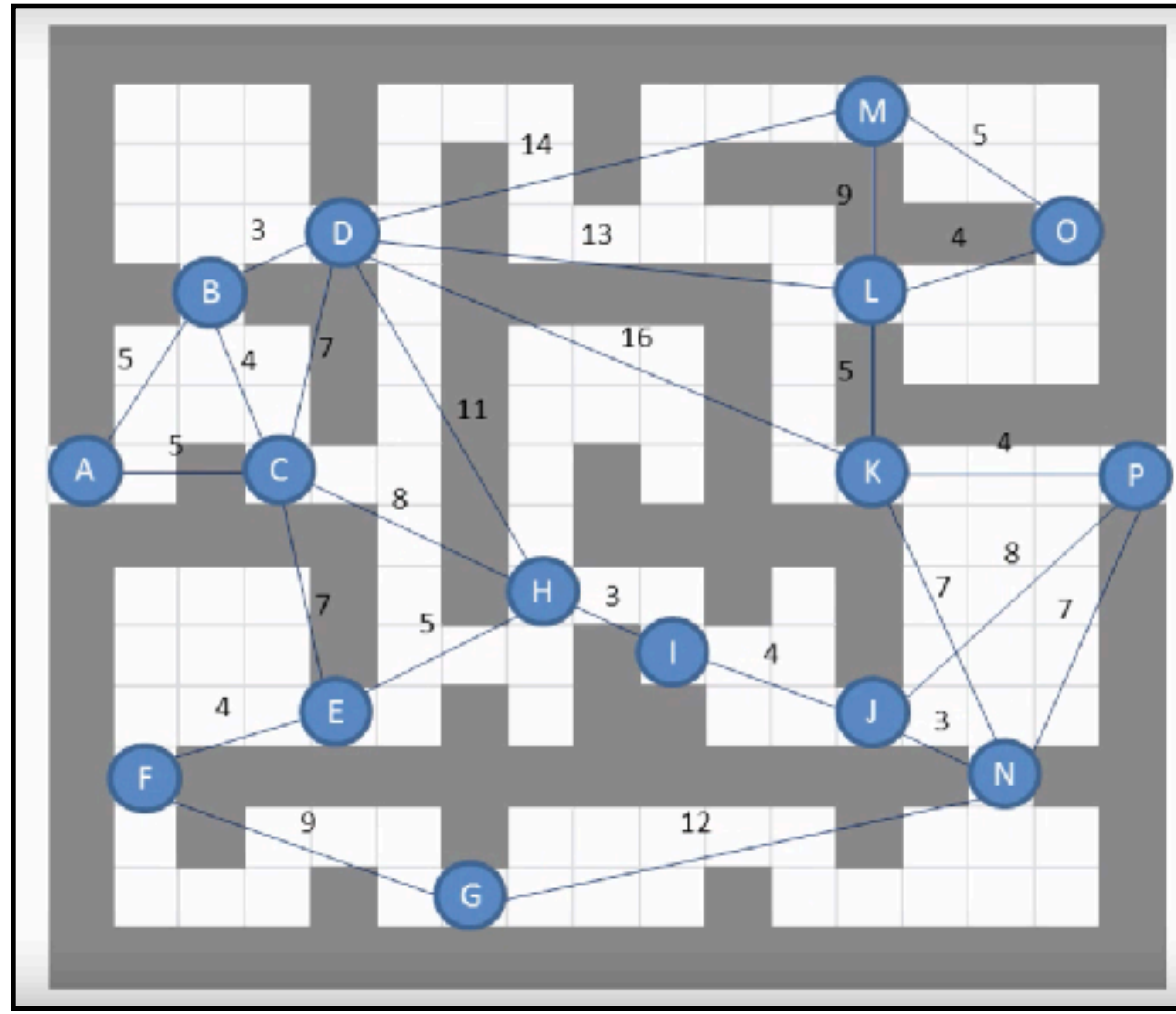
Introduction to A*

- A* search algorithm was invented by Hart, Nilsson and Raphael in 1968 to find paths through graphs [Hart et al. 1968].
- Example:
  - Navigation through a maze [Kevin Drumm 2017]



