Controls, Not Shocks:
Estimating Dynamic Causal Effects in Macroeconomics

Seminar, U. Glasgow

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Our Question

- **Common Issue in Macro**:
  - *Goal*: Policy $z \overset{?}{\Rightarrow}$ Outcome variable y
  - *Challenge*: $z = f(\mathbf{x}) + \varepsilon$ where $\mathbf{x} \Rightarrow y$

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- **Common Solution**: identify 'shock' first with two steps (e.g., Romer and Romer, 2004)
  1. Regress z on $\mathbf{x}$ and call residual the 'shock' (e.g., z change in FFR, $\mathbf{x}$ FOMC Greenbook forecasts)
  2. Regress y on constructed shock (e.g., y CPI)

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What is the difference between these two approaches for range of estimators?

Our Answer

- **Starting Point**: one- and two-step are two sides of same coin (Frisch-Waugh-Lovell)
  - **BUT**...only about coefficients (not standard errors) and a special case (OLS, one set of controls $\mathbf{x}$)

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1. ...over-estimation of standard errors (OLS/IV without controls)
2. ...'genuine bias' (OLS/IV/VAR with controls, QR)

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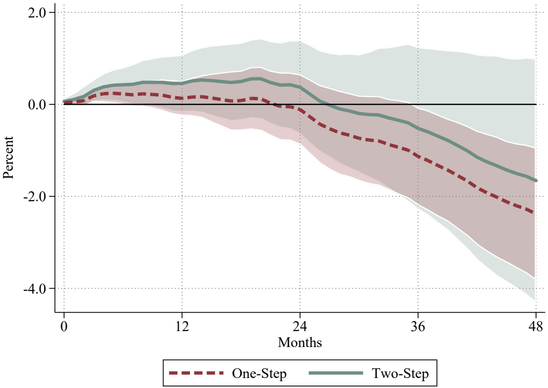
⋆ **Matters in Practice**: headline results can differ between one- and two-step approaches

– Revisit estimated effects of monetary policy controlling for central-bank information
– Simple resolution to price puzzle

Differences Matter in Practice: A Price-Puzzle Teaser

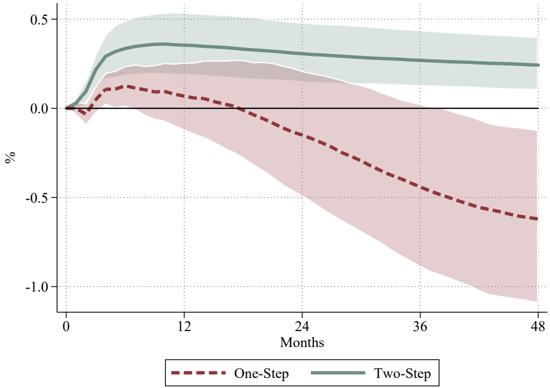
US $\ln(CPI)$ responses to Romer-Romer-style shock, controlling for lags of CPI, IP and U/E

Local Projection



Note: OLS-LP, US 1972m1-2007m12. 90% confidence bands from Newey-West s.e.

VAR with Internal Instrument



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Differences Have Broad Implications in Macro: Two-Step Widely Used

*"Any exercise in dynamic causal inference involves **two conceptually distinct steps**: 1) the construction of the shocks, and 2) the specification used to construct an impulse response once one has the shocks in hand"* [Nakamura & Steinsson 2018, JEP]

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- Used in studies of **average causal effects**:
  - Monetary policy [Romer & Romer 2004, AER; Coibion 2012, AEJ: Macro; Coibion et al. 2017, JME; Cloyne et al. 2020, REStud; Holm et al. 2021, JPE; Miranda-Agripinno & Ricco 2021, AEJM; Bauer & Swanson 2022, AER]
  - Fiscal policy [Auerbach & Gorodnichenko 2013, AER; Miyamoto et al. 2018, AEJ: Macro]
  - Macroprudential policy [Ahnert et al. 2021, JFE; Chari et al. 2022, JIE]
  - Non-policy, e.g.: Oil Prices [Kilian 2009, AER]; Sentiment [Al-Amine & Willems 2023, EJ]; etc.

