Fast Population Maximization in Designating Areas of Substantial Unemployment (ASUs) in Utah with A* Search and Binary Integer Programming

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Designation of ASUs

- **Input**
  - Graph $G = (V, E)$
  - Each vertex has population, employment, unemployment
  - Threshold = 6.5%

Threshold = 0.0645 for rounding to thousandths

- **Problem Definition**
  - Maximize $\sum_{i=1}^{k} p_i$, here p_i is the i-th ASU's population
  - s.t. each ASU is contiguous
  - the unemployment rate of each ASU $\geq 6.5\%$

- **Motivation**
  - Federal Funding is based on the total population of the k ASUs

Dataframe

- **Data**
  - Shape Data (tl_2016_49_tract.shp)
  - Attribute Data (UT_asu_exampleData.csv)
- **Dataframe**

Index	…	Geoid	Geometry	EMP	UNEMP	POP
0		49057210900	<pysal obj>	3638	77	6813
1		49045131200	<pysal obj>	1393	75	3515
2		49045131100	<pysal obj>	4053	205	7987
3		49045130600	<pysal obj>	1241	37	2704

Introduction to Linear Programming Example from Wikipedia:

Index	Name	Fertilizer kg/km^2	Pesticide kg/km^2	Price $\$/km^2$	Decision Var km^2
1	Wheat	F1	P1	S1	X1
2	Barley	F2	P2	S2	X2
Farmer has:		F (kg)	P (kg)		L(km^2)

- A farmer has a land of size L for Wheat and Barley … (https://en.wikipedia.org/wiki/Linear_programming)

- Step 1: Identify decision variables
  - X1 the area of land planted with wheat
  - X2 the area of land planted with barley

- Step 2: Identify objective
  - Farmer wants to maximize the revenue
  - Maximize $S_1x_1 + S_2x_2$

- Step 3: Identify constraints
  - Area constraints: $x_1 + x_2 \leq L$
    $x_1, x_2 \geq 0$
  - Fertilizer limit: $F_1x_1 + F_2x_2 \leq F$
  - Pesticide limit: $P_1x_1 + P_2x_2 \leq P$

BIP Step 1: Identify Decision Variables

Index	…	Geoid	Geometry	EMP	UNEMP	POP
0		49057210900	<pysal obj>	3638	77	6813
1		49045131200	<pysal obj>	1393	75	3515
2		49045131100	<pysal obj>	4053	205	7987
3		49045130600	<pysal obj>	1241	37	2704

- For each census tract in the dataframe, we need to make a decision.
  - Do we choose census tract 0?
  - Do we choose census tract 1?
  - …
- In Math,

$$x_0 = \begin{cases} 1 & \text{if we decide to choose tract } 0 \\ 0 & \text{otherwise} \end{cases}$$

Decision Variable

$$x_1 = \begin{cases} 1 & \text{if we decide to choose tract } 1 \\ 0 & \text{otherwise} \end{cases}$$

BIP Step 1: Identify Decision Variables (cont.)

Index	…	Geoid	Geometry	EMP	UNEMP	POP
0		49057210900	<pysal obj>	3638	77	6813
1		49045131200	<pysal obj>	1393	75	3515
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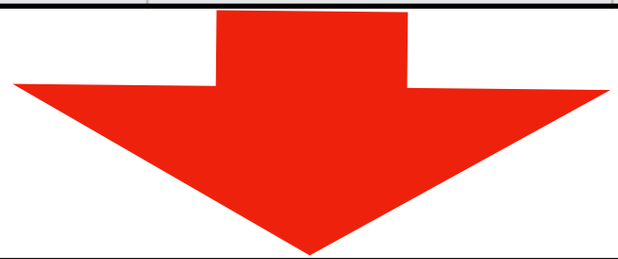
3		49045130600	<pysal obj>	1241	37	2704



Decision Number	Yes or No (1 or 0)	Decision Variables	EMP	UNEMP	POP
0	Choose 0?	x_0	3638	77	6813
1	Choose 1?	x_1	1393	75	3515
2	Choose 2?	x_2	4053	205	7987
3	Choose 3?	x_3	1241	37	2704

BIP Step 2: Identify Objective

Decision Number	Yes/No (1 or 0)	Decision Variables	EMP	UNEMP	POP
0	Choose 0?	x_0	3638	77	6813
1	Choose 1?	x_1	1393	75	3515
2	Choose 2?	x_2	4053	205	7987
3	Choose 3?	x_3	1241	37	2704

- We want to maximize the total population of these decision variables
- In math, $\text{maximize } 6813x_0 + 3515x_1 + 7987x_2 + 2704x_3$

If this decision variable is 1, then the population is 7,987

If this decision variable is 0, then the population is 0

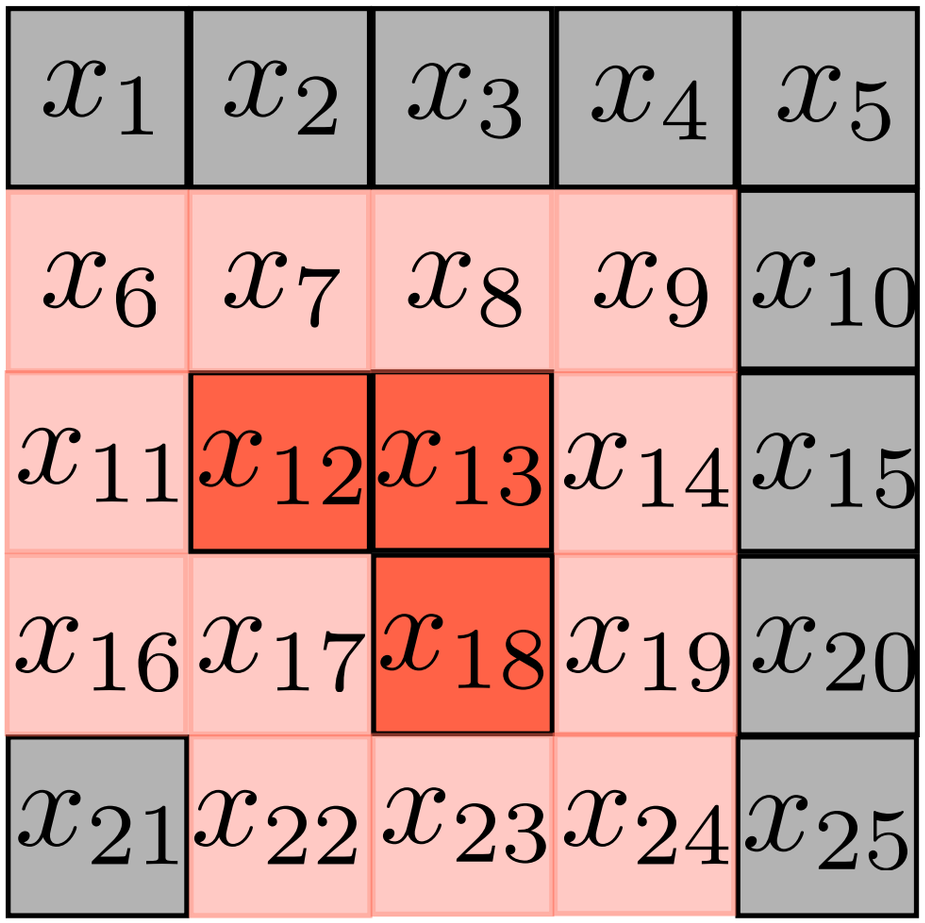
BIP Step 3: Identify Constraints (1)

Decision Number	Yes/No (1 or 0)	Decision Variables	EMP	UNEMP	POP
0	Choose 0?	x_0	3638	77	6813
1	Choose 1?	x_1	1393	75	3515
2	Choose 2?	x_2	4053	205	7987
3	Choose 3?	x_3	1241	37	2704

- The average unemployment rate of an ASU should be >= 6.5%.
- In Math,

$$\frac{x_0 77 + x_1 75 + x_2 205 + x_3 37}{x_0(3638 + 77) + x_1(75 + 1393) + x_2(205 + 4053) + x_3(37 + 1241)} \geq 6.5\%$$