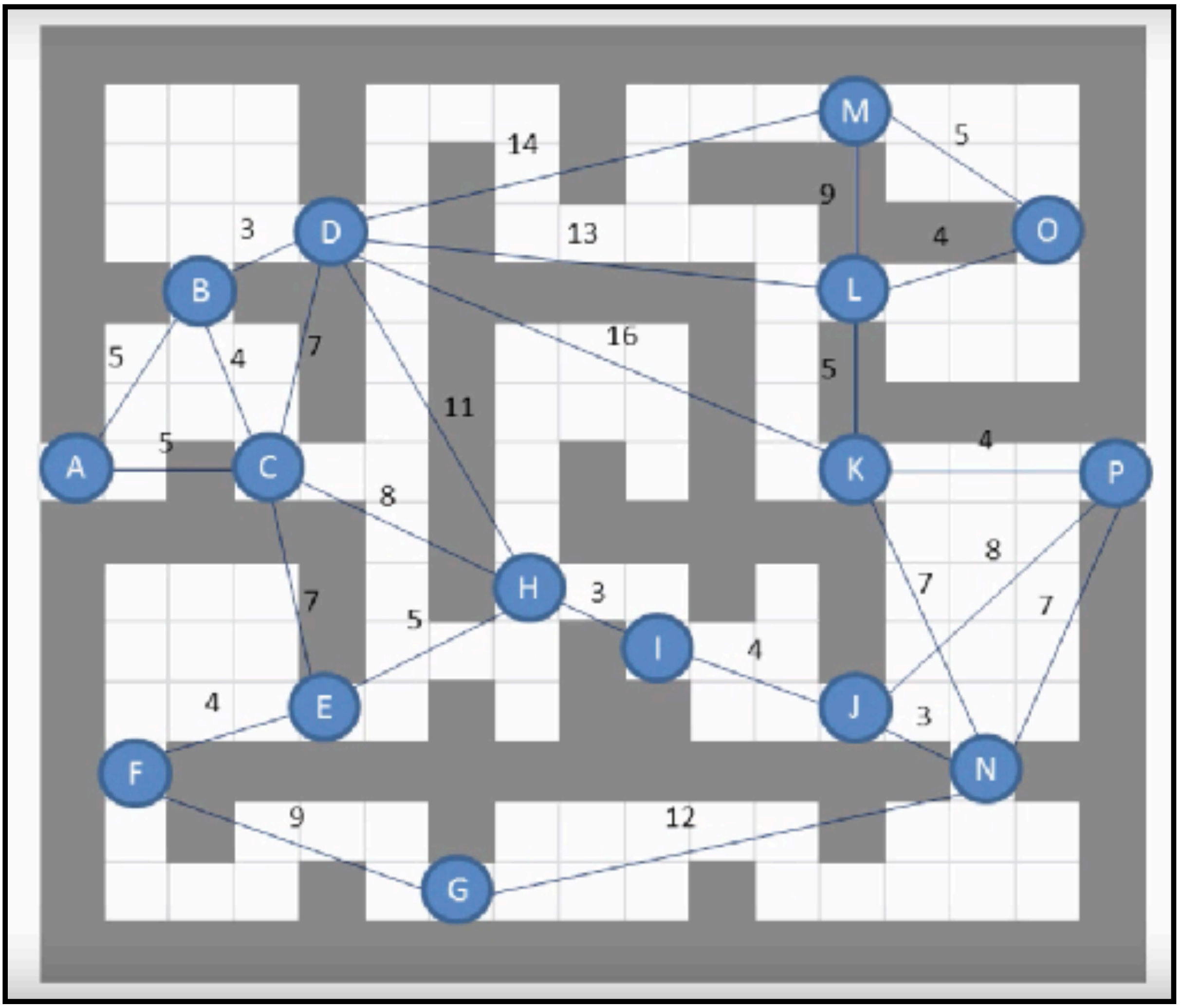
Components of A*

- **State Space:** the state space forms a search tree where nodes are states, edges are actions.
- **State:** nodes in the search tree
- **Initial State:** A
- **Gaol State:** P

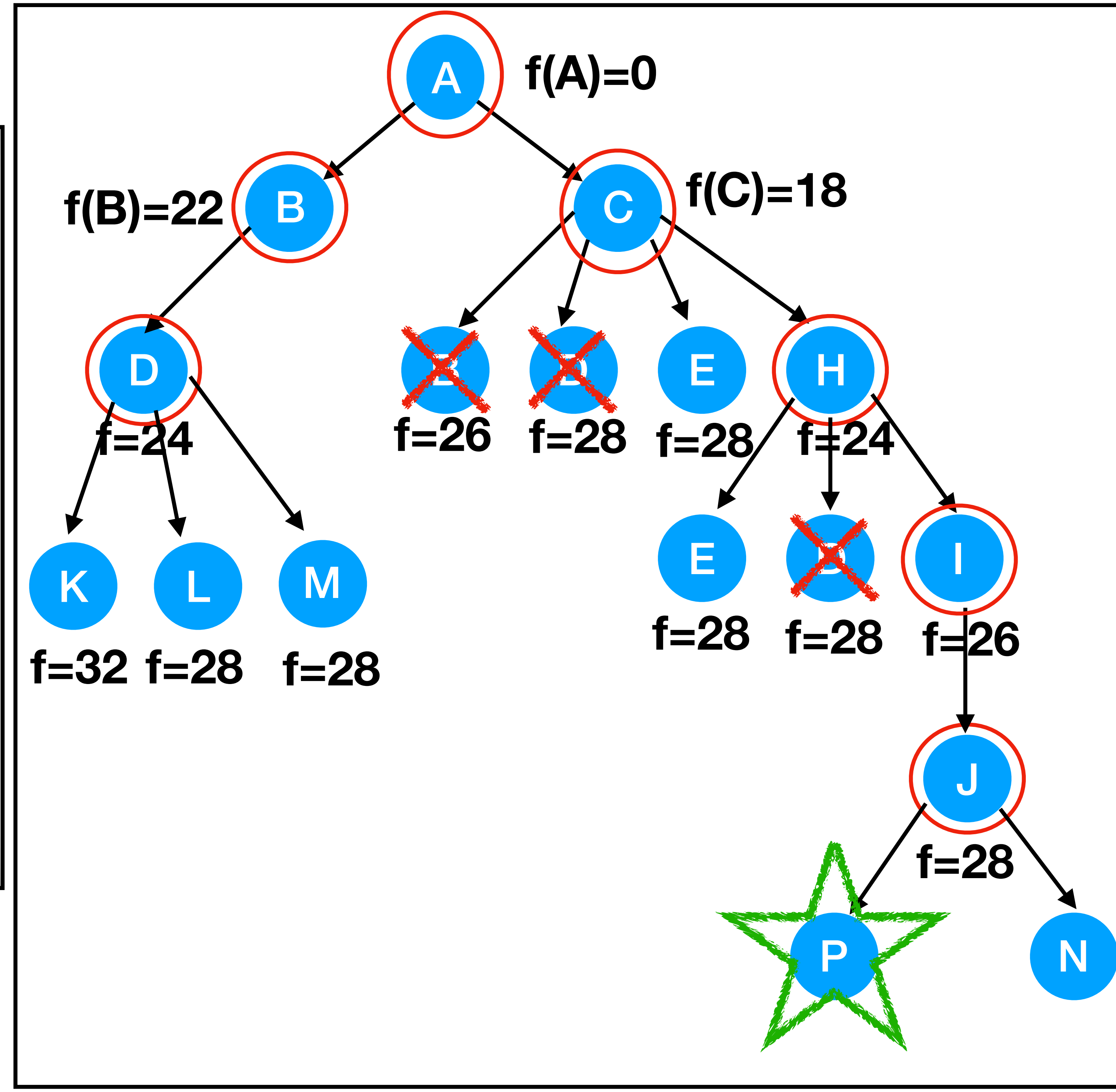


- **G-cost:** the cost to reach the current state from initial state
- **H-cost:** the cost to reach the goal state from the current state
- **F-cost:** the cost to reach the goal state from initial state, F = G + H