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  - Non-policy, e.g.: Oil Prices [Kilian 2009, AER]; Sentiment [Al-Amine & Willems 2023, EJ]; etc.
- And studies of causal effects **across quantiles**:
  - Fiscal policy [Linnemann & Winkler 2016, OEP]
  - Macroprudential policy [Brandão-Marques et al. 2021, IMF WP]
  - Capital-flow measures [Gelos et al. 2022, JIE]

Plan for Today

1. Setup
2. Key Insight: An ‘Omitted-Variable Bias’ (OVB) Result
3. Implications and Applications: OVB in Different Settings
    - Ordinary Least Squares and SVARs with Internal Instruments
    - Instrumental Variables and SVARs with External Instruments
    - Quantile Regression

    Throughout, applications focus on transmission of US monetary policy controlling for central-bank information [Romer and Romer, 2004; Miranda-Agrippino and Ricco, 2021]

Related Literature

Not aware of other literature explicitly highlighting the drawbacks of the two-step approach popular in the macroeconomics literature, but most clearly related to:

#1. Various literature stresses shock-identification strategies relying on orthogonalisation **equivalent** to regression-control approach

- Angrist & Kuersteiner (2011), Jordà & Taylor (2016), Angrist, Jordà & Kuersteiner (2018), Barnichon & Brownlees (2019), Jordà, Schularick & Taylor (2020), Plagborg-Møller & Wolf (2021)

→ **This Paper**: They are **not** (always) equivalent—highlight drawbacks of two-step approach

#2. Generated regressors in macroeconomics

- Pagan (1984), Murphy & Topel (2002)

→ **This Paper**: We propose two-step approach amounts to mis-specification, driven by omission of relevant variables from outcome regression

Setup

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Focus: Effect of some variable z on outcome variable of interest y

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Two-Step Approach:

#1 $z_t = \mathbf{x}'_{1,t}\boldsymbol{\delta} + \varepsilon_t$, where $\mathbf{x}_1$ captures endogenous drivers of z, and ε captures 'shock'

#2 $y_t = \mathbf{x}'_{2,t}\pi + \varepsilon_t\beta_{2S} + u_t$, where $\mathbf{x}_2$ other drivers of y, and β_{2S} is 'two-step' coefficient

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Setup *very* general:

- Not OLS-specific: only require coefficients to be unique solution to minimisation of some function of the residuals
- Nests Frisch-Waugh-Lovell benchmark
- Broad identification: $\mathbf{x}_t$ can contain numerous types of variables

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How does β_{2S} compare to β_{1S}?

Key Insight

Key Insight: An ‘Omitted Variable Bias’ Result

Proposition: OVB Result

Difference between β_{1S} and β_{2S} can be expressed in terms of OVB in β_{2S}:

$$\beta_{2S} = \beta_{1S} + \Omega_{2S}$$

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Sketch Proof:

$$y_t = \mathbf{x}'_{2,t}\pi + \varepsilon_t \beta_{2S} + u_t \quad \text{(Two-Step)}$$

$$y_t = \mathbf{x}'_t\theta + z_t \beta_{1S} + e_t \quad \text{(One-Step)}$$

$$y_t = \mathbf{x}'_t\phi + \epsilon_t \beta_{hyb} + \eta_t \quad \text{(Hybrid)}$$

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- Since $\varepsilon_t = z_t - \mathbf{x}'_{1,t}\boldsymbol{\delta}$ and $\mathbf{x}_1 \subset \mathbf{x}$, one-step and hybrid estimators equivalent: $\beta_{1S} = \beta_{hyb}$
- But, relative to hybrid estimator, two-step excludes $\mathbf{x}_1$. So difference can be expressed as: $\beta_{2S} = \beta_{1S} + \Omega_{2S}$ where Ω_{2S} is OVB from exclusion of $\mathbf{x}_1$ from second-stage regression

Implications and Applications

OLS and SVARs with Internal Instruments

Simple OLS Case: No Auxiliary Controls

Setup:

$$\begin{aligned} y_t &= \beta_{2S} \overbrace{\varepsilon_t}^{z_t = \delta \mathbf{x}_{1,t} + \varepsilon_t} + u_t && \text{(2S)} \\ y_t &= \beta_{1S} z_t + \gamma \mathbf{x}_{1,t} + e_t && \text{(1S)} \end{aligned}$$

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Results:

1. Point estimates equivalent (Frisch-Waugh-Lovell): $\beta_{2S} = \beta_{1S}$ and $\hat{\beta}_{2S} = \hat{\beta}_{1S}$
2. Population standard errors identical: $V(\hat{\beta}_{2S}) = V(\hat{\beta}_{1S})$
3. Two-step leads to over-estimation of standard errors in sample: $\hat{V}(\hat{\beta}_{2S}) > \hat{V}(\hat{\beta}_{1S})$

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Intuition: $\mathbf{x}_1$ explains some variation in z ($\uparrow$ s.e.), but also has explanatory power for y ($\downarrow$ s.e.) $\Rightarrow$ two-step standard errors only reflect first force