BIP Step 3: Identify Constraints (2)



Seed (tract added)

1st-order Neighbor

2nd-order Neighbor

Radius = 2

- An ASU should be a contiguous area.
- The way we do: if we decide to choose a tract only if at least one of its previous order neighbors has been chosen.
- Take x_{21} for example
- In Math, $x_{21} \leq x_{16} + x_{17} + x_{22}$

The real contiguous constraint is as long as there is a path from the tract to the seed

For example, when x_{21} = 1, at least one of x_{16}, x_{17}, x_{22} =1 such that the inequality is valid

BIP Put Everything Together

- **First, we construct a region around the seed with a specified radius**

x_1	x_2	x_3	x_4	x_5
x_6	x_7	x_8	x_9	x_{10}
x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
x_{16}	x_{17}	x_{18}	x_{19}	x_{20}
x_{21}	x_{22}	x_{23}	x_{24}	x_{25}

Seed (tract added)

1st-order Neighbor

2nd-order Neighbor

Radius $= 2$

- **Second, we formulate the BIP model**

$$\text{maximize} \sum_{i=1}^{25} p_i x_i$$

$$\text{s.t.} \frac{\sum_{i=1}^{25} u_i x_i}{\sum_{i=1}^{25} (u_i + e_i) x_i} \geq 0.0645$$

p_i : pop of the i-th tract

u_i : unemp of the i-th tract

e_i : emp of the i-th tract

c : order of neighborhood

q : queen neighbors

$$x_{12} = 1$$

$$x_{13} = 1$$

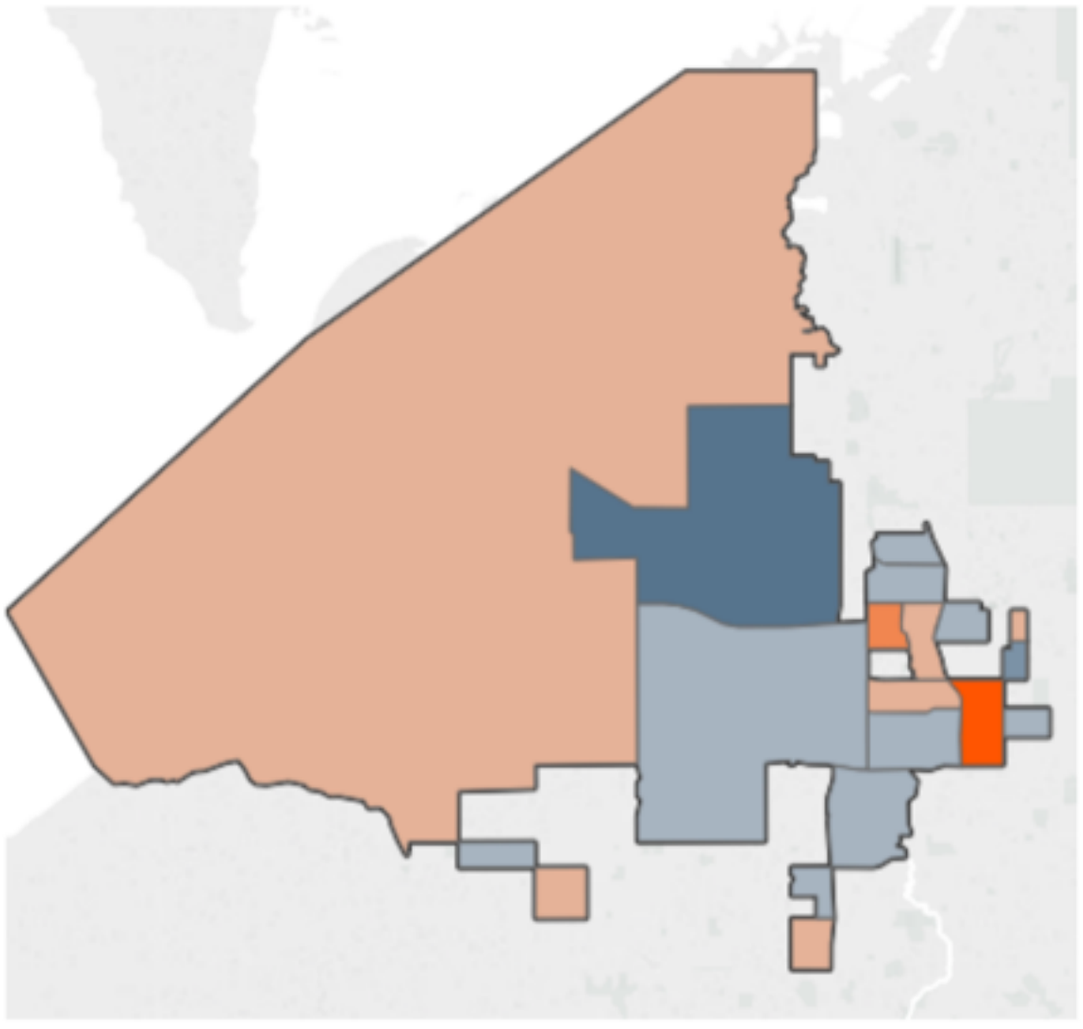
$$x_{18} = 1$$

$$x_i^c \leq \sum_q x_j^{c-1}$$

$$x_i = \text{binary}$$

- **Third, we use Python and PuLP to solve it.**

Marginal Results - BIP



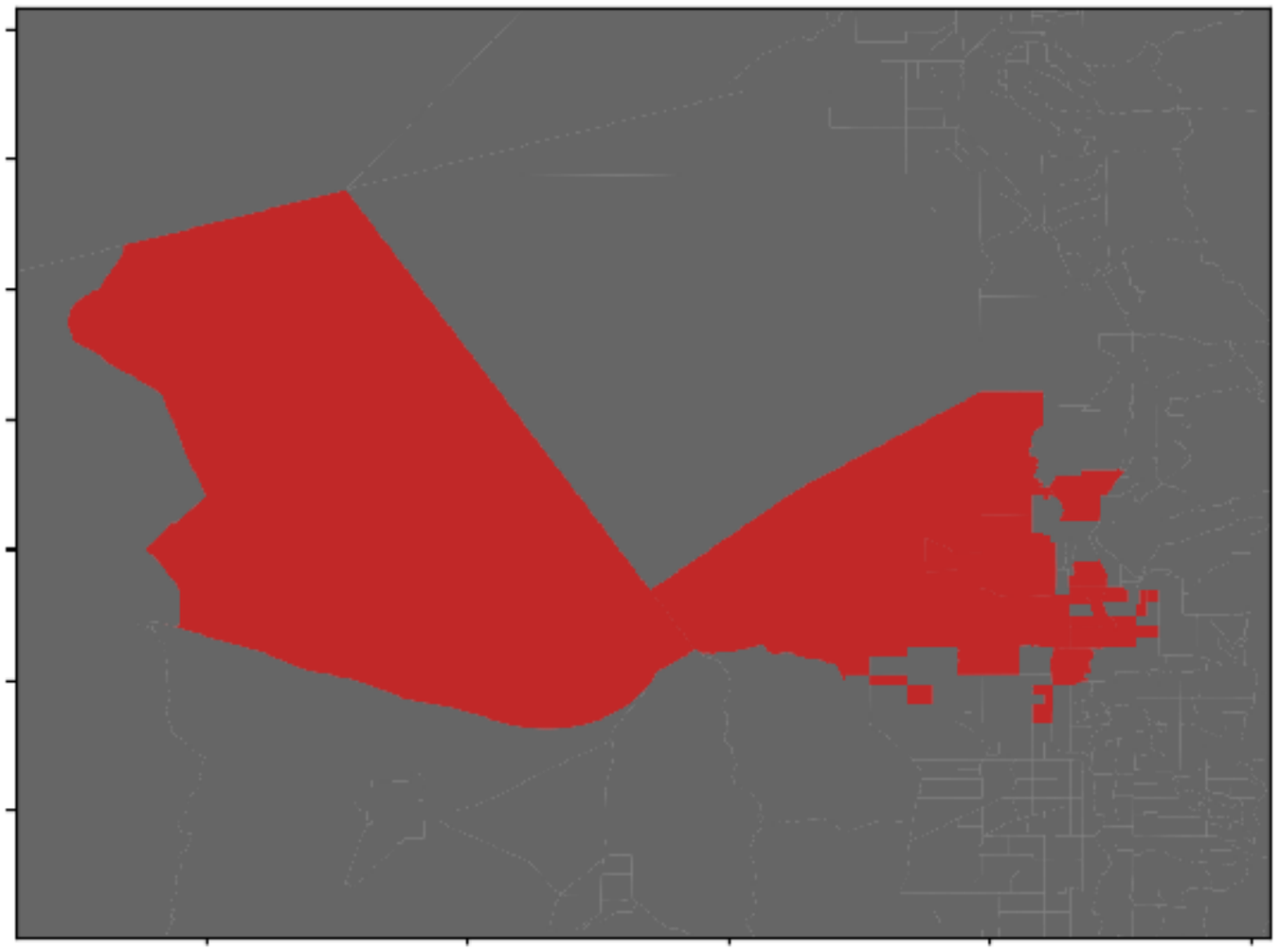
[Workforce Services, 2017]

