Introduction to Local Projections

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September 2024

The views expressed here do not necessarily reflect the position of the Bank of England.

These slides draw on material from Òscar Jordà and Karel Mertens' *Techniques of Empirical Macroeconomics* course, part of the AEA's 2023 Continuing Education Webcast (available here).

Common ‘Complaints’ About VARs

- VARs not robust to misspecification
  - $\Rightarrow$ If you have the ‘wrong’ VAR, you have the wrong IRFs
- Not easy to account for *non-linearities* within VARs
  - VARs are best, linear one-step ahead predictors
  - $\Rightarrow$ Quality of h-step-ahead responses from VARs depends on quality of 1-step-ahead responses, which are iterated upon
- Inference around VAR-implied impulse responses is complex
  - Standard errors around VAR IRFs are highly non-linear functions of estimated parameters
  - $\Rightarrow$ Practitioners often have to resort to complex bootstrap methods for inference

Enter...Local Projections

Òscar Jordà (SF Fed, UC Davis)

Key Advantages

- Estimated by single-equation OLS, with standard regression packages
- Simple, analytic, joint inference for impulse responses
- More robust to misspecification
- Experimentation with non-linearities straightforward

Enter...Local Projections

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But not as different to VARs as you might think

[Plagborg-Møller & Wolf, 2021]

Basic Idea

Impulse Response Definition: A Comparison of Two Averages

$$\mathcal{R}(h) = \mathbb{E}\left[\mathbb{E}[y_{t+h}|s_t = s + \delta, \mathbf{x}_t] - \mathbb{E}[y_{t+h}|s_t = s, \mathbf{x}_t]\right]$$

where:

- y_{t+h}: outcome
- s_t: intervention
    - s: baseline, e.g., $s = 0$
    - δ: 'dose', e.g., $\delta = 1$, $\delta =$ var$(\varepsilon)^{1/2}$,...
- $\mathbf{x}_t$: vector of exogenous and predetermined variables

Estimation by Local Projection

For $h = 1, 2, ..., H$, can estimate:

$$y_{t+h} = \alpha_h + \beta_h s_t + \boldsymbol{\gamma}_h \mathbf{x}_t + v_{t+h} \quad \text{where} \quad \underbrace{v_{t+h} = u_{t+h} + \psi_1 u_{t+h-1} + ... + \psi_h u_t}_{\text{will see later why this residual is MA}(h)}$$

As long as s_t, $\mathbf{x}_t$ exogenous w.r.t. v_t, then $\hat{\beta}_h \to \beta_h$ (identification) and then:

$$\mathcal{R}(h) = \mathbb{E}[y_{t+h}|s_t = s_1, \mathbf{x}_t] - \mathbb{E}[y_{t+h}|s_t = s_0, \mathbf{x}_t] = \beta_h(s_1 - s_0)$$

Remarks

- **Single equation estimation**: generalises to panel; easy to extend to non-linear setup
- **Effects 'local' to each h**: no cross-period restrictions ▸ In Practice
- **Errors serially correlated**: needs fixing (more on this later)
- Can consider binary 'dose' or continuous policy 'treatment'

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▸ In Practice

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Even so, still many assumptions implicit in *linear* formulation:

- **Symmetry**: increase in dose has same effects as decrease
- **Scale independence**: double the dose, double the effect
- **State independence**: the $\mathbf{x}_t$ do not affect $\mathcal{R}(h)$
- **Treatment does not affect covariate effects**: γ^h invariant to s_t
- $\delta|\mathbf{x}_t$ should be **randomly assigned**

Though, these assumptions can be generalised

A STATA Illustration

`LP_example.do`

- Simple illustration of different variable transformations:
  - Levels vs. Differences (e.g., price index vs. inflation)
  - Levels = Long-differences = Cumulation of differences

$$
\begin{aligned}
\Delta y_{t+h} + \ldots + \Delta y_t &= y_{t+h} - y_{t+h-1} + y_{t+h-1} - y_{t+h-2} + \ldots + y_t - y_{t-1} \\
&= y_{t+h} - y_{t-1} = \Delta^h y_{t+h}
\end{aligned}
$$

- Code shows a simple way to construct the loop and plot LPs

- Code can be built upon...as we will do later on!

Relation to VARs

Without Identification

Propagation in AR(1)

Suppose:

$$(y_t - \mu) = \psi(y_{t-1} - \mu) + u_t$$

By recursive substitution, we have:

$$(y_{t+h} - \mu) = \psi^{h+1}(y_{t-1} - \mu) + \underbrace{u_{t+h} + \psi u_{t+h-1} + ... + \psi^h u_t}_{\text{intrinsic MA residuals}}$$

Suppose intervention is $u_t = \delta$ ($u_{t+1} = ... = u_{t+h} = 0$; $y_{t-1} = y^*$)

$$\begin{aligned}
\mathcal{R}(h) &= \mathbb{E}\left[\mathbb{E}[y_{t+h}|u_t = \delta, y_{t-1} = y^*] - \mathbb{E}[y_{t+h}|u_t = 0, y_{t-1} = y^*]\right] \\
&= \mathbb{E}\left[\left(\psi^{h+1}(y^* - \mu) + \psi^h\delta\right) - \psi^{h+1}(y^* - \mu)\right] \\
&= \mathbb{E}[\psi^h\delta] = \psi^h\delta
\end{aligned}$$

