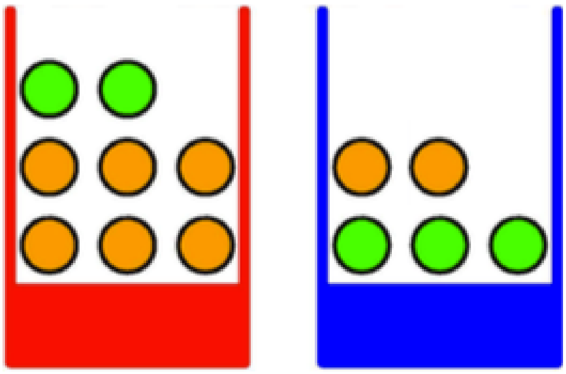

Figure 1: Two boxes (red/blue) with two types of balls (green/orange).

1 Basic probability

Exercise 1.1 *According to Figure 1, which is the probability of selecting the red box?*

Exercise 1.2 *According to Figure 1, if you want to run out of balls both boxes at the same time, how often should you select the blue box?*

Exercise 1.3 *According to Figure 1, which is the probability of picking up a green ball if you are using the red box?*

Exercise 1.4 *According to Figure 1, which is the probability of picking up an orange ball?*

Exercise 1.5 *According to Figure 1, given that you picked up a green ball, which is the probability of having selected the red box?*

Exercise 1.6 *According to Figure 1, given that you picked up an orange ball, which is the probability of having selected the red box?*

Exercise 1.7 *Let D be a disease which the 1% of the population has. A physician has developed a test T that identifies the 90% of cases (people who really have it) but, unfortunately, it also says identifies as sick a 15% of healthy patients (false positives). Given a new patient, they are tested and it turns out to be positive. Which is the probability of the patient really having the disease?*

Exercise 1.8 *Given a joint probability distribution $p(X_1, X_2, X_3, X_4, X_5, X_6)$, how can it factorize taking advantage of the following conditional independence statements?*

a) $X_1 \perp\!\!\!\perp X_2 | X_3$

b) $X_5 \perp\!\!\!\perp X_3 | X_6$

c) $X_1 \perp\!\!\!\perp X_2 | X_3$ *and* $X_4 \perp\!\!\!\perp X_5 | X_6$

d) $X_1 \perp\!\!\!\perp X_2 | X_3, X_4$ *and* $X_2 \perp\!\!\!\perp X_3 | X_4$

e) $X_1 \perp\!\!\!\perp X_2 | X_3, X_4, X_5$ *and* $X_3 \perp\!\!\!\perp X_4 | X_5, X_6$

f) $X_i \perp\!\!\!\perp X_j | X_1, \forall i, j : i \neq j \land i \neq 1 \land j \neq 1$

g) $X_i \perp\!\!\!\perp X_{i+1} | \{X_j\}_{j=i+2}^{n}, \forall i \in \{1, \ldots, n-2\}$ *($n = 6$)*

Exercise 1.9 *Show that the number of parameters required to encode the non-factorized joint probability distribution $p(X_1, \ldots, X_n)$ and to encode the equivalent factorized joint probability distribution $\prod_{i=1}^{n} p(X_i | X_1, \ldots, X_{i-1})$ is the same. Assume that all the variables have v possible values.*

a)	$p(X_6 \mid X_1, X_2, X_3, X_4, X_5)p(X_5 \mid X_1, X_2, X_3, X_4)p(X_4 \mid X_1, X_2, X_3)p(X_1 \mid X_3)p(X_3 \mid X_2)p(X_2)$
b)	$p(X_1 \mid X_2, X_3, X_4, X_5, X_6)p(X_2 \mid X_3, X_4, X_5, X_6)p(X_4 \mid X_3, X_5, X_6)p(X_5 \mid X_6)p(X_6 \mid X_3)p(X_3)$
c)	$p(X_1 \mid X_3, X_4, X_5, X_6)p(X_2 \mid X_3, X_4, X_5, X_6)p(X_3 \mid X_4, X_5, X_6)p(X_4 \mid X_6)p(X_6 \mid X_5)p(X_5)$
d)	$p(X_6 \mid X_1, X_2, X_3, X_4, X_5)p(X_5 \mid X_1, X_2, X_3, X_4)p(X_1 \mid X_3, X_4)p(X_2 \mid X_4)p(X_4 \mid X_3)p(X_3)$
e)	$p(X_1 \mid X_3, X_4, X_5, X_6)p(X_2 \mid X_3, X_4, X_5, X_6)p(X_3 \mid X_5, X_6)p(X_4 \mid X_5, X_6)p(X_5 \mid X_6)p(X_6)$
f)	$p(X_2 \mid X_1)p(X_3 \mid X_1)p(X_4 \mid X_1)p(X_5 \mid X_1)p(X_6 \mid X_1)p(X_1)$
g)	$p(X_1 \mid X_3, X_4, X_5, X_6)p(X_2 \mid X_4, X_5, X_6)p(X_3 \mid X_5, X_6)p(X_4 \mid X_6)p(X_5 \mid X_6)p(X_6)$

Table 1: Factorizations

Answers

Ex. 1.1: 0.5

Ex. 1.2: 0.38

Ex. 1.3: 0.25

Ex. 1.4: 0.575

Ex. 1.5: 0.294

Ex. 1.6: 0.652

Ex. 1.7: 0.057

Ex. 1.8: See Table 1

Ex. 1.9: $v^n - 1$ vs. $\sum_{i=1}^{n}((v-1) \cdot v^{i-1})$