Capital Controls and Trade Policy*

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Abstract

How does optimal capital-flow management change with prevailing trade policies? We study the joint optimal determination of capital controls and trade tariffs in a two-country, two-good model with trade in goods and assets. Because countries are large in both markets, a country-planner can achieve higher domestic welfare by departing from free trade in addition to levying capital controls, despite the cooperative optimal allocation being efficient. However, time variation in the optimal tariff induces households to over- or under-borrow through its effects on the path of the real exchange rate. As a result, optimal capital controls are generally smaller when trade policy is constrained (i.e., by a Free-Trade Agreement), but, absent retaliation, can be larger depending on the paths of underlying fundamentals.

Key Words: Capital-Flow Management; Free-Trade Agreements; Ramsey Policy; Tariffs; Trade Policy.

JEL Codes: F13, F32, F33, F38.

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1 Introduction

Trade and capital-flow management have long been key topics of macroeconomic policy and have, once more, come into sharp focus. Following at least two decades of integration (Baier and Bergstrand, 2007), the process of trade liberalization has stalled, with events like the US-China trade war contributing to substantially heightened uncertainty around world trade (Ahir, Bloom, and Furceri, 2022), a decline in the number of new regional-trade agreements and a deceleration of global value chain integration (see D'Aguanno, Davies, Dogan, Freeman, Lloyd, Reinhardt, Sajedi, and Zymek, 2021, for further discussion). Alongside this, financial liberalization too has slowed and the International Monetary Fund has partially revised their 'institutional view' to emphasize a role for managing capital flows in specific circumstances (Ghosh, Ostry, and Qureshi, 2016) opening the way for 'macroprudential' foreign-exchange interventions targeting cross-border flows (e.g., Ahnert, Forbes, Friedrich, and Reinhardt, 2020). However, despite discussions on trade and capital-flow policies both growing in prominence, academics and policymakers typically consider these measures separately. Trade-policy discussions often balance economic forces (e.g., monopoly power, comparative advantage) with political factors (e.g., de-industrialization, trade sanctions), while recent debates about capital controls have centred on their role in insulating countries from large and volatile cross-border flows.

In this paper, we provide a unifying framework to study the joint optimal determination of trade policy and capital-flow management, in a model where both instruments are driven by a common motive: to exploit a country's monopoly power in markets, often referred to as terms-of-trade manipulation. We show that active trade policy influences optimal capital-flow management through its effects on the path for the real exchange rate and, in turn, private incentives to borrow in international financial markets. This mechanism is closely related to the Harberger-Laursen-Metzler effect (Harberger, 1950; Laursen and Metzler, 1950), who argued that an appreciation of the terms of trade can improve savings in the economy, and is still prominent in policy discussions (see Eichengreen, 2019). We then extend our framework to allow for strategic interactions between countries and we assess the implications of international trade and financial arrangements for global welfare and the likelihood that trade and capital-control wars emerge.

The starting point for our analysis is a canonical two-country, two-good endowment economy model, absent nominal or financial frictions. Households make an inter-temporal consumption-savings decision and intra-temporally choose their optimal consumption bundle. In the laissez-faire or decentralized allocation, relative consumption growth across countries is proportional to the relative decrease in price levels—i.e., the rate of real exchange rate depreciation—a relationship often referred to as the Backus and Smith (1993) condition (see also Kollmann, 1995). However, households do not internalize the effect of their actions on relative prices. These pecuniary externalities, described in Geanakoplos and Polemarchakis (1986), imply that a country planner can improve on domestic welfare by manipulating the inter- and intra-temporal terms of trade—i.e., world interest rates and relative goods prices, respectively—even though the laissez-faire allocation is optimal from a global perspective. Within this setup, Costinot, Lorenzoni, and Werning (2014), building on Obstfeld and Rogoff (1996), consider optimal capital controls which trade off the incentive to drive down the world interest rate with second-best effects on relative goods prices.[1] However, these papers rule out trade policy through an implicit or explicit Free-Trade Agreement (FTA).

Our key contribution is to relax the trade-policy constraint imposed on the planner and assess the interactions between optimal capital controls and trade tariffs within a tractable environment, using the primal approach of Lucas and Stokey (1983). We begin by analyzing the problem of a country-planner acting unilaterally to maximize domestic welfare, without retaliation from abroad. Our headline finding is that optimal capital controls are generally smaller when trade policy is constrained (i.e., by a FTA), but can be larger depending on the paths of underlying fundamentals which governs the alignment of inter- and intra- temporal incentives to manipulate the terms of trade. When the Foreign planner is allowed to retaliate, capital controls are be smaller under a FTA, regardless of the underlying path for fundamentals, due to strategic interactions *across* instruments.

Consider a scenario in which domestic households borrow between two periods, driven by a temporarily low endowment of the good consumed with home bias (the 'domestic good'). The planner will seek to delay aggregate consumption inter-temporally by taxing capital inflows, but also has an intra-temporal incentive to reduce consumption of the relatively expensive domestic good by levying a temporary subsidy on the second good ('foreign good'). This puts pressure on the real exchange rate to depreciate which, all else equal, results in an adjustment in optimal capital controls (see, e.g., Jeanne and Son, 2023). Intuitively, the exchange-rate depreciation encourages 'over-borrowing' by households because their domestic consumption bundle becomes cheaper today relative to the future. Therefore, a larger capital-flow tax is required to induce a constrained-efficient path for consumption. Quantitatively, this interaction between instruments can be significant; in our baseline calibration, capital controls are one-third larger when the tariff is optimally chosen.

Fluctuations in the foreign good, consumed without home bias, involve a different mix of inter- and intra-temporal incentives, which may better capture the trade-offs faced by exporting nations. When the domestic endowment of the foreign good is temporarily low, incentives to delay aggregate consumption are the same, but the optimal unilateral tariff puts pressure on the real exchange rate to appreciate. Absent further action, this incentivizes under-borrowing and therefore a smaller capital inflow tax is required.

We provide intuitive expressions for the optimal instruments based on foreign export-supply elasticities. We show that the optimal capital-inflow tax depends on the sum of trade deficits for each good, across subsequent periods, weighted by the corresponding inverse elasticity of foreign export supply—echoing findings in the optimal taxation literature (e.g., Atkinson and Stiglitz, 1980; Chari and Kehoe, 1999), where the planner taxes inelastic commodities more. Regardless

[1] When domestic households borrow between two periods, the planner tends to levy capital-inflow taxes to delay consumption relative to the decentralized allocation. Capital controls introduce a wedge to the Backus-Smith condition, so consumption growth can be slower than the rate of exchange-rate depreciation. This wedge can also be understood as a measure of exchange-rate misalignment induced by the planner at the optimal allocation (e.g., Corsetti, Dedola, and Leduc, 2023).

of trade policy, the planner taxes goods more heavily when the foreign export-supply elasticity is low—since these goods will be relatively more expensive at the margin. When trade policy is optimally set, tariffs correct for the relative weighted deficits across goods varieties. Given this, capital controls depend on the deficits weighted only by the *partial* elasticity with respect to the untaxed good (i.e., the domestic good). This distinction underlies the interactions between capital controls and trade policy in our framework.

In the limiting case of unitary elasticities of inter- and intra-temporal substitution, studied in Cole and Obstfeld (1991) (henceforth 'CO'), the planner uses capital controls only in response to variation in the domestic-good endowment. In response to variation in the endowment of the foreign good, the planner only uses tariffs when trade policy can be set optimally. In contrast, when trade policy is constrained (i.e., by a FTA), the planner uses both instruments in response to variation in the endowment of either good. So, in this knife-edge case, trade policy fully substitutes for the use of capital controls.

Our analysis applies to more general environments, including where capital-flow management is driven by a demand-management motive (via an aggregate-demand externality) and where foreign-exchange interventions (FXI) are effective. When there are nominal rigidities, the planner faces an additional incentive to bring forward (delay) consumption when the economy is demand-constrained (overheating), as in Farhi and Werning (2016). Generally, within our framework, the planner targets an optimal wedge for the Backus-Smith condition, and an optimal relative-demand wedge which are interrelated in equilibrium. In an extension that allows for segmented financial markets, as in Gabaix and Maggiori (2015), we show that FXI can be used to target the same wedge as capital controls and interact similarly with trade policy.[2] Policy incentives and interactions also persist in small-open economies where an individual country's ability to manipulate the world interest rate disappears (although we show this is not the case with financial segmentation). In this case, in the CO limit, when inter- and intra-temporal incentives are aligned, the optimal capital-inflow tax is invariant to the size of the economy because the tax needed to address the inter-temporal margin exactly coincides with that required to address intra-temporal incentives.

Returning to our baseline model, we analyze a strategic setting with retaliation, where both country planners set policy as a mutual best response—specifically, an open-loop Nash equilibrium. Policy wars follow the same 'inverse elasticity rule'. The total wedge introduced by capital controls in the Backus-Smith condition is larger when the elasticity of inter-temporal substitution is low, whereas the total wedge in relative demand introduced by tariffs is larger when the intra-temporal elasticity of substitution between goods is low. In the Nash equilibrium, we find that capital controls tend to be smaller when trade policy is constrained by a FTA, in response to fluctuations in either goods' endowment. Interestingly, this arises because capital controls in the Home country respond to the distortion in the path for exchange rates due to tariffs levied by the Foreign planner whose effect dominates.

[2]Our insights on the interaction of capital-flow management and policy interventions will also apply to any policies that do not directly induce a wedge in the condition equalizing marginal-utility growth across countries, the Backus-Smith condition (e.g., monetary policy, fiscal policy).

Finally, within the strategic framework, we show that when countries optimally choose tariffs, the incentives to depart from a 'free-financial-flows agreement' (FFFA) and engage in welfare-costly capital-control wars are heightened, providing a novel argument in favor of free trade. Constraints on trade policy, like a FTA, reduce incentives for an individual country to levy capital controls (potentially either subsidies or taxes), which could prompt retaliation, because tariffs distort the path of real exchange rates over time. In short: retaining openness in trade can help to sustain financial openness.

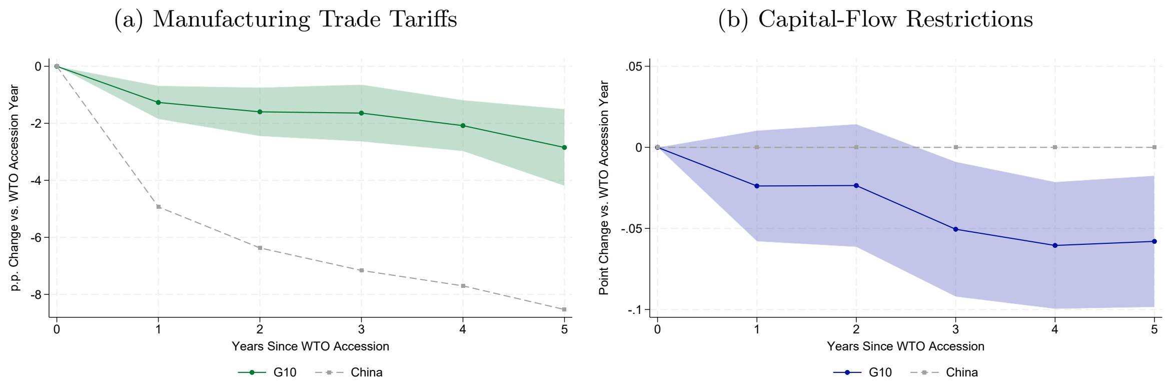
Policy in Practice. Although our analysis is theoretically grounded, our results may speak to observed policy patterns. Since being set up in 1995, advanced G10 economies acceding to the World Trade Organization (WTO) have seen average tariff rates for manufactured goods decline in the years after accession (Figure 1a), even when controlling for country- or time-specific factors. At the same time, these economies have seen their average capital-flow restrictions (measured using IMF data from Fernández, Klein, Rebucci, Schindler, and Uribe, 2016) decline relative to non-WTO members (Figure 1b). These findings are consistent with the results from our strategic framework, in which constraints on trade policy reduce incentives to levy capital-flow restrictions, and are indicative of some complementarity between trade and financial liberalization in these countries.

However, these trends are less clear-cut for developing economies acceding to the WTO, where some have seen lower tariffs alongside *tighter* capital-inflow restrictions relative to non-WTO members. This suggests that for some countries, trade and financial liberalization have been more substitutable—potentially depending on the size and income profiles of countries, a factor which our model sheds light on. In the case of China, where average tariffs fell by around 8pp in the 5 years after acceding to the WTO in 2001, there is some evidence from the People's Bank of China that restrictions on capital flows were loosened (Bank for International Settlements, 2008)—even if headline capital-inflow statistics in Figure 1b do not pick them up.

Related Literature. Our work builds on Costinot et al. (2014) who study the role of capital controls as a means of dynamic terms-of-trade manipulation in large-open endowment economies. We depart from their assumption of free trade and, in doing so, our work combines analyses of inter-temporal incentives to manipulate the terms of trade via the use of capital controls (see e.g. Rebucci and Ma, 2020; Bianchi and Lorenzoni, 2022), with intra-temporal incentives, for which tariffs are regularly applied in practice (Broda, Limao, and Weinstein, 2008). Like us, Ju, Shi, and Wei (2013) analyze the relationship between inter- and intra-temporal trade but within a Heckscher-Ohlin model. While Jeanne (2012) and Farhi, Gopinath, and Itskhoki (2014) analyze the effects of tariffs on real exchange rates. We show that trade policy itself can give rise to incentives to levy capital controls, through their impact on real exchange rates and, in turn, incentives to over-/under-borrow.

Our analysis also contributes to the broader literature on capital controls. Notably, we show the mechanisms underpinning the interaction between optimal trade and financial policy persist in settings with aggregate-demand externalities in models with nominal rigidities (see,

Figure 1: Average Manufacturing Trade Tariffs and Capital-Flow Restrictiveness for G10 Economies Around Countries' WTO Accession



Notes: Estimated change in manufacturing tariffs (Panel a) and capital-flow restrictiveness (Panel b) in the years following accession to WTO. Manufacturing tariffs measured in % for 1990-2019 (annual frequency) from World Development Indicators (World Bank). Metric used here uses weighted mean applied tariff for manufacturing products, which measures the average of effectively applied tariffs weighted by the product import shares corresponding to each partner country. Capital-flow restrictions are a 0-1 index for 1995-2019 (Fernández et al., 2016). Estimates for G10 attained from regression of h-year policy change $y_{i,t+h} - y_{i,t-1}$, in country i at year t, on a WTO accession dummy ($WTO_{i,t} = 1$ in year of accession, 0 otherwise) and country and time fixed effects (f_i and f_t). Shaded area shows 90% confidence bands implied by standard errors, which are clustered by country. Line for China plots raw changes in tariffs and capital-flow restrictions following WTO accession in 2001.

e.g., Farhi and Werning, 2014, 2016; Schmitt-Grohé and Uribe, 2016; Marin, 2022). In addition, while we focus on a deterministic setting, we show that anticipated shocks engender preemptive policy interventions, akin to the precautionary motives driving capital controls in (see, e.g., Mendoza, 2002; Bianchi, 2011). We also draw parallels to a literature on FXI. Fanelli and Straub (2021) show that FXI and capital controls are isomorphic when there is partial segmentation in international markets, up to implementation costs. They share our focus on an endowment economy and pecuniary externalities, but, unlike us, abstract from trade policy.

The literature on trade tariffs has predominantly focused on environments with no trade in assets, albeit with a richer supply-side setup with monopolistic (and often heterogeneous) firms (see, e.g., Demidova and Rodriguez-Clare, 2009; Caliendo, Feenstra, Romalis, and Taylor, 2021). We contribute to this literature by evaluating the scope for tariffs as second-best instruments to manipulate the cost of borrowing in a dynamic setting.

Finally, our paper contributes to a growing literature assessing the joint role of trade and stabilization policies. Bergin and Corsetti (2023) study the response of monetary policy to tariff shocks. Auray, Devereux, and Eyquem (2020) study the scope for trade wars and currency wars in a New-Keynesian small-open economy model but their model features balanced trade, so there is no scope for capital controls. Jeanne (2021) studies monetary policy and the accumulation of foreign reserves, emphasizing the distinction between a demand-constrained 'Keynesian regime' and a 'classical regime' where tariffs are used to manipulate the terms of trade.

Outline. The remainder of the paper is structured as follows. Section 2 describes the model environment. Section 3 characterizes the optimal unilateral planning allocation. Section 4 discusses policy implementation, macroeconomic outcomes and model generalizations. Section 5 studies strategic cross-country interactions. Section 6 considers welfare. Section 7 concludes.

2 Basic Environment

There are two countries, Home H and Foreign F, each populated by a continuum of identical households. Time is discrete and infinite, $t = 0, 1, ...$, and there is no uncertainty. The preferences of the representative Home consumer are denoted by the time-separable utility function:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$$

where C_t is aggregate Home consumption and $u(C)$ is a twice continuously differentiable, strictly increasing and strictly concave function with $\lim_{C\to 0} u'(C) = \infty$. $\beta \in (0, 1)$ is the discount factor. The preferences of the representative Foreign consumer are analogous, with asterisks denoting Foreign variables.

Consumers derive utility from two goods, good 1 and good 2. We denote the representative Home consumer's consumption of good 1 and good 2 by $c_{1,t}$ and $c_{2,t}$, respectively, and group them into the vector $\mathbf{c}_t = [c_{1,t}\ c_{2,t}]'$. Home aggregate consumption is defined by the aggregator $C_t \equiv g(\mathbf{c}_t)$, where $g(\cdot)$ is a function that is twice continuously differentiable, strictly increasing, concave and homogeneous of degree one. We define the Jacobian of $g(\mathbf{c}_t)$ by $\nabla g(\mathbf{c}_t) = [g_{1,t}\ g_{2,t}]'$, where $g_{i,t} = \frac{\partial g(\mathbf{c}_t)}{\partial c_{i,t}}$ for $i = 1, 2$, while second derivatives are written as $g_{ij,t} = \frac{\partial^2 g(\mathbf{c}_t)}{\partial c_{i,t}\partial c_{j,t}}$ for $i, j = 1, 2$. The aggregator for the representative Foreign consumer is written as $C_t^* \equiv g^*(\mathbf{c}_t^*)$, with analogously defined derivatives.

The Home (Foreign) consumer's period-t endowments of goods 1 and 2 are denoted by $y_{1,t}$ ($y_{1,t}^*$) and $y_{2,t}$ ($y_{2,t}^*$), respectively, and are weakly positive in all periods. Throughout, without loss of generality, we assume that Home consumers have a 'home bias' for good 1, and we therefore describe this as the 'domestic good'. Defining the time-t Home expenditure share on domestic goods as α_t, then 'home bias' implies $\alpha_t > 0.5$. Likewise, Foreign consumers prefer good 2 (the 'foreign good') and we assume $\alpha_t^* = \alpha_t$. The total world endowment of goods 1 and 2 are $Y_{1,t} \equiv y_{1,t} + y_{1,t}^*$ and $Y_{2,t} \equiv y_{2,t} + y_{2,t}^*$, respectively.

The inter-temporal budget constraint for the Home household expressed as:

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t - \mathbf{y}_t) \leq 0 \tag{1}$$

where $\mathbf{p}_t = [p_{1,t}\ p_{2,t}]'$ denotes the vector of period-t world goods prices and $\mathbf{y}_t = [y_{1,t}\ y_{2,t}]'$ is the vector of Home endowments.

We define two additional quantities. First, the terms of trade is given by $S_t = p_{2,t}/p_{1,t}$ and, since good 1 is the 'domestic good' and good 2 the 'foreign good', we refer to an increase in S_t

as a deterioration of the Home terms of trade. Second, the real exchange rate is given by the ratio of consumer price indices $Q_t = P_t^*/P_t$, where $P_t^{(*)} \equiv \min_{\mathbf{c}_t^{(*)}} \{\mathbf{p}_t \cdot \mathbf{c}_t^{(*)} : \ g^{(*)}(\mathbf{c}_t^{(*)}) \geq 1\}$. An increase in Q_t corresponds to a depreciation of the Home real exchange rate.

Specific Functional Forms. For our numerical exercises, we use a constant relative risk aversion (CRRA) specification for per-period utility $u(C) \equiv \frac{C^{1-\sigma}-1}{1-\sigma}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion. The aggregate consumption of the representative agent is given by the Armington (1969) aggregator:

$$C_t \equiv g(\mathbf{c}_t) = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{2}$$

where $\phi > 0$ is the elasticity of substitution between good 1 and 2. With this specification for preferences, expenditure shares are constant: $\alpha_t^{(*)} = \alpha^{(*)}$ for all t.

An interesting special case arises when $\sigma = \phi = 1$. This corresponds to the parametrization studied in Cole and Obstfeld (1991), the 'CO case', which we revisit in Section 4.3.

3 Unilateral Planning Allocation

We begin by considering an equilibrium in which the Home planner maximizes domestic welfare, while the Foreign planner is passive—i.e., does not levy taxes in response to Home policy. The optimality conditions for the representative Foreign household act as a constraint for the Home planner and are given by:

$$\beta^t u^{*\prime}(C_t^*) \nabla g^*(\mathbf{c}_t^*) = \lambda^* \mathbf{p}_t \tag{3}$$

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) = 0, \tag{4}$$

where λ^* denotes the Lagrange multiplier on the Foreign inter-temporal budget constraint.

The Home planner maximizes the discounted lifetime utility of the Home representative consumer by choosing consumption of each individual good variety $\{c_{1,t}, c_{2,t}\}_{t\geq 0}$, subject to: (i) the representative Foreign consumer's utility maximization at world prices; (ii) market clearing in each period; and (iii) any constraints imposed on trade policy. Conditions (i) and (ii) can be summarized independently of (iii), in a single implementability condition (Lucas and Stokey, 1983) described in the following lemma.

Lemma 1 (Implementability for Unilateral Planner) *When the Foreign country is passive, an allocation $\{\mathbf{c}_t, \mathbf{c}_t^*\}$, together with world prices $\mathbf{p}_t$, form part of an equilibrium if they satisfy:*

$$\sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0, \tag{IC}$$

where $\boldsymbol{\rho}(C_t) \equiv u^{*\prime}(C^*(C_t)) \nabla g^*(\mathbf{c}_t^*(\mathbf{c}_t))$ denotes the price of consumption at each t.

Proof: See Appendix A.1. □

Absent constraints on trade policy, the Home planner's problem is:

$$\max_{\{c_{1,t},c_{2,t}\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-nFTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$C_t = g(\mathbf{c}_t) \tag{nFTA}$$

where the third line (nFTA) reflects that aggregate consumption C_t can be backed out of the consumption aggregator $g(\mathbf{c}_t)$. Following Costinot et al. (2014), we assume that $\boldsymbol{\rho}(g(\mathbf{c}_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t]$ is strictly convex to guarantee a unique solution.