Remarks

- **Iterative approach** for AR(1): from $\hat{\psi}$ obtain $\hat{\psi}^h$
  - Here, inference on $\hat{\psi}^h$ would need to be based on delta method

$$H_0: \ \psi = 0 \implies H_0: \ ATE(h) = \mathcal{R}(h) = \psi^h = 0$$

- **Direct approach** with local projection:

$$y_{t+h} = \alpha_{h+1} + \psi_{h+1} y_{t-1} + v_{t+h}, \ h = 0, 1, ...$$

  - where: $v_{t+h} = u_{t+h} + \psi u_{t+h-1} + ... + \psi^h u_t$

$\Rightarrow$ Hence $\mathbb{E}[y_{t-1} v_{t+h}] = 0 \implies \hat{\psi}_{h+1} \xrightarrow{p} \psi^{h+1}$

  - Here, inference should correct error serial correlation (we will see how):

$$H_0: \ ATE(h) = \mathcal{R}(h) = \psi_h = 0$$

Extends to Propagation in a VAR(2)

$$\underset{k\times 1}{\mathbf{y}_t} = \underset{k\times k}{\mathbf{A}_1}\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \mathbf{u}_t$$

By recursive substitution, we have:

$$\mathbf{y}_{t+1} = \left(\mathbf{A}_1^2 + \mathbf{A}_2\right)\mathbf{y}_{t-1} + \mathbf{A}_1\mathbf{A}_2\mathbf{y}_{t-2} + \mathbf{u}_{t+1} + \mathbf{A}_1\mathbf{u}_t$$

and, then:

$$\begin{aligned}\mathbf{y}_{t+2} = &\left(\mathbf{A}_1^3 + \mathbf{A}_2\mathbf{A}_1 + \mathbf{A}_1\mathbf{A}_2\right)\mathbf{y}_{t-1} + \left(\mathbf{A}_1^2\mathbf{A}_2 + \mathbf{A}_2^2\right)\mathbf{y}_{t-2} \\ &+ \mathbf{u}_{t+2} + \mathbf{A}_1\mathbf{u}_{t+1} + \left(\mathbf{A}_1^2 + \mathbf{A}_2\right)\mathbf{u}_t\end{aligned}$$

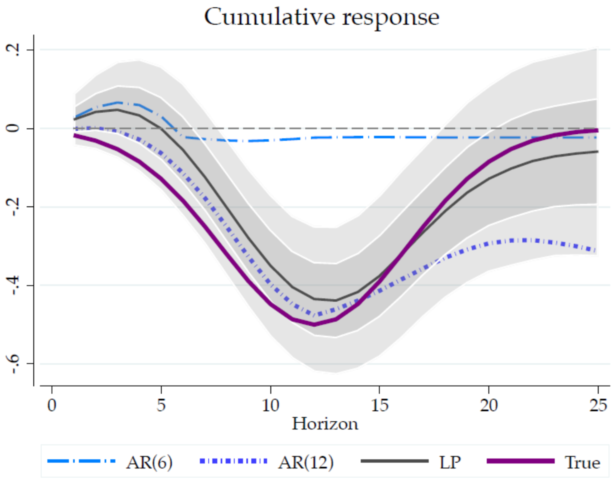
Takeaway: $\mathcal{R}(h)$ a complicated function of $\mathbf{A}_i$

A Note on Lag Lengths in VARs and LPs

- Iterated VAR-based forecasts need *correct specification*
  - If not, responses will be biased
  - $\mathcal{R}(h)$ will be consistent only if in VAR(p) s.t. $p \to h$ as $h \to \infty$
- Instead, local projections are approximations
  - No correct specification is assumed
  - Smaller lag lengths are fine for consistency under mild assumptions
  - However, lag-augmentation can be helpful for inference (more later)
- E.g.: Using VAR(p) to estimate VMA(∞), consistency of VAR(p) only guaranteed up to $h = p$
  - For $h \leq p$: VARs and LPs estimate same response [Plagborg-Møller & Wolf, 2021]
  - For $h > p$: VARs biased, but LPs are not (under certain conditions) [Jordà, Singh & Taylor, 2020]

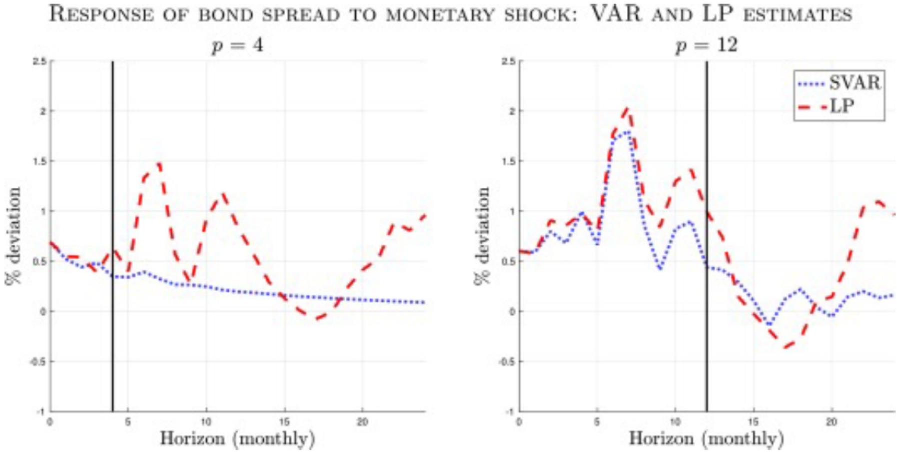
Example I of VAR vs. LP Bias

Based on MA(24) Model



Example II of VAR vs. LP Bias

Plagborg-Møller & Wolf (2021)