Aggregate URate

6.53%

Aggregate 2015 Pop

91,202



Obtained using BIP

Aggregate URate

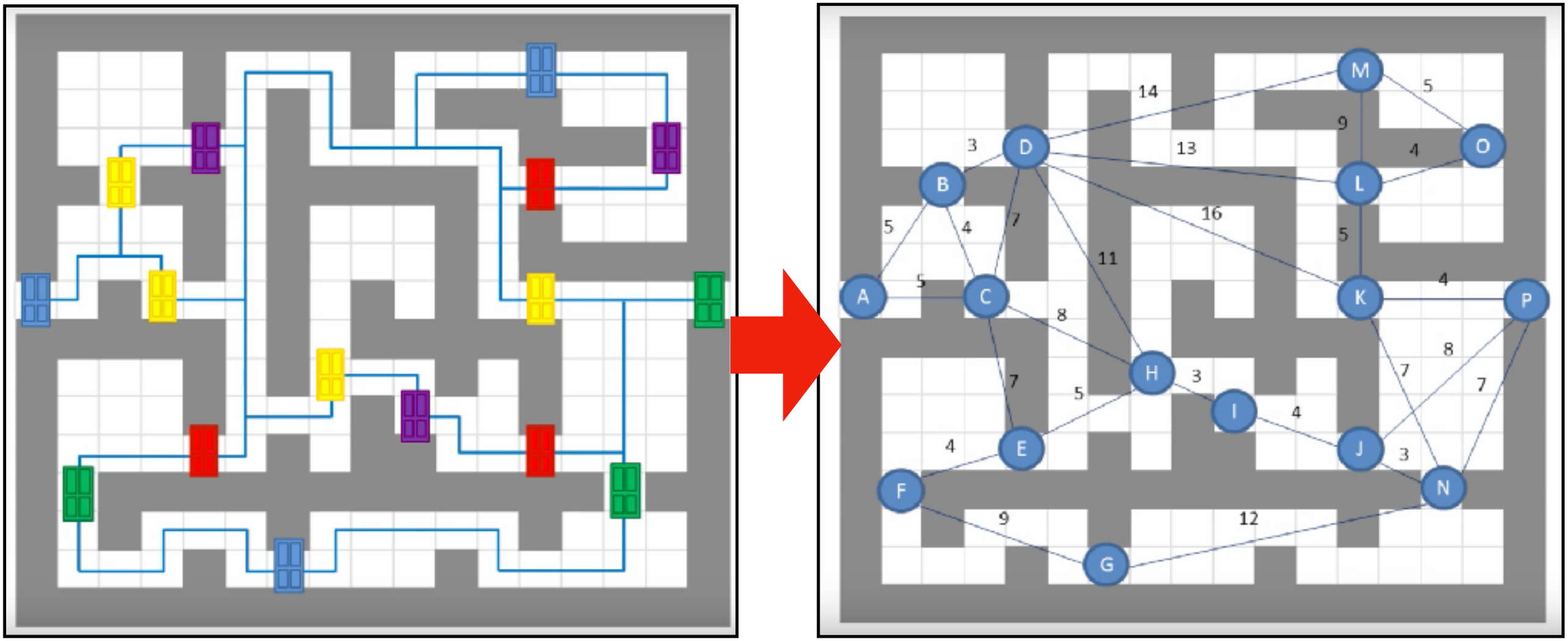
6.46%

Aggregate 2015 Pop

101,000

Introduction to A*

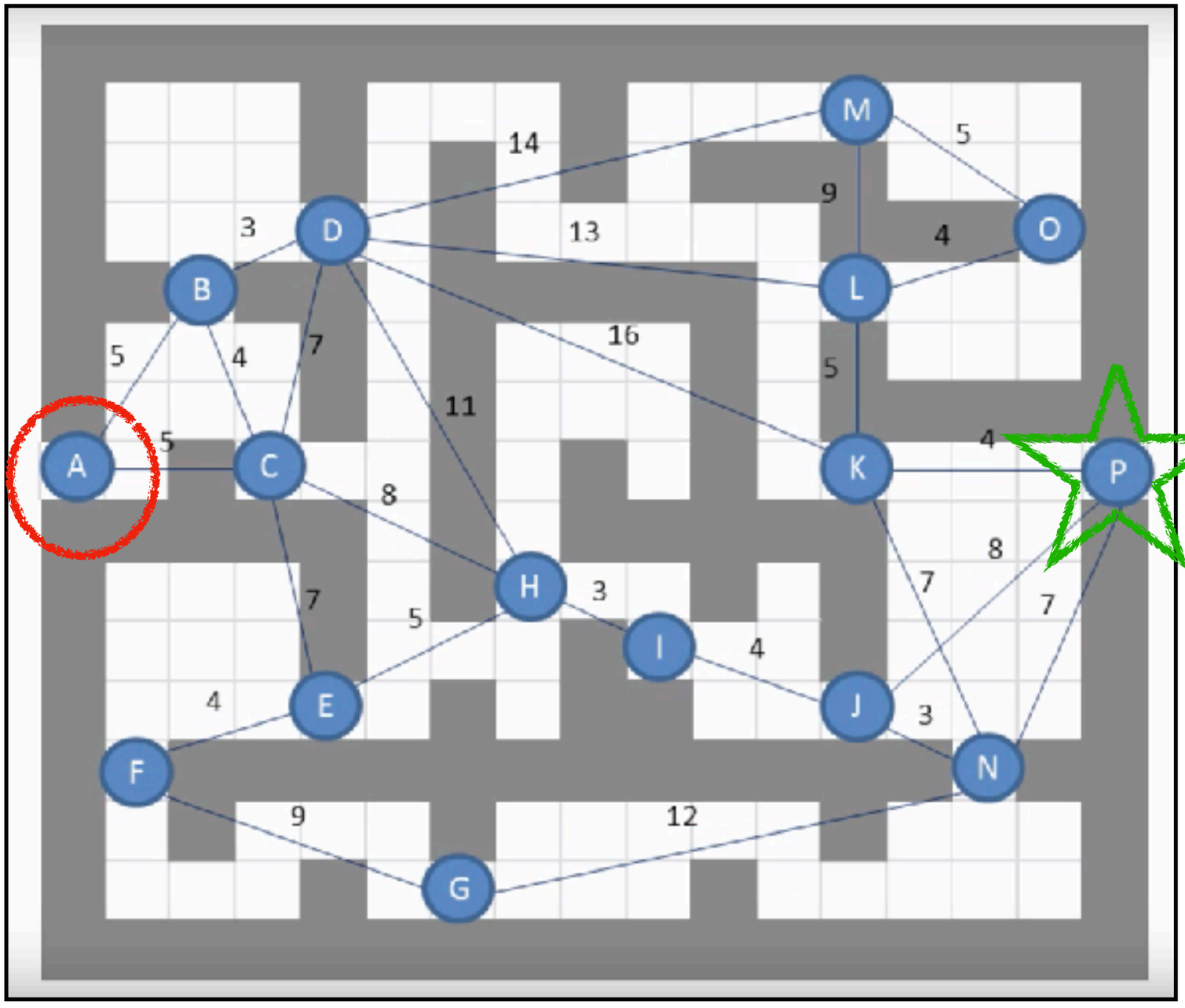
- A* search algorithm was invented by Hart, Nilsson and Raphael in 1968 to find paths through graphs [Hart et al. 1968].
- Example:
  - Navigation through a maze [Kevin Drumm 2017]