A* Maze Example



Maze as Graph



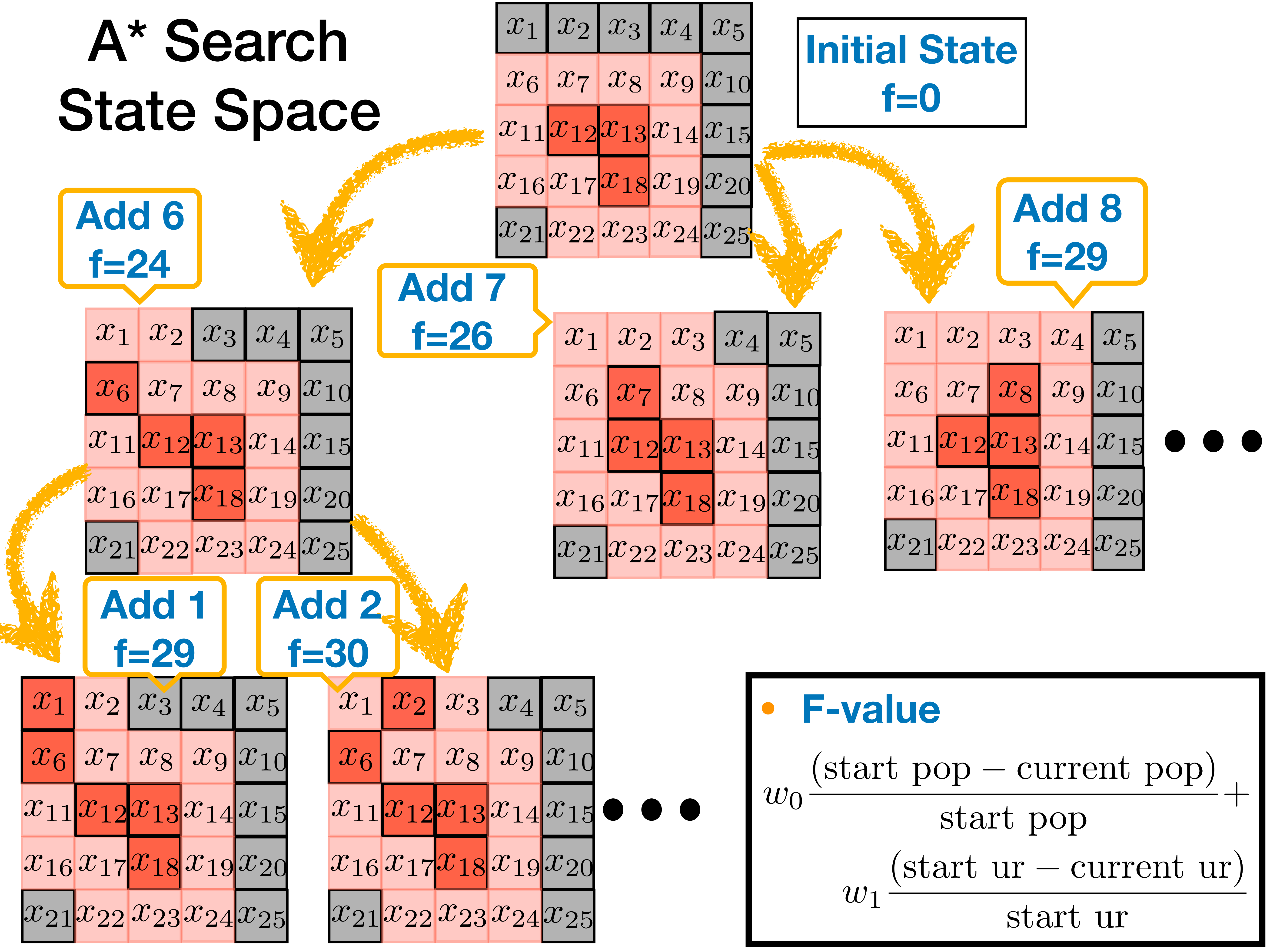
Search Tree

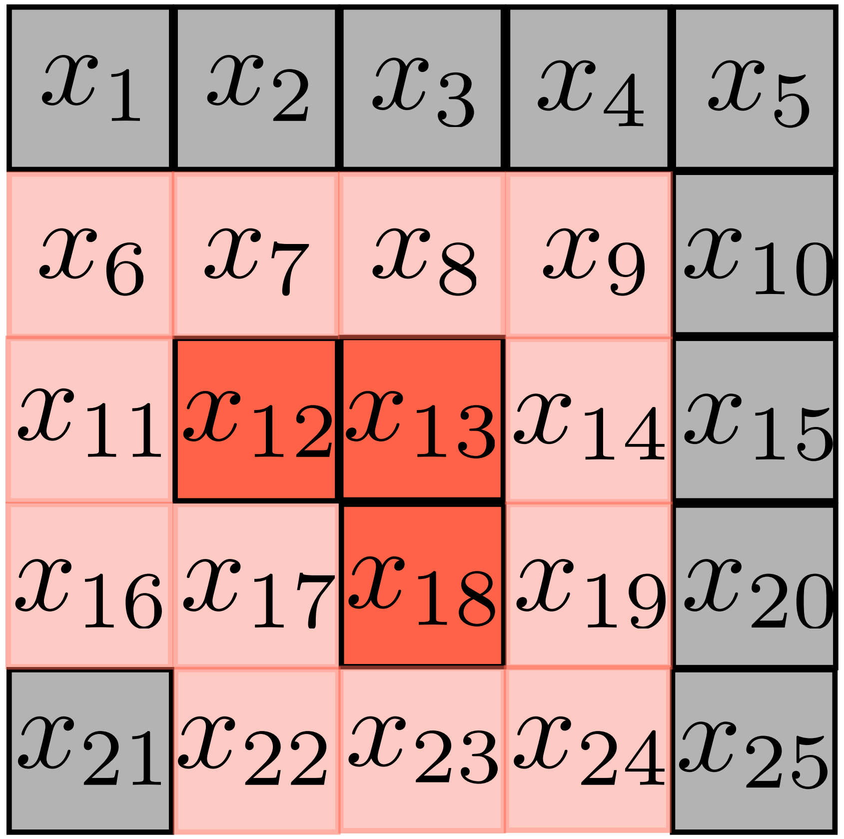
A* Components in the ASU Problem

- **Initial State:** the seed
- **Gaol State:** an ASU with goal pop and goal ur
- **G-cost:** $G = w_0 \dfrac{(\text{start pop} - \text{current pop})}{\text{start pop}} + w_1 \dfrac{(\text{start ur} - \text{current ur})}{\text{start ur}}$
- **H-cost:** $H = 0$
- **F-cost:** $F = G + H = w_0 \dfrac{(\text{start pop} - \text{current pop})}{\text{start pop}} + w_1 \dfrac{(\text{start ur} - \text{current ur})}{\text{start ur}}$

- How do we set the goal for an ASU grown from a seed?
  - The goal ur is 6.5%
  - The goal pop is estimated using BIP

- Why do we set the F-cost that way?
  - F is a weighted summation of percentage change in pop and ur
  - When choose the next node to expand
    - We want pop to increase as much as possible
    - We want ur to decrease as little as possible


A* Search
State Space
Initial State
f=0
Add 6
f=24
Add 7
f=26
Add 8
f=29
Add 1
f=29
Add 2
f=30
F-value
$w_0 \frac{(\text{start pop} - \text{current pop})}{\text{start pop}} +$
$w_1 \frac{(\text{start ur} - \text{current ur})}{\text{start ur}}$