Local Projections in Practice

So You Want To Run a Local Projection? What Next?

$$\Delta^h y_{t+h} = \alpha_h + \beta_h \varepsilon_t + \boldsymbol{\gamma}_h \mathbf{x}_t + v_{t+h}$$

#1 Where do I get my shocks ε from?

#2 Once I have my shocks, how should I use them?

#3 What else should I control $\mathbf{x}$ for?

#4 What standard errors should I use?

#5 How can I assess the contribution of shocks to the variance of outcome variables?

#6 How can I test and account for non-linearities?

#7 How do things change if I decide to use panel data?

#8 What non-OLS estimators can I use?

#1. Fantastic Shocks and Where To Find Them

Where do I get my shocks ε from?

Unbiasedness of LP estimator β_h depends critically on 'exogeneity' of shock ε conditional on observables $\mathbf{x}$—i.e., $\varepsilon_t|\mathbf{x}_t$ must be randomly assigned.

Fortunately, a huge literature has developed a range of shocks (constructed in various ways):

- **Monetary Policy**:
  - US: Romer and Romer (1989, 2004) [Extended by Wieland and Yang (2020)]; Gürkaynak, Sack and Swanson (2005); Gertler and Karadi (2015); Miranda-Agrippino and Ricco (2021); Bauer and Swanson (2023)
  - UK: Cloyne and Hürtgen (2016); Cesa-Bianchi, Thwaites and Vicondoa (2020); Braun, Miranda-Agrippino and Saha (2023)
  - EA: Altavilla, Brugnolini, Gürkaynak, Motto and Ragusa (2019); Jaroncinski and Karadi (2020)
  - RoW: Choi, Willems and Yoo (2023)

  Amongst others

#1. Fantastic Shocks and Where To Find Them

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- **Tax**: Romer and Romer (2010); Mertens and Ravn (2013, 2014); Cloyne and Surico (2017); Cloyne, Martinez, Mumtaz and Surico (2022); Cloyne, Postel-Vinay and Dimsdale (2023)
- **Government Spending**: Ramey (2011); Owyang, Ramey and Zubairy (2014); Ramey and Zubairy (2018)
- **Macroprudential Policy**: Richter, Schularick and Shim (2019); Fernández-Gallardo, Lloyd and Manuel (2023); Bluwstein and Patozi (2023)
- **General Aggregate Demand or Supply**: Angeletos, Collard and Dellas (2020)
- **Oil Shocks**: Känzig (2021)
- **Credit Shocks**: Gilchrist and Zakrejsek (2012)
- **FX Shocks**: Bippus, Lloyd & Ostry (2023)

#2. Handle With Care: Shock-Handling Instructions

Once I have my shocks, how should I use them?

Once you have your shock ε, you can in principle use it in two ways:

#1 **Use Shock Directly in OLS**: Estimate the following OLS regression:

$$\Delta^h y_{t+h} = \alpha_h + \beta_h \varepsilon_t + \boldsymbol{\gamma}_h \mathbf{x}_t + v_{t+h}$$

Assuming: Shock observed without error!

#2 **Use Shock as Instrument in 2SLS**: Use shock ε as an instrument for z in following IV regression:

$$\Delta^h y_{t+h} = \alpha_h + \beta_h z_t + \boldsymbol{\gamma}_h \mathbf{x}_t + v_{t+h}$$

Requirement: No weak instruments (i.e., first-stage $F > 10$)

In practice, two should be comparable. Preference may be a function of interpretation.

#2. Handle With Care: Shock-Handling Instructions

Once I have my shocks, how should I use them?

Conditions for LP-IV

[Stock and Watson, 2018]

- Suppose ε_t a vector of instruments for z_t and denote $\varepsilon_t^p = \varepsilon_t - \mathcal{P}(\varepsilon_t | \mathbf{w}_t)$ where $\mathbf{w}_t$ collects all controls in the LP
  - #1 **Relevance**: $\mathbb{E}\left[\mathbf{z}_{i,t}^{p} \varepsilon_t^{p'}\right] = \boldsymbol{\alpha}' \neq 0$
  - #2 **Basic Exogeneity**: $\mathbb{E}\left[\mathbf{z}_{-i,t}^{p} \varepsilon_t^{p'}\right] = 0$, $j \neq 1$
  - #3 **Lead-Lag Exogeneity**: $\mathbb{E}\left[\mathbf{z}_{-i,t+h}^{p} \varepsilon_t^{p'}\right] = 0$, $\forall j, h \neq 0$
- Usual IV conditions, except lead-lag exogeneity due to dynamics
- A STATA Illustration: `LPIV_example.do`

#3. Controlling Behaviour

What else should I control $\mathbf{x}$ for?