Local-Projection Application: No Auxiliary Controls

Romer-Romer-style identification Detail

- US data, converted from meeting to monthly frequency, 1972m1-2007m12
- Two step:
  1. Regress ΔFFR_t (i.e., z) on Greenbook forecasts (i.e., $\mathbf{x}_1$), save residual $\hat{\varepsilon}_t$
  2. Regress $\ln(CPI)_{t+h}$ (i.e., y) on $\hat{\varepsilon}_t$
- One step: Regress $\ln(CPI)_{t+h}$ on ΔFFR_t and Greenbook forecasts

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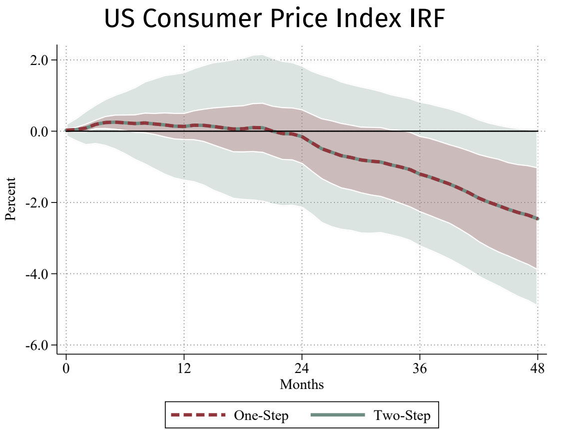

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OLS with Auxiliary Controls

Setup:

$$y_t = \beta_{2S}\varepsilon_t + \boldsymbol{\pi}\overbrace{\mathbf{x}_{2,t}}^{\text{e.g., lagged macro controls}} + u_t \tag{2S}$$

$$y_t = \beta_{1S}z_t + \gamma\mathbf{x}_{1,t} + \boldsymbol{\theta}\mathbf{x}_{2,t} + e_t \tag{1S}$$

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Results:

1. OVB has the form: $\beta_{2S} = \beta_{1S} + \underbrace{\mathbb{E}\left[\varepsilon_t \mathbf{x}'_{2,t}\right] \mathbf{A}^{-1} \mathbb{E}\left[\mathbf{x}_{2,t}\mathbf{x}'_{1,t}\right] \boldsymbol{\phi}_1}_{\equiv OVB}$
2. When $\left[\varepsilon_t \mathbf{x}'_{2,t}\right] = 0$: two-step consistent, but inefficient ($V(\hat{\beta}_{2S}) > V(\hat{\beta}_{1S})$)
3. When $\left[\varepsilon_t \mathbf{x}'_{2,t}\right] \neq 0$: two-step inconsistent

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Intuition: one-step avoids bias, as coefficient estimated 'as if' shock was constructed to be exogenous w.r.t. $\mathbf{x}_{1,t}$ *and* $\mathbf{x}_{2,t}$ $\Rightarrow$ so two-step less robust to misspecification

Local-Projection Application: With Auxiliary Controls

Romer-Romer-style identification

- Let $\mathbf{x}_{2,t}$ include: lagged CPI, IP and U/E
- Two step:
  1. Regress ΔFFR_t (i.e., z) on Greenbook forecasts (i.e., $\mathbf{x}_1$), save residual $\hat{\varepsilon}_t$
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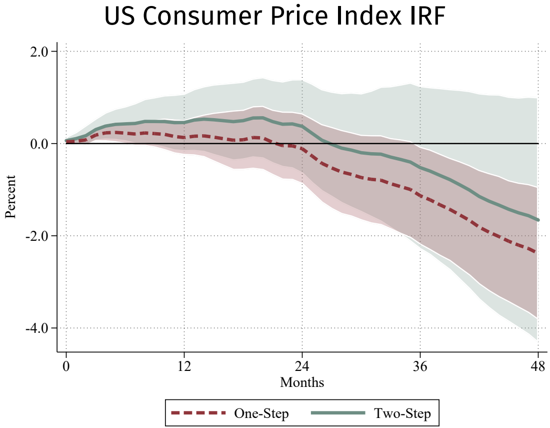

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VAR with Internal Instruments

- **Two-Step Internal-Instruments Approach**: include ε_t in VAR with $\mathbf{w}_t = [\varepsilon_t, \mathbf{y}_t]'$ and put first in recursive ordering

$$y_t = \varepsilon_t \beta_{2S} + \underbrace{\sum_{j=1}^{p} \mathbf{\Gamma}_j^{2S} \mathbf{y}_{t-j} + \sum_{j=1}^{p} \lambda_j^{2S} \varepsilon_{t-j}}_{= \boldsymbol{\pi} \mathbf{x}_{2,t}} + u_t^{2S}$$