3.1 Optimal Allocation

The Lagrangean associated with the planning problem is:

$$\max_{\{c_{1,t},c_{2,t}\}} \quad \mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(C_t) - \mu \left\{ \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] \right\} \tag{5}$$

The first-order conditions—with respect to $c_{1,t}$ and $c_{2,t}$, respectively—are given by:

$$u'(C_t) g_{1,t} = \mu \mathcal{MC}_{1,t}^{nFTA} \tag{6}$$

$$u'(C_t) g_{2,t} = \mu \mathcal{MC}_{2,t}^{nFTA} \tag{7}$$

where μ denotes the Lagrange multiplier on the implementability constraint and $\mathcal{MC}_{i,t}^{nFTA}$ denotes the marginal cost associated with giving up a unit of good $i = 1, 2$:

$$\begin{aligned}
\mathcal{MC}_{1,t}^{nFTA} \equiv & u^{*\prime}(C_t^*) g_1^*(\mathbf{c}_t) + u^{*\prime\prime}(C_t^*) g_1^*(\mathbf{c}_t^*) \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*) \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
\mathcal{MC}_{2,t}^{nFTA} \equiv & u^{*\prime}(C_t^*) g_2^*(\mathbf{c}_t) + u^{*\prime\prime}(C_t^*) g_2^*(\mathbf{c}_t^*) \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*) \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]
\end{aligned}$$

Equations (6) and (7) define the consumption allocations chosen by the planner. They equate the marginal benefit from a unit of good-specific consumption to its marginal cost for goods 1 and 2, respectively, for the representative Home consumer. Consider equation (6). The first term in $\mathcal{MC}_{1,t}^{nFTA}$ reflects the price for a Home household purchasing one unit of good 1. If this were the only term on the right-hand side, the planning allocation would coincide with the decentralized (or laissez-faire) allocation.

However, each additional unit purchased leads to infra-marginal changes in prices which households do not take into account—leading to over-borrowing. In contrast, the planer internalizes how prices vary with consumption allocations, akin to a monopolist. The second term in $\mathcal{MC}_{1,t}^{nFTA}$ reflects the inter-temporal margin—how the cost of borrowing changes with aggregate consumption. The final term, captures the intra-temporal margin—how each good-specific price changes with an additional unit of good 1 consumption c_1.

3.2 How Does the Optimal Allocation Vary with Trade Policy?

When trade policy is constrained, constraint (nFTA) is replaced, and the equilibrium allocation can vary. Within their two-good environment, Costinot et al. (2014) study a setting in which trade policy is constrained by a FTA. In this case, consumption allocations are Pareto efficient (from an individual-household perspective) and can be summarized by:

$$C^*(C_t) = \max_{\mathbf{c}_t, \mathbf{c}_t^*} \{g^*(\mathbf{c}_t^*) \quad \text{s.t.} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \text{ and } g(\mathbf{c}_t) \geq C_t\} \tag{8}$$

for some C_t, where $\mathbf{Y}_t = [Y_{1,t}\ Y_{2,t}]'$. The Pareto frontier defines efficient combinations of consumption $\{c_{1,t}, c_{2,t}\}$ for a given level of aggregate consumption C_t, which coincides with the contract curve when there are no goods-specific taxes. The Home and Foreign Pareto frontiers are defined as $\mathbf{c}(C)$ and $\mathbf{c}^*(C^*)$, which reflect individual households' optimization of consumption bundles given an aggregate consumption C and are reported in Supplementary Materials S.1.2. Therefore, the planner is restricted to choosing a sequence for aggregate consumption $\{C_t\}_{t\geq 0}$, as opposed to consumption varieties $\{c_{1,t}, c_{2,t}\}_{t\geq 0}$. We denote the planning problem when trade policy is constrained by the FTA in (8) by (P-Unil-FTA), reported in Supplementary Materials S.2.1.

In a two-country model, like the one we study, (P-Unil-FTA) amounts to a setting where tariffs are ruled out. In a more general model, however, a FTA member could have zero tariffs with countries participating in the FTA, but non-zero tariffs *vis-á-vis* other countries. Henceforth, for presentational ease, we refer to quantities that result from the allocation in which trade policy is constrained by (8) with the superscript FTA, and the case in which trade policy is unconstrained and set optimally with $nFTA$. Proposition 1 illustrates the relation between the planner's problem with and without constraints on trade policy.

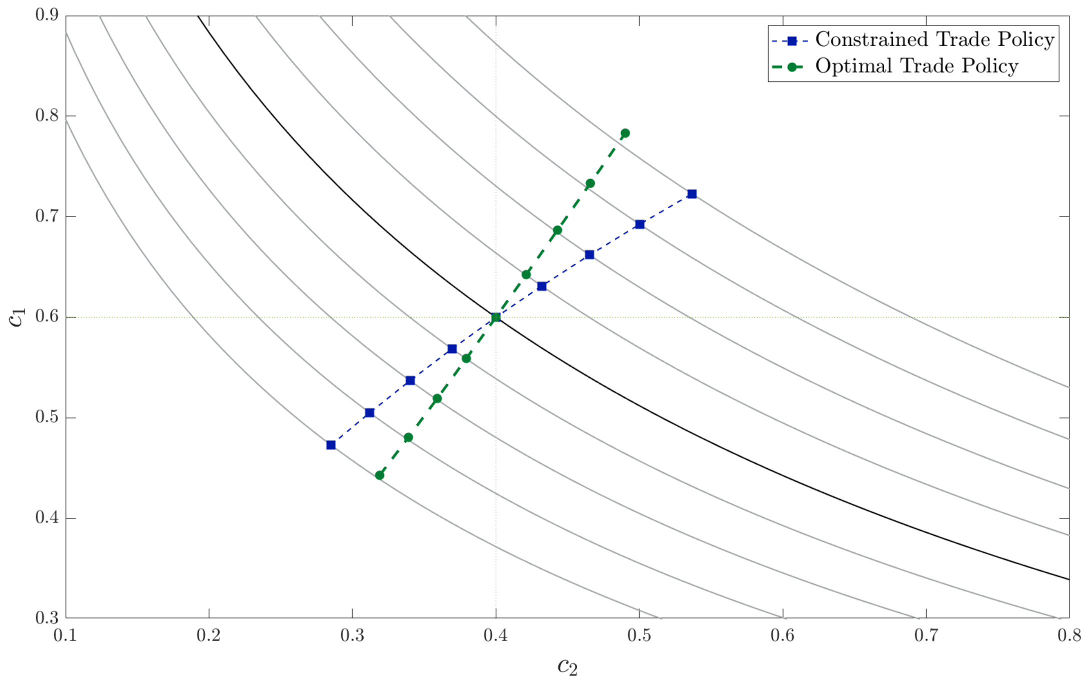
Proposition 1 (Optimal Unilateral Allocations and Trade Policy) *The planners' optimal allocations when trade policy is constrained by a FTA, as defined in* (8)*, and without trade policy restrictions are related by the following condition:*

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = \frac{\partial \mathcal{L}}{\partial c_1} c_1'(C) + \frac{\partial \mathcal{L}}{\partial c_2} c_2'(C)$$

The solution to (P-Unil-FTA) *is defined by* $\frac{d\mathcal{L}}{dC} = 0$*, whereas the solution to* (P-Unil-nFTA) *is defined by* $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2} = 0$.

Proof: The Pareto Frontier (8) implies $c_1'(C), c_2'(C)$ are non-zero, therefore the maximization

Figure 2: Optimal Allocations and the Pareto Frontier



Notes: Plot of optimal consumption allocations for Home consumer from capital flow taxation (i) with a constraint on trade policy from a FTA (blue circles, i.e., the Pareto frontier) and (ii) when trade policy is set optimally (green crosses) at different Home endowments. We use nine equally-spaced allocations for $y_1 \in [\alpha-0.25, \alpha+0.25]$, with $y_1^* = 1 - y_1$, $y_2 = 1 - \alpha$ and $y_2^* = \alpha$. Other model parameters are: $\beta = 0.96$, $\sigma = 2$, $\phi = 1.5$, and $\alpha = 0.6$. Grey/black lines denote loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (black for $C = 1$, grey otherwise). Horizontal (vertical) dotted lines denote α $(1 - \alpha)$, and intersect at the 'no-trade' point.

(P-Unil-FTA) is a constrained version of (P-Unil-nFTA). See Appendix A.2. □

Proposition 1 shows that when trade policy is constrained by a FTA, the unilateral planning allocation is constrained optimal. In particular, since $c_1'(C)$ and $c_2'(C)$ are positive and increasing functions, then it must be that, in general, $sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}) = -sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}) \neq 0$ so there is an incentive to adjust consumption across varieties. In contrast, when trade policy is optimally chosen, the planner sets $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2} = 0$, resulting in $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = 0$.

Figure 2 illustrates these observations. The blue line maps the Pareto frontier: the efficient combinations of $\{c_1(C), c_2(C)\}$ for different levels of long-run aggregate consumption C, consistent with (8). When tariffs are set optimally, the planner achieves a (weakly) higher level of consumption. For $y_1 > \alpha$—the area above the black line, where good 1 is abundant—the allocation absent a FTA is more biased towards c_1. Whereas for $y_1 < \alpha$—the area below the black line, where good 1 is scarce—the allocation is more biased towards c_2. The constrained and unconstrained allocations only coincide in the case $y_1 = y_2^* = \alpha$.

4 Policy and Macro Outcomes at the Optimal Allocation

In this section, we describe the implementation of the optimal allocation and highlight how policy instruments interact. We then contrast the macroeconomic dynamics at the planning allocation, with and without free trade, to the decentralized case.

4.1 Implementation

We consider an implementation where policy instruments map directly to wedges in the Euler and relative goods demand equations. We assume households can trade in non-contingent bonds, denominated in each good variety. The Home planner can impose the same proportional tax θ_t on the gross returns to net lending in all bond markets. So, the per-period budget constraint for the Home consumer can be written as:

$$\mathbf{p}_{t+1} \cdot \mathbf{a}_{t+1} + \tilde{\mathbf{p}}_t \cdot \mathbf{c}_t = \mathbf{p}_t \cdot \mathbf{y}_t + (1 - \theta_{t-1}) (\mathbf{p}_t \cdot \mathbf{a}_t) + T_t$$

where $\tilde{\mathbf{p}}_t = \mathbf{p}_t$ when the FTA constraint is in place, $\mathbf{a}_t$ denotes the vector of asset positions and T_t is a lump-sum rebate. Given a no-Ponzi condition, $\lim_{t\to\infty} \tilde{\mathbf{p}}_t \cdot \mathbf{a}_t \geq 0$, the first-order conditions associated with Home households' utility maximization are given by:

$$u'(C_t)g_i(\mathbf{c}_t) = \beta(1 - \theta_t)(1 + r_{i,t})u'(C_{t+1})g_i(\mathbf{c}_{t+1}) \tag{9}$$

for $i = 1, 2$, where $r_{i,t} \equiv \frac{p_{i,t}}{p_{i,t+1}} - 1$ is a good-specific interest rate. Combining this with the analogous Foreign Euler equation, and using $g_{i,t}/p_{i,t} = 1/P_t$, yields the Backus and Smith (1993) condition with a wedge reflecting capital-flow taxation:

$$(1 - \theta_t) = \frac{u'(C_t)}{u'(C_{t+1})} \frac{u^{*\prime}(C^*_{t+1})}{u^{*\prime}(C_t)} \frac{Q_t}{Q_{t+1}} \tag{10}$$

A tax on capital inflows (or a subsidy for outflows) is captured by $\theta_t < 0$, which can also be interpreted as a tax on current consumption relative to future consumption.

Without constraints on trade policy, the Home planner can additionally levy a proportional tax τ_t on good 2, which we refer to as a tariff, so that $\tilde{\mathbf{p}}_t = \boldsymbol{\tau}_t \cdot \mathbf{p}_t$ where $\boldsymbol{\tau}_t = [1 \;\; \tau_t]'$. A negative value denotes a subsidy ($\tau_t < 0$) and is equivalent to taxing good 1 instead. The representative Home household faces a distorted price for good 2, $p_{2,t}(1 + \tau_t)$, so their relative demand is given by:

$$\frac{c_{1,t}}{c_{2,t}} = \frac{\alpha}{1 - \alpha} \left(\frac{1}{S_t(1 + \tau_t)} \right)^{-\phi} \tag{11}$$

Alternative Instruments. While we focus on an implementation using capital controls and tariffs, the policy problem solves for the optimal wedges in equations (10) and (11). So, consistent with the public finance literature (see, e.g., Chari and Kehoe, 1999), any instruments which map to these wedges can be used. In Section 4.6, we detail an extension of the model with segmented markets and consider FXI (see, e.g., Fanelli and Straub, 2021; Bianchi and

Lorenzoni, 2022). Similarly, time-variation in the optimal relative-demand wedge may reflect manipulation of non-tariff barriers or regulation, evidenced in Broda et al. (2008).[3]

4.2 Interactions Between Optimal Trade and Financial Policy

To build intuition on the interactions between the capital-flow taxes and tariffs, we decompose the log of equation (10), imposing CRRA preferences, into the following two wedges:

$$\ln(1-\theta_t) \approx -\theta_t = \underbrace{-\sigma\left(\hat{C}_t - \hat{C}_{t+1} + \hat{C}^*_{t+1} - \hat{C}^*_t\right)}_{\text{Consumption Wedge}} + \underbrace{\left(\hat{Q}_t - \hat{Q}_{t+1}\right)}_{\text{RER Wedge}} \tag{12}$$

where $\hat{x}$ denotes the natural logarithm of x. The 'consumption wedge' component captures the target relative consumption growth for the planner. The 'RER wedge' reflects capital-flow taxation incentives related to the evolution of the real exchange rate Q. A higher RER wedge (corresponding to a depreciated exchange rate) implies that a larger capital-inflow tax is required to implement a given consumption allocation—so it induces over-borrowing.

Faced with a higher stream of endowments in the future, Home households will borrow to smooth consumption. However, each additional unit of consumption brought forward raises the cost of borrowing (i.e., inter-temporal margin)—so households generally over-borrow because they do not take into account the effect of their choices on prices. Then, consumption growth is too low relative to foreign, so the planner can correct this by levying capital inflow taxes $\theta_t < 0$.

The interaction between optimal capital-flow management and trade policy hinges on *which* good is relatively scarce. If $y_{1,t} < y_{1,t+1}$, good 1 is scarce. Then, the Home household additionally buys relatively more units of good 1 from abroad, at a time when it is relatively more expensive to do so. The optimal tariff on good 2 is negative (i.e., a subsidy) which will depreciate the real exchange rate, raising the RER wedge, and (12) implies a larger capital inflow tax is required. Inter- and intra-temporal manipulation incentives are aligned for the planner, and capital controls are larger absent a FTA. If instead $y_{2,t} < y_{2,t+1}$, good 2 is relatively scarce. In this case, the optimal tariff is positive which will appreciate the real exchange rate, lowering the RER wedge. Inter- and intra-temporal incentives are misaligned, and capital controls are smaller absent a FTA.

4.3 Optimal Instruments with Unitary Elasticities of Substitution

We first focus on the CO limit, defined by $\sigma \to 1, \phi \to 1$. We present results for general preferences in the next sub-section. In the CO limit, we can provide a sharp characterization of households' consumption behaviour, summarized in the following Lemma.

Lemma 2 (Cyclicality of Consumption in the CO Limit) *In the limit as $\sigma \to 1$ and $\phi \to 1$, when there is no aggregate income variation ($Y_{i,t} = \overline{Y}_i$ for $i = 1, 2$ and for all t),*

[3] In addition, De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) show that exporters charge variable markups over marginal costs in foreign markets, suggesting that trade agreements do not necessarily constrain relative prices.

consumption and endowments co-vary as follows.

i. *Decentralized Allocation: consumption* $C_t^{(*)}$ *is invariant to domestic endowments* $y_{i,t}^{(*)}$.

ii. *Planning Allocation without trade policy restrictions: consumption of each good is procyclical with respect to each domestic endowment, but independent from other good endowments* $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{i,t}} > 0$ *and* $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{j,t}} = 0$ *for* $i, j = 1, 2$ *and* $i \neq j$.

iii. *Planning Allocation with FTA (8): consumption is procyclical with respect to domestic endowments,* $\frac{\mathrm{d}C_t}{\mathrm{d}y_{i,t}} > 0$ *for* $i = 1, 2$.

Proof: See Appendix A.3. □

Comparing (i) with (ii) or (iii) shows that households over-borrow in equilibrium. To correct the pecuniary externalities associated with this, the planner (regardless of constraints on trade policy) will choose to lean against capital inflows when income is growing, delaying consumption. The planner's specific response depends on trade-policy constraints. Absent a FTA, a separation arises at the CO limit between the optimal consumption of goods 1 and 2. When the endowment from good i grows (i.e., $y_{i,t} < y_{i,t+1}$), the planner delays consumption in that specific good i (i.e., $c_{i,t} < c_{i,t+1}$) only. However, when trade policy is constrained by a FTA, the planner delays aggregate consumption (i.e., $C_t < C_{t+1}$), if the endowment of either good is growing.

The following Proposition details the instruments chosen in the CO limit.

Proposition 2 (Optimal Instruments in the CO Limit) *The optimal instruments for the Home unilateral Ramsey planner without constraints on trade policy, in the limit as* $\sigma \to 1$ *and* $\phi \to 1$*, are given by:*

$$\theta_t^{nFTA} = 1 - \frac{1 + \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}}}{1 + \frac{c_{1,t+1} - y_{1,t+1}}{\overline{Y}_1 - c_{1,t+1}}}, \qquad \tau_t^{nFTA} = \frac{1 + \frac{c_{2,t} - y_{2,t}}{\overline{Y}_2 - c_{2,t}}}{1 + \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}}} - 1$$

In contrast, when trade policy is constrained by a FTA, as defined by (8):

$$\theta_t^{FTA} = 1 - \frac{1 + \omega_{1,t} \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}} + \omega_{2,t} \frac{c_{2,t} - y_{2,t}}{\overline{Y}_2 - c_{2,t}}}{1 + \omega_{1,t+1} \frac{c_{1,t+1} - y_{1,t+1}}{\overline{Y}_1 - c_{1,t+1}} + \omega_{2,t+1} \frac{c_{2,t+1} - y_{2,t+1}}{\overline{Y}_2 - c_{2,t+1}}}$$

where $\omega_{1,t} \equiv (1-\alpha)\frac{c_1^{*\prime}(C_t^*)}{c_{1,t}^*}$ *and* $\omega_{2,t} \equiv \alpha \frac{c_2^{*\prime}(C_t^*)}{c_{2,t}^*}$.

Proof: See Appendix A.4. □

Consider the case with trade policy constrained by a FTA. Proposition 2 shows that the optimal capital-inflow tax at the CO limit, θ_t^{FTA}, depends on the growth of a weighted average of excess demand for both goods, with weights $\omega_{i,t}$, which depend on the foreign export-supply elasticities. In general, if households are borrowing more at time t than at $t+1$, $\theta_t^{FTA} < 0$ indicating an inflow tax (or a tax on borrowing at t). Tariffs are constrained to zero.

Absent a FTA, the optimal tariff τ_t^{nFTA} depends on the relative excess demand across goods (i.e., $c_{2,t} - y_{2,t}$ relative to $c_{1,t} - y_{1,t}$) weighted by the inverse elasticity of foreign export supply for each good, given by $(\overline{Y}_i - c_{i,t})^{-1}$. The lower the consumption of good i by Foreign households, the lower the price elasticity and the larger the price change when Home purchases a unit of good i. For example, if there is higher excess demand for good 2, given elasticities, a positive tariff is levied on good 2 so that households consume less of the relatively expensive good.

The corresponding optimal capital-inflow tax θ_t^{nFTA} depends on the growth rate of excess demand for good 1 only (i.e., $c_{1,t} - y_{1,t}$ relative to $c_{1,t+1} - y_{1,t+1}$) weighted by the inverse of the corresponding partial elasticity of foreign export supply. This result relies on the fact that taxes are levied on good 2 only,[4] and reflects the separation of allocations implied by Lemma 2. If there is higher excess demand for good 1 at time t, a capital-inflow tax is levied $\theta_t^{nFTA} < 0$. A special case for the interaction between the optimal instruments arises in response to variation in the endowment of good 2.

Corollary 1 (Trade Policy as a Substitute for Capital Controls) *In the limit as $\sigma \to 1$ and $\phi \to 1$, when $y_{2,t} < y_{2,t+1}$ ($y_{2,t} \geq y_{2,t+1}$), the Home unilateral planner will tax (subsidize) capital inflows $\theta_t^{FTA} \leq 0$ ($\theta_t^{FTA} \geq 0$) under a FTA, as defined by (8). Absent a FTA, the capital-inflow tax is zero, $\theta_t^{nFTA} = 0$, and a tariff τ^{nFTA} is used instead.*

Proof: Follows from Lemma 2 and Proposition 2. □

Through the lens of our decomposition (12), this special case arises because the RER wedge moves to perfectly offset changes in the consumption wedge, so trade policy is a perfect substitute for capital controls. While optimal capital controls will be non-zero under a FTA, they will be zero when this is relaxed—replaced instead with an optimal time-varying tariff. Our numerical simulations in Section 4.5 verify that trade policy can substitute for the use of capital controls for a wider range of $\{\sigma, \phi\}$—but only in response to good-2 endowment fluctuations, and only absent retaliation from abroad.

4.4 Optimal Instruments in the General Case

Next, we generalize the optimal instrument formulas to arbitrary preferences.

Proposition 3 (Optimal Instruments) *The optimal instruments for the Home unilateral Ramsey planner without constraints on trade policy are given by:*

$$\theta_t^{nFTA} = 1 - \frac{1 + (\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_{1,t}^G) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 + (\boldsymbol{\epsilon}_{t+1}^T + \boldsymbol{\epsilon}_{1,t+1}^G) \cdot [\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]}, \qquad \tau_t^{nFTA} = \frac{1 + (\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_{2,t}^G) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 + (\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_{1,t}^G) \cdot [\mathbf{c}_t - \mathbf{y}_t]} - 1$$

[4] If taxes were levied on good 1, then capital-inflow taxes would depend only on good-2 excess demand.

where $\boldsymbol{\epsilon}_t^T = -\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*)$ *and* $\boldsymbol{\epsilon}_{1,t}^G = -\frac{1}{g_{1,t}^*}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}^*}$, $\boldsymbol{\epsilon}_{2,t}^G = -\frac{1}{g_{2,t}^*}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}^*}$ *are inverse inter- and intra-temporal price elasticities. Instead, when constrained by the FTA, defined by* (8)*:*

$$\theta_t^{FTA} = 1 - \frac{1 + (\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_t^G)\cdot[\mathbf{c}_t - \mathbf{y}_t]}{1 + (\boldsymbol{\epsilon}_{t+1}^T + \boldsymbol{\epsilon}_{t+1}^G)\cdot[\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]},$$

where $\boldsymbol{\epsilon}_t^G = -\frac{\mathrm{d}\nabla g^*(\mathbf{c}_t^*)}{\mathrm{d}C_t^*}$.

Proof: See Appendix A.5. □

Once again, consider the incentives to tax capital inflows when trade policy is constrained by a FTA. Taking elasticities as given, the planner taxes inflows ($\theta_t < 0$) if households are borrowing relatively more at time t than $t+1$ (i.e., $c_t - y_t > c_{t+1} - y_{t+1}$), as before. More generally, however, the planner will tax more heavily commodities that are inelastically supplied. The inverse foreign export-supply elasticity is given by $(\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_t^G)$ and consists of an inter-temporal term relating to the foreign elasticity of inter-temporal substitution and an intra-temporal term relating to substitution between goods varieties. Inter-temporally, the capital-inflow tax at time t is higher if either $\boldsymbol{\epsilon}_t^T$ or $\boldsymbol{\epsilon}_t^G$ is high (elasticity is low) relative to their $t+1$ values.