- **Simple Answer**: control for anything that could be correlated by ε and y

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- **But**: common to construct shock in a first-stage regression (e.g., of policy z on x)
  - See: Lloyd and Manuel (2024), “Controls, Not Shocks: Estimating Dynamic Causal Effects in Macroeconomics,” Bank of England Staff Working Paper No. 1079.

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What else should I control x for?

- **Simple Answer**: control for anything that could be correlated by ε and y
- **But**: common to construct shock in a first-stage regression (e.g., of policy z on x)
  - See: Lloyd and Manuel (2024), "Controls, Not Shocks: Estimating Dynamic Causal Effects in Macroeconomics," Bank of England Staff Working Paper No. 1079.
- So what's the story then?
  - If adding controls to improve 'exogeneity', you could be undermining the exogeneity of the shock ε you picked in the first place
  - If 'shock' constructed with controls, use same controls in LP (better still: do it in one step)
  - If you do include controls, you're more reasonably doing it to improve fit ($\uparrow R^2$) and $\downarrow$ s.e.

#4. Error-Free Error Bands

What standard errors should I use?

Why is inference different with LPs? The MA structure of the residuals

- Recall the AR(1) example: $y_t = \rho y_{t-1} + u_t$. By recursive substitution:

$$y_{t+h} = \rho^{h+1} y_{t-1} + u_{t+h} + \rho u_{t+h-1} + \dots + \rho^h u_t$$

- So, in a LP:

$$y_{t+h} = \beta_{h+1} y_{t-1} + v_{t+h}; \quad v_{t+h} = u_{t+h} + \rho u_{t+h-1} + \dots + \rho^h u_t$$

In general, we do not know the MA structure!

⇒ Jordà (2005) recommends HAC standard errors (e.g., Newey & West, 1987), but this is an active area of research (see, e.g., ‘significance bands’ by Inoue, Jordà and Kuersteiner, 2023)

#4. Error-Free Error Bands

What standard errors should I use?

Lag augmentation offers a simpler, more elegant solution [Montiel-Olea & Plagborg-Møller, 2021]

The Logic in a Simple Example

- *DGP*: $y_t = \rho y_{t-1} + u_t$, where u_t strictly stationary s.t. $\mathbb{E}[u_t | \{u_s\}_{s \neq t}] = 0$
- *LP*: $y_{t+h} = \beta_h y_t + v_{t+h}$ where $v_{t+h} \sim MA(h)$
- *Plug DGP into LP*: $y_{t+h} = \beta_h u_t + \gamma y_{t-1} + v_{t+h}$
- *FWL Logic*: obtain β_h by regressing $y_{t+h} - \gamma_h y_{t-1}$ on $y_t - \rho y_{t-1}$

$$
\begin{aligned}
\hat{\beta}_h =& \frac{\sum_{t=1}^{T-h} (y_{t+h} - \gamma_h y_{t-1})(y_t - \rho y_{t-1})}{\sum_{t=1}^{T-h} (y_t - \rho y_{t-1})^2} = \frac{\sum_{t=1}^{T-h} (\beta_h u_t + v_{t+h}) u_t}{\sum_{t=1}^{T-h} (y_t - \rho y_{t-1})^2} \\
=& \beta_h + \frac{\sum_{t=1}^{T-h} v_{t+h} u_t}{\sum_{t=1}^{T-h} u_t^2}
\end{aligned}
$$

#4. Error-Free Error Bands

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- Recall:

$$\hat{\beta}_h = \beta_h + \frac{\sum_{t=1}^{T-h} v_{t+h} u_t}{\sum_{t=1}^{T-h} u_t^2} \quad \rightarrow \quad \hat{\sigma}^2(\hat{\beta}_h) = \frac{\sum_{t=1}^{T-h} \hat{v}_{t+h}^2 \hat{u}_t^2}{\left(\sum_{t=1}^{T-h} \hat{u}_t^2\right)^2}$$

- Although $v_{t+h} \sim MA(h)$, note that $v_{t+h}u_t \sim MA(0)$ since for any $s < t$:

$$\begin{aligned}
\mathbb{E}[v_{t+h}u_t v_{s+h}u_s] =& \mathbb{E}[\mathbb{E}[v_{t+h}u_t v_{s+h}u_s | u_{s+1}, u_{s+2}, \ldots]] \\
=& \mathbb{E}[v_{t+h}u_t v_{s+h} \underbrace{\mathbb{E}[u_s | u_{s+1}, u_{s+2}, \ldots]]}_{=0}
\end{aligned}$$

$\Rightarrow$ Do lag-augmented LP with White-corrected errors (no need for Newey-West), *or* wild bootstrap procedure (see paper for more)

#5. Shocking Importance

How can I assess the contribution of shocks to the variance of outcome variables?