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- **Alternative One-Step Approach**: include $\mathbf{x}_{1,t}$ and z_t, ordered first and second respectively, such that $\mathbf{w}_t = \left[\mathbf{x}_{1,t}', z_t, \mathbf{y}_t\right]'$

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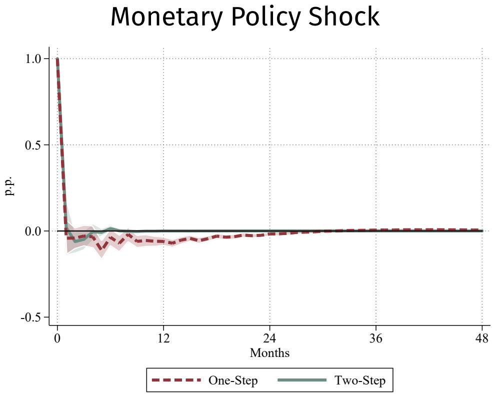
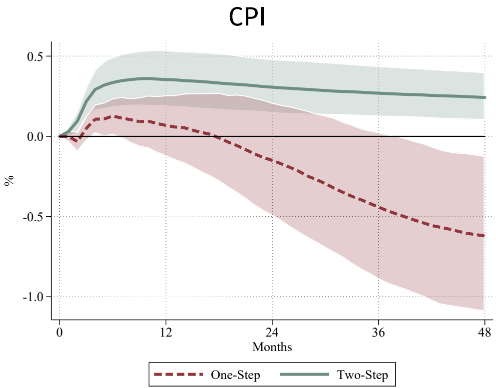
- OVB now captures omission of contemporaneous *and lagged* $\mathbf{x}_{1,t}$: $\Omega_{2S} = \mathbb{E}\left[\varepsilon_t \mathbf{x}_{2,t}'\right] \mathbf{A}^{-1} \mathbb{E}\left[\mathbf{x}_{2,t} \mathbf{x}_{1,t}^{p}\right] \mathbf{\Phi}^p$, where $\mathbf{x}_{1,t}^{p} = \left[\mathbf{x}_{1,t}', \mathbf{x}_{1,t-1}', \ldots, \mathbf{x}_{1,t-p}'\right]'$ and $\mathbf{\Phi}^p$ collects coefficients from hybrid regression

VAR with Internal Instruments Application

Two-step: $\mathbf{w}_t = \left[\hat{\varepsilon}_t^{RR}, \mathbf{y}_t\right]'$ where $\mathbf{y}_t = \mathbf{x}_{2,t}$ including CPI, IP, U/E

One-step: $\mathbf{w}_t = \left[\mathbf{x}_{1,t}', \Delta FFR_t, \mathbf{y}_t'\right]'$ where $\mathbf{x}_{1,t}$ is Greenbook forecasts





Note: VAR(4), US 1972m1-2007m12. 68% confidence bands from wild bootstrap (1000 reps)

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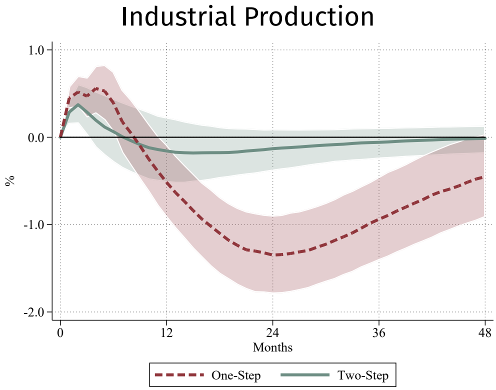

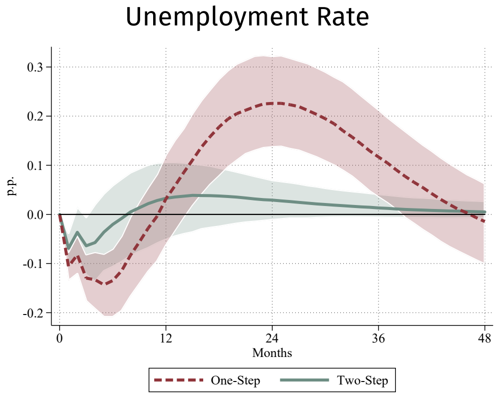

Note: VAR(4), US 1972m1-2007m12. 68% confidence bands from wild bootstrap (1000 reps)