Absent a FTA, the optimal tariff will be higher if the intra-temporal elasticity of supply for good 2 is lower than that for good 1 ($\boldsymbol{\epsilon}_{2,t}^G > \boldsymbol{\epsilon}_{1,t}^G$). The expression for the optimal capital-inflow tax now has an important difference relative to the constrained case. With optimal tariffs in play, only the partial elasticity of supply for the untaxed good 1 ($\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_{1,t}^G$) matters instead of the total elasticity, although deficits in both goods are relevant. This difference drives the interaction between policy instruments, which we investigate numerically in the next section.

4.5 Model Simulation

To illustrate our findings, we describe two simulation scenarios which focus on fluctuations in the good-1 and good-2 endowments in turn, explaining the macroeconomic dynamics and implementation of the optimal allocations that result from each. We focus on a deterministic setting, specifying initial and terminal values for endowments, and constructing the full sequence by assuming they follow a first-order autoregressive process:

$$y_{i,t+1}^{(*)} = \left(1 - \rho_i^{(*)}\right)\overline{y}_i^{(*)} + \rho_i^{(*)} y_{i,t}^{(*)}, \quad \forall t > 0 \text{ and } i = 1,2,$$
$$\mathbf{y}_0 = [y_{1,0}\ y_{2,0}]', \quad \mathbf{y}_0^* = \left[y_{1,0}^*\ y_{2,0}^*\right]'$$

For simplicity we assume $\rho_1 = \rho_2 = \rho_1^* = \rho_2^*$. In both scenarios, we assume a constant aggregate endowment ($Y_{1,t} = \overline{Y}_1$ and $Y_{2,t} = \overline{Y}_2$ for all t). This is a useful benchmark in which households are perfectly able to smooth their consumption smoothing in the decentralized allocation.

Based on CRRA utility and the Armington (1969) aggregator for consumption, the model calibration is detailed in Table 1. In each scenario, we compare the decentralized allocation, the unilateral Ramsey planning allocation in which trade policy is constrained by the FTA (8), and one in which trade policy is set optimally. To focus on the dynamic implications of the three variants in a consistent manner, we equalize the long-run equilibrium of each model by using a constant tariff for the Home country in the decentralized case and when a FTA (8) is in place—an approach that follows the New-Keynesian literature studying allocations where the steady state is first best (or constrained first best).

Table 1: Benchmark Model Calibration

Parameter	**Value**	**Description**
β	0.96	Discount factor, annual frequency
σ	2	Coefficient of relative of risk aversion
ϕ	1.5	Elasticity of substitution between goods 1 and 2
α	0.6	Share of good 1 (good 2) in Home (Foreign) consumption basket
ρ	0.8	Persistence of endowments

4.5.1 Scenario 1: Temporarily Low Endowment of Domestic Good

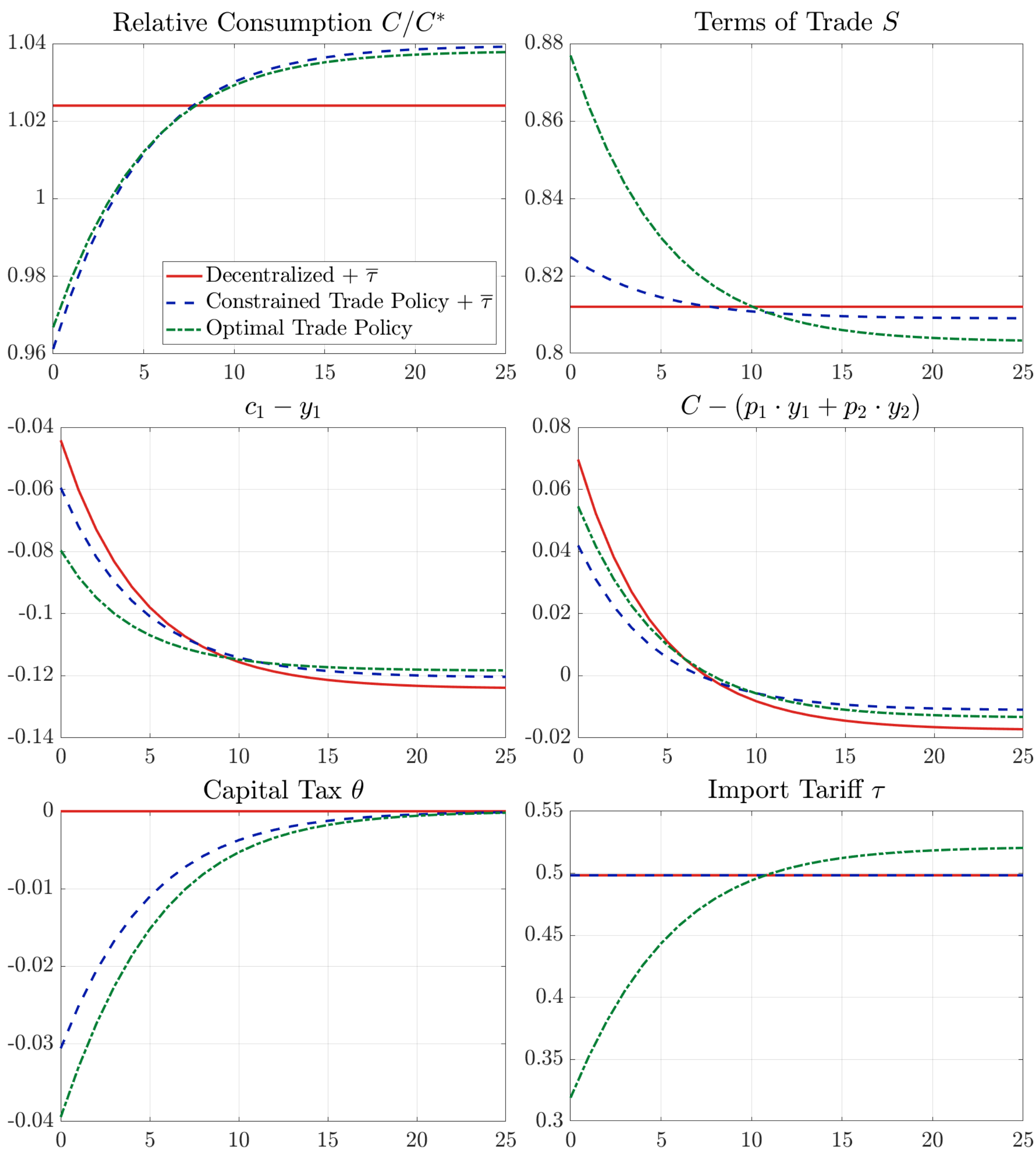
Consider a scenario in which the Home economy is recovering from a domestic downturn, or is growing more quickly than its Foreign counterpart. Specifically, the Home country's endowment of good 1 is low in the near term, and grows towards its long-run level. Denoting initial endowment values by $y_{i,0}^{(*)}$ and long-run levels by $\overline{y}_i^{(*)}$ for $i = 1, 2$, we assume that $y_{1,0} = 0.9\overline{y}_1$ and $y_{2,0} = \overline{y}_2$. To ensure there is no aggregate uncertainty: $y_{1,0}^* = 1 - y_{1,0}$ and $y_{2,0}^* = 1 - y_{2,0}$.

Faced with a higher stream of endowments in the future, Home households borrow to perfectly smooth consumption in the decentralized allocation—as demonstrated by the flat line in the top-left panel of Figure 3. However, households over-borrow relative to the planning allocations, illustrating that the results from Lemma 2 apply more generally. The planner chooses a lower level of near-term aggregate consumption C, but delivers higher consumption in the long run by allocating consumption to periods when it is relatively cheaper. The optimal policy, regardless of constraints on trade policy, involves leaning against capital flows to delay consumption, as illustrated by the evolution of the balance of payments (middle right panel in Figure 3). When trade policy is constrained by a FTA, the required capital-inflow tax is around 3% in the near term and approaches zero as the endowment returns to its long-run level.

Because the Home endowment of good 1—the good consumed with home bias domestically—is initially lower, the planner has an additional incentive to restrict the excess demand for the relatively expensive good 1. As a result, when the good-1 endowment deviates from its long-run level, the planner's inter- (pertaining to the cost of borrowing) and intra-temporal (pertaining to relative goods prices) incentives to manipulate the terms of trade are aligned. The planner chooses to both delay aggregate consumption *and* consumption of good 1, in expectation that the future price of C and c_1 will fall (middle-left panel in Figure 3).

When trade policy is unconstrained, the planner can additionally restrict the excess demand for good 1 by lowering the tariff on good 2 in the near term, but tariffs have second-best effects on the terms of trade (and real exchange rate). The planner sets a rising path for tariffs over time—starting at around 30% (a subsidy relative to the constant tariff under a FTA) but

Figure 3: Time Profile of Optimal Allocations as the Home Endowment of Good 1 Rises in Scenario 1



Notes: Time profile for allocations in Scenario 1, simulated for 100 periods. See Table 1 for calibration details. "Constrained (Optimal) Trade Policy" refers to allocation arising from a Home planner acting unilaterally with (without) a trade-policy constraint from a FTA. The "Decentralized" and Constrained Trade Policy allocations include a constant tariff $\overline{\tau}$ to ensure that long-run allocations replicate the unconstrained case.

increasing to over 50% in the long term. This leads to a near-term depreciation of the terms of trade (upper right panel in Figure 3). Since this implies that consumption is relatively cheap for Home households, all else equal, this would lead to further over-borrowing, consistent with the Harberger-Laursen-Metzler effect. As a consequence, the capital-inflow tax is roughly one-third larger absent a FTA, at 4% in the near term—a difference that is even larger absent the constant tariff in the free-trade case.

Figure S1 in Supplementary Materials S.2.2 details the decomposition of the optimal inflow tax according to (12). Since inter- and intra-temporal incentives are aligned, the consumption and RER wedges move in the same direction. While capital controls are predominantly driven by the consumption wedge, the difference across regimes is driven by differences in the RER wedge.

4.5.2 Scenario 2: Temporarily Low Endowment of Foreign Good

Next, consider the case in which the Home endowment of the foreign good (good 2) starts at a low value relative to its long-run level. We assume that $y^*_{2,0} = 1.1\overline{y}^*_2$ and $y^*_{1,0} = \overline{y}^*_1$.

As in scenario 1, households borrow in the near term, anticipating their endowment will increase in the future. However, the net supply of good 1 that Home sells abroad rises, because c_1 falls while y_1 is unchanged. The planner wants to delay aggregate consumption C inter-temporally, but has an incentive to act monopolisticaly and drive up the price of good 1 (intra-temporally), middle-left panel in Figure 4. When policy is constrained by a FTA, the planner levies a capital-inflow tax in the near term, which implies disproportionately lower consumption of good 1, trading off inter- and intra-temporal incentives to manipulate the terms of trade.

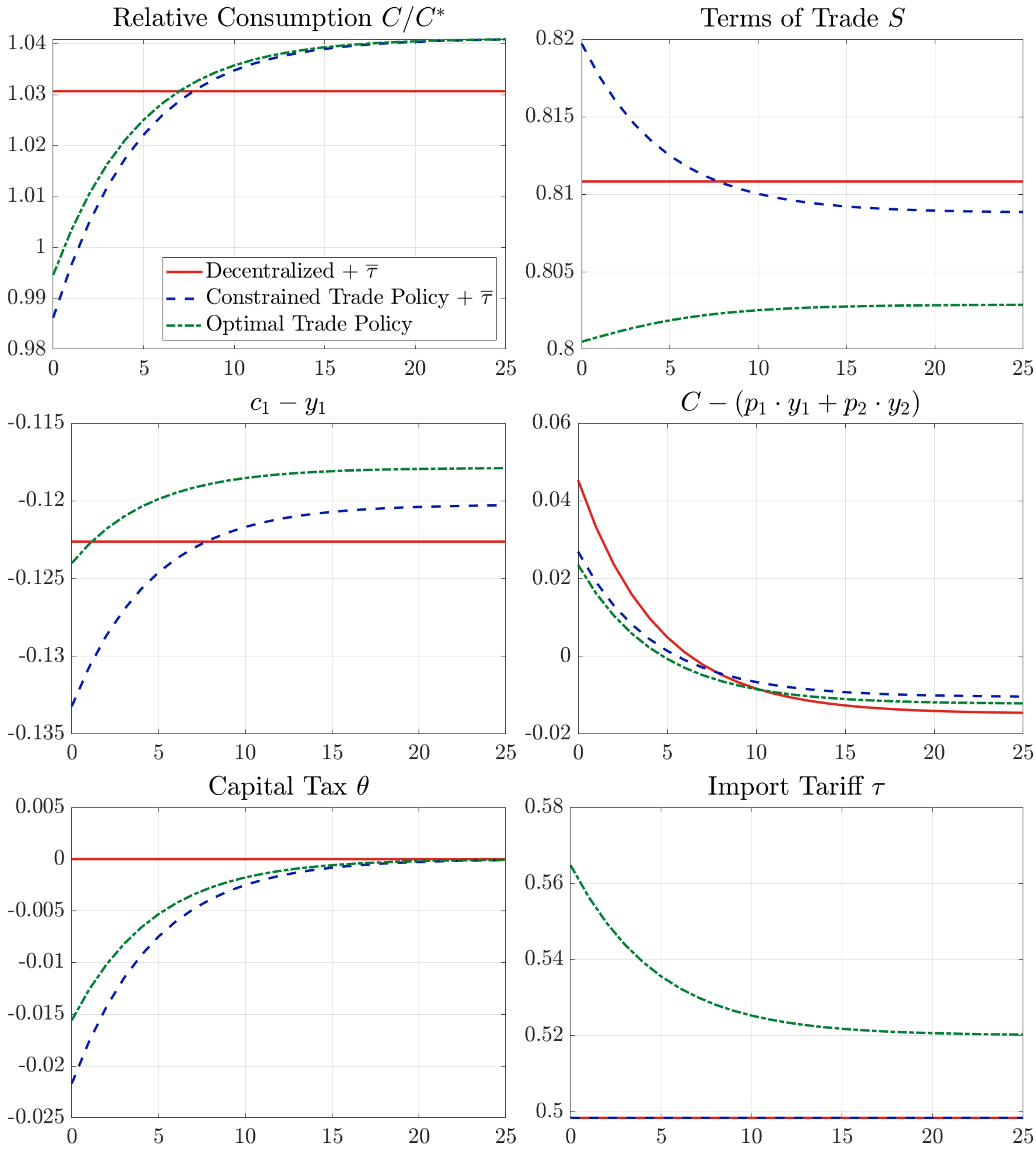
Absent free trade, the planner additionally levies a relatively high tariff in the near term to increase Home demand for good 1, c_1, and drive up its relative price. The declining path for tariffs, all else equal, implies that the terms of trade will depreciate over time (upper right panel in Figure 4)—making Home consumption relatively expensive in the near-term and discouraging borrowing. As a result, time-varying tariffs act as a substitute for the capital-inflow tax, which is lower in the no-FTA case at 1.5% as opposed to over 2% on impact. At the CO limit (Corollary 1), the capital-inflow tax is about 1% on impact with a FTA in place, but falls to zero when tariffs are optimally chosen.

Figure S2 in Supplementary Materials S.2.2 plots the corresponding decomposition (12). In contrast to scenario 1, absent a FTA, the RER wedge moves in the opposite direction to the consumption wedge. This reflects the fact that, generally, when inter- and intra-temporal incentives are misaligned, high near-term tariffs appreciate the real exchange rate disincentivizing consumption and trade policy can partly (or fully in the CO limit) substitute for capital-flow management.

4.5.3 Robustness

Before proceeding, we summarize some additional considerations pertaining to our simulations.

Figure 4: Time Profile of Optimal Allocations as the Foreign Endowment of Good 2 Falls in Scenario 2



Notes: Time profile for allocations in Scenario 2, simulated for 100 periods. See Table 1 for calibration details. "Constrained (Optimal) Trade Policy" refers to allocation arising from a Home planner acting unilaterally with (without) a trade-policy constraint from a FTA. The "Decentralized" and Constrained Trade Policy allocations include a constant tariff $\overline{\tau}$ to ensure that long-run allocations replicate the unconstrained case.

Comparative Statics. Two parameters influence the size of inter- and intra-temporal motives: the respective elasticities of substitution (see Proposition 3). We explore these comparative statics in Supplementary Materials S.2.3. When the elasticity of inter-temporal substitution $1/\sigma$ is low (i.e., σ is high), the planner levies larger capital controls in an attempt to reallocate consumption inter-temporally. When the intra-temporal (trade) elasticity ϕ is low, the planner sets larger tariffs. These factors influence the quantitative difference in optimal capital controls when trade policy is constrained vs. unconstrained. In our baseline simulation of scenario 1, in Figure 3, optimal capital controls are about one-third larger when trade policy is set optimally. When the intra-temporal elasticity is halved ($\phi = 0.75$), the increase in tariffs leads to an even larger increase in capital controls—by around 50%.

Commitment and Time Consistency. As in Lucas and Stokey (1983) and Costinot et al. (2014), the optimal policy is time consistent if the planner at time 0 has a full maturity structure of debt instruments available to them. The planner can structure debt in such a way so as to induce future planners to stick to the consumption plan. In Supplementary Materials S.2.4, we illustrate that this reasoning continues to hold with multiple goods and when trade policy is unconstrained.

Anticipated Changes in Endowments. The mechanisms that underpin optimal allocations in scenarios 1 and 2 also carry over to instances in which changes in endowments are anticipated. In these cases, optimal trade and financial policy involves preemptive action in advance of the shock itself—similar to Mendoza (2002) and Bianchi (2011). For instance, when a temporary fall in the good-1 endowment is anticipated, the Home planner will subsidize capital-inflows, only one period prior to the shock, to facilitate borrowing and help smooth consumption. Absent a FTA, the planner only employs tariffs to subsidize consumption of the relatively abundant good contemporaneously to the fall in endowment. We discuss this further in Supplementary Materials S.2.5.

Ruling out Capital Controls. We also consider a setting where the planner optimally chooses tariffs while capital controls are ruled out by a Free Financial Flows Agreement (FFFA). This case serves both as a useful benchmark to evaluate the welfare consequences of policy interventions, but also illustrates how tariffs can be used as a second-best instrument to manipulate the cost of borrowing over time. The key takeaway is that when inter- and intra-temporal motives are aligned (e.g. scenario 1), the variation in optimal tariffs is smaller under a FFFA –since they lead to inefficiently high borrowing– but larger when incentives are misaligned (e.g. scenario 2) since they correct existing borrowing inefficiencies. Full details in Supplementary Materials S.2.6.

4.6 Generality of Results

Next, we briefly discuss how our results apply to more general environments.

Production, Nominal Rigidities and Aggregate-Demand Externalities. We first consider a variation of the model where policy is driven by alternative *incentives*, specifically demand management. We extend the model to feature production of non-traded goods (denoted with subscript NT), endogenous labor supply and nominal-wage rigidities. Non-traded goods are produced with a linear production technology $y_{NT,t} = A_t L_t$ under perfect competition (where A_t denotes productivity and L_t labor). The associated firm maximization yields $p_{NT,t} = \frac{w_t}{A_t}$, where we assume the nominal wage is perfectly rigid $w_t = \overline{w}$, and $c_{NT,t} = y_{NT,t}$ in equilibrium. A full model exposition is presented in Supplementary Materials S.3.1.

In this setting, the marginal benefit to the unilateral Home planner from a unit of tradable consumption $c_{T,t}$ can be written as: $u'(C_t)g_{T,t}\left(1 + \frac{\omega}{1-\omega}\tau_t^L\right)$, where $g_{T,t}$ is the derivative of the aggregate consumption aggregator with respect to the tradable good, ω represents the expenditure share on non-tradable goods, and τ_t^L is the labor wedge, given by:

$$\tau_t^L = 1 + \frac{1}{A_t}\frac{v_{L,t}}{u'(C_t)g_{T,t}}$$

where $\nu_{L,t}$ represents the marginal disutility of labor supply for the household at time t. It is positive when the economy is demand constrained and households are involuntarily unemployed. The marginal benefit of a unit of tradable consumption is higher when the economy is demand constrained, generating an additional incentive for a planner to bring forward consumption.

Returning to the planner's problem, the implementability constraint (Lemma 1) is unchanged. Absent constraints on trade policy, the first-order conditions with respect to goods 1 and 2 are given by:

$$\begin{aligned} u'(C_t)g_{1,t}\left(1 + \frac{\omega}{1-\omega}\tau_t^L\right) &= \mu\mathcal{MC}_{1,t} \\ u'(C_t)g_{2,t}\left(1 + \frac{\omega}{1-\omega}\tau_t^L\right) &= \mu\mathcal{MC}_{2,t} \end{aligned}$$

Moreover, tariffs affect the path of the exchange rate for tradables in the same way as in the baseline setup. As such, faced with constrained demand and unemployment ($\tau_t^L > 0$), the planner brings consumption forward with an optimal mix of capital-inflow subsidy or an import subsidy which puts pressure on the exchange rate to depreciate, as in the baseline model.

Segmented Markets and Quantity Interventions. We also consider how our results generalize when alternative *instruments* are used to deliver the optimal allocation. Importantly, we show that a similar outcome can be achieved if the planner uses quantity interventions (e.g., open-market operations or FXI) in place of capital controls. Consider an extension of the model with non-traded goods and segmented financial markets, detailed in Supplementary Materials S.3.2. There is a single asset in each economy, denominated in units of the domestic non-traded good, which households trade with financial intermediaries—where positions are denoted by

a_{t+1} and a^I_{t+1}, respectively.[5] The intermediation problem implies one additional equilibrium condition:

$$\left[R^{*\ -1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t}\right] = \Gamma a^I_{t+1} \tag{13}$$

where $R^{(*)}_{NT,t}$ is the cost of borrowing in the Home (Foreign) and $\mathcal{E}_t = p^*_{NT,t}/p_{NT,t}$. The parameter Γ captures how binding limits to arbitrage are, with $\Gamma \to 0$ being the frictionless limit.

The planner can take a position a^G_{t+1} in domestic assets, financed by selling foreign assets, and market clearing requires: $a_{t+1} + a^G_{t+1} + a^I_{t+1} = 0$. Through this, the planner can affect intertemporal consumption smoothing via the balance sheet of financial intermediaries a^I_{t+1}. If $a^G_{t+1} = 0$, (13) indicates that when households are borrowing ($a_{t+1} < 0$) they face higher borrowing costs ($R_{NT,t+1}$ rises) because financiers facing limits to arbitrage must be compensated for taking the opposite position ($a^I_{t+1} > 0$). Planner intervention (e.g., in the form of FXI) can reduce the size of imbalances that need to be intermediated and so the spread narrows. Proposition C2 in Supplementary Materials S.3.2 formalizes that FXI can target a similar allocation to capital controls when markets are segmented.