- Note that we can always write:

$$y_{t+h} = \hat{\mathbb{E}}_t[y_{t+h}] + \hat{v}_{t+h}$$

- Then R^2 from regression of $\hat{v}_{t+h}$ on $\varepsilon_{t+h}, ..., \varepsilon_t$ measures percent of forecast-error variance explained by shock
- If multiple shocks, can run this exercise shock by shock

[Gorodnichenko and Lee, 2020; Plagborg-Møller and Wolf, 2022]

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

An Example From Applied Micro

- Think of observed y as coming from latent mixture:

$$y = (1-\varepsilon)y_0 + \varepsilon y_1 = y_0 + \varepsilon(y_1 - y_0), \quad \varepsilon = 0, 1$$

- Assumption:

$$y_i \sim f(\mu_j, \sigma_j), \quad j = 0, 1 \quad \text{unobserved random variables}$$

- We would like: $\mathbb{E}[y_1 - y_0]$, the average treatment effect ATE

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

Assume linear model for latent variables: y_j, $j = 0, 1$

- Let $y_j = \mu_j + v_j$, where $\mathbb{E}[v_j] = 0$, $j = 0, 1$ and v_j captures heterogeneity
- Let $v_j = \mathbf{x} - \boldsymbol{\mu}_x)\boldsymbol{\gamma}_j + e_j$ with $\mathbb{E}[e_j] = 0$ and $\mathbb{E}[e_j|\mathbf{x}] = 0$, then:

$$\underbrace{\mathbb{E}_x\left[\mathbb{E}[y_1|\varepsilon = 1, \mathbf{x}] - \mathbb{E}[y_0|\varepsilon = 0, \mathbf{x}]\right]}_{ATE} = \left(\mu_1 + \mathbb{E}_x\left[\mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x|\varepsilon = 1]\right]\boldsymbol{\gamma}_1\right)$$

$$- \left(\mu_0 + \mathbb{E}_x\left[\mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x|\varepsilon = 0]\right]\boldsymbol{\gamma}_0\right)$$

- Add/subtract counterfactual: $\mathbb{E}_x\left[\mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x|\varepsilon = 1]\right]\boldsymbol{\gamma}_1$:

$$\begin{aligned} ATE =& (\mu_1 - \mu_0) + \mathbb{E}_x\left[\mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x|\varepsilon = 1]\right](\boldsymbol{\gamma}_1 - \boldsymbol{\gamma}_0) \\ & + \mathbb{E}_x\left[\mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x|\varepsilon = 1] - \mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x|\varepsilon = 0]\right]\boldsymbol{\gamma}_0 \end{aligned}$$

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

Kitagawa-Oaxaca-Blinder Decomposition Components

$$ATE = \underbrace{(\mu_1 - \mu_0)}_{\text{Direct}} + \underbrace{\mathbb{E}_x\left[\mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x | \varepsilon = 1]\right](\boldsymbol{\gamma}_1 - \boldsymbol{\gamma}_0)}_{\text{Indirect}}$$
$$+ \underbrace{\mathbb{E}_x\left[\mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x | \varepsilon = 1] - \mathbb{E}[\mathbf{x} - \boldsymbol{\mu}_x | \varepsilon = 0]\right]\boldsymbol{\gamma}_0}_{\text{Composition}}$$

- **Direct**: ATE under random assignment
- **Indirect**: Treatment spillovers on covariates
- **Composition**: Failure of random assignment? Small-sample bias

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

Null Hypotheses in Linear Case

$$y_i = \mu_0 + (\mathbf{x}_i - \overline{\mathbf{x}}_0)\,\boldsymbol{\gamma}_0 + \varepsilon_i\,[\beta + (\mathbf{x}_i - \overline{\mathbf{x}}_1)\,\boldsymbol{\theta}] + \omega_i$$

Noting: $\beta = \mu_1 - \mu_0$, $\boldsymbol{\theta} = \boldsymbol{\gamma}_1 - \boldsymbol{\gamma}_0$ and $\omega_i = e_{0,i} + \varepsilon_i\,(e_{1,i} - e_{0,i})$

- H_0: $\beta = 0$ null of no *direct* treatment effect
- H_0: $\boldsymbol{\theta} = 0$ null of no *indirect* treatment effect
- H_0: $\mathbb{E}[\mathbf{x}|\varepsilon = 1) - \mathbb{E}[\mathbf{x}|\varepsilon = 0] = 0$ null of no *composition* effect
- H_0: $\boldsymbol{\gamma}_0 = 0$ null of no *random assignment* (hence no composition effect possible)

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

What does this mean for local projections?

- Let $\mathbf{y}_t = [y_t, y_{t+1}, ..., y_{t+H}]$ and $\mathbf{y}$ denote associated random variable
- Assume conditional-mean independence, let $\mathbb{E}[y_s] = \mu_s$ for $s \in \{0, 1\}$, wlog $y_s = \mu_s + v_s$
- Under linearity: $v_s = (\mathbf{x} - \boldsymbol{\mu}_x)\mathbf{b}_s + e_s$, then:

$$\mathbb{E}[y_s|\mathbf{x}] = \mu_s, \quad \mathbb{E}[v_s] = 0, \quad \mathbb{E}[e_s|\mathbf{x}] = 0, \quad s \in \{0, 1\}$$