A* ASU Example

(a) Initial State $\{12, 13, 18\} : f = 0$

(b) After expanding $\{12, 13, 18\}$

$\{12,13, 18\}$
$f = 0$

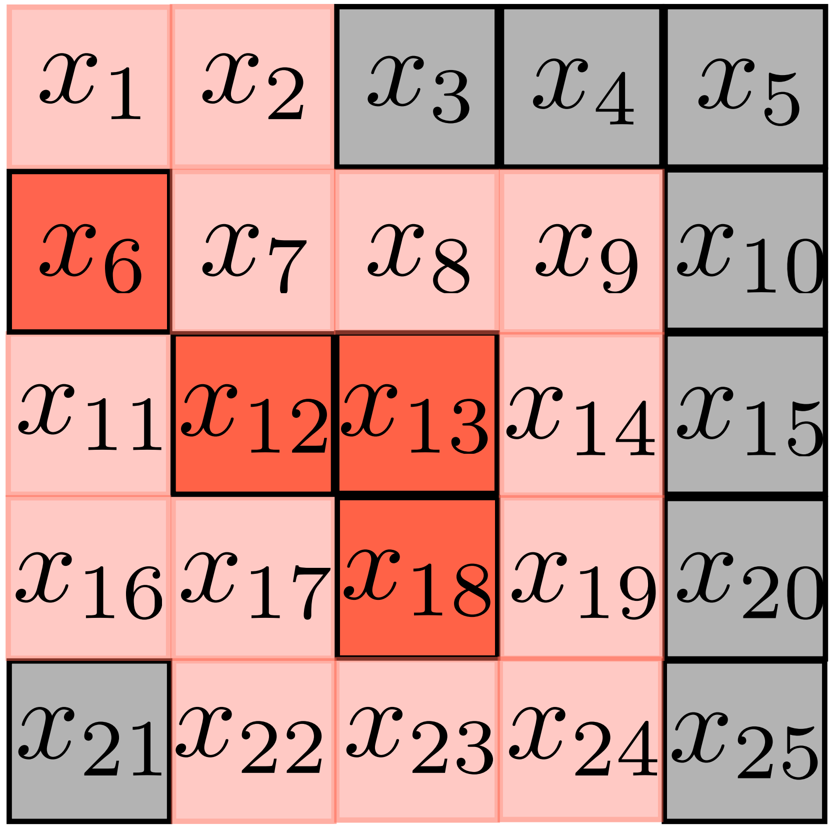
$\{6,12, 13, 18\}$ $f = 24$

$\{7,12, 13, 18\}$ $f = 26$

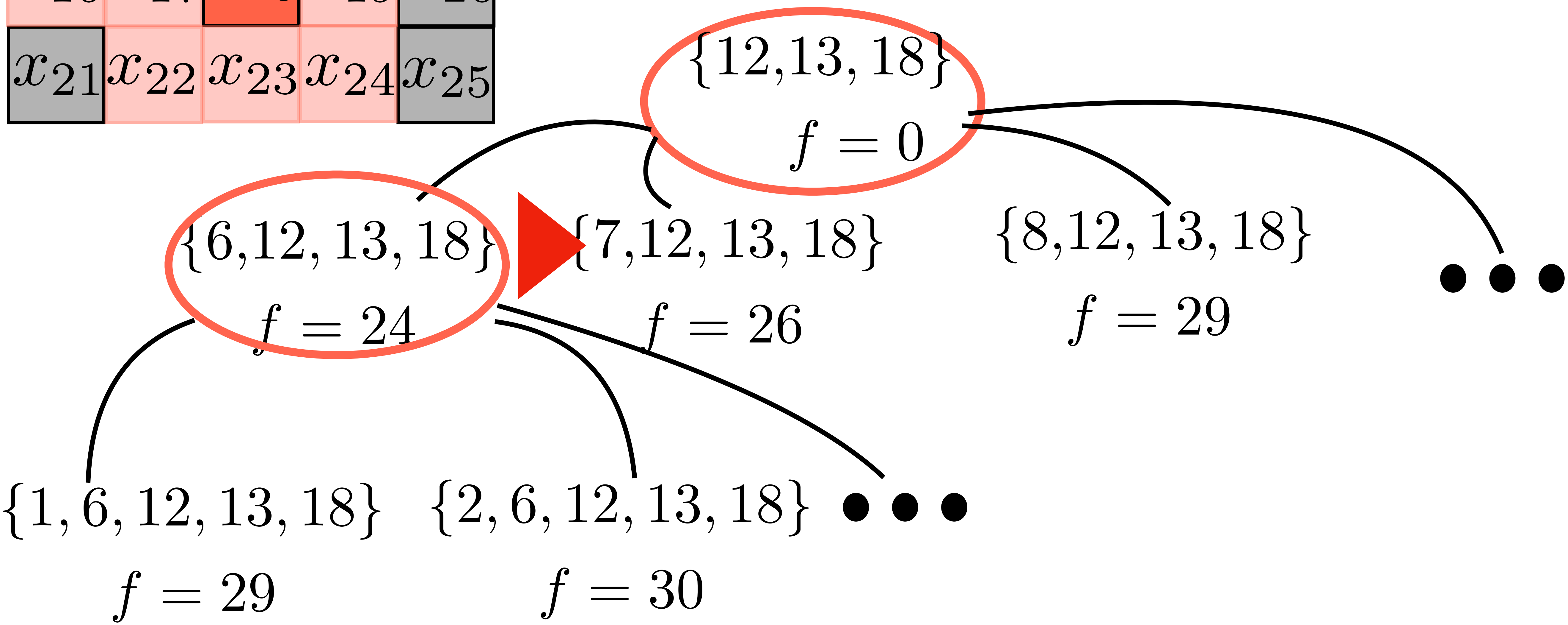
$\{8,12, 13, 18\}$ $f = 29$

...