Implications and Applications

IV and SVARs with External Instruments

Simple IV Case: No Auxiliary Controls

Setup:

$$y_t = \beta_{2S} \overbrace{z_t}^{\text{IV with } m_t^{\perp \mathbf{x}_{1,t}}} + u_t \tag{2S}$$

$$y_t = \beta_{1S} \underbrace{z_t}_{\text{IV with } m_t} + \gamma \mathbf{x}_{1,t} + e_t \tag{1S}$$

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Results:

1. Point estimates still equivalent (Frisch-Waugh-Lovell): $\beta_{2S} = \beta_{1S}$ and $\hat{\beta}_{2S} = \hat{\beta}_{1S}$
2. Two-step leads to over-estimation of standard errors in sample: $\hat{V}(\hat{\beta}_{2S}) > \hat{V}(\hat{\beta}_{1S})$
3. First-stage F-stats from two-step underestimated

Intuition: IV the ratio of two OLS coefficients, so Frisch-Waugh-Lovell benchmark still applies

LP-IV Application: Romer-Romer Example

Romer-Romer-style identification

- Two step:
  1. Regress ΔFFR_t (i.e., m) on Greenbook forecasts (i.e., $\mathbf{x}_1$), save residual $\hat{\varepsilon}_t$
  2. Regress $\ln(CPI)_{t+h}$ (i.e., y) on one-year yield (i.e., z) using $\hat{\varepsilon}_t$ as instrument
- One step: Regress $\ln(CPI)_{t+h}$ on one-year yield and Greenbook forecasts, using ΔFFR_t as instrument

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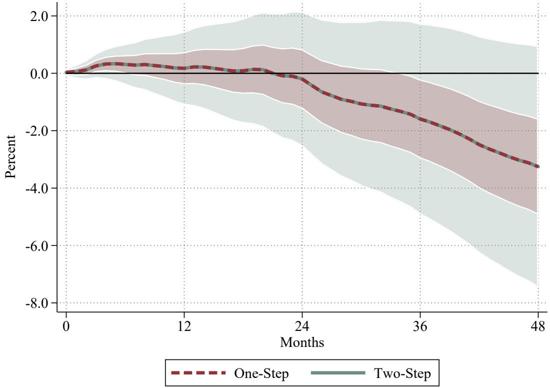
US Consumer Price Index IRF



Note: OLS-LP, US 1972m1-2007m12. 90% confidence bands from Newey-West s.e.

LP-IV Application with Auxiliary Controls: High-Frequency Surprises

Results with auxiliary controls (i.e., $\mathrm{x}_{2,t}$)
analogous to OLS

LP-IV Application with Auxiliary Controls: High-Frequency Surprises

Results with auxiliary controls (i.e., $\mathbf{x}_{2,t}$) analogous to OLS

Miranda-Agrippino-Ricco-style identification

- Two step: Detail
  1. Regress high-frequency surprise m_t on Greenbook forecasts $\mathbf{x}_1$), save residual $\hat{\varepsilon}_t$
  2. Regress $\ln(CPI)_{t+h}$ (i.e., y) on one-year yield (i.e., z) and $\mathbf{x}_{2,t}$ using $\hat{\varepsilon}_t$ as instrument
- One step: Regress $\ln(CPI)_{t+h}$ on one-year yield, Greenbook forecasts and $\mathbf{x}_{2,t}$, using m_t as instrument

LP-IV Application with Auxiliary Controls: High-Frequency Surprises

Results with auxiliary controls (i.e., $\mathbf{x}_{2,t}$) analogous to OLS

Miranda-Agrippino-Ricco-style identification

- Two step: ▸ Detail
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- One step: Regress $\ln(CPI)_{t+h}$ on one-year yield, Greenbook forecasts and $\mathbf{x}_{2,t}$, using m_t as instrument

First-Stage F-Statistics

	Two-Step	One-Step
Without Aux. Controls	11.76	19.57
With Aux. Controls	11.27	19.58

VAR with External Instruments

- **Two-Step Approach**: let $\mathbf{w}_t = [m_t, \mathbf{y}_t]'$, estimate contemporaneous responses via

$$\mathbf{w}_t = \boldsymbol{\beta}_{2S}^0 \overbrace{m_t}^{\text{IV with } \varepsilon_t} + \boldsymbol{\Pi}(L)\mathbf{w}_{t-1} + \mathbf{u}_t^{2S}$$

and estimate IRFs via: $\boldsymbol{\beta}_{2S}^h = \mathbf{C}^h \boldsymbol{\beta}_{2S}^0$

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  and estimate IRFs via: $\boldsymbol{\beta}^h_{2S} = \mathbf{C}^h \boldsymbol{\beta}^0_{2S}$