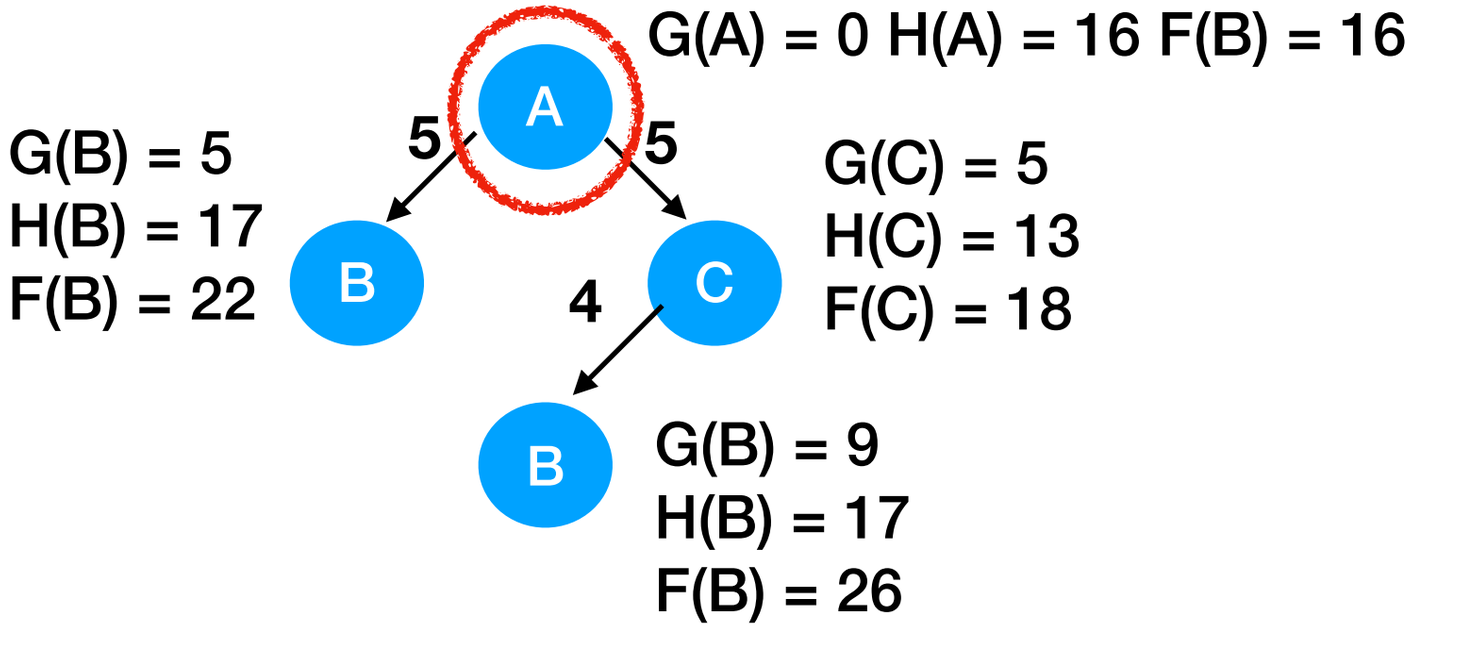


Components of A*

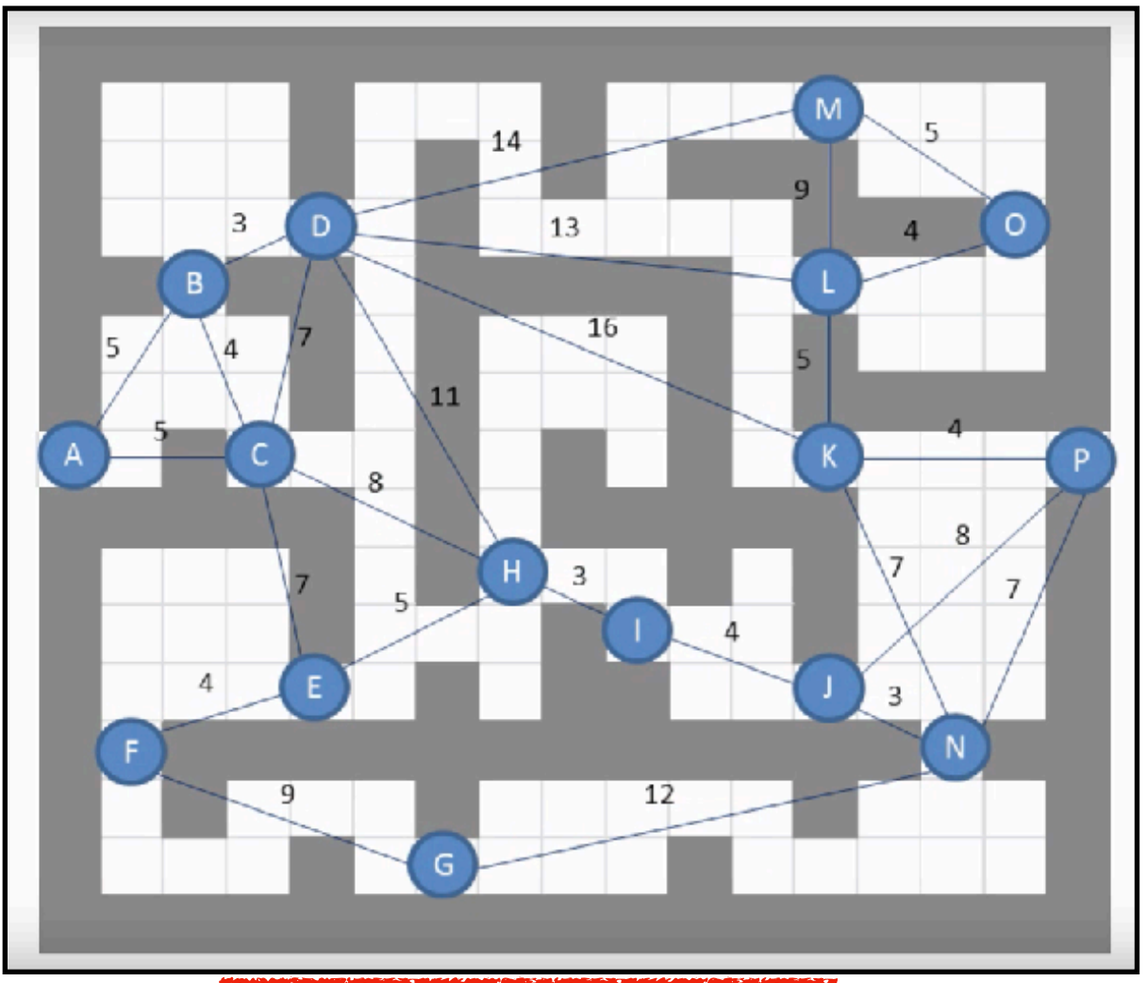
- **Initial State:** A
- **Goal State:** P
- **G-cost:** the cost from initial state (A) to current, e.g. time
- **H-cost:** the cost from current state to goal (P) e.g. straight line distance
- **F-cost:** the cost from Initial state (A) to current to goal (P)
- F = G + H