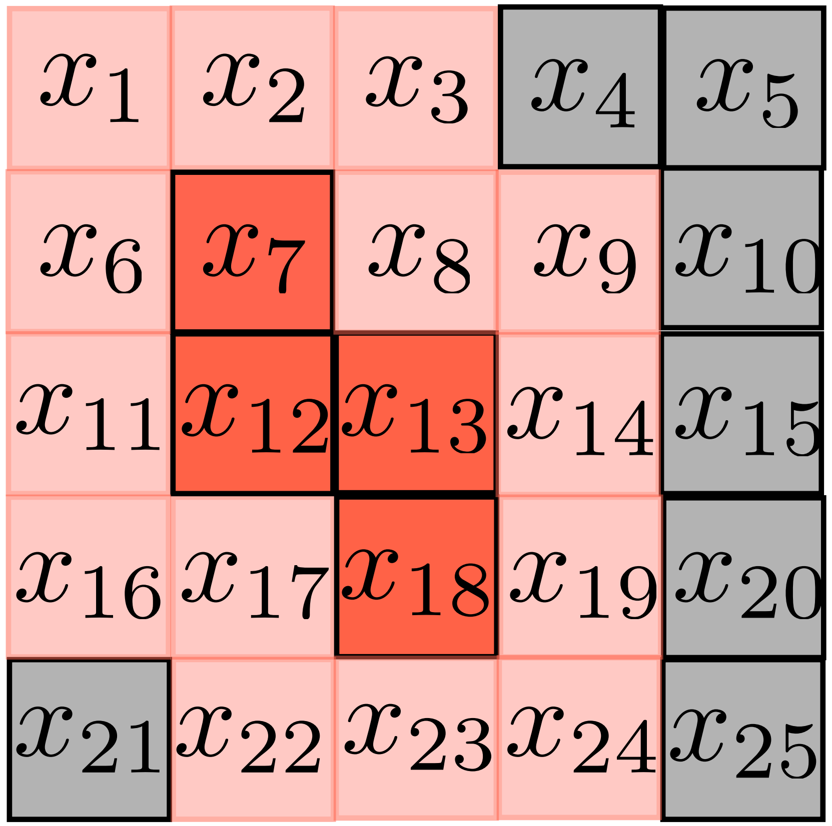
A* ASU Example



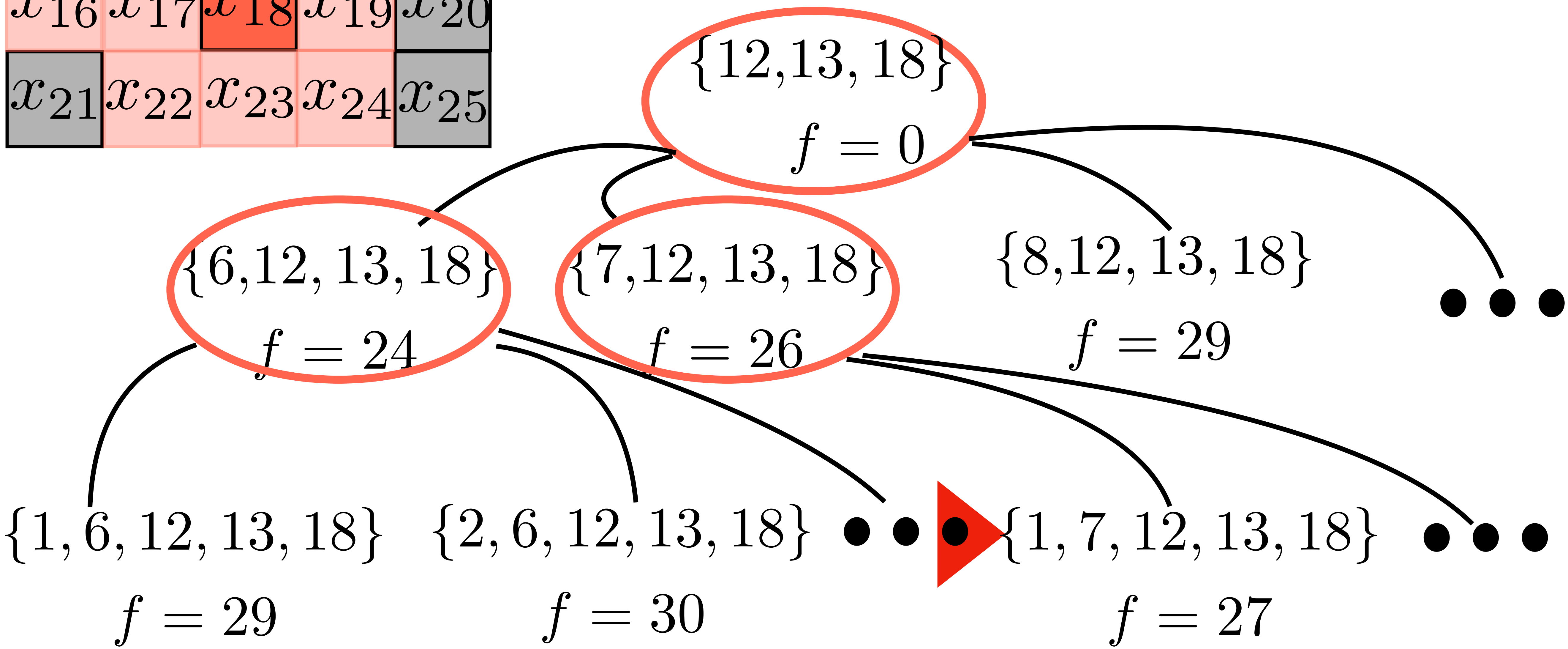
(c) After expanding $\{6, 12, 13, 18\}$



A* ASU Example

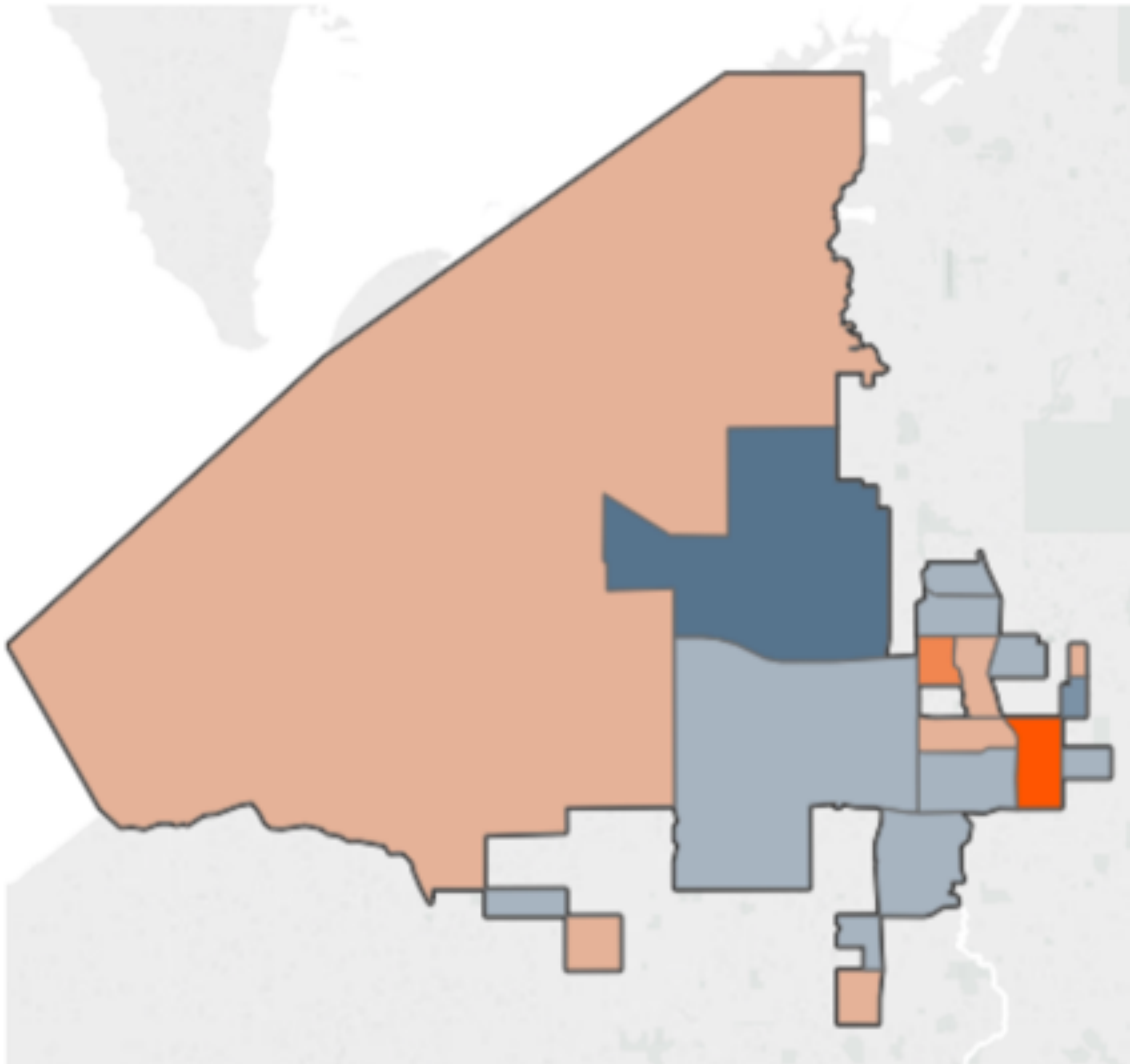