- **Alternative One-Step Approach**: now estimate contemporaneous responses via:

$$\mathbf{w}_t = \boldsymbol{\beta}^0_{1S} \overbrace{m_t}^{\text{IV with } z_t} + \boldsymbol{\gamma}\mathbf{x}_{1,t} + \boldsymbol{\Theta}(L)\mathbf{w}_{t-1} + \mathbf{u}^{1S}_t$$

  and estimate IRFs via: $\boldsymbol{\beta}^h_{1S} = \mathbf{C}^h \boldsymbol{\beta}^0_{1S}$

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  and estimate IRFs via: $\boldsymbol{\beta}_{1S}^h = \mathbf{C}^h \boldsymbol{\beta}_{1S}^0$

- OVB now captures omission of contemporaneous $\mathbf{x}_{1,t}$ scaled by $\mathbf{C}^h$:

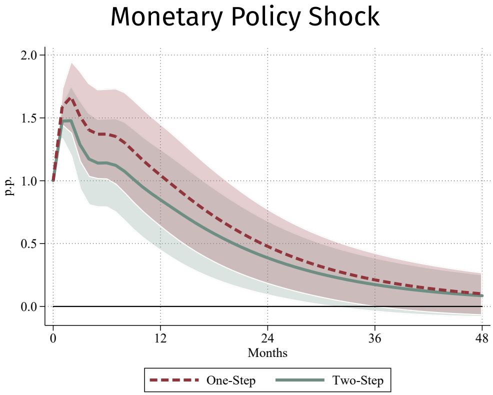
$$\Omega_{2S} = \mathbf{C}^h \frac{\mathbb{E}\left[\varepsilon_t \mathbf{y}_{t-1}'\right] \mathbf{A}_y^{-1} \mathbb{E}\left[\mathbf{y}_{t-1}\mathbf{x}_{1,t}'\right] \boldsymbol{\phi}_y}{\mathbb{E}\left[\varepsilon_t \mathbf{y}_{t-1}'\right] \mathbf{A}_m^{-1} \mathbb{E}\left[\mathbf{y}_{t-1}\mathbf{x}_{1,t}'\right] \boldsymbol{\phi}_m}$$

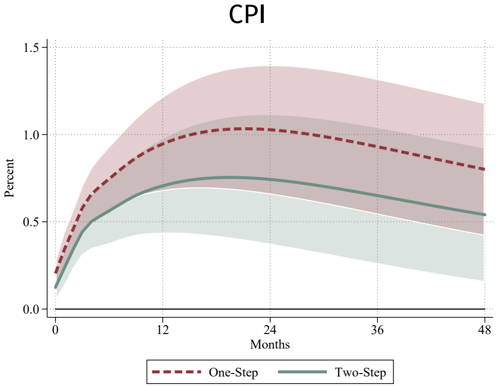
  Intuition: failure of exogeneity in two-step since instrument (potentially) correlated with $\mathbf{x}_{2,t}$ lags that affect the outcome variable

VAR with External Instruments Application

Two-step: $\mathbf{y}_t = [\Delta FFR_t, \mathbf{x}_{2,t}']'$ where $\hat{\varepsilon}_t^{RR}$ used as external IV for ΔFFR_t

One-step: $\mathbf{y}_t = [\mathbf{x}_{1,t}', \Delta FFR_t, \mathbf{x}_{2,t}']'$ with $\hat{\varepsilon}_t^{RR}$ as external IV



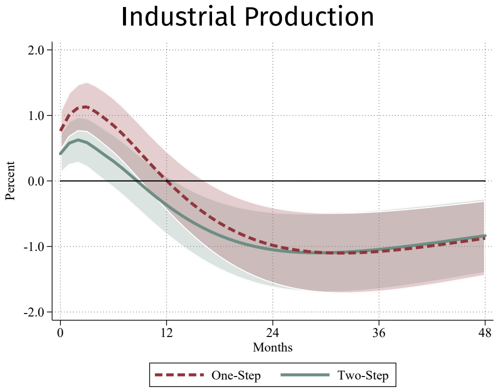


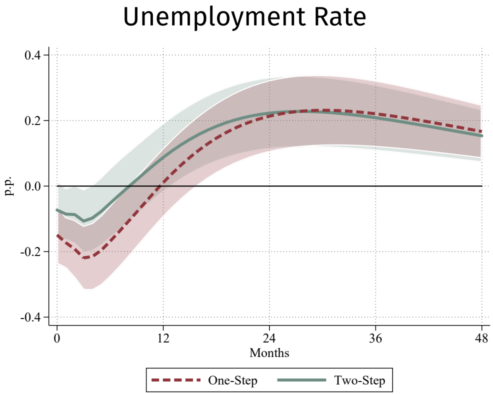
Note: VAR(4), US 1972m1-2007m12. 68% confidence bands from block bootstrap (1000 reps)