Suppose SLD(AP) = 16
SLD(BP) = 17
SLD(CP) = 13

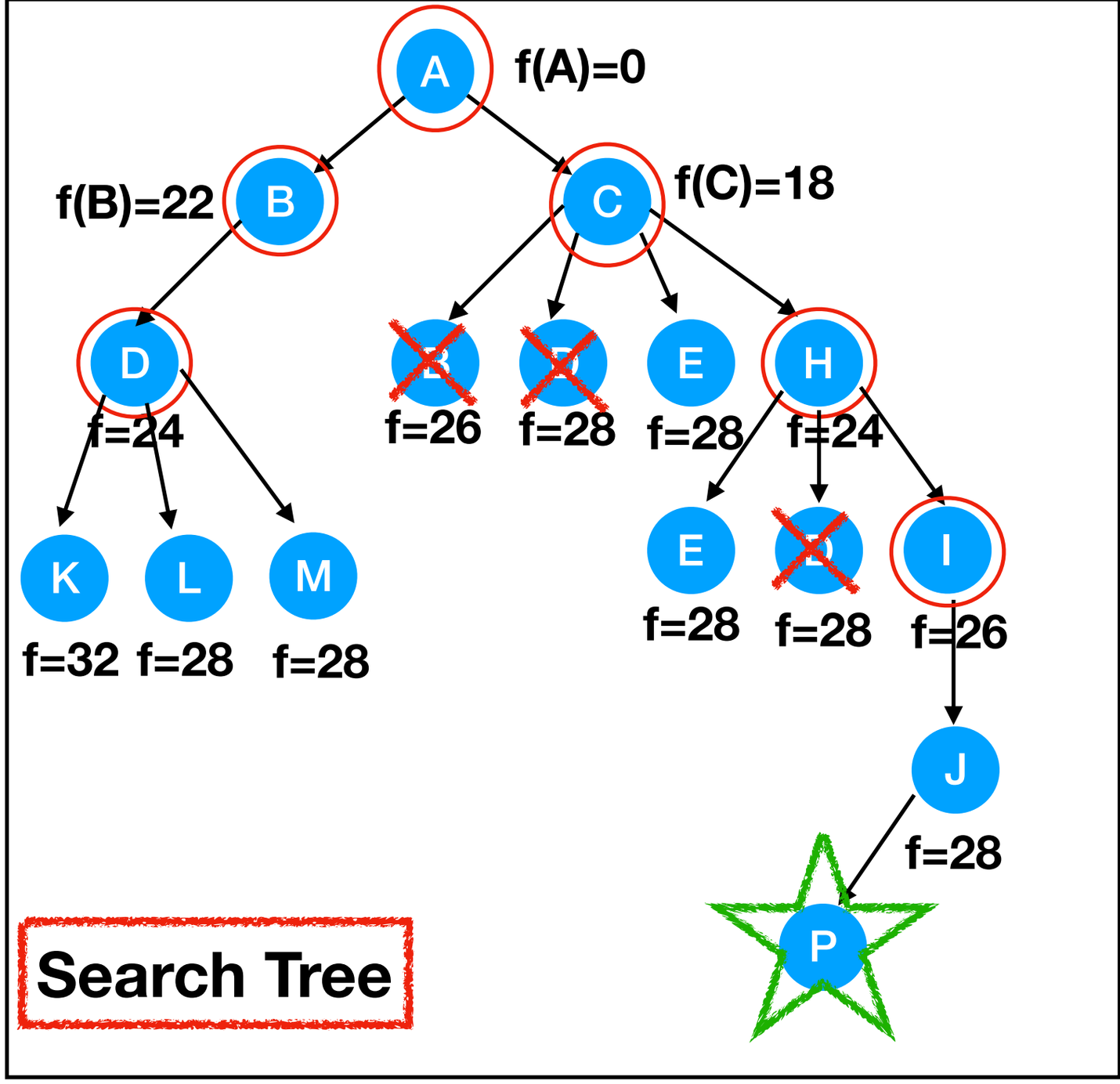




A* Maze Example



Maze as Graph



Search Tree

- **State Space:** search tree
  - all possible locations that can be reached from A
- **State:** nodes in the search tree
  - one possible location in State Space, for example, E
- **Action:** edges in the search tree
  - at a location (say J), choose which way to go, for example, P

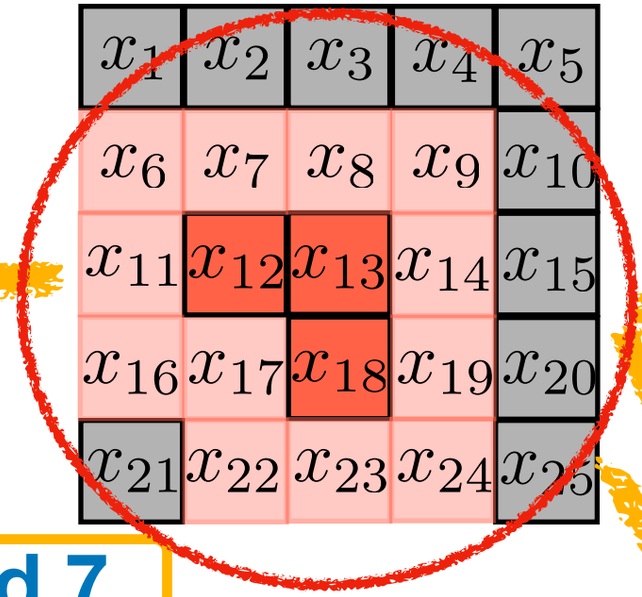
A* Components in the ASU Problem

- **Initial State:** the seed
- **Goal State:** an ASU with goal pop and goal ur
- **G-cost:** $G = w_0 \frac{(\text{start pop} - \text{current pop})}{\text{start pop}} + w_1 \frac{(\text{start ur} - \text{current ur})}{\text{start ur}}$
- **H-cost:** $H = 0$
- **F-cost:** $F = G + H = w_0 \frac{(\text{start pop} - \text{current pop})}{\text{start pop}} + w_1 \frac{(\text{start ur} - \text{current ur})}{\text{start ur}}$

- How do we set the goal for an ASU grown from a seed?
  - The goal ur is 6.5%
  - The goal pop is estimated using BIP

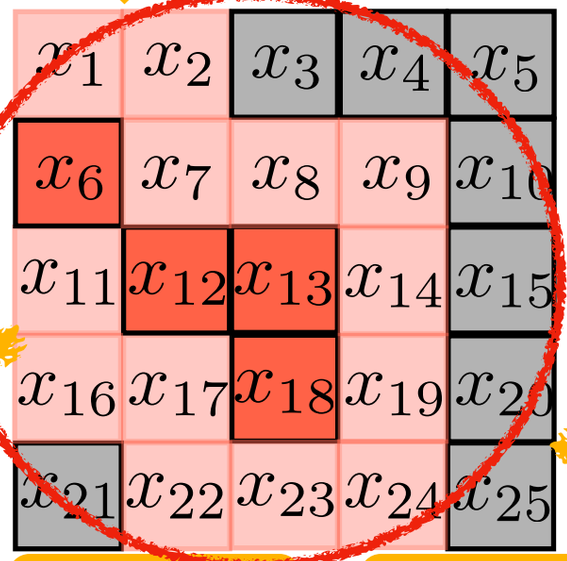
- Why do we set the F-cost that way?
  - F is a weighted summation of percentage changes in pop and ur
  - When choosing the next node to expand
    - We want pop to increase as much as possible
    - We want ur to decrease as little as possible