Moreover, even as countries become small in financial markets (discussed next), the incentive for the planner to manipulate borrowing remains when there are intermediation frictions and profits are not fully rebated to Home households.

Country Size. Finally, we describe how our results generalize with respect to *country size*. Within our two-country model, countries are large in goods *and* financial markets. So planners internalize the effects of domestic allocations on both goods prices and the world real interest rate. Supplementary Materials S.3.3 details a small-open economy setting, with $N \to \infty$ foreign countries. While there are a range of outcomes in the small-open economy setting, an interesting knife-edge case arises when $\sigma = \phi = 1$. Here, the required size of capital controls for inter- and intra-temporal incentives is the same: in scenario 1, as $N \to \infty$, the optimal size of capital controls in both the FTA and no-FTA case is unchanged. Moreover, even though the optimal tariff falls, it is always non-zero since Home goods are scarce. Moving away from this limiting case, when $\sigma > \phi$, the size of capital controls will fall as N rises since the inter-temporal motive dominates, while the opposite is true for $\sigma < \phi$. In scenario 2, consistent with Corollary 1, the optimal capital-inflow tax is 0 absent an FTA.

5 Strategic Planning Allocation

We next consider an open-loop Nash equilibrium, where each planner chooses allocations taking the other's tax sequence $\{\theta^{(*)}_t, \tau^{(*)}_t\}$ as given, where τ^*_t denotes Foreign tariffs levied on good 1. Players cannot observe the actions of their opponents and therefore do not respond optimally

[5] Since the model is deterministic, the exact denomination does not affect the spanning properties of the asset. Without a non-traded good, trade in real bonds would imply interest-rate equalization by the law of one price.

to each others' change in strategy (see, e.g., Fudenberg and Levine, 1988). The strategic equilibrium counterpart to equations (6)-(7) indicate that the ratio of marginal costs from bringing forward a unit of consumption of each good, in each country, should be proportional to the bargaining power of each country.

To see this, let $\boldsymbol{\tau}_t^* \equiv [(1+\tau_t^*)^{-1} \; 1]'$ denote the vector of Foreign goods-specific tariffs. The Home planning problem, accounting for the optimal response by the Foreign planner, is then:

$$\max_{\{\mathbf{c}_t\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Nash-nFTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right] \beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \leq 0 \tag{IC-Nash-nFTA}$$

$$C_t \equiv g(\mathbf{c}_t) \tag{nFTA}$$

which differ from to the unilateral problem (P-Unil-nFTA) by the additional terms in the implementability constraint reflecting the Foreign capital-flow tax θ_t^* and tariff τ_t^*.

5.1 Optimal Strategic Allocation

Problem (P-Nash-nFTA) yields the optimality conditions:

$$u'(C_t)g_1(\mathbf{c}_t) = \mu \hat{\mathcal{MC}}_{1,t}^{nFTA} \tag{14}$$

$$u'(C_t)g_2(\mathbf{c}_t) = \mu \hat{\mathcal{MC}}_{2,t}^{nFTA} \tag{15}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}
\hat{\mathcal{MC}}_{1,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)(1+\tau_t^*)^{-1}g_1^*(\mathbf{c}_t^*) + u^{*\prime\prime}(C_t^*)g_1^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
\hat{\mathcal{MC}}_{2,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t^*) + u^{*\prime\prime}(C_t^*)g_2^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\
& + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1} \cdot \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]
\end{aligned}$$

The Foreign planner undertakes an analogous maximization. Combining the optimality conditions of the Home and Foreign planners yields the equilibrium allocation, summarized in the following proposition.

Proposition 4 (Capital Controls and Tariff Wars) *In a Nash equilibrium where each country chooses optimal capital controls $\{\theta_t, \theta_t^*\}$ and tariffs $\{\tau_t, \tau_t^*\}$ for all $t \geq 0$, the allocations $\{\mathbf{c}_t, \mathbf{c}_t^*\}$ satisfy:*

$$\frac{\hat{\mathcal{MC}}_{1,t}^{nFTA}}{\hat{\mathcal{MC}}_{1,t}^{*nFTA}} = \alpha_{1,0}^{nFTA} \qquad \frac{\hat{\mathcal{MC}}_{2,t}^{nFTA}}{\hat{\mathcal{MC}}_{2,t}^{*nFTA}} = \alpha_{2,0}^{nFTA} \tag{16}$$

where

$$\alpha_{i,0}^{nFTA} \equiv \frac{\hat{\mathcal{MC}}_{i,0}^{nFTA}}{\hat{\mathcal{MC}}_{i,0}^{*nFTA}} \quad \textit{for } i = 1, 2$$

Proof: See Appendix A.6. □

The ratio of the marginal cost of a unit of consumption for the planner across countries, for each good variety, is equal to a constant. The constants $\{\alpha_{i,0}^{nFTA}\}$ reflect the bargaining power of the Foreign country relative to the Home with respect to each good and depend on initial conditions. The interpretation of the marginal cost terms is consistent with that in Section 3. Similar arguments to Lemma 2 can be used to evaluate over-borrowing externality in this environment, but to streamline our discussion we focus on a numerical investigation next.

5.2 Numerical Simulations

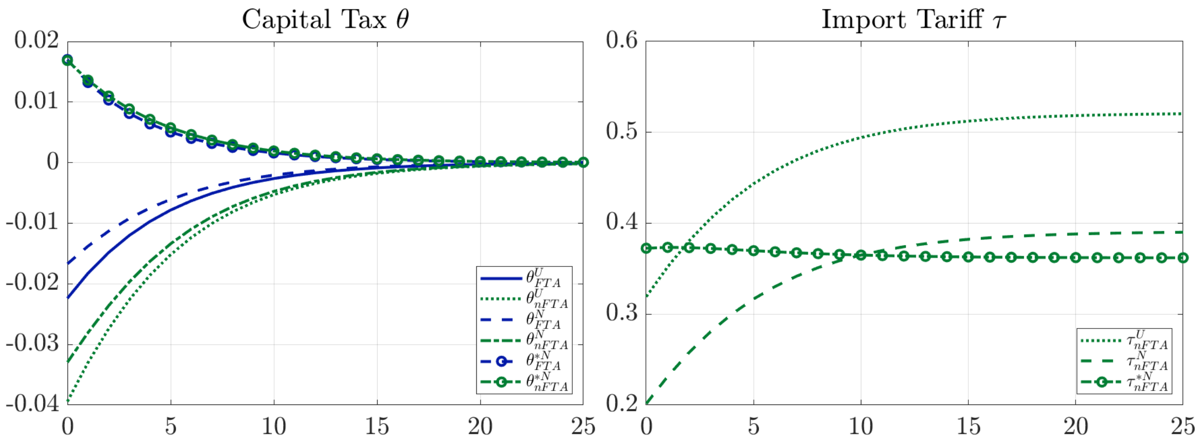
We revisit the two scenarios from Section 4.5 to assess how strategic interactions affect macroeconomic allocations and policy outcomes. The headline result is that capital controls are smaller in both scenarios when trade policy is constrained by a FTA.

Scenario 1. Figure 5 presents the optimal capital-inflow taxes and tariffs for scenario 1, comparing the cases where trade policy is constrained (blue) and unconstrained (green) in the strategic (dashed) and unilateral (solid) settings. As in the unilateral case, Home households over-borrow in the decentralized setting. In the strategic setting, the Home planner will delay consumption using a capital-inflow tax, while the Foreign planner brings forward consumption with a capital-inflow subsidy, regardless of trade policy constraints. The required capital-inflow tax set by the Home planner in the strategic setting is smaller than that in the unilateral case, since Foreign policy is helps to tilt near-term consumption to Foreign households.

The Home planner still has an incentive to delay consumption of good 1, so—unconstrained by a FTA—sets an increasing path for tariffs today. However, while the Home planner's inter- and intra-temporal incentives are aligned, they are opposed for the Foreign planner, so Foreign tariffs decline somewhat over time. Overall, since the Home tariff varies more, trade policy implies a relative depreciation of the terms of trade. So, as in the with the unilateral case, the capital-inflow tax is larger absent a FTA, consistent with the real-world findings in Figure 1, which suggests that joining the WTO supported reductions in capital controls.

Scenario 2. Figure 6 presents the corresponding figures for scenario 2. Here, strategic interactions make a difference in the outcome. In the Foreign country, where good 2 is relatively abundant in the near term, the planner seeks to increase the price of good 2 and does so by setting a declining path for tariffs on good 1. Because the Foreign country is large in the market for good 2, this effect dominates the Home planner's incentive to manipulate relative prices. Consequently, the Home planner faces a real exchange rate depreciation (relative to the constrained trade-policy case) which encourages Home households to borrow further—requiring a larger capital-inflow tax—changing the result relative to the unilateral case.

Figure 5: Optimal Capital-Inflow Taxes and Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 1



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home, solid lines). 'N' denotes Nash outcome for Home (dashed lines) and Foreign (dashed lines with circle markers).

Comparative Statics. As in the unilateral case, the size of capital controls and tariffs depend on the values of the inter- and intra-temporal elasticities of substitution. Lower values for the elasticity of inter-temporal substitution $1/\sigma$ are associated with capital-control wars, while lower trade elasticities ϕ are associated with tariff wars, discussed further in Supplementary Materials S.4.4.

6 Welfare and Policy Games

Finally, we analyze the welfare costs of capital controls and trade tariffs and show that capital-control wars are less likely to emerge when a FTA (8) is in place.

6.1 Global-Cooperation Benchmark

As a starting point, consider the world planning problem maximizing joint (world) welfare:

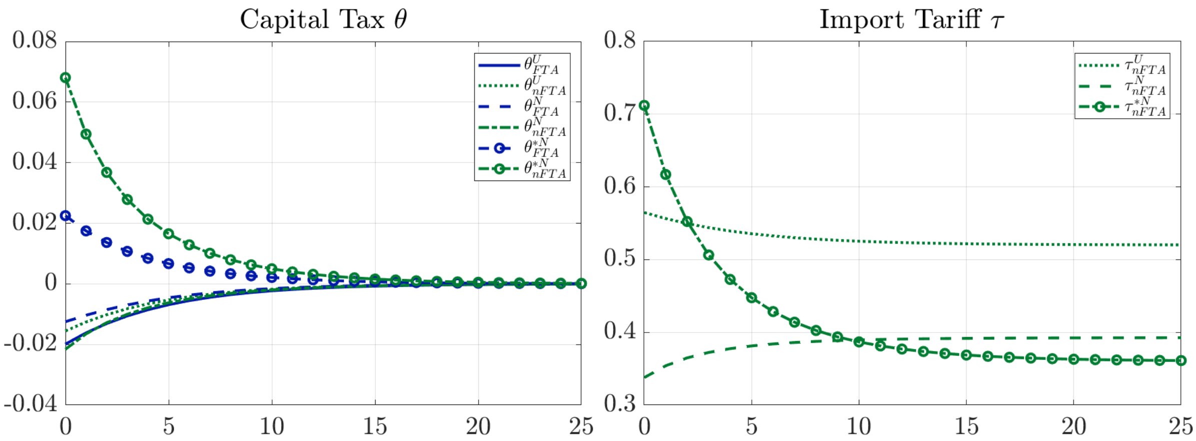
$$\max_{\{\mathbf{c}_t, \mathbf{c}^*_t\}} \quad \sum_{t=0}^{\infty} \beta^t \Big[u(g(\mathbf{c}_t)) + \kappa u(g^*(\mathbf{c}^*_t)) \Big] \tag{P-Coop}$$

$$\text{s.t.} \quad \mathbf{c}_t + \mathbf{c}^*_t = \mathbf{Y}_t \tag{RC}$$

$$\mathbf{c} = \mathbf{c}(C), \quad \mathbf{c}^* = \mathbf{c}^*(C) \tag{FTA}$$

where κ is the relative weight attributed to Foreign welfare. By the First Welfare Theorem, since there are no frictions in the global economy, the cooperative first-best allocation coincides with the laissez-faire equilibrium (no intervention). Therefore, relaxing constraints on trade policy will have no impact on the global planning allocation—as the following Proposition summarizes.

Figure 6: Optimal Capital-Inflow Taxes and Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 2



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home, solid lines). 'N' denotes Nash outcome for Home (dashed lines) and Foreign (dashed lines with circle markers).

Proposition 5 (Global Cooperation Allocation) *In the cooperative allocation resulting from* (P-Coop)*, there exists* κ *such that no capital controls are optimal,* $\theta_t = \theta_t^* = 0$*. Relaxing the FTA, as defined in* (8)*, will have no effect on this allocation since* $\tau_t = \tau_t^* = 0$*.*

Proof: See Appendix A.7. □

The weight κ which ensures no intervention is optimal is given by the ratio of Lagrange multipliers on households' budget constraints, determined by their initial assets, and is equal to 1 in the symmetric case. Since the cooperative outcome is first best, Corollary 2 follows.

Corollary 2 (Negative Spillovers) *Any policy intervention which improves welfare in one country necessarily reduces global welfare by disproportionately worsening welfare in the other.*

Proof: Follows directly from Proposition 5. □

To analyze the global welfare implications, we revisit scenarios 1 and 2 and consider the allocations arising in both the unilateral and strategic settings, with and without the FTA and the FFFA. We compare welfare by assessing consumption-equivalent variation relative to the global-cooperative allocation (i.e., one of no intervention) in Table S1 of Supplementary Materials S.4.5. Country-level and global welfare costs from policy wars are disproportionately larger when countries depart from the FTA since introducing distortions along the intra-temporal margin will exacerbate over-/under-borrowing through the impact of tariffs on the real exchange rate.

6.2 Trade-Policy Constraints and Prospects for Capital-Control Wars

But can commitment to a trade policy constraint, like a FTA, discourage costly capital-control wars in the first place? To answer this question, we consider a dynamic setting in which country planners begin in an equilibrium without capital controls (FFFA), either with a trade-policy constraint (FTA) in place or with optimal tariffs instead. We assess the incentive for the Home planner to deviate from the FFFA and levy capital controls.

To do this we assume that the Foreign planner initially sets no tariffs in the FTA case and the optimal tariff absent a FTA. They assume that the Home planner will adopt the same trade policy (i.e., either none or the optimal) and will never levy capital controls. The Home planner deviates and sets capital-flow taxes (and tariffs in the unconstrained case) assuming the Foreign planner remains passive. However, we allow the Foreign planner to retaliate after $\bar{t}$ periods by re-optimizing and choosing capital controls (and tariffs in the no FTA case)—a 'Grim Trigger' strategy (see, e.g., Friedman, 1971). For our experiments, we use $\bar{t} = 5$, but this parameter does not have important implications for the economics. At this stage, the game is the same as in the strategic allocation: planners choose mutual best responses.

We present the paths for instruments and consumption, in both the constrained and unconstrained cases for both scenarios in Appendix S.4.6. In all cases, the Home planner attains a higher consumption level in the first $\bar{t}$ periods, which comes at the cost of Foreign consumption. After $\bar{t}$ periods, allocations coincide with the Nash outcome. Using these simulations, we calculate the consumption-equivalent welfare gains for the Home and Foreign country to contrast how the incentive to levy capital controls varies with trade policy constraints. Table S2 demonstrates that a (credible) commitment to a FTA (8) reduces the incentive for a country to deviate and levy capital controls. In scenario 1, levying capital controls increases Home welfare by 0.134% with a FTA in place compared to 0.188% without one; in scenario 2, the welfare gains are over three-times larger without a trade-policy constraint. Moreover, the costs for countries that do not deviate are significantly larger when trade policy is unconstrained. In scenario 1, Foreign losses are over 50% larger without a constraint; in scenario 2, the losses are over three-times larger. Intuitively, if the initial equilibrium is one with competition over tariffs, countries face a distorted path for aggregate consumption and, therefore, the welfare gains from levying capital controls are larger.

7 Conclusion

In this paper, we provide a unified framework for the analysis of capital-flow management and trade policy. We show that introducing tariffs distorts the cost of borrowing over time which, in turn, gives rise to a novel motive for managing capital flows. When tariffs are optimally chosen without retaliation from abroad, whether optimal capital controls are larger or smaller depends on whether the inter- and intra-temporal incentives to manipulate the terms of trade are aligned or misaligned. From a technical perspective, we show this is because, when tariffs are optimally chosen, optimal capital controls depend only on the partial elasticity of foreign export supply, with respect to the domestic (untaxed) good, as opposed to the total elasticity which matters when trade policy is ruled out by a FTA.

Allowing the Foreign planner to retaliate, our simulations suggest that capital controls are always smaller under a FTA. This chimes with the experience of G10 economies since countries' accession to the WTO from 1995 onwards. Finally, we conduct a policy experiment and show that commitment to a FTA can reduce incentives to levy capital controls. Since capital-control wars are costly for global welfare, our analysis highlights a novel argument in favor of FTAs: retaining openness in trade can help to sustain financial openness.

An important step for future research is to investigate the role of uncertainty and incomplete markets. While policy cannot improve upon the cooperative allocation absent additional frictions when financial markets are complete, this is not the case with financial-market incompleteness. Additionally, the interaction between capital-flow taxes and tariffs will then depend on the currency denomination of debt which can alter the balance between inter- and intra-temporal incentives facing the planner due to the desire to the inflate away debt obligations.

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Appendix

A.1 Proof to Lemma 1

Foreign households maximize their discounted lifetime utility subject to their inter-temporal budget constraint, given world prices $\mathbf{p}_t$:

$$\max_{\{\mathbf{c}_t\}} \quad U_0^* = \sum_{t=0}^{\infty} \beta^t u^*(g^*(\mathbf{c}_t)) \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0$$

The first-order condition is given by (3) where λ^* is the Lagrange multiplier on the Foreign inter-temporal budget constraint (4). Lemma 1 follows by substituting (3) into (1).

A.2 Proof to Proposition 1

Taking a total derivative of $\frac{d\mathcal{L}}{dC}$ implies: $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = \frac{\partial \mathcal{L}}{\partial c_1} c_1'(C) + \frac{\partial \mathcal{L}}{\partial c_2} c_2'(C)$. Substituting optimality conditions under the FTA into the left-hand side, and those from without an FTA into the right-hand side, verifies the equality.

Next, we prove that (P-Unil-FTA) is a constrained version of (P-Unil-nFTA). The solution to (P-Unil-FTA) satisfies $\frac{d\mathcal{L}}{dC} = 0$ at the (constrained) optimal allocation. Since $c_1'(C)$ and $c_2'(C)$ are positive and increasing functions detailed in Appendix S.1.2, generally $sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}) = -sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2})$ indicating an incentive to adjust consumption across varieties remains at the constrained-optimal allocation. The solution to (P-Unil-nFTA) given by (6) and (7) implies $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2} = 0$ which implies aggregate consumption is (unconstrained) optimal as well. Formally, denote:

$$\overline{C} = \{C : \max \quad \mathcal{L}(C) \mid c_1(C), c_2(C) \text{ on Pareto frontier}\}, \tag{A1}$$

where $\overline{C}$ is a scalar because $\mathcal{L}$ is strictly concave in the region of interest. Then note that $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_1(\overline{C}),c_2(\overline{C})}, \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. If, $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_1(\overline{C}),c_2(\overline{C})} > 0$, then $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} < 0$ and there exists an ϵ perturbation such that a $c_1(\overline{C}) \pm \epsilon, c_2(\overline{C}) \pm \epsilon$ are preferred. The same is true for $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}_{|c_1(\overline{C}),c_2(\overline{C})} < 0$, and $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}_{|c_1(\overline{C}),c_2(\overline{C})} > 0$. □

A.3 Proof to Lemma 2

(i) The decentralized allocation is given by:

$$u'(C_t) g_1(\mathbf{c}_t) = \mu u^{*\prime}(C_t^*) g_1^*(\mathbf{c}_t), \qquad u'(C_t) g_2(\mathbf{c}_t) = \mu u^{*\prime}(C_t^*) g_2^*(\mathbf{c}_t)$$

Evaluating the equations above as $\sigma \to 1$ and $\phi \to 1$ yields: $\frac{\alpha}{c_{1,t}} = \mu \frac{1-\alpha}{c^*_{1,t}}$ and $\frac{\alpha}{c_{2,t}} = \mu \frac{1-\alpha}{c^*_{2,t}}$, which can be rewritten using market clearing as: $\frac{\alpha}{c_{1,t}} = \mu \frac{1-\alpha}{Y_{1,t}-c_{1,t}}$ and $\frac{\alpha}{c_{2,t}} = \mu \frac{1-\alpha}{Y_{2,t}-c_{2,t}}$. Keeping total world endowment of each good constant ($Y_{i,t} = \overline{Y_i}$ for $i = 1, 2$, for all t), consumption of each good variety is constant at the decentralized allocation.

(ii) Evaluate equations (6) and (7) as $\sigma \to 1$ and $\phi \to 1$. In this case, (6) can be written as:

$$\begin{aligned}\frac{\alpha}{c_{1,t}} =& \mu(1-\alpha)\frac{1}{c^*_{1,t}}\left\{1 + \left[(1-\alpha)\frac{1}{c^*_{1,t}}(c_{1,t}-y_{1,t}) + \alpha\frac{1}{c^*_{2,t}}(c_{2,t}-y_{2,t})\right]\right. \\ &\left. + \left[\left(\frac{1}{c^*_{1,t}} - (1-\alpha)\frac{1}{c^*_{1,t}}\right)(c_{1,t}-y_{1,t}) - \alpha\frac{1}{c^*_{2,t}}(c_{2,t}-y_{2,t})\right]\right\}\end{aligned}$$

The inter- and intra-temporal partial price adjustments on good 1 partly cancel out, but the inter- and intra-temporal price adjustments on good 2 fully cancel out. This then simplifies to:

$$\frac{\alpha}{c_{1,t}}\left(\mu\frac{1-\alpha}{\overline{Y}_1 - c_{1,t}}\right)^{-1} = 1 + \frac{c_{1,t}-y_{1,t}}{\overline{Y}_1 - c_{1,t}},$$

Analogously, (7) simplifies to :

$$\frac{1-\alpha}{c_{2,t}}\left(\mu\frac{\alpha}{\overline{Y}_2 - c_{2,t}}\right)^{-1} = 1 + \frac{c_{2,t}-y_{2,t}}{\overline{Y}_2 - c_{2,t}}$$

The left-hand side of these last two expressions are decreasing in $c_{1,t}$ and $c_{2,t}$, respectively.

Similarly, the right-hand side is increasing in $c_{i,t}$ and decreasing in $y_{i,t}$ (for $i = 1, 2$, respectively), but $y_{j,t}$ does not feature in the first-order condition for $c_{i,t}$ for $j \neq i$. From this it follows that $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{i,t}} > 0$ and $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{j,t}} = 0$, verifying the lemma.