- Hence

$$y_{t+h} = \underbrace{\mu_0^h + (\mathbf{x}_t - \overline{\mathbf{x}})\gamma_0^h + \varepsilon_t \beta^h}_{\text{usual local projection}} + \underbrace{\varepsilon_t (\mathbf{x}_t - \overline{\mathbf{x}})\boldsymbol{\theta}^h}_{\text{Kitagawa term}} + \omega_{t+h}$$

for $h = 0, 1, ..., H$, $t = h, ..., T$

#6. Impulse Response Heterogeneity

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Kitagawa-Oaxaca-Blinder Decomposition Components

$$y_{t+h} = \underbrace{\mu_0^h + (\mathbf{x}_t - \overline{\mathbf{x}})\gamma_0^h + \varepsilon_t \beta^h}_{\text{usual local projection}} + \underbrace{\varepsilon_t (\mathbf{x}_t - \overline{\mathbf{x}})\boldsymbol{\theta}^h}_{\text{Kitagawa term}} + \omega_{t+h}$$

for $h = 0, 1, ..., H$, $t = h, ..., T$

- **Direct Effect**: $\hat{\mu}_1^h - \hat{\mu}_0^h = \hat{\beta}^h$
- **Indirect Effect**: $(\overline{\mathbf{x}}_1 - \overline{\mathbf{x}})(\hat{\gamma}_1^h - \hat{\gamma}_0^h) = (\overline{\mathbf{x}}_1 - \overline{\mathbf{x}})\hat{\boldsymbol{\theta}}^h$
- **Composition Effect**: $(\overline{\mathbf{x}}_1 - \overline{\mathbf{x}}_0)\hat{\gamma}_0^h$

Ergodicity: needed to ensure $\overline{\mathbf{x}} \to \boldsymbol{\mu}_x$

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

Implications: State-Dependence Suppose $\mathbf{x} = \mathbf{x}^*$ then total response is:

$$
\begin{aligned}
\mathbb{E}[y_1|\mathbf{x}^*, \varepsilon = \delta] - \mathbb{E}[y_0|\mathbf{x}^*, \varepsilon = 0] \\
&= \delta\mu_1 + \delta\left[\mathbf{x}^* - \mathbb{E}[\mathbf{x}]\right]\boldsymbol{\gamma}_1 - \left[\mu_0 + (\mathbf{x}^* - \mathbb{E}[\mathbf{x}])\boldsymbol{\gamma}_0\right) \\
&= \delta\beta + \delta(\mathbf{x}^* - \mathbb{E}[\mathbf{x}])\boldsymbol{\theta}
\end{aligned}
$$

- Dependence on $\mathbf{x}^*$ is only partial equilibrium
- Need identification (instruments) for $\mathbf{x}$
- Usual single-variable stratification omits other terms in $\mathbf{x} \rightarrow$ bias

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

State-Dependence Example: Monetary Policy

- How effective was monetary policy in...
  - #1 November 1987 (Post-Stock Market Crash)
    - Stocks 23% lower by end-October
    - Fed lowered funds rate by $50bo$
  - #2 February 1996 (Middle of a Long Expansion)
    - Middle of stable funds rate
- **Idea**: two different scenarios, but similar policy paths $\rightarrow$ differences not due to policy

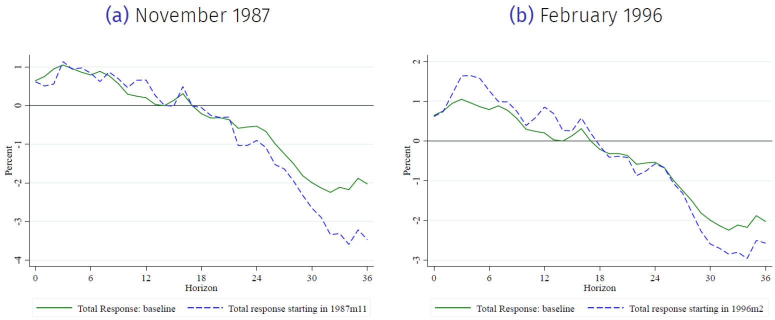
#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

State-Dependence Example: Monetary Policy

Funds rate paths nearly identical

Federal funds rate



Baseline = average over sample

#6. Impulse Response Heterogeneity

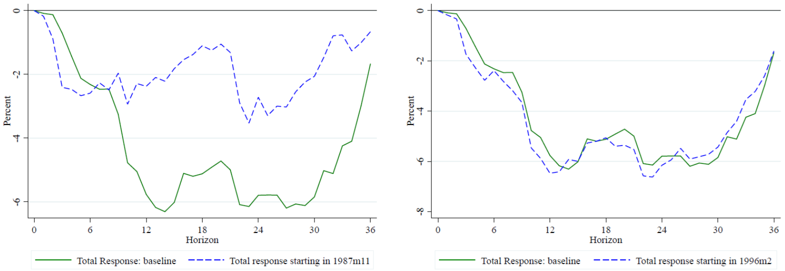
How can I test and account for non-linearities?