A* Search State Space

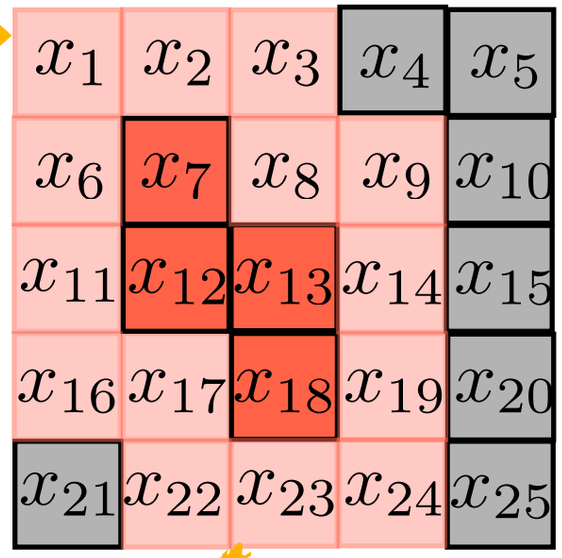


Initial State
f=0

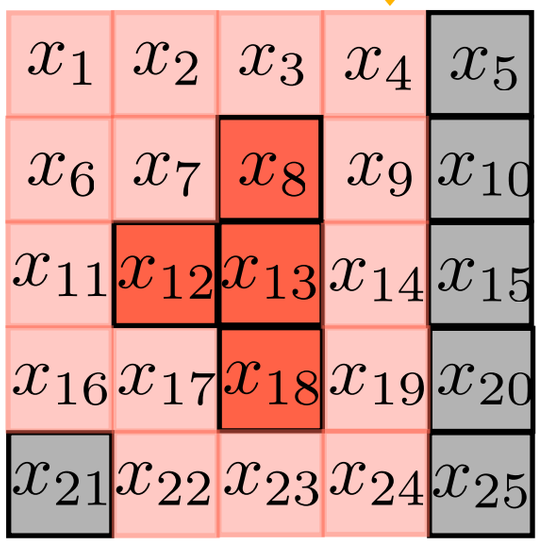
Add 6
f=24



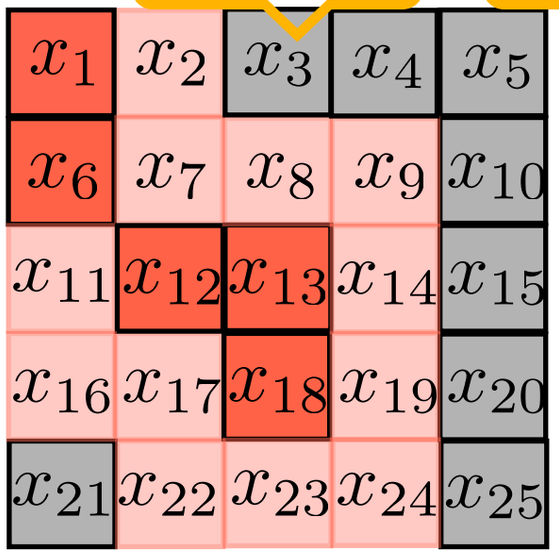
Add 7
f=26



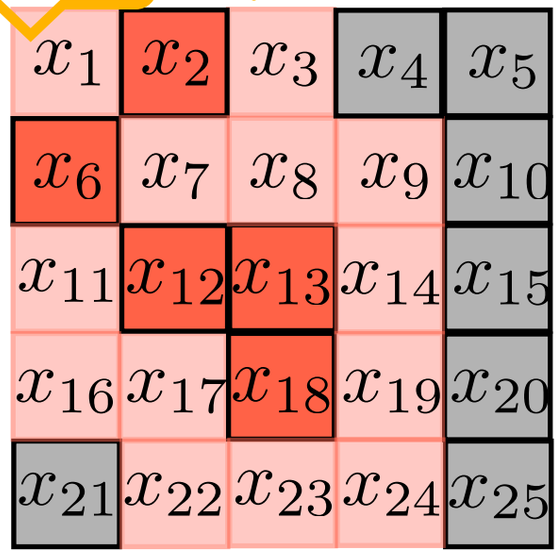
Add 8
f=29



Add 1
f=29

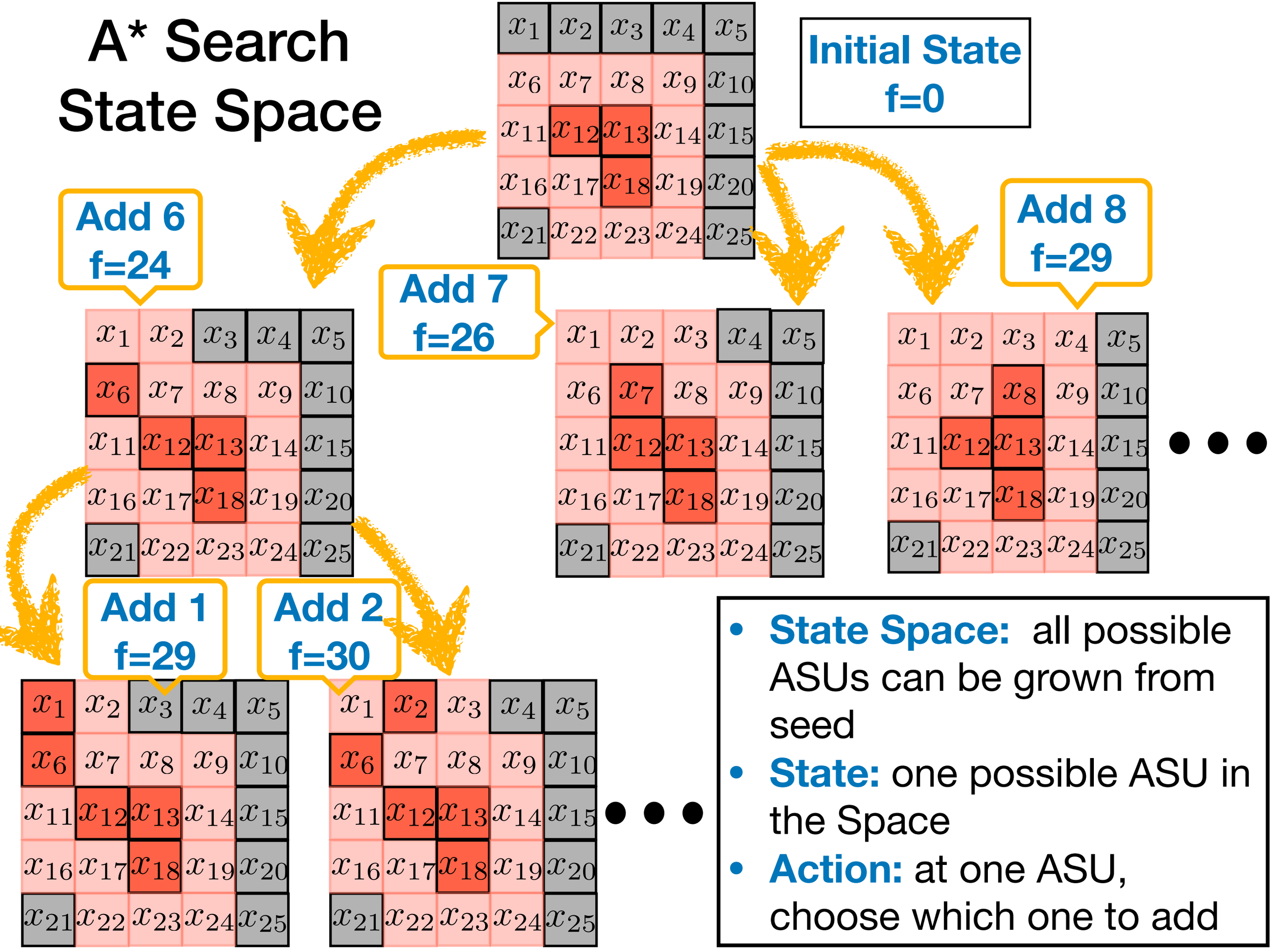


Add 2
f=30



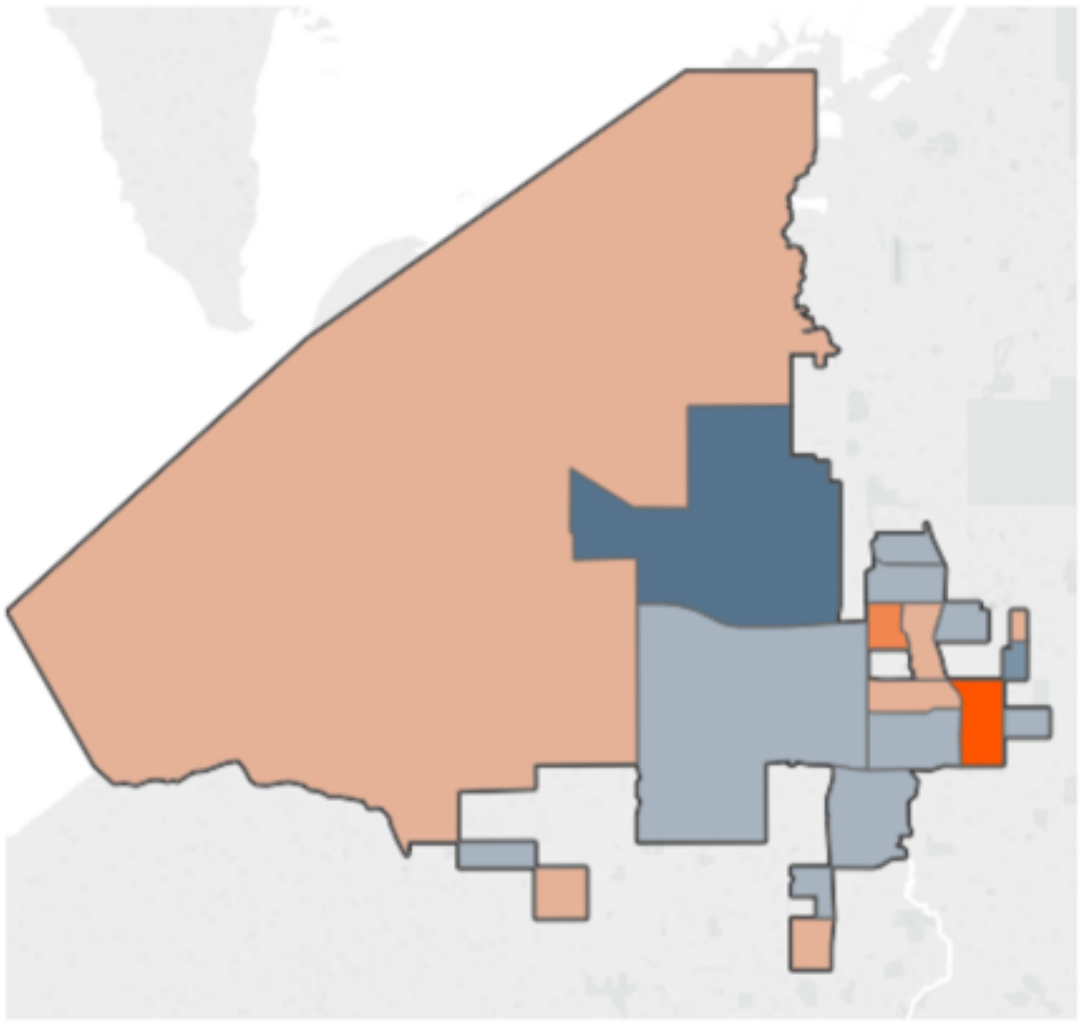