(iii) Evaluate (S4) as $\sigma \to 1$ and $\phi \to 1$ and simplifying:

$$\begin{aligned}C_t^{-1} =& \mu\left[\frac{(1-\alpha)C^*_t}{c^*_{1,t}}c'_1(C_t) + \frac{\alpha C^*_t}{c^*_{2,t}}c'_2(C_t)\right]C_t^{*-1}\left\{1 - \right. \\ &\left.\left[\left(\frac{(1-\alpha)C^*_t}{c^*_{1,t}}c_1^{*\prime}(C^*_t)\right)\frac{c_{1,t}-y_{1,t}}{c^*_{1,t}} + \left(\frac{\alpha C^*_t}{c^*_{2,t}}c_2^{*\prime}(C^*_t)\right)\frac{c_{2,t}-y_{2,t}}{c^*_{2,t}}\right]\right\}\end{aligned}$$

As in Costinot et al. (2014), taking the total derivative of this with respect to $\mathbf{y}_t$, yields $\mathrm{d}C_t > 0$ if $\sum_i \frac{\mathrm{d}u'(C^*_t)\nabla g^*_{i,t}}{\mathrm{d}C_t}\mathrm{d}y_{i,t} > 0$. Evaluating this in the CO limit yields:

$$1 + \frac{P_t C_t}{P^*_t C^*_t} > \frac{2\alpha - 1}{\alpha} \tag{A2}$$

The right-hand side attains a maximum at $\alpha = 1$, so the inequality is trivially satisfied. $\square$

A.4 Proof to Proposition 2

This follows from the above after substituting for CRRA per-period utility and the Armington aggregator as $\sigma \to 1$ and $\phi \to 1$. □

A.5 Proof to Proposition 3

nFTA: Rearranging (6), combining with equation (10), and using $Q_t = \frac{g_{1,t}}{g^*_{1,t}}$, yields the optimal capital-inflow tax:

$$\theta_t^{nFTA} = 1 - \frac{1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*) + \frac{1}{g^*_{1,t}}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c^*_{1,t}}\right)\cdot[\mathbf{c}_t - \mathbf{y}_t]}{1 - \left(\frac{u^{*\prime\prime}(C_{t+1}^*)}{u^{*\prime}(C_{t+1}^*)}\nabla g^*(\mathbf{c}_{t+1}^*) + \frac{1}{g^*_{1,t+1}}\frac{\partial \nabla g^*(\mathbf{c}_{t+1}^*)}{\partial c^*_{1,t+1}}\right)\cdot[\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]}$$

Using Home and Foreign relative-demand expressions, the optimal tariff can then be expressed as $1 + \tau_t^{nFTA} = \frac{g_{2,t}/g_{1,t}}{g^*_{2.t}/g^*_{1,t}}$. Then, using equations (6) and (7):

$$\tau_t^{nFTA} = \frac{1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*) + \frac{1}{g^*_{2,t}}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c^*_{2,t}}\right)\cdot[\mathbf{c}_t - \mathbf{y}_t]}{1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*) + \frac{1}{g^*_{1,t}}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c^*_{1,t}}\right)\cdot[\mathbf{c}_t - \mathbf{y}_t]} - 1$$

FTA: Rearranging equation (S4) using $Q_t^{-1} = -\frac{\mathrm{d}C_t^*}{\mathrm{d}C_t} = \nabla g^*(\mathbf{c}_t^*)\mathbf{c}_t'(C_t) = -\nabla g^*(\mathbf{c}_t^*)\mathbf{c}_t^{*\prime}(C_t)$ yields:

$$\frac{u'(C_t)}{\mu u^{*\prime}(C_t^*)}Q_t = 1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*(C_t)) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_t^*(C_t))}{\mathrm{d}C_t^*}\right)\cdot[\mathbf{c}_t - \mathbf{y}_t]$$

Combining this with equation (10) yields:

$$\theta_t^{FTA} = 1 - \frac{1 - \left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*(C_t)) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_t^*(C_t))}{\mathrm{d}C_t^*}\right)\cdot[\mathbf{c}_t - \mathbf{y}_t]}{1 - \left(\frac{u^{*\prime\prime}(C_{t+1}^*)}{u^{*\prime}(C_{t+1}^*)}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1})) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1}))}{\mathrm{d}C_{t+1}^*}\right)\cdot[\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]}$$

Substituting the definitions for elasticities yields the expressions in Proposition 3. □

A.6 Proof to Proposition 4

We derive mutual best responses, for each good. Dividing (14) by its $t+1$ analogue yields:

$$\frac{C_t^{-\sigma}g_{1,t}}{C_{t+1}^{-\sigma}g_{1,t+1}} = \frac{1}{1-\theta_t^*}\frac{\hat{MC}_{1,t}}{\hat{MC}_{1,t+1}}$$

Introduce $1-\theta_t$ using the Home Euler (9) and substitute out $\frac{1}{1-\theta_t^*}$ using the Foreign Euler equation. This yields the expression for the optimal tax on capital flows levied by Home:

$$1-\theta_t = \frac{1+\sigma C_t^{*-1}\left[\ g^*_{1,t}(c_{1,t}-y_{1,t})+g^*_{2,t}(1-\tau^*_t)^{-1}(c_{2,t}-y_{2,t})\ \right]- \frac{1}{g^*_{1,t}}\left[\ g^*_{11,t}(c_{1,t}-y_{1,t})+g^*_{21,t}(1-\tau^*_t)^{-1}(c_{2,t}-y_{2,t})\ \right]}{1+\sigma C_{t+1}^{*-1}\left[\ g^*_{1,t+1}(c_{1,t+1}-y_{1,t+1})+g^*_{2,t+1}(1-\tau^*_{t+1})^{-1}(c_{2,t+1}-y_{2,t+1})\ \right]- \frac{1}{g^*_{1,t+1}}\left[\ g^*_{11,t+1}(c_{1,t+1}-y_{1,t+1})+g^*_{21,t+1}(1-\tau^*_{t+1})^{-1}(c_{2,t+1}-y_{2,t+1})\ \right]} \tag{A3}$$

Abroad, following the analogous steps as for (A3) yields the expression for the optimal tax on capital flows levied by the Foreign country $(1-\theta^*_t)$. Finally, combine (A3) and the expression for $1-\theta^*_t$ and substitute out τ_t and τ^*_t:

$$\frac{C_t^{*-\sigma}g^*_{1,t}+\sigma C_t^{*-\sigma-1}g^*_{1,t}\left[\ g^*_{1,t}(c_{1,t}-y_{1,t})+g^*_{1,t}S_t(c_{2,t}-y_{2,t})\ \right]- C_t^{*-\sigma}\left[\ g^*_{11,t}(c_{1,t}-y_{1,t})+g^*_{21,t}\frac{g^*_{1,t}}{g^*_{2,t}}S_t(c_{2,t}-y_{2,t})\ \right]}{C_t^{-\sigma}g_{1,t}+\sigma C_t^{-\sigma-1}g_{1,t}\left[\ g_{1,t}(c^*_{1,t}-y^*_{1,t})+g_{1,t}S_t(c^*_{2,t}-y^*_{2,t})\ \right]- C_t^{-\sigma}\left[\ g_{11,t}(c^*_{1,t}-y^*_{1,t})+g_{21,t}\frac{g_{1,t}}{g_{2,t}}S_t(c^*_{2,t}-y^*_{2,t})\ \right]}=\alpha_{1,0}$$

The constant $\alpha_{1,0}$ is given by the same equation evaluated at $t=0$. Analogous steps for the good-2 first-order conditions (Home and Foreign) yield the second equilibrium condition. □

A.7 Proof to Proposition 5

When trade policy is constrained by a FTA, the optimal cooperative allocation satisfies:

$$u'(g(\mathbf{c}_t))+\kappa u'(g(\mathbf{c}^*_t))\frac{\mathrm{d}C^*}{\mathrm{d}C}=0 \tag{A4}$$

where $\frac{\mathrm{d}C^*_t}{\mathrm{d}C_t}=-\frac{P_t}{P^*_t}$, yielding the decentralized condition (10) with $\kappa=\frac{u'(g(\mathbf{c}_{t-1}))}{u'(g(\mathbf{c}^*_{t-1}))}\frac{P^*_{t-1}}{P_{t-1}}$ implying $\theta_t=0$. Removing constraints on trade policy does not change the optimal allocation (since goods taxes are zero at the optimal). We get two first-order conditions,

$$u'(g(\mathbf{c}_t))g_1+\kappa u'(g(\mathbf{c}^*_t))g^*_1\frac{\mathrm{d}c^*_1}{\mathrm{d}c_1}=0,\quad u'(g(\mathbf{c}_t))g_2+\kappa u'(g(\mathbf{c}^*_t))g^*_2\frac{\mathrm{d}c^*_2}{\mathrm{d}c_2}=0 \tag{A5}$$

Note that $\frac{g_1}{g^*_1}=\frac{\mathrm{d}C}{\mathrm{d}c_1}\frac{\mathrm{d}c^*_1}{\mathrm{d}C^*}=\frac{\mathrm{d}C}{\mathrm{d}C^*}\frac{\mathrm{d}c^*_1}{\mathrm{d}c_1}=-\frac{\mathrm{d}C}{\mathrm{d}C^*}$, therefore both of the above conditions imply (A4), as in the FTA case. □

Supplementary Materials

S.1 Model Preliminaries

S.1.1 Derivatives of the Consumption Aggregator

In this appendix, we define the derivatives of the Armington (1969) aggregator which arise in the Ramsey-planning first-order conditions. We present the expressions for the representative Home consumer only, but they are analogous for the representative Foreign consumer. The first derivatives of the Home aggregator are given by:

$$
\begin{aligned}
g_1(\mathbf{c}_t) &\equiv \frac{\partial g(\mathbf{c}_t)}{\partial c_{1,t}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}} \\
g_2(\mathbf{c}_t) &= \frac{\partial g(\mathbf{c}_t)}{\partial c_{2,t}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}
\end{aligned}
$$

The second derivatives are:

$$
\begin{aligned}
g_{11}(\mathbf{c}_t) =& -\frac{1}{\phi}\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} \\
&+ \frac{1}{\phi}\alpha^{\frac{2}{\phi}} c_{1,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}} \\
g_{12}(\mathbf{c}_t) =& \frac{1}{\phi}\alpha^{\frac{1}{\phi}}(1-\alpha)^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}} \\
g_{21}(\mathbf{c}_t) =& g_{12}(\mathbf{c}_t) \\
g_{22}(\mathbf{c}_t) =& -\frac{1}{\phi}(1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} \\
&+ \frac{1}{\phi}(1-\alpha)^{\frac{2}{\phi}} c_{2,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}}
\end{aligned}
$$

S.1.2 Derivation of the Pareto Frontier

This appendix provides derivations for the Pareto frontier defined in Section 3.2. The Pareto frontier summarizes combinations of consumption allocations $\{c_{1,t}, c_{2,t}\}$ which are Pareto efficient, given a level of aggregate consumption C_t.

The Home representative household chooses their consumption by minimizing expenditure, for a given level of aggregate consumption $\overline{C}$: $\min_{c_{1,t},c_{2,t}} \quad p_{1,t}c_{1,t} + p_{2,t}c_{2,t} \quad \text{s.t.} \quad \overline{C} = g(\mathbf{c}_t)$. The first-order conditions for this problem yield the Home relative demand equation: $\frac{g_{1,t}}{g_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c_{2,t}}{c_{1,t}}\right)^{\frac{1}{\phi}}$, where $p_{1,t}/p_{2,t} \equiv 1/S_t$ and S_t refers to the terms of trade.

To derive the Pareto frontier, note that the analogous Foreign relative demand curve is

$\frac{g^*_{1,t}}{g^*_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c^*_{2,t}}{c^*_{1,t}}\right)^{\frac{1}{\phi}}$ and equate relative prices across countries to attain:

$$\frac{c^*_{2,t}}{c^*_{1,t}} = \left(\frac{\alpha}{1-\alpha}\right)^2 \frac{c_{2,t}}{c_{1,t}} \tag{S1}$$

This expression for optimal relative consumption must be consistent with goods market clearing ($Y_{i,t} = c_{i,t} + c^*_{i,t}$ for $i = 1, 2$). Combining (S1) with goods market clearing, we attain the following expressions for consumption:

$$c_{1,t} = \frac{b c_{2,t} Y_{1,t}}{Y_{2,t} - (1-b)c_{2,t}}, \qquad c_{2,t} = \frac{c_{1,t} Y_{2,t}}{bY_{1,t} + (1-b)c_{1,t}} \tag{S2}$$

where $b \equiv \left(\frac{\alpha}{1-\alpha}\right)^2$.

Solving for $\mathrm{d}c_i(C)/\mathrm{d}C$**.** Rearranging the Armington aggregator, we can show that:

$$c_{1,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}}{\alpha^{\frac{1}{\phi}}}\right]^{\frac{\phi}{\phi-1}}, \qquad c_{2,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}}}{(1-\alpha)^{\frac{1}{\phi}}}\right]^{\frac{\phi}{\phi-1}} \tag{S3}$$

Equating (S2) with (S3) yields:

$$\left[C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} (bY_{1,t} + (1-b)c_{1,t}(C_t)) = c_{1,t}(C_t) Y_{2,t} \,(1-\alpha)^{\frac{1}{\phi-1}}$$

Totally differentiating this expression and rearranging yields:

$$\frac{\mathrm{d}c_{1,t}(C_t)}{\mathrm{d}C_t} = \frac{C_t^{-\frac{1}{\phi}} (1-\alpha)^{-\frac{1}{\phi}} c_{2,t}^{\frac{1}{\phi}} (bY_{1,t} + (1-b)c_{1,t}(C_t)}{Y_{2,t} - c_{2,t}(C_t)(1-b) + \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{-\frac{1}{\phi}} (1-\alpha)^{-\frac{1}{\phi}} c_{2,t}^{\frac{1}{\phi}} (bY_{1,t} + (1-b)c_{1,t}C_t)}$$

The expression for $\mathrm{d}c_{2,t}(C_t)/\mathrm{d}C_t$ can be derived analogously.

S.1.3 Derivation of Price Indices

Repeating the expenditure minimization exercise in Appendix S.1.2 while allowing for tariffs:

$$\min_{c_{1,t}, c_{2,t}} p_{1,t} c_{1,t} + p_{2,t} c_{2,t}(1+\tau_t) \quad \text{s.t. } \overline{C} = g(\mathbf{c}_t)$$

yields the relative demand condition (11). Substituting this into total expenditure yields:

$$c_{1,t} = \frac{\hat{y}_t p_1^{-\phi} \alpha^{-1}}{\alpha p_{1,t}^{1-\phi} + (1-\alpha) p_{2,t}^{1-\phi} (1 + \tau_t^{1-\phi})}$$

$$c_{2,t} = \frac{\hat{y}_t p_2^{-\phi}(1-\alpha)^{-1}}{\alpha p_{1,t}^{1-\phi} + (1-\alpha)p_{2,t}^{1-\phi}(1+\tau_t^{1-\phi})}$$

where $\hat{y}_t$ denotes the sum of endowment income and lump-sum transfers to the household.

Finally, substituting this into the constraint of the minimization above and setting $\overline{C} = 1$ and replace $\overline{y}_t$ by P_t:

$$P_t = \left[\alpha p_{1,t}^{1-\phi} + (1-\alpha)p_{2,t}^{1-\phi}(1+\tau_t)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

Solving an analogous problem for the foreign country yields:

$$P_t^* = \left[(1-\alpha)p_{1,t}^{1-\phi}(1+\tau_t^*)^{1-\phi} + \alpha p_{2,t}^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

The real exchange rate is defined as the ratio P^*/P. This coincides with the ratio of CPI, rather than the PPI, and thus includes sales taxes.

S.2 Unilateral Planning Allocation

S.2.1 The Planning Problem with a FTA

The Home planner's problem when trade policy is constrained by a FTA coincides with that studied in Costinot et al. (2014). We reproduce it below:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$\mathbf{c}_t = \mathbf{c}(C_t), \quad \mathbf{c}_t^* = \mathbf{c}^*(C_t) \tag{FTA}$$

where the third line (FTA) summarizes the Pareto frontier constraint imposed by the presence of a FTA. After substituting (FTA) into (IC), we assume that $\boldsymbol{\rho}(C_t) \cdot [\mathbf{c}(C_t) - \mathbf{y}_t]$ is a strictly convex function of C_t to guarantee a unique solution to (P-Unil-FTA).

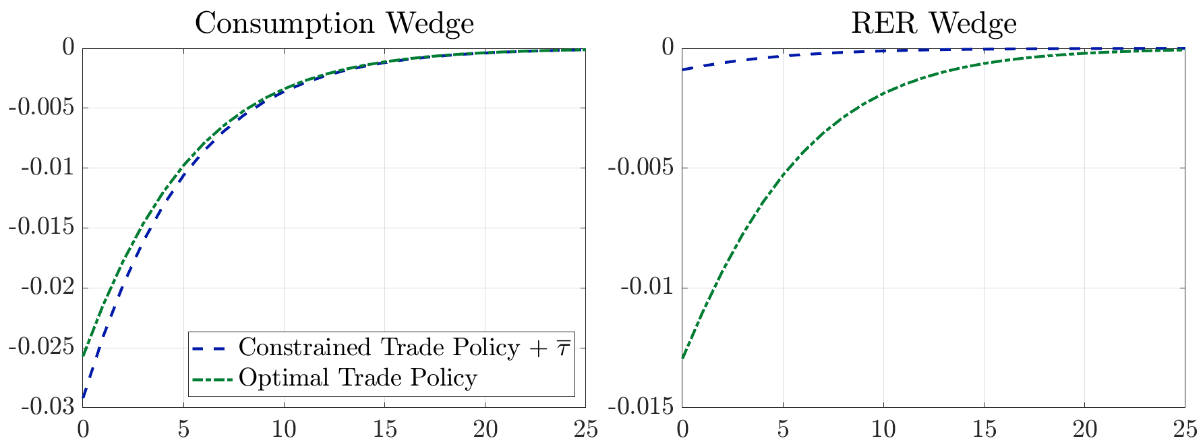
Optimal Allocation. Since utility is time-separable, the first-order condition is given by:

$$u'(C_t) = \mu \mathcal{MC}_t^{FTA} \tag{S4}$$

where μ is the multiplier on the implementability constraint and:

$$\begin{aligned}\mathcal{MC}_t^{FTA} \equiv & u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*(C_t)) \cdot \mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*)C^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*(C_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t(C_t))}{\partial C_t} \cdot [\mathbf{c}_t - \mathbf{y}_t]\end{aligned}$$

Figure S1: Decomposition of Optimal Capital-Flow Taxes for Scenario 1



Notes: Time profile for Home capital-flow tax components in Scenario 1, simulated for 100 periods. See Table 1 for calibration details. "Constrained (Optimal) Trade Policy" refers to allocation arising from a Home planner acting unilaterally with (without) constraint on trade policy from a FTA.

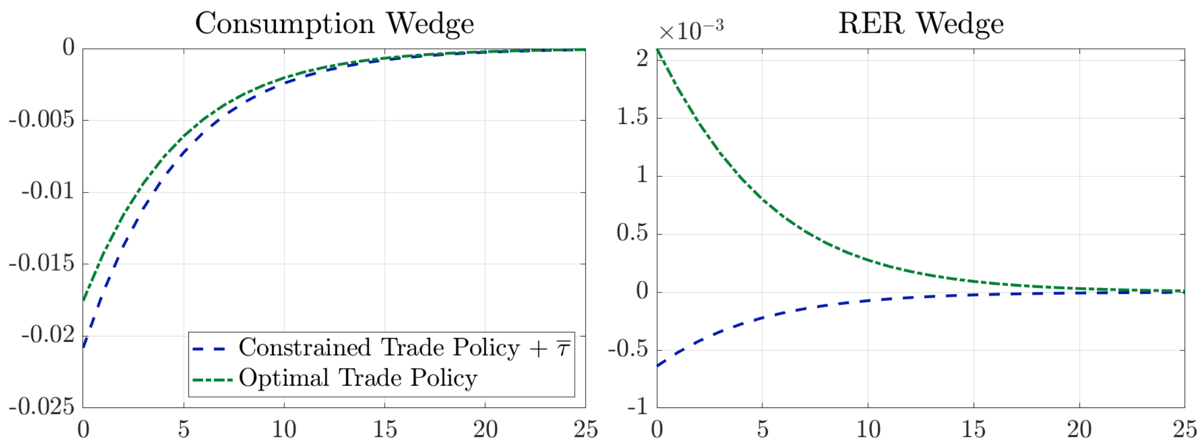
The left-hand side of equation (S4) is the marginal utility from one additional unit of aggregate consumption for the representative Home consumer. The right-hand side represents the marginal cost of that unit of consumption, captured by $\mathcal{MC}_t^{FTA}$. The first term in $\mathcal{MC}_t^{FTA}$ is the price of one unit of consumption, which can be shown to be equal to $u^{*\prime}(C_t^*)Q_t^{-1}$. The second term reflects how the inter-temporal price of consumption changes when importing one additional unit of consumption, for given relative goods prices. The final term reflects how relative goods prices change with aggregate consumption. If endowments and consumption outcomes coincide, $\mathbf{c}_t = \mathbf{y}_t$, (S4) collapses to $u'(C_t) = \mu u^{*\prime}(C_t^*)Q_t^{-1}$, which corresponds to the decentralized allocation.

S.2.2 Wedges in Unilateral Simulations

Figure S1 plots the decomposition of optimal capital-flow taxes in scenario 1 of Section 4.5 using equation (12). The left-hand plot indicates that while the consumption wedge explains a substantial portion of the overall variation in the capital-flow tax θ but is very similar across the constrained and unconstrained cases. In contrast, the RER wedge is significantly more negative in the case where trade policy is unconstrained, shown in the right-hand panel, and this drives the increase in the capital inflow tax. Nevertheless, both the consumption and RER wedge have the same sign, reflecting the alignment of inter- and intra-temporal incentives in this scenario.

Figure S2 plots the wedges corresponding to scenario 2. Once again the consumption wedge explains the majority of overall variation in the capital-flow tax but the differences between the constrained and unconstrained cases are small. However, in contrast to scenario 1, the right-hand panel demonstrates that the RER wedge has the opposite sign for the planner when there is no constraint on trade policy. This reflects the misalignment of inter- and intra-temporal incentives in this scenario. As a consequence of this, when the planner levies tariffs to monop-

Figure S2: Decomposition of Optimal Capital Flow Taxes for Scenario 2



Notes: Time profile for Home capital-flow tax components in Scenario 2, simulated for 100 periods. See Table 1 for calibration details. "Constrained (Optimal) Trade Policy" refers to allocation arising from a Home planner acting unilaterally with (without) constraint on trade policy from a FTA.

olistically drive the price of good 1 up, at the same time, this appreciates the terms of trade in the near term, which discourages households from borrowing and reduces the need for a capital-inflow tax.

S.2.3 Comparative Statics

Figure S3 demonstrates the comparative statics with respect to the intra-temporal trade elasticity for scenario 1 —although the 'inverse elasticity rule' holds in both scenarios. As the right-hand figure shows, optimal tariffs are both larger and vary more over time when the trade elasticity is lower. These intra-temporal incentives interact with the optimal capital-flow taxes too, which are higher for lower trade elasticities, regardless of the prevailing trade agreement.