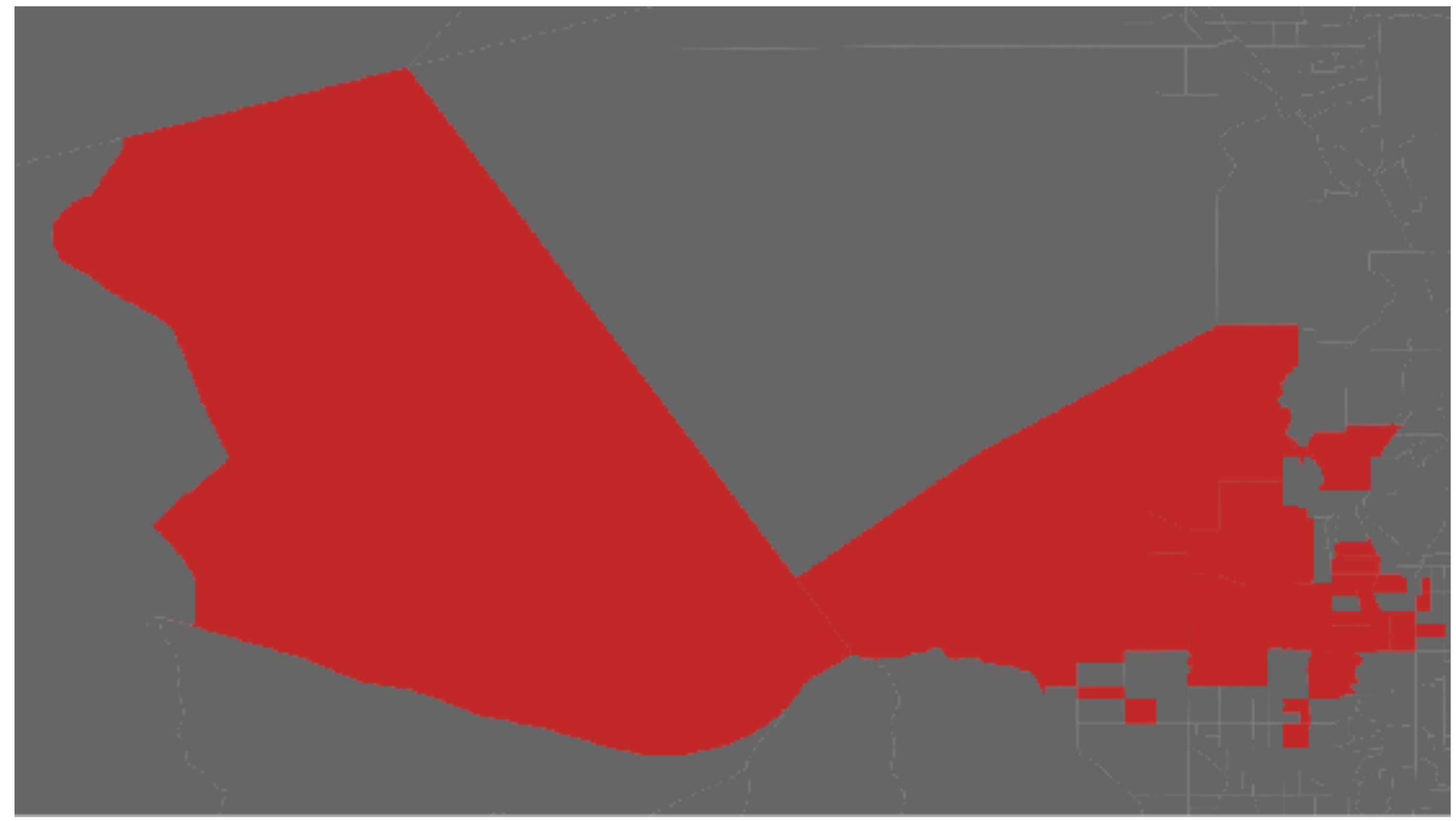
(d) After expanding $\{7, 12, 13, 18\}$



We repeat this process until a goal node is reached.
The goal node is returned as the resulting ASU.