State-Dependence Example: Monetary Policy

Policy unable to boost activity post-1987 crash

Industrial production

(a) November 1987 (b) February 1996



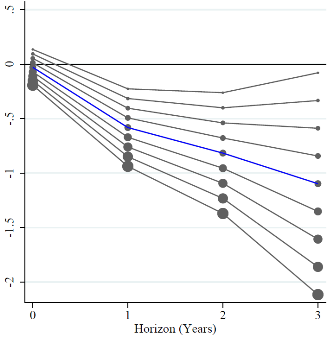
Baseline = average over sample

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

State-Dependence Example: Fiscal Policy

GDP response to fiscal policy varies with monetary stance



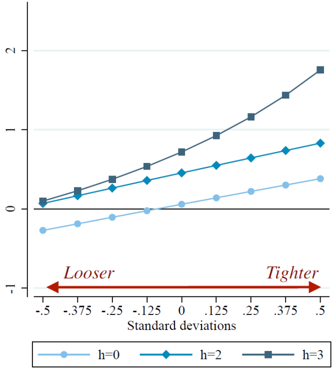
Cloyne, Jordà and Taylor (2023)

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

State-Dependence Example: Fiscal Policy

Variation in the multiplier by horizon and stance



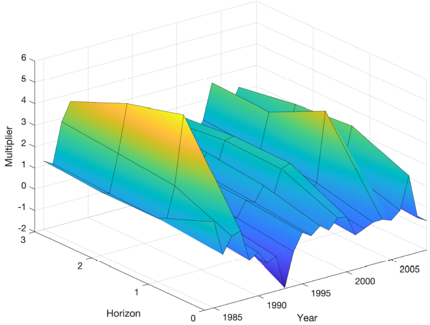
Cloyne, Jordà and Taylor (2023)

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

State-Dependence Example: Fiscal Policy

Time-varying estimates of the multiplier



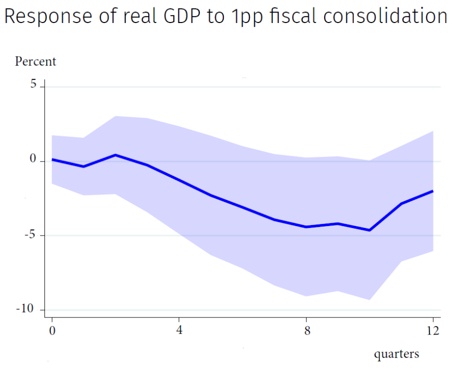
Cloyne, Jordà and Taylor (2023)

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

A STATA Illustration: Usual LPIV

`kob_example.do`



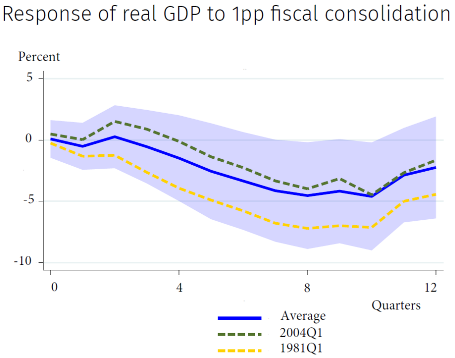
90% error bands

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

A STATA Illustration: Choosing Two Dates

`kob_example.do`



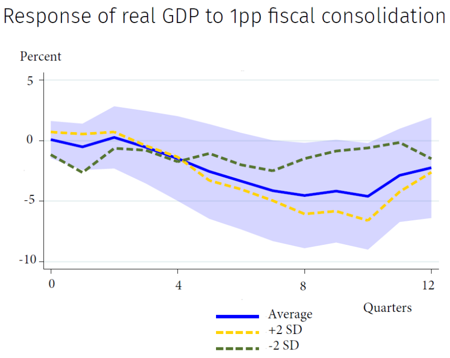
90% error bands

#6. Impulse Response Heterogeneity

How can I test and account for non-linearities?

A STATA Illustration: Monetary Offset

`kob_example.do`



90% error bands

#7. LPs in Panel Data

How do things change if I decide to use panel data?

$$y_{i,t+h} = \alpha_i + \delta_t + \varepsilon_{i,t}\beta_h + \mathbf{x}_{i,t}\boldsymbol{\gamma}_h + v_{i,t+h}, \quad i = 1, ..., N, \ t = 1, ..., T$$

- α_i: unit fixed effects
- δ_t: time fixed effects
- $\mathbf{x}_{i,t}$: exogenous and pre-determined variables
- $\varepsilon_{i,t}$: treatment variable
- β_h: response coefficient of interest

A STATA Illustration: `LP_example_panel.do`

#7. LPs in Panel Data

How do things change if I decide to use panel data?

Remarks: Usual panel data issues, plus:

- LP is costly in short panels (lose time dimension)
- But cross-section brings more power
- Incidental parameter issues (fixed effects)
  - Beware of high autocorrelation and low T [Alvarez and Arellano, 2003]
  - May need Arellano-Bond or similar estimator
- Inference
  - N, T large: Two-way clustering helps MA(h) and heteroskedasticity
  - N large, T small: Cluster by unit helps with MA(h)
  - T large, N small: Cluster by time helps heteroskedasticity
  - Else Driscoll-Kraay s.e. are like Newey-West s.e. for panel data (`xtscc` in STATA)
  - When clustering with small N, T, may need bootstap (`summclust` and `boottest` in STATA)