VAR with External Instruments Application

Two-step: $\mathbf{y}_t = [\Delta FFR_t, \mathbf{x}'_{2,t}]'$ where $\hat{\varepsilon}_t^{RR}$ used as external IV for ΔFFR_t

One-step: $\mathbf{y}_t = [\mathbf{x}'_{1,t}, \Delta FFR_t, \mathbf{x}'_{2,t}]'$ with $\hat{\varepsilon}_t^{RR}$ as external IV





Note: VAR(4), US 1972m1-2007m12. 68% confidence bands from block bootstrap (1000 reps)

Implications and Applications

Quantile Regression

Quantile-Regression Setting: No Auxiliary Controls

Setup:

$$y_t = \beta_{2S}(\tau) \overbrace{\varepsilon_t}^{z_t^{\perp \mathbf{x}_{1,t}}} + u_t(\tau) \quad \text{(2S)}$$

$$y_t = \beta_{1S} z_t + \gamma(\tau)\mathbf{x}_{1,t} + e_t(\tau) \quad \text{(1)}$$

where $\beta_{2S}(\tau)$, $\beta_{1S}(\tau)$ and $\gamma(\tau)$ are QR coefficients for $\tau \in [0, 1]$

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where $\beta_{2S}(\tau)$, $\beta_{1S}(\tau)$ and $\gamma(\tau)$ are QR coefficients for $\tau \in [0,1]$

Results: even without a without auxiliary controls, two-step inconsistent due to QR-OVB:

$$\beta_{2S} = \beta_{1S} + \underbrace{\phi_1(\tau)\frac{\mathbb{E}[w_\tau(\mathbf{x})\epsilon_t \mathbf{x}_{1,t}]}{\mathbb{E}[w_\tau(\mathbf{x})\epsilon_t^2]}}_{\equiv OVB}$$

[Angrist et al. 2006, Ecta]

OVB term more complex with auxiliary controls

Quantile-Regression Setting: No Auxiliary Controls

Setup:

$$y_t = \beta_{2S}(\tau)\overbrace{\varepsilon_t}^{z_t^{\perp \mathbf{x}_{1,t}}} + u_t(\tau) \tag{2S}$$

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OVB term more complex with auxiliary controls

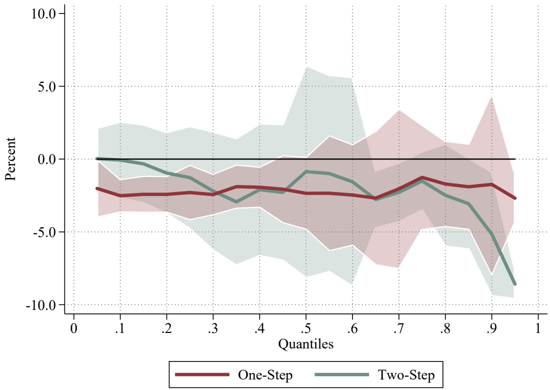
Intuition: $\mathbb{E}[\varepsilon_t \mathbf{x}_{1,t}] = 0$ by construction in two-step, but does not imply $\mathbb{E}[w_\tau(\mathbf{x})\varepsilon_t \mathbf{x}_{1,t}] = 0$
Hard to find reasonable assumptions under which bias is zero ($\phi_1(\tau) = 0$ or $w_\tau(\mathbf{x})$ constant)

Quantile Application: Effect of Monetary Policy Across Quantiles

Figure: Quantile Response of $\ln(CPI)$ after 4 years

Romer-Romer-style identification

- Two step:
  1. OLS of ΔFFR_t (i.e., z) on Greenbook forecasts (i.e., $\mathbf{x}_1$), save residual $\hat{\varepsilon}_t$
  2. QR of $\ln(CPI)_{t+h}$ (i.e., y) on $\hat{\varepsilon}_t$
- One step: QR of $\ln(CPI)_{t+h}$ on ΔFFR_t and Greenbook forecasts



Note: 90% CI from bootstrapped s.e.