Marginal Results





Obtained using A*

[Workforce Services, 2017]

Aggregate URate

6.53%

Aggregate 2015 Pop

91,202

Aggregate URate

6.49%

Aggregate 2015 Pop

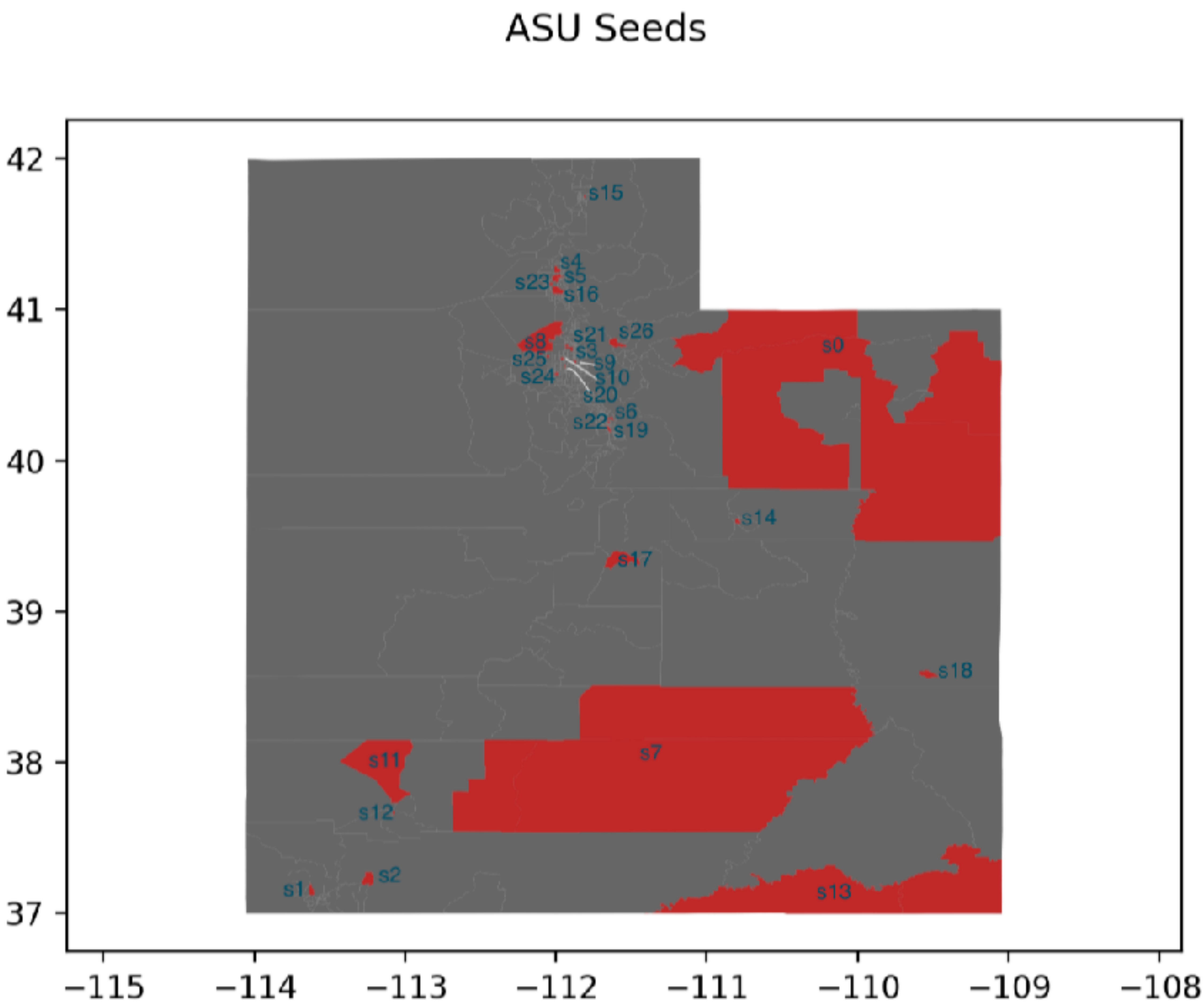
98,424

Find all ASUs

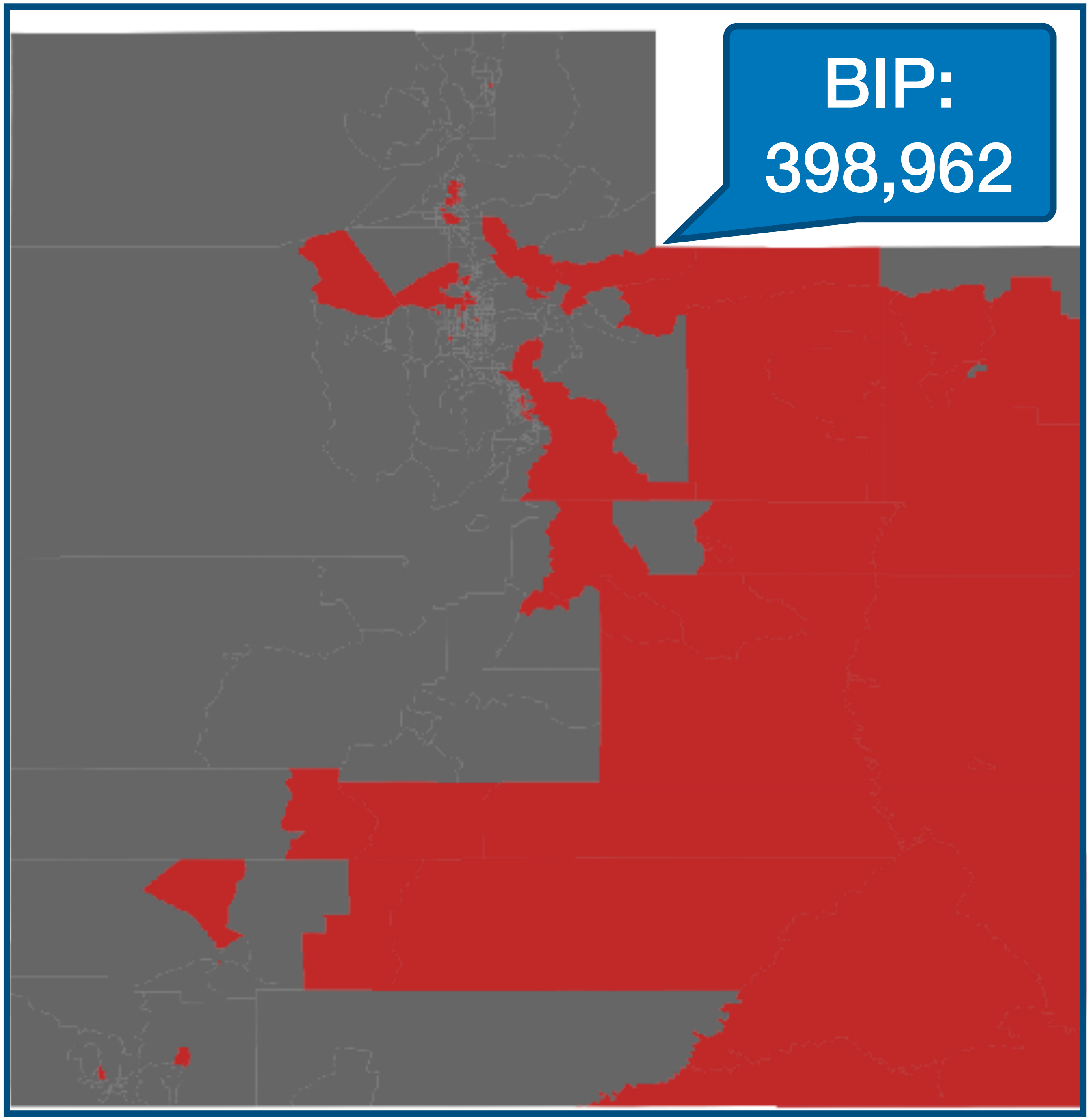
- So far, we have been discussing how to find a single ASU using BIP or A*