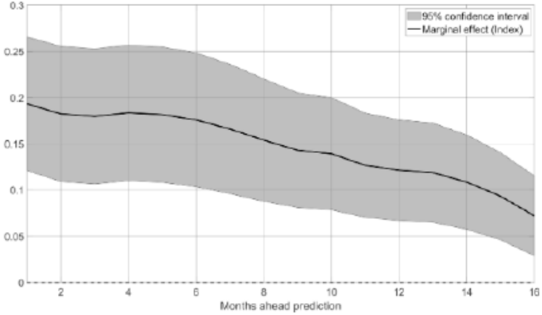
#8. Making Things Funky

What non-OLS estimators can I use?

$$\Delta^h y_{t+h} = \alpha_h + \beta_h \varepsilon_t + \boldsymbol{\gamma}_h \mathbf{x}_t + v_{t+h}$$

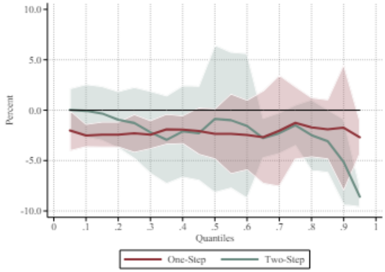
Don't *only* need to use OLS to estimate this!

Binary Events: Response of recession prob. to newspaper index



Quantiles: Response infl. distbn. to mon. pol.

(d) Quantile Response at 4-year Horizon from QR-LP



Relation to VARs*

With Identification

Short-Run Restrictions

Recall the Cholesky decomposition from VARs: $\mathbf{\Sigma} = \mathbf{P}\mathbf{P}'$ with $\mathbf{P}$ lower triangular

- This always exists, is unique, and is trivial to implement but:
  - Different ordering of variables implies different $\mathbf{P}$
  - Implied 0 restrictions need not be correct
  - Just-identification $\Rightarrow$ ordering can't be tested

- **Interpretation**:
  - $y_{(1),t}$ does not contemporaneously depend on others
  - $y_{(2),t}$ only depends on $y_{(1),t}$ contemporaneously
  - $y_{(3),t}$ only depends on $y_{(1),t}, y_{(2),t}$ contemporaneously
  - and so on...

Recursive Identification in LPs

- Let $\mathbf{y}_t$ be an $n \times 1$ vector, and decide causal ordering
- Include contemporaneous values of variables causally-ordered first:

$$y_{j,t+h} = \mu_j^h + \beta_{j,1}^h y_{1,t} + \ldots + \beta_{j,i-1}^h y_{i-1,t} + \beta_{j,i}^h y_{i,t} + \sum_{k=0}^{p} \mathbf{c}_{j,k}^h \mathbf{y}_{t-k} + v_{j,t+h}$$

- Structural LP estimate: $\hat{\mathcal{R}}_{ij}(h) = \hat{\beta}_{j,t}^h$, $h = 0, 1, \ldots, H$, $i, j \in \{1, \ldots, n\}$
- Good idea to order treatment variable $y_{i,t}$ last $\rightarrow$ variation cannot be explained by observables

Long-Run Restrictions

Two-Step Procedure in LPs

Blanchard and Quah (1989) Example: $\mathbf{y}_t = [x_t, u_t]'$ where x_t log real GDP and u_t unemp. rate

#1 Long-Run LP, where H is large

$$x_{t+H} - x_{t-1} = \alpha_H + \boldsymbol{\delta}_{x,H}\mathbf{y}_t + \sum_{k=1}^{p} \mathbf{c}_{x,k}^{H}\mathbf{y}_{t-k} + v_{x,t+H}$$

$\boldsymbol{\delta}_{x,H}$: linear combination that best explains long-run GDP (i.e., supply shock)

#2 Regress:

$$y_{j,t+h} = \mu_h + \beta_{j,h}\left(\hat{\boldsymbol{\delta}}_{x,H}\mathbf{y}_t\right) + \sum_{k=1}^{p} \mathbf{x}_{j,k}^{h}\mathbf{y}_{t-k} + v_{j,t+h}, \quad j = x, u,\ h = 0, 1, ..., H$$

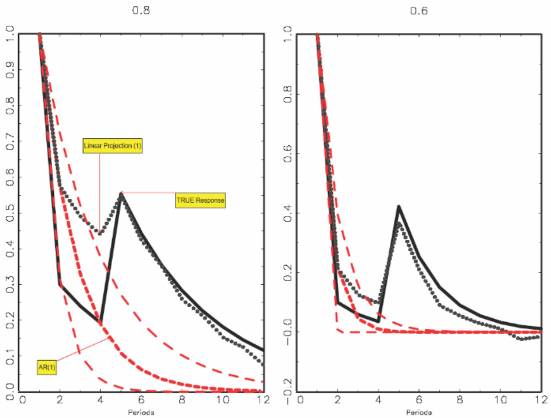
$\beta_{j,h}$ is response of jth variable to supply shock in period h

Little guidance on how to choose H, try different value

Could generalise idea in a number of ways (e.g., medium-run identification)

Appendix

Example: AR(1) vs. Local Projection



$y_t = \rho y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1} + 0.4\varepsilon_{t-4}$, $T = 180$, 100 replications $\rho = 0.8, 0.6$

Back