We repeat this process until a goal node is reached. The goal node is returned as the resulting ASU.

A* Search State Space



- **State Space:** all possible ASUs can be grown from seed
- **State:** one possible ASU in the Space
- **Action:** at one ASU, choose which one to add

Marginal Results - A*



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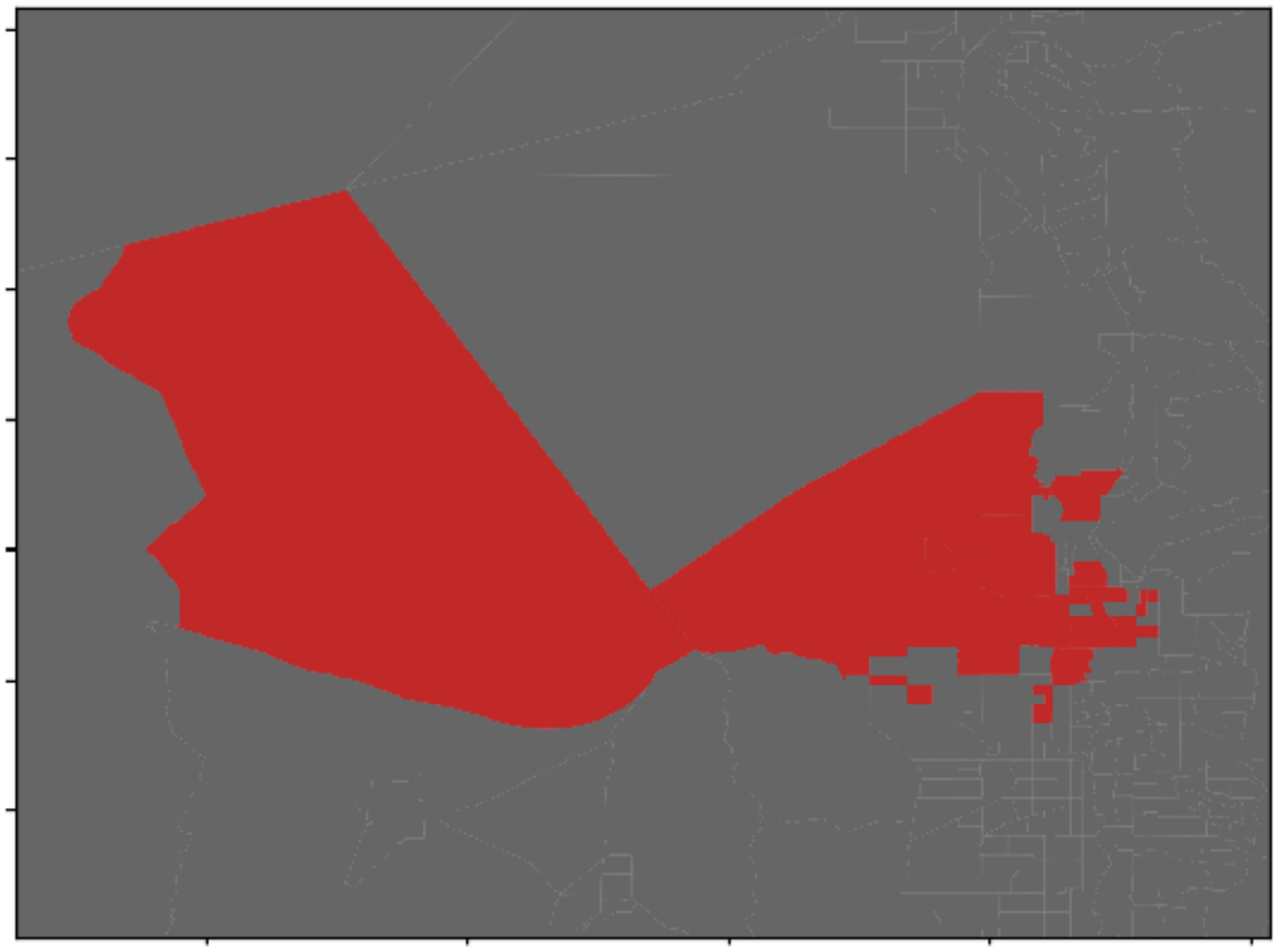