Conclusions

- Widely-used two-step shock-first approach is problematic for identification and inference
  - OLS and IV: over-estimation of standard errors + inconsistency (with auxiliary controls)
  - Inconsistency in QR even absent auxiliary controls
- Alternative one-step estimation procedure will circumvent OVB, yielding unbiased and efficient estimates with same (claimed) identification

Why 2S? More on 1S

- Differences between two- and one-step approaches matter in practice
  - Simple resolution to price puzzle from one-step, when revisiting estimated effects of monetary policy controlling for central-bank information

Appendix

Empirical Setup

▸ Back

- Response of US CPI to US interest rate i, coontrolling for Greenbook forecasts of GDP growth Δy^e, inflation π^e and unemployment u^e
- Estimate at monthly frequency (rather than meeting frequency) for 1972m1-2007m12
- Two-step approach:

$$\begin{aligned}\Delta i_t =& \delta_0 + \delta_1 i_{t-1} + \sum_{i=-1}^{2} \left[\delta_{2,i} \Delta y^e_{t,i} + \delta_{3,i}\left(\Delta y^e_{t,i} - \Delta y^e_{t-1,i}\right) + \delta_{4,i}\pi^e_{t,i} + \delta_{5,i}\left(\pi^e_{t,i} - \pi^e_{t-1,i}\right)\right] \\ &+ \delta_6 u^e_{t,0} + \epsilon_t \qquad \text{(Step \#1)} \\ \ln(CPI_{t+h}) =& \beta^h_{2S}\epsilon_t + \mathbf{x}'_{2,t}\boldsymbol{\pi}^h + \varepsilon_{t+h} \qquad \text{(Step \#2)}\end{aligned}$$

- One-step approach:

$$\begin{aligned}\ln(CPI_{t+h}) =& \theta^h_0 + \beta^h_{1S}\Delta i_t + \theta^h_2 i_{t-1} + +\mathbf{x}'_{2,t}\boldsymbol{\theta}^h_3 \\ &+ \sum_{i=-1}^{2}\left[\theta^h_{4,i}\Delta y^e_{t,i} + \theta^h_{5,i}\left(\Delta y^e_{t,i} - \Delta y^e_{t-1,i}\right) + \theta^h_{6,i}\pi^e_{t,i} + \theta^h_{7,i}\left(\pi^e_{t,i} - \pi^e_{t-1,i}\right)\right] + \theta^h_8 u^e_{t,0} + e_{t+h}\end{aligned}$$

Two-Step Approach in IV: Motivation

- Intuition: interested in identifying effects of monetary policy using high-frequency 'surprises'
$\rightarrow$ Surprises best thought of as an instrument for 'true' shock $\Rightarrow$ use IV to estimate effects to avoid issues of measurement error [Stock & Watson 2018]
- HF surprises may not be fully exogenous (including due, e.g., Fed information effect)
$\rightarrow$ Regress surprises on CB forecasts or private-sector forecasts and use residual as instrument [Gertler & Karadi 2015; Miranda-Agrippino & Ricco 2021; Bauer & Swanson 2022]
- Other papers similarly use OLS residuals as instruments:
  - Monetary-policy shock identification based on intnl. finance 'trilemma' [Jorda et al. 2020]
  - Romer and Romer monetary policy shocks as external instruments in a VAR [Barnichon & Mesters 2020]
  - High-frequency oil-price shock identification [Känzig 2021]

Back

So, Why Use Two-Step Approach?

Back

A few explanations:

#1. **Plotting and Interpreting Shock**

– Hybrid regression $\Rightarrow$ no reason not to plot shock (with appropriate controls $\mathbf{x}_{1,t}$ *and* $\mathbf{x}_{2,t}$)

$\rightarrow$ Inference on causal coefficients should be carried out on one-step estimates

#2. **Mixed Frequencies**

– Shocks observed at different frequency to macro variables

$\rightarrow$ Possible to run one-step LP with T at shock frequency and H at data frequency

#3. **Accounting for Additional Controls in Step #2**

– Easy to assess robustness by adding controls $\mathbf{x}_{2,t}$

$\rightarrow$ Unless $\mathbb{E}_t[\epsilon_t \mathbf{x}'_{2,t}] = 0$, additional controls should be reflected in step #1 too

$\rightarrow$ Publish shocks *and* controls

Implications of Results for Different Estimation Approaches

Back

Results have implications for range of techniques used in applied literature

#1. **Local Projections**

- Issues for s.e. calculation arise when using generated residual in LP, and can get inconsistency when shock is correlated with auxiliary controls in LP

→ Simple multivariate LP using confounders as controls avoids these issues

#2. **External Instruments**

- Using generated residuals as instrument in LP-IV or Proxy-SVAR runs into similar issues for s.e. calculation, efficiency, and/or consistency

→ Can avoid these issues by including confounders as exogenous variables

#3. **Quantile Regression** (or other 'non-linear' estimators)

- Using orthogonalised shock in QR (or other non-OLS settings) generates inconsistency in general

→ Always use one-step estimator, controlling for confounding factors