How does RSA work?

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TWEET THIS

Hey guys , I wanted to write a little bit about **RSA** cryptosystem .

RSA is an **asymmetric** system , which means that a key pair will be generated (we will see how soon) , a **public** key and a **private** key , obviously you keep your private key secure and pass around the public one.

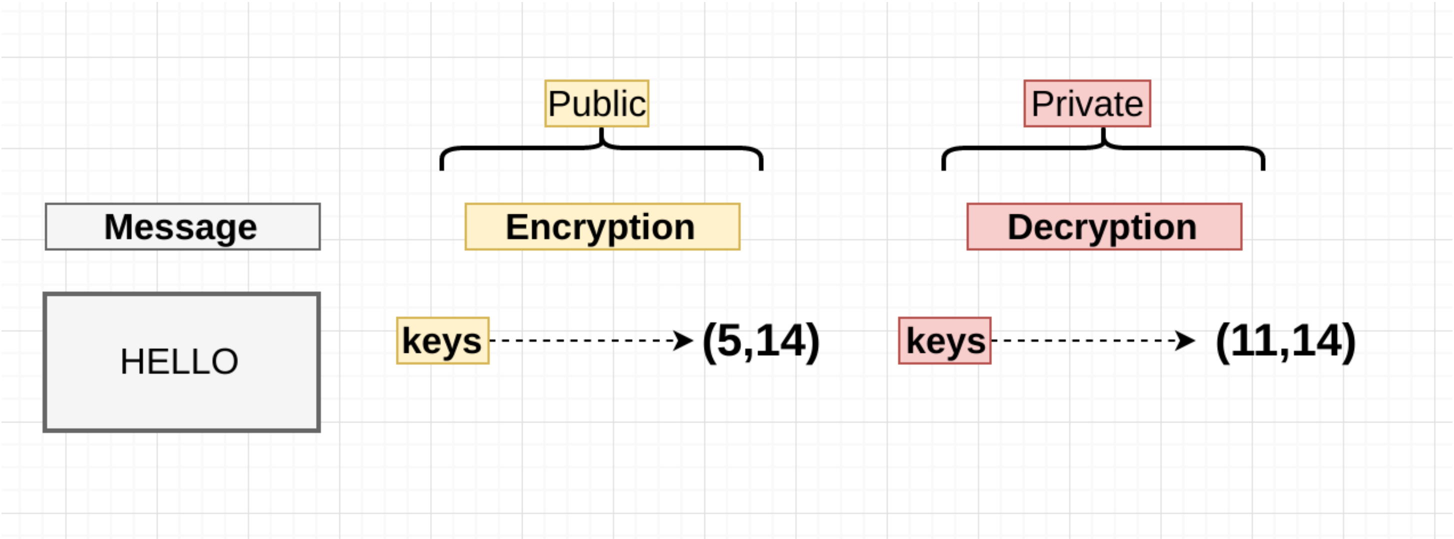
The algorithm was published in the 70's by Ron **R**ivest, Adi **S**hamir, and Leonard **A**dleman, hence RSA , and it sort of implement's a trapdoor function such as Diffie's one.

RSA is rather slow so it's hardly used to encrypt data , more frequently it is used to encrypt and pass around **symmetric** keys which can actually deal with encryption at a **faster** speed.

How does it work?

As in D-H I'm gonna be using rather small numbers, but keep in mind that the real value of most of **mod(p)** based algorithms happens when huge primary numbers are used.

The first part i'll show how the trapdoor function works , and then i'll explain why it works , so bare with me.