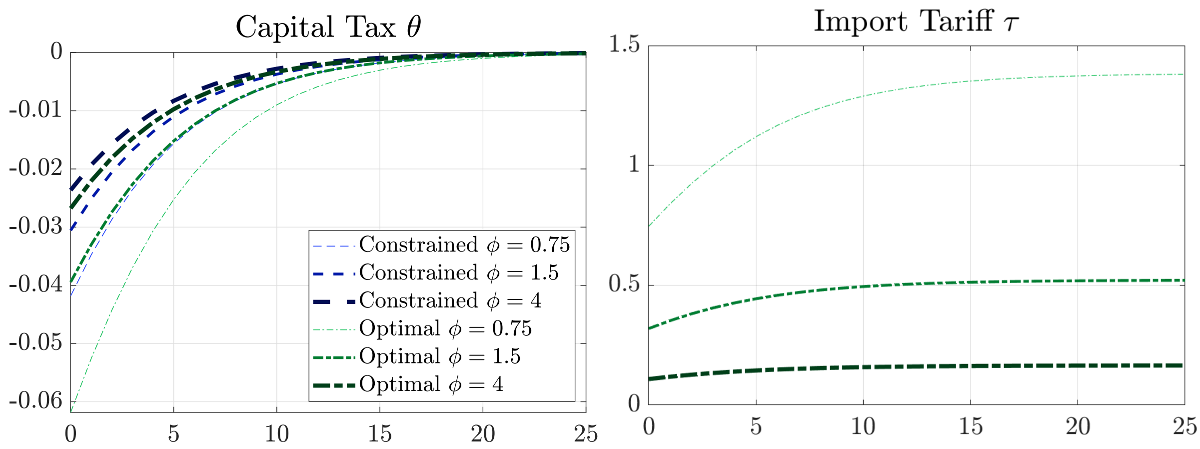
Similarly, Figure S4 shows that optimal capital-flow taxes are larger when the inter-temporal elasticity of substitution is lower (i.e., higher coefficient of relative risk aversion σ). In turn, variation in tariffs is larger when σ is high.

S.2.4 Commitment and Time Consistency

Consider the problem of a planner at time $s \geq t$:

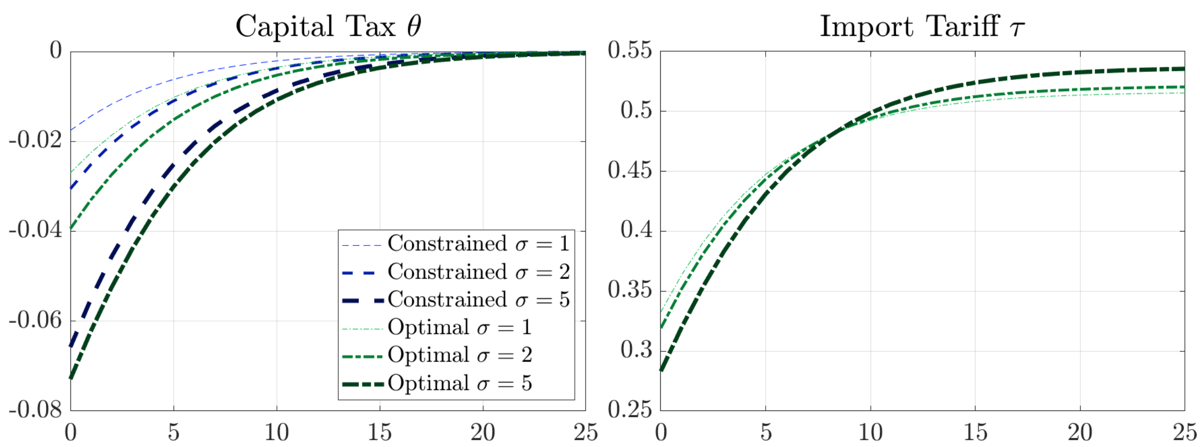
$$\begin{aligned} \max_{\{c_{1,s},c_{2,s}\}} \quad & \sum_{s=t}^{\infty} \beta^s u(C_s) \\ \text{s.t.} \quad & \sum_{s=t}^{\infty} \beta^s \boldsymbol{\rho}(C_s) \cdot [\mathbf{c}_s - \mathbf{y}_s - \mathbf{a}_{t,s}] = 0 \\ & C_s = g(\mathbf{c}_s) \end{aligned}$$

Figure S3: Comparative Statics of Optimal Capital-Flow Taxes and Tariffs with Respect to the Intra-temporal Trade Elasticity ϕ in Scenario 1



Notes: Time profile for Home capital-flow tax and tariff in scenario 1, simulated for 100 periods, with three different values of intra-temporal elasticity of substitution between goods 1 and 2 ϕ. See Table 1 for calibration details. "Constrained (Optimal)" refers to allocation arising from a Home planner acting unilaterally with (without) a constraint on trade policy from a FTA. The constrained model includes a steady-state tariff to ensure that the steady-state allocation replicates the unconstrained case.

Figure S4: Comparative Statics of Optimal Capital-Flow Taxes and Tariffs with Respect to the Coefficient of Relative Risk Aversion σ (Inverse Inter-temporal Elasticity of Substitution) in Scenario 1



Notes: Time profile for Home capital-flow tax and tariff in scenario 1, simulated for 100 periods, with three different values of the coefficient of relative risk aversion σ (i.e., inverse inter-temporal elasticity of substitution). See Table 1 for calibration details. "Constrained (Optimal)" refers to allocation arising from a Home planner acting unilaterally with (without) a constraint on trade policy from a FTA. The constrained model includes a steady-state tariff to ensure that the steady-state allocation replicates the unconstrained case.

where $a_{t,s}$ denotes assets issued at time t which mature at time s and market clearing requires $\mathbf{a}_{t,s} = -\mathbf{a}^*_{t,s}$ for all s. Importantly, as in the main body, we assume there are assets denominated in each good variety. The planner takes the assets they enter the period with as given. Consider the first-order conditions implied by planner choosing consumption paths at time $t = 0$:

$$u'(C_t)g_{1,t} = \mu_0 \left\{ u^{*\prime}(C^*_t)g^*_1(\mathbf{c}_t) + \left(u^{*\prime\prime}(C^*_t)g^*_1(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{1,t}} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t - \mathbf{a_{0,t}}] \right\}$$
$$u'(C_t)g_{2,t} = \mu_0 \left\{ u^{*\prime}(C^*_t)g^*_2(\mathbf{c}_t) + \left(u^{*\prime\prime}(C^*_t)g^*_2(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{2,t}} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t - \mathbf{a_{0,t}}] \right\}$$

and another planner choosing paths at time $t = 1$.

$$u'(C_t)g_{1,t} = \mu_1 \left\{ u^{*\prime}(C^*_t)g^*_1(\mathbf{c}_t) + \left(u^{*\prime\prime}(C^*_t)g^*_1(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{1,t}} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t - \mathbf{a_{1,t}}] \right\}$$
$$u'(C_t)g_{2,t} = \mu_1 \left\{ u^{*\prime}(C^*_t)g^*_2(\mathbf{c}_t) + \left(u^{*\prime\prime}(C^*_t)g^*_2(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{2,t}} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t - \mathbf{a_{1,t}}] \right\}$$

Critically the time-0 planner faces a multiplier μ_0 whereas the time-1 planner faces μ_1.

The plan is time consistent if the time-1 planner does not revise the time-0 allocation. This is the case if:

$$\mu_0 \left\{ u^{*\prime}(C^*_t)g^*_i(\mathbf{c}_t) + \left(u^{*\prime\prime}(C^*_t)g^*_i(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{i,t}} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t - \mathbf{a_{0,t}}] \right\} =$$
$$\mu_1 \left\{ u^{*\prime}(C^*_t)g^*_i(\mathbf{c}_t) + \left(u^{*\prime\prime}(C^*_t)g^*_i(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{i,t}} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t - \mathbf{a_{1,t}}] \right\}$$

for $i = 1, 2$ and for all t. This requires:

$$\begin{aligned}
&\left(u^{*\prime\prime}(C^*_t)g^*_i(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{i,t}} \right) \cdot \mathbf{a_{1,t}}(\mu_1) = \\
&\left\{ u^{*\prime}(C^*_t)g^*_i(\mathbf{c}_t) + \left(u^{*\prime\prime}(C^*_t)g^*_i(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{i,t}} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t] \right\} - \\
&\frac{\mu_0}{\mu_1} \left\{ u^{*\prime}(C^*_t)g^*_i(\mathbf{c}_t) + \left(u^{*\prime\prime}(C^*_t)g^*_i(\mathbf{c}^*_t)\nabla g^*(\mathbf{c}^*_t) + u^{*\prime}(C^*_t)\frac{\partial \nabla g^*(\mathbf{c}^*_t)}{\partial c_{i,t}} \right) \cdot [\mathbf{c}_t - \mathbf{y}_t - \mathbf{a_{0,t}}] \right\}
\end{aligned} \tag{S5}$$

for $i = 1, 2$ and for all t. This can be solved as a system of equations in $\mathbf{a_{1,t}} = [a_{1,1,t}\ a_{2,1,t}]$ and $\mathbf{a_{0,t}} = [a_{1,0,t}\ a_{2,0,t}]$, where $a_{i,t,s}$ denotes assets denominated in good i, issued at time t and maturing at time s. Standard arguments detailed in Costinot et al. (2014) can be used to show that if $\mu_0 > 0$, $\mu_1 > 0$.[6] The implementability condition (IC) is satisfied by construction, so the above conditions constitute part of an equilibrium.

Since (S5) is cumbersome, we illustrate the debt structure chosen at $t = 0$ which makes the policy plan time consistent at $t = 1$ in the CO limit. The expressions for $\{a_{1,1,t}, a_{2,1,t}\}$ are then

[6] Note that the positivity of μ_1 is not true in general, as shown in Debortoli et al. (2021) who revisit the environment in Lucas and Stokey (1983).

given by:

$$a_{1,1,t}(\mu_1) = \left(\frac{1}{c^*_{1,t}}\right)^{-1} \left\{1 + \frac{1}{c^*_{1,t}}(c_{1,t} - y_{1,t})\right\} - \frac{\mu_0}{\mu_1}\left\{1 + \frac{1}{c^*_{1,t}}(c_{1,t} - y_{1,t} - a_{1,0,t})\right\},$$
$$a_{2,1,t}(\mu_1) = \left(\frac{1}{c^*_{2,t}}\right)^{-1} \left\{1 + \frac{1}{c^*_{2,t}}(c_{2,t} - y_{2,t})\right\} - \frac{\mu_0}{\mu_1}\left\{1 + \frac{1}{c^*_{2,t}}(c_{2,t} - y_{2,t} - a_{2,0,t})\right\}$$

S.2.5 Anticipated Changes in Endowments

In this appendix, we discuss how optimal capital-flow taxes and tariffs are levied in the face of anticipated changes in endowments. To operationalize this within our deterministic simulations, we assume that at $t = 0$, a change in the endowment at some time period $\bar{t}$ is fully and accurately anticipated by all agents in the economy.

As an example, Figure S5 plots the optimal policy instruments from the unilateral setting when the dynamics from scenario 1 are anticipated to occur at $\bar{t} = 5$ (rather than on impact). Concretely, initial endowments are defined as: $y^{(*)}_{i,t} = \bar{y}^{(*)}_i$ for $i = 1, 2$ and $t = 0, 1, 2, 3, 4$. Period-5 endowments at Home are given by $y_{1,5} = 0.9\bar{y}_1$ and $y_{2,5} = \bar{y}_2$, and to ensure no aggregate uncertainty $y^*_{1,5} = 1 - y_{1,5}$ and $y^*_{2,5} = 1 - y_{2,5}$. From period 5 onwards, endowments are assumed to return to their long-run values gradually. So, in essence, this example represents an anticipated negative, but temporary, endowment shock for the Home country.

We choose $\bar{t} = 5$ as an example. Since capital-flow taxes depend only on income in adjacent periods (i.e., t and $t + 1$), a key insight from Costinot et al. (2014), this is sufficient to capture how anticipated shocks can generate preemptive policy action more generally.

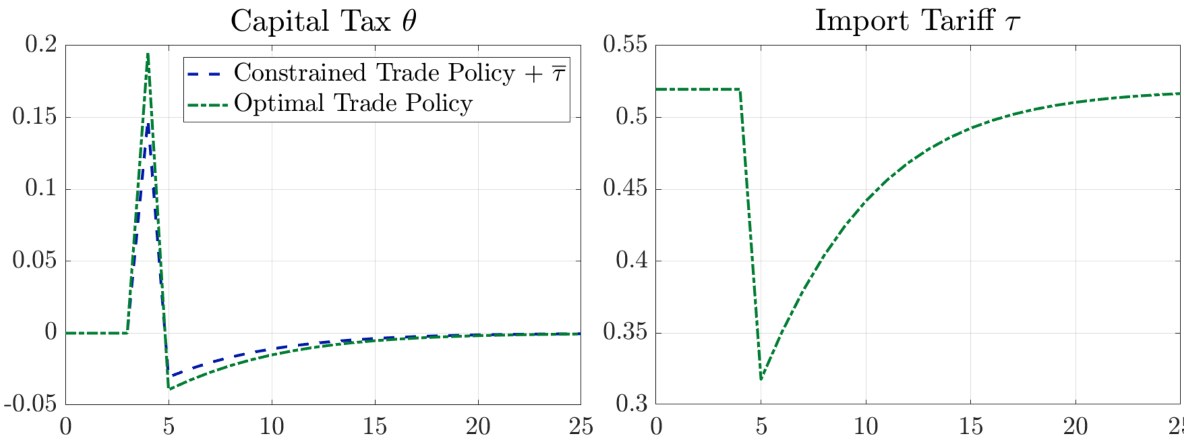
Consistent with the logic in the main body of the paper, optimal trade and financial policy involves action in advance of the shock—akin to the 'precautionary' motives for intervention in small-open economy models with borrowing constraints (see, e.g., Mendoza, 2002; Bianchi, 2011). In Figure S5, the Home unilateral planner subsidizes capital-inflows in period 4, prior to the shock, both with and without constraints on trade policy, since good 1 is relatively abundant at that time. This facilitates borrowing to help smooth Home consumption. Thereafter, since good 1 becomes relatively scare, the Home planner taxes capital-inflows—as in scenario 1. Without any constraints on trade policy, the planner is able to smooth Home consumption by more, relative to the case where trade policy is constrained, by employing tariffs alongside preemtive capital-flow taxes. Tariffs are only employed contemporaneously (i.e. at $t = 5$).

S.2.6 Ruling Out Capital Controls

We also consider the case where the planner optimally chooses tariffs, but capital controls are contractually ruled out—i.e., by a 'free-financial-flows agreement' (FFFA). To rule out capital controls, the allocation must satisfy:

$$\frac{u'(C_{t+1})}{u'(C_t)}\frac{u^{*\prime}(C_t)}{u^{*\prime}(C^*_{t+1})} = \frac{Q_t}{Q_{t+1}}$$

Figure S5: Time Profile of Optimal Instruments for Anticipated and Temporary Fall in Home Endowment of Good 1 (Scenario 1, Anticipated at $\bar{t} = 5$)



Notes: Time profile for Home capital-flow tax and tariff in anticipated variant of scenario 1, simulated for 100 periods. See Table 1 for calibration details. "Constrained (Optimal) Trade Policy" refers to optimal instruments for Home planner acting unilaterally with (without) constraints on trade policy from a FTA.

which corresponds to the Backus and Smith (1993) condition. While this condition rules out capital-flow taxation, it can allow for tariffs, which can be seen by rewriting it as follows:

$$\frac{u'(C_{t+1})}{u'(C_t)} \frac{u^{*\prime}(C_t)}{u^{*\prime}(C^*_{t+1})} \frac{1+\tau^*_t}{1+\tau^*_{t+1}} = \frac{g_{1,t}}{g^*_{1,t}} \left(\frac{g_{1,t+1}}{g^*_{1,t+1}} \right)^{-1}$$

We further impose that this holds period-by-period:

$$u'(C_t) g_{i,t} = \kappa u'(C^*_t) g^*_{i,t} \frac{1}{1+\tau^*_t} \quad \forall t \tag{S6}$$

where κ is a constant, calculated in an equilibrium with the optimal tariffs in place to ensure no transfers are needed.

Considering a setting absent a FTA, but with a FFFA. The first-order conditions for a unilateral Home planner with respect to $c_{1,t}$ and $c_{2,t}$ when capital controls are ruled out become:

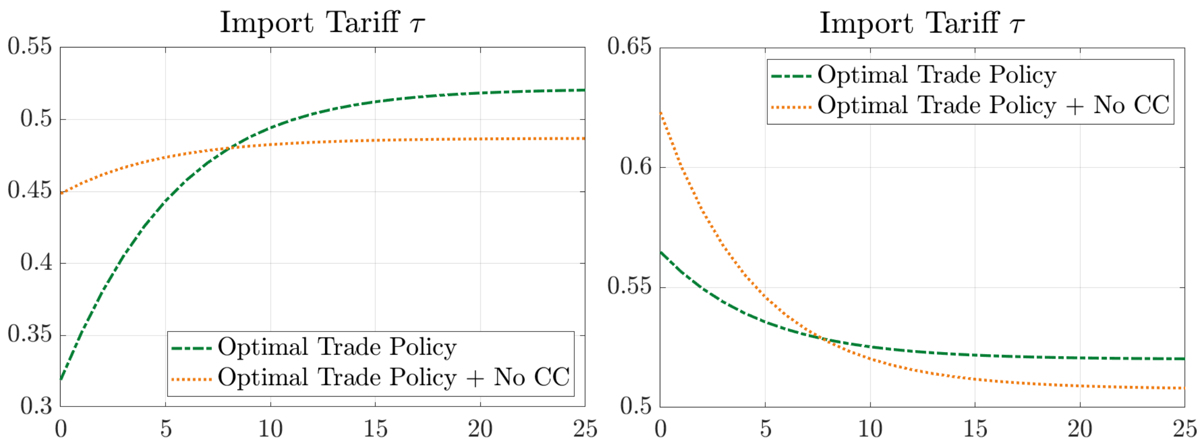
$$u'(C_t) g_{1,t} = \mu \mathcal{MC}^{nFTA}_{1,t} + \zeta_t \mathcal{RS}_{1,t}$$
$$u'(C_t) g_{2,t} = \mu \mathcal{MC}^{nFTA}_{2,t} + \zeta_t \mathcal{RS}_{2,t}$$

where ζ_t is the multiplier on condition (S6) and, for $i = 1, 2$:

$$\mathcal{RS}_{i,t} = u''(C_t) g_{i,t} - \kappa u''(C^*_t) g_{i,t} \frac{g^*_{1,t}}{g_{1,t}} \frac{1}{1+\tau^*_t} - \kappa u'(C^*_t) \frac{-g^*_{1i,t} g_{1,t} - g^*_{1,t} g_{1i,t}}{g^2_{1,t}} \frac{1}{1+\tau^*_t}$$

Intuitively, the planner now internalizes the effect of an additional unit of consumption of good 1 and 2 respectively on the intertemporal consumption smoothing. An increase in C_t is only

Figure S6: Optimal Tariffs when Capital Controls are Ruled Out by a FFFA: Tariffs Acting as Second-Best Instrument in Scenarios 1 (Left) and 2 (Right)



Notes: Time profile for optimal tariffs in Scenario 1 (left) and 2 (right), simulated for 100 periods. See Table 1 for calibration details. "Optimal Trade Policy" refers to allocation arising from a Home planner acting unilaterally without constraints on trade policy from a FTA. This is compared to the "Optimal Trade Policy + No CC" allocation, in which capital controls are ruled out by a FFFA.

permitted if the allocation of c_1 and c_2 is such that there is a sufficient depreciation in the real exchange rate.

The paths for the optimal tariffs when capital controls are ruled out are displayed in Figure S6. In scenario 1, the path for tariffs is less variable with a FFFA, compared to the no-FTA case. This occurs because good 1 is relatively scarce in the near term, so the optimal path for tariffs (on good 2) is increasing. All else equal, this would incentivise over-borrowing in the near-term, with knock-on effects for optimal capital controls in the no-FTA case. Consequently, in the absence of capital controls, variation in the optimal tariff is smaller.

In contrast, the optimal path for tariffs in the no-FTA case for scenario 2 will, all else equal, disincentivise over-borrowing because inter- and intra-temporal incentives oppose. As a consequence, the path for tariffs is more variable in this case, with a larger optimal tariff in the near term than in the no-FTA case. In this instance, tariffs in effect act as a second-best instrument to stabilise borrowing.

S.3 Model Extensions and Generalizations

S.3.1 Production and Nominal Rigidities

In this appendix, we illustrate the incentives to manipulate the terms of trade remain in a model with non-traded goods, endogenous labour supply and nominal wage rigidities. Specifically, the planner has an additional motive to bring forward consumption with policy interventions when output is demand constrained due to the presence of an aggregate-demand externality.

Setup. We consider a minimal model of production and nominal rigidities. Households consume non-traded goods NT in addition to traded T goods 1 and goods 2 as in the baseline model. Their instantaneous period-by-period utility function is given by:

$$\mathcal{U} = u(c_1, c_2, c_{NT}) + v(L)$$

where u is CRRA with risk aversion σ and v represents captures disutility from labor supply L. Aggregate consumption C_t takes a nested CES form:

$$C_t = \left[(1-\omega)^{\frac{1}{\phi}} c_{T,t}^{\frac{\phi-1}{\phi}} + \omega^{\frac{1}{\phi}} c_{NT,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

$$c_{T,t} = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

Non-traded goods are produced with a linear production function $y_{NT,t} = A_t L_t$ under perfect competition. Firm maximization yields $p_{NT,t} = \frac{w_t}{A_t}$ and we assume wages are perfectly rigid, $w_t = \overline{w}$.

The budget constraint for the Home representative household is given by:

$$\tilde{\mathbf{p}}_t \cdot \mathbf{c}_t - \mathbf{p}_t \mathbf{y}_t + \mathbf{p}_{T,t+1} \cdot \mathbf{a}_{T,t+1} \leq \overline{w} L_t + \mathbf{p}_{T,t} \cdot \mathbf{a}_{T,t} + T_t \tag{S7}$$

where $\tilde{\mathbf{p}}_t$ captures prices of goods after taxes. We assume households trade in good 1 and good 2 denominated bonds and earn wages. The consolidated present-value budget constraint, assuming no initial assets, a no-Ponzi condition, substituting in profits, and market clearing $y_{NT,t} = c_{NT,t}$ can be written as:

$$\sum_{t=0}^{\infty} \mathbf{p}_T \cdot [\mathbf{c}_T - \mathbf{y}_T] \leq 0$$

The indirect utility function is given by:

$$V\left(c_{T,t}, \frac{p_{T,t}}{p_{NT}}\right) = u\left(c_{T,t}, \frac{\omega}{1-\omega}\left(\frac{p_{T,t}}{p_{NT}}\right)^{\phi} c_{T,t}\right) + v\left(\frac{1}{A_t}\frac{\omega}{1-\omega}\left(\frac{p_{T,t}}{p_{NT}}\right)^{\phi} c_{T,t}\right)$$

The marginal benefit to the planner for a unit of c_T can be expressed as:

$$\frac{\partial V_t}{\partial c_{T,t}} = u'(C_t) g_{T,t}\left(1 + \frac{\omega}{1-\omega}\tau_t^L\right)$$

where τ_t^L is the labor wedge, defined as:

$$\tau_t^L = 1 + \frac{1}{A_t}\frac{v_{L,t}}{u'(C_t) g_{T,t}}$$

The labor wedge is positive when the economy is demand constrained and households are involuntarily unemployed. The marginal benefit of a unit of tradable consumption is higher when the economy is demand constrained.