Obtained using A*

Aggregate URate

6.46%

Aggregate 2015 Pop

101,000

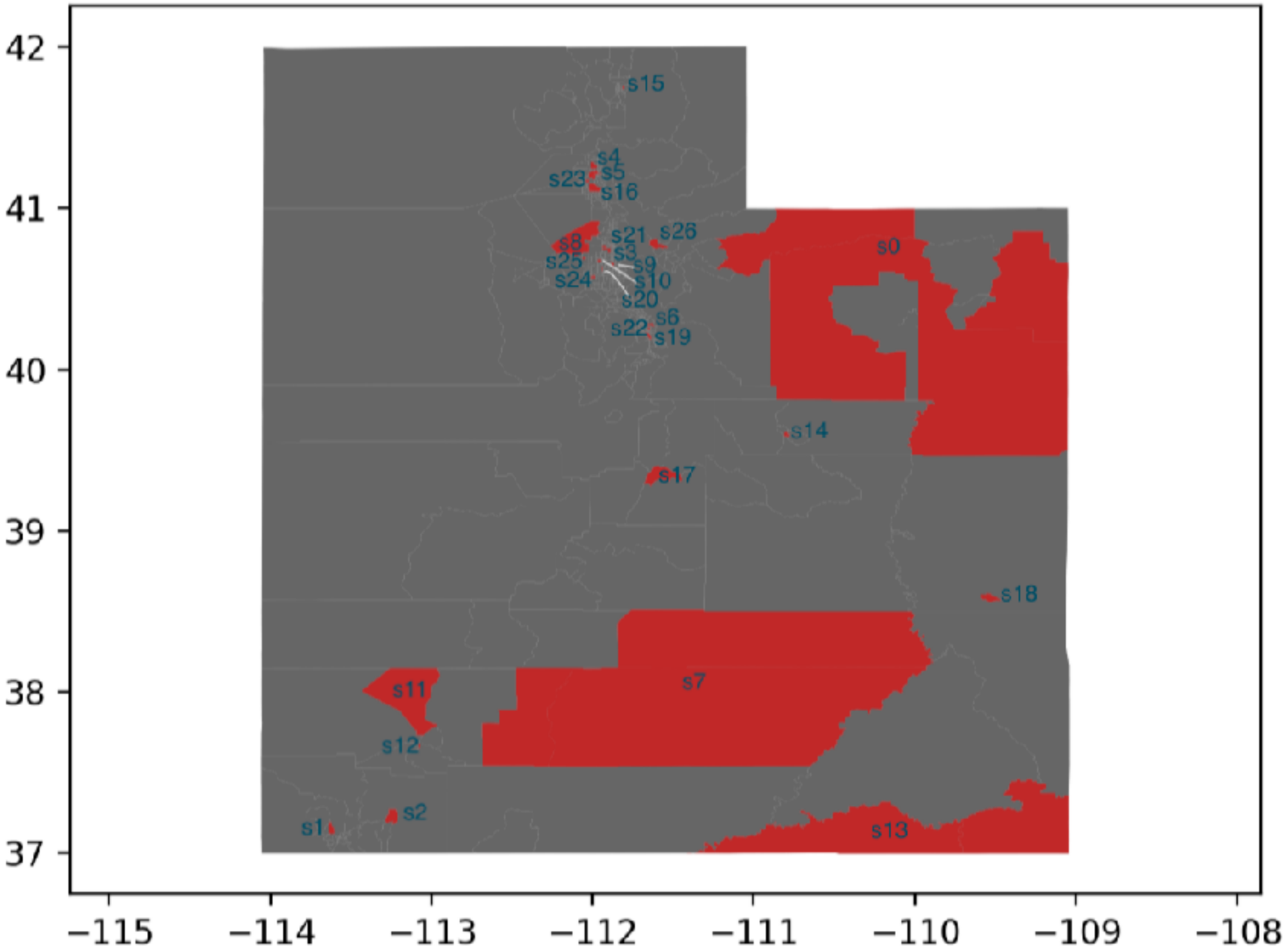
Find all ASUs

- So far, we have been discussing how to find a single ASU using BIP or A*

- Now, we will explain how to find all ASUs in the state of Utah
  - Initialize a result set A
  - Store all census tracts whose ur >= 6.5% in a seed set S
  - For each seed in S
    - If the seed is not in A
      - Grow an ASU from the seed using BIP or A* without repetition
      - Add the resulting ASU to A
  - After all seeds in S are processed, we return A

- In the next slide, we will show the seed set S on a map
- In the slide after the next, we will show all ASUs (the result set A) found by BIP and A* respectively

ASU Seeds


42
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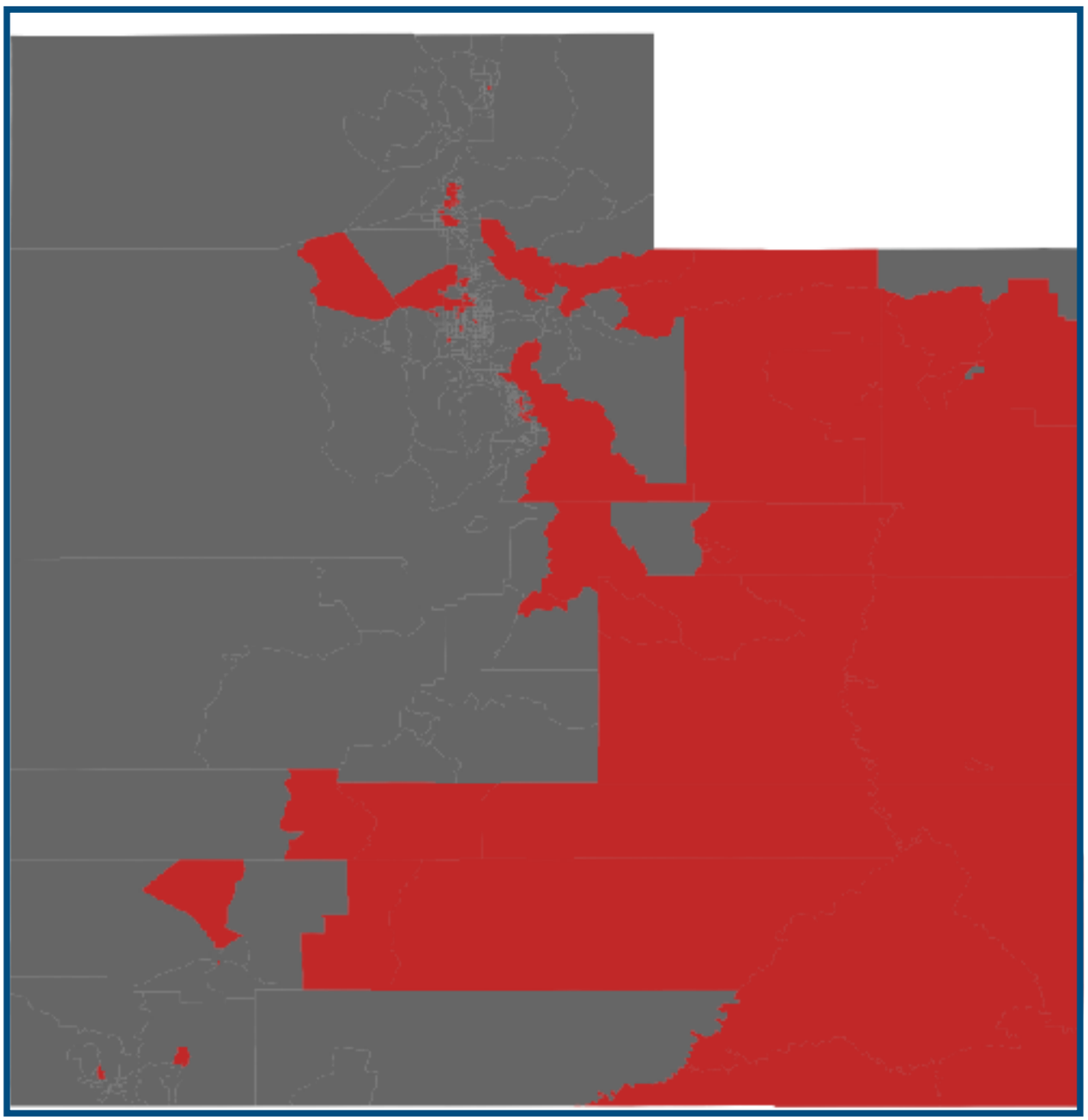
Joint Results from BIP and A*



Obtained using BIP

Total 2015 Pop

398,962

Obtained using A*

Total 2015 Pop

407,900

Code and References

- **Code**
  - https://github.com/splashmountain/asu

- **Tools**
  - PySAL for Spatial Data Analysis (http://pysal.readthedocs.io/en/latest/users/installation.html)
  - PuLP for Linear Programming (https://pythonhosted.org/PuLP/)
  - Geopandas for Spatial Data Plot (https://automating-gis-processes.github.io/2016/Lesson2-geopandas-basics.html)

- **Papers**
  - Redistricting Using Constrained Ploygonal Clustering [Joshi et al. 2012]
  - Contiguity Constraints for Single-Region Site Search Problems [Cove and Church 2000]
  - A Formal Basis for the Heuristic Determination of Minimum Cost Paths [Hart et al. 1968]