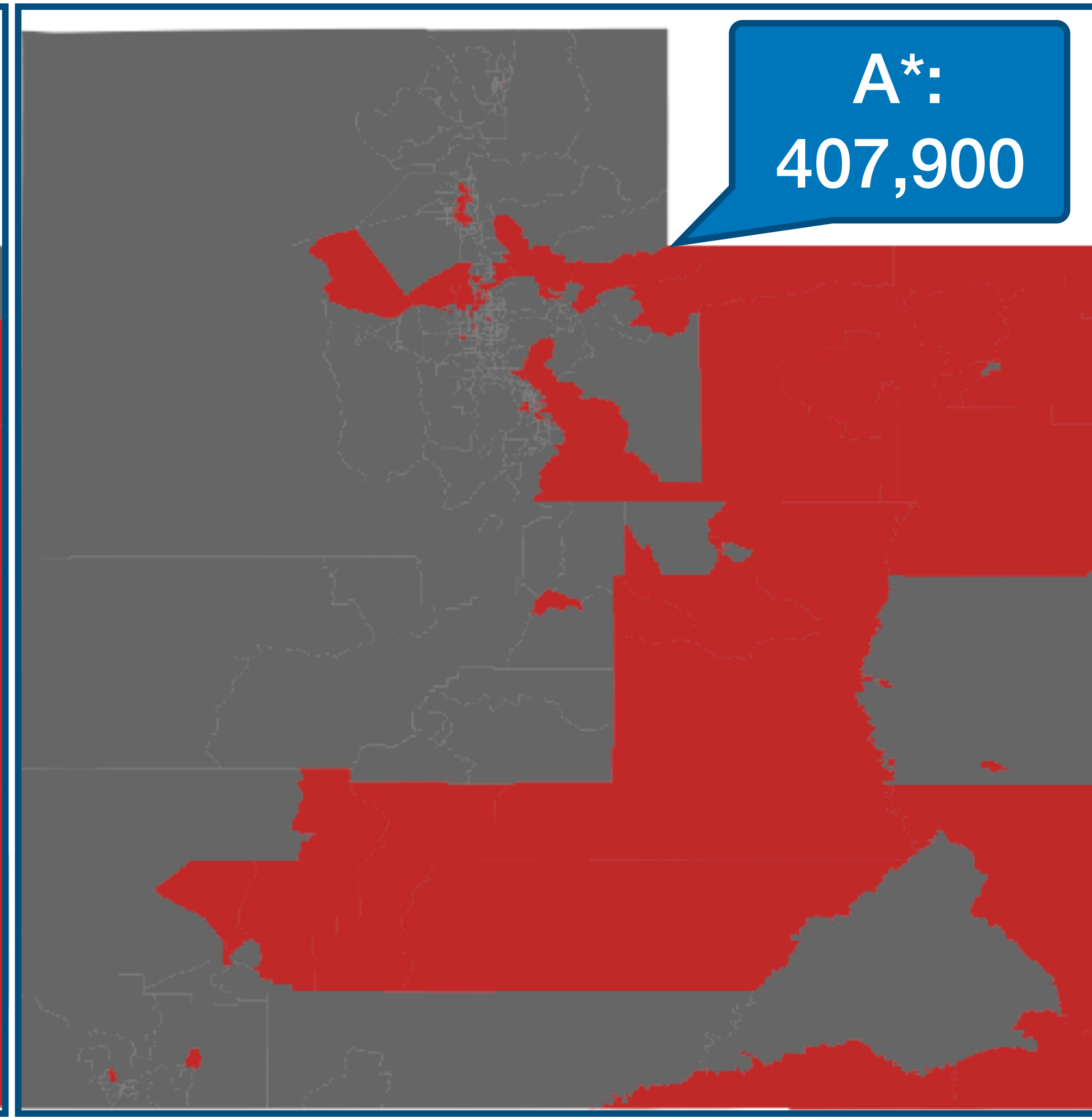
- Now, we will explain how to find all ASUs in the state of Utah
  - Initialize a result set A
  - Store all census tracts whose ur >= 6.5% in a seed set S
  - for each seed in S if the seed is not in A
    - Grow an ASU for the seed using BIP or A* without repetition
    - Add the resulting ASU to A
  - After all seeds in S are processed, we return A

- In the next slide, we will show the seed set S on a map
- In the slide after the next, we will show all ASUs (the result set A) found by BIP and A* respectively


ASU Seeds
42
41
40
39
38
37
−115
−114
−113
−112
−111
−110
−109
−108
s15
s4
s5
s23
s16
s21
s26
s8
s3
s25
s9
s24
s10
s20
s6
s22
s19
s0
s14
s17
s18
s11
s7
s12
s2
s1
s13

Joint Results from BIP and A*





Code and References

- **Code**
  - https://github.com/splashmountain/asu

- **Tools**
  - PySAL for Spatial Data Analysis (http://pysal.readthedocs.io/en/latest/users/installation.html)
  - PuLP for Linear Programming (https://pythonhosted.org/PuLP/)
  - Geopandas for Spatial Data Plot (https://automating-gis-processes.github.io/2016/Lesson2-geopandas-basics.html)

- **Papers**
  - Redistricting Using Constrained Ploygonal Clustering [Joshi et al. 2012]
  - Contiguity Constraints for Single-Region Site Search Problems [Cove and Church 2000]
  - A Formal Basis for the Heuristic Determination of Minimum Cost Paths [Hart et al. 1968]