Returning to the planner's problem, the implementability constraint is unchanged. Absent a FTA, the first-order conditions with respect to goods 1 and 2 are given by:

$$u'(C_t)g_{T,t}\left(1+\frac{\omega}{1-\omega}\tau_t^L\right)\frac{g_{1,t}}{g_{T,t}} = \mu\mathcal{MC}_{1,t} \tag{S8}$$

$$u'(C_t)g_{T,t}\left(1+\frac{\omega}{1-\omega}\tau_t^L\right)\frac{g_{2,t}}{g_{T,t}} = \mu\mathcal{MC}_{2,t} \tag{S9}$$

Suppose $\tau_t^L > 0$ because the economy is demand constrained. The planner now has an additional inter-temporal incentive to bring forward consumption to stimulate employment, as reflected by a higher marginal benefit form a unit of tradable consumption.[7]

Policy Instruments. Capital-flow taxes are given by:

$$(1-\theta_t) = \frac{u'(C_t)g_{T,t}}{u'(C_{t+1})g_{T,t+1}}\frac{u'(C_{t+1}^*)g_{T,t+1}}{u'(C_t^*)g_{T,t}}\frac{Q_{T,t}}{Q_{T,t+1}}$$

where $Q_{T,t} = P_{T,t}^*/p_{T,t}$ and $p_{T,t}$ has the same form as the aggregate price level in the baseline model with only goods 1 and 2.

Proposition S1 *The capital-flow tax, absent constraints to trade policy, is given by:*

$$\theta_t^{nFTA} = 1 - \frac{\left(1+\frac{\omega}{1-\omega}\tau_t^L\right)\left(1-\left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*) + \frac{1}{g_{1,t}^*}\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}^*}\right)\cdot[\mathbf{c}_t - \mathbf{y}_t]\right)}{\left(1+\frac{\omega}{1-\omega}\tau_{t+1}^L\right)\left(1-\left(\frac{u^{*\prime\prime}(C_{t+1}^*)}{u^{*\prime}(C_{t+1}^*)}\nabla g^*(\mathbf{c}_{t+1}^*) + \frac{1}{g_{1,t+1}^*}\frac{\partial \nabla g^*(\mathbf{c}_{t+1}^*)}{\partial c_{1,t+1}^*}\right)\cdot[\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]\right)}$$

and the optimal tariff formula is unchanged. When a FTA (8) is in place, the optimal capital-flow tax is:

$$\theta_t^{FTA} = 1 - \frac{\left(1+\frac{\omega}{1-\omega}\tau_t^L\right)\left(1-\left(\frac{u^{*\prime\prime}(C_t^*)}{u^{*\prime}(C_t^*)}\nabla g^*(\mathbf{c}_t^*(C_t)) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_t^*(C_t))}{\mathrm{d}C_t^*}\right)\cdot[\mathbf{c}_t - \mathbf{y}_t]\right)}{\left(1+\frac{\omega}{1-\omega}\tau_{t+1}^L\right)\left(1-\left(\frac{u^{*\prime\prime}(C_{t+1}^*)}{u^{*\prime}(C_{t+1}^*)}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1})) + \frac{\mathrm{d}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1}))}{\mathrm{d}C_{t+1}^*}\right)\cdot[\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]\right)}$$

Proof: Follows from the Proof to Proposition 2, replacing the first-order conditions with equation (S8), (S9). □

Since the risk-sharing condition is unchanged, tariffs affect the path of the exchange rate for tradables in the same way as in the baseline model. Consistent with this, tradables consumption can be brought forward either with a capital-inflow tax or an import subsidy which puts pressure on Q_T to depreciate, as in the baseline model.

[7] Jeanne (2021) considers an environment with tradables production and shows that when the economy is demand constrained there is an incentive to use trade policy to stimulate demand for the domestic good through a substitution argument. Here, we emphasise trade policy can be used to stimulate aggregate demand as a substitute for a capital-inflow subsidy.

S.3.2 Segmented Markets and Quantity Interventions

In this appendix, we explain how a similar outcome to our baseline model (with capital controls and tariffs) can be achieved if the planner uses quantity interventions (e.g., open-market operations or FXI) in place of capital controls.

Setup. We present a model with non-traded goods and financial intermediation with international financial market segmentation. We allow for tariffs but not capital controls. The budget constraint for the Home representative household is given by:

$$\tilde{\mathbf{p}}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] + p_{NT,t+1} a_{t+1} \leq p_{NT,t} a_t + \Pi_t^f + T_t$$

where Π_t^f are rebated profits from financial intermediaries and T_t is the lump-sum rebate from the government. Normalising $p_{NT,t} = 1$ yields:

$$\tilde{\mathbf{p}}_t \cdot \mathbf{c}_t - \mathbf{p}_t \mathbf{y}_t + R_{NT,t}^{-1} a_{t+1} \leq a_t + \Pi_t^f + T_t$$

where $R_{NT,t}^{-1}$ is the price of an asset highlighting that the NT good is the numèraire in the economy. We define $\mathcal{E}_t = \frac{p_{NT,t}^*}{p_{NT,t}}$ as the exchange rate, as in Gabaix and Maggiori (2015).

The households' maximization yields the following Euler equation for non-traded goods:

$$\beta \frac{u'(C_{t+1}) g_{NT,t+1}}{u'(C_t) g_{NT,t}} = R_{NT,t}^{-1}$$

Moreover, the relative demand equation is given by,

$$\frac{g_{NT,t}}{g_{i,NT}} = \frac{p_{NT,t}}{p_{i,t}(1+\tau_{i,t})} \tag{S10}$$

where $g_{NT,t} = \frac{\partial C_t}{\partial c_{NT,t}}$. Foreign households undertake an analogous maximization.

Monetary Authority. The planner, in this case a monetary authority, can take a position $p_{NT,t+1} a_{t+1}^G$ in domestic assets. We assume this is financed by an exactly opposition position $p_{NT,t+1}^* a_{t+1}^{G\,*}$ in foreign assets. If the monetary authority cannot borrow in foreign assets, there must be sufficient reserves to sell and carry out the operation. The monetary authority also provides a lump-sum transfer T_t to households.

Financial Intermediaries. A continuum of financial intermediaries indexed by $k \in [0, \overline{k}]$ trade in one-period assets with households in both countries. Each financier starts with no initial capital, faces a participation cost k and position limits $\{\overline{\alpha}, \underline{\alpha}\}$. The variable k corresponds to both the financiers' cost of participating and their index. Financiers choose a position in the asset $\alpha_{t+1}^I(k)$, financed by a position $-\alpha_{t+1}^I(k)\mathcal{E}_t$ in the foreign asset to maximize profits earned at t, subject to breaking even at $t+1$. The $t+1$ break-even condition is $\alpha_{t+1}^I(k) + \mathcal{E}_{t+1}^* \alpha_{t+1}^{*\,I}(k) = 0$.

The problem of an individual financier, indexed by k, at time t can be summarized as:

$$\max_{\{\alpha^I_{t+1}(k))\in[\overline{\alpha},\underline{\alpha}]\}} \left[R^{*-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t}\right]\alpha^I_{t+1}(k) - k$$

In equilibrium, a measure $\boldsymbol{k} = |R^{*-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t}|\overline{\alpha}$ participates. The total position taken up by financiers is given by $\alpha^I_{t+1} = \boldsymbol{k}\overline{\alpha}$. Defining $\Gamma = \frac{1}{\overline{\alpha}^2}$ yields:

$$\left[R^{*-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R^{-1}_{NT,t}\right] = \Gamma\alpha^I_{t+1}$$

Market clearing requires:

$$a_{NT,t+1} + a^G_{NT,t+1} + \alpha^I_{t+1} = 0 \tag{S11}$$

Due to limited participation by financiers and limits to arbitrage, the cost of borrowing is not equalised across countries. The more constrained the position that financiers can take, the higher the Γ and the larger the gap in the cost of borrowing when there are imbalances. Specifically, if the Home country is a net borrower, $\alpha^I_{t+1} > 0$, and the cost of borrowing for Home households $R_{NT,t}$ will be relatively high.

Substituting in the Euler equations for $R_{NT,t}$ and $R^*_{NT,t}$, yields a modified risk-sharing condition:

$$\left[\beta\frac{u^{*\prime}(C^*_{t+1})g^*_{NT,t+1}}{u^{*\prime}(C^*_t)g^*_{NT,t}}\frac{p^*_{NT,t}}{p_{NT,t}}\frac{p_{NT,t+1}}{p^*_{NT,t+1}} - \beta\frac{u'(C_{t+1})g_{NT,t+1}}{u'(C_t)g_{NT,t}}\right] = \Gamma\alpha^I_{t+1} \tag{S12}$$

Using the relative demand (S10), home and abroad, market clearing for assets (S11), and simplifying:

$$\left[\frac{\frac{p_{NT,t+1}}{P_{t+1}}}{\frac{p_{NT,t}}{P_t}}\right]\left[\beta\frac{u'(C_{t+1})}{u'(C_t)} - \beta\frac{u^{*\prime}(C^*_{t+1})}{u^{*\prime}(C^*_t)}\frac{Q_{t+1}}{Q_t}\right] = \Gamma(a_{NT,t+1} + a^G_{NT,t+1}) \tag{S13}$$

Relationship Between Instruments. Suppose Home households are borrowing $a_{NT,t+1} < 0$. By taking an opposing position and purchasing these assets $a^G_{NT,t+1} > 0$, funded by selling Foreign reserves ($a^{G*}_{NT,t+1} < 0$), the planner reduces the size of the imbalance that needs to be intermediated. As a result, this lowers the cost of borrowing for Home households. Below, we illustrate that such an intervention in a model with $\Gamma_t > 0$ can target the same wedge in risk-sharing as a capital-inflow tax in the baseline model.

Proposition S2 (Capital Controls and Quantity Intervention Equivalence) *Any path for risk-sharing wedges $\frac{u'(C_t)}{\mu u'(C^*_t}Q_t - 1$ implemented with capital controls in the model with perfect financial markets can be implemented by FXI in the model with international financial frictions.*

Proof: To see the relationship between capital controls θ_t and open-market interventions

$a^G_{NT,t+1}$, we first define a risk-sharing wedge as in Costinot et al. (2014):

$$\psi_t = \frac{u'(C_t)}{\mu u'(C_t^*)} Q_t$$

In the baseline model, capital controls (on assets denominated in traded varieties) can implement a desired risk-sharing wedge through the following mapping:

$$\theta_t = 1 - \frac{1+\psi_{t+1}}{1+\psi_t}$$

The risk-sharing condition, allowing for capital-flow taxes, can be written as:

$$\left[\beta \frac{u'(C_{t+1})}{u'(C_t)} - \beta \frac{u^{*\prime}(C_t)}{u^{*\prime}(C^*_{t+1})} \frac{Q_{t+1}}{Q_t}\right] = \theta_t \beta \frac{u'(C_{t+1})}{u'(C_t)}, \quad \text{for } i = \{1,2\} \tag{S14}$$

Combining the definition of the risk-sharing wedge and (S13) suggests that, in the model with non-traded goods and financial frictions, FXI can implement a desired risk-sharing wedge through the following mapping:

$$a^G_{NT,t} = \frac{1}{\Gamma}\left[\left(1 - \frac{1+\psi_{t+1}}{1+\psi_t}\right) \beta \frac{u'(C_{t+1})}{u'(C_t)}\right]\left[\frac{\frac{p_{NT,t+1}}{P_{t+1}}}{\frac{p_{NT,t}}{P_t}}\right] - a_{NT,t+1} \tag{S15}$$

To ensure the two models yield equivalent allocations is to determine lump-sum transfers and allocate profits when $\Gamma > 0$. Financiers earns $\Gamma(\alpha^I_{t+1})^2$ total profits. which we assume are fully rebated to Home households.[8] The monetary authority earns $-\Gamma\alpha^I_{t+1}a^G_{NT,t+1}$ on its FXI, which is potentially a loss. We assume these losses are imposed on households through lump-sum transfers.

Finally, we can rewrite the consolidated budget constraint, summing up the position of Home households, the monetary authority and financial intermediaries. Substituting $\Pi^f_t = \Gamma(\alpha^I_{t+1})^2$, $T_t = \tau_{2,t}p_{2,t}c_{2,t} + \Gamma\alpha^I_{t+1}a^G_{NT,t+1}$, imposing a no-Ponzi condition ($p_{NT,\infty}a_\infty \to 0$) and assuming zero initial assets ($p_{NT,0}a_0 + \Pi^f_t = 0$) yields the budget constraint:

$$\mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] + R^{*\,-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} a_{NT,t+1} \leq a_{NT,t}$$

where $R^{*\,-1}_{NT,t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} = \left(\frac{p^*_{NT,t+1}}{p^*_{NT,t}}\right)\left(\frac{p^*_{NT,t+1}}{p_{NT,t+1}}\right)^{-1} p^*_{NT,t} = p_{NT,t+1}$. Iterating this forward yields:

$$\sum_{t=1}^{\infty} \mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] \leq 0$$

Since $c_{NT,t} = y_{NT,t}$ $\forall t$, the present-value budget constraint is unchanged relative to the baseline two-good model with trade in bonds denominated in units of goods 1 and 2. As a

[8] Relaxing this condition would create a quadratic-cost term as in Fanelli and Straub (2021). This would provide an additional motive for the monetary authority to narrow the spread.

result, the planning problem's implementability condition is also unchanged. □

Interaction Between Financial and Trade Policy. Inspecting (S15) yields two key insights. First, the interaction between trade and financial policy persists, since the path for real exchange rates is contained in $\left(1 - \frac{1+\psi_{t+1}}{1+\psi_t}\right)$. Second, the interaction now also depends on the evolution for the ratio of the price of non-traded goods to the aggregate price level.

Consider the special case where aggregators are Cobb-Douglas and utility has a logarithmic form, then:

$$a^G_{NT,t} = \frac{1}{\Gamma}\left[\left(1 - \frac{1+\psi_{t+1}}{1+\psi_t}\right)\right]\frac{\frac{\chi_{t+1}}{y_{NT,t+1}}}{\frac{\chi_t}{y_{NT,t}}} - a_{NT,t+1}$$

where χ_t is the share of expenditure spent on non-tradables. If $\chi_t = y_{NT,t}$ in every period such that variations in the marginal utility of tradables is neutralised, as assumed in Gabaix and Maggiori (2015), then our results on the direction of interactions between capital controls and trade policy go through for the case of open-market operations.

Financial Terms of Trade Manipulation. The parametrization above ensures that intermediation frictions leave the country as a whole unchanged, and the planner does not distinguish between profits going to households or financiers due to lump-sum rebates. Suppose now that only a share $\Omega < 1$ of profits is rebated to Home households. Then the present-value budget constraint can be written as:

$$\sum_{t=1}^{\infty} \mathbf{p}_t \cdot [\mathbf{c}_t - \mathbf{y}_t] + (1-\Omega)\Gamma a^2_{I,t+1} \leq 0$$

Remember that, assuming no intervention ($a^G_{NT,t+1} = 0$), $a_{I,t+1} = -a_{NT,t+1}$ financiers are the counter-parties to households borrowing. Suppose that $y_{i,t} < y_{i,,t+1}$, $a_{NT,t+1} < a_{NT,t}$, such that households borrow in period t. Then, a^I_{t+1} rises and there is an additional incentive to delay consumption and manipulate the terms of trade, even at the SOE limit ($dC^*_t/dC_t \to 0$), because intermediation is costly.[9]

S.3.3 Country Size

In this appendix, we explain how incentives to manipulate relative prices remain for a small-open economy, as they remain large in goods markets. We then focus on an interesting knife-edge case in which the required size of capital controls for inter- and intra-temporal motives is the same in both the FTA and no-FTA case.

[9] While the model is in principle symmetric, if the government issues debt but does not make purchases this ensures $a^I_{NT,t} \geq 0$.

Setup. We adopt the small-open economy limit of Costinot et al. (2014). To do so, we define aggregate consumption for the rest of the world as:

$$C^* = \frac{{c_1^*}^{\frac{1}{N}} {c_2^*}^{1-\frac{1}{N}}}{N-1}$$

where N is the number of countries. In the Home (small) economy, aggregate consumption is given by:

$$C = c_1^{\frac{1}{2}} c_2^{\frac{1}{2}}$$

The expenditure minimization problem used to derive the price index is analogous to Section S.1.3. Taking the ratio of the two price indices yields the real exchange rate:

$$Q_t = (N-1)\, \frac{p_{2,t}}{p_{1,t}}^{\frac{1}{2}-\frac{1}{N}} \left[2 \left(\frac{1}{N}\right)^{\frac{1}{N}} \left(1 - \frac{1}{N}\right)^{1-\frac{1}{N}} \right] \tag{S16}$$

The market-clearing conditions are given by: $c_1 + c_1^* = y_1$ and $c_2 + c_2^* = y_2 + (N-1)y_2^*$.

In the limit $N \to \infty$, the Home country becomes a small-open economy and $C_t^* \to c_{2_t}^* = Y_{2,t}$ resulting in $\frac{\mathrm{d}C^*}{\mathrm{d}C} \to 0$. Moreover, as before, $\frac{\mathrm{d}C_t^*}{\mathrm{d}C_t} = -\frac{1}{Q_t} \to 0$ as $Q_t \to \infty$. The Home small-open economy planner maximizes utility subject to:

$$\sum_t (N-1)\; u^{*\prime}(C_t^*) \nabla \mathbf{g}_t^* \cdot [\mathbf{c}_t - \mathbf{y}_t] \tag{S17}$$

with the $(N-1)$ appearing because C^* is defined as per-country aggregate consumption. The first-order conditions are derived analogously as in Section 3.

Optimal Policy and Country Size. While there are a range of outcomes in the small-open economy setting, an interesting knife-edge case arises in the CO limit ($\sigma = \phi \to 1$). At this parametrization, the required size of capital controls for inter- and intra-temporal incentives is the same. In Figure S7, we plot the optimal size of capital controls in both the FTA and no-FTA cases as $N \to \infty$, as well as tariffs in the no-FTA case for scenario 1.

S.4 Strategic Planning Allocation

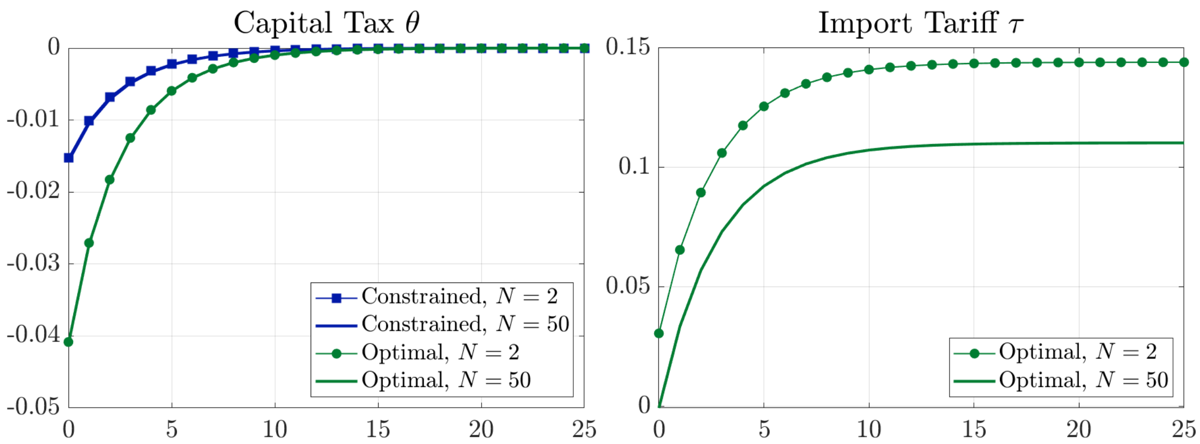
S.4.1 Derivation of Strategic Planning Allocation Without Free Trade

The following lemma details the implementability constraint for the Home planner. Existence of a Nash equilibrium is established in Costinot et al. (2014).

Lemma S3 (Implementability for Nash Planner without FTA) *The Home allocation forms part of an equilibrium absent a FTA if it satisfies:*

$$\sum_{t=0}^{\infty} \left[\prod_{s=0}^{t-1} (1 - \theta_s^*) \right] \beta^t u^{*\prime}(C_t^*) \boldsymbol{\tau}_t^{*-1} \cdot \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \leq 0 \tag{IC-Nash-nFTA}$$

Figure S7: Time Profile for Optimal Taxes and Tariffs in a Small-Open Economy as the Home Endowment of Good 1 Rises in Scenario 1



Notes: Time profile for optimal capital-flow taxes and tariffs in Scenario 1, simulated for 100 periods, for two-country case ($N = 2$) and small-open economy case ($N = 50$). "(Un)constrained" refers to allocation arising from a Home planner acting unilaterally with (without) constraints on trade policy from a FTA. See Table 1 for additional calibration details.

Foreign Planner's Problem.

$$\max_{\{\mathbf{c}_t^*\}} \quad \sum_{t=0}^{\infty} \beta^t \; u\left(g(\mathbf{c}_t^*)\right) \tag{$P1^{*\;Nash}$}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \beta^t u'(g(\mathbf{c}_t))\boldsymbol{\tau}_t^{-1} \boldsymbol{\nabla} g(\mathbf{c}_t) \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0 \tag{$IC^{*\;Nash}$}$$

where:

$$\boldsymbol{\tau}_t = \begin{bmatrix} 1 & 0 \\ 0 & (1-\tau_t) \end{bmatrix}$$

The first-order conditions for the Foreign country with respect to $c_{1,t}^*$ and $c_{2,t}^*$ are given by:

$$C_t^{*\;-\sigma} g_{1,t}^* = \mu \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \left\{ C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \begin{bmatrix} g_{1,t}(c_{1,t}^* - y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} - \right.$$
$$\left. C_t^{-\sigma} \begin{bmatrix} g_{11,t}(c_{1,t}^* - y_{1,t}^*)+ \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} \right\} \tag{S18}$$

such that:

$$C_t^{*\;-\sigma} g_{1,t}^* = \mu \; \hat{\mathcal{MC}}_{1,t}^*$$

and:

$$C_t^{*\;-\sigma} g_{2,t}^* = \mu \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \left\{ C_t^{-\sigma} g_{2,t}(1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1} g_{2,t} \begin{bmatrix} g_{1,t}(c_{1,t}^* - y_{1,t}^*)+ \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} - \right.$$

$$C_t^{-\sigma}\left[\begin{array}{c} g_{12,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{22,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{array}\right]\Bigg\}$$

such that

$$C_t^{-\sigma}g^*_{2,t} = \mu\ \hat{\mathcal{MC}}^*_{2,t}$$

S.4.2 With Trade-Policy Constraints

Next, we present the details of the strategic planning allocation when trade policy is constrained with a FTA. Focusing on the Home planning problem, we can characterize the optimal allocation with a FTA in place, taking the sequence of Foreign capital flow taxes $\{\theta^*_t\}$ as given. The Foreign optimality conditions, for $i = 1, 2$ can be written:

$$u^{*\prime}(C^*_t)g^*_i(\mathbf{c}^*_t) = \beta(1-\theta^*_t)(1+r_{i,t})u^{*\prime}(C^*_{t+1})g^*_i(\mathbf{c}^*_t) \tag{S19}$$

The Foreign optimality conditions, the Home inter-temporal budget constraint and the market-clearing conditions yield an implementability condition for the Home planner, described below.