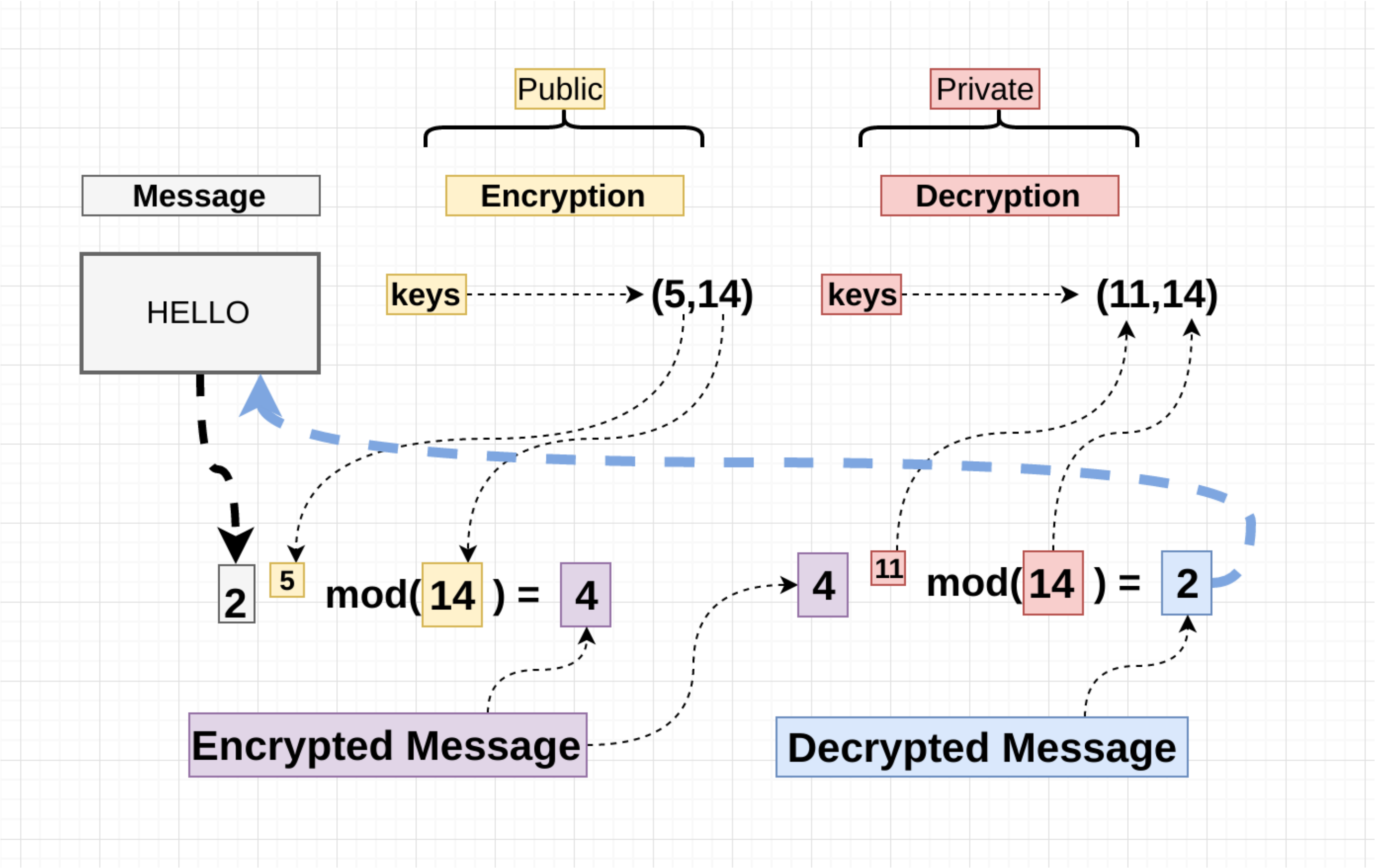


Essentials

We’ve got a message (“HELLO”) , and we’ve picked two tuples with two numbers each (I will explain how these came about later). Obviously there’s no arithmetic operation we can perform with strings , so the message has to be convert it to something , so let’s say “HELLO” converts using some conversion algo to “**2**”

Normally , in production , a lot of different techniques are used to encode the message and padding is also used

Ok Fantastic we have everything we need , let’s run the message through the functions that will in theory encrypt this for us:



mini rsa

So there we have the basics of the RSA algorithm , and how the trapped function is laid out.

The interesting bit is how we come about those numbers , and how (5,14) is related to (11,14), and this is the interesting part i believe , let's start:

The details of the Decryption/Encryption pair:

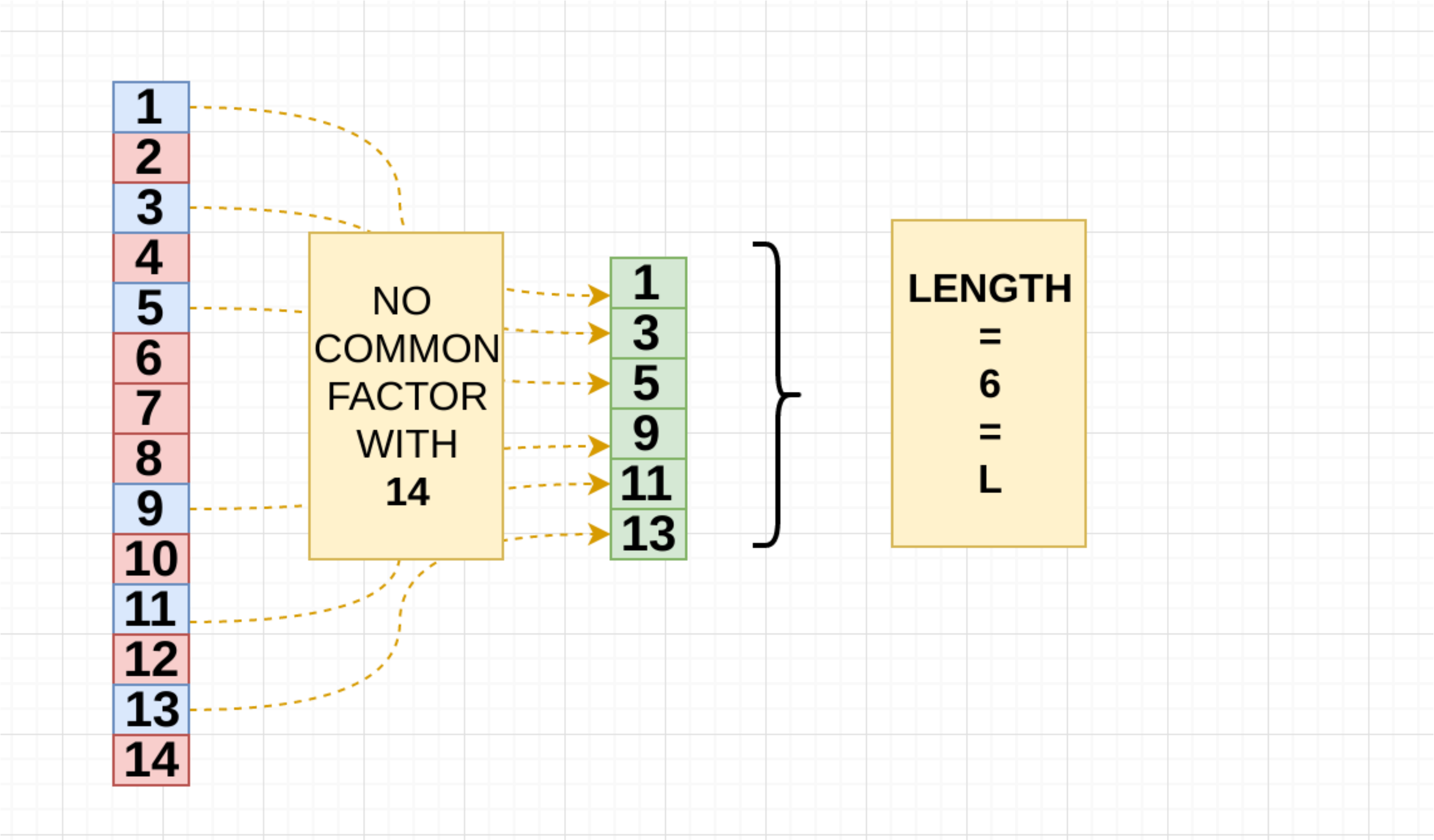
1. Pick two prime numbers , I will pick **2** and **7 ,** lets call them p and q

```
P = 2 and Q = 7
```

2. Multiply **P** and **Q** , and that becomes the modulus

```
N = P * Q = 14
```

3. Make a list between **1** and 14 and remove the common factors:



Now there’s an easy way to get this and that is:

```
( Q - 1 ) * ( P - 1) = L (7 - 1 ) * ( 2 -1 ) = 6
```

Great let’s save this number , let’s call it “**L**”

4. Now we get to pick the encryption key , in the example was (**5,14**) , we know **14** is the modulus.

So for the encryption key there’s a few rules:

- it’s got to be between **1** and **L**

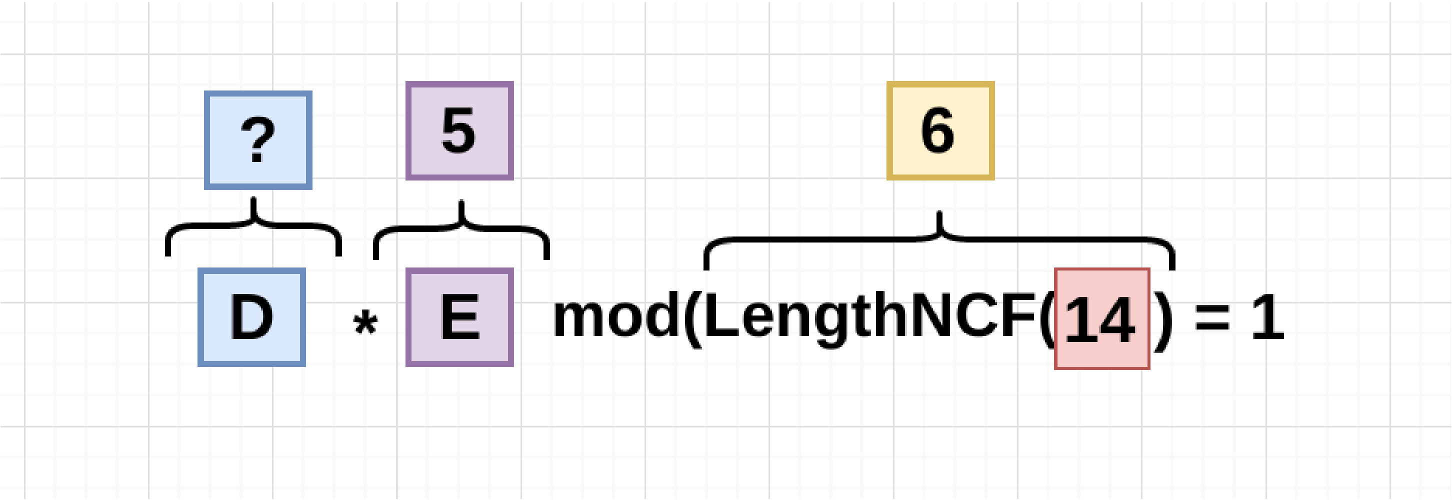
```
[2,3,4,5]
```

- Coprime with **L** (6) and the **Modulus** (14) , the answer is **5 ,** there's no other possibility .

So there we came to a conclusion of why we picked (**5,14**)

5. The Decryption part , In the example we've picked (**11,14**) , again 14 is the modulus but where does 11 come from?? , from now on let's call it **D ,** let's find out why **D** is 11:

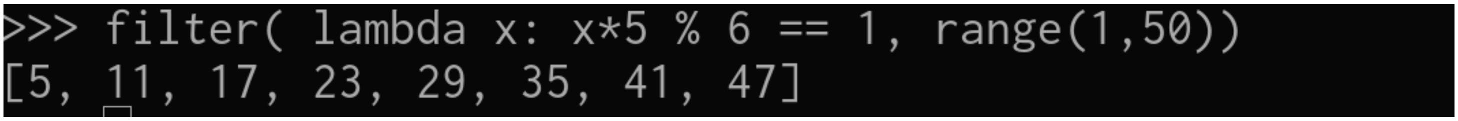
D has to follow one rule and this is it:



So the Decryptor(11) multiplied by the Encryptor(5) modulus the length of the non coMmon factor with the modulus(14) has to be equals to 1.

```
D * E % LNCF(N) = 1
11 * 5 % 6 = 1
```

So we know if we multiply D * E and E is 5 , D will need to be a common factor of 5 , so:

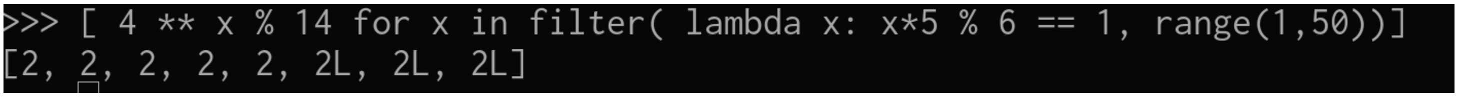


```
>>> filter( lambda x: x*5 % 6 == 1, range(1,50))
[5, 11, 17, 23, 29, 35, 41, 47]
```

So I’ve made a list of numbers from 1 to **50** and , filtered the ones that when multiplied by E and **moduled** by **LNCF**(N) are equals 1 , so let’s see if those can decrypt the message :) , Remember the encrypted message is “**4**”,and the decrypted message is “**2**” so the function should be something like:

```
4 ** D % 14 = 2 <---Decrypted message
```

So let’s do a little list comprehension to see if the rule works:



```
>>> [ 4 ** x % 14 for x in filter( lambda x: x*5 % 6 == 1, range(1,50))]
[2, 2, 2, 2, 2, 2L, 2L, 2L]
```

Great ! it worked , you see how applying the function to all the Decryption keys that follow the rule (**D** * **E** % **LNCF**(N) = **1) ,** decrypted the message successfully .

I hope this didn’t get too complex , it certainly felt a bit complex but i tried to keep it as simple as possible .