Lemma S4 (Implementability for Nash Planner with Free Trade) *When the Foreign planner chooses $\{\mathbf{c}^*_t\}$ to maximize domestic welfare, the Home allocation $\{\mathbf{c}_t\}$ forms part of an equilibrium if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta^*_s)\right]\beta^t u^{*\prime}(C^*_t)\nabla g^*(\mathbf{c}^*_t)\cdot[\mathbf{c}_t-\mathbf{y}_t]\le 0 \tag{IC-Nash-FTA}$$

The Home planning problem, accounting for the optimal response by the Foreign planner, is given by:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty}\beta^t u(C_t) \tag{P-Nash-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta^*_s)\right]\beta^t u^{*\prime}(C^*_t)\nabla g^*(\mathbf{c}^*_t)\cdot[\mathbf{c}_t-\mathbf{y}_t]\le 0 \tag{IC-Nash-FTA}$$

$$\mathbf{c}_t=\mathbf{c}(C_t), \quad \mathbf{c}^*_t=\mathbf{c}^*(C_t) \tag{FTA}$$

which relative to the unilateral problem (P-Unil-FTA) features an additional term in the implementability constraint reflecting the sequence of Foreign capital flow taxes $\{\theta^*_t\}$.

Optimal Allocation. Problem (P-Nash-FTA) yields the optimality condition:

$$u'(C_t)=\mu\left[\prod_{s=0}^{t-1}(1-\theta^*_s)\right]\hat{\mathcal{MC}}_t^{FTA} \tag{S20}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}\hat{\mathcal{MC}}_t^{FTA} \equiv & u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot\mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*)C^{*\prime}(C_t)\nabla g^*(\mathbf{c}^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\\ &+ u^{*\prime}(C_t^*)\frac{\partial\nabla g^*(\mathbf{c}_t^*)}{\partial C_t}\cdot[\mathbf{c}_t-\mathbf{y}_t]\end{aligned}$$

Taking the ratio of t and $t+1$ optimality conditions further implies that:

$$\frac{u'(C_t)}{u'(C_{t+1})} = \frac{1}{1-\theta_t^*}\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_{t+1}^{FTA}} \tag{S21}$$

Combining equation (S21) with the Foreign Euler equations (S19) and the analogous Home Euler equations, yields an expression for $1-\theta_t$. The planning problem of the Foreign government is symmetric, so an analogous expression for $1-\theta_t^*$ can be derived. After some simplification, the combination of these expressions yields a mutual best response function, given by:

$$\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_t^{*FTA}} = \alpha_0^{FTA} \tag{S22}$$

where

$$\alpha_0^{FTA} \equiv \frac{\hat{\mathcal{MC}}_0^{FTA}}{\hat{\mathcal{MC}}_0^{*FTA}}$$

This is the strategic counterpart of equation (S4). In the Nash setup, α_0^{FTA} can be interpreted as the bargaining power of the Foreign country relative to the Home. Then the allocations C_t, C_t^* in a Nash equilibrium must satisfy:

$$\frac{\begin{array}{c}C_t^{*-\sigma}(g_{1,t}^*c_{1,t}'(C_t)+g_{2,t}^*c_{2,t}'(C_t))+\\ \sigma C_t^{*\ -\sigma-1}C_t^{*\prime}(C_t)\left[g_{1,t}^*(c_{1,t}-y_{1,t})+g_{2,t}^*(c_{2,t}-y_{2,t})\right]+\\ C_t^{*-\sigma}\left[(g_{11,t}^*+g_{21,t}^*)c_{1,t}'(C_t)(c_{1,t}-y_{1,t})+(g_{12,t}^*+g_{22,t}^*)c_{2,t}'(C_t)(c_{2,t}-y_{2,t})\right]\end{array}}{\begin{array}{c}C_t^{-\sigma}(g_{1,t}c_{1,t}^{*\prime}(C_t)+g_{2,t}c_{2,t}^{*\prime}(C_t))+\\ \sigma C_t^{-\sigma-1}C_t'(C_t^*)\left[g_{1,t}(c_{1,t}^*-y_{1,t}^*)+g_{2,t}(c_{2,t}^*-y_{2,t}^*)\right]+\\ C_t^{-\sigma}\left[(g_{11,t}+g_{21,t})c_{1,t}^{*\prime}(C_t^*)(c_{1,t}^*-y_{1,t}^*)+(g_{12,t}+g_{22,t})c_{2,t}^{*\prime}(C_t^*)(c_{2,t}^*-y_{2,t}^*)\right]\end{array}} = \alpha_0^{FTA}$$

Optimal capital controls levied by the Home country are given by:

$$1-\theta_t = \frac{\begin{array}{c}(g_{1,t}^*c_{1,t}'(C_t)+g_{2,t}^*c_{2,t}'(C_t))+\\ \sigma C_t^{*\ -1}C_t^{*\prime}(C_t)\left[g_{1,t}^*(c_{1,t}-y_{1,t})+g_{2,t}^*(c_{2,}-y_{2,t})\right]+\\ \left[\begin{array}{c}(g_{11,t}^*+g_{21,t}^*)c_{1,t}'(C_t)(c_{1,t}-y_{1,t})+(g_{12,t}^*+\\ g_{22,t}^*)c_{2,t}'(C_t)(c_{2,t}-y_{2,t})\end{array}\right]\end{array}}{\begin{array}{c}(g_{1,t+1}^*c_{1,t+1}'(C_{t+1})+g_{2,t+1}^*c_{2,t+1}'(C_{t+1}))+\\ \sigma C_{t+1}^{*\ -1}C_{t+1}^{*\prime}(C_{t+1})\left[g_{1,t+1}^*(c_{1,t+1}-y_{1,t+1})+g_{2,t+1}^*(c_{2,t+1}-y_{2,t+1})\right]+\\ \left[\begin{array}{c}(g_{11,t+1}^*+g_{21,t+1}^*)c_{1,t+1}'(C_{t+1})(c_{1,t+1}-y_{1,t+1})+\\ (g_{12,t+1}^*+g_{22,t+1}^*)c_{2,t+1}'(C_{t+1})(c_{2,t+1}-y_{2,t+1})\end{array}\right]\end{array}}$$

with an analogous condition for the Foreign.

S.4.3 Optimal Nash Tariffs

To derive the optimal tariffs, divide the Foreign by the Home optimality condition for good 1 and use the Euler to substitute in the Home optimal tariff on the left-hand side. Use the Foreign Euler to substitute out the Foreign optimal tariff:

$$1-\tau_t = \frac{1}{S_t}\frac{C_t^{*-\sigma}g_{1,t}^*S_t + \sigma C_t^{*-\sigma-1}g_{2,t}^*\begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^*S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma}\begin{bmatrix} g_{12,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^*\dfrac{g_{1,t}^*}{g_{2,t}^*}S_t(c_{2,t}-y_{2,t}) \end{bmatrix}}{C_t^{*-\sigma}g_{1,t}^* + \sigma C_t^{*-\sigma-1}g_{1,t}^*\begin{bmatrix} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^*S_t(c_{2,t}-y_{2,t}) \end{bmatrix} - C_t^{*-\sigma}\begin{bmatrix} g_{11,t}^*(c_{1,t}-y_{1,t})+ \\ g_{21,t}^*\dfrac{g_{1,t}^*}{g_{2,t}^*}S_t(c_{2,t}-y_{2,t}) \end{bmatrix}}$$

and then:

$$1-\tau_t^* = \frac{1}{S_t}\frac{C_t^{-\sigma}g_{1,t}S_t + \sigma C_t^{-\sigma-1}g_{2,t}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t}S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma}\begin{bmatrix} g_{12,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{22,t}\dfrac{g_{1,t}}{g_{2,t}}S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}}{C_t^{-\sigma}g_{1,t} + \sigma C_t^{-\sigma-1}g_{1,t}\begin{bmatrix} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t}S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma}\begin{bmatrix} g_{11,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{21,t}\dfrac{g_{1,t}}{g_{2,t}}S_t(c_{2,t}^*-y_{2,t}^*) \end{bmatrix}}$$

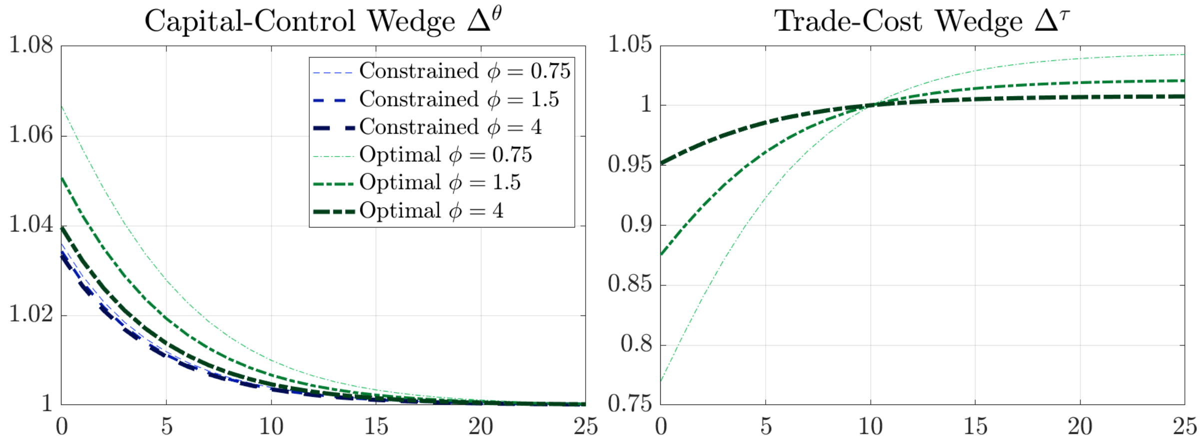
S.4.4 Comparative Statics in Nash Setting

To analyze the comparative statics in the Nash equilibrium, it is useful to define two quantities to capture the difference in the cost of borrowing in the Home *vis-à-vis* the Foreign country, and the relative ratio of tariffs at Home *vis-à-vis* Foreign:

$$\Delta^\theta = \frac{1-\theta_t}{1-\theta_t^*} \quad \text{and} \quad \Delta^\tau = \frac{1+\tau_t}{1+\tau_t^*}$$

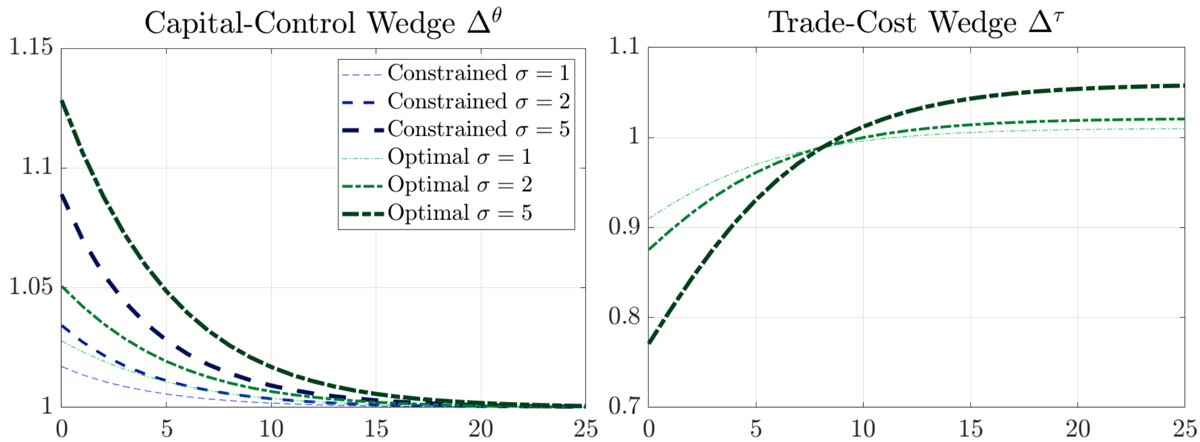
The distance of these quantities from unity captures the total distortion to the inter- and intratemporal margins, respectively. Figures S8 and S9 demonstrate the 'inverse elasticity' relationship between the inter- and intra-temporal wedges and the corresponding inter- and intra-temporal elasticities of substitution.

Figure S8: Comparative Statics of Wedges in Strategic Allocation with Respect to the Intra-Temporal Elasticity of Substitution ϕ in Scenario 1



Notes: Time profile of capital-flow and tariff wedges in scenario 1, simulated for 100 periods, with three different values of intra-temporal elasticity of substitution between goods 1 and 2 ϕ. See Table 1 for calibration details. "Constrained (Optimal)" refers to allocation arising from strategic allocation with (without) constraints on trade policy from a FTA.

Figure S9: Comparative Statics of Wedges in Strategic Allocation with Respect to the Coefficient of Relative Risk Aversion σ (Inverse Inter-temporal Elasticity of Substitution) in Scenario 1



Notes: Time profile of capital-flow and tariff wedges in scenario 1, simulated for 100 periods, with three different values of the coefficient of relative risk aversion σ (i.e., inverse inter-temporal elasticity of substitution). See Table 1 for calibration details. "Constrained (Optimal)" refers to allocation arising from strategic allocation with (without) constraints on trade policy from a FTA.

Table S1: Welfare and Spillovers: % Consumption-Equivalent Welfare Losses from Alternative Planning Allocations

	H	F	Global $\sum_{H,F}$
Scenario 1			
Unilateral-Home Allocation:			
with trade constraints (FTA)	-0.020	0.032	0.006
optimal trade policy	-1.992	3.442	0.815
(from dynamics)	*(-0.052)*	*(0.007)*	-
optimal trade policy, FFFA	-1.944	3.350	0.777
(from dynamics)	*(-0.003)*	*(-0.081)*	-
Nash Allocation:			
with trade constraints (FTA)	0.009	0.017	0.014
optimal trade policy	1.757	1.534	1.668
optimal trade policy, FFFA	1.751	1.709	1.725
Scenario 2			
Unilateral-Home Allocation:			
with trade constraints (FTA)	-0.015	0.025	0.004
optimal trade policy	-2.277	3.956	0.964
(from dynamics)	*(-0.018)*	*(0.169)*	-
optimal trade policy, FFFA	-2.271	3.943	0.959
(from dynamics)	*(-0.012)*	*(0.157)*	-
Nash Allocation:			
with trade constraints (FTA)	0.025	0.003	0.015
optimal trade policy	2.269	1.653	2.007
optimal trade policy, FFFA	2.137	2.153	2.133

Notes: Table presents the % of extra consumption that a country (or the world) would require in the planning allocation to deliver the same welfare as in the decentralized allocation in scenarios 1 and 2. A positive (negative) number represents a welfare loss (gain) in the planning allocation relative to the decentralised allocation. Results come from 100-period simulation of scenarios 1 and 2. Home (Foreign) consumption-equivalent expressed in units of Home (Foreign) aggregate consumption. Global consumption-equivalent expressed in units of PPP-weighted world aggregate consumption.

S.4.5 Welfare in the Nash Equilibrium

Table S1 presents the consumption-equivalent welfare losses for each country and globally.[10] Three results stand out. First, the capital-flow taxes and tariffs levied by the Home planner are distortionary and change consumption paths in a manner that is inefficient for the Foreign country. Consistent with corollary 2, unilateral policy does not simply reallocate consumption across borders: the Home welfare gain is small in comparison to the welfare costs to the Foreign country for both scenarios 1 and 2. So, world welfare is lower. Second, departing from a FTA can generate larger welfare gains for the Home country relative to the FTA case, both in levels (i.e., without equalizing steady states with a constant tax) and dynamically (i.e., with a steady-

[10]Home (Foreign) consumption-equivalents are expressed in units of Home (Foreign) aggregate consumption. Global consumption-equivalent expressed in units of PPP-weighted world aggregate consumption.

state tax). Third, comparing the FTA with the difference between the no FTA and FFFA, we see that welfare gains from capital controls are larger in the absence of a FTA in scenario 1 but smaller in scenario 2, consistent with our analysis in Section 4.

S.4.6 Allocations from Dynamic Policy Game

Figures S10 to S13 plot the allocations from the policy games discussed in Section 6.2. Table S2 below illustrates welfare outcomes for the policy game.

Figure S10: Scenario 1: Allocations and Policy Instruments when Home Deviates from FFFA with Trade-Policy Constraints and Foreign Retaliates $\bar{t} = 5$ Periods Later

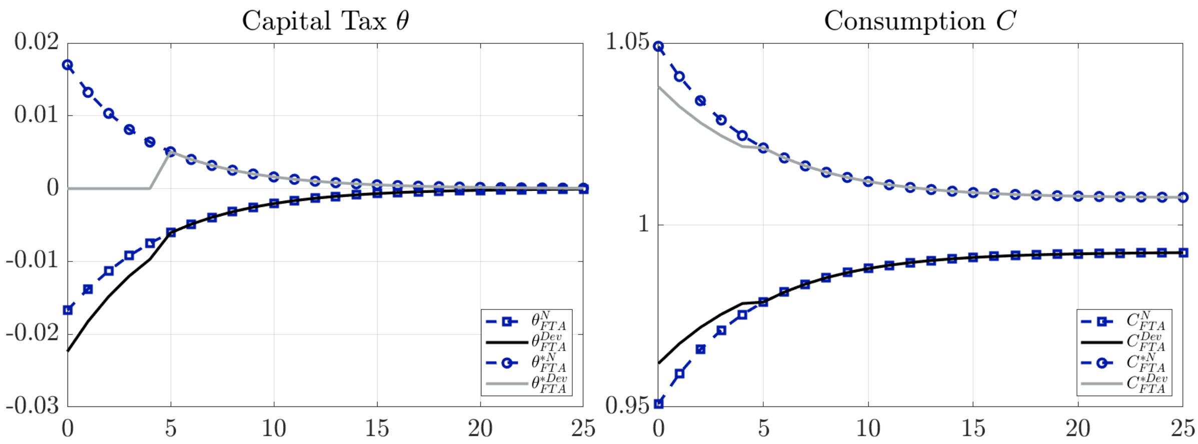


Figure S11: Scenario 2: Allocations and Policy Instruments when Home Deviates from FFFA with Trade-Policy Constraints and Foreign Retaliates $\bar{t} = 5$ Periods Later

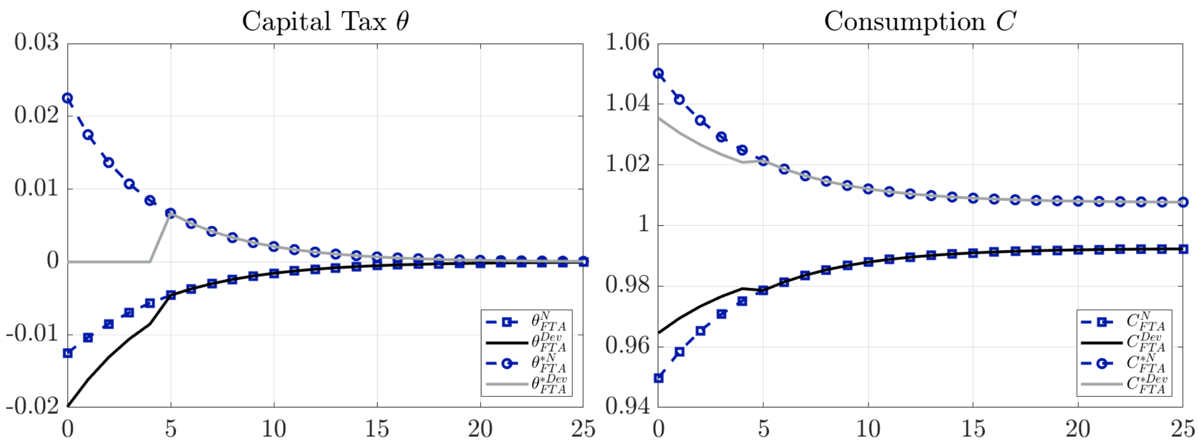


Figure S12: Scenario 1: Allocations and Policy Instruments when Home Deviates from FFFA without Trade-Policy Constraints and Foreign Retaliates $\bar{t} = 5$ Periods Later

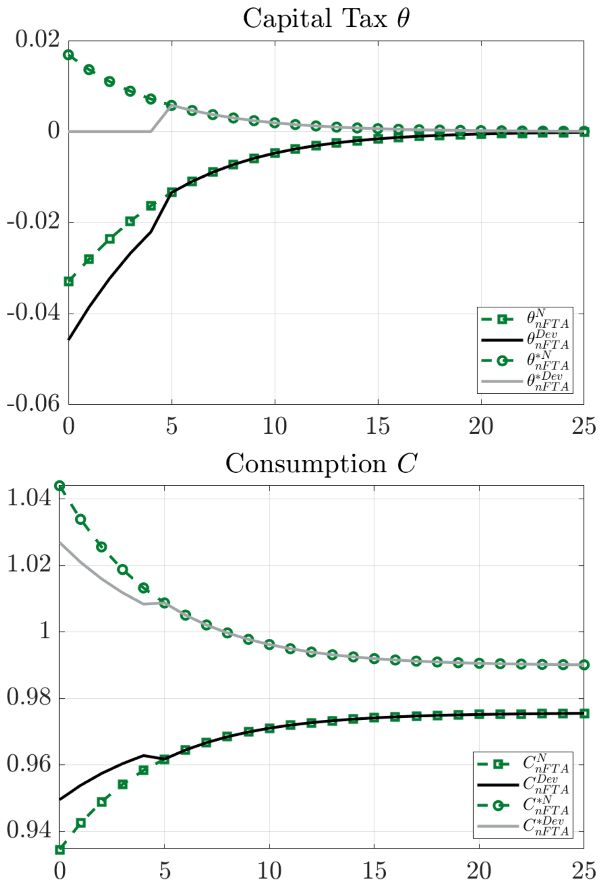


Table S2: Welfare Losses (% Consumption Equivalent) when Home Deviates from a Free Financial Flows aggrement FFFA (with and without Trade-Policy Constraints) and Foreign Retaliates $\bar{t} = 5$ Periods Later

	Scenario 1		**Scenario 2**	
	H	F	H	F
Constrained Pol.	-0.134	0.119	-0.178	0.159
Optimal Trade Pol.	-0.188	0.188	-0.535	0.483

Notes: Table presents the % of extra consumption that a country would require to deliver the same welfare as in the strategic allocation in scenarios 1 and 2. A positive (negative) number represents a welfare loss (gain) in the planning allocation relative to the strategic allocation. Results come from 100-period simulation of scenarios 1 and 2. Home (Foreign) consumption-equivalent expressed in units of Home (Foreign) aggregate consumption.

Figure S13: Scenario 2: Allocations and Policy Instruments when Home Deviates from FFFA without Trade-Policy Constraints and Foreign Retaliates $\bar{t} = 5$ Periods Later